Rare charm decays

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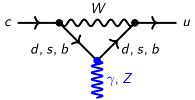
Rare charm decays are special!

1. **Window to test FCNCs in the up-sector!**
2. **Strong non-perturbative dynamics** $\rightarrow$ **"Null tests"** $\mathcal{O} \pm \delta\,\mathcal{O}$
   - Use SM symmetries: $\mathcal{O}_{\rm SM} = \mathbf{0}$,
   - Small uncertainties: $\mathcal{O}_{\rm SM} \gg \delta\,\mathcal{O}_{\rm SM}$,
   - Use large hadronic effects to enhance NP contributions,
   - Construct $\mathcal{O}$ sensitive to specific NP,
   - Use $SU(3)_F$-flavor symmetry, ...
3. **Very efficient GIM mechanism:** $\sum_i \lambda_i = 0$ with $\lambda_i \equiv V_{ci}^* V_{ui}$.



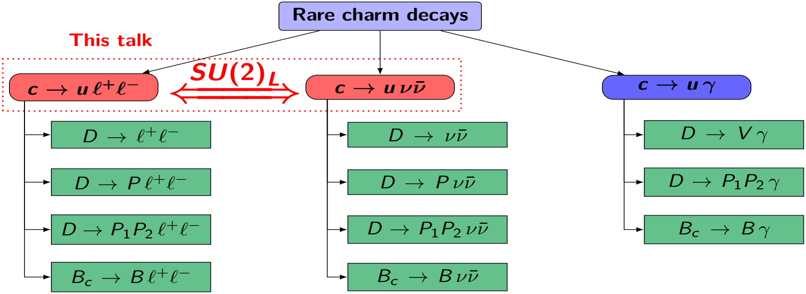
$$= \sum_{i=d,s,b} \lambda_i\, f_i \;=\; \lambda_s \left[(f_s - f_d) + \frac{\lambda_b}{\lambda_s} (f_b - f_d) \right]$$

$f_i \sim \frac{m_i^2}{(4\pi)^2 M_W^2}$, $\mathrm{Im}(\lambda_b/\lambda_s) \sim 10^{-3}$

BRs ($A_{\rm CP}$) are loop-(CKM-) suppressed!

Excellent place to search for BSM physics!

A sketch of the playground



Observables:

Branching ratios
Angular observable
LU ratios& LFV
CP asymmetries

⇑

Theory:

2011.12856, 2004.01206, 1909.11108,
1510.00311, 1606.05521, 1703.08799,
1705.02251, 1705.05891, 1707.00988,
1510.00311, 0810.4858 , 1409.0181,
1208.0759 , 2004.01206, 1805.08516,
2202.02331, 2107.13010, 1712.05783,
1805.02516, 2101.02421, ... ,

Experiment:

1305.5059 , 1512.00322, 1107.4465 ,
2011.00217, 1802.09752, 1510.08367,
1310.2535 , 1707.08377, 2111.03327,
1712.07938, 1806.10793, ...

Observables:

Branching ratios

⇑

Theory:

2007.05001, 2010.02225, 2012.15847,
2101.10712, 2107.07291, ...

Experiment:

2112.14236, 1611.09455.

Ideally:

Theory + Experiment

⇓

Optimized observables!

Observables:

Branching ratios
Angular observables
CP asymmetries

⇑

Theory:

2104.08287, 2009.14212, 1812.04679,
1905.00907, 1802.02769, 1702.05059,
1701.06392, 2203.14982, ...

Experiment:

1603.03257, 0808.1838, 0808.1838,

For a review:
Mod.Phys.Lett.A 36 (2021) 04, 2130002

EFT approach to charm physics (1)

1. **Dynamical fields ϕ_i at μ_{EW}: $\phi_i^{\text{SM}} = q_i, \ell_i, A_\mu, \ldots$**

2. **Symmetries to build all $O_j(\phi_i)$ up to desired dimension ($D = 6$):**

$$\mathcal{H}_{\text{eff}} \sim \frac{4\,G_F}{\sqrt{2}} \frac{\alpha_e}{4\pi} \sum_i C_i\, O_i$$

$$O_1^q = (\bar{u}_L \gamma_\mu T^a q_L)(\bar{q}_L \gamma^\mu T^a c_L)\,, \quad O_2^q = (\bar{u}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu c_L)\,,\ q = d, s\,,$$

$$O_7^{(\prime)} = \frac{m_c}{e} (\bar{u}_{L\,(R)}\, \sigma_{\mu\nu}\, c_{R\,(L)})\, F^{\mu\nu}\,, \quad O_{9\,(10)}^{(\prime)} = (\bar{u}_{L\,(R)} \gamma_\mu c_{L\,(R)})(\bar{\ell}\, \gamma^\mu (\gamma_5)\, \ell)\,,$$

