Antiscreening Effects in the Nuclear Pairing

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Outline:

- *Motivation*
- *BCS theory of pairing in nuclear matter*
- *Medium polarization :*
  screening and anti-screening
  s
- *nn spin singlet vs. np triplet pairing*

Pairing in Nuclear Matter

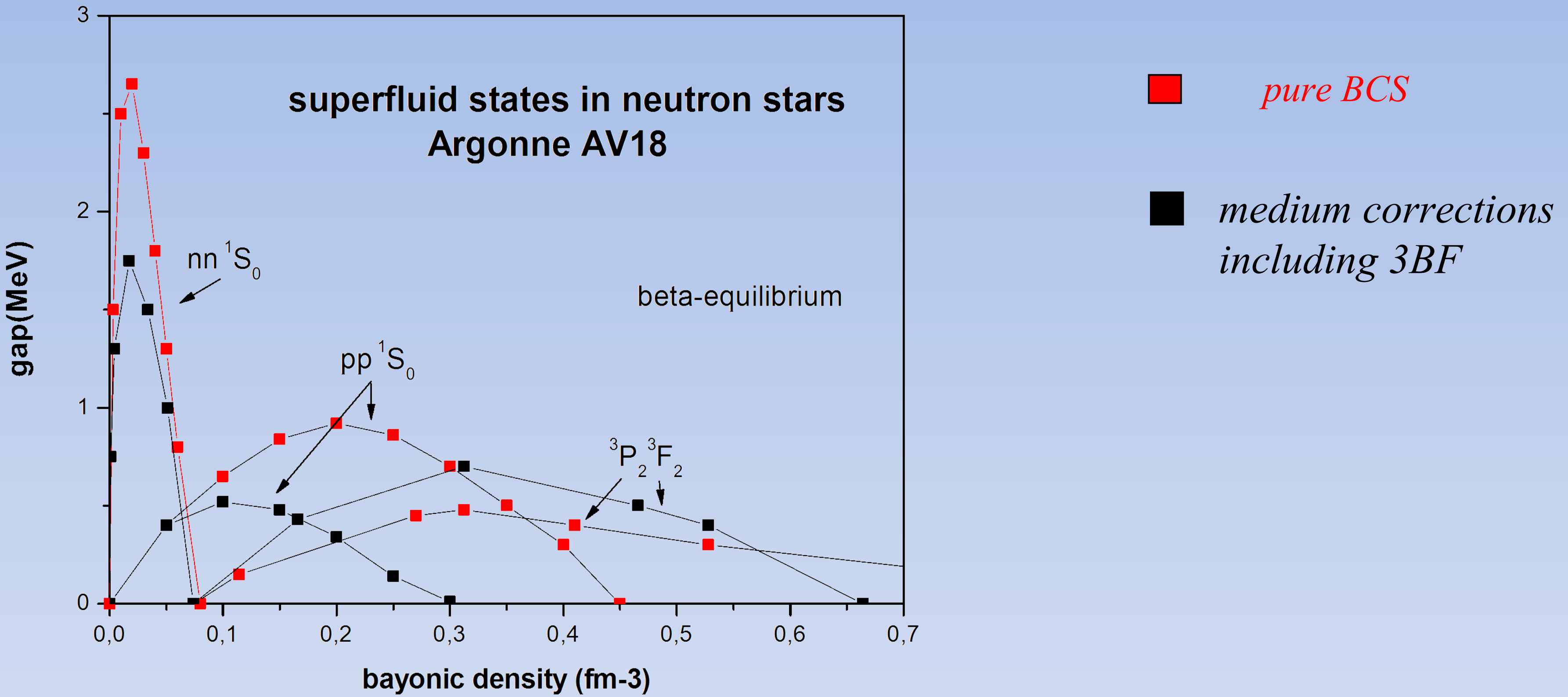
Motivations:

Pairing calculations with realistic interaction could provide for additional constrain to the interaction (e.g. non-locality, scalar,vector , tensor force, …)

Applications to rotational (glitches) and thermal properties of neutron stars (CasA cooling). Three kinds of superfluidity

