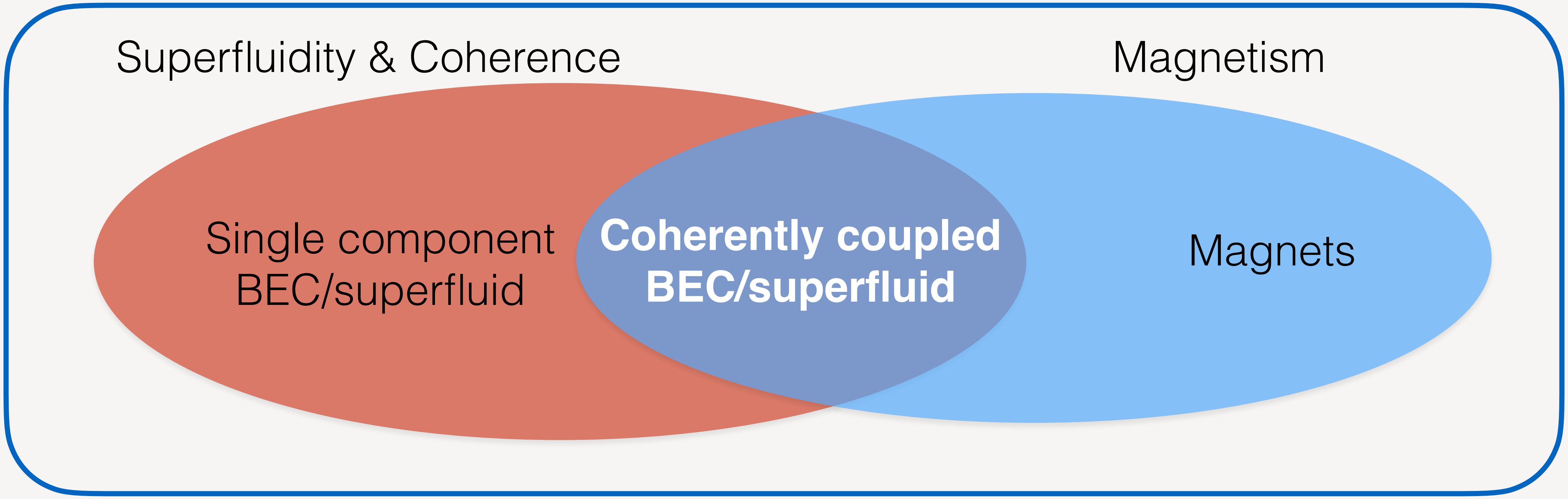
Ferromagnetism in coherently coupled atomic mixtures



Alessio Recati

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CNR INO ISTITUTO NAZIONALE DI OTTICA CONSIGLIO NAZIONALE DELLE RICERCHE

Conference on Quantum-Many-Body Correlations in memory of Peter Schuck

FISh Fundamental Interactions Simulations with quantum gases
PROVINCIA AUTONOMA DI TRENTO
Q@TN
MIUR MINISTERO DELL' ISTRUZIONE, DELL'UNIVERSITÀ E DELLA RICERCA

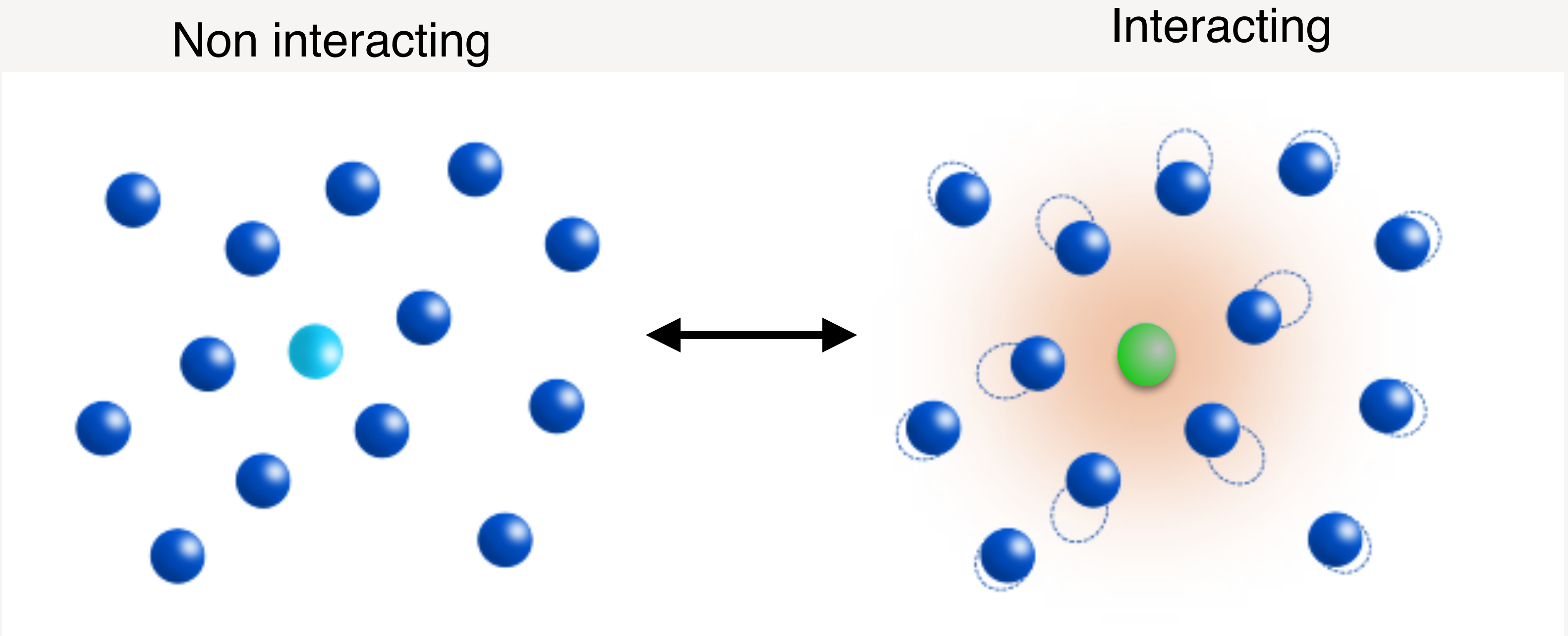
New states in ultra-cold matter

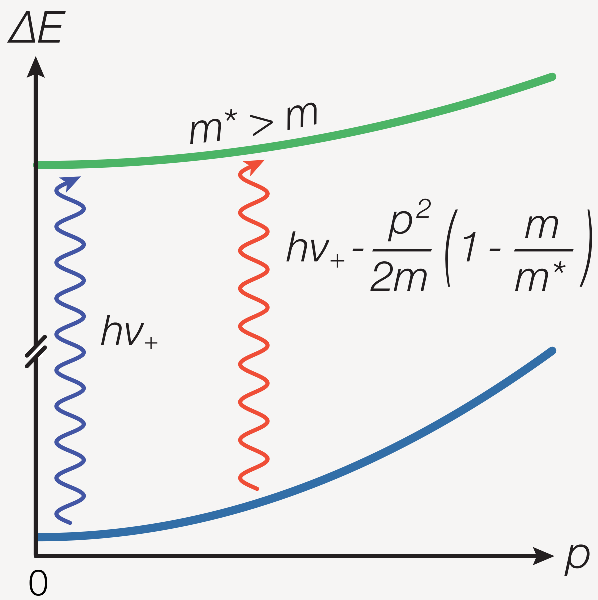
[Pfau's group, Nature (2015)]

New states in ultra-cold matter

Polaron problem (and the Normal Phase for highly polarisation)

Strongly interacting problem and out of equilibrium problem





What happens under a coupling which
Transform a non-interacting impurity in a strongly interacting one.
How does the system react?
(Time dependent variational Ansatz/ ***semiclassical derivation of dressed Bloch equation***)

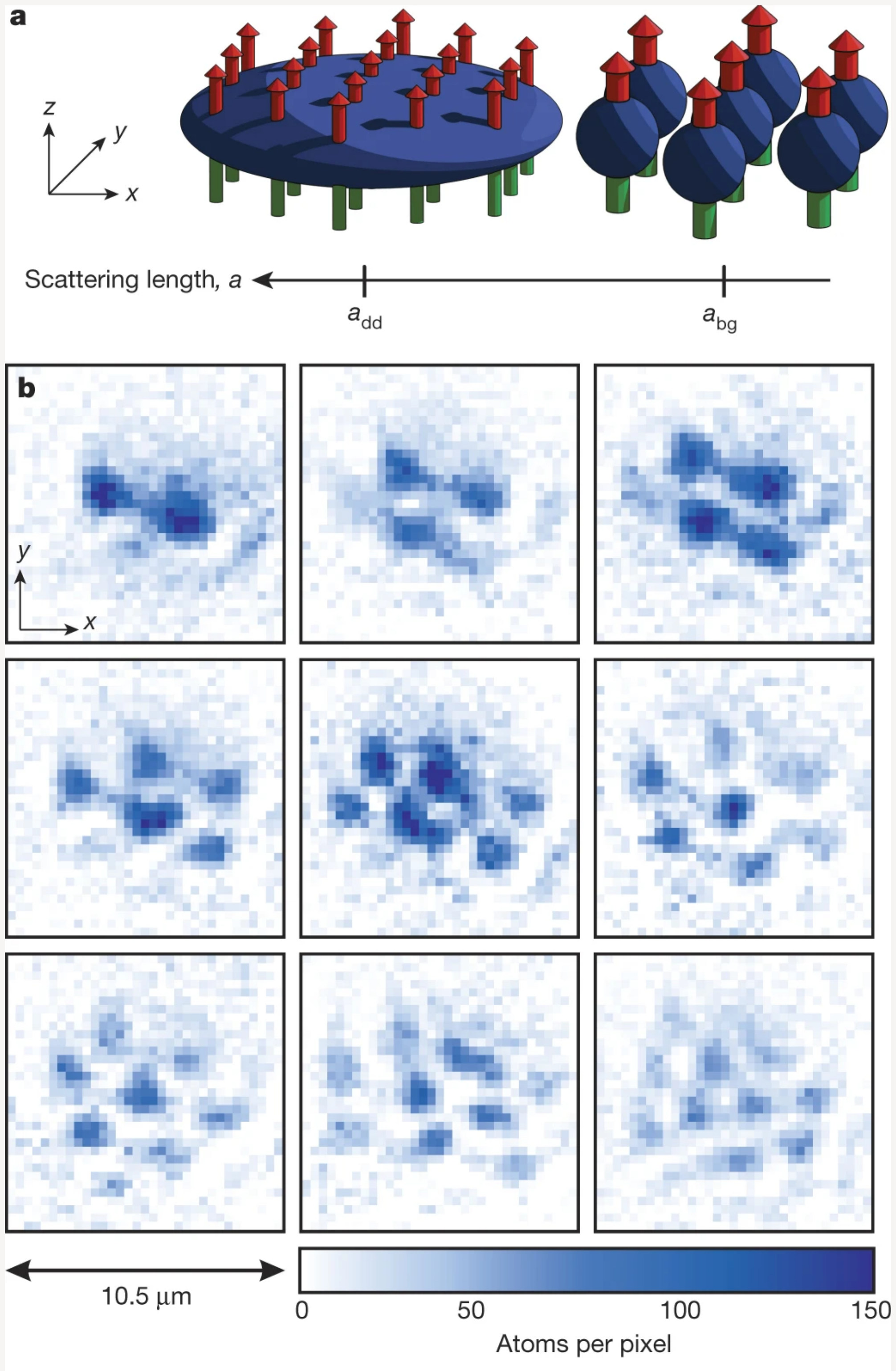
$$\partial n_2 + i\frac{\Omega}{2}(f_{23} - f_{23}^*) = 0$$

$$\partial n_\alpha - iZ_\alpha\frac{\Omega}{2}(f_{23} - f_{23}^*) = I_{\rm coll}^\alpha$$

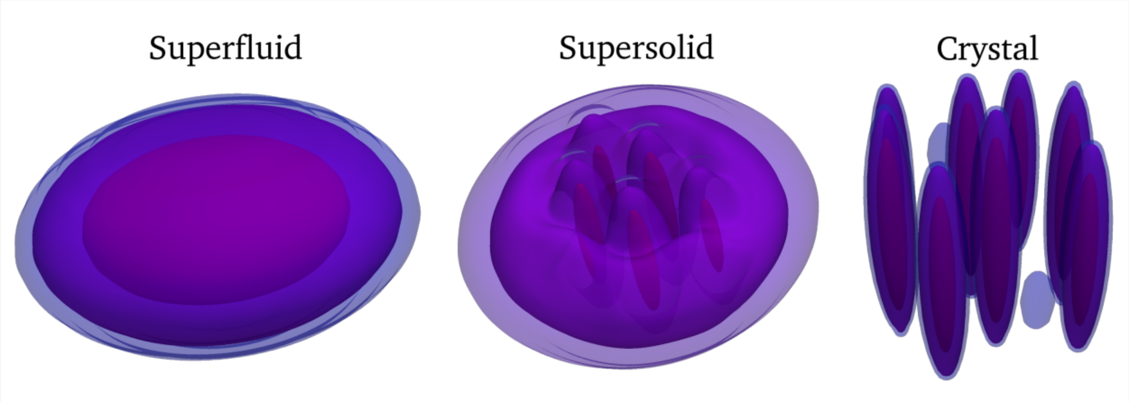
$$\partial f_{23} + i\tilde{Z}_\alpha\delta_\alpha f_{23} + i\frac{\Omega}{2}(n_\alpha - n_2) = -\frac{\Gamma_\alpha^{\rm dec}}{2}f_{23}$$

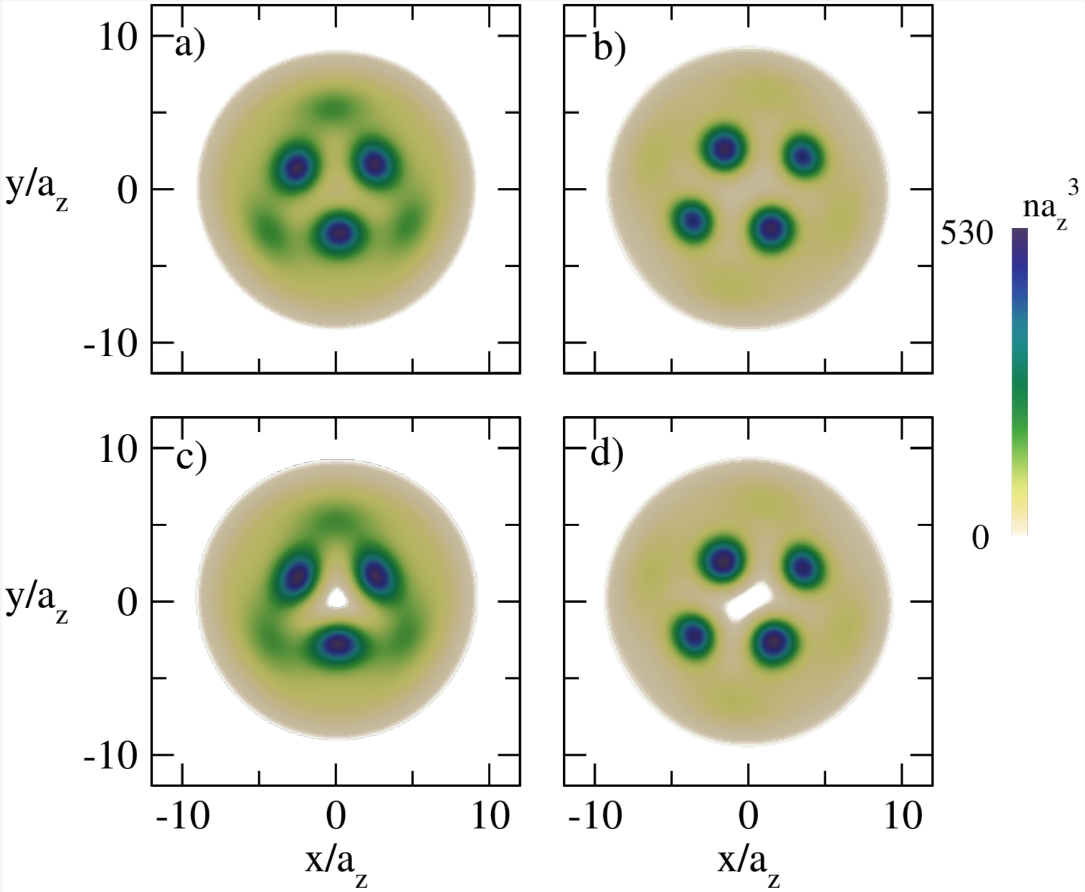
New states in ultra-cold matter

Liquid droplets and supersolidity in dipolar Bose gases (beyond mean field equation of state)



[Pfau's group, Nature (2015)]

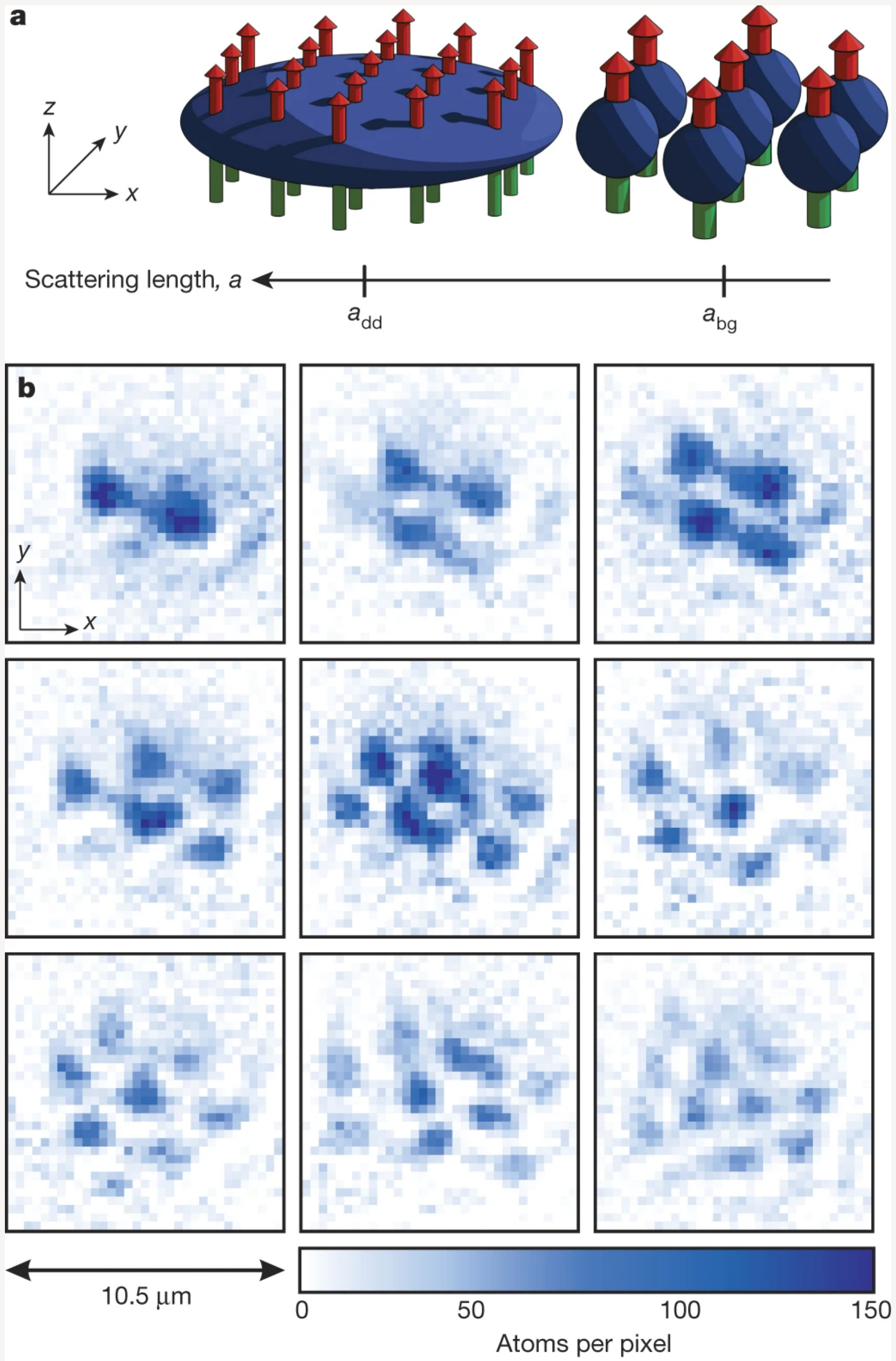




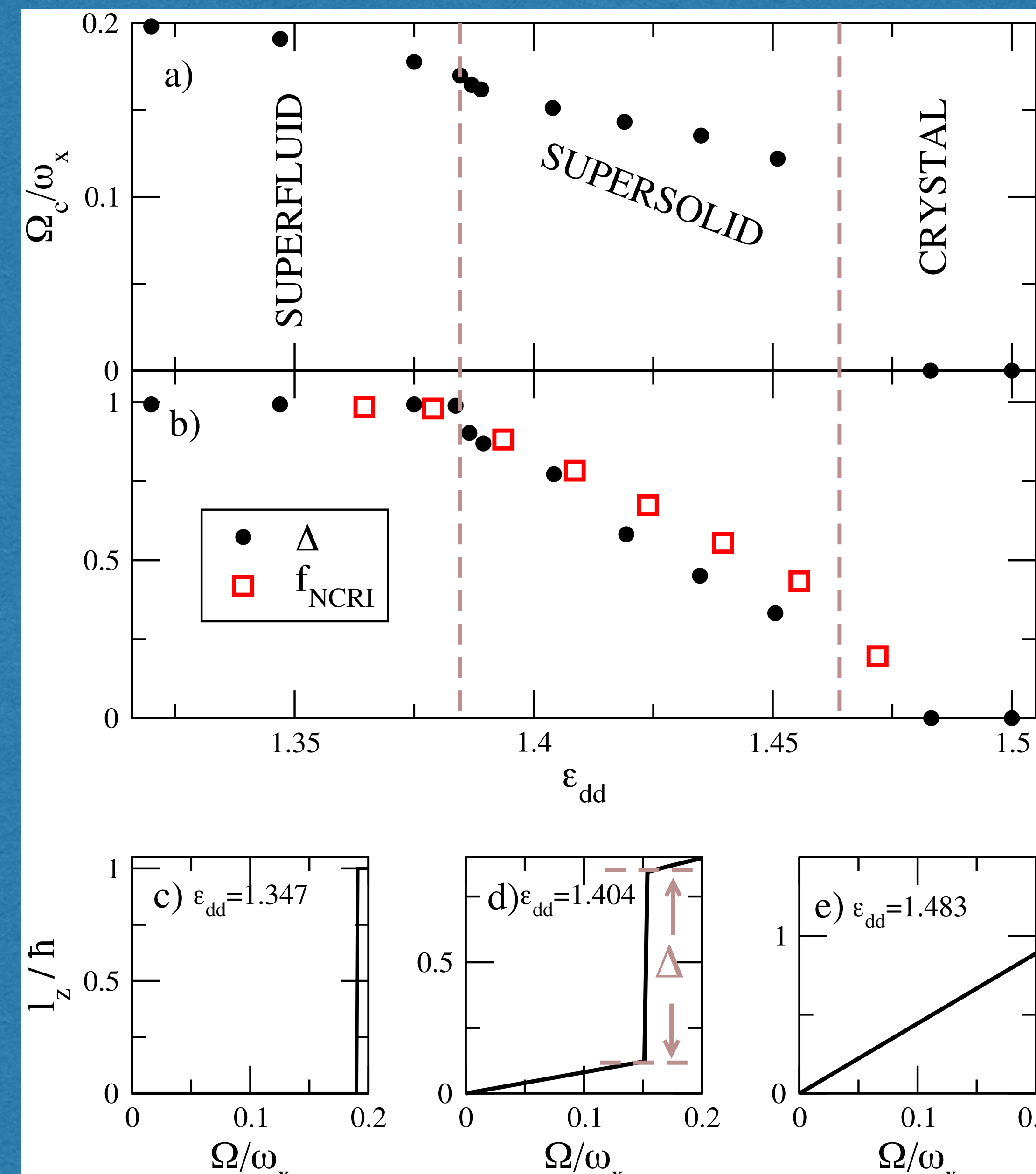
[Gallemì, Roccuzzo, Stringari, AR, PRA (2020)]

New states in ultra-cold matter

Liquid droplets and supersolidity in dipolar Bose gases



[Pfau's group, Nature (2015)]

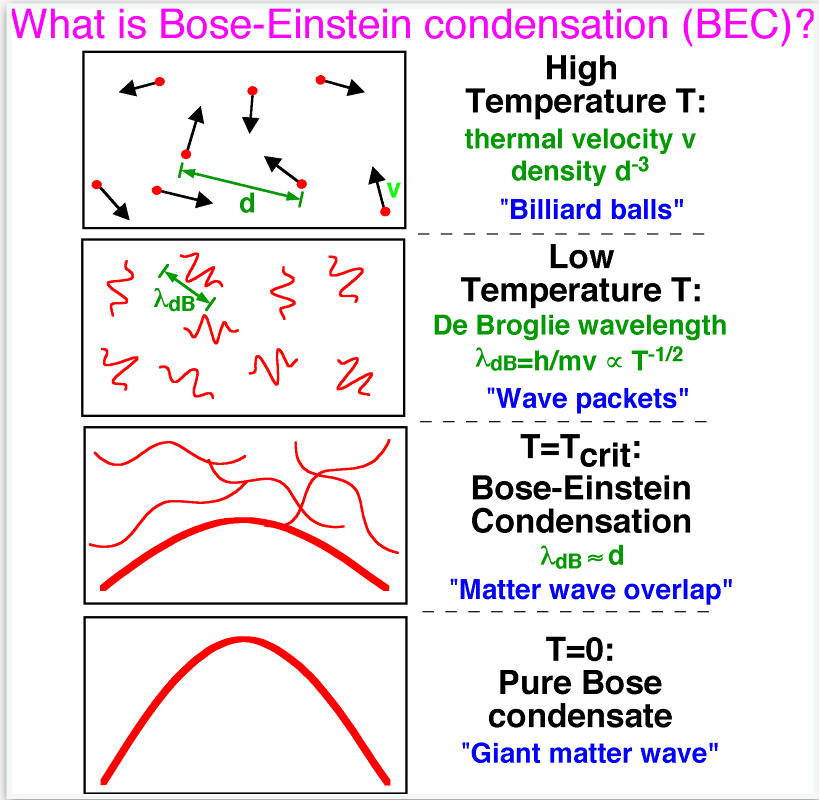


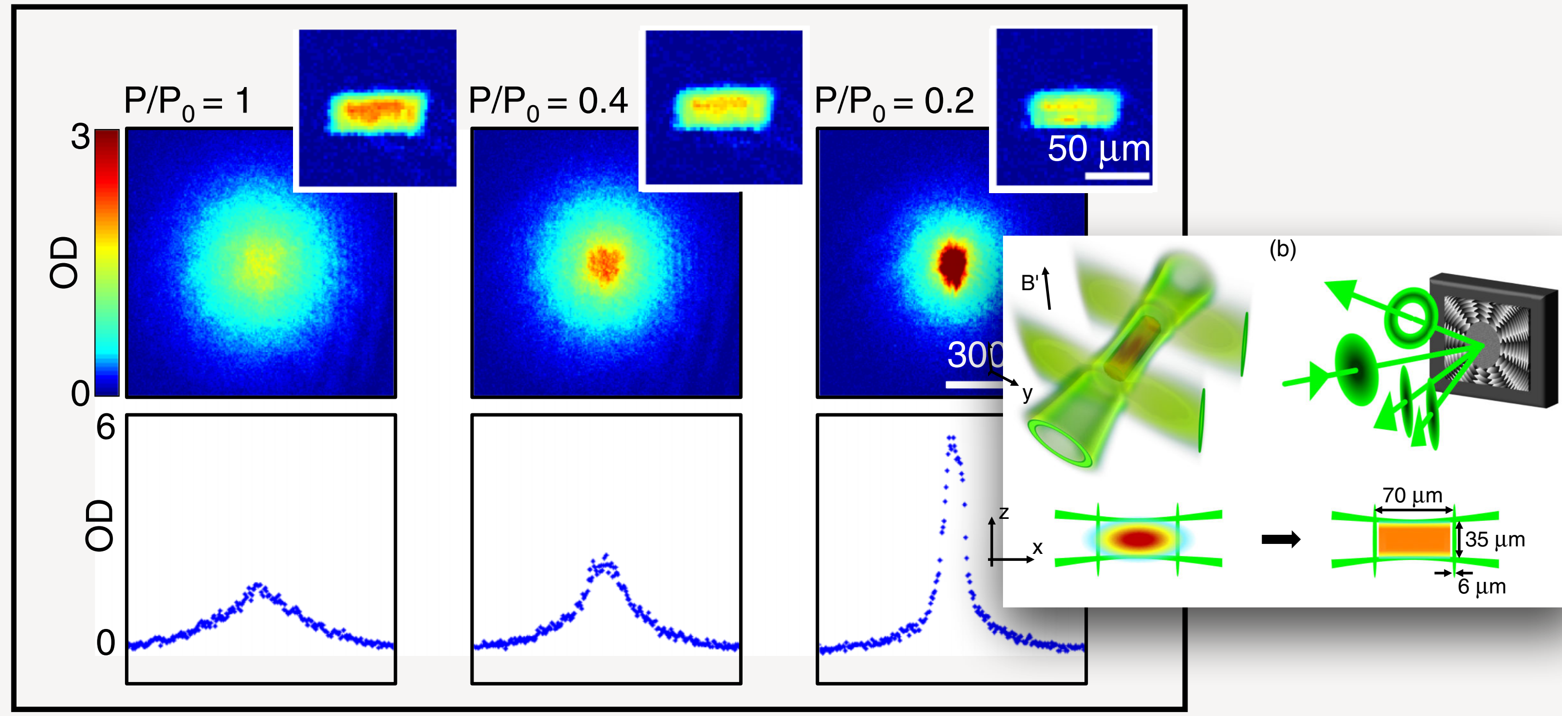
$\Delta \propto n_s/n$

is the angular momentum carried by the vortex

[Gallemì, Roccuzzo, Stringari, AR, PRA (2020)]

