SM uncertainties for $b \to s\ell\ell$ decays

Beauty 2023 – Clermont-Ferrand

Méril Reboud

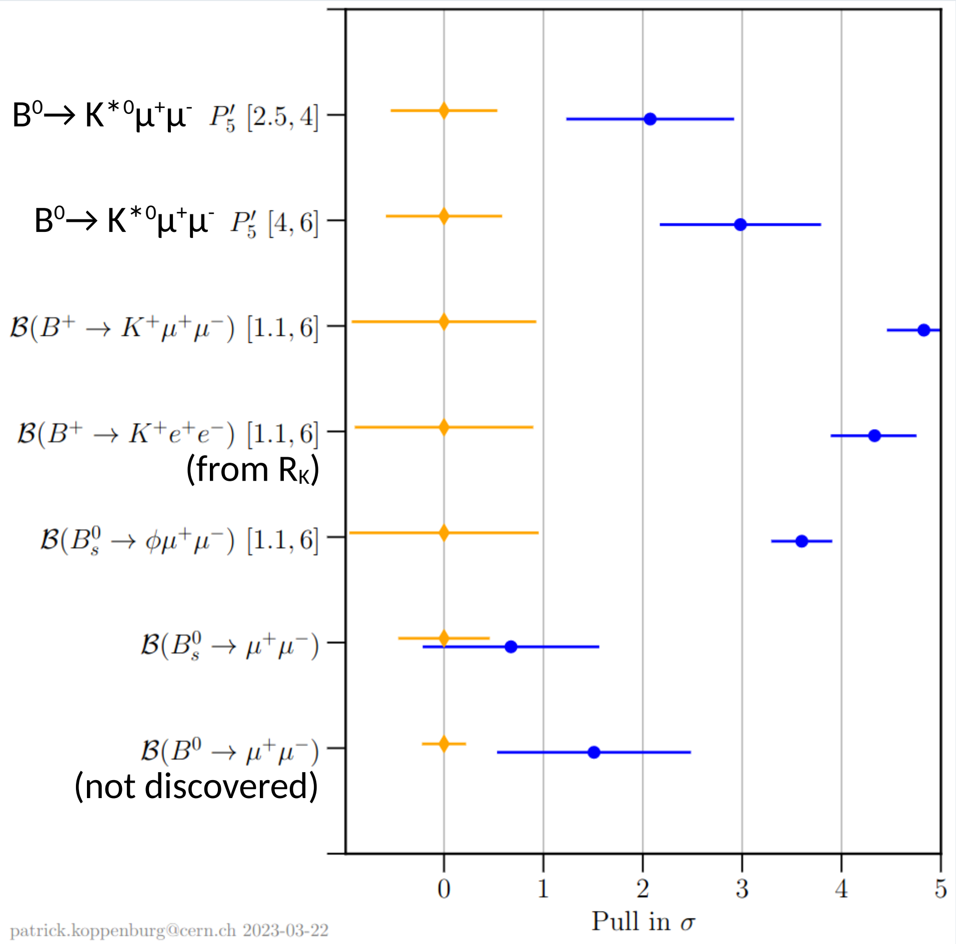
Based on:

- Gubernari, van Dyk, Virto 2011.09813
- Gubernari, MR, van Dyk, Virto 2206.03797, 2305.06301

Durham University

IP3

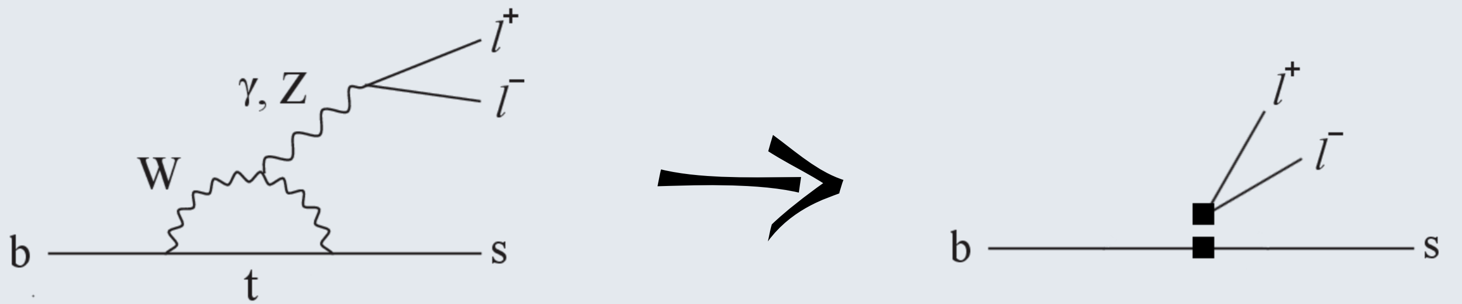
Experimental "Summary plot"



- Many **measurements** of $b \to s\ell\ell$ transitions are in tension with **theory**
- This is a *advertising plot:*
  - It only shows the observables featuring a tension
  - Observables and predictions are correlated
- This talk: **theory uncertainties**

Weak Effective Theory

- These processes take place at a scale $m_b < m_W, m_t$



- Allows for a model independent interpretation of the anomalies

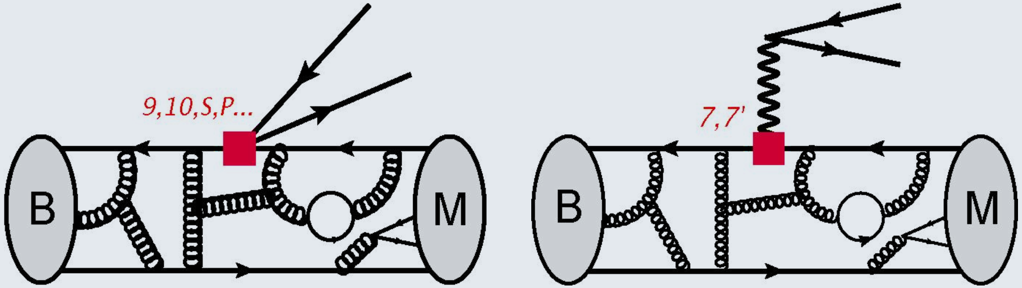
$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2} \left(\bar{s}_L \gamma_\mu b_L\right) \left(\bar{\ell} \gamma^\mu (\gamma_5) \ell\right)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} \left(\bar{s}_L \sigma_{\mu\nu} b_R\right) F^{\mu\nu}$$