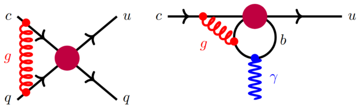
$$O_{S\,(P)}^{(\prime)} = (\bar{u}_{L\,(R)} c_{R\,(L)})(\bar{\ell}\, (\gamma_5)\, \ell)\,, \quad O_{T\,(T5)} = \frac{1}{2} (\bar{u}\, \sigma_{\mu\nu}\, c)(\bar{\ell}\, \sigma^{\mu\nu}\, (\gamma_5)\, \ell)\,.$$

3. **Compute $C_i(\mu_{\text{EW}})$ to avoid large $\alpha_s(\mu_{\text{low}}) \log(\mu_{\text{low}}^2/\mu_{\text{EW}}^2)$.**

$$m_{q_{\text{light}}} = 0 + \text{GIM mechanism} \Longrightarrow \boxed{C_{7,9,10}^{\text{SM}}(\mu_{\text{EW}}) = 0!}$$

EFT approach to charm physics (2)

4. **RGEs to go down $\mu_{\text{low}} \approx m_c$ (2-step matching at μ_{EW} and m_b).**
   - **Penguins generated at $\mu = m_b$**
   - **$O_{7,9}$ mix with $O_{1,2}$:**



$$|C_7^{\text{eff}}(\mu_c)| \lesssim 0.004 \,\&\, |C_9^{\text{eff}}(\mu_c)| \lesssim 0.01$$

   - **BUT NOT all other SM WCs:**

$$C_i'^{\text{SM}} = C_S^{\text{SM}} = C_T^{\text{SM}} = C_{T5}^{\text{SM}} = C_{10}^{\text{SM}} = 0$$

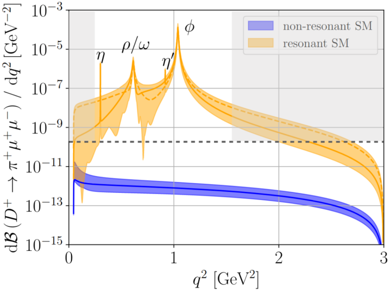
Rock stars of charm physics!
Any observable proportional to these WCs is a null test!

5. **$\langle O_i(\mu_{\text{low}})\rangle$ from non-perturbative techniques (Lattice, LCSR, ...)**

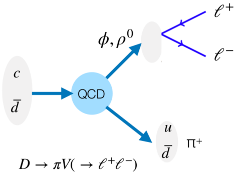
6. **Include resonances: Breit–Wigner distributions + exp. data.**

$C_{10}^{\text{QED}}(\mu_c) < 0.01 C_9(\mu_c) < 10^{-4}$ (S. de Boer, PhD thesis, TU Dortmund, 2017)

Rare semileptonic charm $c \to u\,\ell^+\ell^-$ decays



Dominated by resonances!



$$\mathcal{B}(D \to \pi\,\ell^+\ell^-)_{\mathrm{SM}} \approx \mathcal{B}(D \to \pi\,V(\to \ell^+\ell^-))$$

- **Current data still allows for large NP effects at large q^2**

$$\mathcal{B}(D^+ \to \pi^+\mu^+\mu^-) < 6.7 \cdot 10^{-8}\ (90\%\ \mathrm{C.L.})$$

- **Allowing to get bounds on WCs ($\mathcal{B}(D^+ \to \mu^+\mu^-) < 2.9 \cdot 10^{-9}$ (90% C.L.))**

$$|C_7| \lesssim 0.3,\ |C_9^{(\prime)}| \lesssim 0.9,\ |C_{10}^{(\prime)}| \lesssim 0.6,\ |C_{S,P}^{(\prime)}| \lesssim 0.06,\ |C_{T,T5}| \lesssim 1.6$$

- **NP searches in BRs are difficult, and clearly are not the way to go!**
- **Can we extract something positive from BRs? YES, with more data model parameters (a_j, δ_j) can be constrained and the model can be improved!**

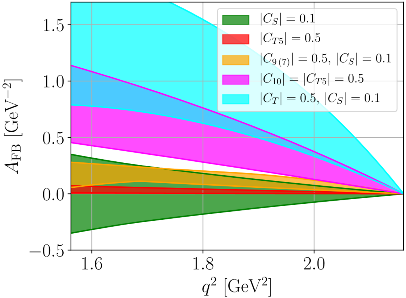
Null tests the way to go! Angular observables

- Lepton forward-backward asymmetry (many more ...)