Ultra-cold Bose Gases: Condensate





Hadzibabic PRL 2013

homogeneous:

$$\hat{a}_{p=0} = a_0$$

$$\hat{\psi} = a_0/\sqrt{V} + \dots$$

$$e(n) = \frac{1}{2} g n^2$$

Dynamics and non-homogeneity- Gross-Pitaevskii equation: $\hat{\psi}(x) = \Psi(x) + \dots$

$$i\hbar \frac{\partial}{\partial t}\Psi = \left[-\frac{\hbar^2 \nabla^2}{2m} + V(x) + g|\Psi|^2 \right] \Psi$$ with $$|\Psi(x)|^2 = n(x)$$

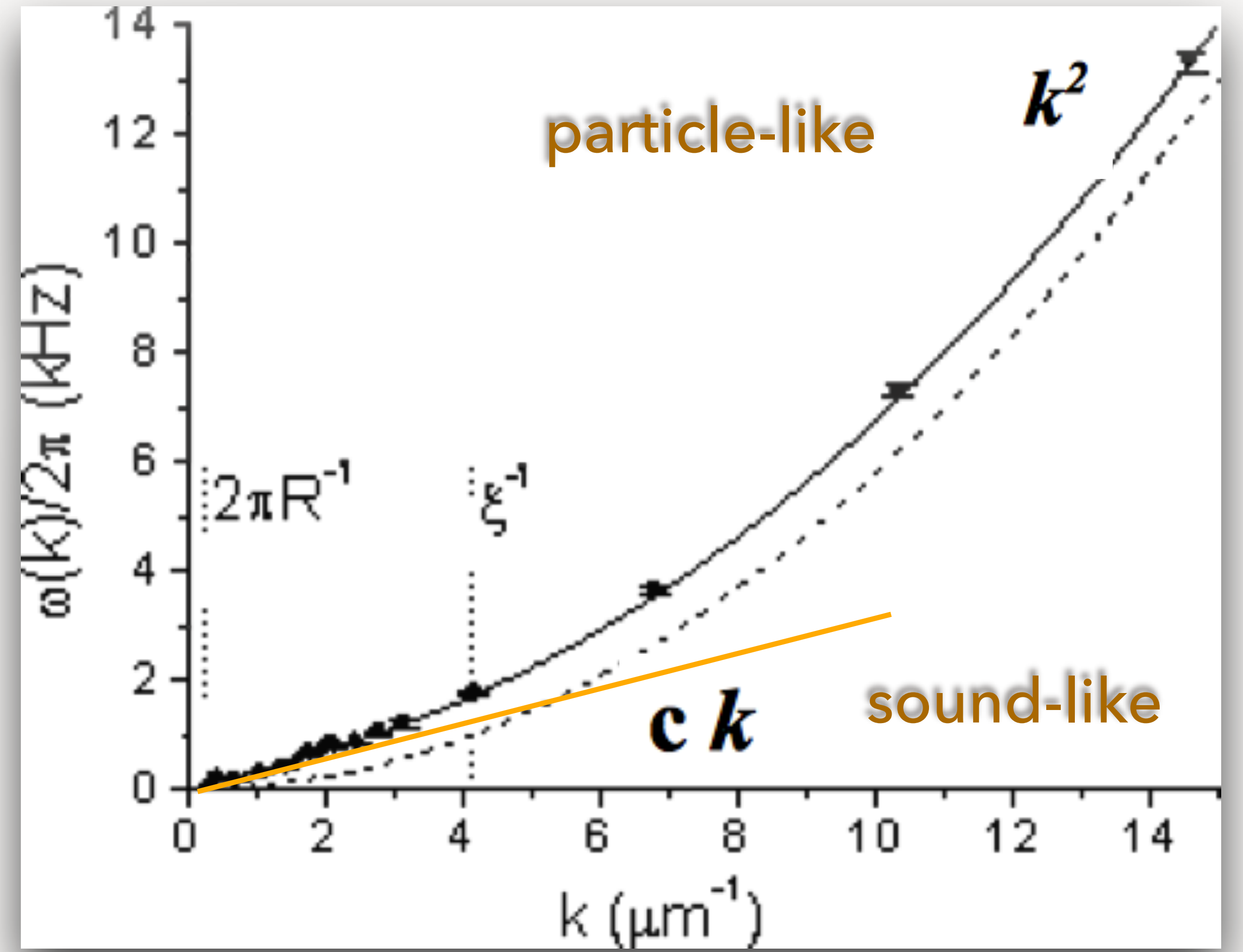
$$g = \frac{4\pi\hbar^2 a_s}{2m_r}$$

← s-wave scattering length

T=0 Bose gases: Elementary excitations

Uniform system (Mean-Field): $$e(n) = \frac{1}{2} g n^2$$

Bogoliubov Spectrum (Goldstone mode of the U(1) broken symmetry)



[MIT 1999]

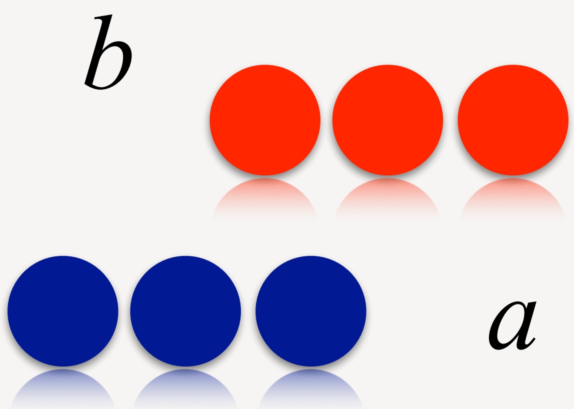
$$\omega_k = \sqrt{\frac{q^2}{2m}\left(2mc^2 + \frac{q^2}{2m}\right)}$$

where the speed of sound is:

$$c^2 = gn/m$$

and the healing length

$$\xi = \frac{\hbar}{\sqrt{2}mc}$$



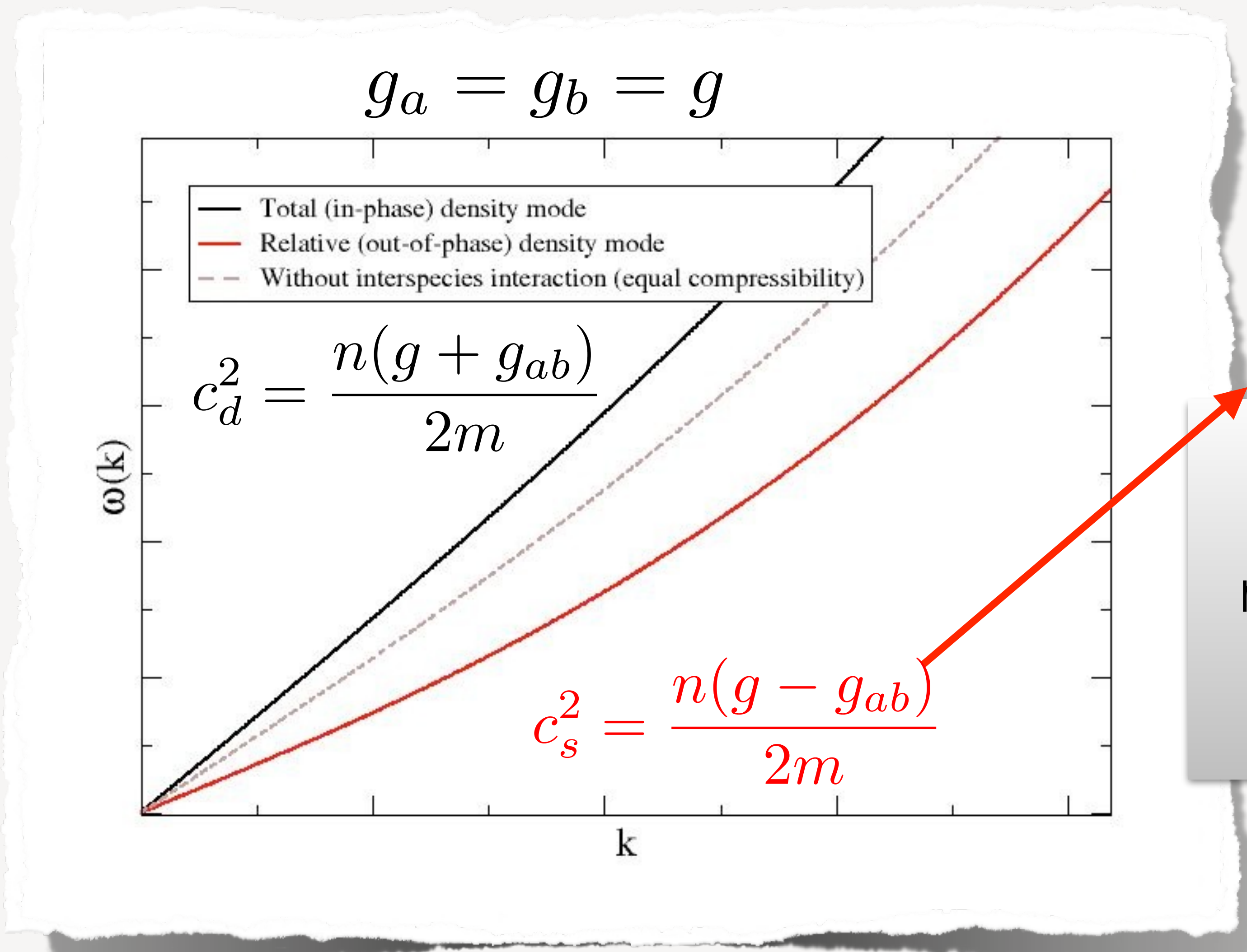
T=0 Bose mixtures

$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b$$

Both Na and Nb are conserved

Elementary excitations

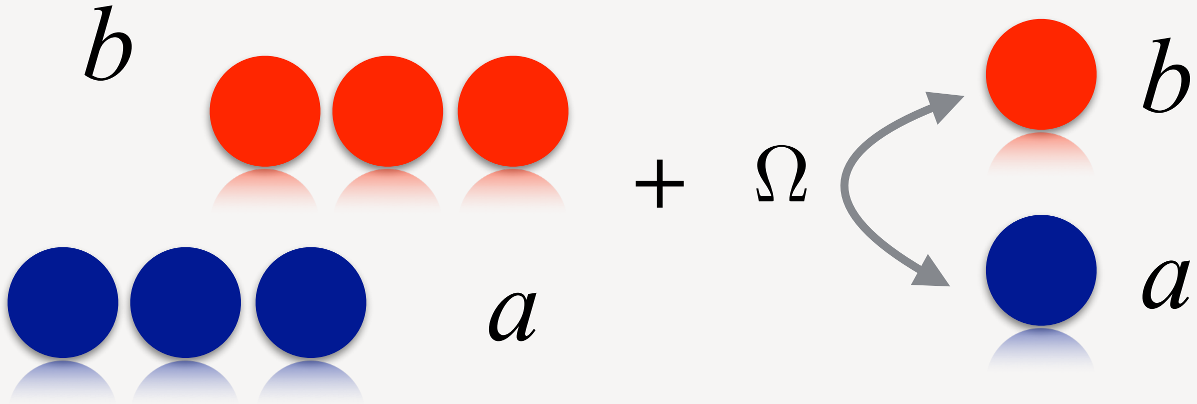
Ground state breaks U(1)xU(1) symmetry: 2 Goldstone modes - coming from no cost to change the global and relative phase of the 2 order parameters



$g = g_{ab}$

Spin mode soft: unstable with respect to phase separation (Immiscibility)

Coherently Coupled Bose Condensates



We consider a two component Bose gas with an interconversion term (Rabi coupling)

$$H = \int_{\mathbf{r}} \left[\sum_{\sigma} \left(\frac{\nabla \psi_\sigma^\dagger \nabla \psi_\sigma}{2m} + V_\sigma \psi_\sigma^\dagger \psi_\sigma \right) - \frac{\Omega}{2} (\psi_a^\dagger \psi_b + \psi_b^\dagger \psi_a) + \sum_{\sigma\sigma'} \frac{g_{\sigma\sigma'}}{2} \psi_\sigma^\dagger \psi_{\sigma'}^\dagger \psi_{\sigma'} \psi_\sigma \right]$$

The cleanest case is: $g_a = g_b = g$ ➡ $U(1) \times \mathbb{Z}_2$

In the weakly interacting regime at T=0 the system forms a BEC well described within mean-field (MF) theory by a spinor order parameter

$$\Psi = e^{i\phi_d/2} \begin{pmatrix} \sqrt{n_\uparrow} e^{i\phi_r/2} \\ \sqrt{n_\downarrow} e^{-i\phi_r/2} \end{pmatrix}$$

The MF energy reads: $\varepsilon_{MF} = \frac{g_{dd}}{2} n^2 + \frac{g_{ss}}{2} s_z^2 - \frac{\Omega}{2} \sqrt{n^2 - s_z^2} \cos(\phi_s)$

$$n = n_a + n_b \qquad \phi_d = \phi_a + \phi_b \qquad g_{dd} = (g + g_{ab})/2$$

$$s_z = n_a - n_b \qquad \phi_s = \phi_a - \phi_b \qquad g_{ss} = (g - g_{ab})/2$$

Para- or ferro-magnetic ground state

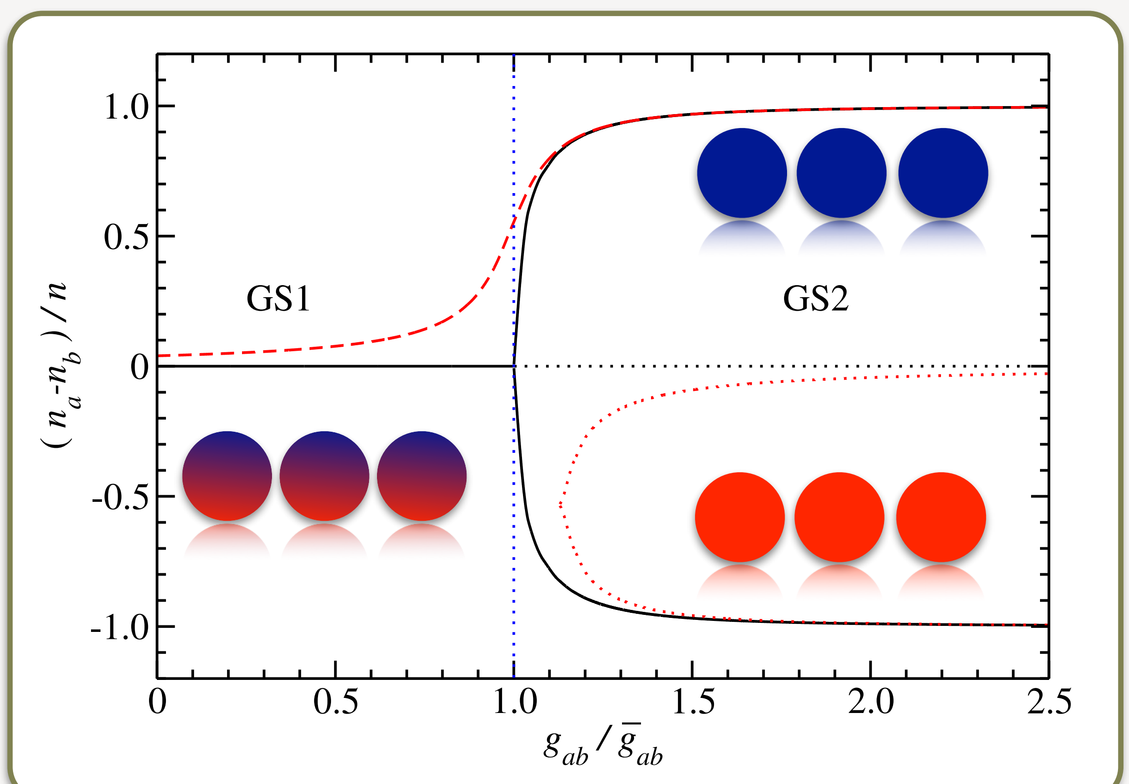
$$\varepsilon_{MF} = \frac{g_{dd}}{2} n^2 + \frac{g_{ss}}{2} s_z^2 - \frac{\Omega}{2}\sqrt{n^2 - s_z^2}\cos(\phi_s)$$

$\phi_s = 0$

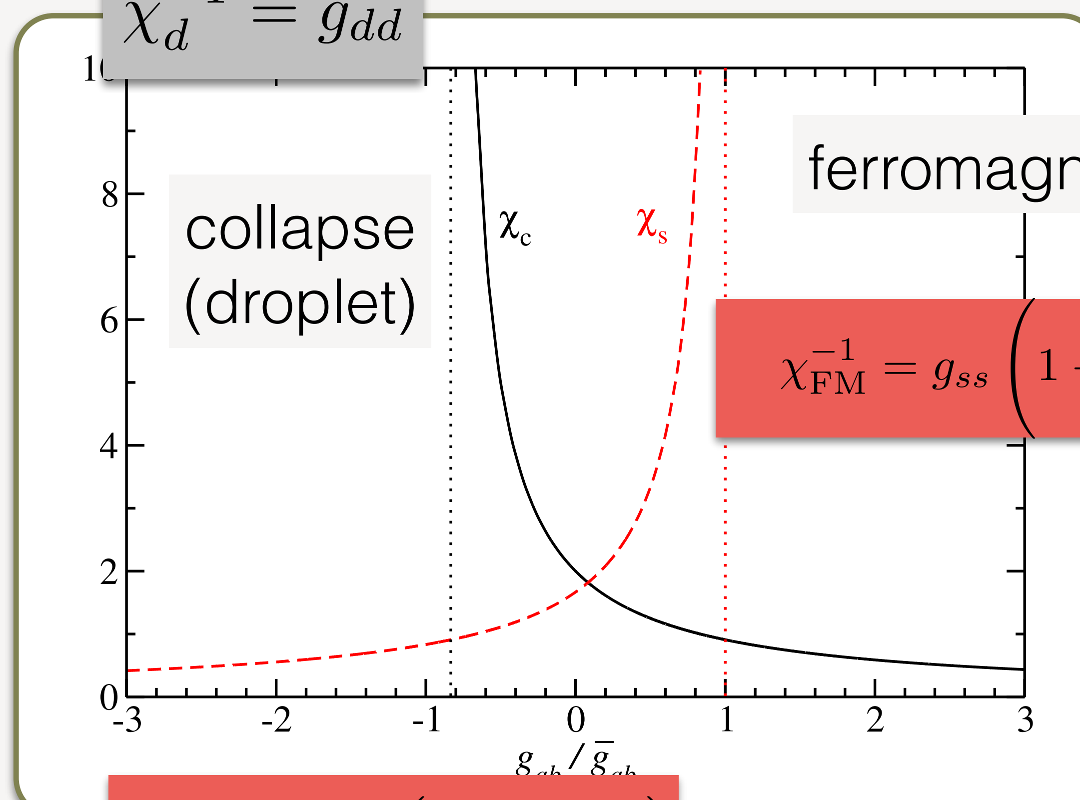
- **para** $s_z = 0$ for $\Omega + 2g_{ss}n > 0$
- **ferro** $s_z = \pm n\sqrt{1 - \left(\frac{\Omega}{2g_{ss}n}\right)^2}$ for $\Omega + 2g_{ss}n < 0.$

Critical condition for the II order phase transition: $\Omega + 2g_{ss}n = 0$

$s_z \propto (-(2g_{ss}n + \Omega))^\beta \qquad \beta = 1/2$



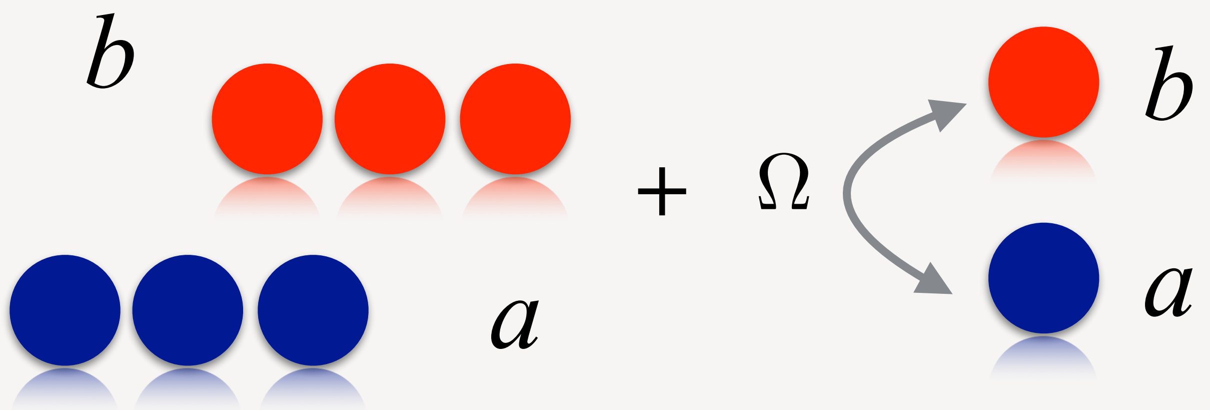
$\chi_d^{-1} = g_{dd}$



$$\chi_{\mathrm{FM}}^{-1} = g_{ss}\left(1 - \left(\frac{2g_{ss}n}{\Omega}\right)^2\right)$$

$$\chi_P^{-1} = g_{ss}\left(1 + \frac{\Omega}{2g_{ss}n}\right)$$

T=0 coherently coupled Bose gases



$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b$$
$$+2|\Omega| \cos(\phi_a - \phi_b)\sqrt{n_a n_b}$$

Only
Na+Nb
is conserved

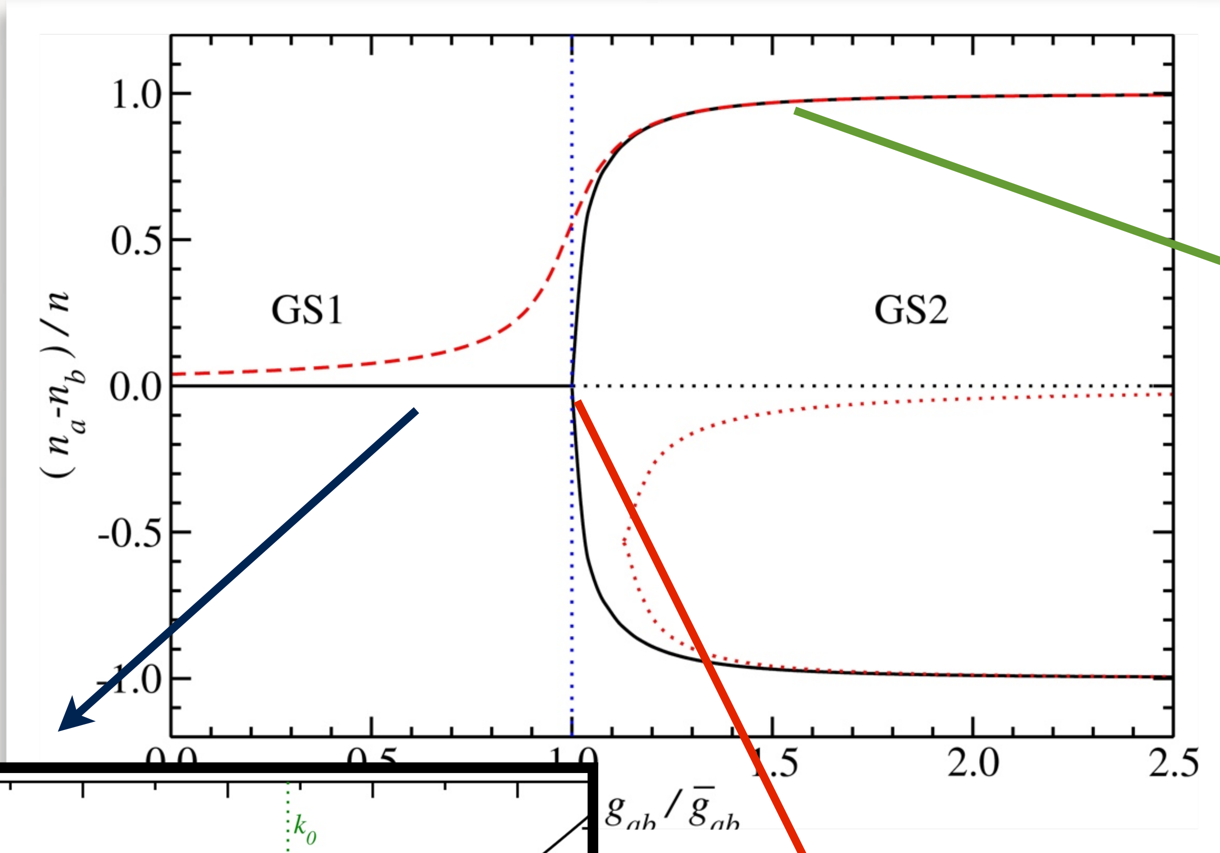
Indeed the system is a single condensate
with a 2-component wave function

Elementary excitations

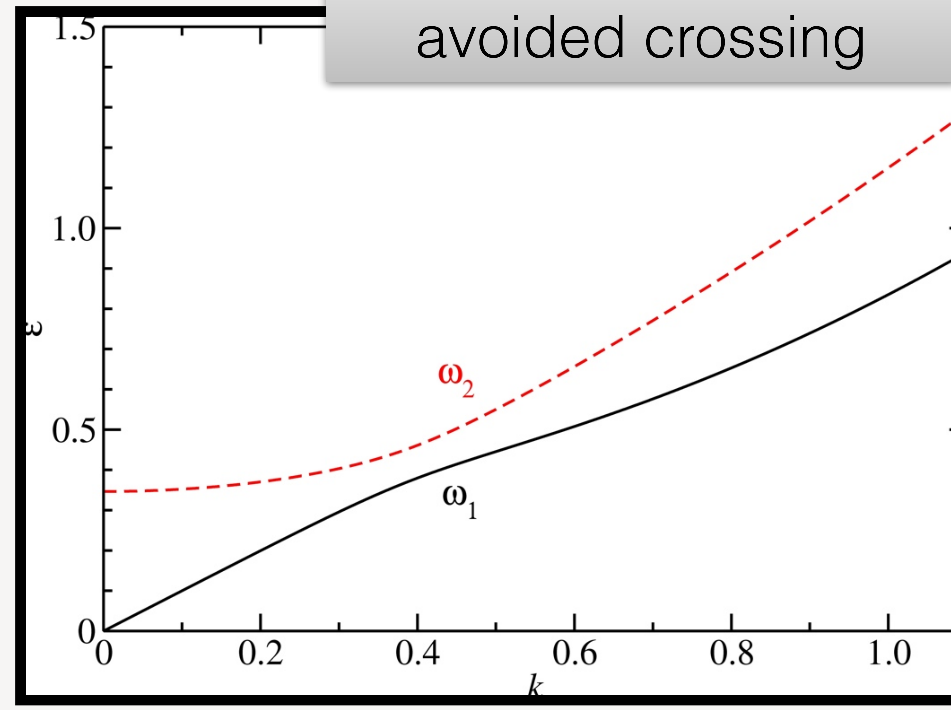
Ground state breaks U(1) symmetry:

1. Goldstone mode - coming from no cost to change the global total phase.
2. A gapped mode - due to the cost of changing the relative phase

