

# Measurements of $\phi_s$ and $\sin 2\beta$ at LHCb

### Peilian Li (on behalf of the LHCb collaboration)





### **CKM** matrix

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5) \sim \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix}$$

• Key test of the SM: Verify unitarity of CKM matrix

- Magnitudes: branching fractions or mixing frequencies
- Phases: CP violation measurement



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- Key test of the SM: Verify unitarity of CKM matrix
  - Magnitudes: branching fractions or mixing frequencies
  - Phases: CP violation measurement
- Sensitive probe for new physics





### Neutral *B* meson oscillation

 $\odot$  Neutral *B* mesons can oscillate through box diagrams



Mixing and decay can be described by Schröding	ger-like equation
$i\frac{d}{dt}\left(\frac{B}{B}\right) = \tilde{\mathbf{H}}\left(\frac{B}{B}\right) = \begin{bmatrix} m - \frac{i}{2}\Gamma\\m_{12}^* - \frac{i}{2}\Gamma_{12}^* \end{bmatrix}$	$ \begin{array}{c} m_{12} - \frac{i}{2} \Gamma_{12} \\ m - \frac{i}{2} \Gamma \end{array} \right] \begin{pmatrix} B \\ \overline{B} \end{pmatrix} $

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• Decay rate of initial B or  $\overline{B}$ 

$$\begin{split} |\langle f|H|B\rangle|^2 &= \frac{1}{2}e^{-\Gamma t}|A_f|^2 \Big\{ D \cosh\left(\frac{\Delta\Gamma}{2}t\right) + A_{\Delta\Gamma} \sinh\left(\frac{\Delta\Gamma}{2}t\right) \\ &\pm C \cos(\Delta mt) \mp S \sin(\Delta mt) \Big\} \\ &\text{direct } CP \qquad CP \text{ in mixing} \end{split}$$

• Mass difference  $\Delta m_{(s)} = M_H - M_L = 2 |M_{12}| \rightarrow \text{oscillation frequency}!$ 

• Decay-width difference  $\Delta \Gamma_{(s)} = \Gamma_L - \Gamma_H = 2 |\Gamma_{12}| \cos \phi_{12}$ 

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### **Opportunities for new physics**



• New physics (NP) short-distance contributions can influence mixing  $m_{12}^q = m_{12}^{SM,q} \cdot \Delta_q^{NP}$ [PRD 86(2012)033008]

 Through B mixing, NP energy scales of up to 20 TeV for tree-level NP or 2 TeV for NP in loops can be probed [PRD 89(2014)033016]



# CP violation in B system

- CP violation requires two interfering amplitudes with different strong and weak phases
- For a  $B_{(s)}^0$  decays to a CP eigenstate *f*, CP-violating effects depend on  $\lambda = \frac{q}{p} \frac{A_f}{A_f}$



*CP* violation in interference of decays with/without mixing

Time-dependent or time-integrated difference of decay rates of initial flavour eigenstates  $\Gamma(B_{(\rightsquigarrow\overline{B})} \to f_{CP})(t) \neq \Gamma(\overline{B}_{(\rightsquigarrowB)} \to f_{CP})(t)$ 

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 $\phi_s \& \sin(2\beta)$ 

•  $\sin(2\beta) \text{ in } B^0 \to \psi K_s^0 \ (\beta \sim 22^\circ)$   $\beta = arg(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*})$ •  $B^0 \to J/\psi(\mu^+\mu^-)K_s^0(\to \pi^+\pi^-)$ •  $B^0 \to J/\psi(e^+e^-)K_s^0(\to \pi^+\pi^-)$ •  $B^0 \to \psi(2S)(\mu^+\mu^-)K_s^0(\to \pi^+\pi^-)$ 



 $\phi_s \& \sin(2\beta)$ 

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$$\sin(2\beta)$$
 in  $B^0 \to \psi K_s^0$  ( $\beta \sim 22^\circ$ )



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$$B^0 \to J/\psi(\mu^+\mu^-)K_s^0(\to \pi^+\pi^-)$$
  
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• 
$$\phi_s \text{ in } B_s^0 \to J/\psi(\mu^+\mu^-)K^+K^-$$
  
 $\phi_s^{SM} \approx -2\beta_s = 2arg(\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*})$ 

(ignoring penguin contribution)

• Tree amplitude dominant



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- Tree amplitude dominant
- $\phi_s^{s\bar{s}s}$  in  $B_s^0 \to \phi(\to K^+K^-)\phi(\to K^+K^-)$ 
  - Penguin dominant





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General purpose detector specialised in beauty and charm hadrons

• Daughters of b & c hadron decays:  $p_T \sim \mathcal{O}(1 \text{ GeV}/c)$ , flight distance L~1mm



LHCb MC

√s = 14 TeV

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# Luminosity



- Run 1(2011+2012): 3 fb<sup>-1</sup> + Run 2 (2015-2018): 6 fb<sup>-1</sup>
- Large number of beauty hadrons: [PRL118(2017)052002]  $\sigma(b\bar{b})(7TeV) = 72.0 \pm 0.3 \pm 6.8 \ \mu b, \ \sigma(b\bar{b})(13TeV) = 144 \pm 1 \pm 21 \ \mu b \ in 2<\eta<5$

## Mass fit

### *sPlot technique* to subtract combinatorial background: $\rightarrow$ perform fits to invariant mass distribution

•  $B^0 \to J/\psi(\mu^+\mu^-)K_s^0$  (85%) •  $B^0 \to J/\psi(e^+e^-)K_s^0$  (12%) •  $B^0 \to \psi(2S)(\mu^+\mu^-)K_s^0$  (6%)