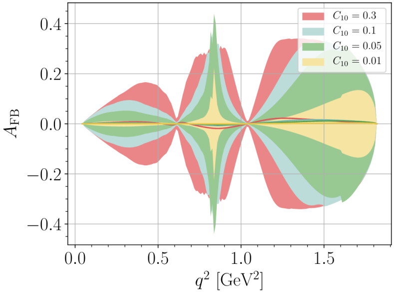
$$A_{\mathrm{FB}}(q^2) \propto \left[\int_0^1 - \int_{-1}^0\right] \frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2 \mathrm{d}\theta_{\ell P}}$$

- Linear dependence with C_i', $C_{S,T,T5,10}$ $\rightarrow$ $A_{\mathrm{FB}}^{\mathrm{SM}}(q^2) \approx 0$

$D_s^+ \to K^+ \mu^+ \mu^-$ (1909.11108)



$\Lambda_c \to p\, \mu^+ \mu^-$ (2107.13010)



Any signal is NP!

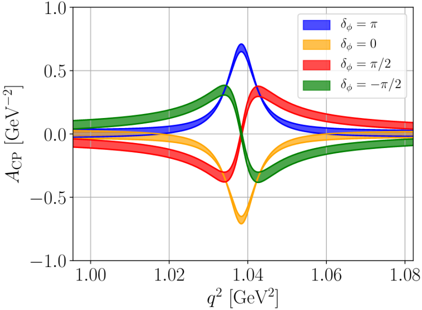
Next stop, CP-asymmetries!

$$A_{CP}(q^2) \propto \frac{d\Gamma}{dq^2} - \frac{d\bar{\Gamma}}{dq^2}$$

Let's get benefit from resonances! (1208.0759)

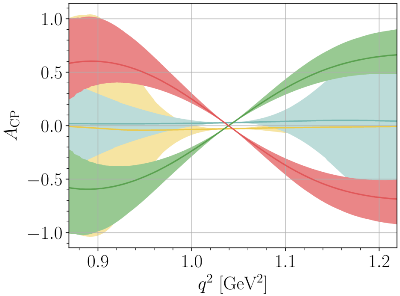
- Linear dependence with $\mathrm{Im}[C_i^{NP}] \times \mathrm{Im}[C_{9,P}^{R}] \rightarrow$ $A_{CP}^{SM}(q^2) \approx 0$

$D_s^+ \rightarrow K^+ \mu^+ \mu^-$ (1909.11108)



$C_9 = 0.1 \exp(i\pi/4)$, $q^2 \in [(m_\phi - 5\,\Gamma_\phi)^2, (m_\phi + 5\,\Gamma_\phi)^2]$

$\Lambda_c \rightarrow p\,\mu^+ \mu^-$ (2107.13010)



$C_9 = 0.5 \exp(i\pi/4)$, $q^2 \in [(m_\phi - 5\,\Gamma_\phi)^2, (m_\phi + 5\,\Gamma_\phi)^2]$

Any signal is CP-violating NP!

LU ratios

- **LU can be probed in $c \to u\,\ell^+\ell^-$ (same as B decays)**

$$R_P^D = \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\mathcal{B}(D \to P\mu^+\mu^-)}{dq^2} dq^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\mathcal{B}(D \to Pe^+e^-)}{dq^2} dq^2}$$

- **Same kinematical limits $\to$ Cancellation of had. uncertainties!**
- **Well control of SM prediction:** $\boxed{R_P^D|_{\rm SM} \approx 1}$
- **e.g. $D^+ \to \pi^+\ell^+\ell^-$ 1909.11108, see 1805.08516 ($D \to P_1 P_2\,\ell^+\ell^-$)**
  - **full q^2:** insensitive to NP.
  - **low q^2:** poor knowledge of resonances $\to$ sizable uncertainties.
  - **high q^2:** induce significant NP effects.

	SM	$\lvert C_9\rvert = 0.5$	$\lvert C_{10}\rvert = 0.5$	$\lvert C_9\rvert = \pm\lvert C_{10}\rvert = 0.5$	$\lvert C_{S(P)}\rvert = 0.1$	$\lvert C_T\rvert = 0.5$	$\lvert C_{T5}\rvert = 0.5$
full q^2	$1.00 \pm \mathcal{O}(10^{-2})$	SM-like	SM-like	SM-like	SM-like	SM-like	SM-like
low q^2	$0.95 \pm \mathcal{O}(10^{-2})$	$\mathcal{O}(100)$	$\mathcal{O}(100)$	$\mathcal{O}(100)$	$0.9\ldots1.4$	$\mathcal{O}(10)$	$1.0\ldots5.9$
high q^2	$1.00 \pm \mathcal{O}(10^{-2})$	$0.2\ldots11$	$3\ldots7$	$2\ldots17$	$1\ldots2$	$1\ldots5$	$2\ldots4$