- Avoids the appearance of large logarithm in the calculations of observables

QCD in b → sℓℓ



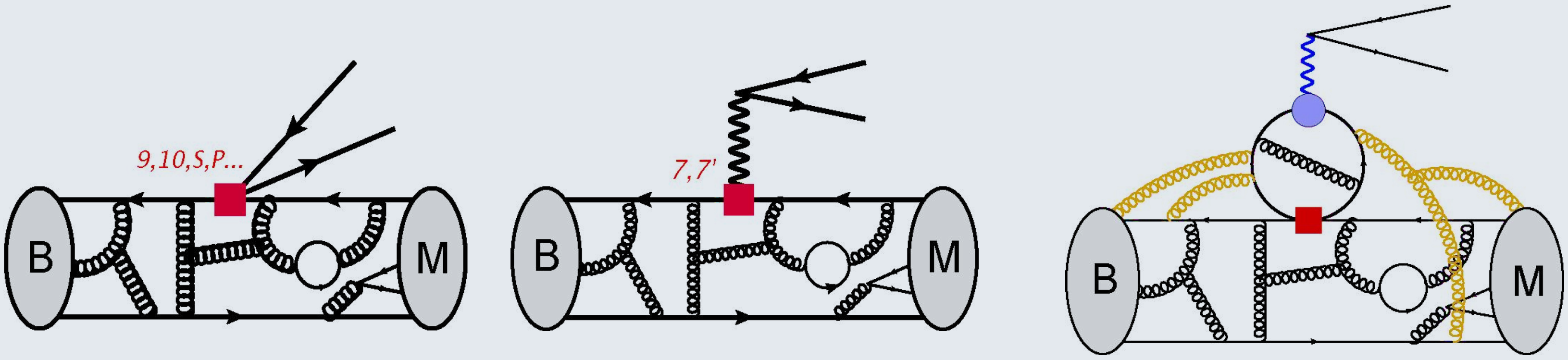
$$\mathcal{A}_\lambda^{L,R}(B\to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) \qquad\qquad \right]\right\}$$

- B → K$^{(*)}$ μμ
- B$_s$ → φ μμ
- Λ$_b$ → Λ$^{(*)}$ μμ

Local form-factors,
involves e.g.

$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$$

QCD in $b \to s\ell\ell$



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$$\mathcal{H}_\mu(k,q) = i \int d^4x\, e^{iq\cdot x} \langle \bar{M}(k) | T\{\mathcal{J}_\mu^{\text{em}}(x), \mathcal{C}_i\mathcal{O}_i\} | \bar{B}(q+k)\rangle$$

Non-local form-factors

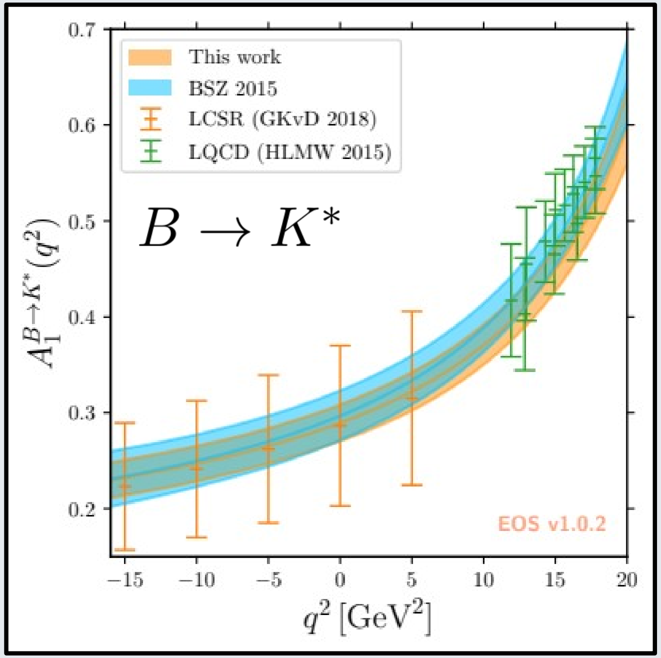
→ Main contributions: the **"charm-loops"** $\mathcal{O}_{2(1)}^c = (\bar{s}_L\gamma_\mu(T^a)c_L)(\bar{c}_L\gamma^\mu(T^a)b_L)$

Local form factors

- Conceptually easy, but still the dominant source of uncertainties
- **2 main approaches**
  - Lattice QCD → most feasible at **large q^2**
  - Light-cone sum rules → most feasible at **small q^2**, 2 possible LCSRs

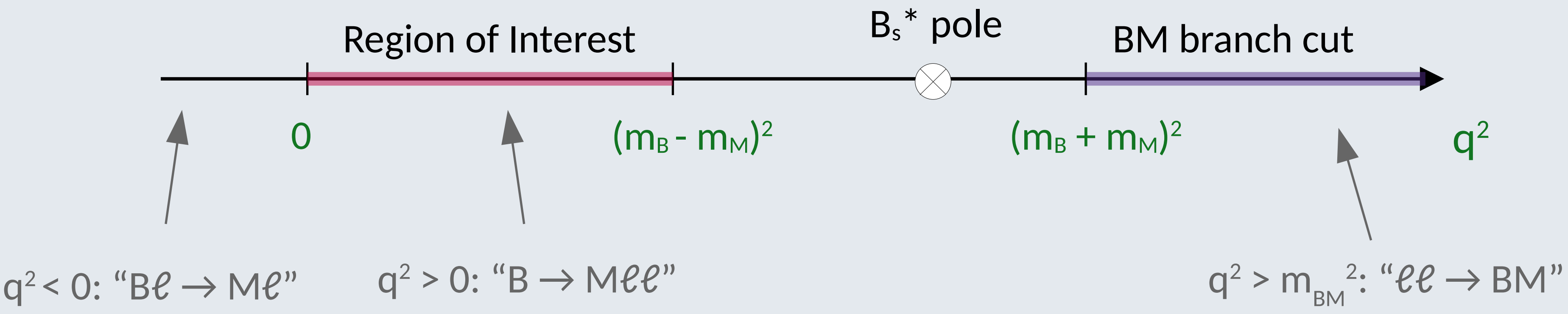
→ **Interpolation/Extrapolation**, depending on the use case

→ How to control extrapolation uncertainties?



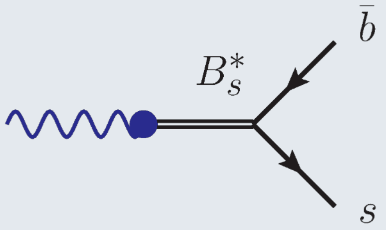
Form Factor Properties

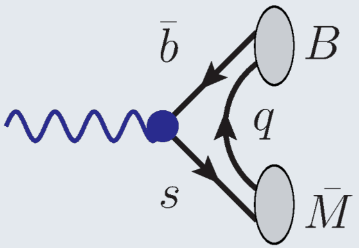
$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$$



Analytic properties of the form factors:

- Pole due to **$\bar{b}s$ bound state**
- **Branch cut** due to on-shell BM production