Hints to understand pairing in nuclei

new scenario

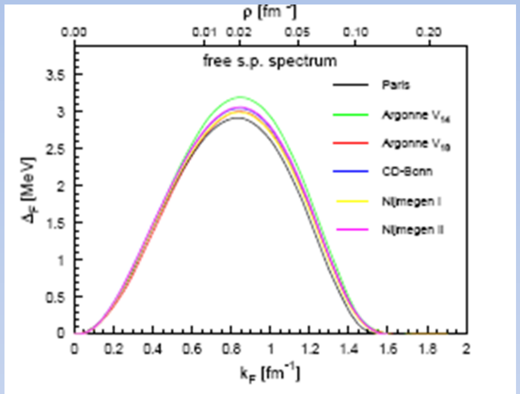


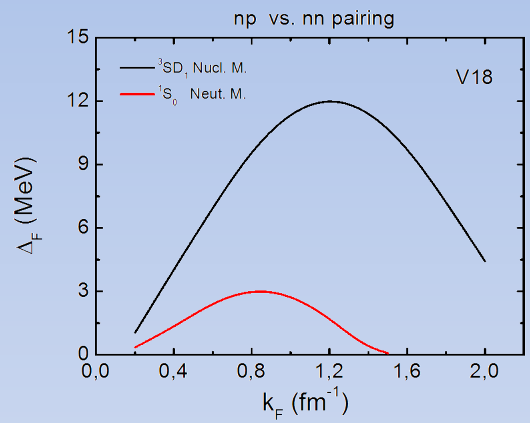
$^1S_0$ nn pairing suppressed by polarization effects
$^1S_0$ pp pairing below the threshold of URCA processes
$^3PF_1$ nn pairing extending up to NS inner core

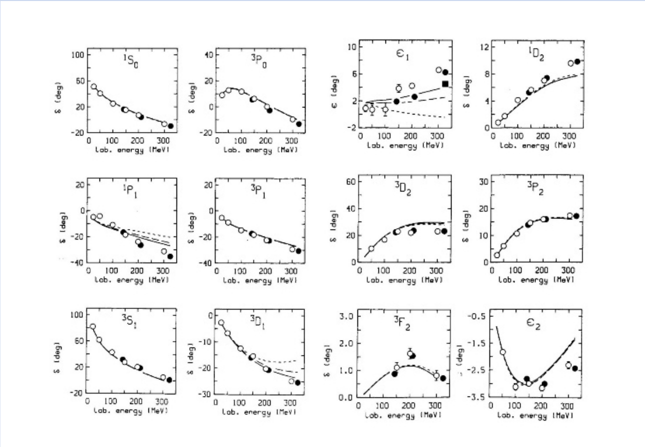
NS cooling program:
Catania-Lanzhou-St Petersbourg

Pairing vs. Interaction

spin-singlet n-n vs. spin-triplet n-p







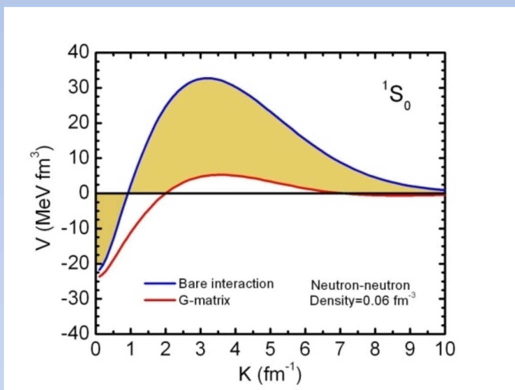
Extremely sensitive to the interaction strength

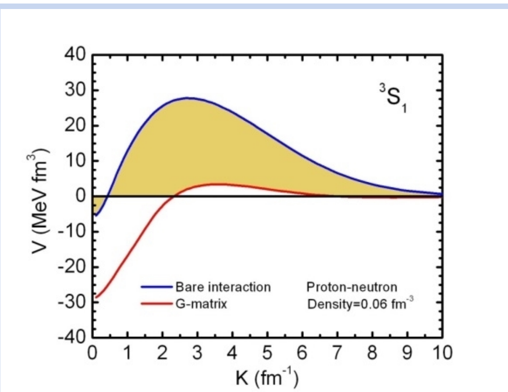
$\Delta \sim e^{-1/V}$

Strong vs. Weak Coupling Approx

$$\tilde{V}_{k,k'} = V_{k,k'} - \int_{k''>k_c} \frac{d^3k''}{(2\pi)^3} V_{k,k''} \frac{1}{2E_{k''}} \tilde{V}_{k'',k'} ,$$

$$\Delta_k = - \int_{k'<k_c} \frac{d^3k'}{(2\pi)^3} \frac{\tilde{V}_{k,k'}}{2E_{k'}} \Delta_{k'} ,$$





$$\frac{1}{\tilde{V}} = \sum_{l<k_{cutoff}} \frac{1}{2\sqrt{(e_{k'} - e_F)^2 + \Delta^2}}$$