• 
$$B_s^0 \to J/\psi K^+ K^-$$

• 
$$B_s^0 \to \phi \phi$$

(a) LHCb  $6 \text{ fb}^{-1}$ 

5500

5600



Total signal candidates ~306090 + 42700 + 23560

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Total signal candidates ~349000

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Total signal candidates ~15840

5400

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### **CP** asymmetry

• Time-dependent CP asymmetry:  $A_{CP}(t) = \frac{\Gamma(\bar{B}^0_{(s)} \to f) - \Gamma(B^0_{(s)} \to f)}{\Gamma(\bar{B}^0_{(s)} \to f) + \Gamma(B^0_{(s)} \to f)} = \eta_f \cdot \sin 2\beta_{(s)} \cdot \sin(\Delta m_{(s)}t)$ • Experimentally

$$A_{CP}(t) \propto \eta_f \cdot e^{-\frac{1}{2}\Delta m_s^2 \sigma_t^2} \cdot (1 - 2\omega) \cdot \sin 2\beta_{(s)} \cdot \sin(\Delta m_{(s)}t)$$

- Tagging of  $B_{(s)}^0$  flavor at production: probability of wrong tag  $\omega$
- Excellent decay-time resolution  $\sigma_t \sim 43$  fs
- CP eigenvalue of the final state  $\eta_f$

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- $B_s^0 \to J/\psi(\to \mu^+\mu^-)\phi(\to K^+K^-) + B_s^0 \to \phi(KK)\phi(KK)$ :
  - → a mixture of CP-even (L = 0,2) & CP-odd (L = 1) components









• Same-side (SS) tagging: Use charge of kaon produced in the fragmentation



- Same-side (SS) tagging: Use charge of kaon produced in the fragmentation
- Opposite-side (OS) tagging: charge of leptons or hadrons from the decay of the other *b* hadrons

tagging efficiency  $\epsilon_{tag}$ 



- Decay time resolution dilutes oscillations,  $\mathcal{D} = exp(-\frac{1}{2}\sigma_{\text{eff}}^2 \Delta m_s^2)$
- Significant for  $B_s^0$  system, negligible for  $B^0$



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  - $B^0 \rightarrow \psi K_S^0$ :  $\sigma_{MC} \sim 60$  fs •  $B_s^0 \rightarrow J/\psi KK \& B_s^0 \rightarrow \phi \phi$

$$\delta_t^2 = (\frac{m}{p})^2 \sigma_L^2 + (\frac{t}{p})^2 \sigma_p^2$$

$$\sim 200 \,\mu \text{m} \quad \sigma_p/p \sim 0.4 \,\%$$



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 $\begin{array}{c} P \downarrow & P \downarrow \\ \sim 200 \,\mu \mathrm{m} & \sigma_p/p \sim 0.4 \,\% \end{array}$ 

Effective Gaussian resolution model:  $\sigma_{eff} \text{ as a function of per-event } \delta_t \text{ (11 bins)}$ 

$$\sigma_{eff} \sim 42(3) \text{ fs} \rightarrow \mathcal{D} = 0.757$$



### Decay time & angular efficiencies

- Reconstruction and selection criteria introduce non-uniform efficiency
- Decay-time efficiencies: Data driven method  $\varepsilon_{\text{data}}^{B_s^0}(t) = \varepsilon_{\text{data}}^{B^0}(t) \times \frac{\varepsilon_{\text{sim}}^{B_s^{\circ}}(t)}{\varepsilon_{\text{sim}}^{B^0}(t)}$ Scaled  $\varepsilon^{B_s^0}_{\text{data}}$ LHCb EPJC79(2019)706  $B_s^0 \to J/\psi KK$ Modelled by cubic splines: 0.8 knots at 0.3, 0.91, 1.96, 9 ps 0.6 10 1 *t* [ps]  $f(t) \propto \varepsilon(t) \cdot e^{-t/\tau} \otimes G(0,\sigma_t)$

# Decay time & angular efficiencies

• Reconstruction and selection criteria introduce non-uniform efficiency



• Angular efficiencies for  $B_s^0$  decays estimated with simulation



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 $\phi_s \text{ in } B_s^0 \to J/\psi KK$ 

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Parameters	Values <sup>1</sup>
$\phi_s$ [rad]	$-0.039 \pm 0.022 \pm 0.006$
$ \lambda $	$1.001 \pm 0.011 \pm 0.005$
$\Gamma_s - \Gamma_d \ [\mathrm{ps}^{-1}]$	$-0.0057  {}^{+ 0.0013}_{- 0.0015} \pm 0.0014$
$\Delta\Gamma_s \ [ \mathrm{ps}^{-1}]$	$0.0846 \pm 0.0044 \pm 0.0024$
$\Delta m_s \; [ { m ps}^{-1}]$	$17.743 \pm 0.033 \pm 0.009$
$ A_{\perp} ^2$	$0.2463 \pm 0.0023 \pm 0.0024$
$ A_0 ^2$	$0.5179 \pm 0.0017 \pm 0.0032$
$\delta_{\perp} - \delta_0$ [rad]	$2.903 {}^{+ 0.075}_{- 0.074} \pm 0.048$
$\delta_{\parallel} - \delta_0 \; [rad]$	$3.146 \pm 0.060 \pm 0.052$

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- The most precise measurement in single channel to date
- Compatible with prediction assuming the SM
- No evidence of CP violation
- Consistent and combined with Run 1 measurement:

 $\phi_s = -0.043 \pm 0.020$  rad

# $\phi_s$ combinations in $b \rightarrow c\bar{c}s$ transition

Previous World Average:  $\phi_s^{c\bar{c}s} = -0.049 \pm 0.019$  rad  $\phi_s^{J/\psi KK} = -0.070 \pm 0.022$  rad New World Average: (preliminary)  $\phi_s^{c\bar{c}s} = -0.050 \pm 0.016 \text{ rad (16\%)}$  $\phi_s^{J/\psi KK} = -0.039 \pm 0.017 \text{ rad (23\%)}$ 

Consistent with the Global fits with SM assumption

 $\phi_s^{\text{CKMFitter}} \approx -2\beta_s = (-0.0368^{+0.0006}_{-0.0009}) \text{ rad} \quad \phi_s^{\text{UTFitter}} = (-0.0370 \pm 0.0010) \text{ rad}$ 



# $\phi_s$ in $b \rightarrow s\bar{s}s$ transition

#### LHCb-PAPER-2023-001

$$\phi_s^{s\bar{s}s} = -0.042 \pm 0.075 \pm 0.009 \text{ rad}$$
$$|\lambda| = 1.004 \pm \pm 0.030 \pm 0.009$$

- The most precise measurement in any penguin dominated B decays
- No polarisation dependence is observed



# $\sin 2\beta$ in $B^0 \to \psi K_S^0$

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0.55

0.60

0.65

0.70

0.75

0.80

0.85

 $S_{\psi K_{
m S}^0}$ 

# $\sin 2\beta$ combinations





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- Consistent with other measurements, still statistical uncertainty limited
- **Dominant contribution** to the World Average

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### Looking at Run 3 and beyond



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- Further precision improvement with more data
- Great opportunities to search for NP indirectly, up to > TeV scale





### Summary

- LHCb dominates the world average of many CPV measurements
- ✓ Flag-ship time-dependent measurements of CP violation in *B*-meson decays with full LHCb data sample, providing the most precise results for
  - $\checkmark \phi_s$  in  $b \rightarrow c\bar{c}s$  transition,  $\sigma(\phi_s) \sim 20$  mrad
  - $\checkmark \phi_s^{s\bar{s}s}$  in penguin dominant *B* decays
  - $\checkmark \sin 2\beta = 0.716 \pm 0.013 (\text{stat.}) \pm 0.008 (\text{syst.})$
- Looking forward to further test of the SM and search for new physics with more data from Upgrade I & II

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Back up slides

- Statistical process, the tag is not always right  $\rightarrow$  knowledge of mistag rate  $\omega$
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- Data-driven method to calibration the performance

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- SS tagging for  $B_s^0: B_s^0 \to D_s^- \pi^+$
- SS tagging for  $B^0: B^0 \to J/\psi K^*$



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# $\phi_s$ polarisation dependent fit

- New physics effects can vary in different polarisation states
  - Allow  $|\lambda|$  and  $\phi_s$  differ in polarisation states
  - Shows no evidence for any polarisation dependence

LHCb-PAPER-2023-016

$ \begin{array}{lll} \phi^0_s \; [{\rm rad}] & -0.034 \pm 0.023 \\ \phi_s^{\parallel} - \phi^0_s \; [{\rm rad}] & -0.002 \pm 0.021 \\ \phi_s^{\perp} - \phi^0_s \; [{\rm rad}] & -0.001 \stackrel{+ \; 0.020}{- \; 0.021} \\ \phi_s^{S} - \phi^0_s \; [{\rm rad}] & 0.022 \stackrel{+ \; 0.027}{- \; 0.026} \\  \lambda^0  & 0.969 \stackrel{+ \; 0.025}{- \; 0.024} \\  \lambda^{\parallel}/\lambda^0  & 0.982 \stackrel{+ \; 0.055}{- \; 0.052} \\  \lambda^{\perp}/\lambda^0  & 1.107 \stackrel{+ \; 0.081}{- \; 0.075} \\ \end{array} $	Parameters	Values (stat. unc. only)
$ \lambda^{3}/\lambda^{0} $ 1.121 + 0.005	$\begin{array}{l} \phi_s^0 \; [rad] \\ \phi_s^{\parallel} - \phi_s^0 \; [rad] \\ \phi_s^{\perp} - \phi_s^0 \; [rad] \\ \phi_s^{\ S} - \phi_s^0 \; [rad] \\  \lambda^0  \\  \lambda^0  \\  \lambda^{\parallel}/\lambda^0  \\  \lambda^{\perp}/\lambda^0  \\  \lambda^S/\lambda^0  \end{array}$	$\begin{array}{c} -0.034 \pm 0.023 \\ -0.002 \pm 0.021 \\ -0.001 \stackrel{+ \ 0.020}{_{- \ 0.021}} \\ 0.022 \stackrel{+ \ 0.027}{_{- \ 0.026}} \\ 0.969 \stackrel{+ \ 0.025}{_{- \ 0.024}} \\ 0.982 \stackrel{+ \ 0.055}{_{- \ 0.052}} \\ 1.107 \stackrel{+ \ 0.081}{_{- \ 0.075}} \\ 1.121 \stackrel{+ \ 0.085}{_{- \ 0.079}} \end{array}$

