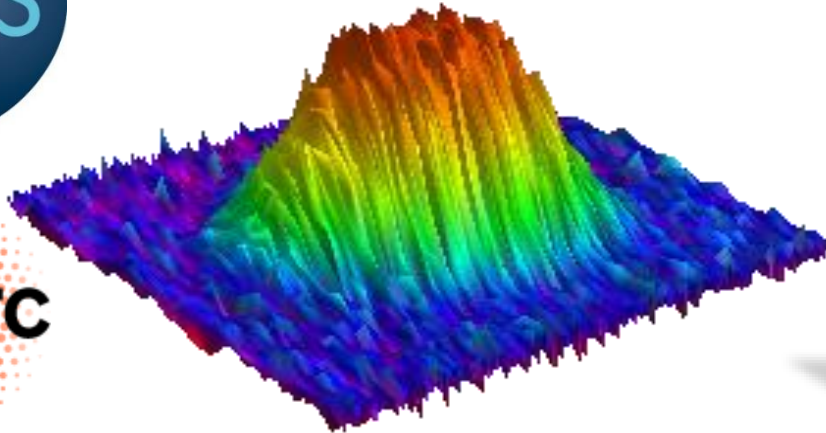


# Ultracold Fermions

## From Nuclear to Atomic Physics



C. Salomon



QMBC- 2023, Orsay  
March 23, 2023



# From nuclear matter to dilute gases

VOLUME 87, NUMBER 19

PHYSICAL REVIEW LETTERS

5 NOVEMBER 2001

## Alpha Cluster Condensation in $^{12}\text{C}$ and $^{16}\text{O}$

A. Tohsaki,<sup>1</sup> H. Horiuchi,<sup>2</sup> P. Schuck,<sup>3</sup> and G. Röpke<sup>4</sup>

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(Received 29 June 2001; published 17 October 2001)

A new  $\alpha$ -cluster wave function is proposed which is of the  $\alpha$ -particle condensate type. Applications to  $^{12}\text{C}$  and  $^{16}\text{O}$  show that states of low density close to the 3 and 4  $\alpha$ -particle thresholds in both nuclei are possibly of this kind. It is conjectured that all self-conjugate  $4n$  nuclei may show similar features.

DOI: 10.1103/PhysRevLett.87.19

There exists an intriguing problem with fermions as bound states of fermions are intrinsically of Bose character of the composite clusters. In this paper, we discuss the fermionic properties of their constituents. In simple terms, we discuss the relevance of  $\alpha$ -cluster correlations in atomic nuclei. Special attention is given to such correlations which correspond to the  $\alpha$ -cluster condensate in low-density symmetric nuclear matter. The Bose-Einstein condensation observed for bosonic atoms such as Rb or Na in trap



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## The BCS–BEC crossover: From ultra-cold Fermi gases to nuclear systems

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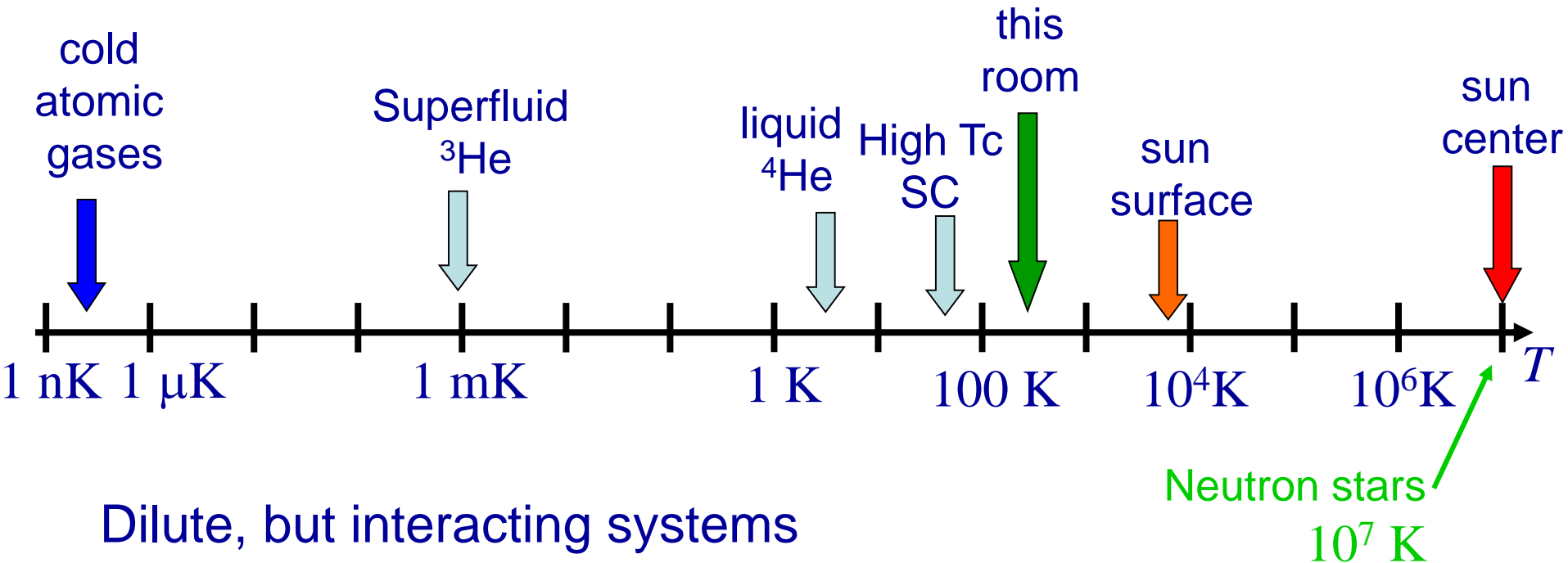
<sup>e</sup> Laboratoire de Physique et de Modélisation des Milieux Condensés, CNRS and Université Joseph Fourier, BP 166, 38042 Grenoble cedex 9, France



# Ultracold quantum gases

- Dilute systems
- Tunable interaction + arbitrary trapping potentials
- Precision measurements through imaging
- Exploring the link between fermionic superfluidity and Bose Einstein condensation
- Global thermodynamic properties: equation of state link with nuclear matter
- Dual Bose-Fermi superfluid mixture: a surprise !

# Temperature scale of cold gases



Dilute, but interacting systems

Typical density:  $\rho = 10^{13}$  to  $10^{15}$  atoms/cm<sup>3</sup>

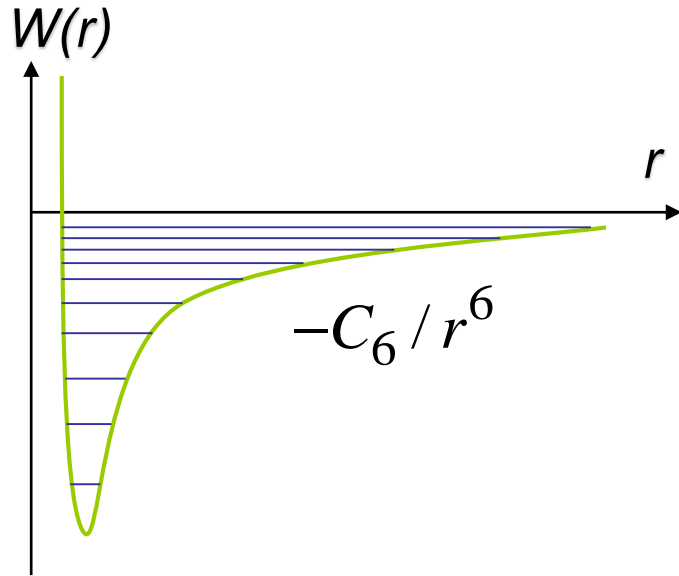
Interatomic distance 0.1 to 0.5  $\mu\text{m} \gg$  range of interatomic potentials

$E_{\text{int}} \gg \hbar\omega$  quantum of motion in the trap or box

$E_{\text{int}} \gg k_B T$  thermal energy

Equilibrium properties and dynamics are governed by interactions

# Atom-atom interactions



The magnitude and sign of  $a$  depend sensitively on the detailed shape of long range potential  
 Importance of position of last bound state

$$a_{\text{vdW}} \ll \lambda_{\text{dB}}$$

$$a_{\text{vdW}} \ll n^{-1/3}$$

Tuning interactions via Fano-Feshbach resonance

At low temperature,  
 only s wave collisions,  $l = 0$

$$\psi(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} - \frac{a}{r} e^{ikr}$$