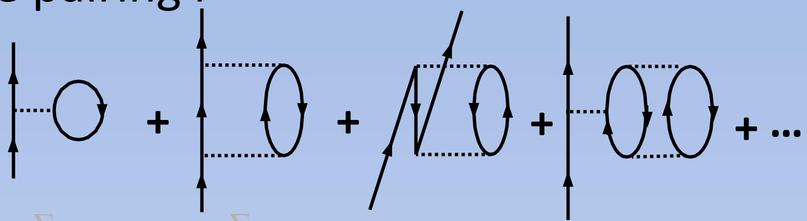
cutoff effect

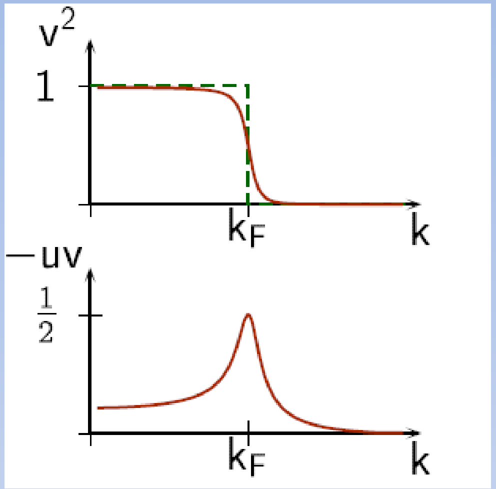
$\eta(fm^{-1})$	$\tilde{V}(MeV\,fm^3)$	$\Delta_F(MeV)$
0.1	-989	2.6
0.2	-607	2.7
0.3	-505	2.9
0.4	-455	3.1
0.6	-418	3.8
0.8	-409	4.8

Self-Energy Corrections

effective mass (m* < m) and Fermi surface depletion conspire to reduce the pairing r

Σ_{EBHF} =



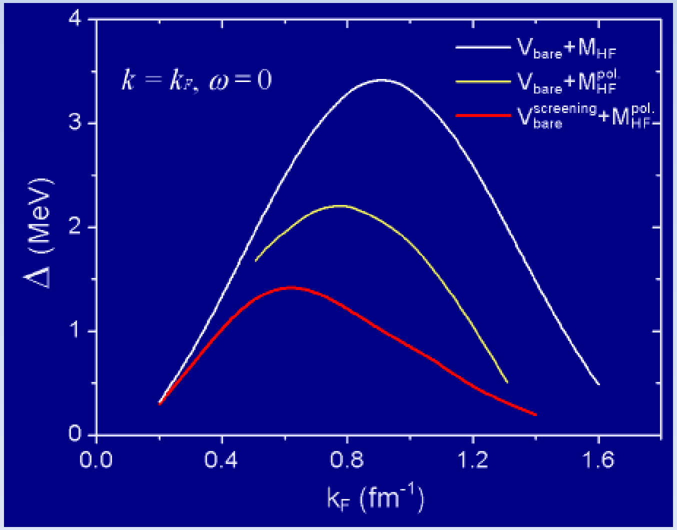


$$m_e(p) = m\left[1 - \left(\frac{\partial \Sigma(p,\omega)}{\partial \omega}\right)\right]_{\omega=\omega_p},$$

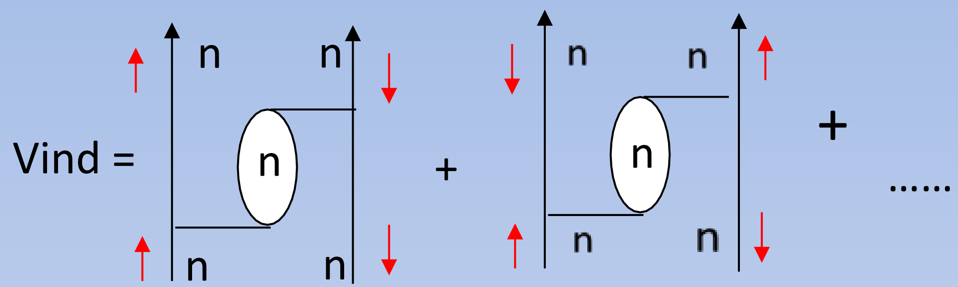
$$m_p(p) = m\left[1 + \frac{m}{p}\left(\frac{\partial \Sigma(p,\omega)}{\partial p}\right)\right]^{-1}_{\omega=\omega_p}.$$