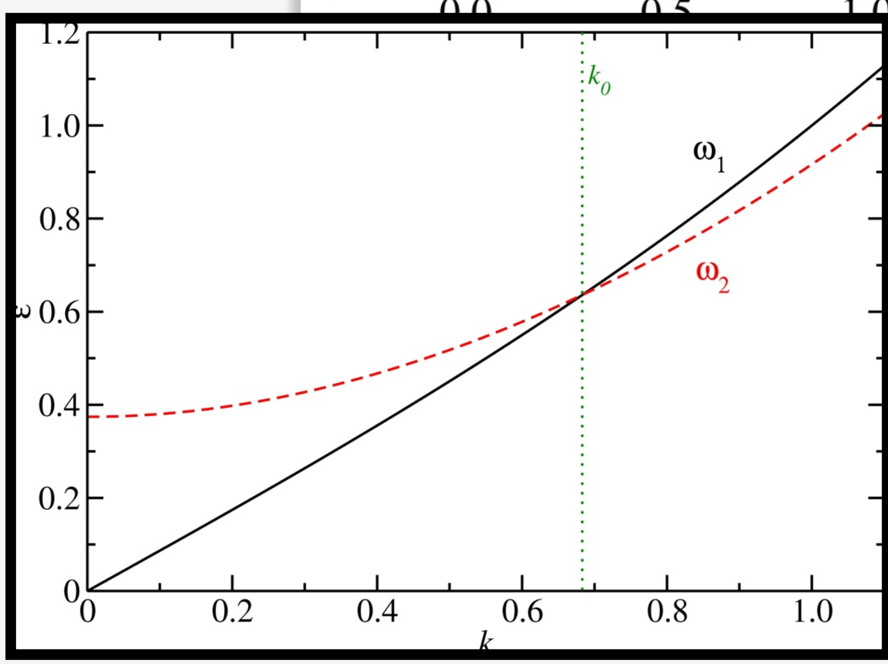
Bogolyubov spectrum



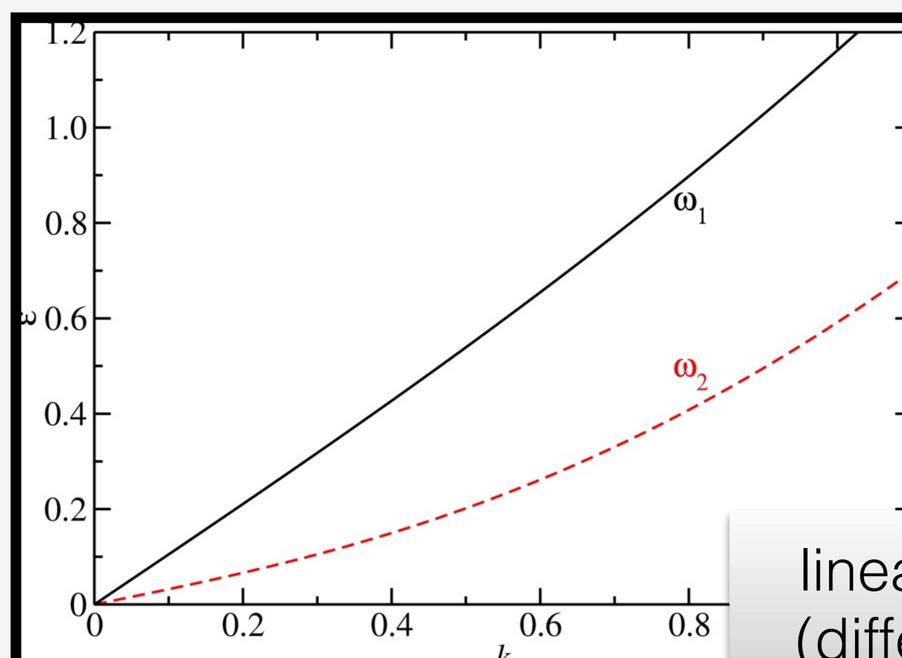
hybridization ⇒ avoided crossing



gap $\Delta_{FM} = \sqrt{\frac{2\Omega^2}{|g_{ss}|}\chi_{FM}^{-1}}$

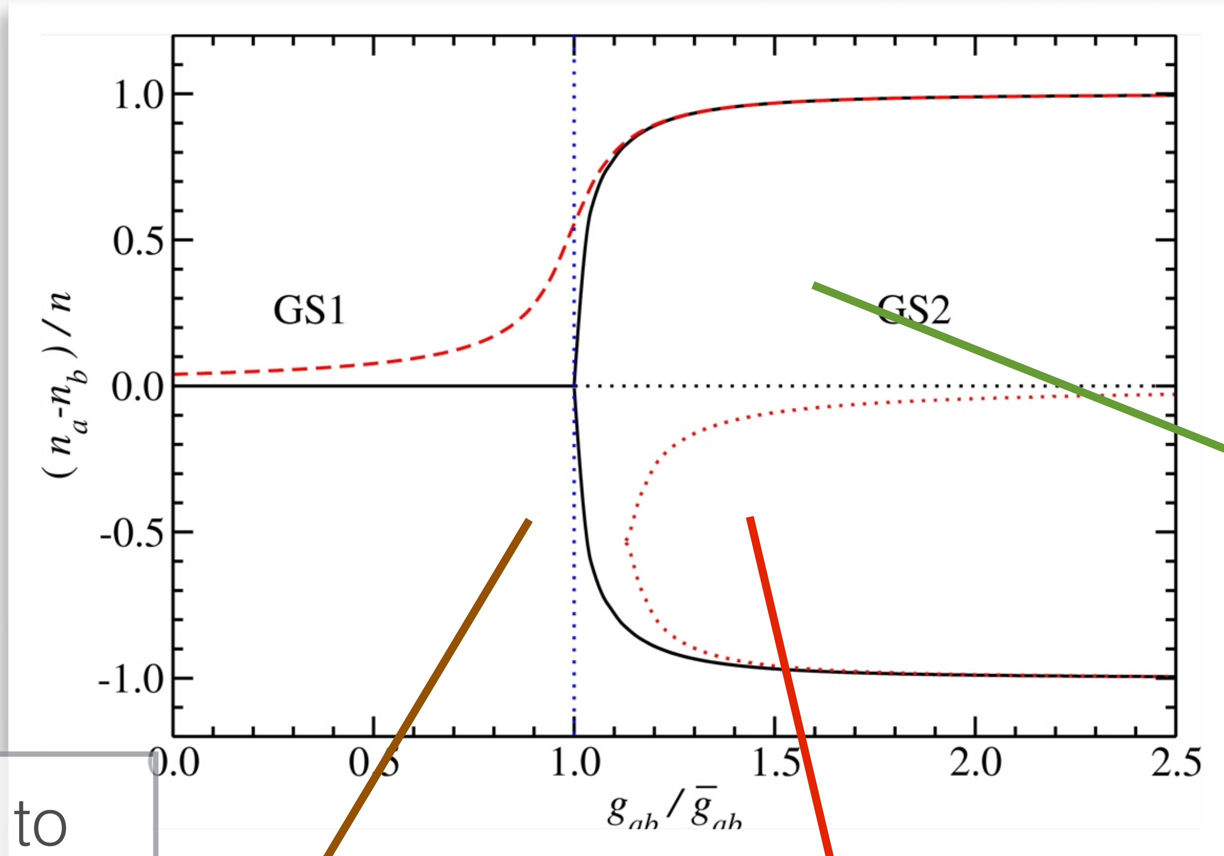


gap $\Delta_P = \sqrt{2n\Omega\chi_P^{-1}}$

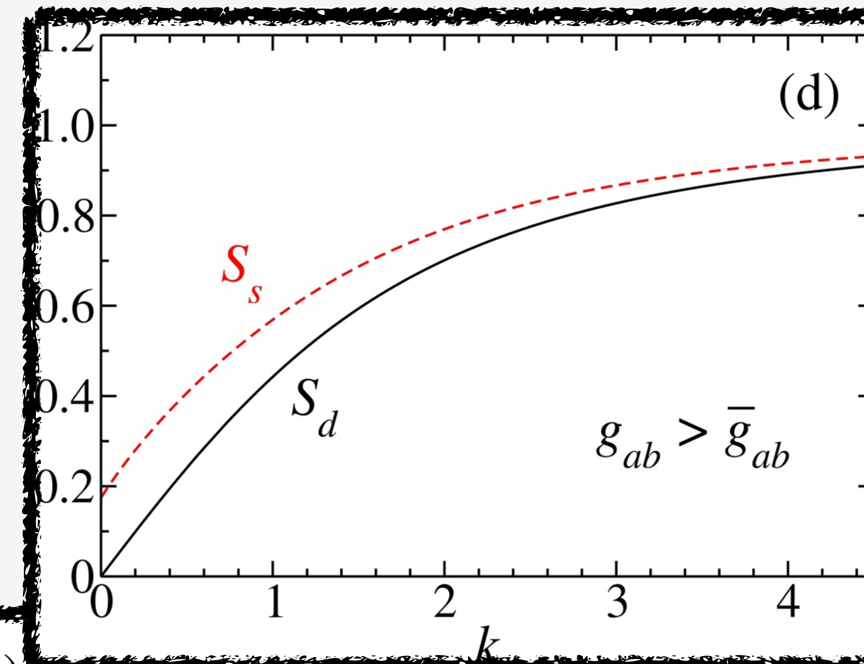


linear gapless modes (different behaviour in mixtures)

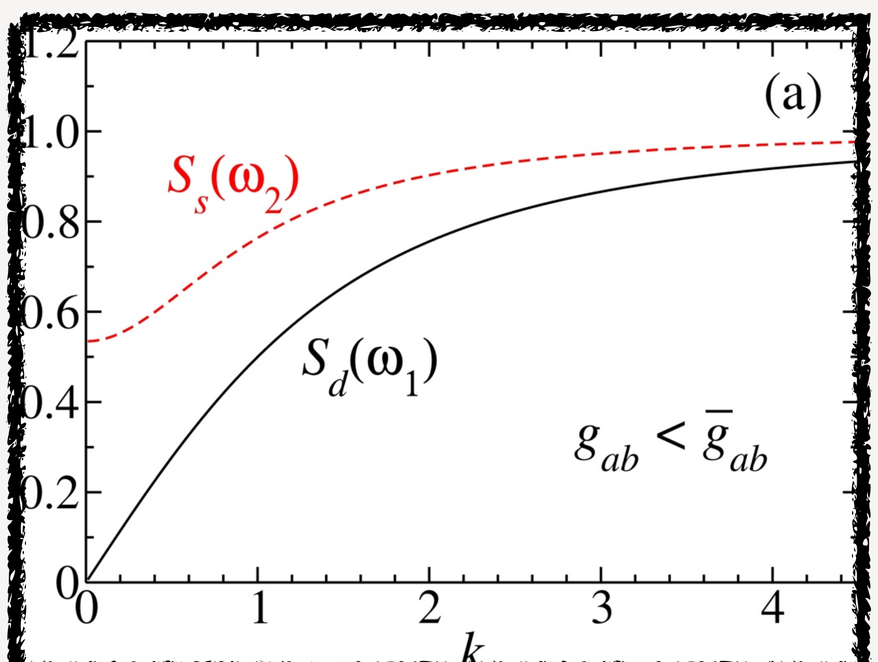
Static structure factor across the transition

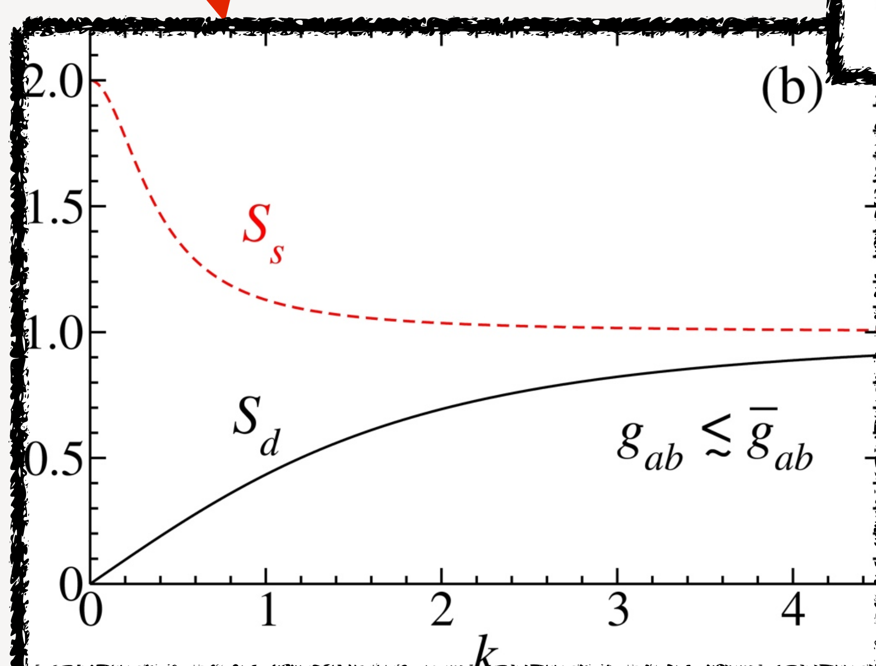


$$S_{d(s)}(k) = S^1_{d(s)}(k) + S^2_{d(s)}(k)$$



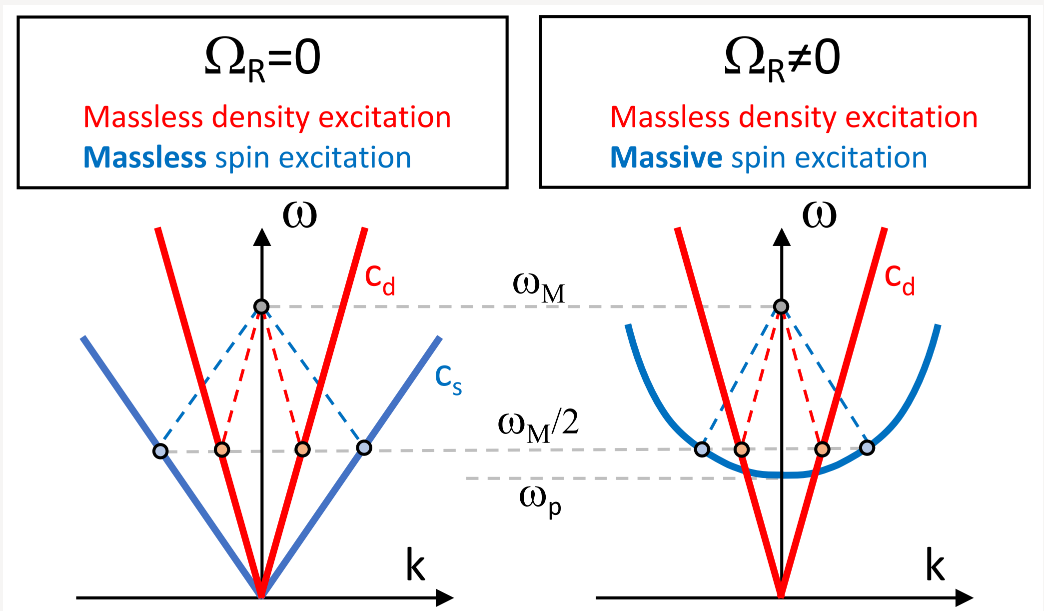
gap due to single-particle

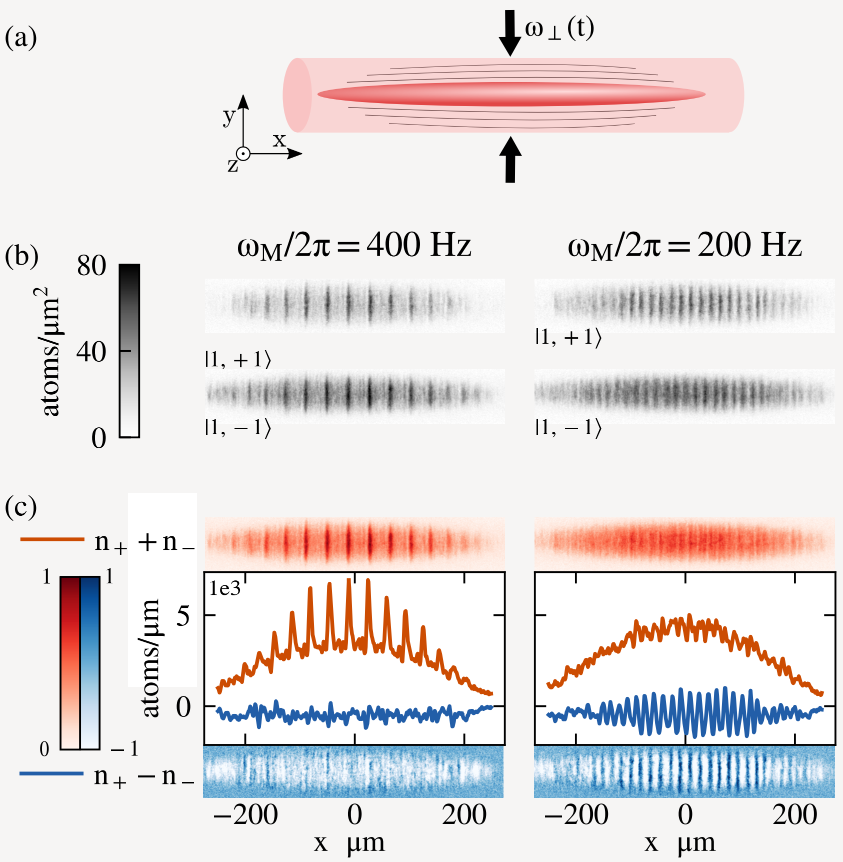


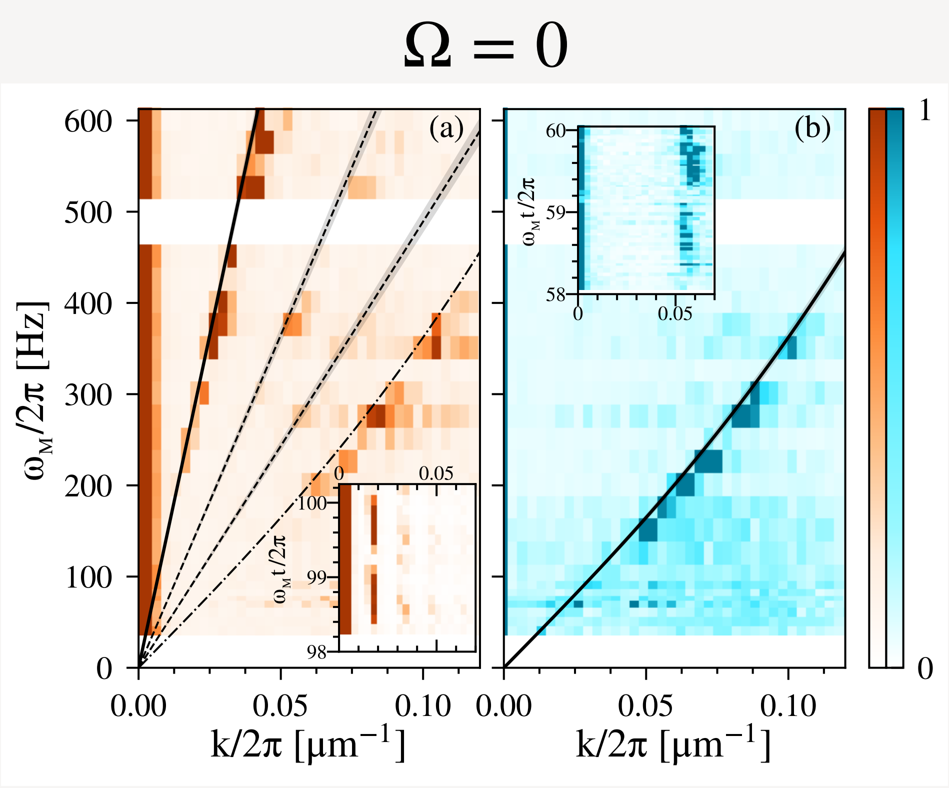


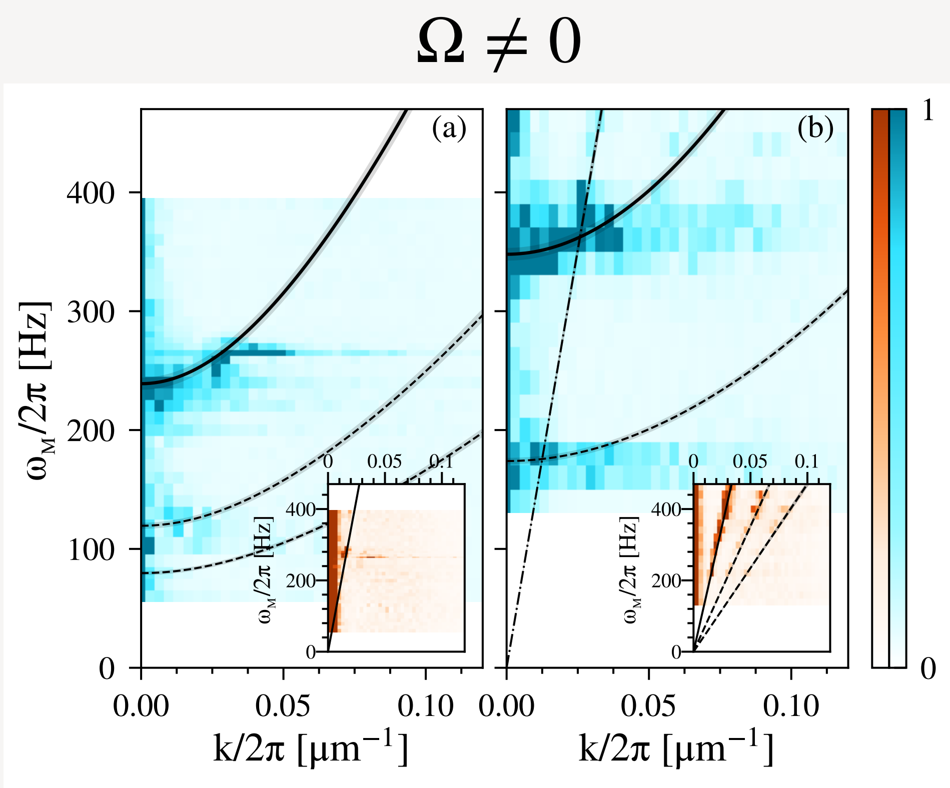
Feynman criterion for the density response

Measuring the dispersion relation via “Faraday wave spectroscopy”









Measuring the dispersion relation via "Faraday wave spectroscopy"

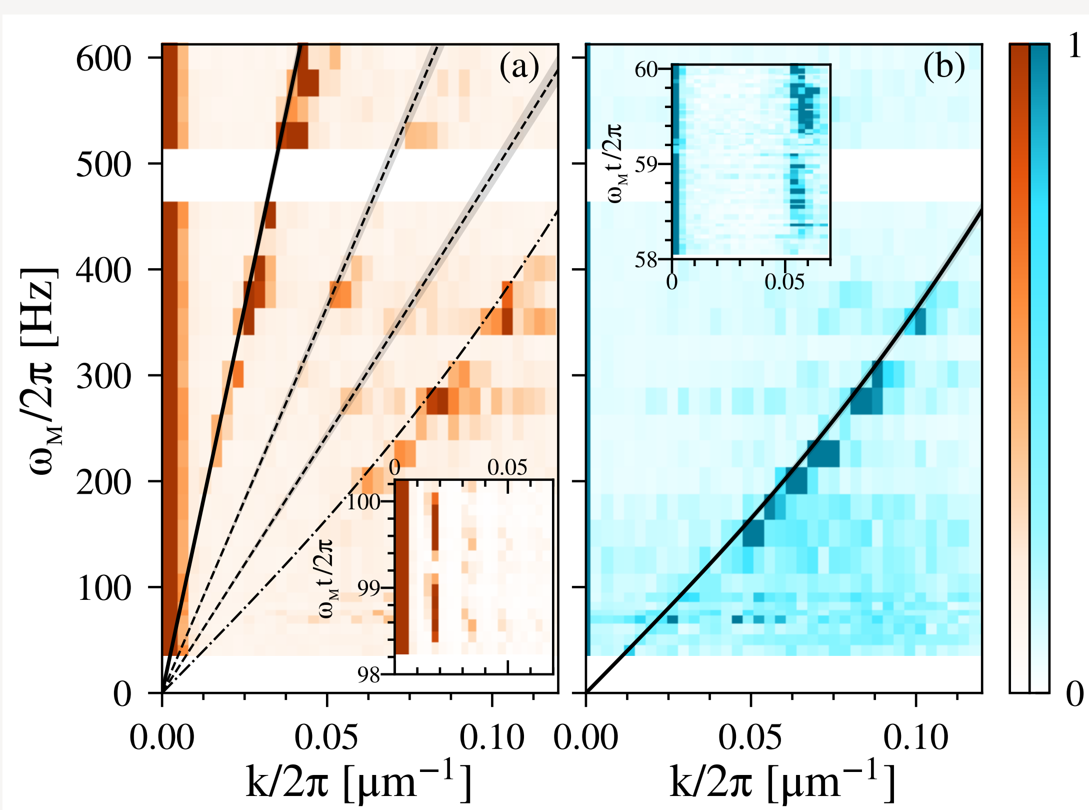
$$\widehat{\mathcal{H}}(t) = \sum_{k\neq 0}\Big(\hbar\omega(k)+F(k,t)\Big)\hat{b}_k^\dagger\hat{b}_k+\sum_{k>0}F(k,t)\Big(\hat{b}_k^\dagger b_{-k}^\dagger+h.c.\Big)$$

$F(k,t) = \mu S(k) f(t)$ with $f(t) = \eta \sin(\omega_M t)$

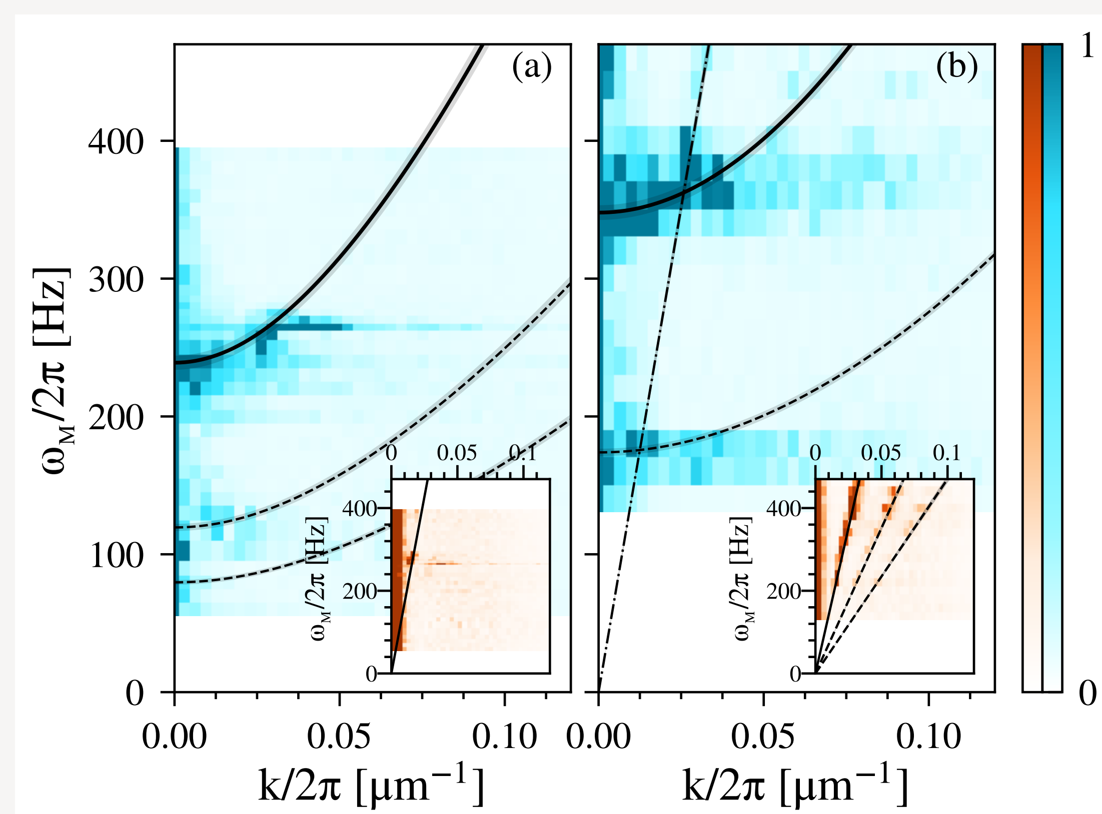
The time evolution of the quasi-particle operator is equivalent to parametrically driven harmonic oscillators described by the Mathieu equation:

$$\ddot{x}_k + \omega(k)\Big[\omega(k) + 2\eta\mu S(k)\sin(\omega_M t)/\hbar\Big]x_k = 0.$$ instabilities for: $\omega_M = 2\omega(k)/l$

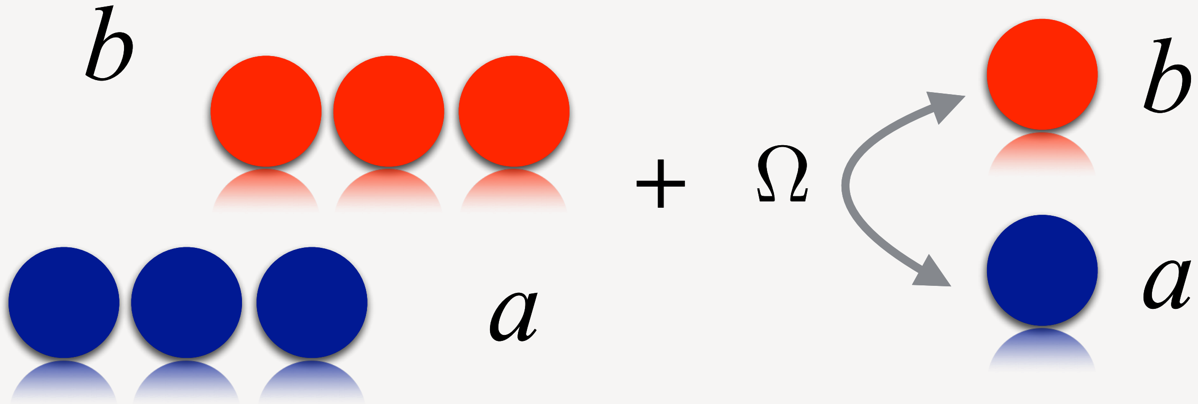
$\Omega = 0$



$\Omega \neq 0$



Coherently Coupled Bose Condensates



$$\Omega(|a\rangle\langle b| + |b\rangle\langle a|) = \Omega\sigma_x$$

We consider a two component Bose gas with an interconversion term (Rabi coupling)

$$H = \int_{\mathbf{r}} \left[\sum_{\sigma} \left(\frac{\nabla\psi_\sigma^\dagger \nabla\psi_\sigma}{2m} + V_\sigma \psi_\sigma^\dagger \psi_\sigma \right) - \frac{\Omega}{2}(\psi_a^\dagger\psi_b + \psi_b^\dagger\psi_a) + \sum_{\sigma\sigma'} \frac{g_{\sigma\sigma'}}{2} \psi_\sigma^\dagger \psi_{\sigma'}^\dagger \psi_{\sigma'} \psi_\sigma \right]$$

The cleanest case is: $g_a = g_b = g$ ➡ $U(1) \times \mathbb{Z}_2$

In the weakly interacting regime at T=0 the system is well described within mean-field theory by a spinor order parameter

$$\Psi = e^{i\phi_d/2} \begin{pmatrix} \sqrt{n_\uparrow} e^{i\phi_r/2} \\ \sqrt{n_\downarrow} e^{-i\phi_r/2} \end{pmatrix}$$

The dynamics of the classical fields are given by *coupled Gross-Pitaevskii equations*. They can be recast in a very convenient as the equation of motion for the relevant variables describing the gas, namely the **density** and the **spin-density**.