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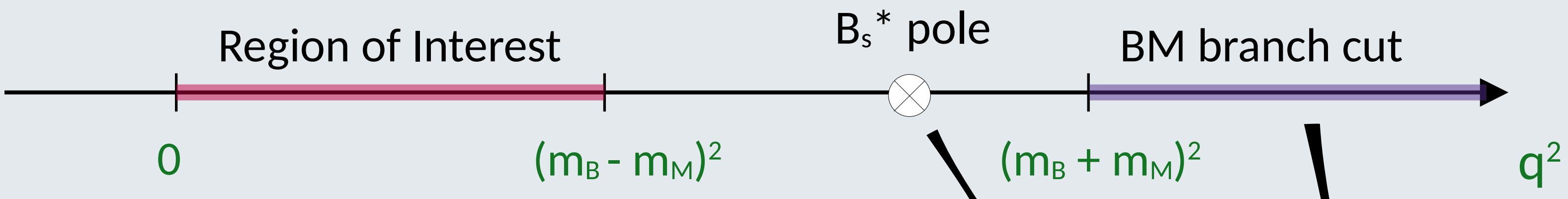

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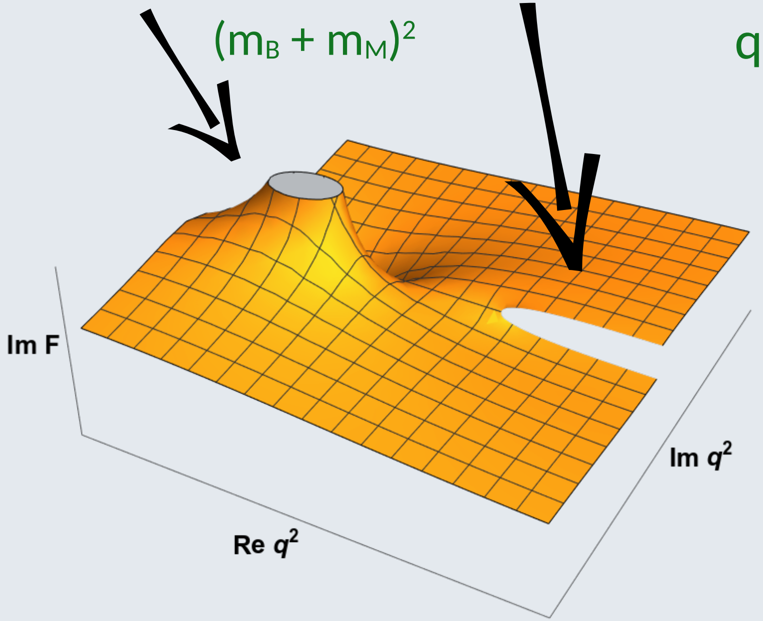
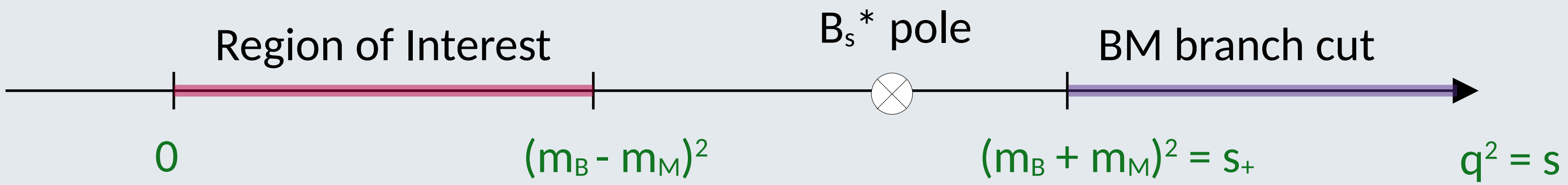


Form Factor Parametrization



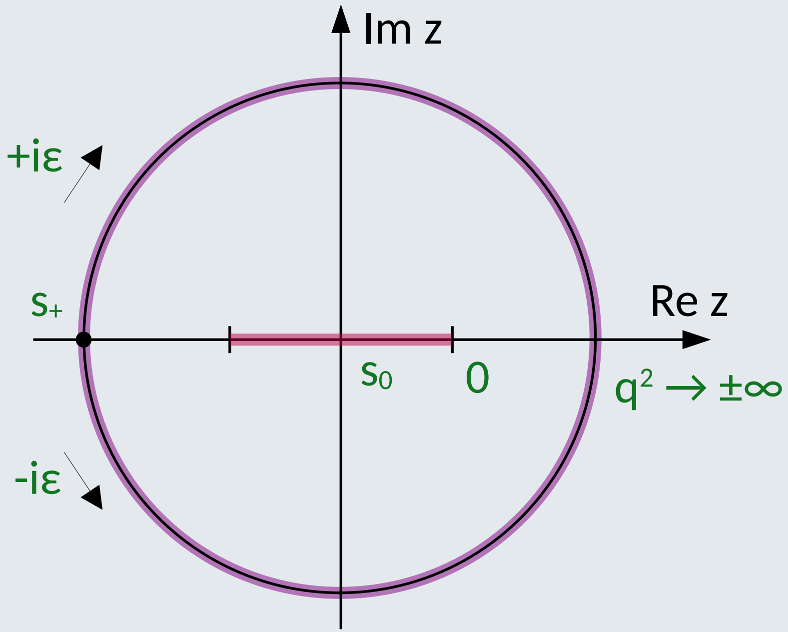
Conformal mapping [Boyd, Grinstein, Lebed '97]

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$

Simplified Series expansion [Bourrely, Caprini, Lellouch, '08; Bharucha, Feldmann, Wick '10]

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{q^2 - m_{B_s^*}^2} \sum_{k=0}^{N} \alpha_{\lambda,k} z^k$$

What is the uncertainty due to the truncation order N?



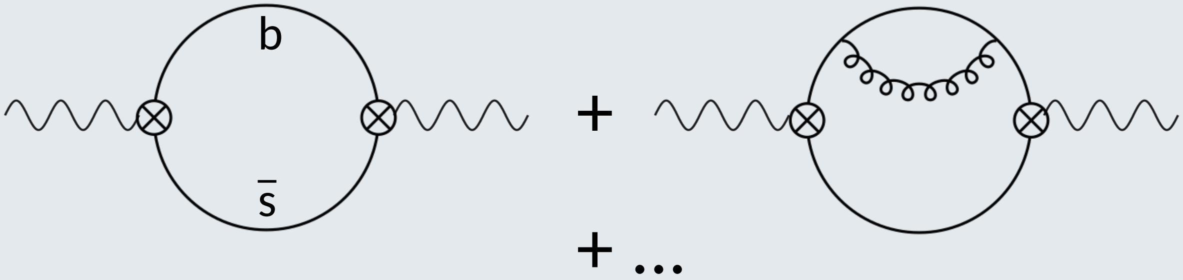
Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick ‘10]

$$\Pi^{\mu\nu}_{\Gamma}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0 | \mathcal{T}\left\{ J^{\mu}_{\Gamma}(x) J^{\dagger,\nu}_{\Gamma}(0) \right\} |0\rangle$$

<u>1) Partonic calculation</u>

Insertion of a scalar, vector or tensor current



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2) Relation to form factors

$$\mathrm{Im}\,\Pi_I^X(q^2) = \frac{1}{2}\sum_\Gamma \int d\rho_\Gamma\, (2\pi)^4\, \delta^4(q-p_\Gamma)\, P_I^{\mu\nu} \langle 0|\, j_\mu^X\, |\Gamma\rangle \langle \Gamma|\, j_\nu^{\dagger X}\, |0\rangle$$

~ |form factor|^2

Sum over all the $\bar{s}b$ states: $\bar{B}_s$, $\bar{B}K$, $\bar{B}K^*$, $\bar{B}K\pi$, ...

Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

- Assuming global quark-hadron duality we have

$$\underbrace{\chi_\Gamma^{(\lambda)}\big|_{\rm OPE} = \chi_\Gamma^{(\lambda)}\big|_{\rm 1pt}}_{\text{Known terms}} + \underbrace{\chi_\Gamma^{(\lambda)}\big|_{\bar{B}K} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}K^*} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}_s\phi} + \dots}_{\text{Sum of positive quantities}} .$$

Further contributions such as $B \to K\pi\pi$ or $\Lambda_b \to \Lambda^{(*)}$.

Any new terms *strengthens* the bound.

Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

- Changing the basis of the *z*-polynomials yields: $$\sum_{\mathcal{F}} \sum_{n\geq 0} \left|a_n^{\mathcal{F}}\right|^2 < 1$$

$$\mathcal{F}_{\Gamma,\lambda}^{B\to M}(z) = \frac{1}{\mathcal{P}_{\mathcal{F}}(z)\phi_{\mathcal{F}}(z)} \sum_{n\geq 0} a_n^{\mathcal{F}}\, p_n(z, \alpha_{\Gamma}^{B\to M})$$

Known functions

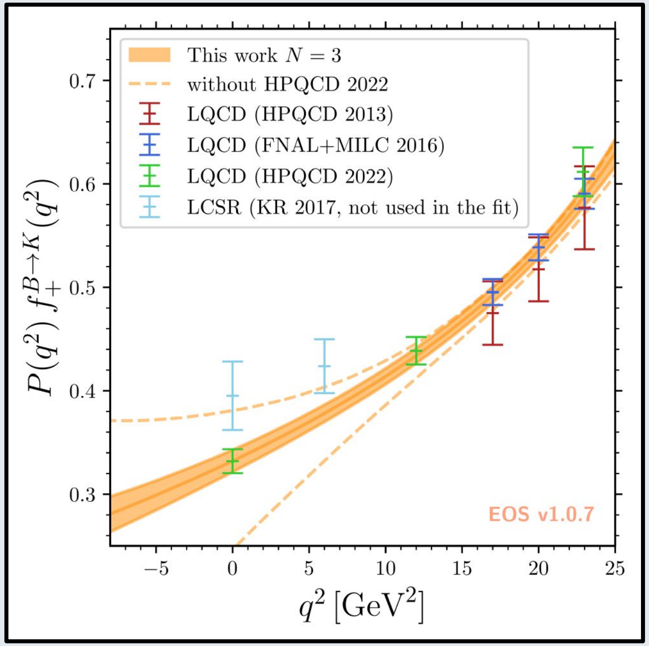
Orthnormal polynomials

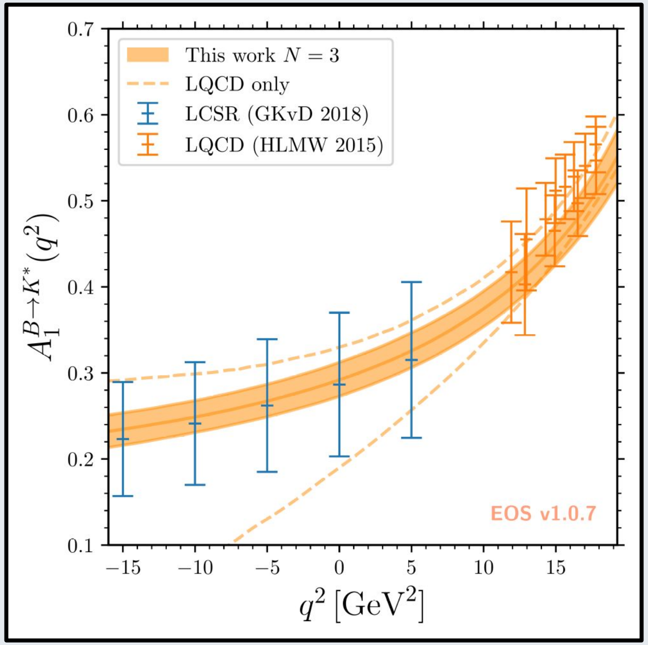
Local form factors fit

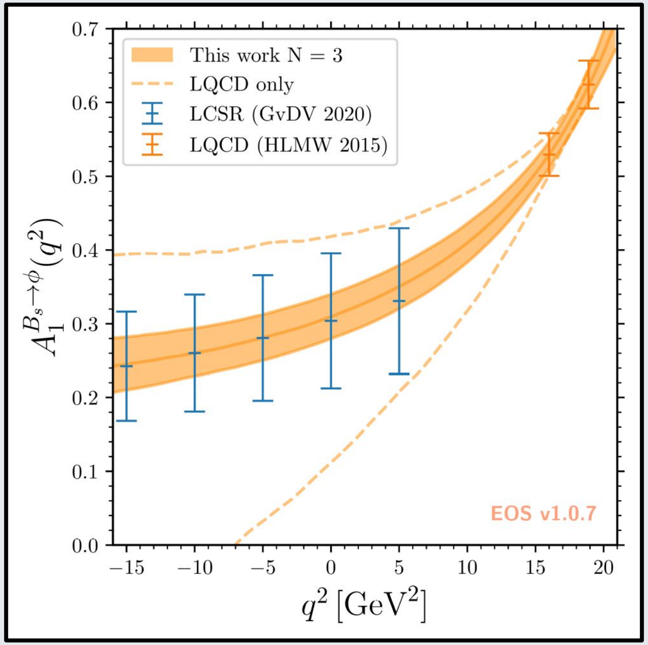
- With this framework we perform a **combined fit** of $B \to K$, $B \to K^*$ and $B_s \to \varphi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD ’13 and ’22; FNAL/MILC ’17]
    - ([Khodjamiriam, Rusov ’17]) → large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, Kokulu, van Dyk ’18] (B-meson LCSRs)
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, van Dyk, Virto ’20] (B-meson LCSRs)
- Adding $\Lambda_b \to \Lambda^{(*)}$ form factors is possible and desirable

Main conclusions:

- Fits are very good already at N = 2 (p-values > 77%)
- LCSR and LQCD combine nicely and still dominate the uncertainties
- Progresses in LQCD will gradually replace LCSR



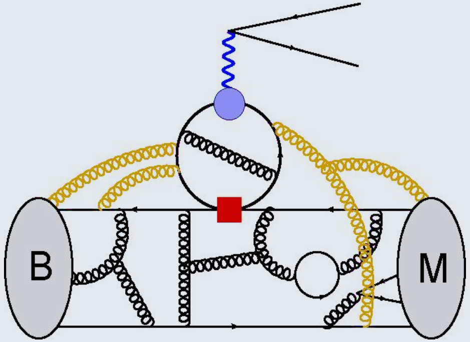




Non-local form factors

$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b}\mathcal{H}_\lambda(q^2)\right]\right\}$$

$$\mathcal{H}_\mu(k,q) = i\int d^4x\, e^{iq\cdot x}\langle \bar{M}(k)|T\{\mathcal{J}_\mu^{\text{em}}(x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$