$$\Delta(k') = -\frac{1}{\pi}\int k^2 dk \frac{Z^2}{E_k} V(k',k)\Delta(k)$$

$E_p = p^2/2m^* - e_F$

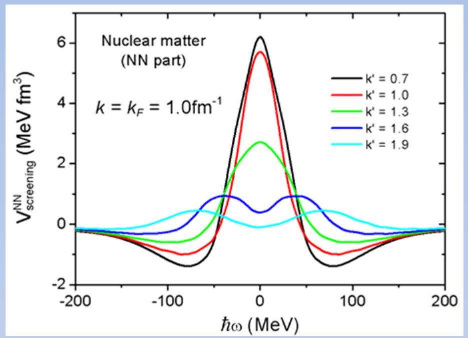


medium-polarization effects in neutron matter
spin-singlet nn pairing

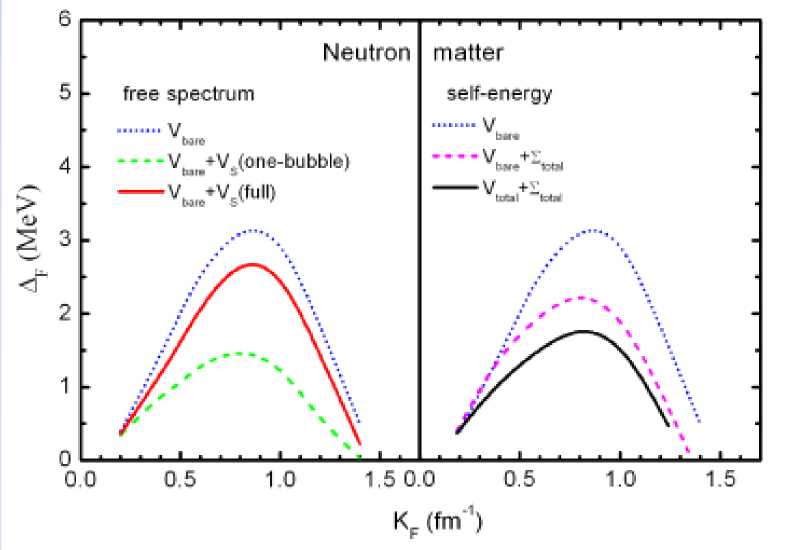


density flucts (S_{ph}=0) spin flucts (S_{ph}=1)

B&B Theory: one-bubble



Pairing gap in B&B Approx



Vind > 0

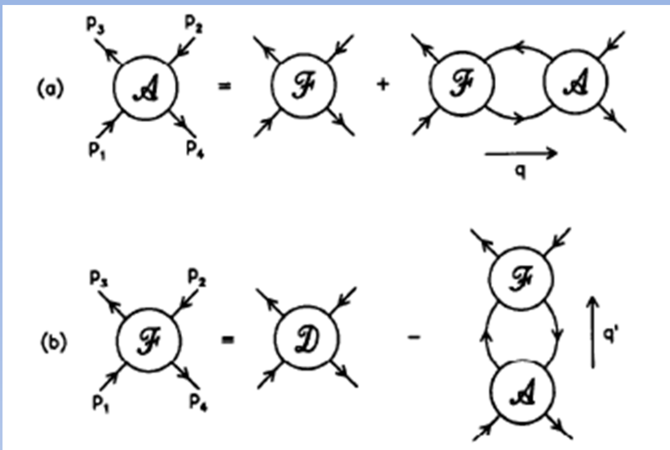
J.Clark et al, 1976 : : $V^{S=0}_{screen}$= 3 $G_0 - F_0$

CT – Liege Coll,, 1996 : $V^{S=0}_{screen}$= 3 V^{ph}_1 - V^{ph}_0

Theory underestimates exp data!

need to move from PNM to SNM

B&B Induced Interaction Model



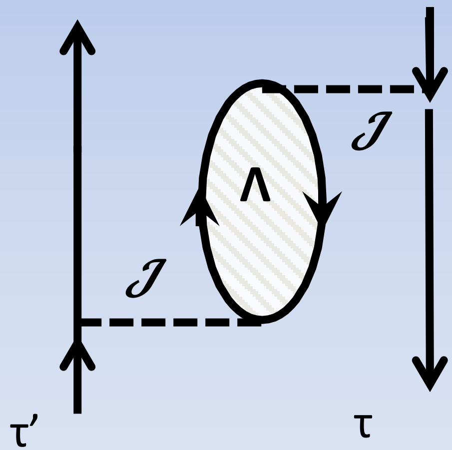
F(p,p';q) **=** D(p,p';q) *-+ Σ' F(p,p'';q) Q(p'',q) A(p'',p';q)*

A(p,p';q) = F(p,p';q) - Σ F(p,p'';q) Q(p'',q) A(p'',p';q)

D(p,p';q) ≈ G_{BHF} $\lambda\,(p'',q) = \dfrac{n\left(p''+\frac{q}{2}\right)-n\left(p''-\frac{q}{2}\right)}{p''\,q\,\cos\theta}$ (Lindhard)

$|p|=|p'|=p_F \qquad 0 < q < 2p_F$

$F^{i}_{ST} =$



$$4F_i = \frac{F_0^2\,U(q,0)}{1+F_0\,U(q,0)} + \frac{3F_0'^2\,U(q,0)}{1+F_0'\,U(q,0)} + \frac{3G_0^2\,U(q,0)}{1+G_0\,U(q,0)} + \frac{9G_0'^2\,U(q,0)}{1+G_0'\,U(q,0)}$$

$$4F_i' = \frac{F_0^2\,U(q,0)}{1+F_0\,U(q,0)} - \frac{F_0'^2\,U(q,0)}{1+F_0'\,U(q,0)} + \frac{3G_0^2\,U(q,0)}{1+G_0\,U(q,0)} - \frac{3G_0'^2\,U(q,0)}{1+G_0'\,U(q,0)}$$

$$4G_i = \frac{F_0^2\,U(q,0)}{1+F_0\,U(q,0)} + \frac{3F_0'^2\,U(q,0)}{1+F_0'\,U(q,0)} - \frac{G_0^2\,U(q,0)}{1+G_0\,U(q,0)} - \frac{3G_0'^2\,U(q,0)}{1+G_0'\,U(q,0)}$$

