Thermodynamics of quark matter with multi-quark clusters in a Beth-Uhlenbeck type approach [1]

David Blaschke

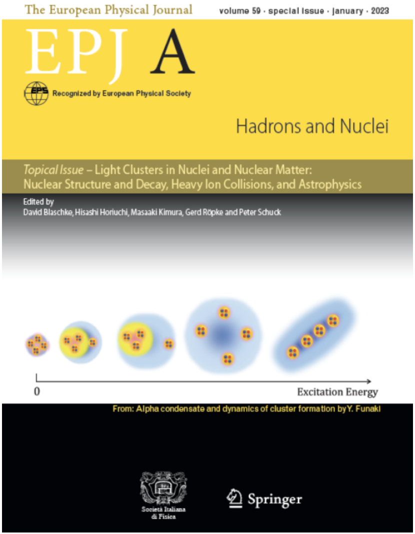
Institute of Theoretical Physics, University Wrocław, Poland
Center for Advanced Systems Understanding (CASUS), Grlitz, Germany
Helmholtzzentrum Dresden-Rossendorf (HZDR), Dresden, Germany

Quantum Many-Body Correlations, IJCLab Orsay, 22 March 2023

[1]Work with: M. Cierniak and G. Röpke, initiated by Peter Schuck.

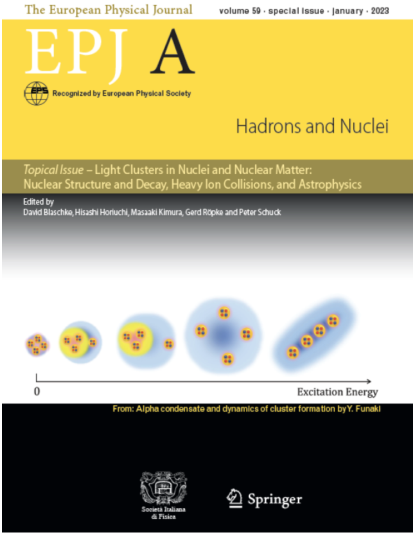
Workshop on "Light Clusters in Nuclei and Nuclear Matter: ..."
ECT* Trento, 2.-6. September 2019

EPJ A Topical Collections (TC)



From ECT* Trento Workshop in 2019

EPJ A Topical Collections (TC)



From ECT* Trento Workshop in 2019

New TC:

"The Nuclear Many-Body Problem"

Devoted to the legacy of Peter Schuck

Topics:

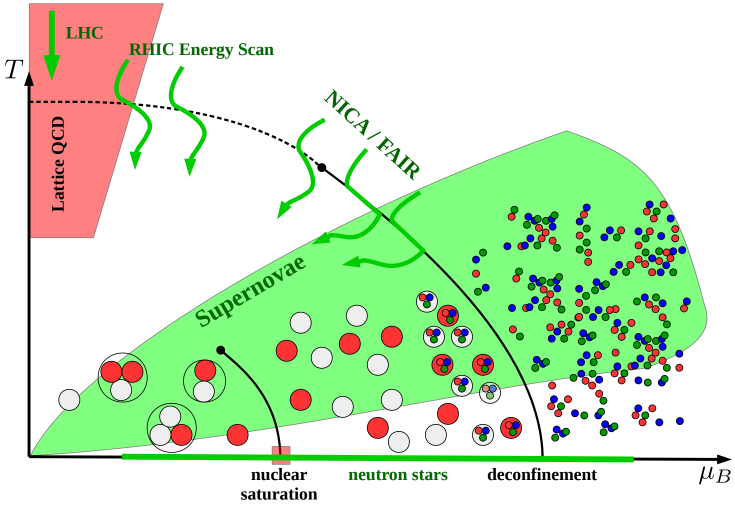
- The interacting boson model and collective phenomena in nuclear systems
- Nuclear energy density functionals
- Equation of motion method and extended RPA
- Quantum condensates and pairing
- Alpha-particle clustering
- Pions and related experiments, astrophysics
- Applications in solid state physics, quartetting in semiconductor layers, etc.