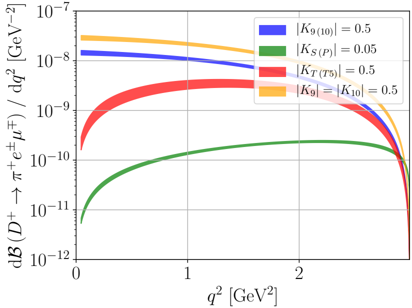
Testing LFV with $c \to u\,\ell^+\ell'^-$ decays

- **Forbidden in SM! Any signal is LFV NP!**
- **Experimental limits (90% C.L.)** (2011.00217, 1107.4465)

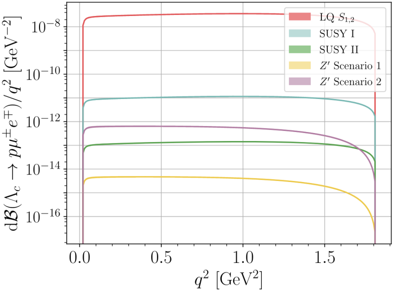
$$\mathcal{B}(D^+ \to \pi^+ e^{+(-)}\mu^{-(+)})_{\text{LHCb}} < 2.1(2.2)\cdot 10^{-7}$$

$$\mathcal{B}(\Lambda_c \to p\, e^{+(-)}\mu^{-(+)})_{\text{Babar}} < 9.9\,(19)\cdot 10^{-6}$$

1909.11108



2107.13010



Dineutrino modes $c \to u\,\nu\bar{\nu}$

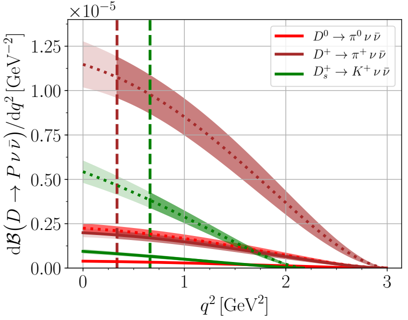
- **Extremely GIM-suppressed in the SM** (hep-ph/0112235, 0908.1174)

$$\boxed{\mathcal{B}(D \to \pi\nu\bar{\nu})_{\text{SM}} \sim 10^{-16}!}$$

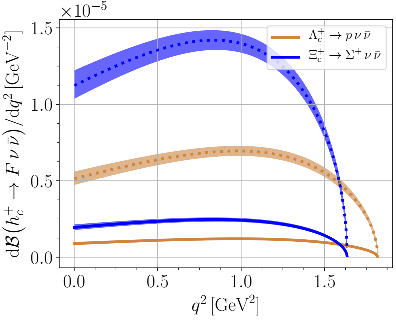
- **Only experimental information on (90% C.L.)** (1611.09455, 2112.14236)

$$\mathcal{B}(D^0 \to \nu\bar{\nu}) < 9.4 \cdot 10^{-5},\ \mathcal{B}(D^0 \to \pi^0\nu\bar{\nu}) < 2.1 \cdot 10^{-4}$$

2010.02225

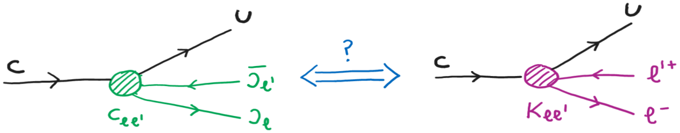


2010.02225



Can we get complementary information on LFV from dineutrino modes?

ℓ and ν_ℓ (with $\ell = e, \mu, \tau$) belong to same **SU(2)$_L$ doublet** in the SM.



$$\begin{pmatrix} c_{ee} & c_{e\mu} & c_{e\tau} \\ c_{\mu e} & c_{\mu\mu} & c_{\mu\tau} \\ c_{\tau e} & c_{\tau\mu} & c_{\tau\tau} \end{pmatrix}$$

Neutrino flavor not tagged!

$$\mathcal{B}(c \to u\,\nu\bar{\nu}) = \sum_{\ell,\ell'} \mathcal{B}(c \to u\,\nu_\ell\bar{\nu}_{\ell'})$$

LU, cLFC or general:

$$\mathcal{B}(c \to u\,\nu\bar{\nu}) \sim \frac{1}{3}\sum_{\ell,\ell'} c_{\ell\ell'}$$

$$\begin{pmatrix} k_{ee} & k_{e\mu} & k_{e\tau} \\ k_{\mu e} & k_{\mu\mu} & k_{\mu\tau} \\ k_{\tau e} & k_{\tau\mu} & k_{\tau\tau} \end{pmatrix}$$

Charged leptons tagged!