Coherently Coupled Bose Condensates

density: $n(\mathbf{r}) = (\Psi^*_\uparrow, \Psi^*_\downarrow) \cdot (\Psi_\uparrow, \Psi_\downarrow)^T = n_\uparrow + n_\downarrow$

spin-density: $s_i(\mathbf{r}) = (\Psi^*_\uparrow, \Psi^*_\downarrow)\sigma_i(\Psi_\uparrow, \Psi_\downarrow)^T$ with $|\mathbf{s}(\mathbf{r})| = n(\mathbf{r})$

They satisfy what can be called collision-less spin hydrodynamic equations [1]:

$$\dot{n} + \mathrm{div}(n\mathbf{v}) = 0,$$

$$m\dot{\mathbf{v}} + \nabla\left(\frac{mv^2}{2} + \mu + \frac{s_z}{n}h + V - \frac{\hbar^2\nabla^2\sqrt{n}}{2m\sqrt{n}} + \frac{\hbar^2|\nabla\mathbf{s}|^2}{8mn^2}\right) = 0,$$

$$\dot{\mathbf{s}} + \sum_{\alpha=x,y,z}\partial_\alpha(\mathbf{j}_{s,\alpha}) = \mathbf{H}(\mathbf{s}) \times \mathbf{s},$$

where the superfluid and the spin currents read:

$$\mathbf{v}(\mathbf{r}) = \frac{\mathbf{j}(\mathbf{r})}{n} = \frac{\hbar}{2mni}\sum_{\sigma=\uparrow,\downarrow}(\Psi^*_\sigma\nabla\Psi_\sigma - \Psi_\sigma\nabla\Psi^*_\sigma) = \frac{\hbar}{2m}(\nabla\phi_d + s_z/n\,\nabla\phi_r)$$

irrotational (remind: Mermin-Ho A-phase He-3)

$$\mathbf{j}_{s,\alpha} = v_\alpha\mathbf{s} - \frac{\hbar}{2m}\left(\frac{\mathbf{s}}{n} \times \partial_\alpha\mathbf{s}\right), \quad \alpha = x, y, z\ ,$$

advection — vectorial nature

and the non-linear magnetic field is $\mathbf{H}(\mathbf{s}) = (-\Omega, 0, (g - g_{\uparrow\downarrow})s_z/\hbar)$

break SU(2) symmetry

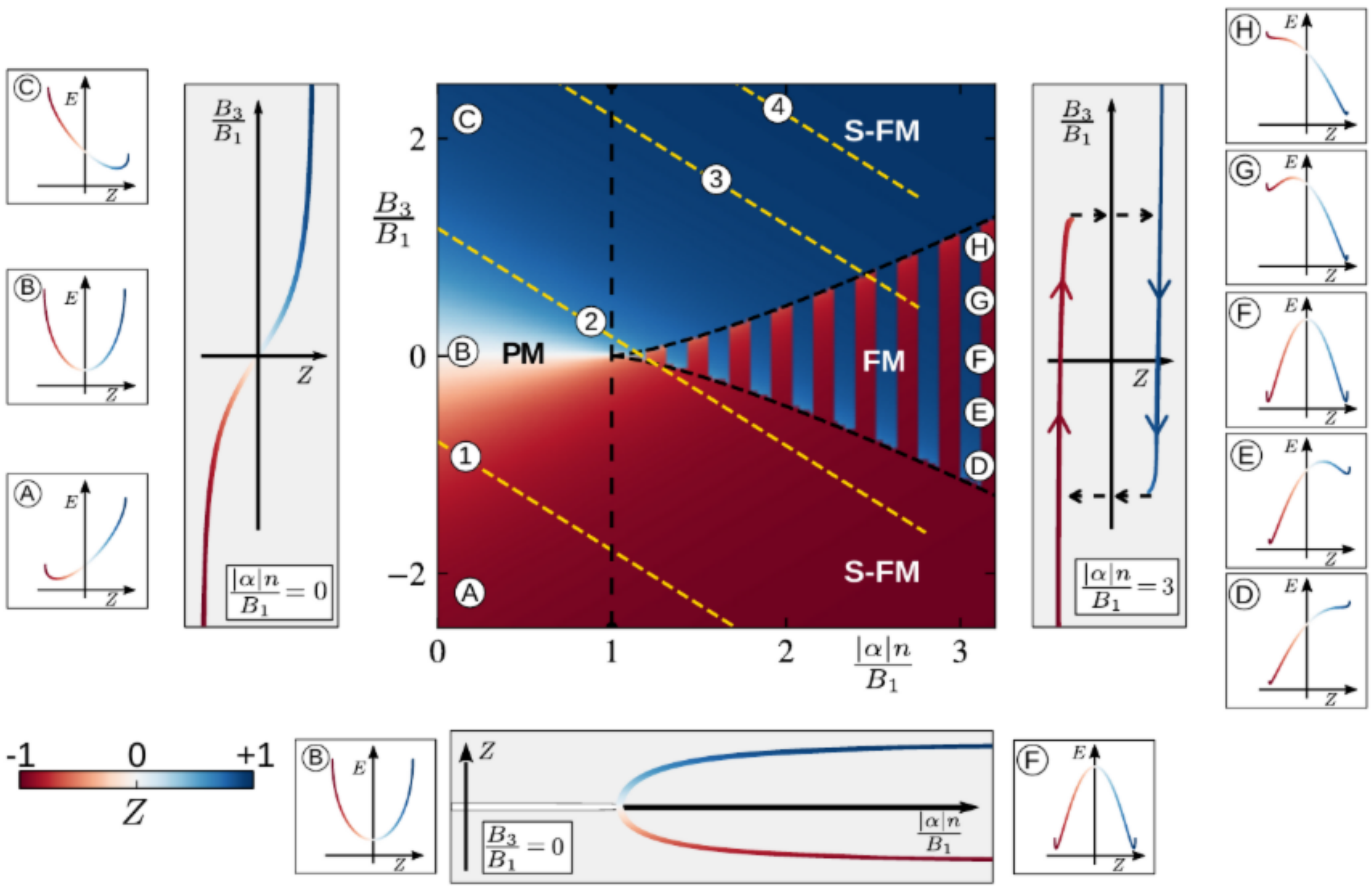
[1] T. Nikuni, J.E. Williams, J. Low Temp. Phys. 133, 323 (2003)

The spin sector and the equivalent magnetic system

A Landau-Lifshitz magnetic functional

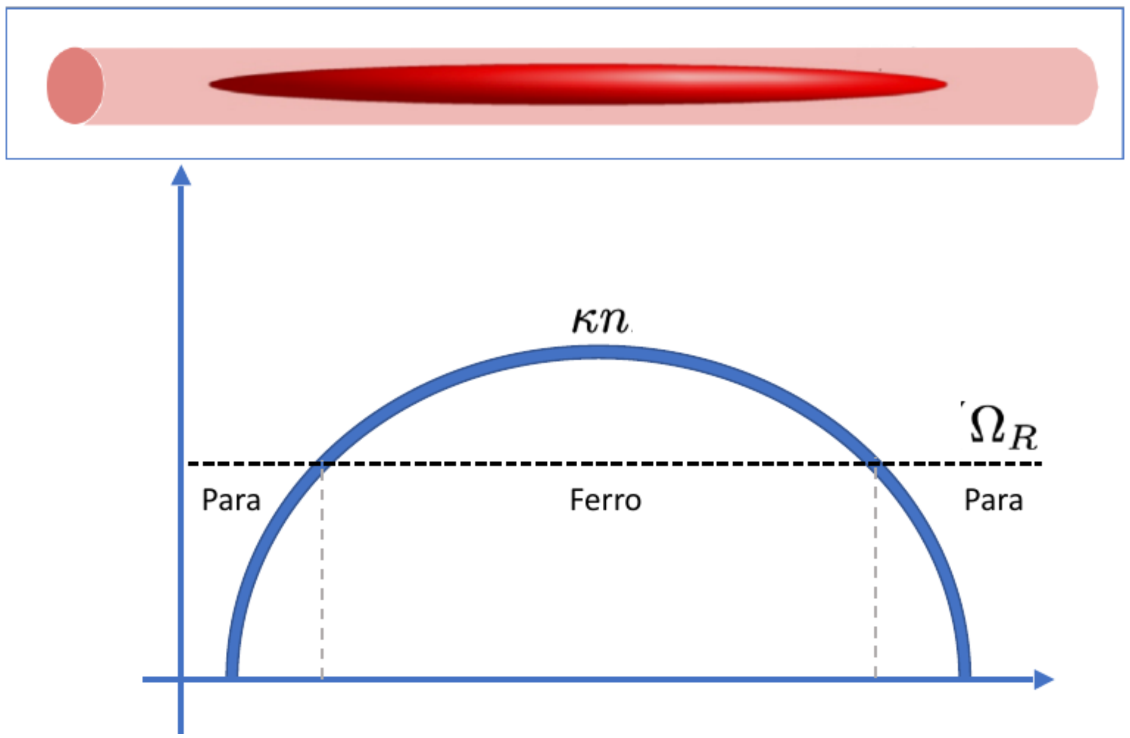
$$E(\mathbf{S}) \propto -\int \left(-\frac{\gamma}{2}|\nabla \mathbf{S}|^2 + \mathbf{B}\cdot\mathbf{S} + \kappa S_z^2\right) dV$$

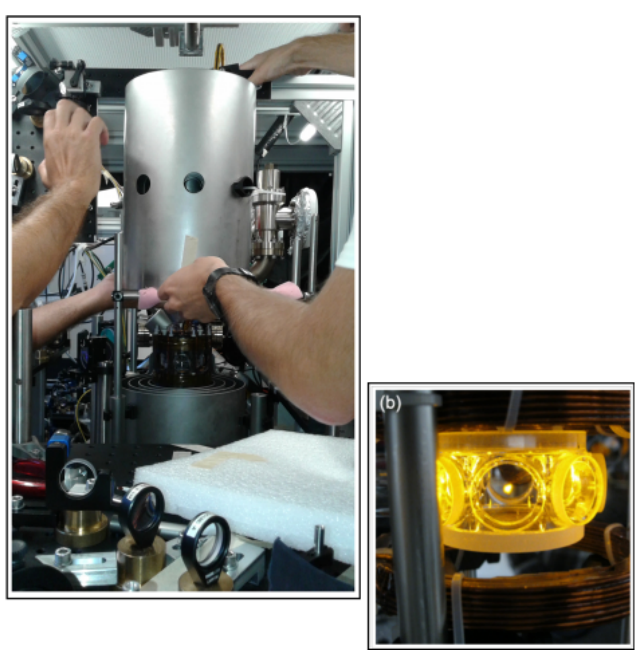
$$\gamma = \frac{\hbar^2}{2mn},\ \mathbf{B} = (\Omega, 0, \delta(n)),\ \kappa \propto (g_{12} - g)$$

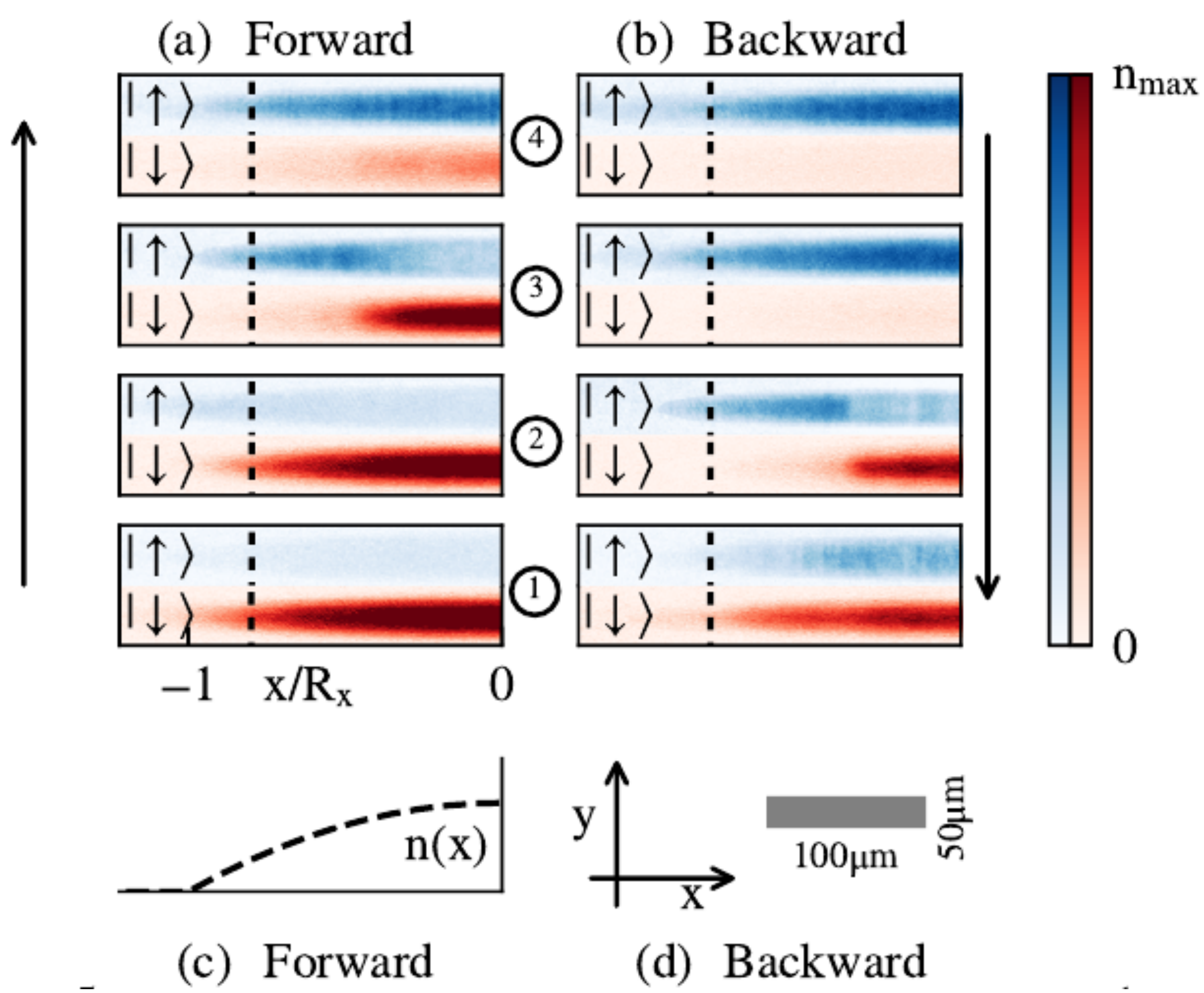


The spin sector and the equivalent magnetic system

Our system is harmonically trapped in an elongated potential



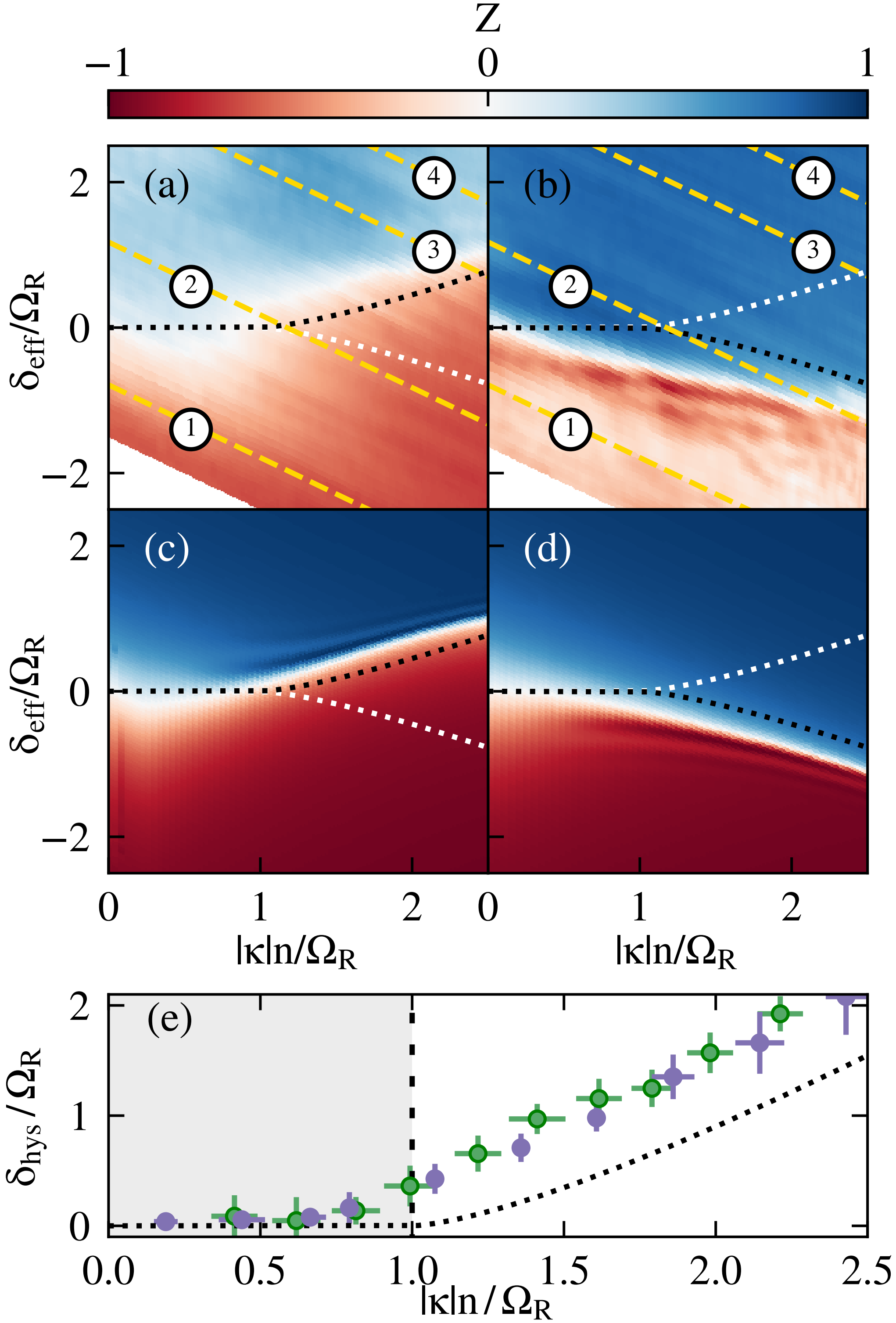




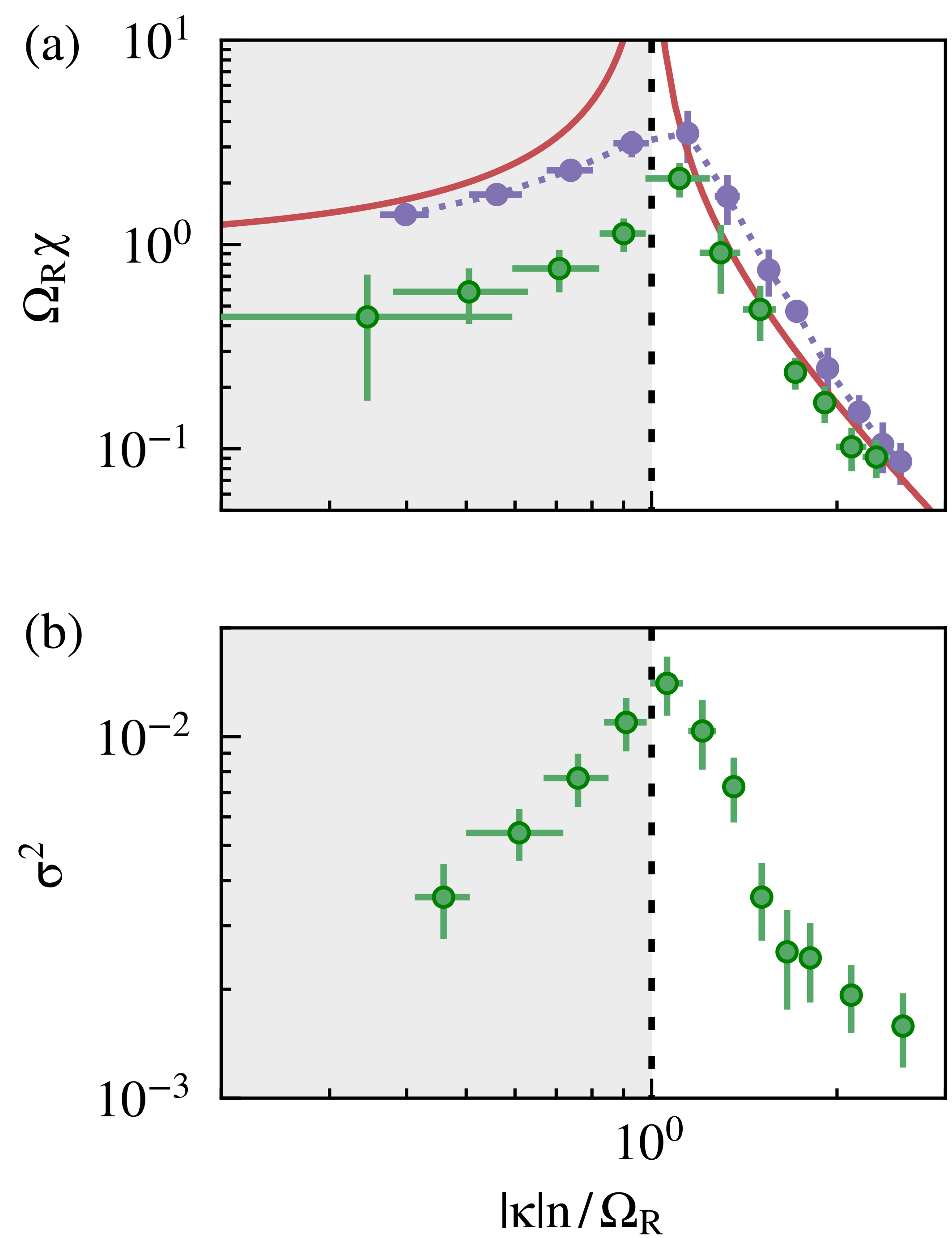
The spin sector and the equivalent magnetic system

From local measurement we can extract the phase diagram

Magnetisation and Hysteresis



Susceptibility and magnetic Fluctuations

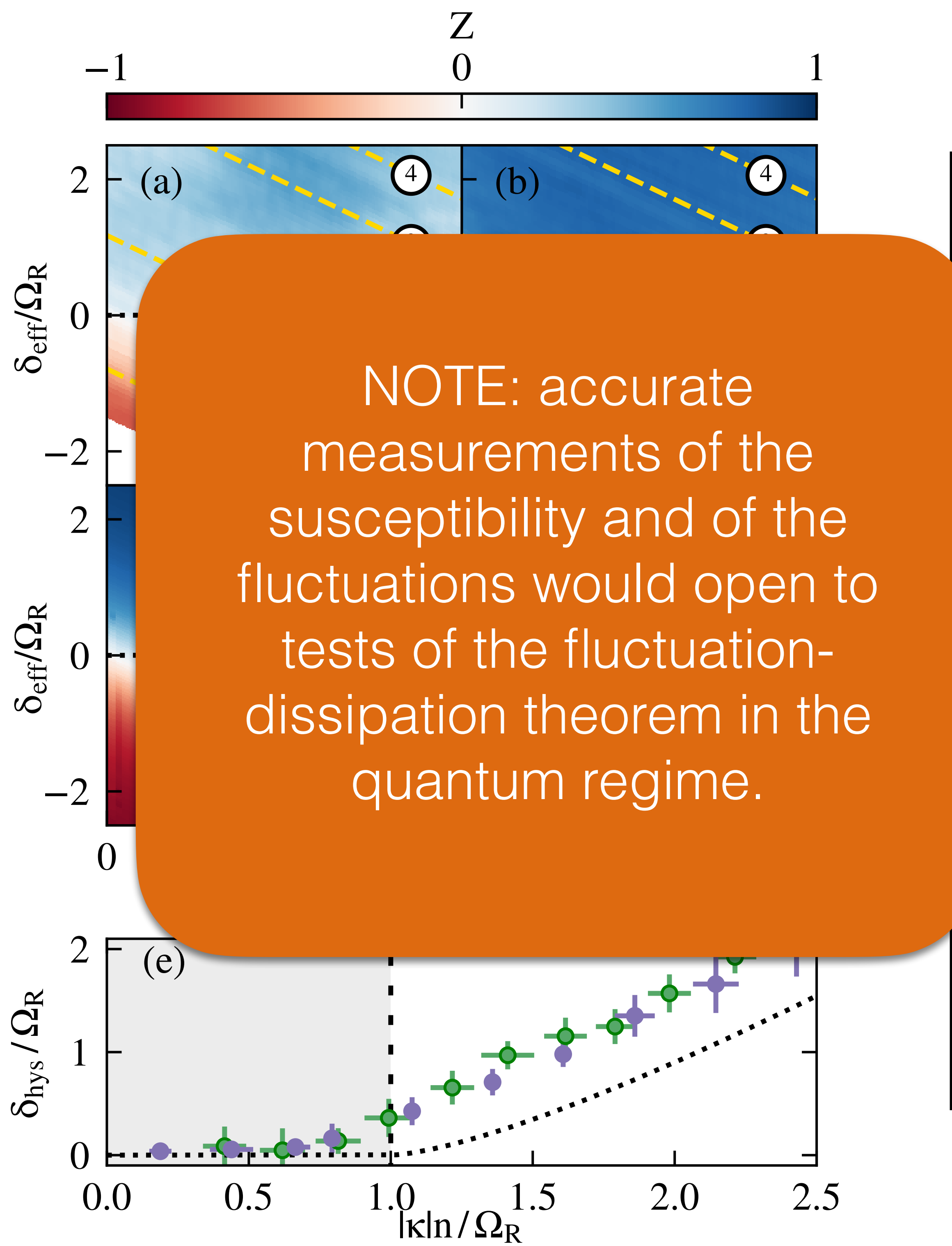


Theory (c)-(d) and purple points: noisy/dissipative GPE - Truncated Wigner

The spin sector and the equivalent magnetic system

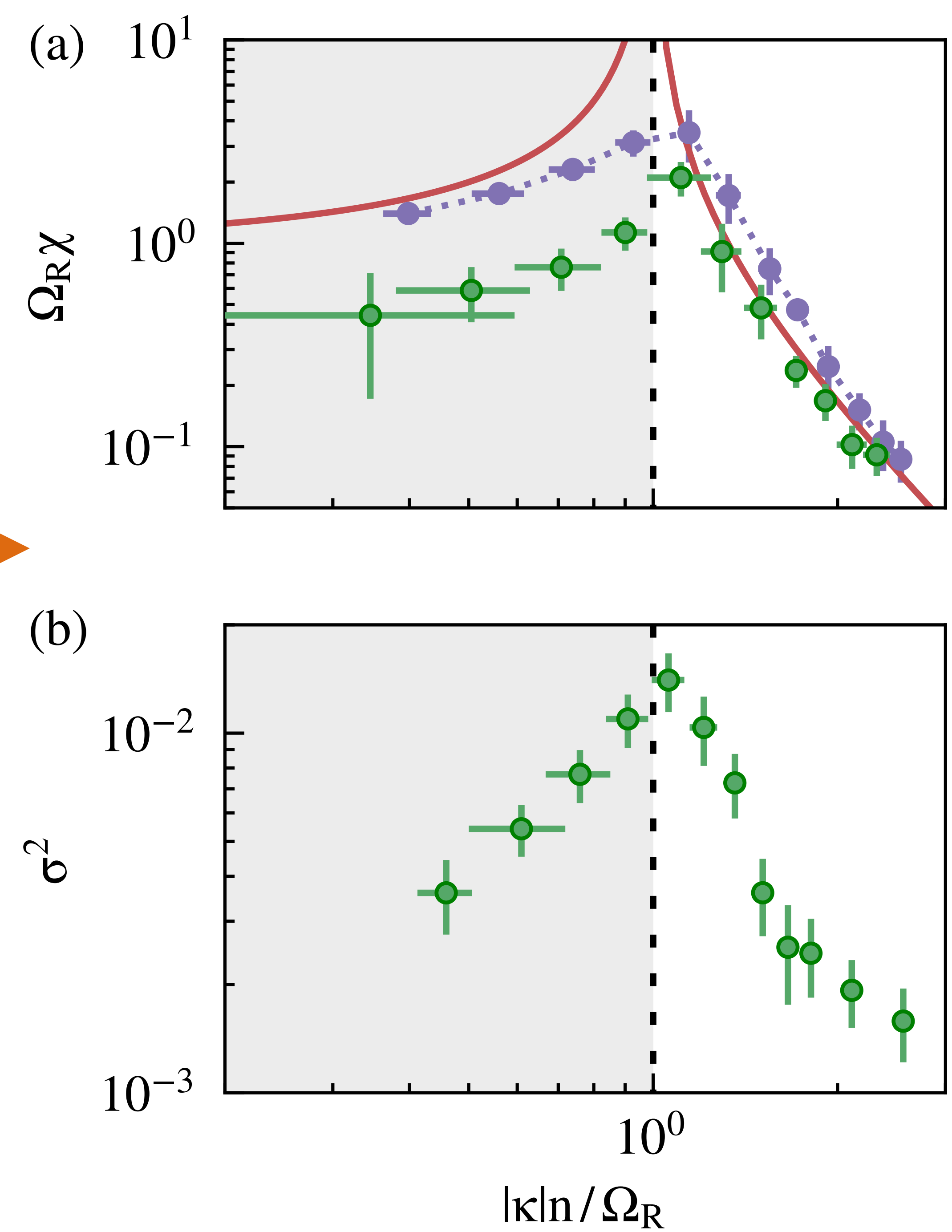
From local measurement we can extract the phase diagram

Magnetisation and Hysteresis



NOTE: accurate measurements of the susceptibility and of the fluctuations would open to tests of the fluctuation-dissipation theorem in the quantum regime.