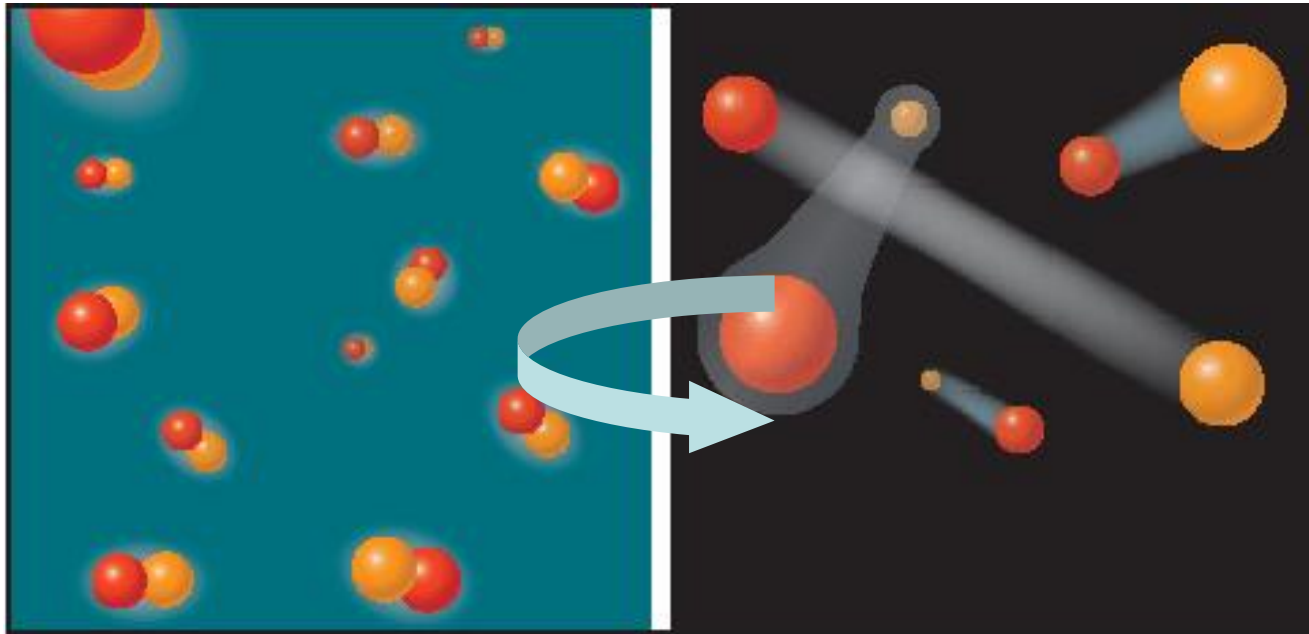
$$a = -\lim_{k \rightarrow 0} \frac{\tan \delta_0(k)}{k}$$

$a$ : scattering length  
 $|a| \sim 1$  to 10 nm

$$V(\vec{r}_1 - \vec{r}_2) = \frac{4\pi\hbar^2 a}{m} \delta(\vec{r}_1 - \vec{r}_2)$$

$a > 0$  : effective repulsive interaction  
 $a < 0$  : effective attractive interaction

# Fermions with two spin states with attractive interaction



$$T_c \approx T_F e^{-\pi/2k_F|a|}$$

BEC of molecules



BCS fermionic superfluid

Bound state

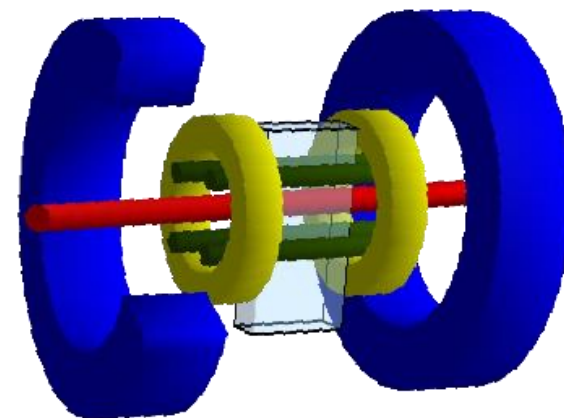
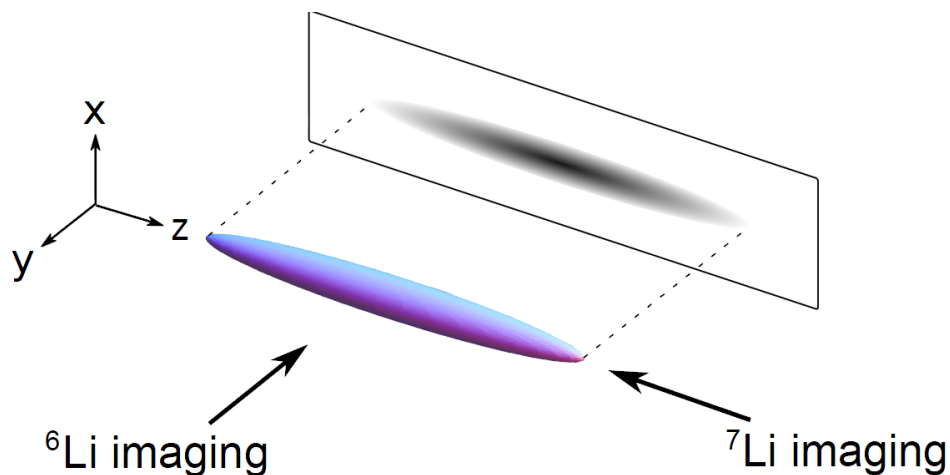
Interaction strength

No bound state

Equation of state in the BEC-BCS crossover

# Spin 1/2 Fermi gas with tunable interaction

- Loading of  ${}^6\text{Li}$  fermions in the optical trap
- Tune magnetic field to Feshbach resonance
- Evaporation of  ${}^6\text{Li}$  to 30 nK
- Image of  ${}^6\text{Li}$  *in-situ*



# Equation of State of Quantum Gases

Equilibrium properties given by **thermodynamic potentials**:

Grand potential

$$\Omega = -PV = E - TS - \mu N$$

Pressure                      Temperature                      Chemical potential  
Volume                      Entropy                      Atom number

Internal energy

We have measured the grand potential  
of tunable Fermi and Bose gases

S. Nascimbène et al., Nature, **463**, 1057, (2010), temperature dependence

N. Navon et al., Science **328**, 729 (2010), ground state in crossover

N. Navon et al., PRL 2011, Lee-Huang-Yang quantum correction in Bose gas

S. Nascimbène et al., Fermi liquid behavior, PRL 2011

M. Horikoshi et al., Science, **327**, (2010),

M. Ku et al., Science, **335**(2012), MIT



# Equation of State of Quantum Gases

Q. Zhou, T.L. Ho,  
Nature Physics, 09

C. Cheng, S.Yip,  
PRB (2007)

The pressure is obtained from *in situ* images

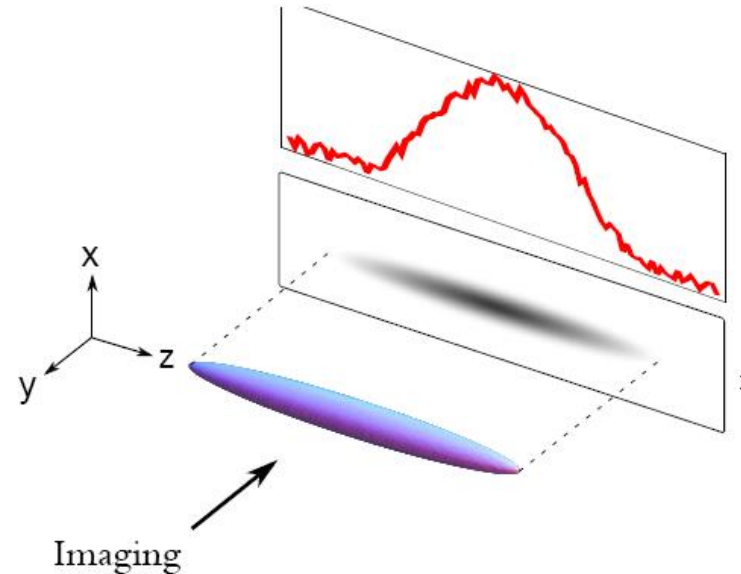
$$P(\mu_z, T) = \frac{m\omega_r^2}{2\pi} \bar{n}(z)$$

$$\bar{n}(z) = \int dx dy n(x, y, z)$$

Doubly-integrated density profile

Local density approx.