LU: $R_H \sim \frac{\mathcal{B}(c\to u\,\mu^+\mu^-)}{\mathcal{B}(c\to u\,e^+e^-)} \sim 1 + k_{\mu\mu} - k_{ee}$

cLFC or general: $\mathcal{B}(c \to u\,\ell'^+\ell^-) \sim k_{\ell\ell'}$

Is there a link between $c_{\ell\ell'}$ and $k_{\ell\ell'}$?

Low-energy $|\Delta c| = |\Delta u| = 1$ EFT description

$c \to u\, \nu_\ell \bar{\nu}_{\ell'}$ $\overset{?}{\Longleftrightarrow}$ $c \to u\, \ell^- \ell'^+$

$$\mathcal{H}_{\rm eff}^{\nu_\ell \bar{\nu}_{\ell'}} = -\frac{4\, G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sum_k \mathcal{C}_k^{U\ell\ell'}\, Q_k^{U\ell\ell'} \qquad \mathcal{H}_{\rm eff}^{\ell^- \ell'^+} = -\frac{4\, G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sum_k \mathcal{K}_k^{U\ell\ell'}\, O_k^{D\ell\ell'}$$

Only two operators (no RH neutrinos like SM) **Further operators non-connected**

$$Q_{\rm L\,(R)}^{U\ell\ell'} = (\bar{u}_{L\,(R)} \gamma_\mu c_{L\,(R)})\, (\bar{\nu}_{\ell'\,L} \gamma^\mu \nu_{\ell\,L}) \qquad O_{\rm L\,(R)}^{U\ell\ell'} = (\bar{u}_{L\,(R)} \gamma_\mu c_{L\,(R)})\, (\bar{\ell}'_L \gamma^\mu \ell_L)$$

...

Dineutrino BR is obtained via an incoherent neutrino flavor sum:

$$\mathcal{B}\,(c \to u\, \nu\bar{\nu}) = \sum_{\ell,\ell'} \mathcal{B}\,(c \to u\, \nu_\ell \bar{\nu}_{\ell'}) \sim \sum_{\ell,\ell'} \left| \mathcal{C}_L^{U\ell\ell'} \pm \mathcal{C}_R^{U\ell\ell'} \right|^2$$

$\mathcal{C}^P$ and $\mathcal{K}^P$ in the mass basis. $P = D$ ($P = U$) $\to$ down-quark sector (up-quark sector).

Correlate neutrinos and charged leptons with $SU(2)_L$

Lowest order $SU(2)_L \times U(1)_Y$–invariant effective theory 1008.4884

$$\mathcal{L}_{\text{SMEFT}}^{\text{LO}} \supset \frac{C_{\ell q}^{(1)}}{v^2} \bar{Q}\gamma_\mu Q \, \bar{L}\gamma^\mu L + \frac{C_{\ell q}^{(3)}}{v^2} \bar{Q}\gamma_\mu \tau^a Q \, \bar{L}\gamma^\mu \tau^a L + \frac{C_{\ell u}}{v^2} \bar{U}\gamma_\mu U \, \bar{L}\gamma^\mu L + \frac{C_{\ell d}}{v^2} \bar{D}\gamma_\mu D \, \bar{L}\gamma^\mu L$$

1. **Writing in $SU(2)_L$-components:** ($C \to$ dineutrinos and $K \to$ dileptons in the gauge basis)

$$C_L^U = K_L^D = \frac{2\pi}{\alpha} \left(C_{\ell q}^{(1)} + C_{\ell q}^{(3)} \right) , \quad C_R^U = K_R^U = \frac{2\pi}{\alpha} C_{\ell U} \, .$$

2. **Mass basis:** $\boxed{\mathcal{C}_L^U = W^\dagger \mathcal{K}_L^D W + \mathcal{O}(\lambda) \, , \qquad \mathcal{C}_R^U = W^\dagger \mathcal{K}_R^U W}$

3. **BR is independent of PMNS matrix!**

$$\mathcal{B}(c \to u \, \nu\bar{\nu}) \sim \sum_{\ell,\ell'} \left| \mathcal{C}_L^{U\ell\ell'} \pm \mathcal{C}_R^{U\ell\ell'} \right|^2 = \text{Tr}\left[(\mathcal{C}_L^U \pm \mathcal{C}_R^U)(\mathcal{C}_L^U \pm \mathcal{C}_R^U)^\dagger \right]$$

$$= \text{Tr}\left[W^\dagger (\mathcal{K}_L^D \pm \mathcal{K}_R^U) W \, W^\dagger (\mathcal{K}_L^D \pm \mathcal{K}_R^U)^\dagger W \right] = \sum_{\ell,\ell'} \left| \mathcal{K}_L^{D\ell\ell'} \pm \mathcal{K}_R^{U\ell\ell'} \right|^2 + \mathcal{O}(\lambda)$$

Prediction of dineutrino rates for different leptonic flavor structures $\mathcal{K}_{L,R}^{\ell\ell'}$ can be probed with lepton-specific measurements!