Susceptibility and magnetic Fluctuations



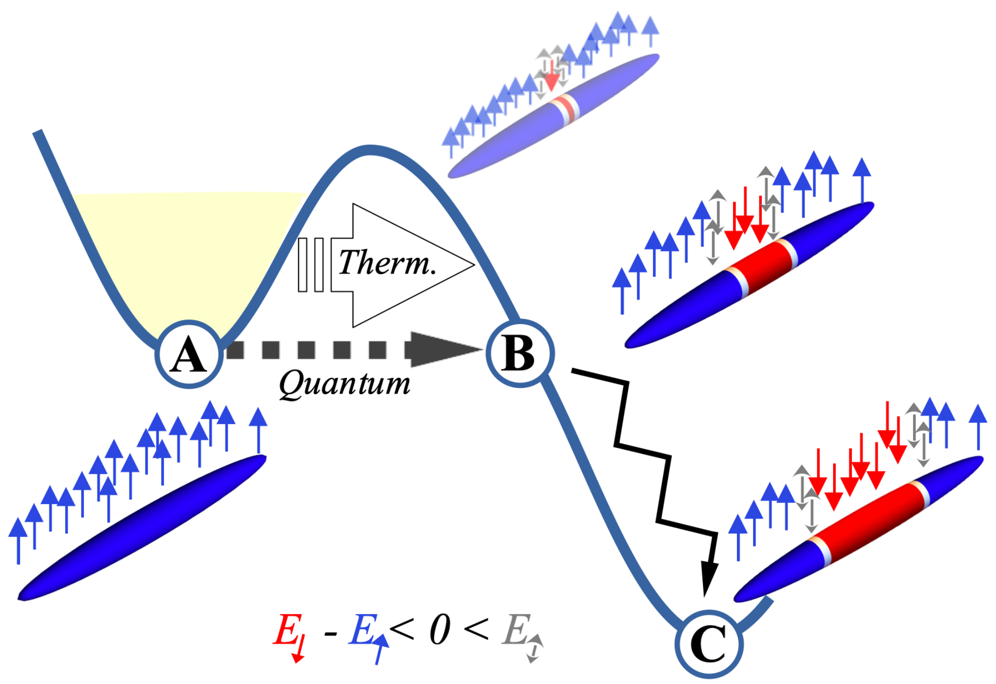
Theory (c)-(d) and purple points: noisy/dissipative GPE - Truncated Wigner

Metastability and Bubble Creation

The magnetic sector can be described by a scalar field theory with (in the ferromagnetic regime) a potential with a local and global minimum

$$E_c = \frac{\hbar n}{4} \int \left\{ \frac{\hbar}{2m} \frac{(\nabla Z)^2}{1 - Z^2} + V \right\} dx$$

$$V = \kappa n Z^2 - 2\Omega (1 - Z^2)^{1/2} - 2\delta_{\text{eff}} Z$$



Is it possible to study the decay to the ground state by starting in the metastable one?
Or to use Coleman wording to study the False vacuum decay?

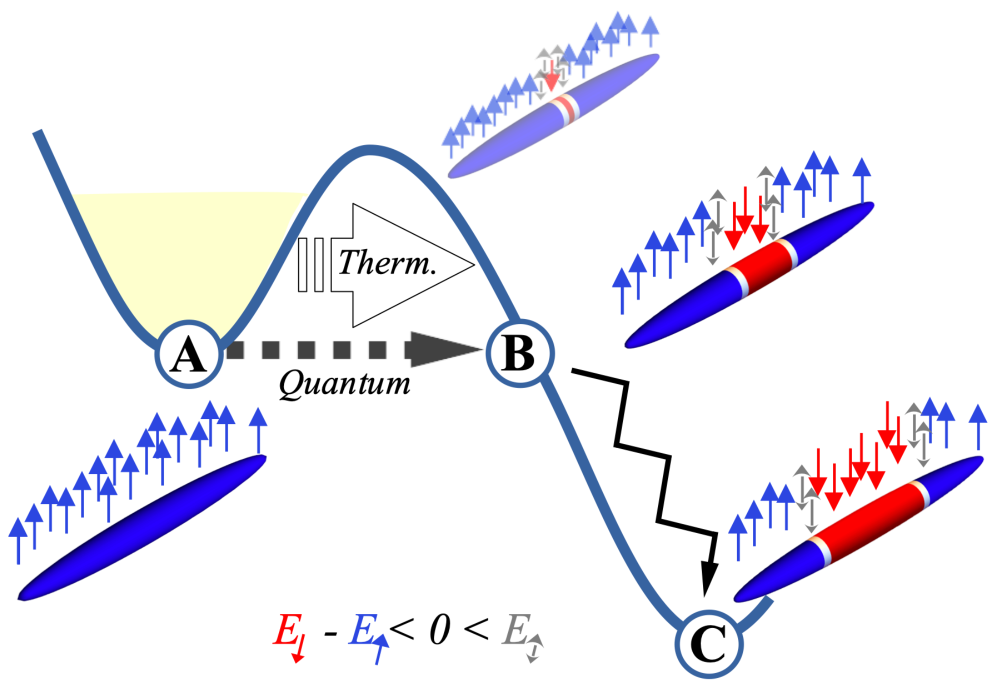
[S. Coleman, PRD 15, 2929 (1977); C. G. Callan and S. Coleman, PRD 16, 1762 (1977); A. D. Linde, Nucl. Phys.B 216, 421 (1983)]

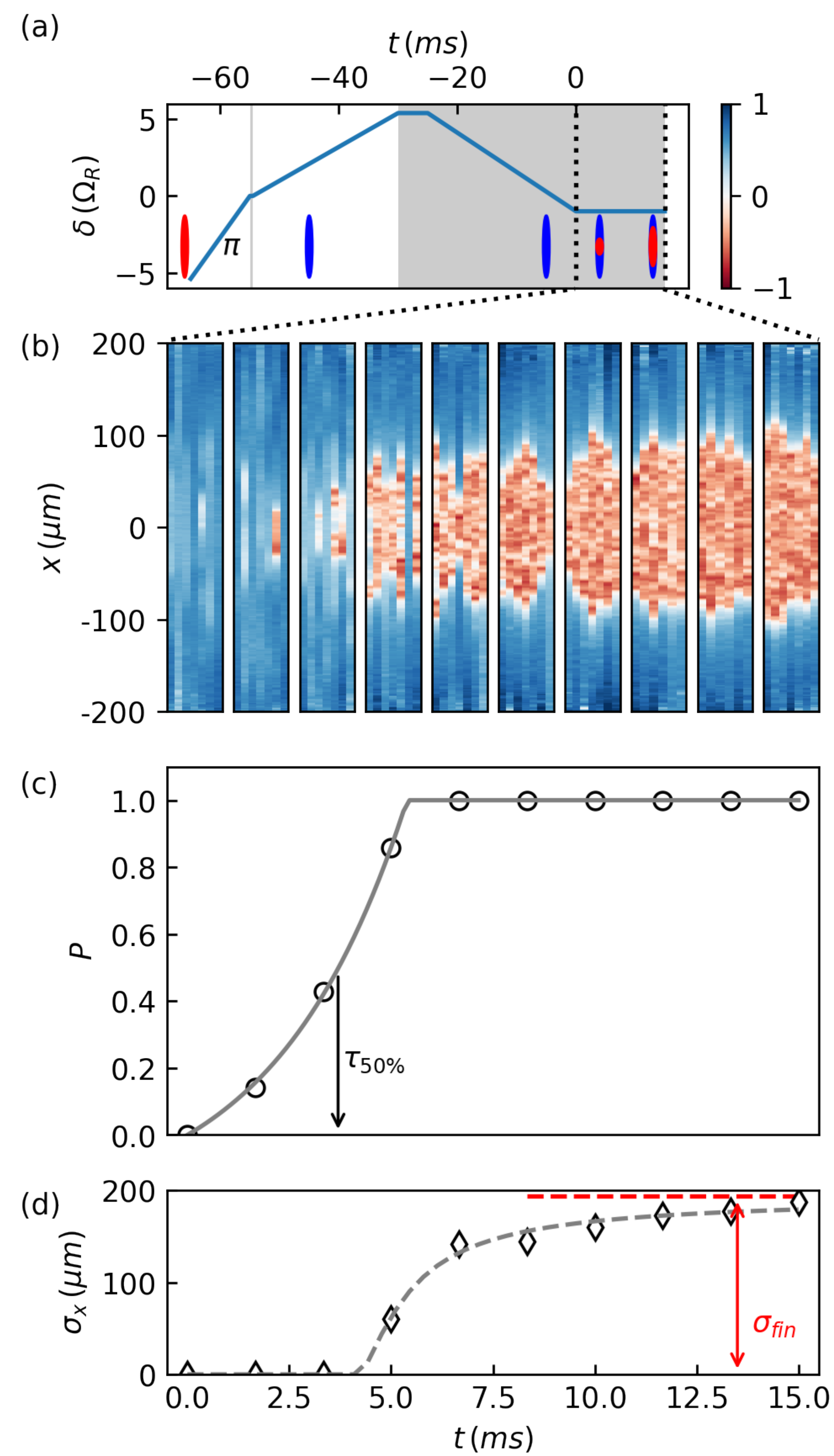
Metastability and Bubble Creation

The magnetic sector can be described by a scalar field theory with (in the ferromagnetic regime) a potential with a local and global minimum

$$E_c = \frac{\hbar n}{4}\int\left\{\frac{\hbar}{2m}\frac{(\nabla Z)^2}{1-Z^2} + V\right\}dx$$

$$V = \kappa n Z^2 - 2\Omega(1-Z^2)^{1/2} - 2\delta_{\rm eff} Z$$





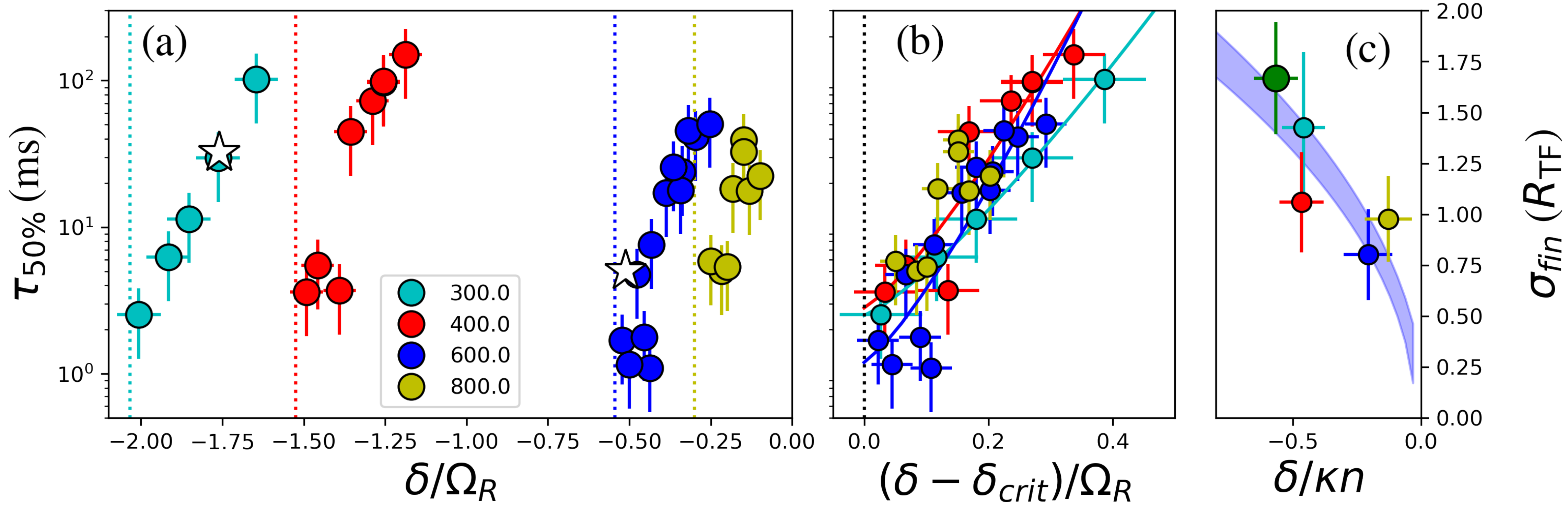
Is it possible to study the decay to the ground state by starting in the metastable one?
Or to use Coleman wording to study the False vacuum decay?

[S. Coleman, PRD 15, 2929 (1977); C. G. Callan and S. Coleman, PRD 16, 1762 (1977); A. D. Linde, Nucl. Phys.B 216, 421 (1983)]

Metastability and Bubble Creation

From the general of False Vacuum Decay the exponential behaviour of the decay rate can be inferred

$1/\tau \sim e^{\tilde{E}c/T}$ with $\tilde{E}_c$ the critical (istanton) bubble field configuration



NOTE:
we indeed observe orders of magnitude change in the bubble formation

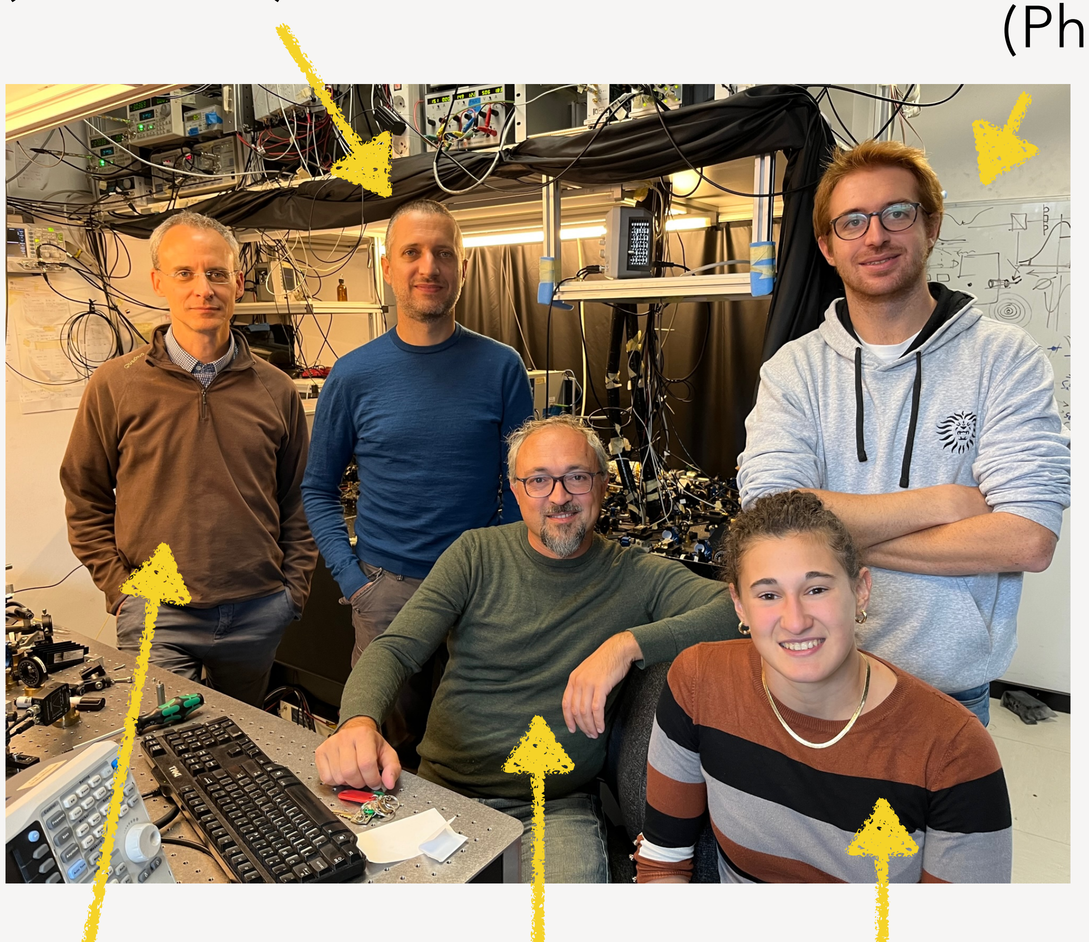
DISCLAIMER:
instanton theory provides only the qualitative behaviour
(Temperature, quantum fluctuations, noise, complex field…still a lot to do)

The Trento team on Coherently Coupled Bose Gases

Experimental group

Alessandro Zenesini (INO-CNR)

Riccardo Cominotti (PhD)



Gabriele Ferrari (UNITN)

Giacomo Lamporesi (INO-CNR)

Chiara Rogora (PhD)

Theory group

Albert Gallemì (postdoc) -> Hannover

Arko Roy (postdoc) -> IIT Mandi

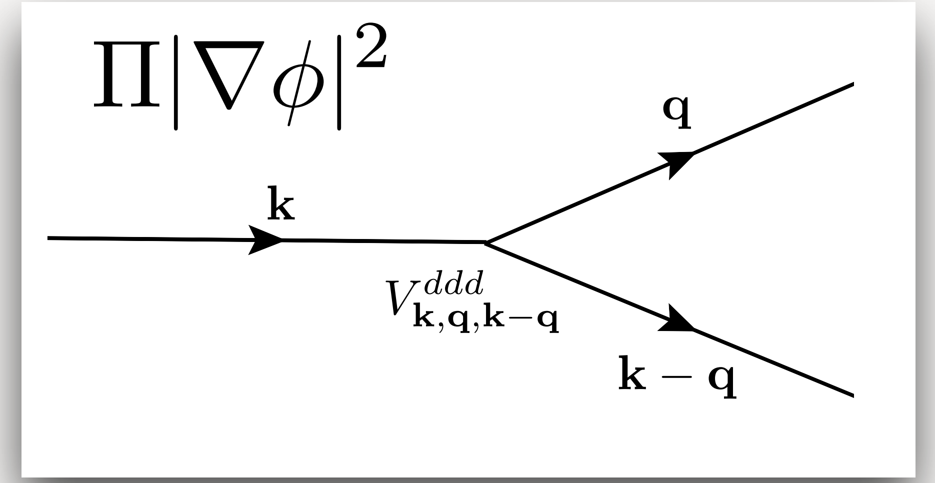
Anna Berti (PhD)

Iacopo Carusotto (INO-CNR)

[Exp+Theory: Nat. Phys. 17, 1359 (2021), PRA 104, 023326 (2021), PRL 128, 210401 (2022), arXiv:2209.13235]

Beyond Mean Field Effect:
anomalous Goldstone mode damping and “tunable” LHY correction

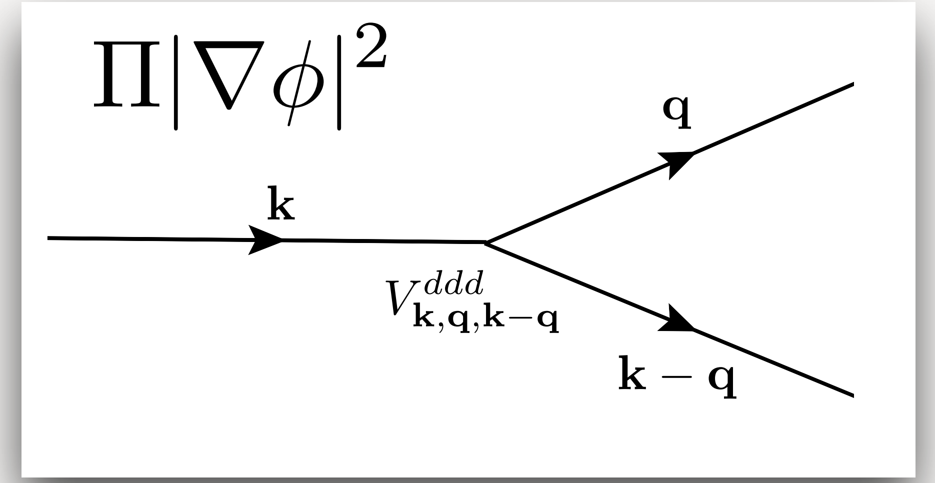
Beliaev Decay across the phase transition



Bogolyubov modes have finite life time.
For a single component gas
the lowest order decaying processes is due to a three phonon vertex leading to Beliaev decay

$$\Gamma(\mathbf{k}) \propto k^5$$

Beliaev Decay across the phase transition



Bogolyubov modes have finite life time.
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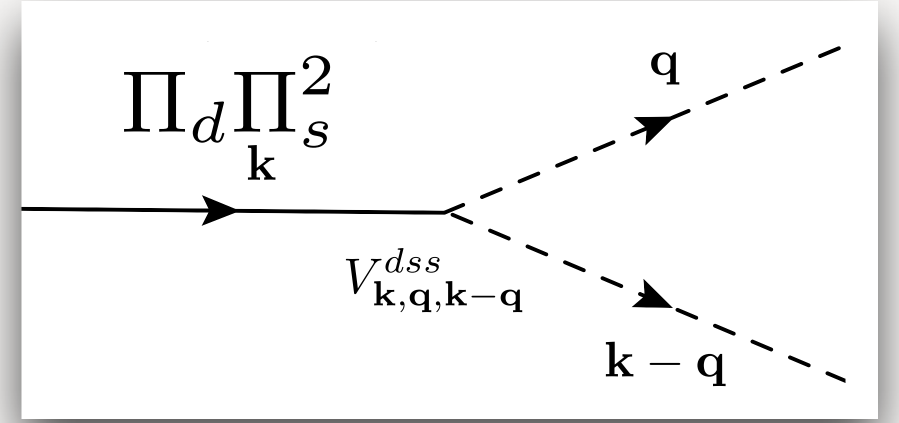
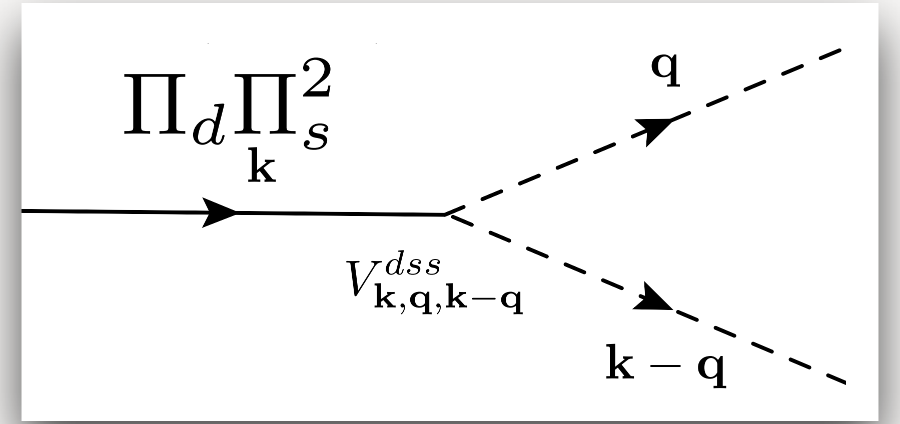
$$\Gamma(\mathbf{k}) \propto k^5$$

For a two-component gas more process are possible.
What happens to the Goldstone (in-phase Bogolyubov mode) close to the FM transition?



Due to symmetries the possible term are not that many and it is easy to find out that the most relevant one is $\Pi_d\Pi_s^2$
(density in 2 spin modes):

Beliaev Decay across the phase transition



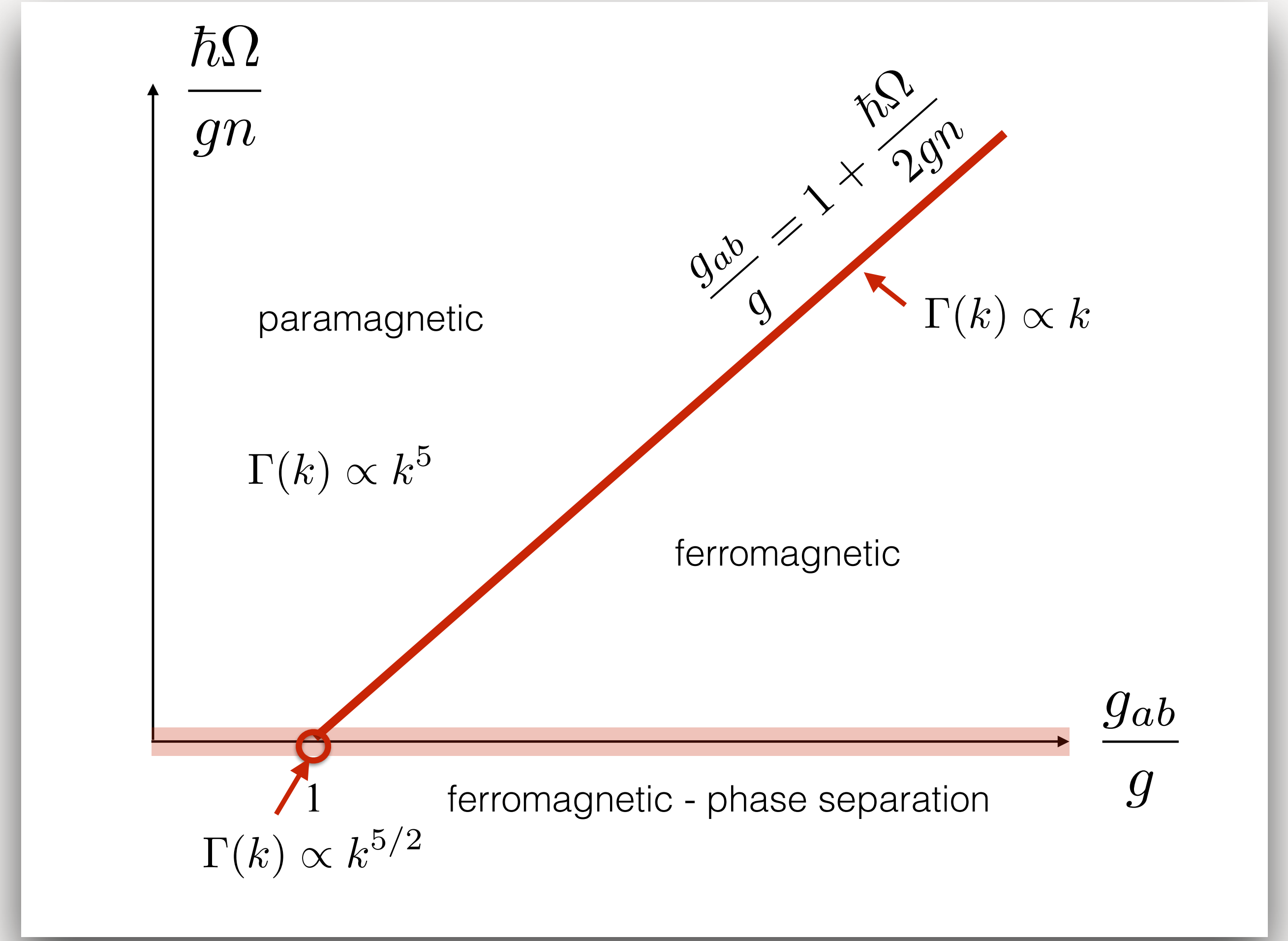
$$\Gamma(\mathbf{k}) = \frac{\mathrm{Pm}_V}{(2\pi)^2}\int d^3\mathbf{q}|V_{\mathbf{k},\mathbf{q},\mathbf{k}-\mathbf{q}}|^2\delta(\omega^d_k - \omega^s_q - \omega^s_{|\mathbf{k}-\mathbf{q}|})$$

For a mixture at the PS point the Goldstone mode is still well defined at low-*k*

$$\Gamma(\mathbf{k}) = \frac{(mc_d k)^{5/2}}{48nm\pi}.$$

For coh. coup. gases at the PT point the Goldstone mode is not well defined anymore at low-*k*

$$\Gamma(\mathbf{k}) = \frac{(mc_s)^4 k}{4nm\pi}$$



[AR and F. Piazza, PRB 99, 064505 (2019)]

LHY corrections

The LHY correction to the mean-field equation is obtained as by expanding the energy functional to second order in the fluctuations around the mean-field solution and considering the zero-point energy:

$$E_{\text{LHY}} = \frac{8}{15\pi^2}\left[\frac{(g+g_{\uparrow\downarrow})n}{2}\right]^{5/2} + \frac{2}{\pi^2}\left[\frac{(g-g_{\uparrow\downarrow})n}{2}\right]^{5/2}\int_0^1 \sqrt{x(1-x)\left[x+\frac{\Omega}{(g-g_{\uparrow\downarrow})n}\right]}\,dx$$

Due to the in-phase fluctuations: like LHY for a single component

Due to the spin fluctuations: the gap introduce a new scaling.

1. Without Rabi one obtains standard LHY just from two independent modes
2. For $\Omega \gg (g-g_{\uparrow\downarrow})n$ strong change. ***From a 2.5-body to 2+3-body*** interaction:

$$\frac{\sqrt{\Omega}}{2\sqrt{2}\pi}\left(\frac{g-g_{ab}}{2}n\right)^2 + \frac{1}{8\sqrt{2\Omega}\pi}\left(\frac{g-g_{ab}}{2}n\right)^3$$

Both terms are exactly equal to a coupled STM calculation for 2 and 3 atoms. LHY correction is much easier and therefore it could be even used to get results for 2 and 3 body interaction strengths in more general cases.