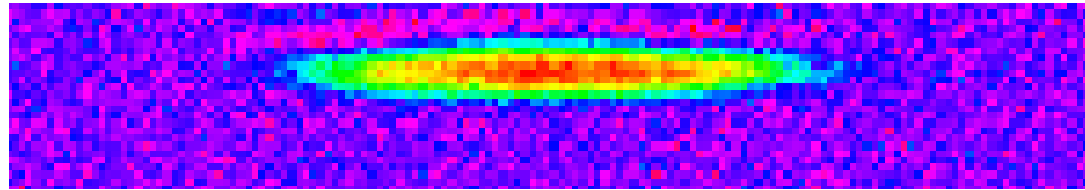
$$\mu(r) = \mu_0 - V(r)$$



$P(\mu_z, T)$  is an Equation of State of the locally homogeneous gas

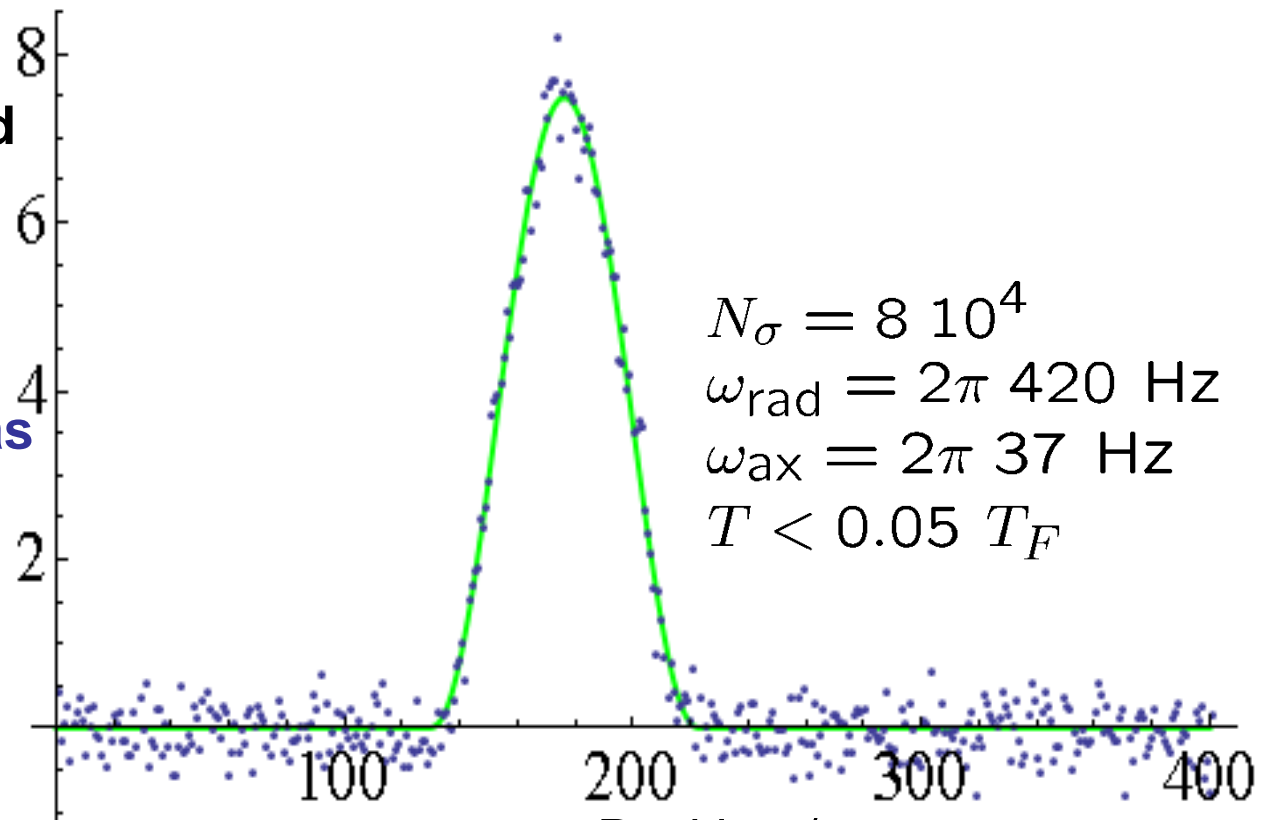
# Unitary Fermi Gas

$$a = \infty$$



Doubly integrated  
Density

Pressure of the  
locally  
homogeneous gas



Position / **Chemical potential  $\mu(z)$**

# The Equation of State at unitarity: temperature dependence

$$1/k_F a = 0$$

Continuous scale invariance  
Thermodynamics is universal

T.L. Ho, E. Mueller, '04

$$\mu = 0.376(5) E_F$$

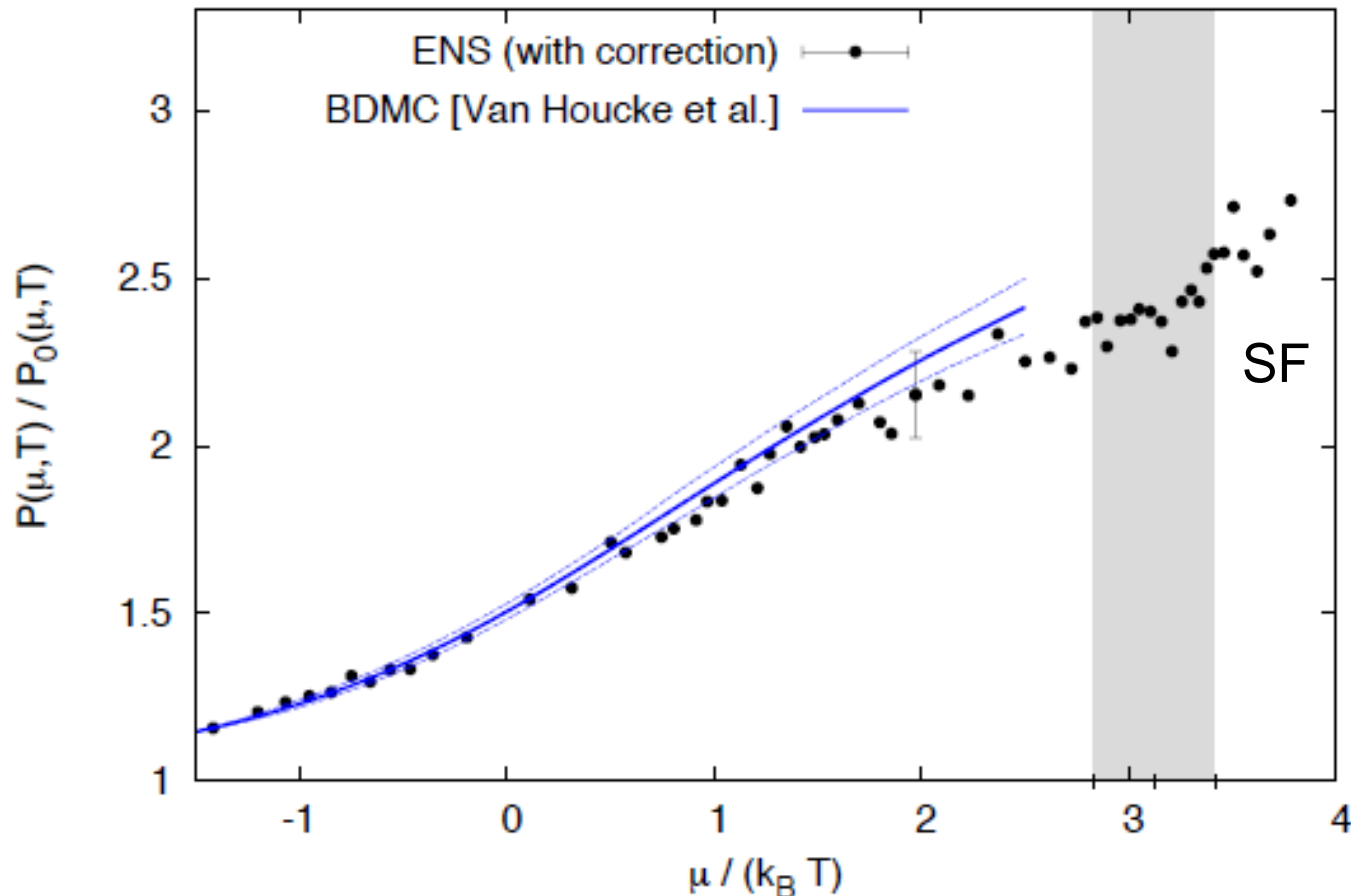
$$T_C = 0.167(15) T_F$$

*MIT 2012, Ku et al.*