### Results of the parameters for S-wave

$$\begin{split} |A_S^1|^2 &= 0.472 \pm 0.024 \pm 0.027, \\ |A_S^2|^2 &= 0.042 \pm 0.005 \pm 0.010, \\ |A_S^3|^2 &= 0.0029^{+0.0013}_{-0.0019} \pm 0.023, \\ |A_S^4|^2 &= 0.0037^{+0.0025}_{-0.0019} \pm 0.032, \\ |A_S^5|^2 &= 0.0508 \pm 0.007 \pm 0.027, \\ |A_S^6|^2 &= 0.151 \pm 0.011 \pm 0.051, \\ \delta_S^1 - \delta_\perp &= 2.05^{+0.12}_{-0.14} \pm 0.19 \text{ rad}, \\ \delta_S^2 - \delta_\perp &= 1.62^{+0.19}_{-0.19} \pm 0.41 \text{ rad}, \\ \delta_S^3 - \delta_\perp &= 1.16^{+0.37}_{-0.15} \pm 0.31 \text{ rad}, \\ \delta_S^5 - \delta_\perp &= -0.637^{+0.068}_{-0.076} \pm 0.17 \text{ rad}, \\ \delta_S^6 - \delta_\perp &= -1.013^{+0.074}_{-0.083} \pm 0.07 \text{ rad}. \end{split}$$

## Systematics for $\phi_s$

* Uncertainties (×0	0.01)	Domina	ted sy	s. S	ub-domin	ated sys.	Stat.	limited	
Source	$ A_0 ^2$	$ A_{\perp} ^2$	$\phi_s$ [rad]	$ \lambda $	$\delta_{\perp} - \delta_0 \ [ ext{rad}]$	$\delta_{\parallel} - \delta_0 \ [ ext{rad}]$	$\Gamma_s - \Gamma_d$ [ps <sup>-1</sup> ]	$\Delta \Gamma_s$ [ps <sup>-1</sup> ]	$\Delta m_s$ [ps <sup>-1</sup> ]
Mass parametrization	0.04	0.03	0.03	0.02	0.15	0.12	0.02	0.04	0.03
Mass: shape statistical	0.04	0.04	0.05	0.09	0.62	0.33	0.02	0.01	0.11
Mass factorization	0.11	0.10	0.42	0.19	0.54	0.60	0.12	0.16	0.18
$B_c^+$ contamination	0.04	0.05	_	0.02	_	0.17	(0.07)	(0.03)	_
D–wave component	0.04	0.04	0.02	_	0.07	0.13	0.01	0.03	0.02
Bkgcat 60	0.07	0.04	0.02	0.10	0.18	0.18	0.02	_	0.01
Multiple candidates	0.01	_	0.27	0.22	0.90	0.41	0.01	0.01	0.24
Particle identification	0.06	0.09	0.27	0.27	1.31	0.51	0.05	0.15	0.46
$C_{\rm SP}$ factors	_	0.01	0.01	0.03	0.73	0.41	—	0.01	0.04
DTR model portability	_	—	0.08	0.03	0.26	0.09	—	—	0.09
DTR calibration	_	—	0.03	0.02	0.11	0.07	—	—	0.05
Time bias correction	0.04	0.05	0.06	0.05	0.77	0.11	0.03	0.05	0.44
Angular efficiency	0.05	0.14	0.25	0.32	0.42	0.44	0.01	0.02	0.13
Angular resolution	0.01	0.01	0.02	0.01	0.02	0.08	_	0.01	0.02
Kinematic weighting	0.24	0.09	0.01	0.01	0.98	0.86	0.02	0.03	0.31
Momentum uncertainty	0.08	0.04	0.04	—	0.07	0.11	0.01	_	0.13
Longitudinal scale	0.07	0.04	0.04	—	0.10	0.09	0.02	_	0.31
Neglected correlations	—	—	—	—	4.20	4.96	—	—	—
Total sys. unc.	0.32	0.24	0.6	0.5	4.8	5.2	0.14	0.24	0.9
Stat. unc.	0.17	0.23	2.2	1.1	7.5	6.0	0.14	0.44	3.3

•  $\phi_s$ ,  $|\lambda|$ ,  $\Delta\Gamma_s$ ,  $\Delta m_s$  are statistically limited

# Systematics for $\phi_s^{s\bar{s}s}$

#### LHCb-PAPER-2023-016

Source	$\phi^{s\overline{s}s}_{s}$	$ \lambda $	$ A_0 ^2$	$ A_{\perp} ^2$	$\delta_{\parallel}-\delta_{0}$	$\delta_{\perp} - \delta_0$
Time resolution	4.9	2.6	0.8	0.8	0.1	3.4
Flavor tagging	4.8	4.7	0.9	1.3	1.2	9.7
Angular acceptance	3.9	4.9	1.4	1.7	4.7	1.2
Time acceptance	2.3	1.7	0.1	0.1	5.6	0.7
Mass fit & factorization	2.2	4.4	1.9	2.3	2.3	2.5
MC truth match	1.1	0.2	0.1	0.1	0.2	0.3
Fit bias	0.8	0.7	0.9	0.3	3.6	0.7
Candidate multiplicity	0.3	0.2	0.1	0.8	0.2	0.1
Total	8.8	8.6	2.7	3.3	8.5	10.7

Parameter	Result
$\phi_s^{s\overline{s}s}$ [rad ]	$-0.042 \pm 0.075 \pm 0.009$
$ \lambda $	$1.004 \pm 0.030 \pm 0.009$
$ A_0 ^2$	$0.384 \pm 0.007 \pm 0.003$
$ A_{\perp} ^2$	$0.310 \pm 0.006 \pm 0.003$
$\delta_{\parallel} - \delta_0 \; \; [ { m rad} \; ]$	$2.463 \pm 0.029 \pm 0.009$
$\delta_{\perp}^{-} - \delta_{0} \; \; [ { m rad} \; ]$	$2.769 \pm 0.105 \pm 0.011$