Possible leptonic flavor structures for $\mathcal{K}_{L,R}^{\ell\ell'}$

$$\mathcal{B}(c \to u\,\nu\bar{\nu}) \sim \sum_{\ell,\ell'} \left|\mathcal{K}_L^{D\ell\ell'} \pm \mathcal{K}_R^{U\ell\ell'}\right|^2$$

i) ***Lepton-universality* (LU).**

$$\begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$

ii) ***Charged lepton flavor conservation* (cLFC).**

$$\begin{pmatrix} k_{ee} & 0 & 0 \\ 0 & k_{\mu\mu} & 0 \\ 0 & 0 & k_{\tau\tau} \end{pmatrix}$$

iii) $\mathcal{K}_{L,R}^{\ell\ell'}$ **arbitrary.**

$$\begin{pmatrix} k_{ee} & k_{e\mu} & k_{e\tau} \\ k_{\mu e} & k_{\mu\mu} & k_{\mu\tau} \\ k_{\tau e} & k_{\tau\mu} & k_{\tau\tau} \end{pmatrix}$$

Dineutrino branching ratios

$$\mathcal{B} = A_+ \, x^+ + A_- \, x^- , \qquad x^\pm = \sum_{\ell,\ell'} \left| \mathcal{C}_L^{U\ell\ell'} \pm \mathcal{C}_R^{U\ell\ell'} \right|^2$$

→ Long-distance dyn. & kinematics $A_\pm$: **LCSR (low q^2) + Lattice (high q^2)**

→ Short-distance dynamics $x^\pm$: **WCs (BSM)**

→ Excellent complementarity $\mathcal{B}$:

- $A_- = 0$ in $D \to P \, \nu\bar{\nu}$ decays.
- $A_- > A_+$ in $D \to P_1 P_2 \, \nu\bar{\nu}$ decays.
- $A_- = A_+$ in inclusive D decays.

$D \to F$	A_+ [10^{-8}]	A_- [10^{-8}]
$D^0 \to \pi^0$	0.9	0
$D^+ \to \pi^+$	3.6	0
$D^0 \to \pi^0\pi^0$	0	0.2
$D^0 \to \pi^+\pi^-$	0	0.4
$D^0 \to X$	2.2	2.2
$D^+ \to X$	5.6	5.6

Upper limits on dineutrino modes can probe LU!

- **Limits from high-p_T & charged dilepton D and K-decays (†):**[1]

	$\lvert\mathcal{K}_A^{P\ell\ell'}\rvert$	ee	$\mu\mu$	$\tau\tau$	$e\mu$	$e\tau$	$\mu\tau$
$s\,d$	$\lvert\mathcal{K}_L^{D\ell\ell'}\rvert$	$5\cdot 10^{-2\dagger}$	$1.6\cdot 10^{-2\dagger}$	6.7	$6.6\cdot 10^{-4\dagger}$	6.1	6.6
$c\,u$	$\lvert\mathcal{K}_R^{U\ell\ell'}\rvert$	2.9	$0.9^{\dagger}$	5.6	1.6	4.7	5.1

- $x^{\pm} < 2\,x\ ,\ x = \sum_{\ell,\ell'}\left(\lvert\mathcal{K}_L^{D\ell\ell'}\rvert^2 + \lvert\mathcal{K}_R^{U\ell\ell'}\rvert^2\right) + \mathcal{O}(\lambda) = \sum_{\ell,\ell'} R^{\ell\ell'} + \mathcal{O}(\lambda)$

$\boldsymbol{x = 3\,R^{\mu\mu} \lesssim 2.6}$, **(Lepton Universality)** **LU is fixed by muons.**

$\boldsymbol{x = R^{ee} + R^{\mu\mu} + R^{\tau\tau} \lesssim 156}$, **(charged Lepton Flavor Conservation)**

$\boldsymbol{x = R^{ee} + R^{\mu\mu} + R^{\tau\tau} + 2\,(R^{e\mu} + R^{e\tau} + R^{\mu\tau}) \lesssim 655.}$

[1] 2002.05684, 2003.12421 & 2007.05001 (†)