Pressure depends only on  $\mu/k_B T$

# Equation of State at unitarity & Comparison with Bold Diagrammatic Monte-Carlo

S. Nascimbène et al., Nature, **463**, 1057, (2010)

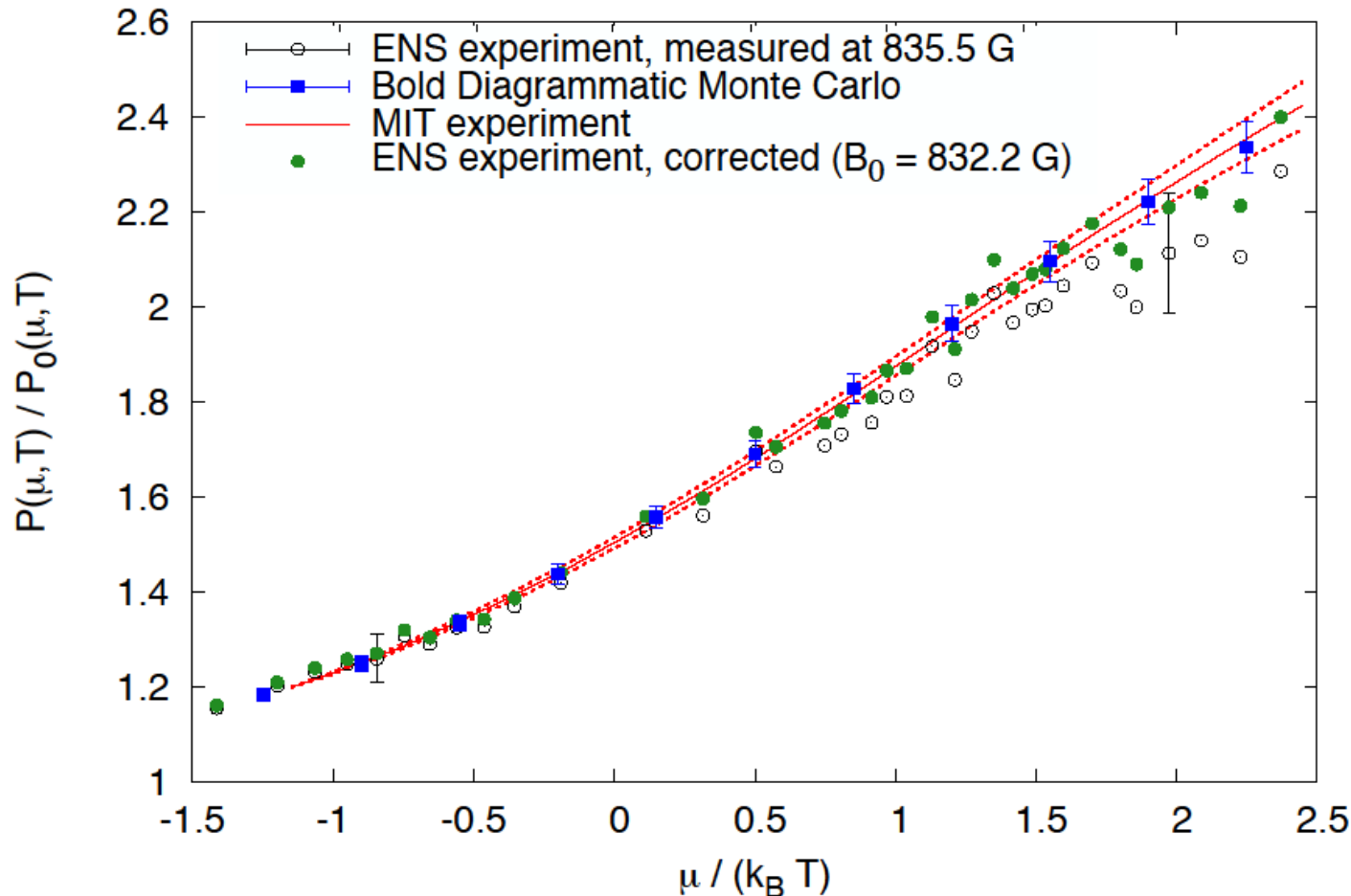


Theory:  
Van Houcke,  
et al.  
Amherst  
MIT  
Nature Phys.,  
2012

5% agreement with a Many-Body theory in strongly interacting regime

# Universal Equation of State at Unitarity

Comparison with MIT 2012 and Bold diag MC simulation

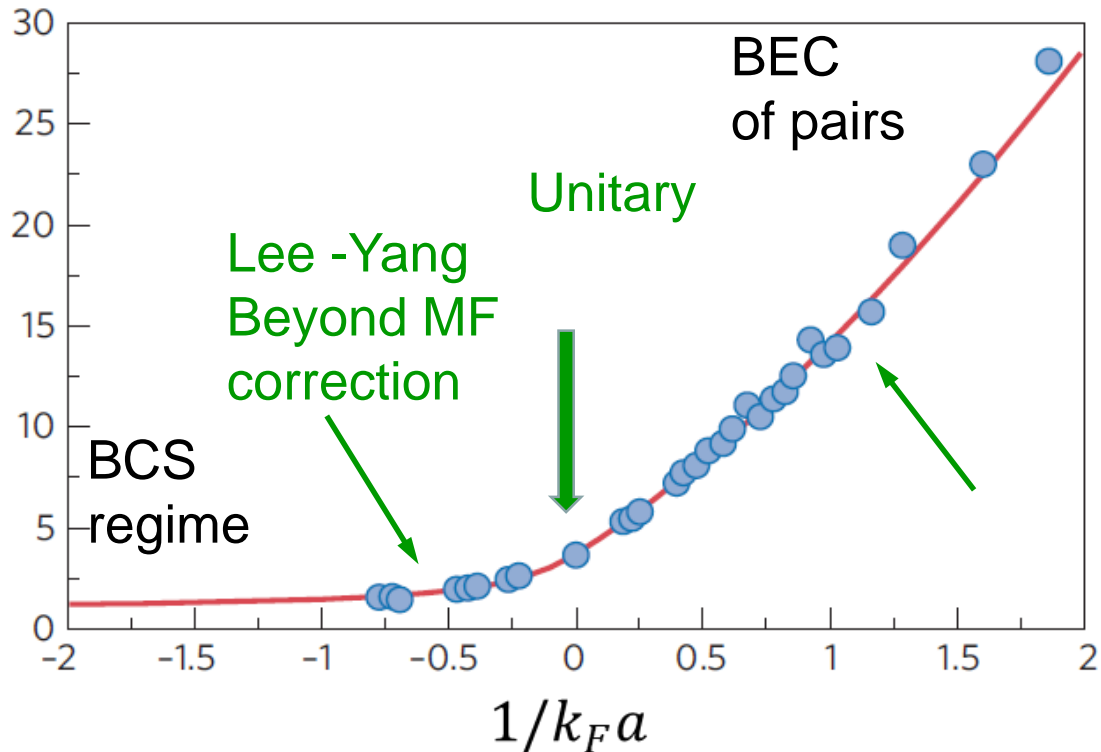


Theory:  
Van Houcke,  
Werner,  
Kosik, Prokof'ev,  
Svistunov,  
Ku, Sommer  
Cheuk, Schirotzek  
Zwierlein  
Nature Phys.,  
2012