# Systematics for $\phi_s^{s\bar{s}s}$

#### LHCb-PAPER-2023-016

Source	$\phi^{s\overline{s}s}_{s}$	$ \lambda $	$ A_0 ^2$	$ A_{\perp} ^2$	$\delta_{\parallel}-\delta_{0}$	$\delta_{\perp} - \delta_0$
Time resolution	4.9	2.6	0.8	0.8	0.1	3.4
Flavor tagging	4.8	4.7	0.9	1.3	1.2	9.7
Angular acceptance	3.9	4.9	1.4	1.7	4.7	1.2
Time acceptance	2.3	1.7	0.1	0.1	5.6	0.7
Mass fit & factorization	2.2	4.4	1.9	2.3	2.3	2.5
MC truth match	1.1	0.2	0.1	0.1	0.2	0.3
Fit bias	0.8	0.7	0.9	0.3	3.6	0.7
Candidate multiplicity	0.3	0.2	0.1	0.8	0.2	0.1
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$\delta_{\perp}^{-} - \delta_{0} \; \; [ { m rad} \; ]$	$2.769 \pm 0.105 \pm 0.011$

# Systematics for $\sin 2\beta$

Source	$\sigma(S)$	$\sigma(C)$
Fitter validation	0.0004	0.0006
$\Delta \Gamma_d$ uncertainty	0.0055	0.0017
FT calibration portability	0.0053	0.0001
FT $\Delta \epsilon_{\text{tag}}$ portability	0.0014	0.0017
Decay-time bias model	0.0007	0.0013

$$\begin{split} S^{\text{Run }2}_{J/\psi(\rightarrow\mu^+\mu^-)K^0_{\text{S}}} &= & 0.714 \pm 0.015 \,(\text{stat}) \pm 0.0074 \,(\text{syst}) \\ C^{\text{Run }2}_{J/\psi(\rightarrow\mu^+\mu^-)K^0_{\text{S}}} &= & 0.013 \pm 0.014 \,(\text{stat}) \pm 0.0025 \,(\text{syst}) \\ S^{\text{Run }2}_{\psi(2S)K^0_{\text{S}}} &= & 0.647 \pm 0.053 \,(\text{stat}) \pm 0.018 \quad(\text{syst}) \\ C^{\text{Run }2}_{\psi(2S)K^0_{\text{S}}} &= -0.083 \pm 0.048 \,(\text{stat}) \pm 0.0053 \,(\text{syst}) \\ S^{\text{Run }2}_{J/\psi(\rightarrow e^+e^-)K^0_{\text{S}}} &= & 0.752 \pm 0.037 \,(\text{stat}) \pm 0.084 \quad(\text{syst}) \\ C^{\text{Run }2}_{J/\psi(\rightarrow e^+e^-)K^0_{\text{S}}} &= & 0.046 \pm 0.034 \,(\text{stat}) \pm 0.0077 \,(\text{syst}) \end{split}$$

# **Time-dependent angular fit** $\mathscr{P}(t, \theta_K, \theta_\mu, \phi_h | \delta_t) \propto \sum_{k=1}^{10} N_k h_k(t) f_k(\theta_K, \theta_\mu, \phi_h) \rightarrow \phi_s, \Delta m_s, \Delta \Gamma_s, \Gamma_s - \Gamma_d$

k=1

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$$\begin{split} \mathscr{P}(t,\theta_{K},\theta_{\mu},\phi_{h}|\delta_{t}) \propto \sum_{k=1}^{10} N_{k}h_{k}(t)f_{k}(\theta_{K},\theta_{\mu},\phi_{h}) &\to \phi_{s}, \Delta m_{s}, \Delta\Gamma_{s}, \Gamma_{s}-\Gamma_{d} \\ & \mathcal{P}\left(t,\Omega|\mathfrak{q}^{\mathrm{OS}},\mathfrak{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}},\delta_{t}\right) \\ & \propto \sum_{k=1}^{10} C_{\mathrm{SP}}^{k}N_{k}f_{k}(\Omega)\varepsilon_{\mathrm{data}}^{B_{s}^{0}}(t) \\ & \cdot\left\{\left[\mathcal{Q}\left(\mathfrak{q}^{\mathrm{OS}},\mathfrak{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}}\right)h_{k}\left(t|B_{s}^{0}\right)\right. \\ & \left.+\bar{\mathcal{Q}}\left(\mathfrak{q}^{\mathrm{OS}},\mathfrak{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}}\right)h_{k}\left(t|\overline{B}_{s}^{0}\right)\right] \otimes \mathcal{R}\left(t-t'|\delta_{t}\right)\right\} \end{split}$$

$$\begin{aligned} \mathcal{P}(t,\theta_{K},\theta_{\mu},\phi_{h}|\delta_{t}) \propto \sum_{k=1}^{10} N_{k}h_{k}(t)f_{k}(\theta_{K},\theta_{\mu},\phi_{h}) & \rightarrow \phi_{s}, \Delta m_{s}, \Delta\Gamma_{s}, \Gamma_{s} - \Gamma_{d} \\ \\ \mathcal{P}\left(t,\Omega|\mathfrak{q}^{\mathrm{OS}},\mathfrak{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}},\delta_{t}\right) & \qquad \text{Angular amplitudes} \\ \approx \sum_{k=1}^{10} C_{\mathrm{SP}}^{k}N_{k}f_{k}(\Omega)\varepsilon_{\mathrm{data}}^{B_{s}^{0}}(t) & \qquad \\ \cdot\left\{\left[\mathcal{Q}\left(\mathfrak{q}^{\mathrm{OS}},\mathfrak{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}}\right)h_{k}\left(t|B_{s}^{0}\right)\right. \\ \left. +\bar{\mathcal{Q}}\left(\mathfrak{q}^{\mathrm{OS}},\mathfrak{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}}\right)h_{k}\left(t|\overline{B}_{s}^{0}\right)\right] \otimes \mathcal{R}\left(t-t'|\delta_{t}\right)\right\} \end{aligned}$$