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Therefore if the interspecies interaction is $g_{ab} = -g < 0$ the mean-field energy as well as the LHY for the in-phase vanishes and the EoS reads:

$$\frac{\sqrt{\Omega}}{2\sqrt{2}\pi}\left(-g_{ab}n\right)^2+\frac{1}{8\sqrt{2\Omega}\pi}\left(-g_{ab}n\right)^3$$

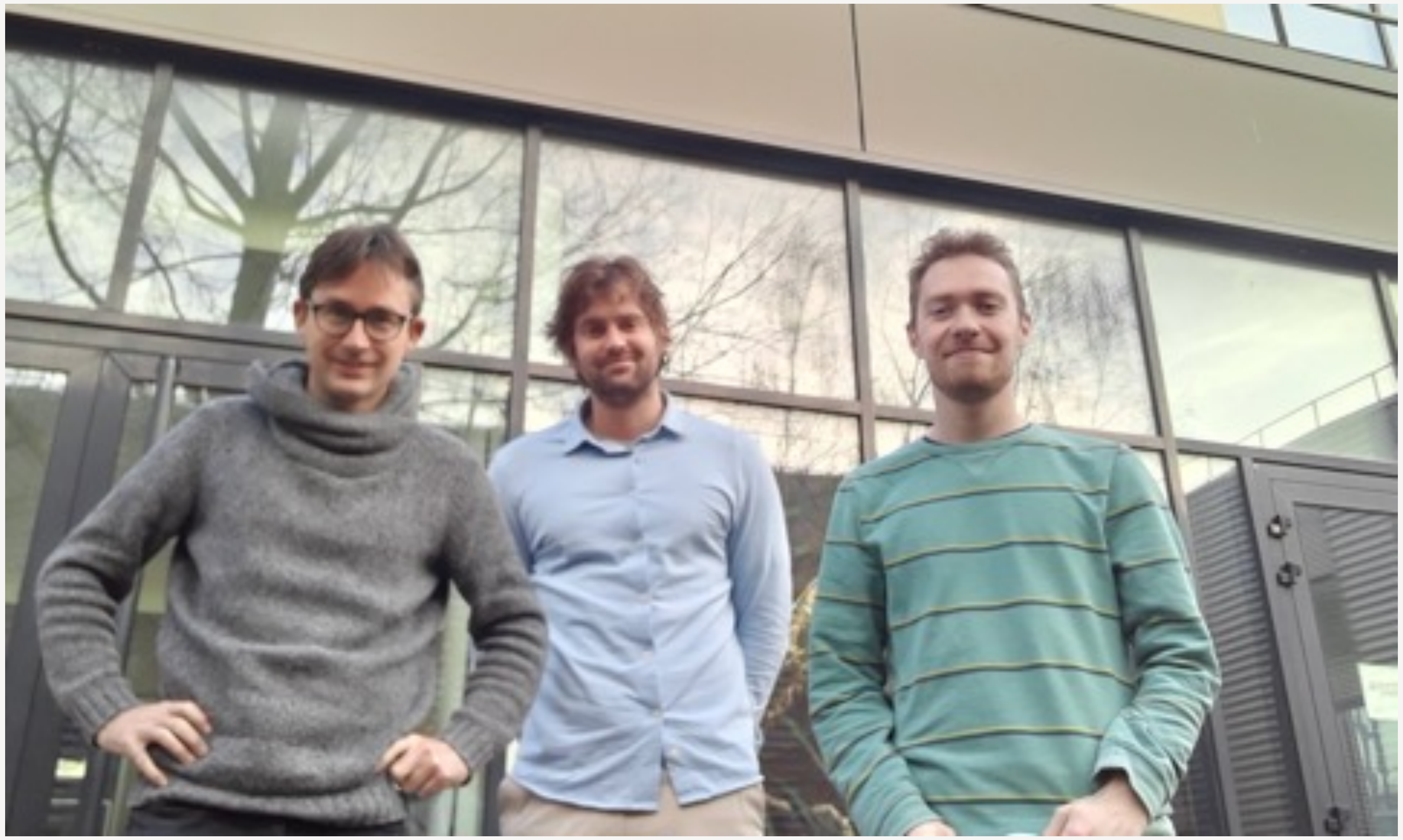
[L. Lavoine, A. Hammond, AR, D. Petrov, T. Bourdel, PRL 127, 203402 (2021)]

Beyond-Mean-Field Correction close to collapse

In collaboration with Dima Petrov (LPTMS, France)

Dmitry Petrov
(Orsay)

and
the Thomas Bourdel K-39 experimental group (Inst. Optique, France)

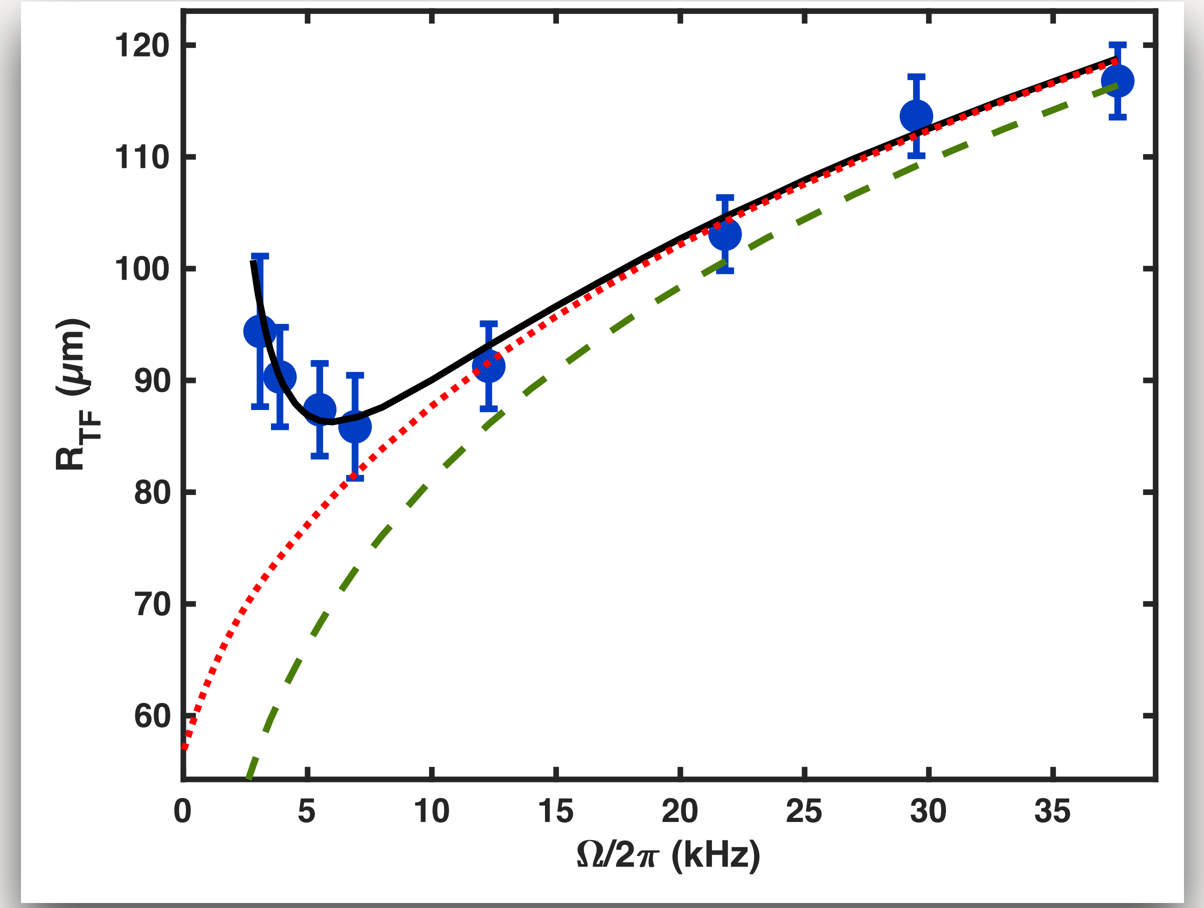


See also A. Cappellaro, F. Macri, G. Bertacco, and L. Salasnich,
Scientic Reports 7, 13358 (2017) for the discussion of the effect of coherent coupling on (Z_2 symmetric) droplets

Beyond-Mean-Field Correction

Experimental results vs extended Gross-Pitaevskii equation (eGPE)

Black line:
full LHY equation of state
in the eGPE
including experimental noise



$$E_{\rm BMF} \approx \frac{\sqrt{\tilde{\Omega}}}{2\sqrt{2}\pi} g_{\uparrow\downarrow}^2 \frac{n^2}{2} + \frac{3}{4\sqrt{2}\pi\sqrt{\tilde{\Omega}}} |g_{\uparrow\downarrow}|^3 \frac{n^3}{6}$$

[L. Lavoine, A. Hammond, AR, D. Petrov, T. Bourdel, PRL 127, 203402 (2021)]

Thanks!

For a recent review on coherently coupled Bose gases including spin-orbit coupling:

Alessio Recati and Sandro Stringari, Ann. Rev. Cond. Matter Physics 13, 407 (2022)

Vortices in coherently coupled BECs

Phase domain walls: simple picture [Son & Stephanov PRA '02]

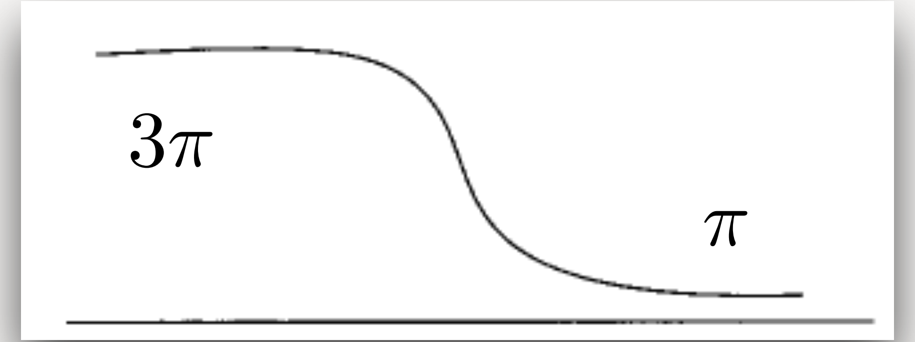
$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b + 2|\Omega| \cos(\phi_a - \phi_b)\sqrt{n_a n_b}$$

For fixed (equal) densities the functional energy of the relative phase reads:

$$E_{spin} = \int \left[\frac{\hbar^2 n}{4m} (\nabla \phi_s)^2 + 2\Omega n \cos(\phi_s) \right]$$

Global minimum for $\phi_s = (2n+1)\pi$

Domain wall or kink is a local minimum solution which connects 2 global minima



The kink surface tension reads

$$\sigma = \sqrt{\frac{8\hbar^3}{m} n \sqrt{\Omega}}$$

Vortices in coherently coupled BECs

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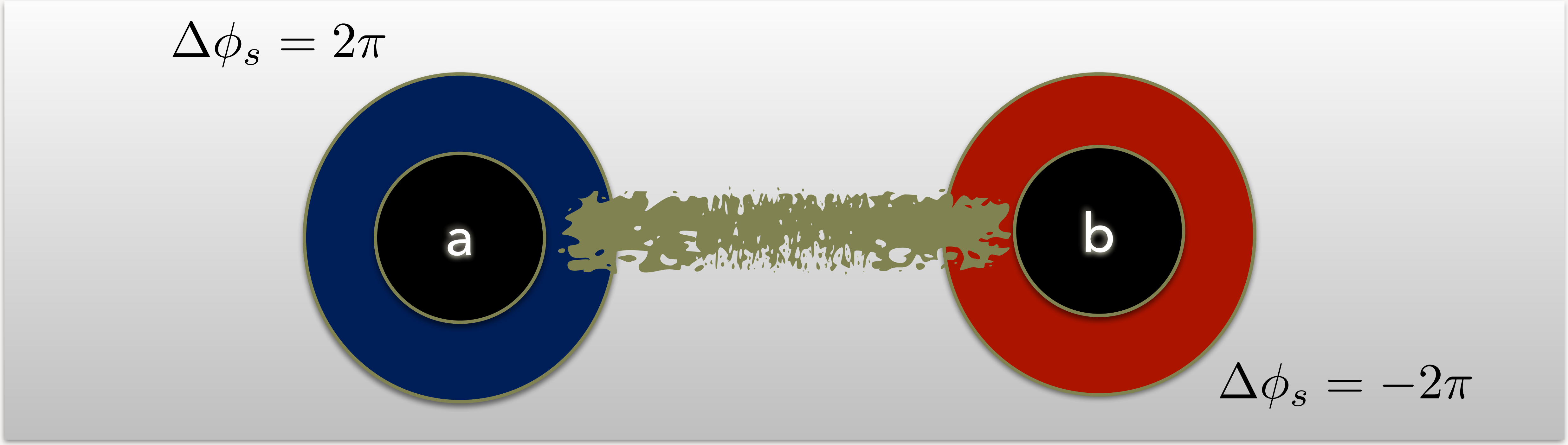

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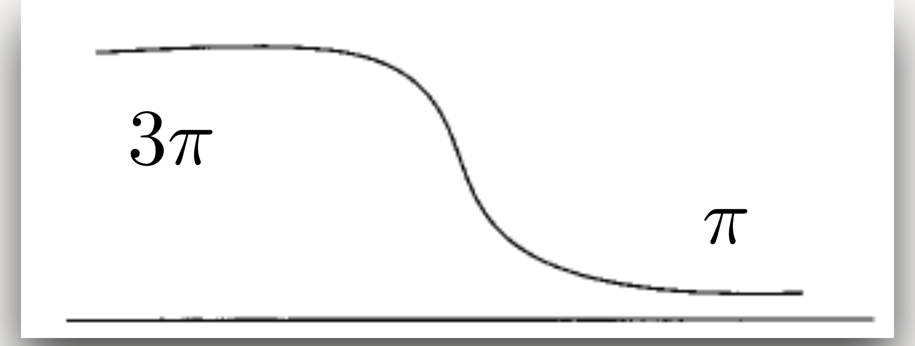

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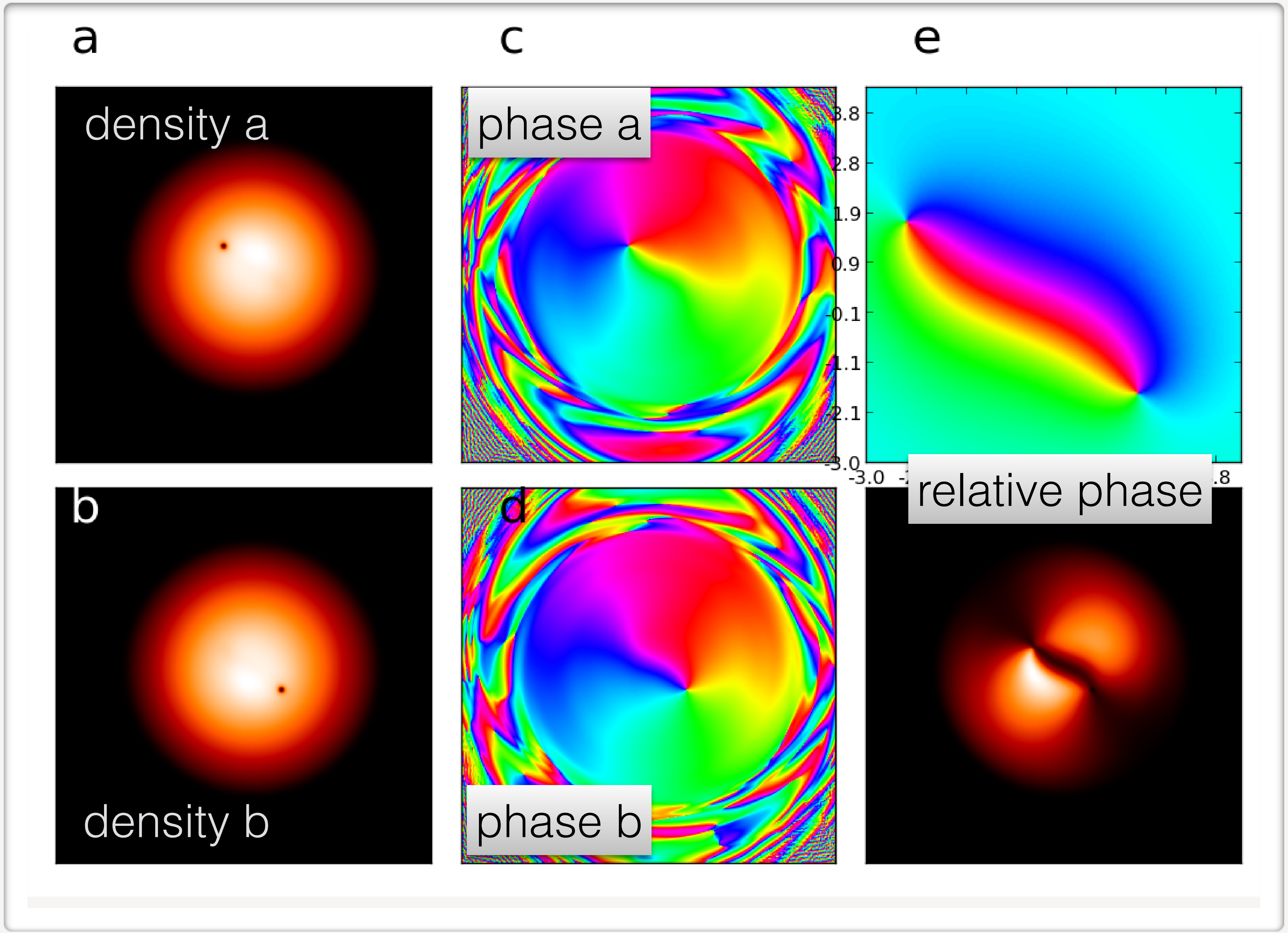
Vortices in coherently coupled BECs: dimers



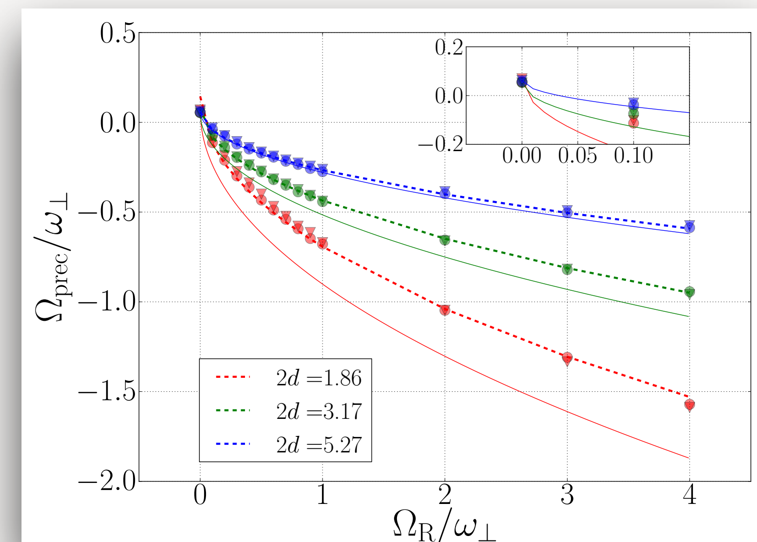
Tylutky, AR, Pitaevskii, Stringari, PRA (2016) -
see also K. Kasamatsu, M. Tsubota, and M. Ueda, PRL 93, 250406 (2004).

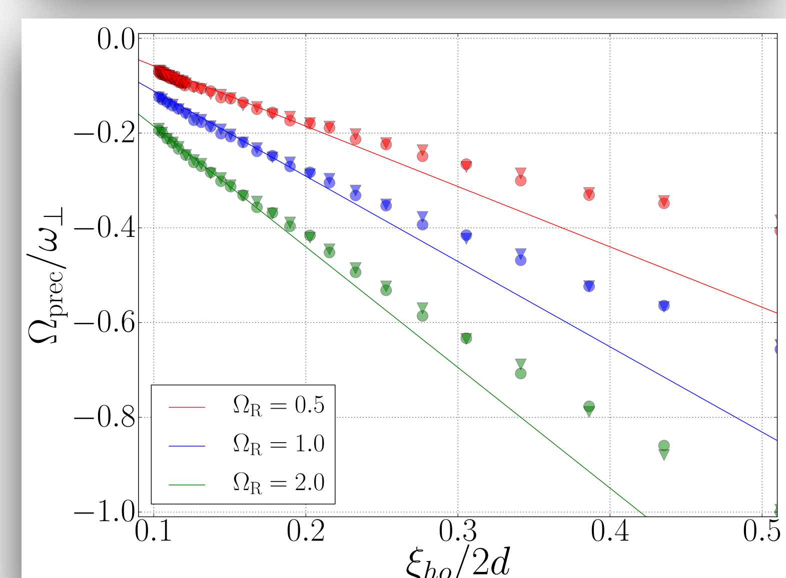
Vortices in coherently coupled BECs: dimers

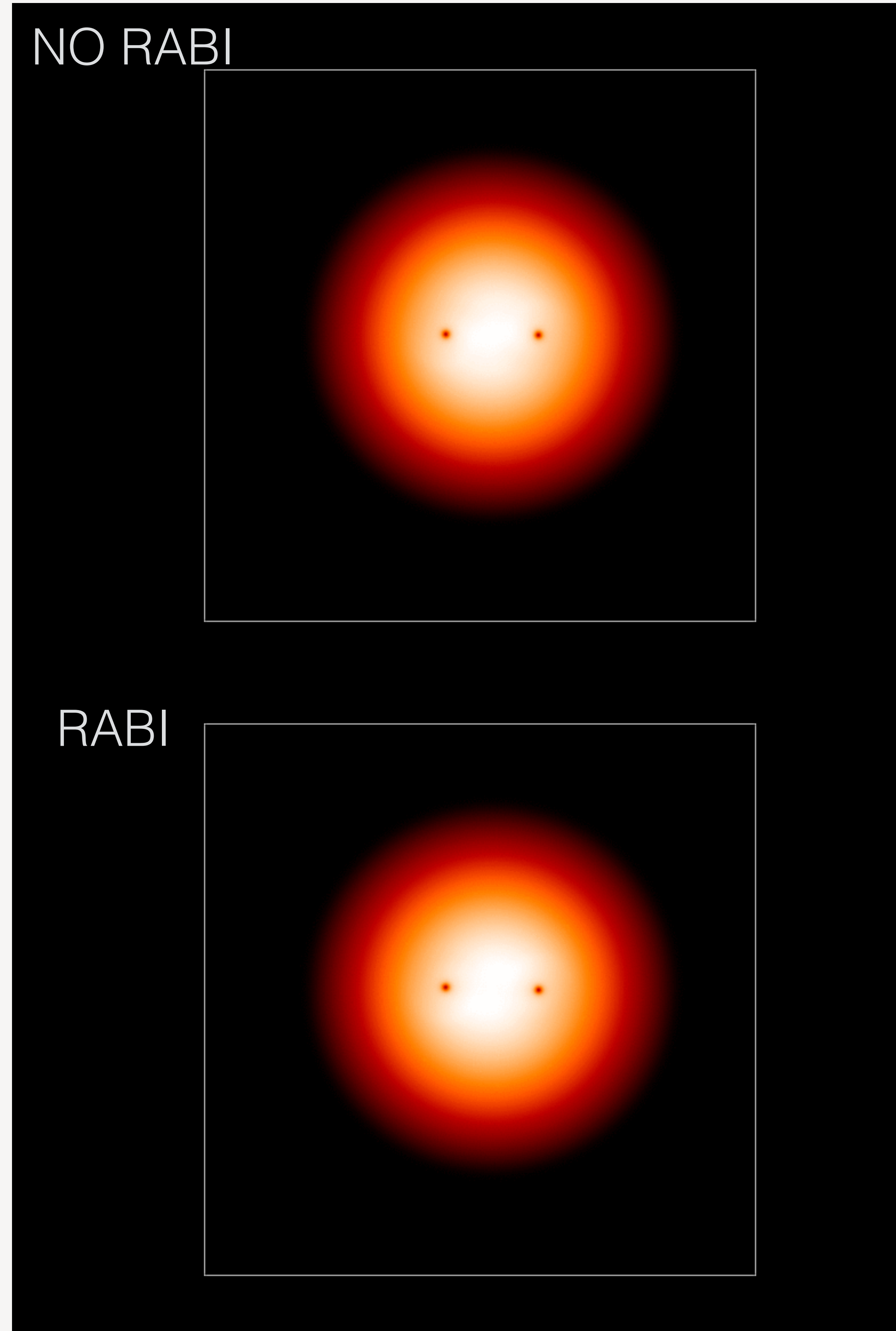
Magnus force = surface tension

$$n_s \mathbf{v}_l \times \kappa = -\sigma \propto -\sqrt{\Omega}$$

$$\omega_{rot} \propto \frac{\sqrt{\Omega}}{d}$$

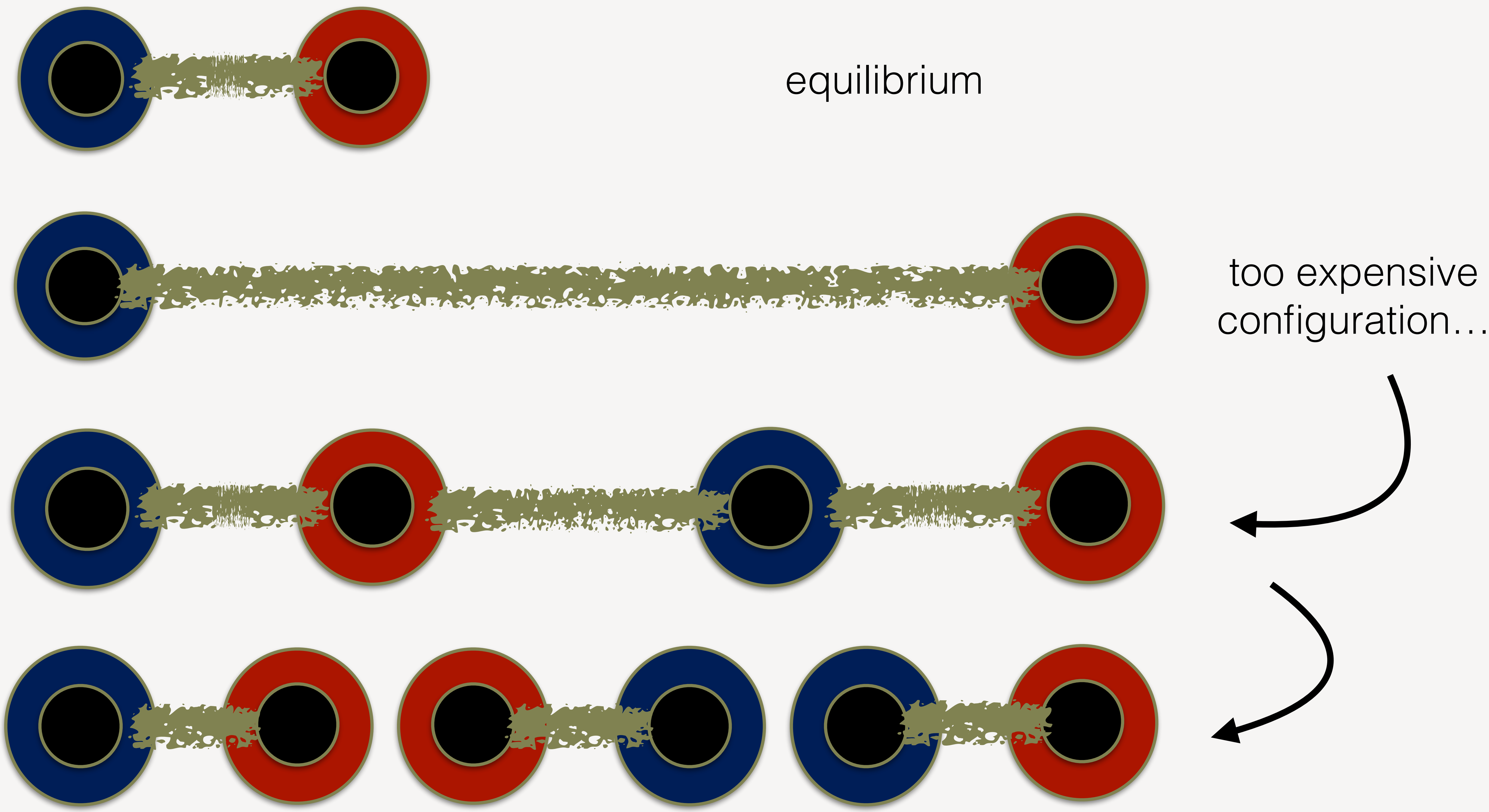




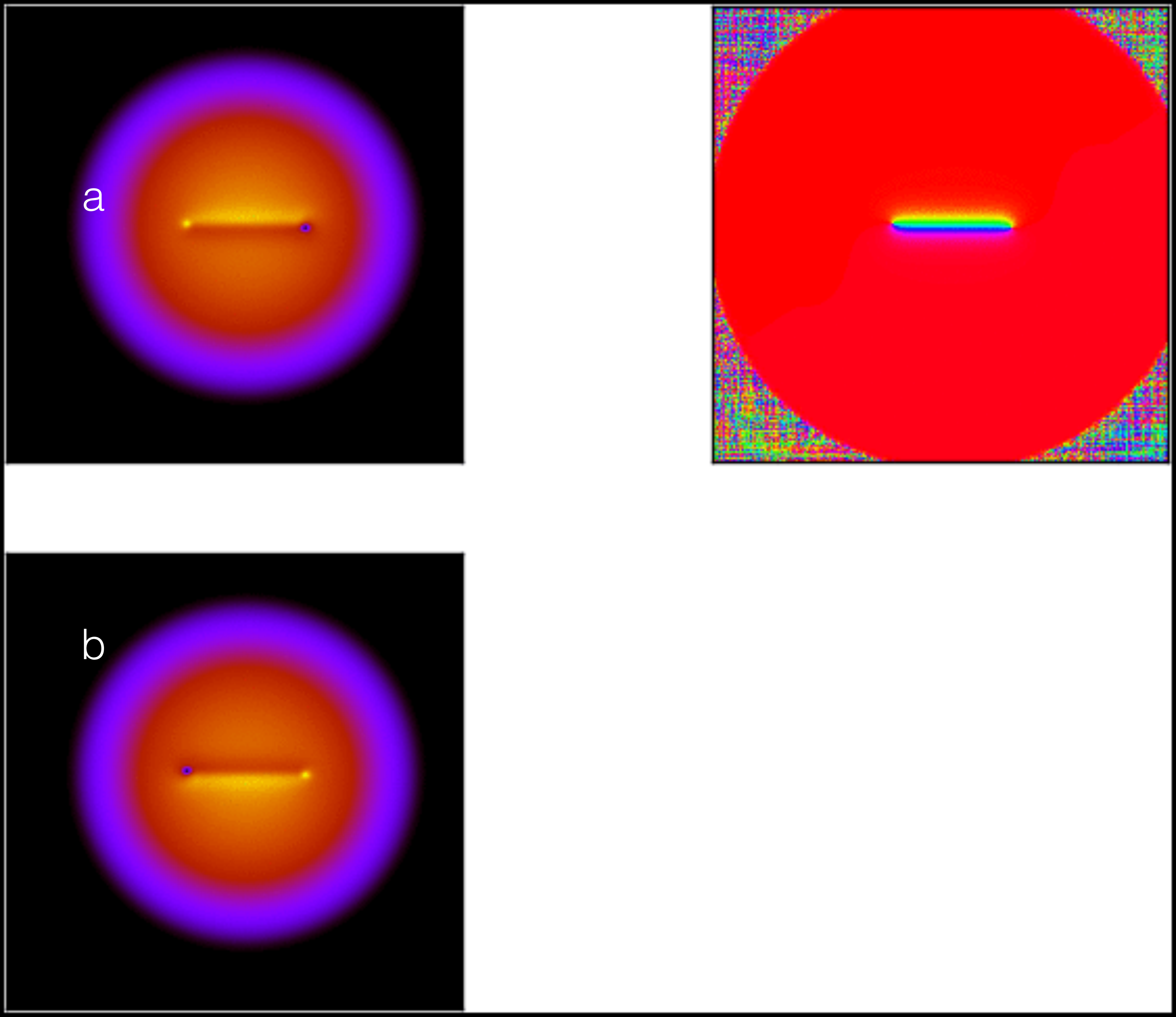


Tylutky, AR, Pitaevskii, Stringari, PRA (2016)