Dineutrino branching ratios upper limits

$h_c \to F$	$\mathcal{B}^{\text{max}}_{\text{LU}}$ $[10^{-7}]$	$\mathcal{B}^{\text{max}}_{\text{cLFC}}$ $[10^{-6}]$	$\mathcal{B}^{\text{max}}$ $[10^{-6}]$
$\mathbf{D^0 \to \pi^0}$	**0.5**	**2.8**	**12**
$D^+ \to \pi^+$	1.9	11	47
$D^0 \to \pi^0\pi^0$	0.1	0.7	2.8
$D^0 \to \pi^+\pi^-$	0.2	1.3	5.4
$\Lambda_c^+ \to p^+$	1.4	8.4	35
$\Xi_c^+ \to \Sigma^+$	2.7	17	70

$\mathcal{B}(D^0 \to \pi^0 \nu\bar{\nu}) < 2.1 \cdot 10^{-4}$ from BES III is about one order of magnitude away from our predictions 2112.14236

Well-suited for Belle II and FCC-ee

- $N(c\bar{c})_{\text{Belle II (FCC-ee)}} = 65 \cdot 10^9\,(5.5 \cdot 10^{11})!$ (Abada:2019lih)
- **How many charm hadrons h_c?**

⋆ $N(h_c) = 2\, f(c \to h_c)\, N(c\bar{c})$

⋆ **Fragmentation fractions** (1509.01061)

$$N(h_c) \sim 10^{10}\,(10^{11})!$$

h_c	$f(c \to h_c)$	$N(h_c)_{\text{Belle II (FCC-ee)}}$
D^0	0.59	$8 \cdot 10^{10}\,(6 \cdot 10^{11})$
D^+	0.24	$3 \cdot 10^{10}\,(3 \cdot 10^{11})$
D_s^+	0.10	$1 \cdot 10^{10}\,(1 \cdot 10^{11})$
Λ_c^+	0.06	$8 \cdot 10^{9}\,(7 \cdot 10^{10})$

- **And translated to branching ratios?**

Relative statistical uncertainty: $\delta\mathcal{B}(h_c) = 1/\sqrt{N^{\text{exp}}}$ with $N^{\text{exp}} = \eta_{\text{eff}}\, N(h_c)\, \mathcal{B}(h_c)$

$$\eta_{\text{eff}}\, \mathcal{B}(h_c) \sim 10^{-9}\,(10^{-10}) \text{ for } \delta\mathcal{B}(h_c) = \frac{1}{5}$$

Conclusions & Outlook

⋆ **Window to explore FCNCs in the up-sector.**

⋆ **Unique phenomenology (strong GIM suppression).**

⋆ **Clean null test observables can probe NP.**

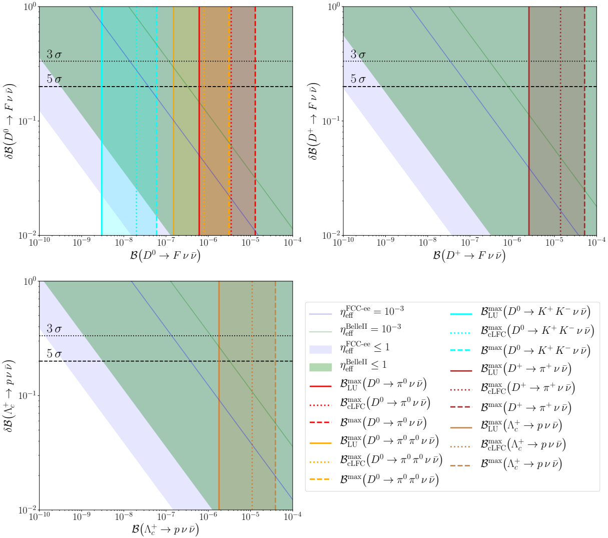
⋆ **Experimentally plenty of room for NP:**

- **Angular observables**
- **CP-asymmetries**
- **LU ratios**
- **LFV BRs**
- **Dineutrino BRs**

Thank you for your attention!

$\delta\mathcal{B}$ vs $\mathcal{B}$: exp. projections and theo. predictions

2010.02225



Estimated future LHCb prospects

2011.09478

Table 9. Estimated upper limits (UL) of selected rare and forbidden decay modes at LHCb for future data sets, taken from Ref. 112. Limits for the decay channels $D^+ \to \pi^+ e^+ e^-$ and $D^+ \to \pi^+ e^+ \mu-$ have been obtained by scaling the observed limits taken from Ref. 48 to $23\,\text{fb}^{-1}$ and $300\,\text{fb}^{-1}$ of integrated luminosity, assuming the upper limit to scale with the square root of the integrated luminosity.