5% agreement with a Many-Body theory in strongly interacting regime

# Equation of State of Fermi gas in the BEC-BCS crossover

Pressure equation of state  $P/P_0 = f(1/k_F a)$

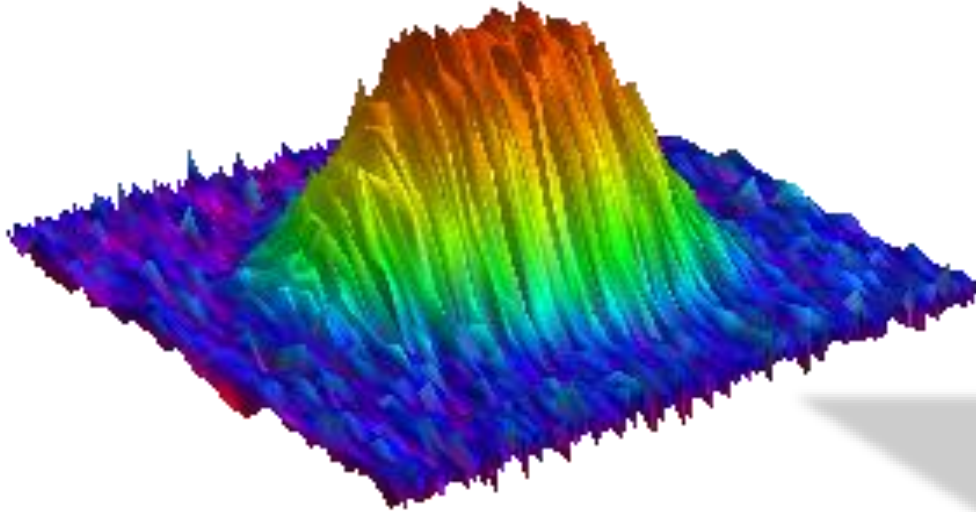


BCS-BEC crossover at  $T \sim 0$

Lee-Huang-Yang Beyond MF correction

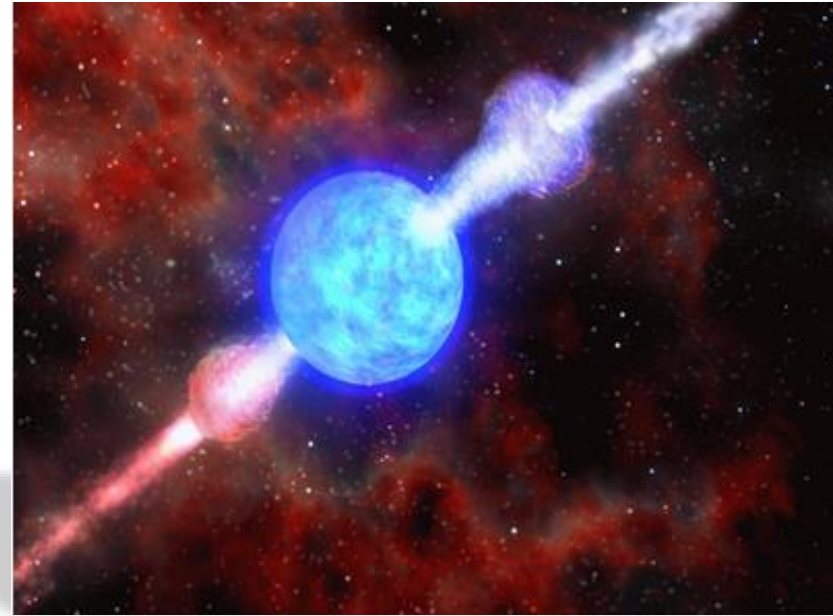
An example of quantum simulation in the strongly correlated regime

# Simulating the Eq. of State of neutron stars low density region



lithium 6 atoms, spin  $\frac{1}{2}$ ,

$n \sim 10^{13} \text{ cm}^{-3}$ ,  $T = 10^{-8}$  Kelvin  
A superfluid 1 million times  
thinner than air !



Neutron star, Spin  $\frac{1}{2}$

$a = -18.6 \text{ fm}$ ,  $n \sim 2 \cdot 10^{36} \text{ cm}^{-3}$

•  $T_c = 10^{10} \text{ K}$ ,  $T = T_F/100$

•  $k_F a \sim -4, -10, \dots$

1000 billion times denser than Earth !

Baym, Carlson, Bertsch,  
Pethick, Schwenk...

Second example

A novel system

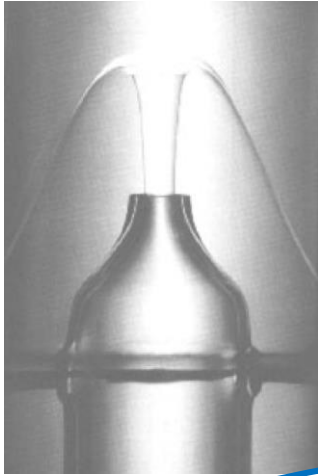
Bose-Fermi superfluid mixture



# 113 years of Quantum Fluids

## Bose Einstein condensate

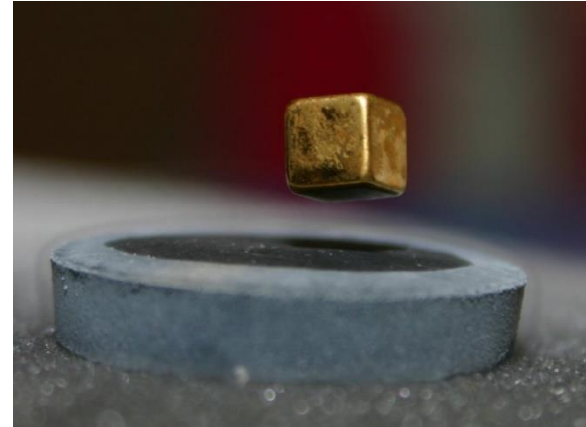
$^4\text{He}$



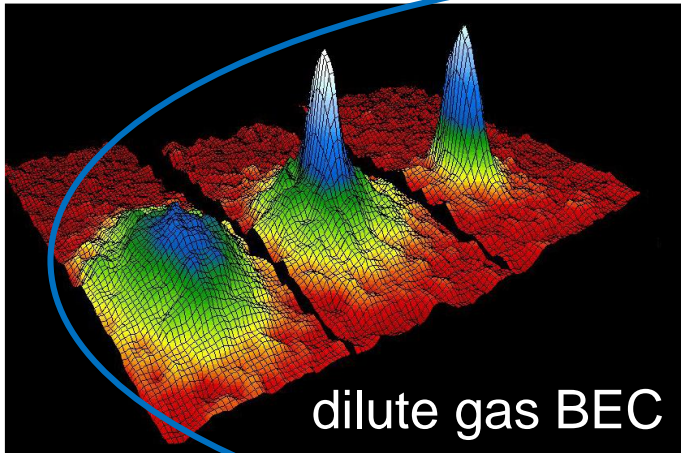
$T \sim 2.2 \text{ K}$

## Superconductors

High  $T_c$   
77 K

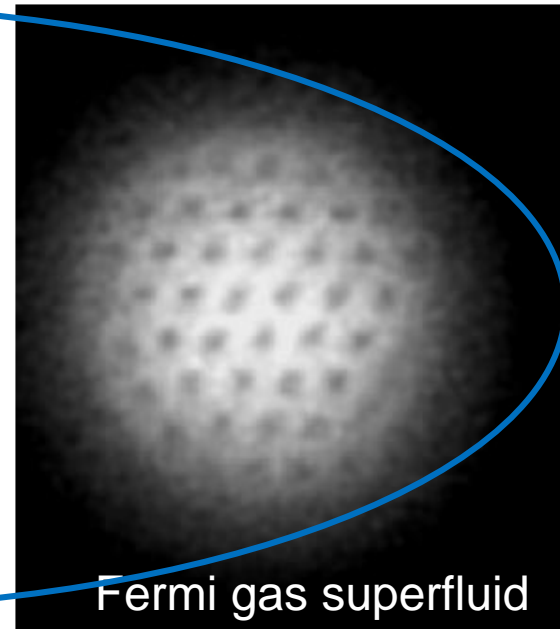


$^3\text{He}$   
2.5 mK



100 nK

Also BEC of light (Bonn)  
and exciton-polariton superfluids



ENS 2001

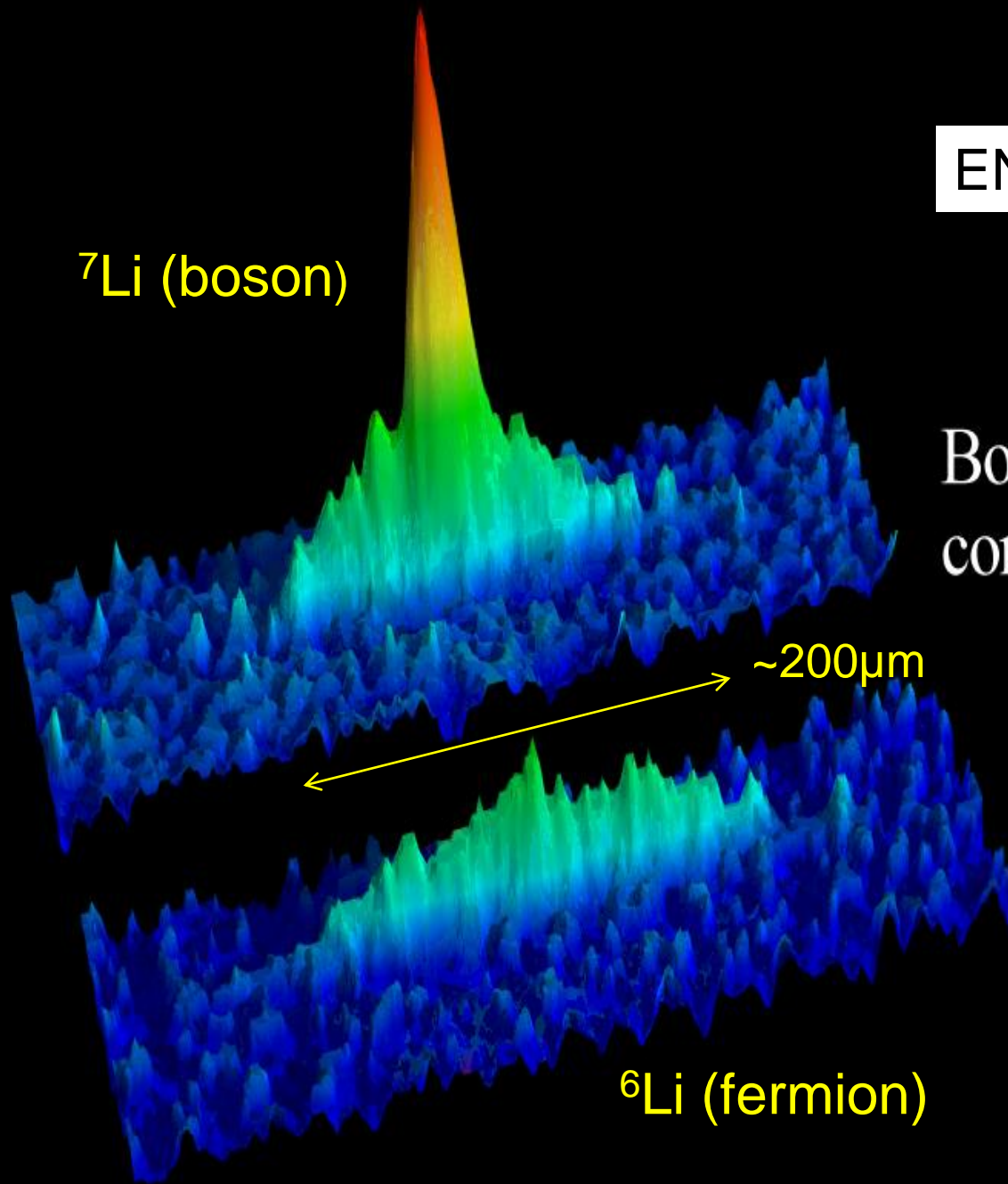
${}^7\text{Li}$  (boson)