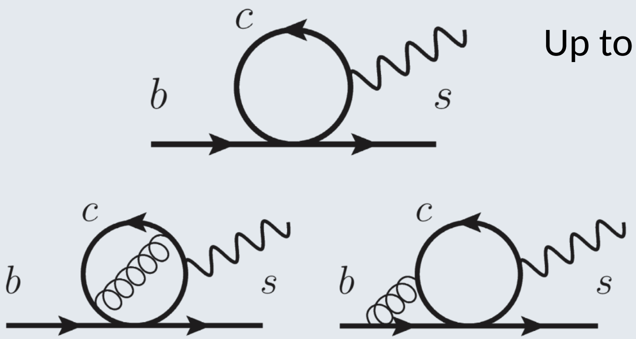


- Problematic because **they can mimic a BSM signal!**
  - $\mathcal{H}_\lambda$ can be interpreted as a shift to C_9 and C_7
  - This shift is lepton-flavour universal i.e. compatible with the measurements
- Notably **harder to estimate**, no lattice computation so far
- **Different parametrizations** are suggested

Theory inputs

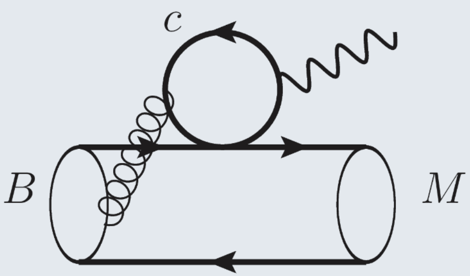
$\mathcal{H}_\lambda$ can still be calculated in **two kinematics regions**:

- **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol '04; Beylich, Buchalla, Feldmann '11]
- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang '10]



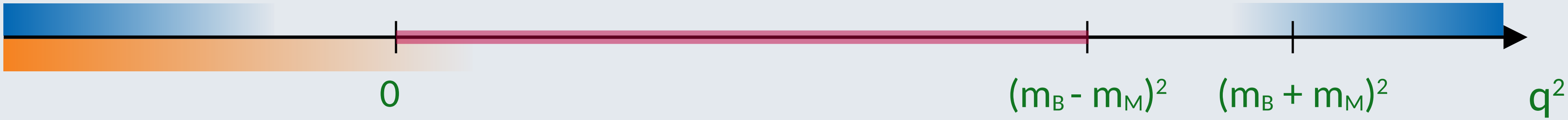
Up to NLO in α_s

[Asatarian, Greub, Virto '19]



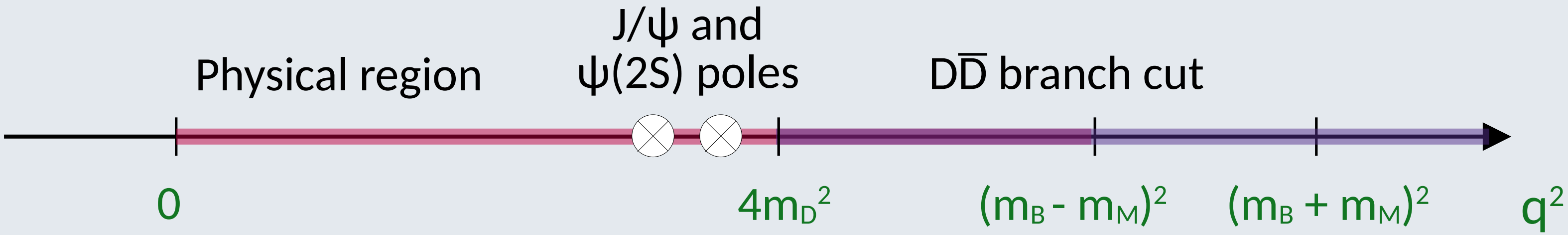
Non-perturbative soft gluon corrections

[Gubernari, van Dyk, Virto '20]



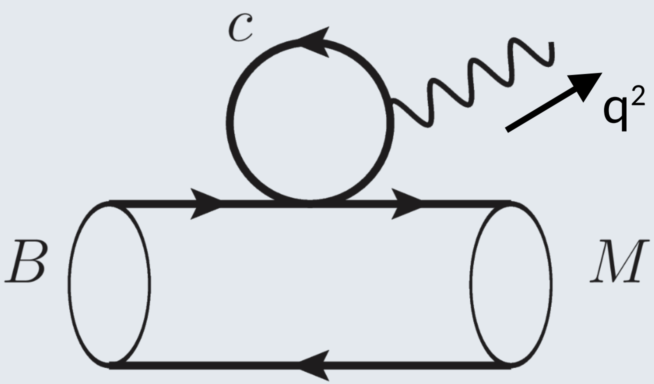
Analyticity properties

$$\mathcal{H}_\mu(k,q) = i \int d^4x\, e^{iq\cdot x} \langle \bar{M}(k) | T\{Q_c[\bar{c}\gamma_\mu c](x), \mathcal{C}_i \mathcal{O}_i\} | \bar{B}(q+k) \rangle$$



Analyticity properties:

- Poles due to **charmonium state**
- **Branch cut** in the physical range due to on-shell D meson production: B → MD$\bar{\text{D}}$
- The coefficients of the z-expansion are now **complex-valued**



Parametrization of the charm loop

- Still focusing on $B \to K$, $B \to K^*$ and $B_s \to \varphi$

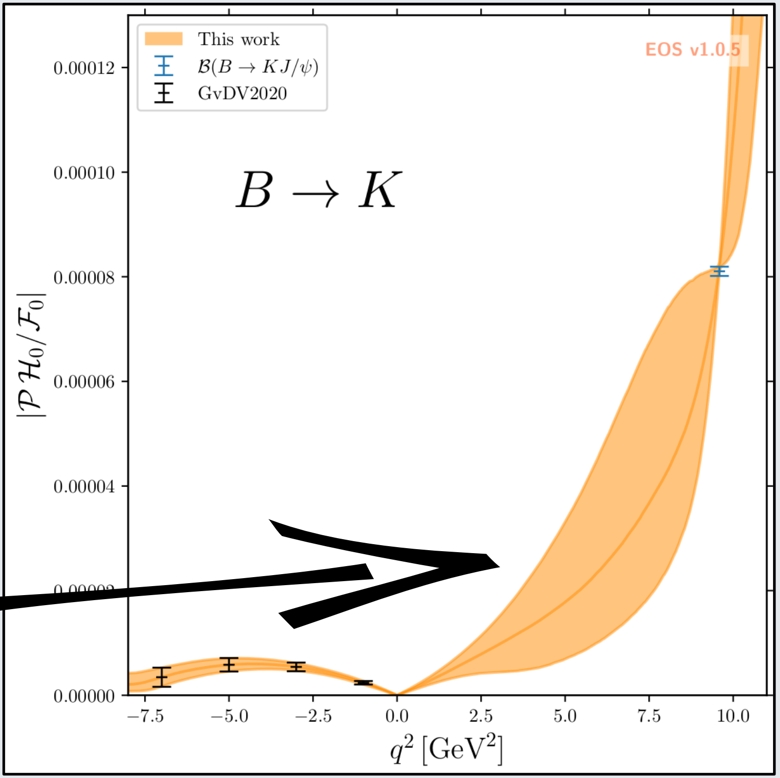
  Inputs:
  - 4 theory point at negative q^2 from the light cone OPE
  - Experimental results at the J/ψ (we keep $\psi(2S)$ for future work)