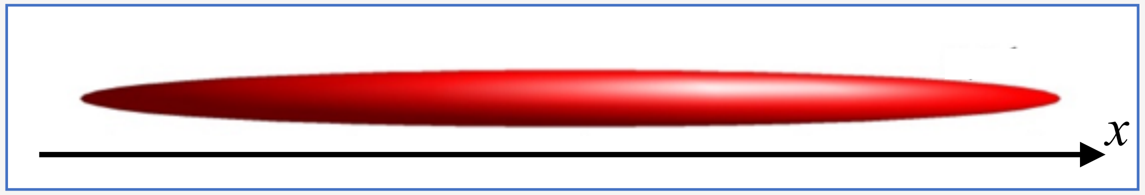
Vortices in coherently coupled BECs: string breaking



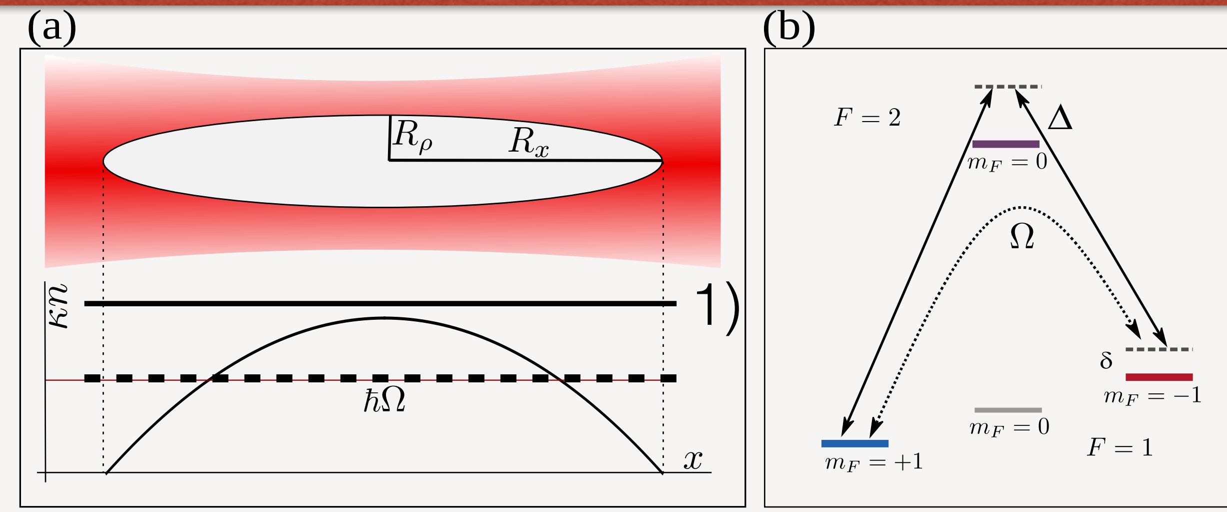
Vortices in coherently coupled BECs: string breaking



An elongated inhomogeneous system:
1) Local Josephson oscillations



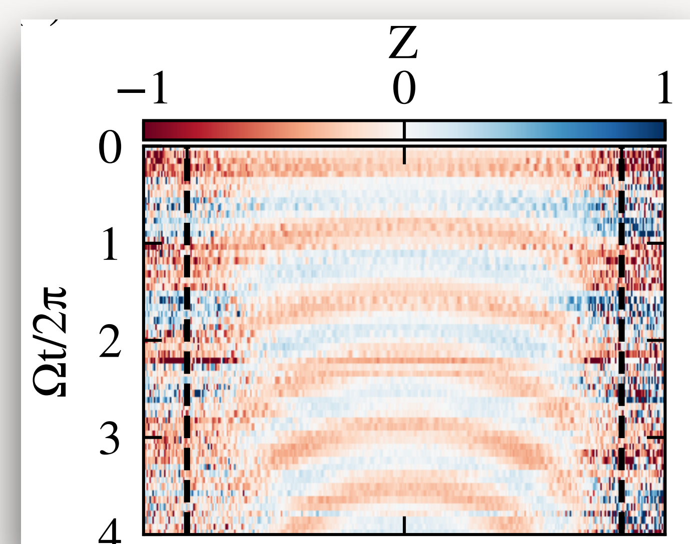
$N = 3 \times 10^6$ ^{23}Na

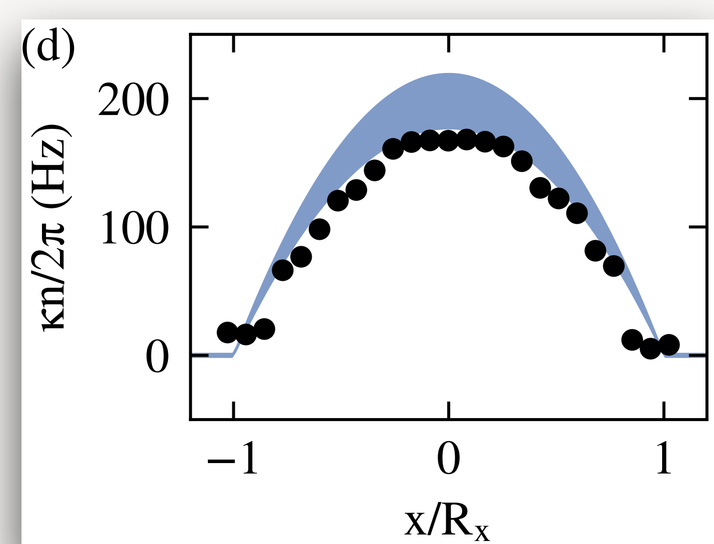


1) We first prove the local BJJ in an elongated inhomogeneous cloud in the Josephson oscillation regime. 1D spin dynamics with Thomas-Fermi profile and "large" Rabi coupling

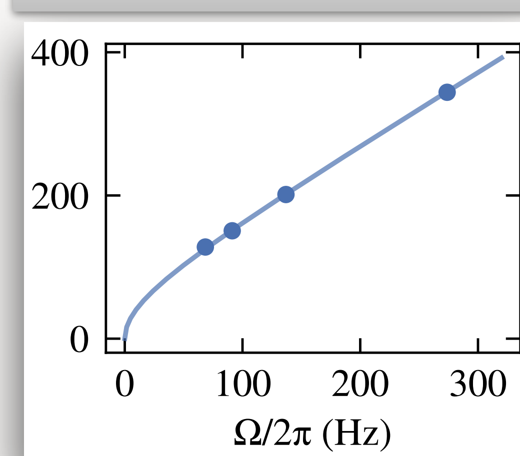
$$\dot{\mathbf{s}}(x) = \begin{pmatrix} \Omega \\ 0 \\ \delta + \kappa s_z(x) \end{pmatrix} \times \mathbf{s}(x)$$ in the Josephson regime $$\omega_J(x) = \sqrt{\Omega(\Omega + \kappa n(x))}$$

The parameter κ is determined experimentally from the local Josephson oscillation frequency (black dots) and from the loa





Central JJ frequency as a function of Rabi coupling

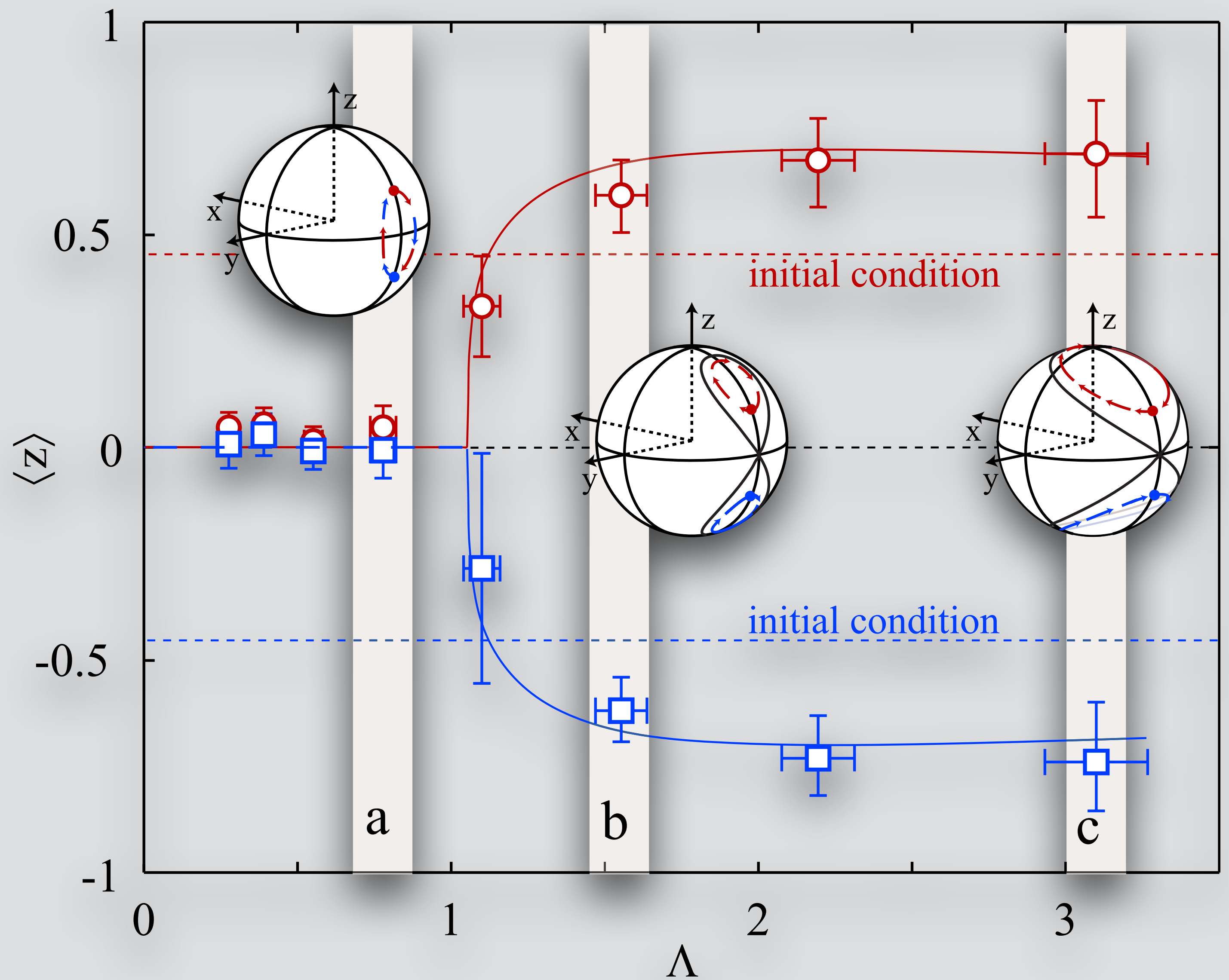


Coherently coupled BEC and Josephson junction dynamics

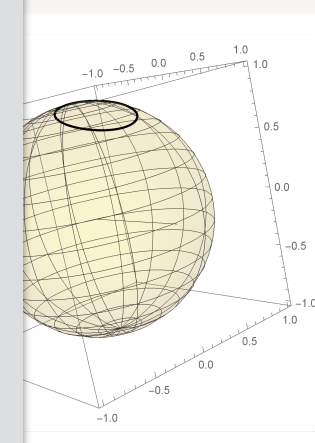
Bose-JJ eqs. $\dot{\mathbf{s}} = \mathbf{H}(\mathbf{s}) \times \mathbf{s}$ $\quad \mathbf{H}(\mathbf{s}) = (\Omega, 0, 2g_{ss}s_z)$

1. the *(non-line*
   oscillation re

2. the so
   self-trapp

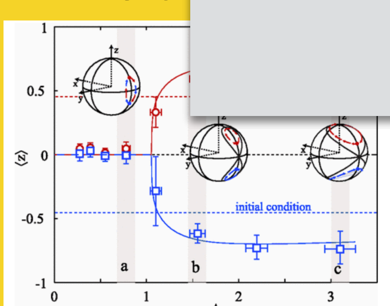


T. Zibold et al. PRL (2010) - in a "0-dimensional" system - Josephson physics

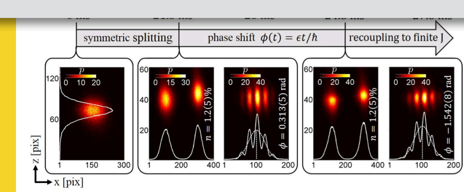


Experimen

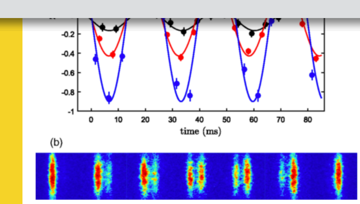
Heide



Phys. Rev. Lett. 95, 010402 (2005)
Phys. Rev. Lett. 105, 204101 (2010).

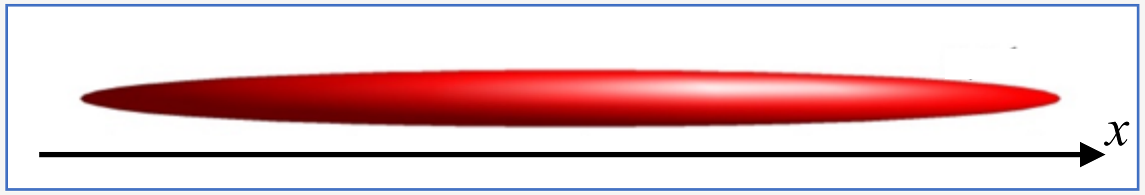


Phys. Rev. Lett. 120,173601 (2018).

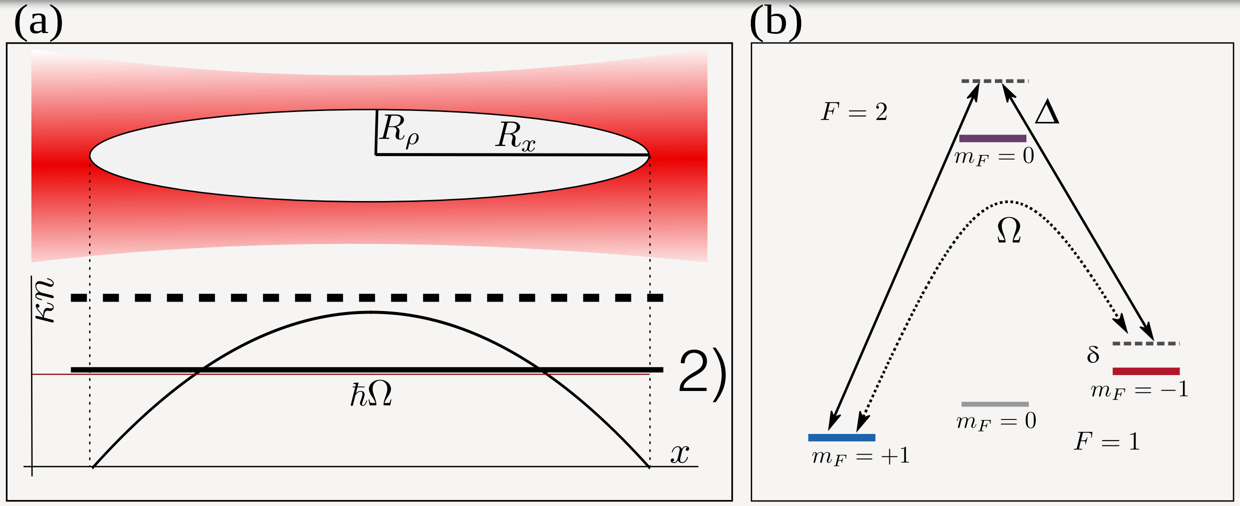


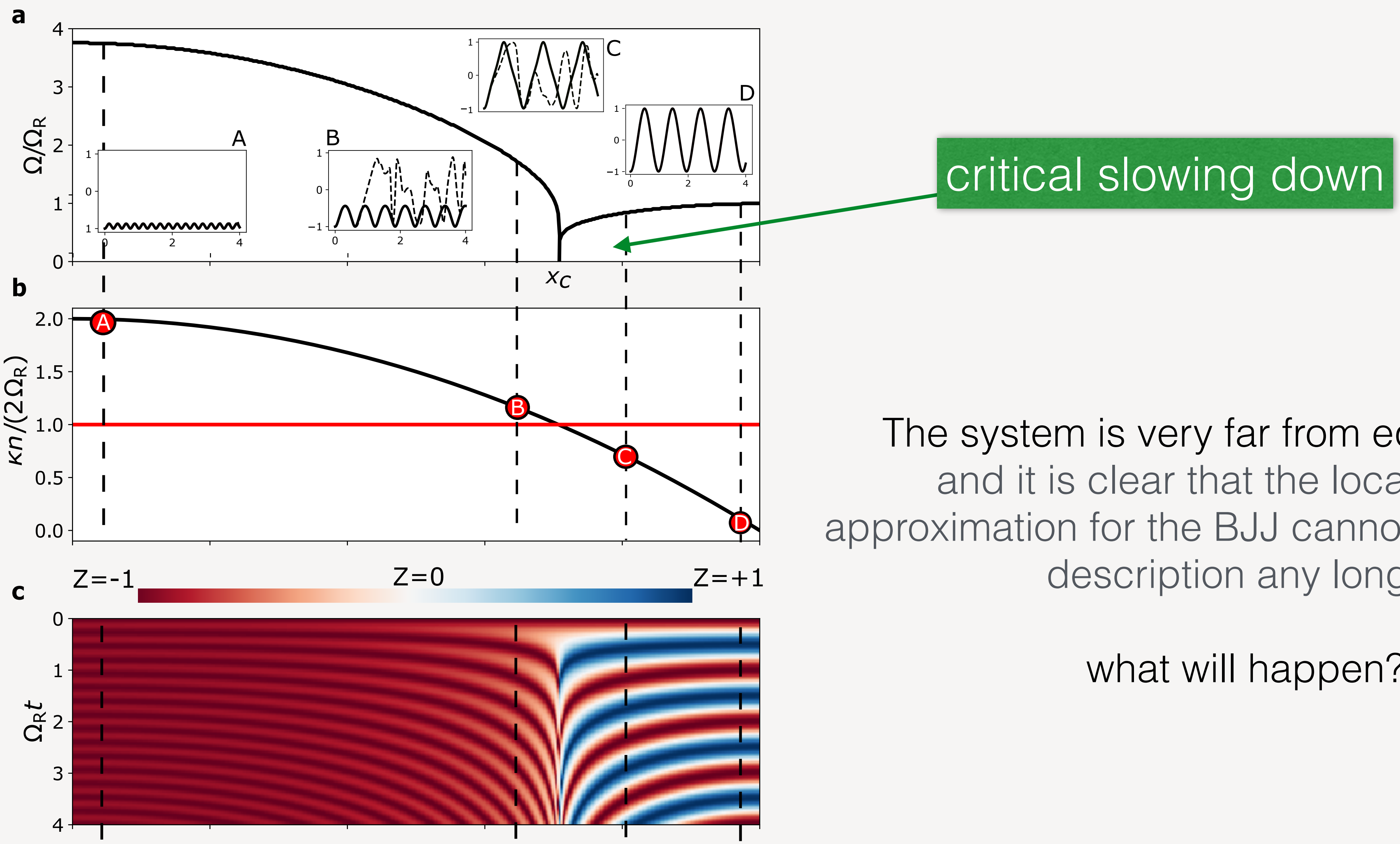
Phys. Rev. Lett. 118, 230403 (2017).

An elongated inhomogeneous system:
2) Critical region and Quantum-torque



^{23}Na

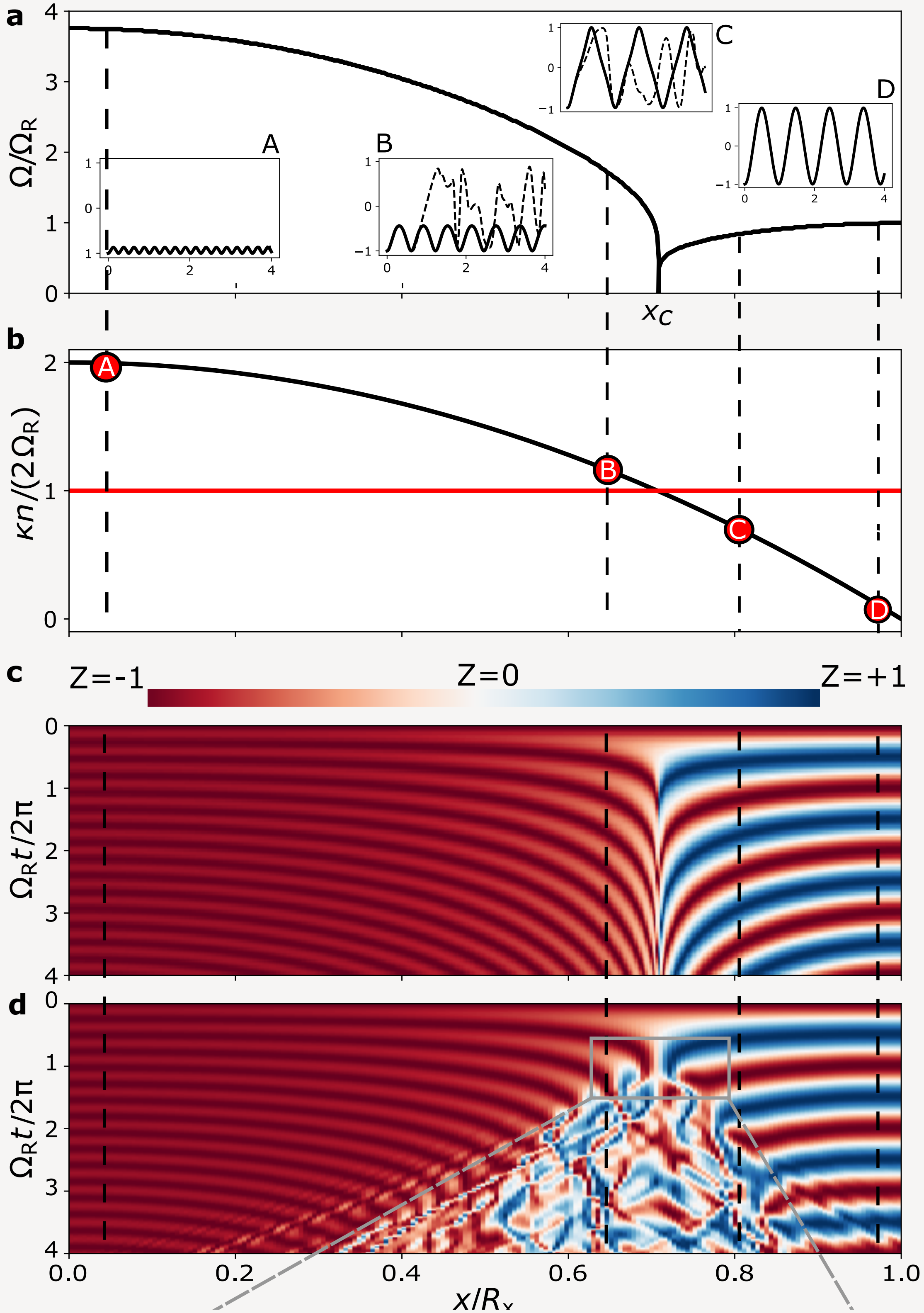




The system is very far from equilibrium…
and it is clear that the local density approximation for the BJJ cannot be the proper description any longer

what will happen?