Bose-Einstein  
condensate

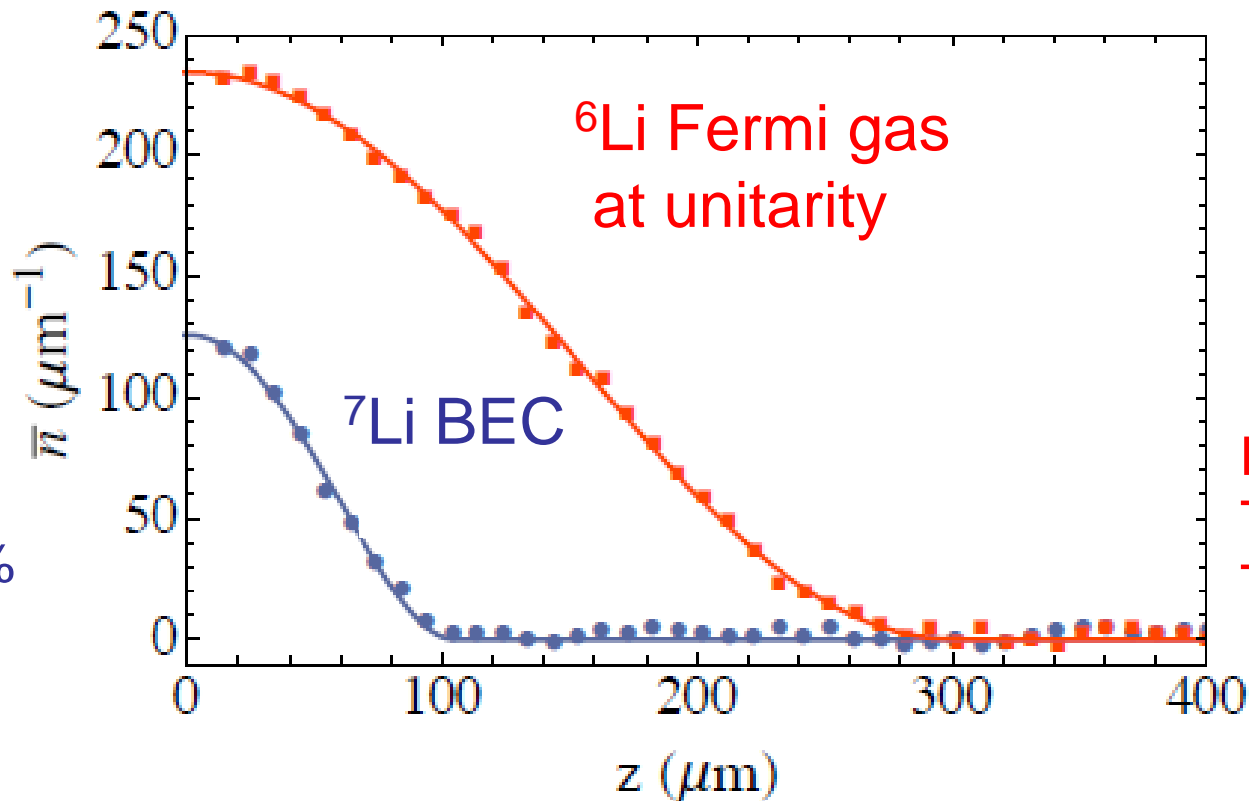
$\sim 200\mu\text{m}$

Fermi sea

${}^6\text{Li}$  (fermion)



# In situ density profiles



$N_F = 2 \cdot 10^5$   
 $T = 80 \text{ nK} \sim T_c/2$   
 $T_F = 800 \text{ nK}$

$N_B = 2 \cdot 10^4$   
 $T = 80 \text{ nK}$   
 $N_0/N_B > 80\%$   
 $T < T_c/2$

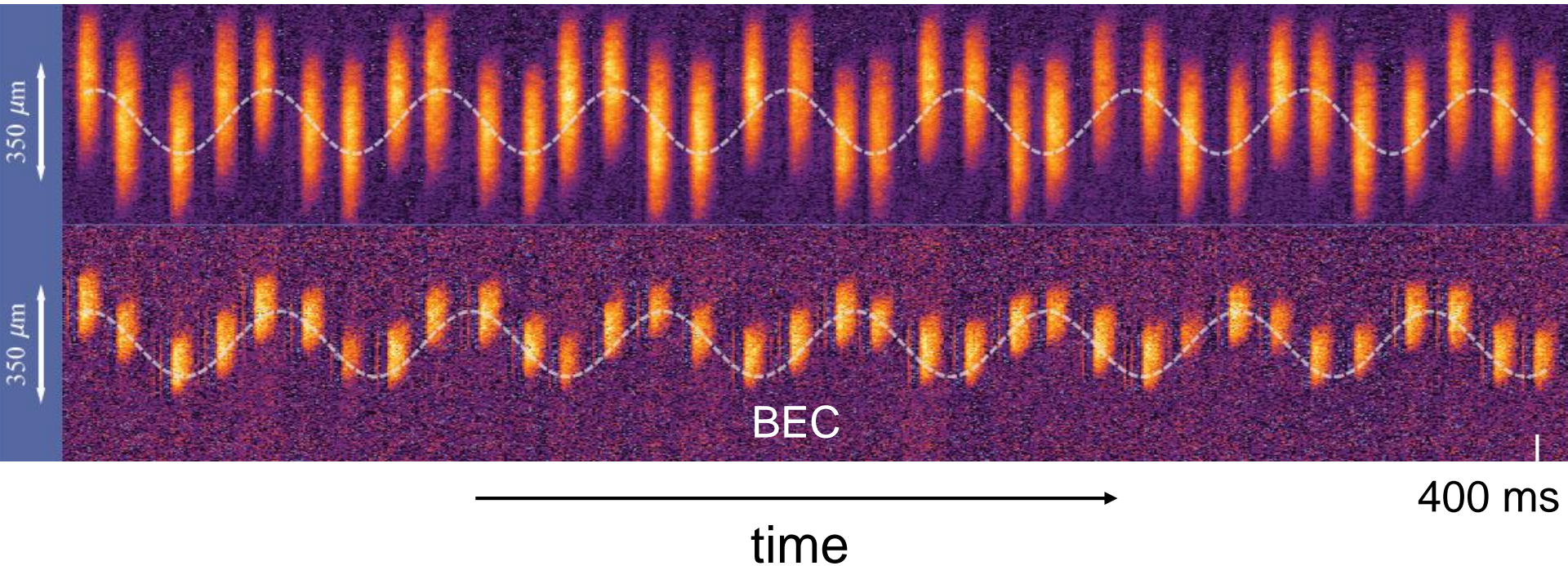
Trap frequencies:  $\nu_z = 15.6 \text{ Hz}$   
for bosons,  $\nu_{\text{rad}} = 440 \text{ Hz}$

Lifetime of mixture: 7s in shallowest trap

Unitary  ${}^6\text{Li}$  Fermi gas can cool any species fulfilling the requirements to BEC  
See also  ${}^6\text{Li}$ - ${}^{41}\text{K}$ , USTC, China, PRL '16, and  ${}^6\text{Li}$ - ${}^{173}\text{Yb}$ , UWash, PRL'17