k	$A_k$	$f_k( heta_\mu, heta_K,arphi_h)$
1	$ A_0 ^2$	$2\cos^2 heta_K\sin^2 heta_\mu$
2	$ A_{\ } ^{2}$	$\sin^2 heta_k(1-\sin^2 heta_\mu\cos^2arphi_h)$
3	$ A_{\perp} ^2$	$\sin^2 heta_k(1-\sin^2 heta_\mu\sin^2arphi_h)$
4	$ A_{\parallel}A_{\perp} $	$\sin^2 heta_k \sin^2 heta_\mu \sin 2arphi_h$
5	$ A_0A_{\parallel} $	$\frac{1}{2}\sqrt{2}\sin 2 heta_k\sin 2 heta_\mu\cos arphi_h$
6	$ A_0A_\perp $	$-\frac{1}{2}\sqrt{2}\sin 2 heta_k\sin 2 heta_\mu\sin arphi_h$
7	$ A_{S} ^{2}$	$rac{2}{3}\sin^2 heta_\mu$
8	$ A_S A_{\parallel} $	$\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\cos\varphi_h$
9	$ A_S A_{\perp} $	$-\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\sin\varphi_h$
10	$ A_S A_0 $	$\frac{4}{3}\sqrt{3}\cos heta_K\sin^2 heta_\mu$

$$\mathcal{P}(t,\theta_K,\theta_\mu,\phi_h | \delta_t) \propto \sum_{k=1}^{10} N_k h_k(t) f_k(\theta_K,\theta_\mu,\phi_h) \to \phi_s, \Delta m_s, \Delta \Gamma_s, \Gamma_s - \Gamma_d$$

$$\mathcal{P}\left(t, \Omega | \mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}, \delta_{t}\right)$$

$$\propto \sum_{k=1}^{10} \mathbb{C}_{\mathrm{SP}}^{k} N_{k} f_{k}(\Omega) \varepsilon_{\mathrm{data}}^{B_{s}^{0}}(t)$$

$$\cdot \left\{ \left[ \mathcal{Q}\left(\mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}\right) h_{k}\left(t | B_{s}^{0}\right) \right. \right. \\ \left. + \bar{\mathcal{Q}}\left(\mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}\right) h_{k}\left(t | \overline{B}_{s}^{0}\right) \right\}$$

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Angular amplitudes  $C_{\text{SP}}^k$  account for the interference between P- and S- wave

k	$A_k$	$f_k( heta_\mu, heta_K,arphi_h)$
1	$ A_0 ^2$	$2\cos^2 heta_K\sin^2 heta_\mu$
2	$ A_{\ } ^{2}$	$\sin^2 heta_k(1-\sin^2 heta_\mu\cos^2arphi_h)$
3	$ A_{\perp} ^2$	$\sin^2 heta_k(1-\sin^2 heta_\mu\sin^2arphi_h)$
4	$ A_{\parallel}A_{\perp} $	$\sin^2 heta_k \sin^2 heta_\mu \sin 2arphi_h$
5	$ A_0A_{\parallel} $	$\frac{1}{2}\sqrt{2}\sin 2 heta_k\sin 2 heta_\mu\cos arphi_h$
6	$ A_0A_\perp $	$-\frac{1}{2}\sqrt{2}\sin 2\theta_k \sin 2\theta_\mu \sin \varphi_h$
7	$ A_{S} ^{2}$	$\frac{2}{3}\sin^2 heta_{\mu}$
8	$ A_S A_{\parallel} $	$\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\cos\varphi_h$
9	$ A_S A_\perp $	$-\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\sin\varphi_h$
10	$ A_S A_0 $	$\frac{4}{3}\sqrt{3}\cos\theta_K\sin^2\theta_\mu$

$$\mathcal{P}(t,\theta_K,\theta_\mu,\phi_h | \delta_t) \propto \sum_{k=1}^{10} N_k h_k(t) f_k(\theta_K,\theta_\mu,\phi_h) \to \phi_s, \Delta m_s, \Delta \Gamma_s, \Gamma_s - \Gamma_d$$

$$\mathcal{P}\left(t, \Omega | \mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}, \delta_{t}\right)$$

$$\propto \sum_{k=1}^{10} \mathcal{C}_{\mathrm{SP}}^{k} N_{k} f_{k}(\Omega) \varepsilon_{\mathrm{data}}^{B_{s}^{0}}(t)$$

$$\cdot \left\{ \left[ \mathcal{Q}\left(\mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}\right) h_{k}\left(t | B_{s}^{0}\right) \right. \right. \\ \left. + \bar{\mathcal{Q}}\left(\mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}\right) h_{k}\left(t | \overline{B}_{s}^{0}\right) \right\}$$