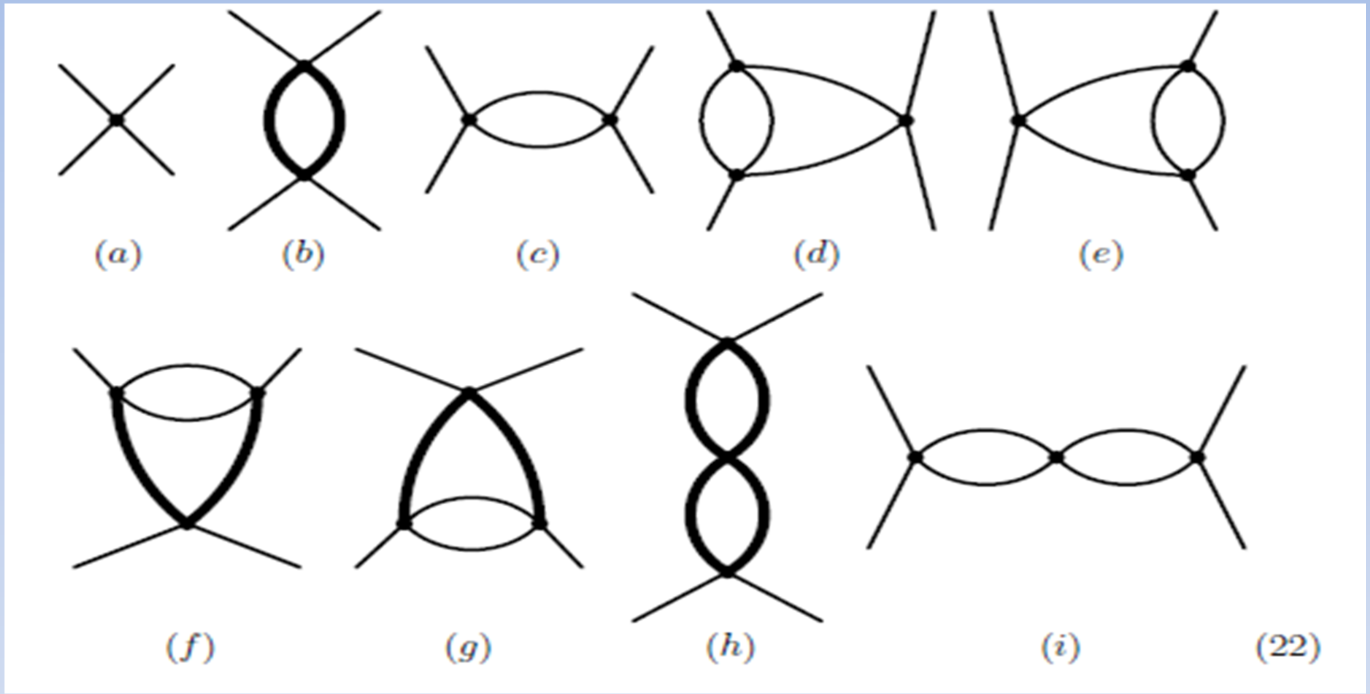
$$4G_i' = \frac{F_0^2\,U(q,0)}{1+F_0\,U(q,0)} - \frac{F_0'^2\,U(q,0)}{1+F_0'\,U(q,0)} - \frac{G_0^2\,U(q,0)}{1+G_0\,U(q,0)} + \frac{G_0'^2\,U(q,0)}{1+G_0'\,U(q,0)}$$

From particle-hole to particle-particle:

spin-singlet $(V_{ind})_{01} = ¼\,(F^{i}_{00}(q) - 3F^{i}_{10}(q)) + ¼\,(F^{i}_{01}(q) - 3\,F^{i}_{11}(q))$

spin-triplet $(V_{ind})_{10} = ¼\,(F^{i}_{00}(q) - 3\,F^{i}_{01}(q)) + ¼\,(F^{i}_{10}(q) - 3\,F^{i}_{01}(q))$

Expansion of the Babu-Brown Induced Interaction



Neutron Matter vs. Nuclear Matter
spin-triplet pairing

P. Schuck, PRC 2006

that, in a nuclear rather than neutron matter environment, the medium polarization of the interaction can favor the formation of Cooper pairs similar to the lattice vibrations in ordinary superconductors. These indications come both from nuclear matter calculations and from finite nuclei. In nuclear matter the medium enhancement of neutron-neutron ${}^1S_0$ pairing is to be traced back to the proton particle-hole excitations [4], and in finite nuclei to the surface vibrations [5]. Milano group

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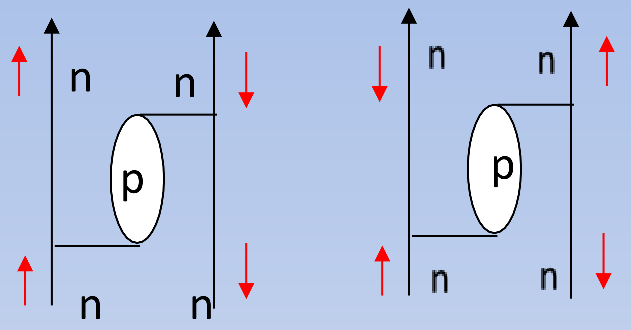