# Long-lived Oscillations of Superfluid Counterflow

## Fermi Superfluid



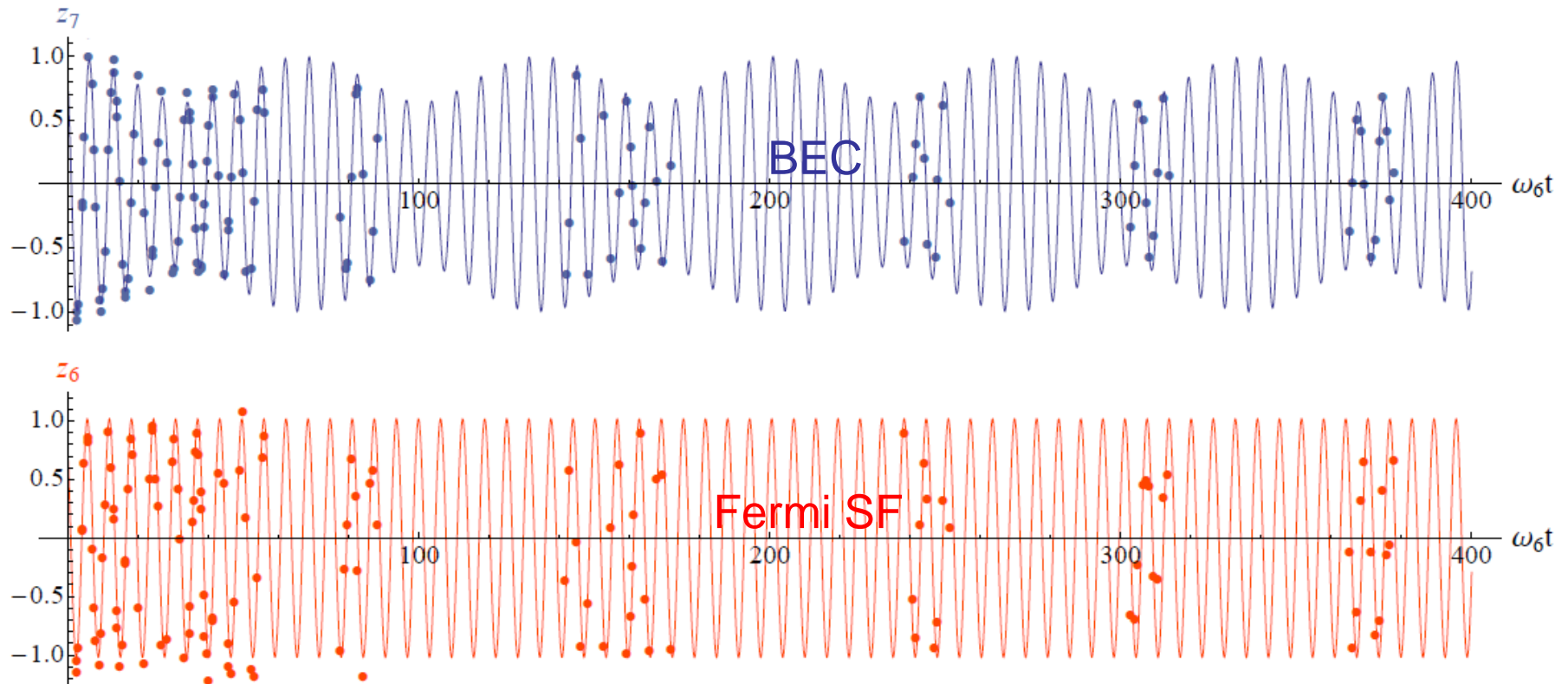
$$\tilde{\omega}_6 = 2\pi \times 17.06(1) \text{ Hz}$$

$$\tilde{\omega}_7 = 2\pi \times 15.40(1) \text{ Hz}$$

I. Ferrier-Barbut et al., *Science*, **345**, 1035, (2014)

Also, C. Hammer *et al* *Phys. Rev. Lett.* **106**, 065302 (2011) for boson-boson superfluid counterflow

# Oscillations of both superfluids



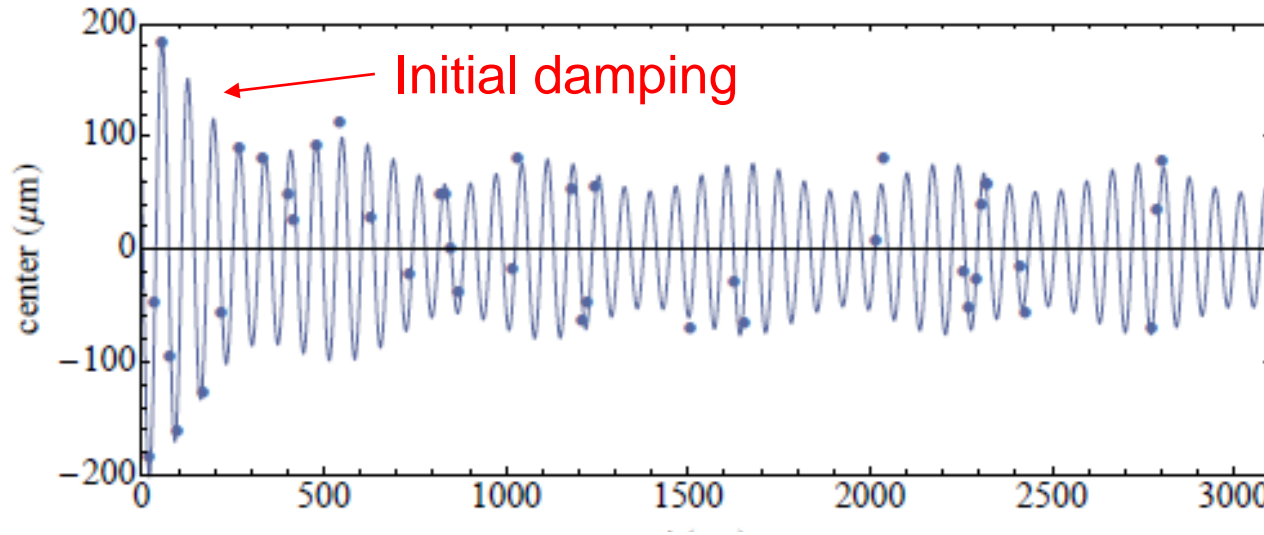
0 Very small damping: superfluid counterflow 4 s

Modulation of the  ${}^7\text{Li}$  BEC amplitude by  $\sim 30\%$  at  $(\tilde{\omega}_6 - \tilde{\omega}_7) / 2\pi$

Coherent energy exchange between the two oscillators

Frequencies can be measured very precisely !

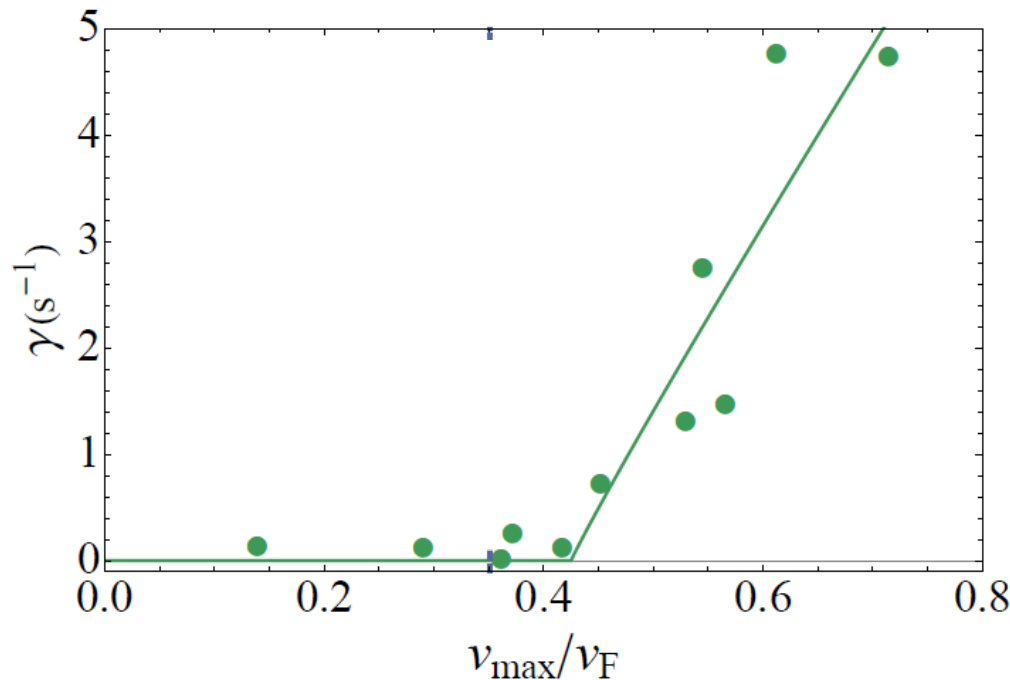
# Critical velocity for superfluid counterflow



$$d = d_0 \exp(-\gamma t) + d'$$

$$\gamma = 3.1 \text{ s}^{-1}$$

Time(ms)