Emulating Landau-Lifshitz equations



$$\dot{\mathbf{s}} = \mathbf{H}(\mathbf{s}) \times \mathbf{s} \qquad \mathbf{H}(\mathbf{s}) = (\Omega, 0, 2g_{ss}s_z)$$

the correct equation reads

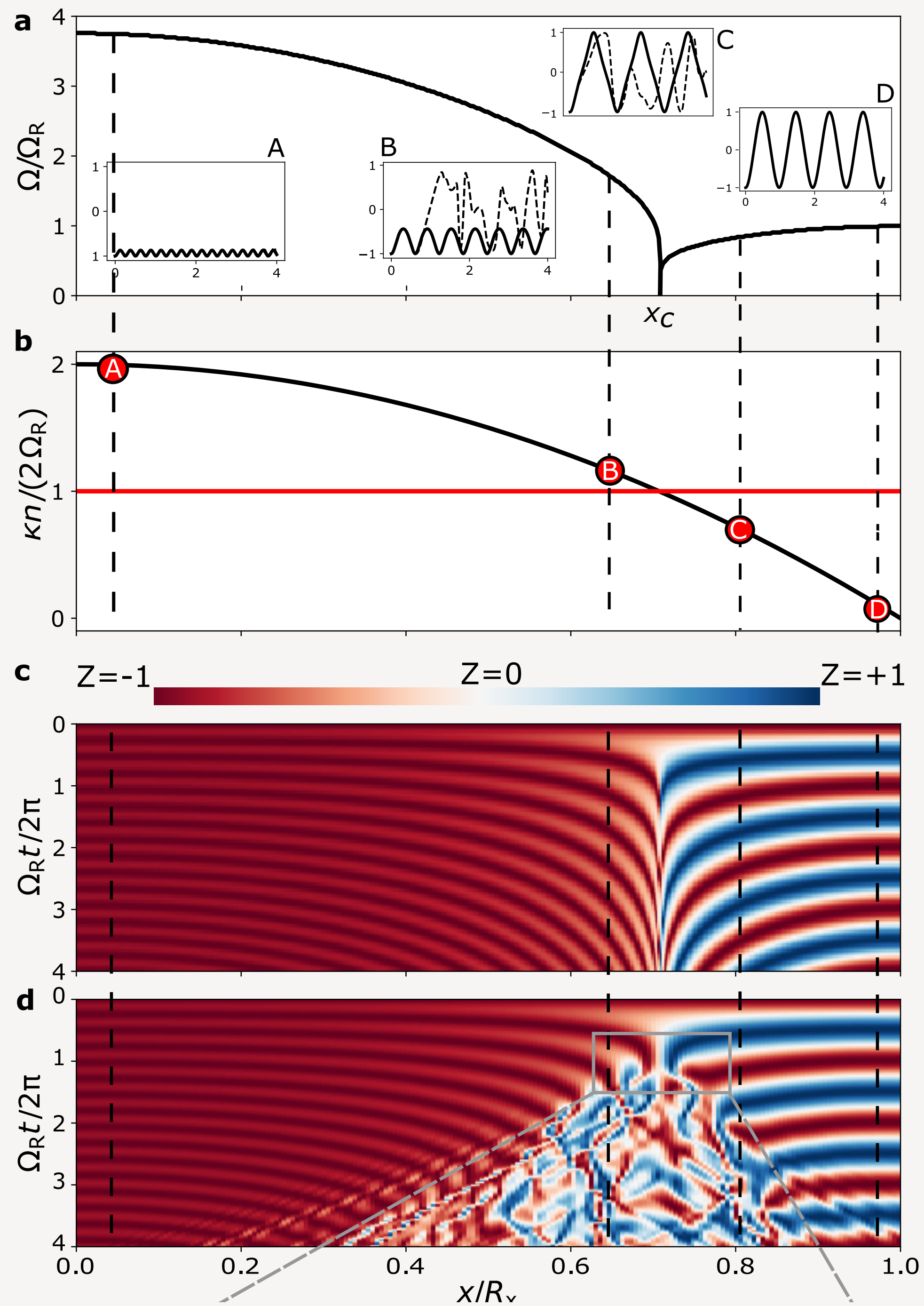
$$\dot{\mathbf{s}} + \partial_x \mathbf{j_s} = \mathbf{H}(\mathbf{s}) \times \mathbf{s}$$

Spin current $\mathbf{j_s} = v\mathbf{s} + \frac{\hbar}{2mn}\partial_x \mathbf{s} \times \mathbf{s}$

Dissipationless Landau Lifshitz Equation (LLE, 1935)

New Exp: An elongated inhomogeneous system

Local JJ



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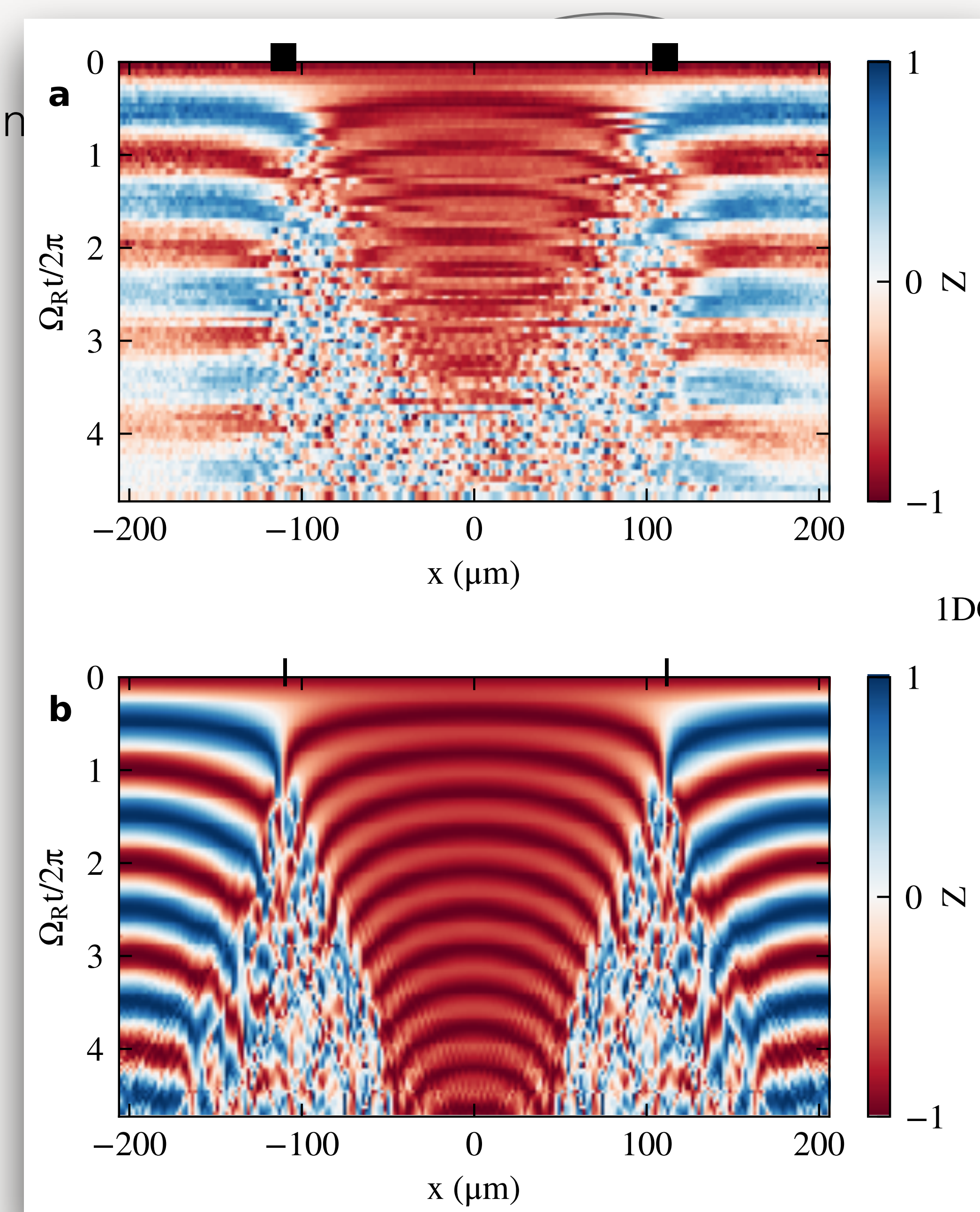
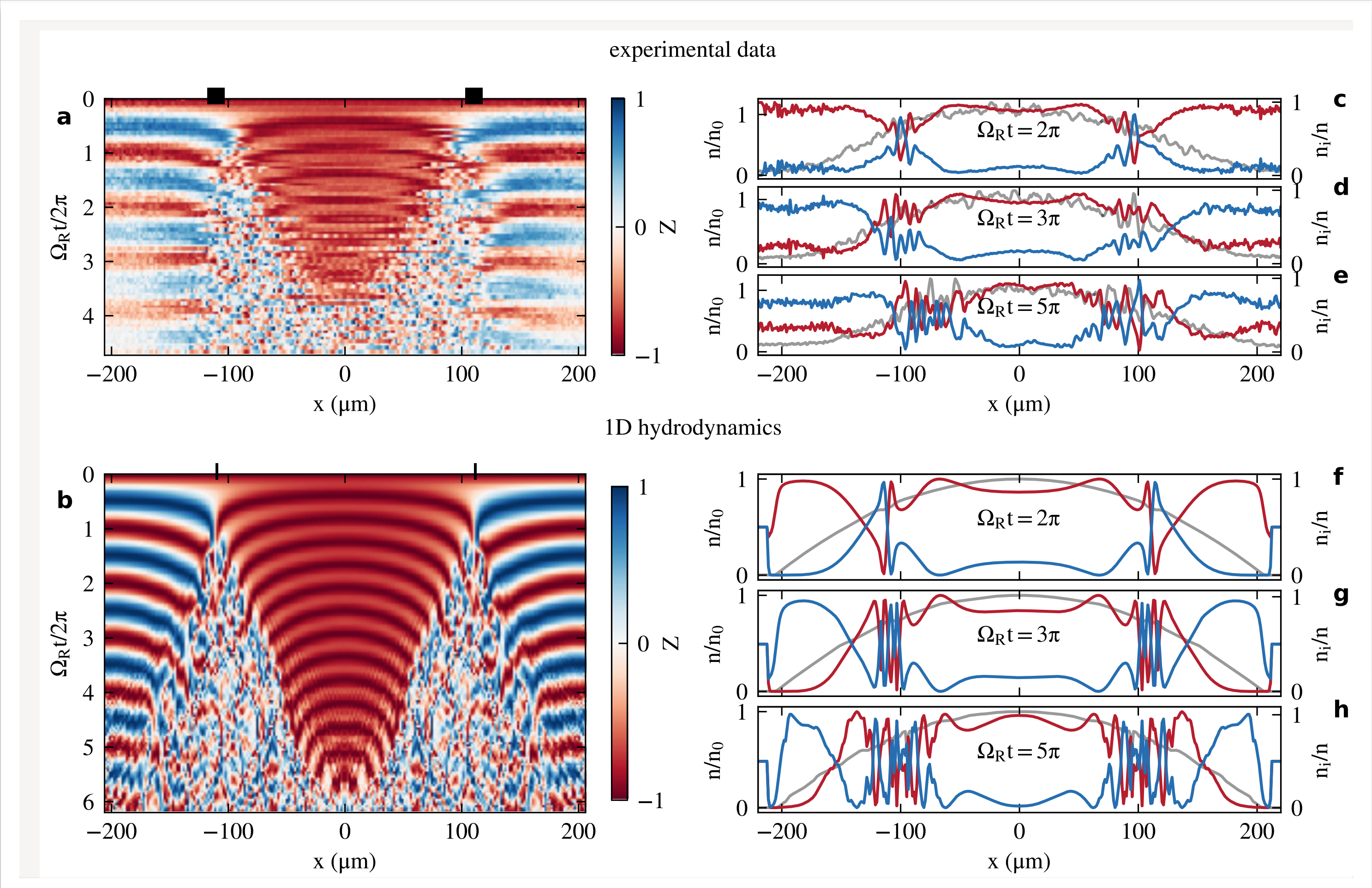
Spin

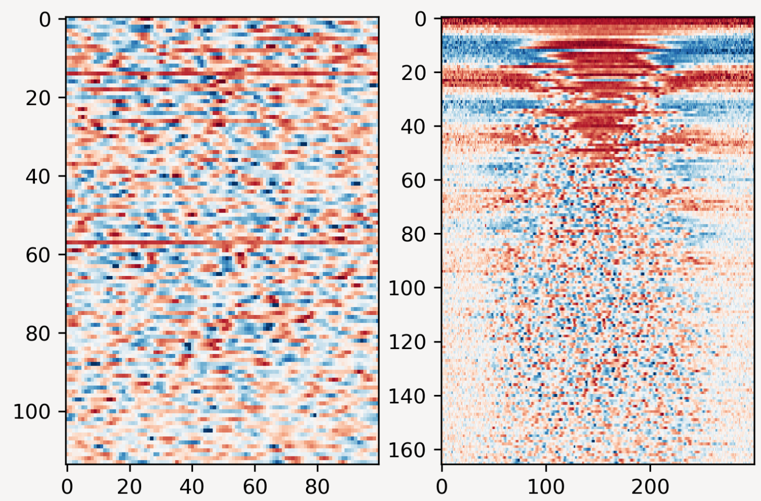


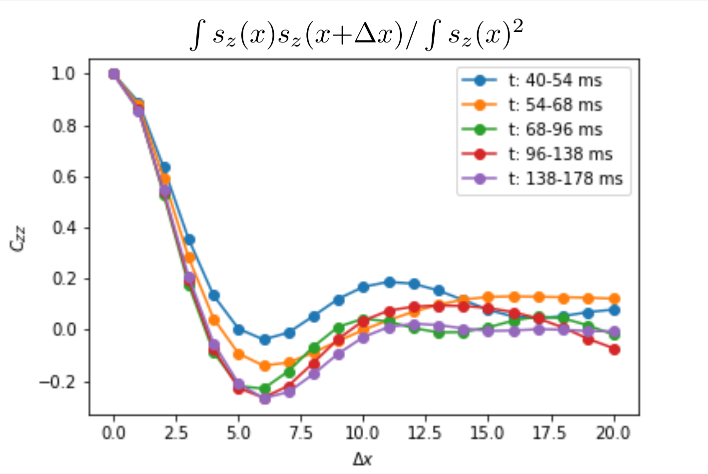
Equation of motion for the "hydrodynamic" quantities



Note: total density (and total current, not shown) pretty much constant, thus the LLE description can be safely applied

Question: What is generated by the torque?
Dispersive magnetic shock wave? (shape and constant front speed)
Turbulence?





The correlation looks very much as for a quenched anti-ferromagnet

Need more theoretical and experimental investigations…

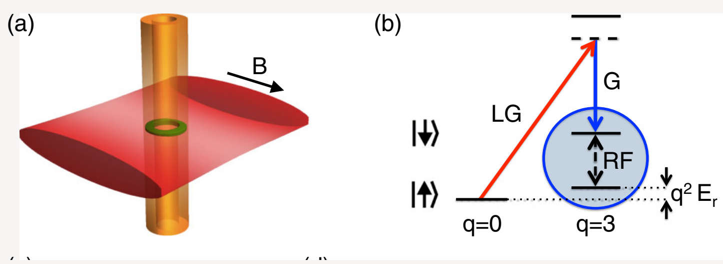
Supercurrent stability

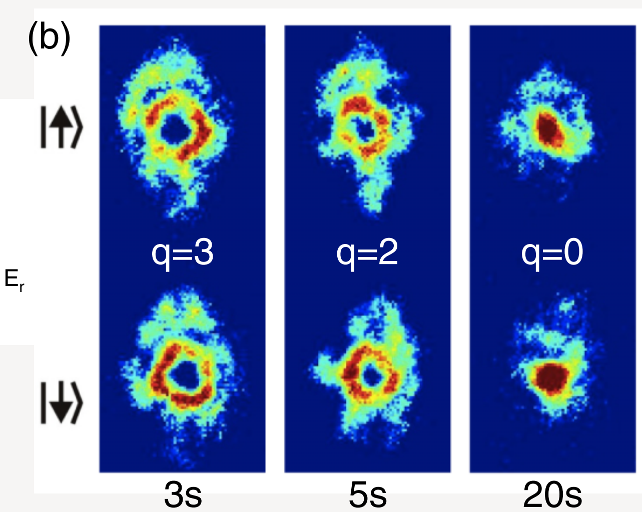
PRL 110, 025301 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

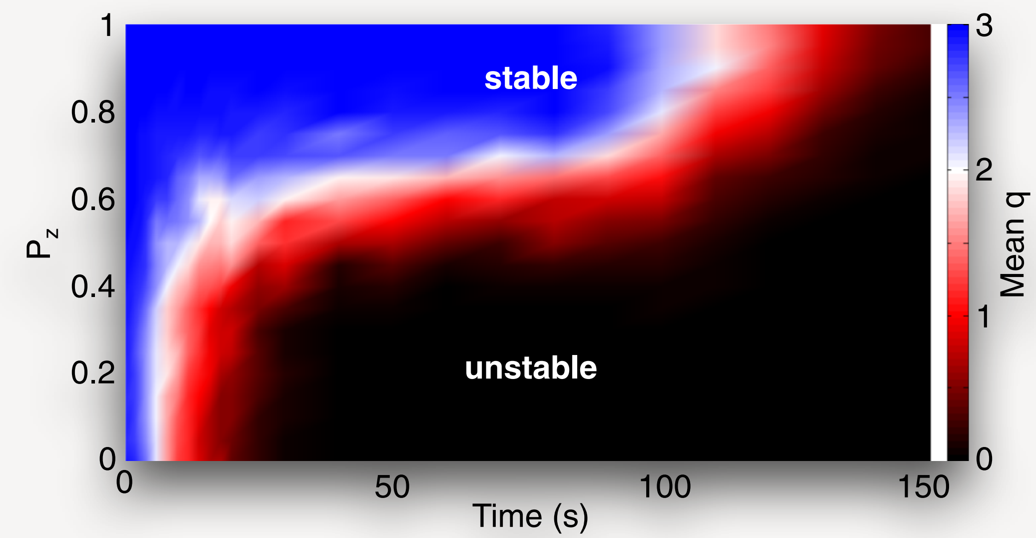
Persistent Currents in Spinor Condensates

Scott Beattie, Stuart Moulder, Richard J. Fletcher, and Zoran Hadzibabic

Experiment

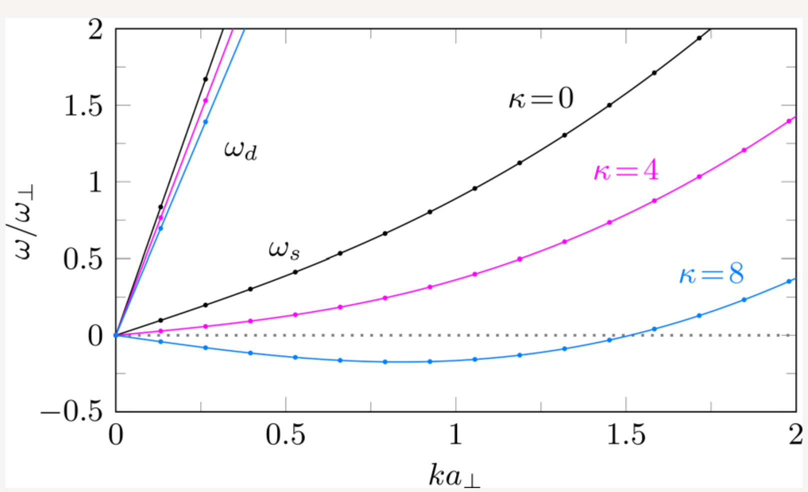




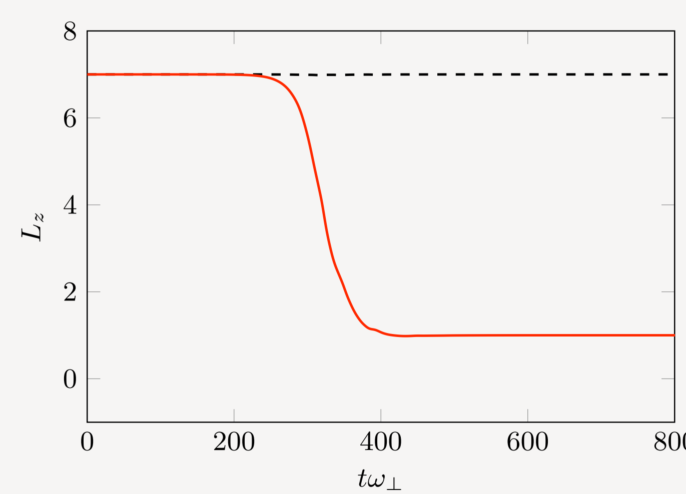


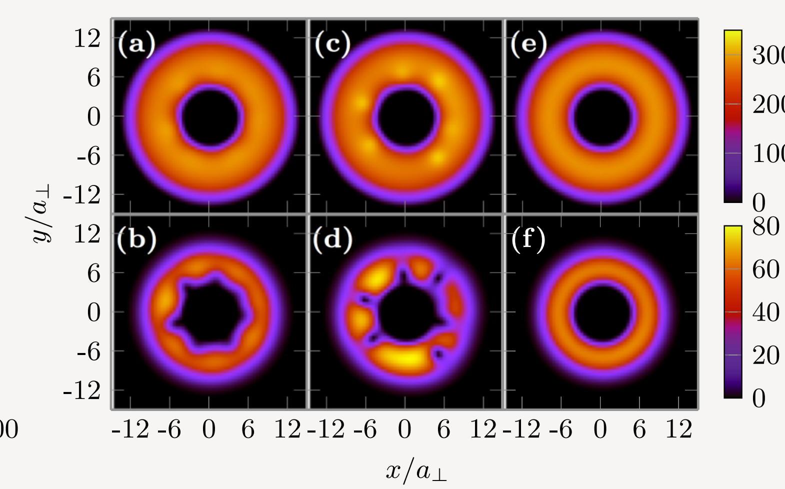
Theory

Bogolyubov dispersion relations



GP real time dynamics (Pz=0.8, κ=7)





phase slips due to vortices in the minority component (spin dominated)

Supercurrent stability

Experiment

PRL 110, 025301 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

Persistent Currents in Spinor Condensates

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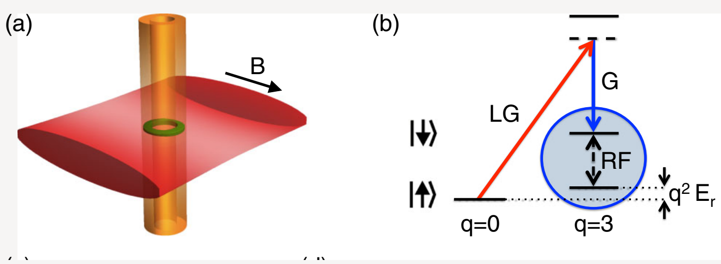

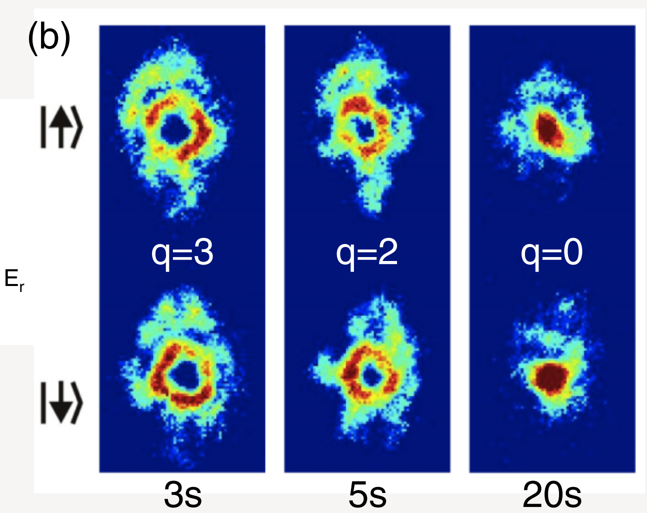

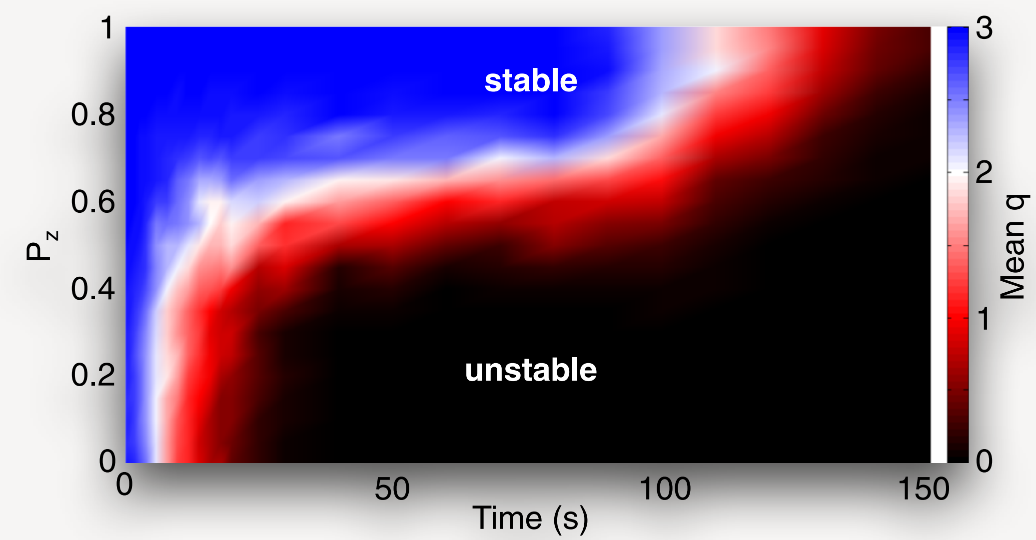

BUT if the RF is kept on the current is stable for over 1 minute

Coherently coupled Bose gas more…

The Rabi coupling strongly modified the physics of two component Bose gas at the few and many body level.

1. ITF-like (or phi^4) Ferromagnetic Transition *(2D not-at-all MF)*
2. Vortex dimer and string breaking (QCD-like) - Y.Shin
3. **(tunable) LHY corrections from 2.5 to 3-body corrections**
4. Peculiar spin collective modes (breaking of f-sum rule)
5. Goldstone mode decay at the FM transition
6. Simulator of Magnetic Models (**Continuous** and Lattice)
7. Massive Hawking Radiation and boomerang effect

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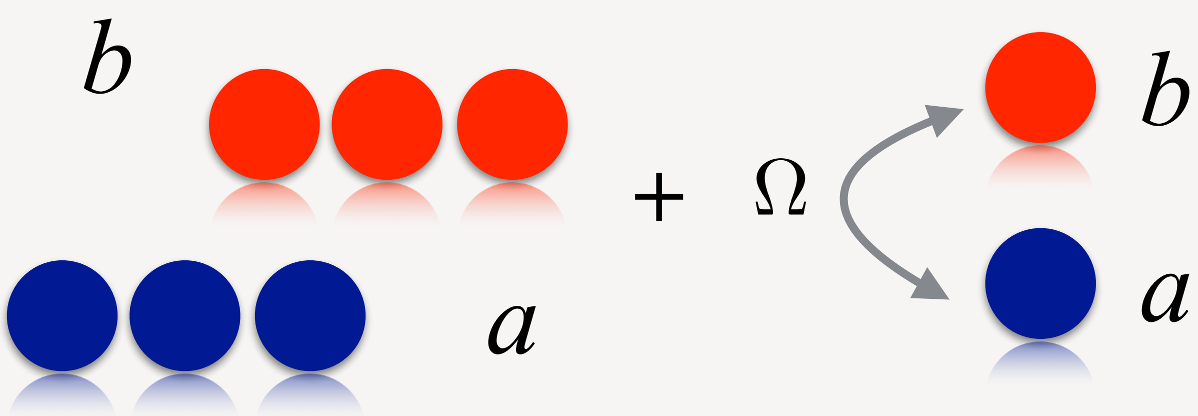
Elementary excitations



$$H = \int_{\mathbf{r}} \left[\sum_{\sigma} \frac{\nabla \psi_\sigma^\dagger \nabla \psi_\sigma}{2m} - \frac{\Omega}{2} (\psi_a^\dagger \psi_b + \psi_b^\dagger \psi_a) + \sum_{\sigma\sigma'} \frac{g_{\sigma\sigma'}}{2} \psi_\sigma^\dagger \psi_{\sigma'}^\dagger \psi_{\sigma'} \psi_\sigma \right]$$

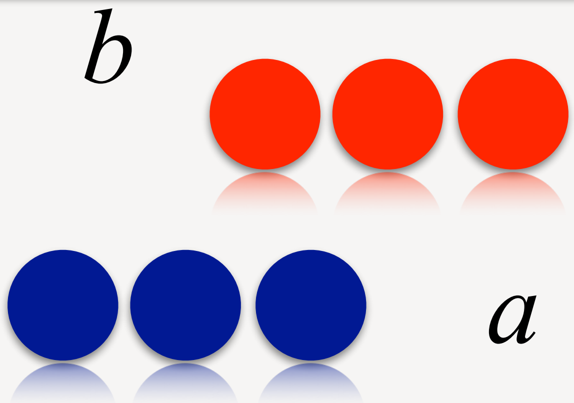
Only Na+Nb is conserved: The system is a *single condensate with a 2-component (spinor) wave function*

Ground state breaks one cont. symmetry U(1):

1 gapless Goldstone modes: no cost to change the phase of the order parameter

1 gapped mode: due to the cost of changing the relative phase
(explicit U(1) symmetry breaking)

Two Component (mixture) BEC

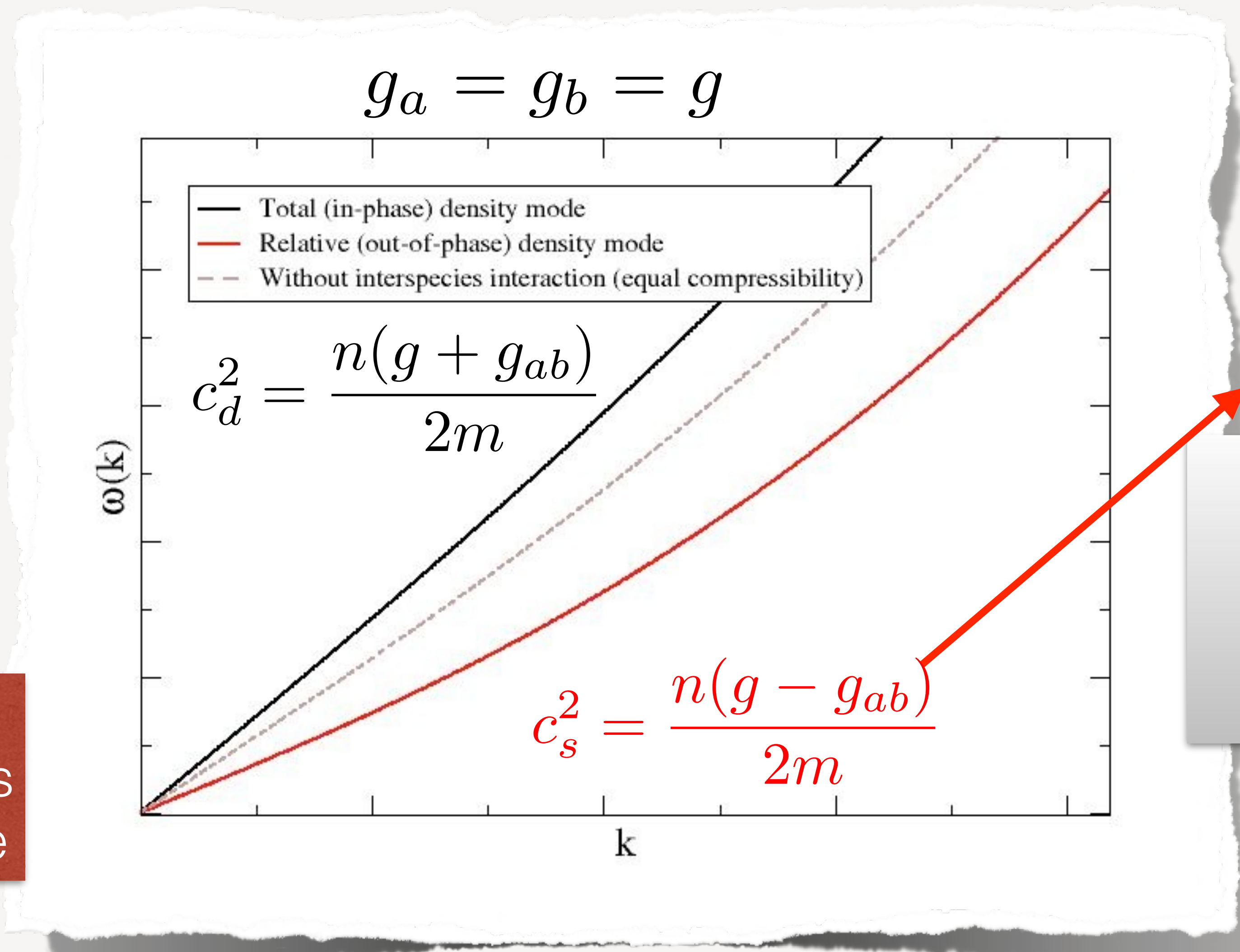


$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b$$

Both Na and Nb are conserved

Elementary excitations

Ground state breaks U(1)xU(1) symmetry: 2 Goldstone modes - coming from no cost to change the global and relative phase of the 2 order parameters



$g = g_{ab}$

Spin mode soft: unstable with respect to phase separation

Note: the static structure factors are qualitatively the same