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Angular amplitudes  $C_{\text{SP}}^k$  account for the interference between P- and S- wave

flavor tagging

k	$A_k$	$f_k( heta_\mu, heta_K,arphi_h)$
1	$ A_0 ^2$	$2\cos^2 heta_K\sin^2 heta_\mu$
2	$ A_{\ } ^{2}$	$\sin^2 heta_k(1-\sin^2 heta_\mu\cos^2arphi_h)$
3	$ A_{\perp} ^2$	$\sin^2 heta_k(1-\sin^2 heta_\mu\sin^2arphi_h)$
4	$ A_{\parallel}A_{\perp} $	$\sin^2 heta_k \sin^2 heta_\mu \sin 2arphi_h$
5	$ A_0A_{\parallel} $	$\frac{1}{2}\sqrt{2}\sin 2 heta_k\sin 2 heta_\mu\cos arphi_h$
6	$ A_0A_\perp $	$-\frac{1}{2}\sqrt{2}\sin 2\theta_k \sin 2\theta_\mu \sin \varphi_h$
7	$ A_{S} ^{2}$	$\frac{2}{3}\sin^2 heta_{\mu}$
8	$ A_S A_{\parallel} $	$\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\cos\varphi_h$
9	$ A_S A_{\perp} $	$-\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\sin\varphi_h$
10	$ A_S A_0 $	$\frac{4}{3}\sqrt{3}\cos\theta_K\sin^2\theta_\mu$

$$\mathscr{P}(t,\theta_K,\theta_\mu,\phi_h|\delta_t) \propto \sum_{k=1}^{10} N_k h_k(t) f_k(\theta_K,\theta_\mu,\phi_h) \to \phi_s, \Delta m_s, \Delta \Gamma_s, \Gamma_s - \Gamma_d$$

$$\mathcal{P}\left(t, \Omega | \mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}, \delta_{t}\right)$$

$$\propto \sum_{k=1}^{10} \mathcal{C}_{\mathrm{SP}}^{k} N_{k} f_{k}(\Omega) \varepsilon_{\mathrm{data}}^{B_{s}^{0}}(t)$$

$$\cdot \left\{ \left[ \mathcal{Q}\left(\mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}\right) h_{k}\left(t | B_{s}^{0}\right) + \bar{\mathcal{Q}}\left(\mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}\right) h_{k}\left(t | \overline{B}_{s}^{0}\right) \right\}$$

Angular amplitudes  $C_{\text{SP}}^k$  account for the interference between P- and S- wave

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flavor tagging time-dependent oscillation

$$h_k(t|B_s^0) = \frac{3}{4\pi} e^{-\Gamma t} \left( a_k \cosh \frac{\Delta \Gamma t}{2} + b_k \sinh \frac{\Delta \Gamma t}{2} + c_k \cosh(\Delta m t) + d_k \sin(\Delta m t) \right),$$
$$+c_k \cos(\Delta m t) + d_k \sin(\Delta m t) \right),$$
$$h_k(t|\bar{B}_s^0) = \frac{3}{4\pi} e^{-\Gamma t} \left( a_k \cosh \frac{\Delta \Gamma t}{2} + b_k \sinh \frac{\Delta \Gamma t}{2} - c_k \cos(\Delta m t) - d_k \sin(\Delta m t) \right),$$

 $a_k, b_k, c_k, d_k$  involve strong and weak phases  $(\delta, \phi_s)$  of each component

k	$A_k$	$f_k( heta_\mu, heta_K,arphi_h)$
1	$ A_0 ^2$	$2\cos^2 heta_K\sin^2 heta_\mu$
2	$ A_{\ } ^{2}$	$\sin^2 heta_k(1-\sin^2 heta_\mu\cos^2arphi_h)$
3	$ A_{\perp} ^2$	$\sin^2 heta_k(1-\sin^2 heta_\mu\sin^2arphi_h)$
4	$ A_{\parallel}A_{\perp} $	$\sin^2 heta_k \sin^2 heta_\mu \sin 2arphi_h$
5	$ A_0A_{\parallel} $	$\frac{1}{2}\sqrt{2}\sin 2 heta_k\sin 2 heta_\mu\cos arphi_h$
6	$ A_0A_\perp $	$-\frac{1}{2}\sqrt{2}\sin 2\theta_k \sin 2\theta_\mu \sin \varphi_h$
7	$ A_{S} ^{2}$	$\frac{2}{3}\sin^2 heta_{\mu}$
8	$ A_S A_{\parallel} $	$\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\cos\varphi_h$
9	$ A_S A_\perp $	$-\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\sin\varphi_h$
10	$ A_S A_0 $	$\frac{4}{3}\sqrt{3}\cos\theta_K\sin^2\theta_\mu$

10

$$\mathcal{P}(t,\theta_{K},\theta_{\mu},\phi_{h}|\delta_{t}) \propto \sum_{k=1}^{10} N_{k}h_{k}(t)f_{k}(\theta_{K},\theta_{\mu},\phi_{h}) \rightarrow \phi_{s}, \Delta m_{s}, \Delta\Gamma_{s}, \Gamma_{s} - \Gamma_{d}$$

$$\mathcal{P}(t,\Omega|\mathfrak{q}^{\mathrm{OS}},\mathfrak{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}},\delta_{t})$$
Angular amplitudes
$$C^{k}$$
account for the interference

$$\propto \sum_{k=1}^{\infty} C_{SP}^{k} N_{k} f_{k}(\Omega) \varepsilon_{data}^{D_{s}}(t)$$

$$\cdot \left\{ \begin{bmatrix} \mathcal{Q} \left( \mathfrak{q}^{OS}, \mathfrak{q}^{SSK}, \eta^{OS}, \eta^{SSK} \right) h_{k} \left( t | B_{s}^{0} \right) \\ + \bar{\mathcal{Q}} \left( \mathfrak{q}^{OS}, \mathfrak{q}^{SSK}, \eta^{OS}, \eta^{SSK} \right) h_{k} \left( t | \overline{B}_{s}^{0} \right) \end{bmatrix} \otimes \mathcal{R} \left( t - t' | \delta_{t} \right) \right\}$$