- Use an under-constrained fit (N = 5) and allows for saturation of the dispersive bound

  → The uncertainties are **truncation order independent**, increasing the expansion order does not change their size

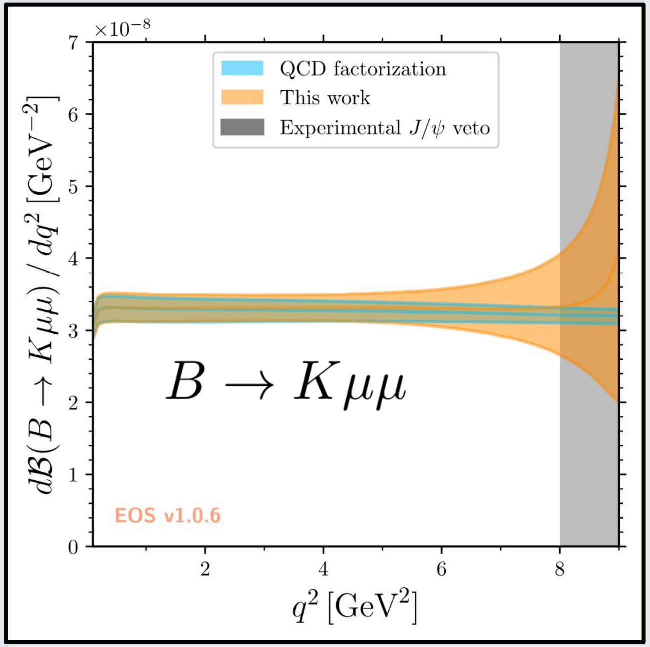
  → All p-values are larger than 11%

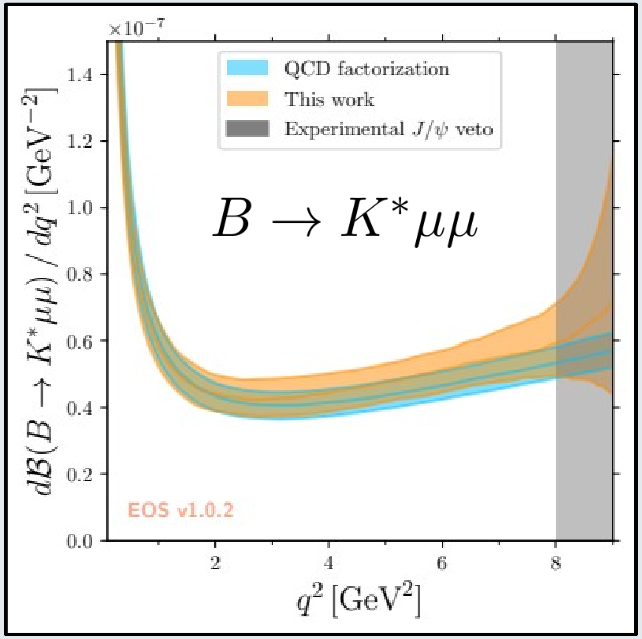
[Gubernari, MR, van Dyk, Virto '22]

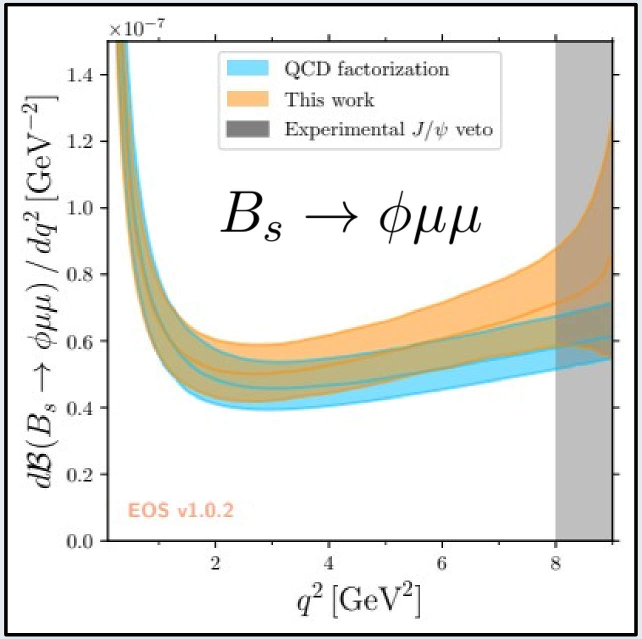


SM predictions

- Good overall agreement with previous theoretical approaches [Beneke, Feldman, Seidel '01 & '04]
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- Larger but controlled uncertainties especially near the J/ψ
  - $\to$ The approach is **systematically improvable** (new channels, $\psi(2S)$ data...)