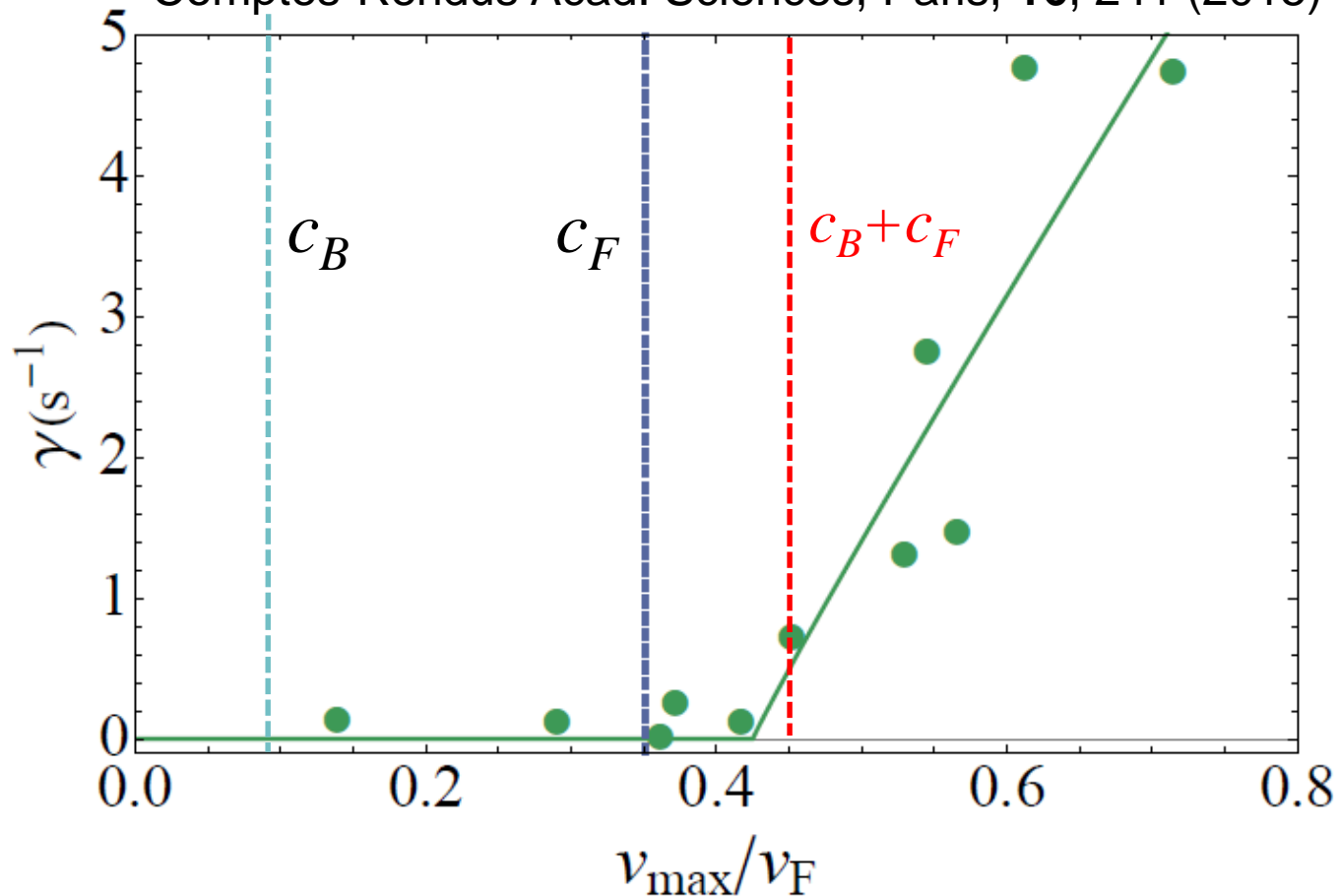


$V_c = 2 \text{ cm/s}$   
is quite high !

# Counter-flow critical velocity

Y. Castin, I. Ferrier-Barbut and C. Salomon

Comptes-Rendus Acad. Sciences, Paris, **16**, 241 (2015)



M. Delehaye, S. Laurent, I. Ferrier-Barbut, S. Jin, F. Chevy, C. Salomon, PRL 2015

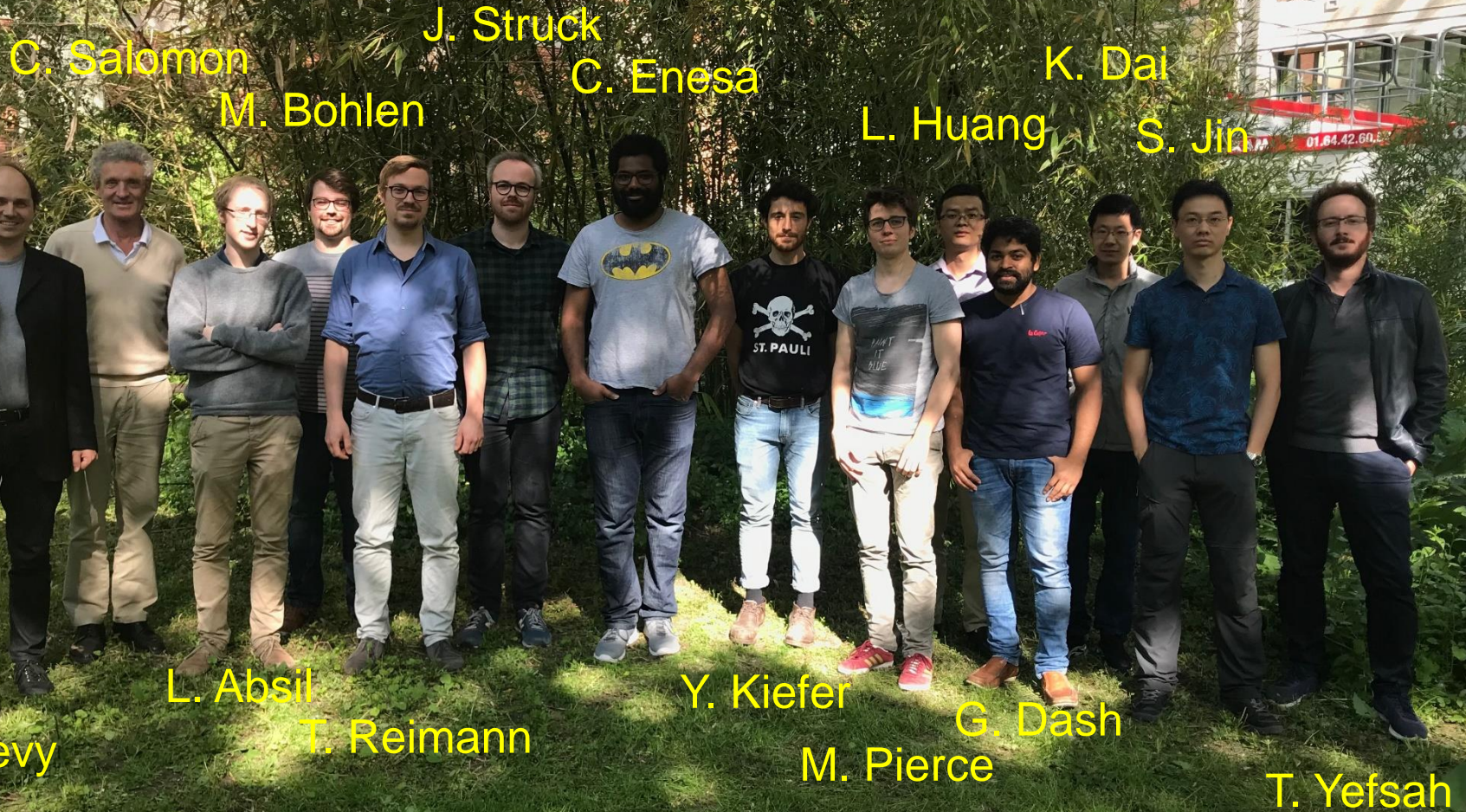
Related studies on Fermi gas at MIT, Miller PRL 2008, Hamburg, Weimer PRL 2015

# Perspectives on quantum gases

- 2 examples of quantum simulation with cold gases
- Explore further the cold atom-condensed matter interface: ex: spin polarization, FFLO phase
- Dynamics of quantum systems: time dependent Hamiltonian, Many-body localization, quantum quenches,.....
- Long range interactions: supersolids, dipole-dipole interaction
- Gauge fields and topological bands
- Spin-orbit coupling, integer quantum Hall states, and fractional QH.
- Mixed dimensions: 3D-2D, 3D-1D, 3D-0D.



# The ENS Fermi gas group



Theory

Y. Castin, F. Werner, X. Leyronas (ENS), S. Stringari (Trento), A. Recati, T. Ozawa, O. Goulko (Amherst), C. Lobo, J. Lau (Southampton), I. Danaila (Rouen)