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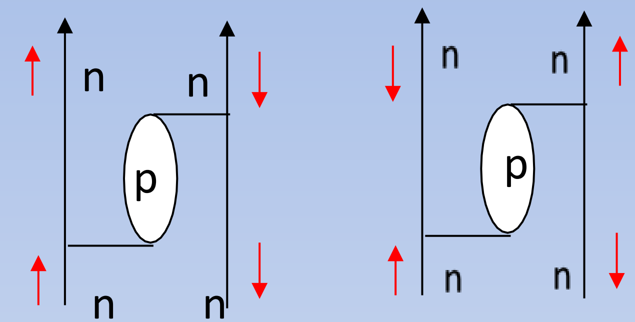
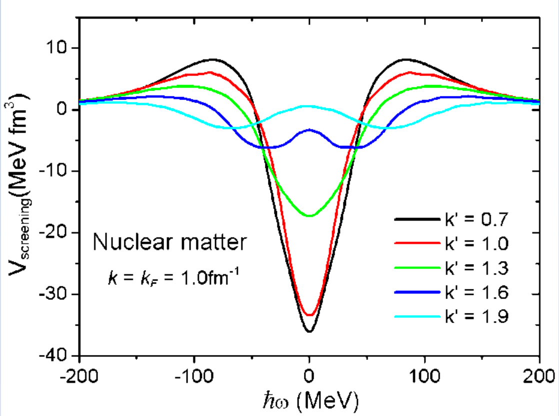
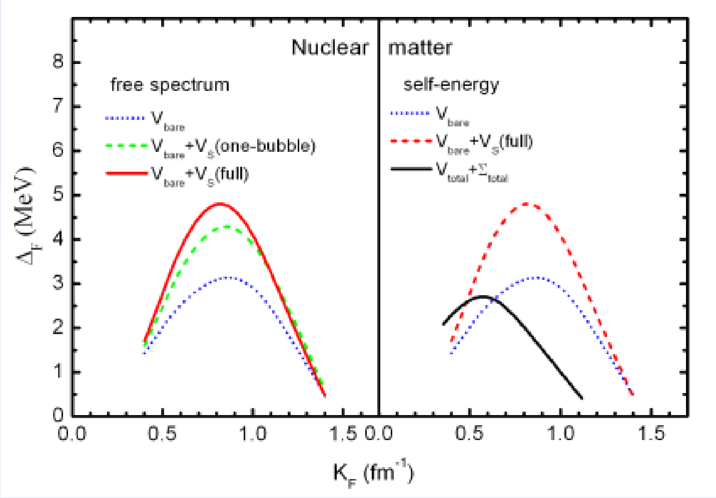




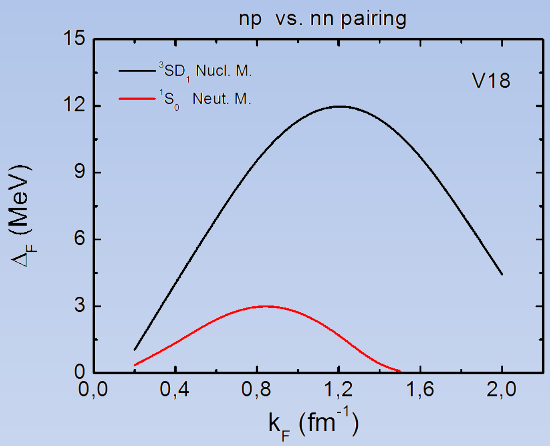
$V_{ind} < 0$
antiscreening



$$4V_{screening} = (V^{ph}_{00} + V^{ph}_{01}) - 3\,(V^{ph}_{10} + V_{11})$$
$$= (F_0 + F_0') - (3G_0 + G_0')$$

theory reconsiles with exp

Nuclear Matter
neutron-proton spin-triplet pairing



- $\Delta_{np} \approx 4\, \Delta_{nn}$ in nuclear matter
- despite $V(^3SD_1)$ is just sligthly stronger than $V(^1S_0)$, but $\Delta = \exp(-1/V)$
- strong Σ-suppression, $\Delta_{np} \approx 6$ MeV, ininfluent at low density $\rho = 0.05\, \rho_0$ (nuclear surface) , but still much bigger than the spin singlet

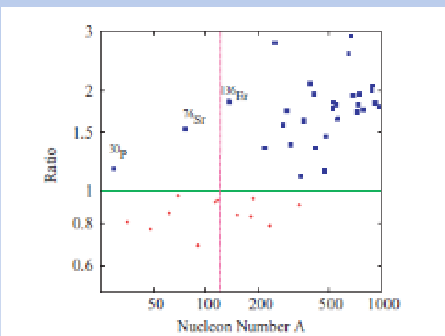
- the question arises: why the np pairing is not observed ?