Decay channel	UL LHCb extrapolation [$23\,\text{fb}^{-1}$]	UL LHCb extrapolation [$300\,\text{fb}^{-1}$]
$D^0 \to \mu^+\mu^-$	$\sim 5.9 \times 10^{-10}$	$\sim 1.8 \times 10^{-10}$
$D^+ \to \pi^+\mu^+\mu^-$	$\sim 1.3 \times 10^{-8}$	$\sim 3.7 \times 10^{-9}$
$\Lambda_c^+ \to p\mu^+\mu^-$	–	$\sim 4.4 \times 10^{-9}$
$D^+ \to \pi^+e^+e^-$	$\sim 4.2 \times 10^{-7}$	$\sim 1.2 \times 10^{-7}$
$D^+ \to \pi^+e^+\mu^-$	$\sim 5.5 \times 10^{-8}$	$\sim 1.5 \times 10^{-8}$

For 23 (300) fb^{-1}:

$$\left|\mathcal{C}_{9,\,10}^{(\mu)(\prime)}\right| \lesssim 0.4\,(0.3)\;, \quad \left|\mathcal{C}_{T,\,T5}^{(\mu)}\right| \lesssim 0.8\,(0.5)\;, \quad \left|\mathcal{C}_{S,P}^{(\mu)(\prime)}\right| \lesssim 0.03\,(0.02)\;,$$

$$\left|\mathcal{C}_{9,\,10}^{(e)(\prime)}\right| \lesssim 2\,(1)\;, \quad \left|\mathcal{C}_{T,\,T5}^{(e)}\right| \lesssim 2\,(1)\;,$$

Corrections to the trace

$$\mathcal{C}_L^U = W^\dagger [V\,\mathcal{K}_L^D\,V^\dagger]\,W\,,\qquad \mathcal{C}_R^U = W^\dagger [\mathcal{K}_R^U]\,W\,. \tag{A1}$$

The $\mathcal{C}_{L,R}^U$ depend on the PMNS matrix, which drops out in the flavor-summed branching ratios (4) due to unitarity. $\mathcal{C}_L^U$ depends on the CKM-matrix that allows for an expansion in the Wolfenstein parameter λ, relevant for $c \to u$ transitions as

$$\mathcal{C}_L^{U_{12}} = W^\dagger \mathcal{K}_L^{D_{12}} W + \lambda\, W^\dagger (\mathcal{K}_L^{D_{22}} - \mathcal{K}_L^{D_{11}}) W + \mathcal{O}(\lambda^2)\,.$$

The superscripts 12, 11 and 22 given explicitly indicate the generations in the quark currents of the operators, *i.e.*, $\bar{u}c$, $\bar{d}s$, $\bar{d}d$ and $\bar{s}s$. In the remainder of this work, which focuses on $c \to u$ transitions, we use $\mathcal{C}_{L,R}^{U_{12}} = \mathcal{C}_{L,R}^U$ to avoid clutter. For x_U, one obtains

$$\begin{aligned} x_U &= \sum_{\nu=i,j} \left(|\mathcal{C}_L^{U ij}|^2 + |\mathcal{C}_R^{U ij}|^2 \right) = \mathrm{Tr}\left[\mathcal{C}_L^U \mathcal{C}_L^{U\dagger} + \mathcal{C}_R^U \mathcal{C}_R^{U\dagger} \right] \\ &= \mathrm{Tr}\left[\mathcal{K}_L^{D_{12}} \mathcal{K}_L^{D_{12}\dagger} + \mathcal{K}_R^{U_{12}} \mathcal{K}_R^{U_{12}\dagger} \right] + \delta x_U + \mathcal{O}(\lambda^2) \\ &= \sum_{\ell=i,j} \left(|\mathcal{K}_L^{D_{12} ij}|^2 + |\mathcal{K}_R^{U_{12} ij}|^2 \right) + \delta x_U + \mathcal{O}(\lambda^2)\,, \end{aligned} \tag{A2}$$

with the $\mathcal{O}(\lambda)$-correction

$$\begin{aligned} \delta x_U &= 2\lambda\, \mathrm{Tr}\left[\mathrm{Re}\left\{ \mathcal{K}_L^{D_{12}} \left(\mathcal{K}_L^{D_{22}\dagger} - \mathcal{K}_L^{D_{11}\dagger} \right) \right\} \right] \\ &= 2\lambda \sum_{\ell=i,j} \mathrm{Re}\left\{ \left(\mathcal{K}_L^{D_{12} ij}\, \mathcal{K}_L^{D_{22} ij *} - \mathcal{K}_L^{D_{12} ij} \mathcal{K}_L^{D_{11} ij *} \right) \right\}. \end{aligned} \tag{A3}$$

$C_L^U = K_L^D$ and $C_R^U = K_R^U$ are broken by RGE corrections from gauge, Yukawa, and QED coupling dependences, the effect is less than 5% for $\Lambda_{NP} = 10\,\mathrm{TeV}$.