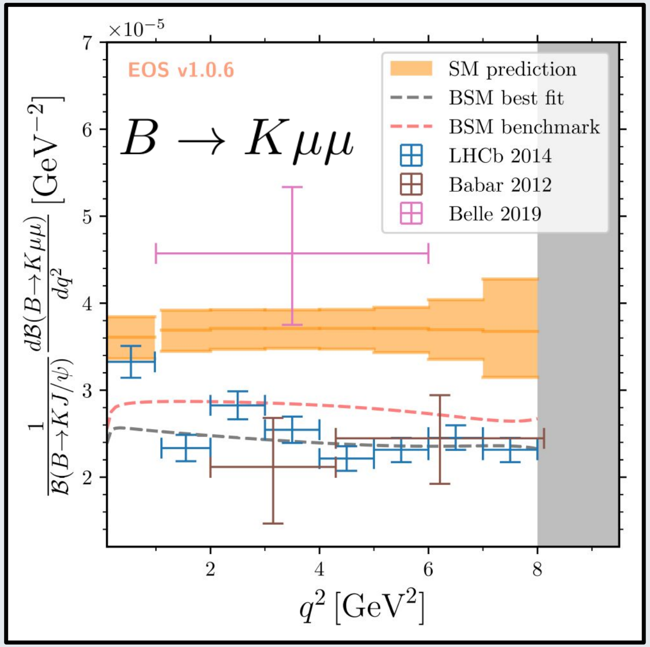


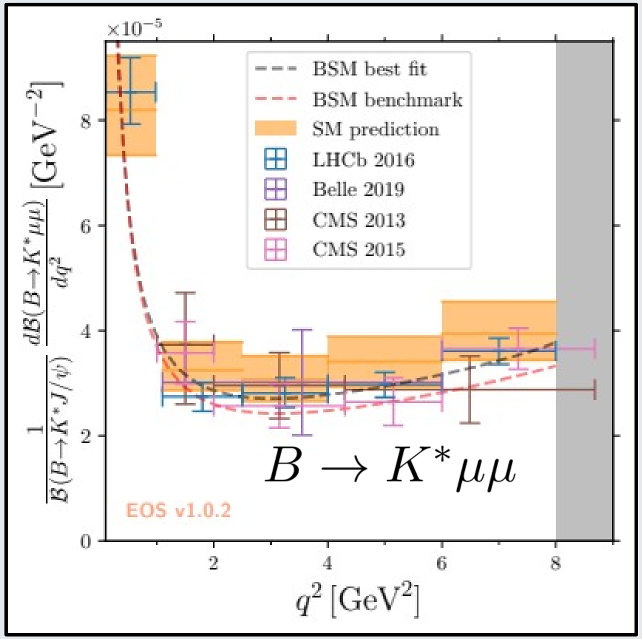


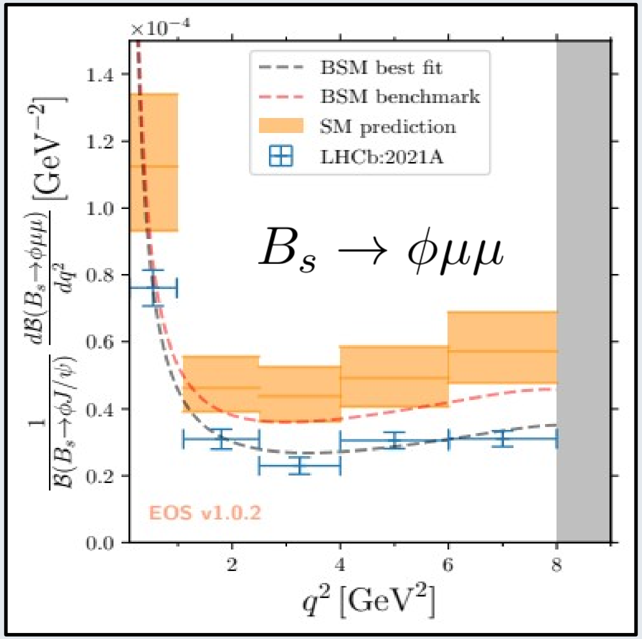
Comparison with data

- Conservatively accounting for the non-local form factors does not solve the b → sμμ anomalies
- The largest source of theoretical uncertainty at low q^2 still comes from local form factors

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]







 Additional plots can be found in the paper: 2206.03797

Conclusion

Discussing BSM models requires a solid understanding of the hadronic physics:

- **Local form factors** uncertainties can be controlled and reduced by using improved dispersive bound and a *appropriate* parametrization
  - This is the first global analysis of $b \to s$ form factors
  - It is reassuring as it confirms channel-specific analyses…
  - … and promising as dispersive effects start to be visible

- Understanding of **non-local form factors** is essential to distinguish BSM from SM interpretation of the measurements, but still requires theory inputs.

  → In both cases:
  - Uncertainties are still large, but controlled by dispersive bounds
  - Our approach is systematically improvable

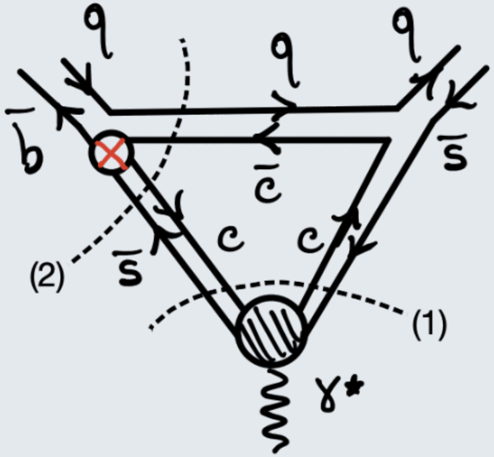
Back-up

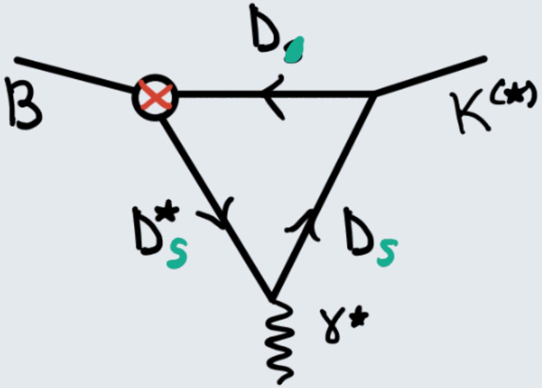
Where to find our results

- All the plots are available here: https://doi.org/10.5281/zenodo.7919635
- We also added
  - the updated posterior distributions for N = 2 in our parametrization and using a SSE as YAML files
  - All the tools/documentation to reproduce our results
- These results are also available in **EOS v1.0.7**:
  - /eos/constraints/B-to-P-form-factors.yaml
  - /eos/constraints/B-to-V-form-factors.yaml

Additional effects

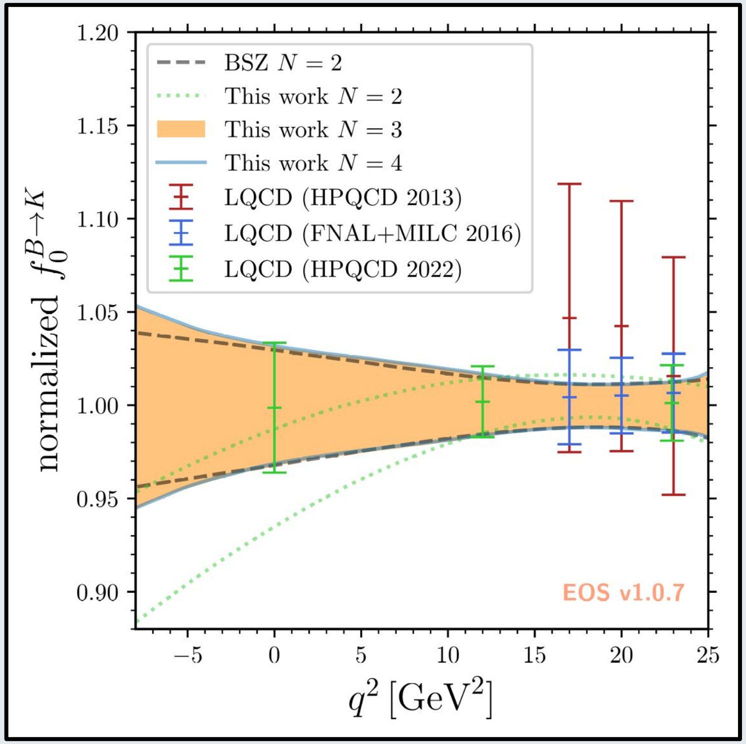
- Rescattering of intermediate hadronic states might spoil the analytic structure of the non-local form-factors [Ciuchini, *et al*, ‘22]