The puzzle of the missing neutron-proton pairing in nuclei
short history

- *A.M. Lane . (Nuclear Theory, Benjamin 1964)*

« The neglect of the neutron-proton interaction is the major weakness of the pairing force theory. This interaction is just as strong as that between a pair of like nucleons. In fact in the T=0 state is stronger»

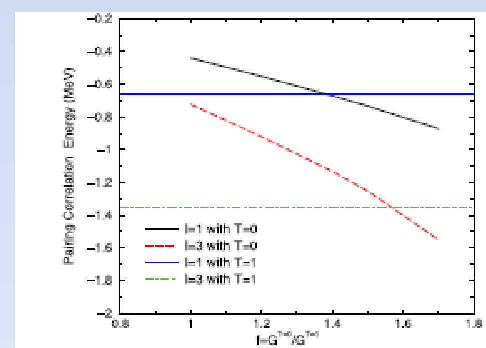
G.F. Bertsch et al (PRC 2010)
Study the effect of ***spin-orbit splitting*** *on the pairing in N=Z nuclei and predict a crossover from spin-singlet to spin-triplet pairing at A ~ 140*



N >> Z Fermi energy splitting prevents np pairing
moving from lighter to heavier nuclei the pairing force quenches down due to the surface dependence of spin-orbit force

$$H_{sp} = \frac{p^2}{2m} + V_{WS} f(r) + \vec{\ell} \cdot \vec{s}\, V_{so} \frac{1}{r} \frac{df(r)}{dr}$$

H. Sagawa et al (Physica Scripta,2014
Study interplay between S=1 np and nn S=0 pairing in pf-shell of N=Z nuclei , based on the pairing w.f. projection on the jj coupling

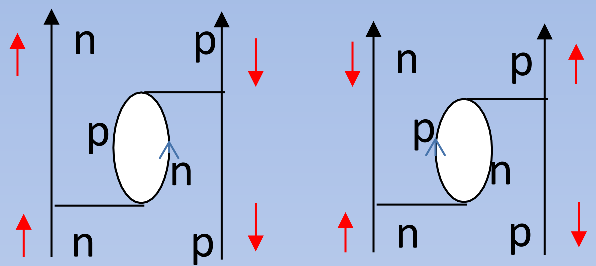


P. Schuck, PRC 2019

In the case of spin-triplet *np* pairing BCS calculations with bare interaction in nuclear matter predict sizable energy gaps of the order of 12 MeV, i.e., four times that of the spin singlet [14]. Even if significant rescaling is expected from the self-energy effects, the energy gap could be still large enough by antiscreening due to the induced interaction [12]. Therefore the predicted effect of the spin-orbit energy splitting could be resized by the large spin-triplet pair correlation energy.

Vertex Corrections

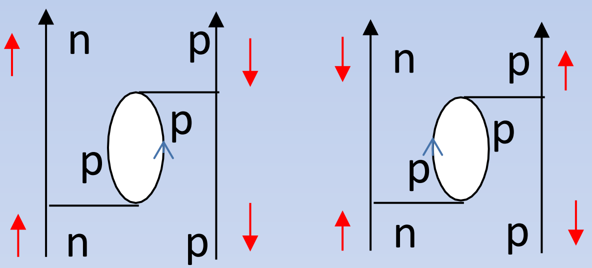
Competition between isoscalar and isovector flcts



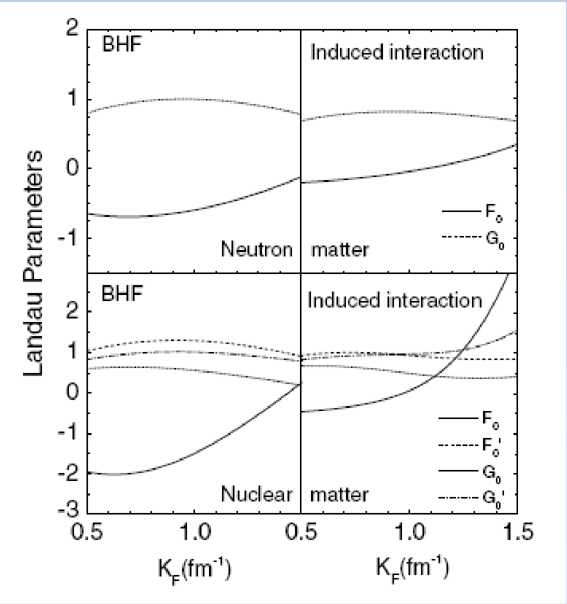
1S0 : $4V_i = (V^{ph}_{00} + V^{ph}_{01}) - 3 (V^{ph}_{10} + V^{ph}_{11})$

3SD1 : $4V_i = (V^{ph}_{00} - 3 V^{ph}_{01}) + (V^{ph}_{10} - 3 V^{ph}_{11})$

$\Delta V_i = - (V^{ph}_{01} - V^{ph}_{10})$ at any density



low density estimate
with BHF Landau par½ $\Delta V_i \approx - 3/4$
with B&B par $\Delta V_i \approx +3/8$



spin-triplet polarization

Calculations of induced interaction need to extend B&B to interaction with tensor forces