**D**0

 $C_{\rm SP}^k$  account for the interference between P- and S- wave

flavor tagging time-dependent oscillation

decay-time resolution

$$h_k(t|B_s^0) = \frac{3}{4\pi} e^{-\Gamma t} \left( a_k \cosh \frac{\Delta \Gamma t}{2} + b_k \sinh \frac{\Delta \Gamma t}{2} + c_k \cosh(\Delta m t) + d_k \sin(\Delta m t) \right),$$
$$+c_k \cos(\Delta m t) + d_k \sin(\Delta m t) \right),$$
$$h_k(t|\bar{B}_s^0) = \frac{3}{4\pi} e^{-\Gamma t} \left( a_k \cosh \frac{\Delta \Gamma t}{2} + b_k \sinh \frac{\Delta \Gamma t}{2} - c_k \cos(\Delta m t) - d_k \sin(\Delta m t) \right),$$

 $a_k, b_k, c_k, d_k$  involve strong and weak phases  $(\delta, \phi_s)$  of each component

k	$A_k$	$f_k( heta_\mu, heta_K,arphi_h)$
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2	$ A_{\ } ^{2}$	$\sin^2 heta_k(1-\sin^2 heta_\mu\cos^2arphi_h)$
3	$ A_{\perp} ^2$	$\sin^2 heta_k(1-\sin^2 heta_\mu\sin^2arphi_h)$
4	$ A_{\parallel}A_{\perp} $	$\sin^2 heta_k \sin^2 heta_\mu \sin 2arphi_h$
5	$ A_0A_{\parallel} $	$\frac{1}{2}\sqrt{2}\sin 2 heta_k\sin 2 heta_\mu\cos arphi_h$
6	$ A_0A_\perp $	$-\frac{1}{2}\sqrt{2}\sin 2\theta_k \sin 2\theta_\mu \sin \varphi_h$
7	$ A_{S} ^{2}$	$\frac{2}{3}\sin^2 heta_{\mu}$
8	$ A_S A_{\parallel} $	$\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\cos\varphi_h$
9	$ A_S A_{\perp} $	$-\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\sin\varphi_h$
10	$ A_S A_0 $	$\frac{4}{3}\sqrt{3}\cos\theta_K\sin^2\theta_\mu$

$$\mathscr{P}(t,\theta_K,\theta_\mu,\phi_h|\delta_t) \propto \sum_{k=1}^{10} N_k h_k(t) f_k(\theta_K,\theta_\mu,\phi_h) \to \phi_s, \Delta m_s, \Delta \Gamma_s, \Gamma_s - \Gamma_d$$

$$\mathcal{P}\left(t, \Omega | \mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}, \delta_{t}\right)$$

$$\propto \sum_{k=1}^{10} \mathcal{C}_{\mathrm{SP}}^{k} N_{k} f_{k}(\Omega) \varepsilon_{\mathrm{data}}^{B_{s}^{0}}(t)$$

$$\cdot \left\{ \left[ \mathcal{Q}\left(\mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}\right) h_{k}\left(t | B_{s}^{0}\right) + \bar{\mathcal{Q}}\left(\mathfrak{q}^{\mathrm{OS}}, \mathfrak{q}^{\mathrm{SSK}}, \eta^{\mathrm{OS}}, \eta^{\mathrm{SSK}}\right) h_{k}\left(t | \overline{B}_{s}^{0}\right) \right\}$$

Angular amplitudes
 C<sup>k</sup><sub>SP</sub> account for the interference between P- and S- wave
 flavor tagging
 time-dependent oscillation
 decay-time efficiency
 decay-time resolution

$$h_k(t|B_s^0) = \frac{3}{4\pi} e^{-\Gamma t} \left( a_k \cosh \frac{\Delta \Gamma t}{2} + b_k \sinh \frac{\Delta \Gamma t}{2} + c_k \cosh(\Delta m t) + d_k \sin(\Delta m t) \right),$$
$$+c_k \cos(\Delta m t) + d_k \sin(\Delta m t) \right),$$
$$h_k(t|\bar{B}_s^0) = \frac{3}{4\pi} e^{-\Gamma t} \left( a_k \cosh \frac{\Delta \Gamma t}{2} + b_k \sinh \frac{\Delta \Gamma t}{2} - c_k \cos(\Delta m t) - d_k \sin(\Delta m t) \right),$$

 $a_k, b_k, c_k, d_k$  involve strong and weak phases  $(\delta, \phi_s)$  of each component

k	$A_k$	$f_k( heta_\mu, heta_K,arphi_h)$
1	$ A_0 ^2$	$2\cos^2 heta_K\sin^2 heta_\mu$
2	$ A_{\ } ^{2}$	$\sin^2 heta_k(1-\sin^2 heta_\mu\cos^2arphi_h)$
3	$ A_{\perp} ^2$	$\sin^2 heta_k(1-\sin^2 heta_\mu\sin^2arphi_h)$
4	$ A_{\parallel}A_{\perp} $	$\sin^2 heta_k \sin^2 heta_\mu \sin 2arphi_h$
5	$ A_0A_{\parallel} $	$\frac{1}{2}\sqrt{2}\sin 2 heta_k\sin 2 heta_\mu\cos arphi_h$
6	$ A_0A_\perp $	$-\frac{1}{2}\sqrt{2}\sin 2\theta_k \sin 2\theta_\mu \sin \varphi_h$
7	$ A_{S} ^{2}$	$\frac{2}{3}\sin^2 heta_{\mu}$
8	$ A_S A_{\parallel} $	$\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\cos\varphi_h$
9	$ A_S A_\perp $	$-\frac{1}{3}\sqrt{6}\sin\theta_k\sin2\theta_\mu\sin\varphi_h$
10	$ A_S A_0 $	$\frac{4}{3}\sqrt{3}\cos heta_K\sin^2 heta_\mu$