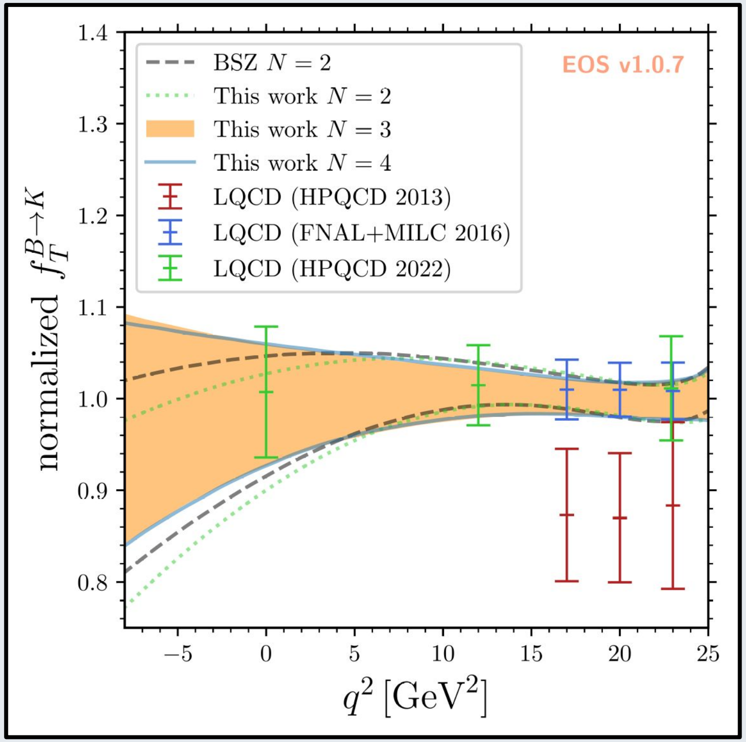




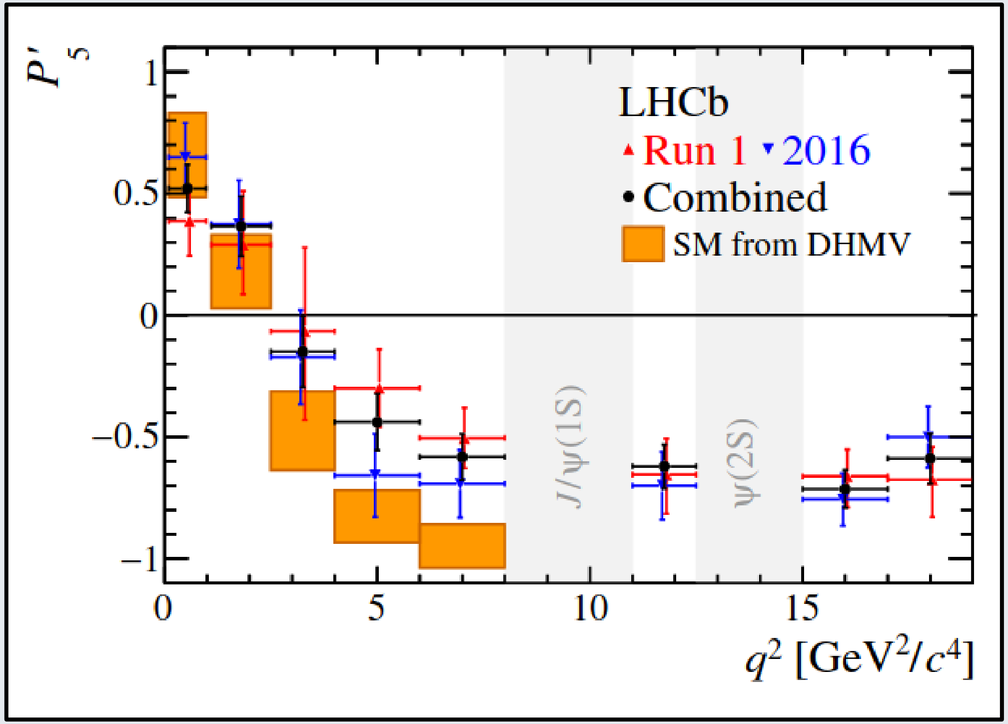
- The effects of the finite width of the K^* amount to a ~10% shift and are accounted for in the fit [Descotes-Genon, Khodjamirian, Virto, ‘19]

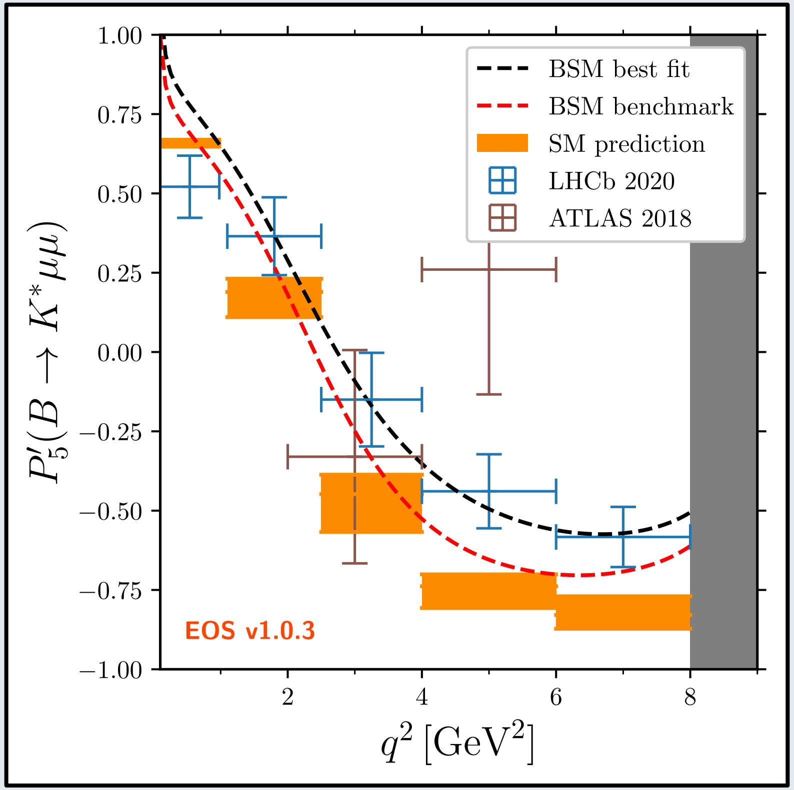
Comparison plots





- Normalizing the form factors to the N = 3 best fit point allows for a model comparison
- All the plots are available here: https://doi.org/10.5281/zenodo.7919635


LHCb
Run 1 2016
Combined
SM from DHMV
P'_5
$J/\psi(1S)$
$\psi(2S)$
q^2 [GeV$^2/c^4$]


BSM best fit
BSM benchmark
SM prediction
LHCb 2020
ATLAS 2018
EOS v1.0.3
$P'_5(B \to K^* \mu\mu)$
q^2 [GeV^2]