PHYSICAL REVIEW C **99**, 014310 (2019)

Medium-polarization effects in $^3SD_1$ spin-triplet pairing

Wenmei Guo,[1,2] U. Lombardo,[1,*] and P. Schuck[3,4]

RPA induced interaction

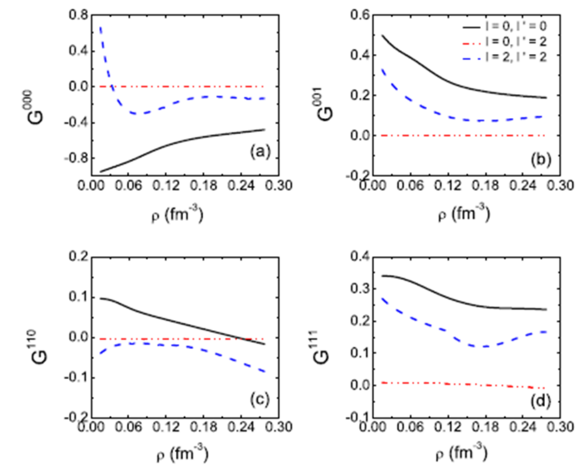
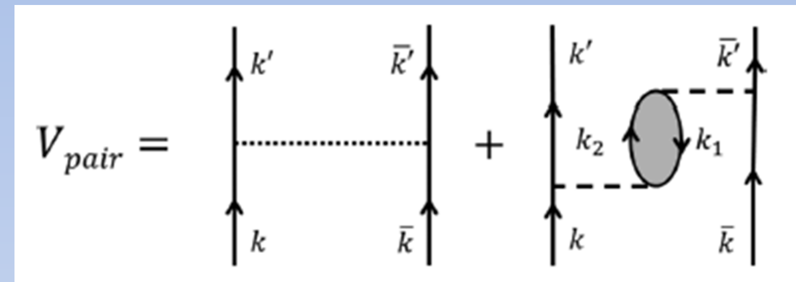
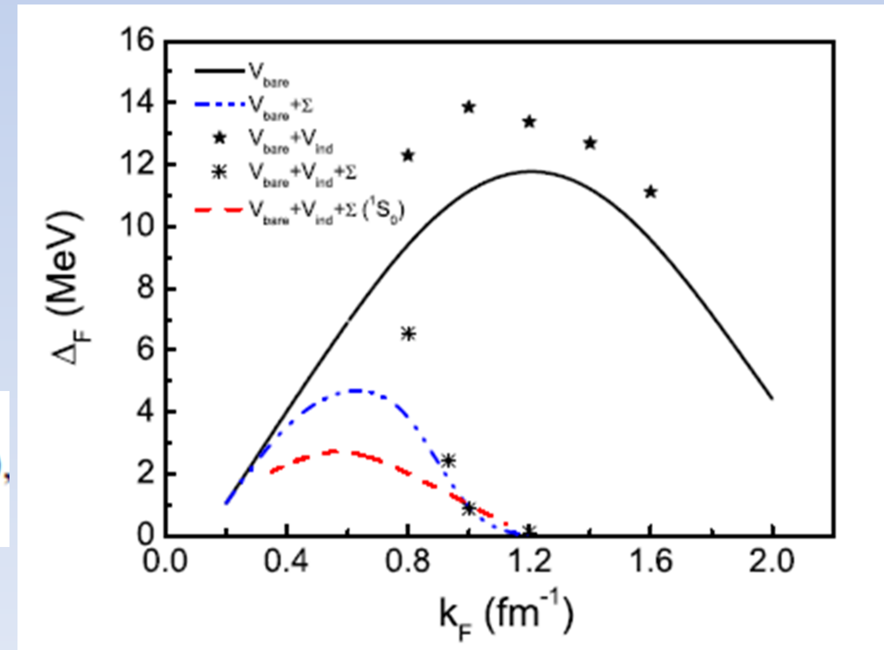


FIG. 3. Driving p-h interaction $\mathscr{G}^{SJT}_{ll'}$ from the G matrix in the SD channel.



L = S,D

$$\Delta^{ST}_{L}(k) = -\frac{Z_F^2}{\pi}\int_0^\infty k'^2 dk' \sum_{L'} \frac{V^{ST}_{LL'}(k,k')}{\sqrt{\varepsilon_k^2 + \Delta(k')^2}} \Delta^{ST}_{L'}(k'),$$



low density instaility !

Conclusions

The medium-polarization effects on the spin-singlet pairing gap have been presented in the framework of the spin B&B theory of the induced interaction. A substantial compensation has been reported between self-energy and vertex corrections according to Peter-Schuck's antiscreening prediction.

RPA calculations of the spin-triplet induced interaction have been presented. The results give indications in support of the same antiscreening for the np pairing.
The competition between spin triplet and spin-singlet pairing in nuclei and nuclear matter is briefly discussed.

My partnership with Peter was among the most beautiful, thanks to his captivating enthusiasm for work, his physical intuition and his deep humanity.

Thank you Peter !