More details on the symposium website and

https://epja.epj.org/epja-open-calls-for-papers

New Deadline: 30. June 2023

QCD Phase Diagram with Clustering Aspects



From: N.-U. Bastian, D.B., et al., Universe 4 (2018) 67; arxiv:1804.10178

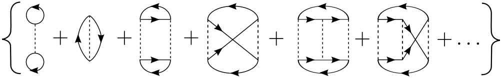
Quantum statistical approach to clustering

$$\Omega = -PV = -T \ln \mathrm{Tr}\, \mathrm{e}^{-(H-\mu N)/T},$$

$$P = \frac{1}{V}\mathrm{Tr} \ln[-G_1^{(0)}] - \frac{1}{2V}\int_0^1 \frac{d\lambda}{\lambda}\mathrm{Tr}\Sigma_\lambda G_\lambda,$$

or

$$P = P_0 - \frac{1}{2V}\int_0^1 \frac{\mathrm{d}\lambda}{\lambda}\Big\{ \ldots \Big\}$$



Alternative approach via density (normalization condition)

$$n_{\tau_1}(T, \mu_p, \mu_n) = \frac{2}{V}\sum_{p_1}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi} f_1(\omega) S_1(1,\omega)\,,$$

$$S_1(1,\omega) = \frac{2\mathrm{Im}\,\Sigma_1(1, \omega - i0)}{(\omega - E^{(0)}(1) - \mathrm{Re}\,\Sigma_1(1,\omega))^2 + (\mathrm{Im}\,\Sigma_1(1, \omega - i0))^2}\,,$$

Cluster Green's function - Ladder approximation

Bethe-Salpeter equation (BSE) for A-particle Green's function

$$G_A^{\rm ladder}(1\dots A;1'\dots A';z_A) = G_A^0(1\dots A;z_A)\delta_{\rm ex}(1\dots A;1'\dots A')$$
$$+\sum_{1''\dots A''} G_A^0(1\dots A;z_A)V_A(1\dots A;1''\dots A'')G_A^{\rm ladder}(1''\dots A'';1'\dots A';z_A)$$

BSE is equivalent to the A-particle wave equation.
Neglecting all medium effects, we get the A-particle Schrödinger equation

$$[E^{(0)}(1)+\dots+E^{(0)}(A)]\psi_{A\nu P}(1\dots A)$$
$$+\sum_{1'\dots A'} V_A(1\dots A;1'\dots A')\Psi_{A\nu P}(1'\dots A') = E^{(0)}_{A,\nu}(P)\Psi_{A\nu P}(1\dots A)$$

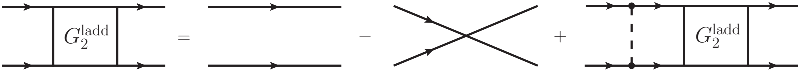


Figure: BSE in ladder approximation. Iteration gives the infinite sum of ladder diagrams for $G_A^{\rm ladder}$, where $A=2$.

Cluster virial expansion for nuclear matter

From cluster decomposition of the nucleon self-energy follows[2]

$$n^{\rm tot}_{n,p}(T,\mu_n,\mu_p) \quad = \quad \frac{1}{V}\sum_{A,\nu,P} N_{n,p} f_{A,Z}[E_{A,\nu}(P;T,\mu_n,\mu_p)], \quad N_n = N,\ N_p = Z$$

$$f_{A,Z}(\omega;T,\mu_n,\mu_p) = \frac{1}{\exp[(\omega - N\mu_n - Z\mu_p)/T] - (-1)^A}$$

Non-degenerate case (Boltzmann distributions)

$$\frac{1}{V}\sum_{\nu,P} f_{A,Z}[E_{A,\nu}(P)] \quad = \quad \sum_c e^{(N\mu_n+Z\mu_p)/T}\int\frac{d^3P}{(2\pi)^3}\sum_{\nu_c} g_{A,\nu_c} e^{-E_{A,\nu_c}(P)/T}$$

$$= \quad \sum_c \int \frac{d^3P}{(2\pi)^3} z_{A,c}(P)$$

Gneralized Beth-Uhlenbeck EoS

$$z_{A,c}(P;T,\mu_n,\mu_p) = \int_{-\infty}^{\infty}\frac{dE}{2\pi}\ e^{\left(-E-P^2/(2M_A)+N\mu_n+Z\mu_p\right)/T} 2\sin^2\delta_c(E)\frac{d\delta_c(E)}{dE}\ .$$

[2] G. Röpke, Phys. Rev. C 92 (2015) 054001

Cluster virial expansion for nuclear matter

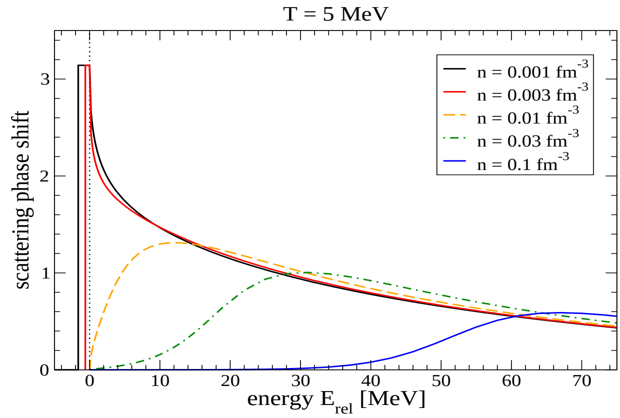
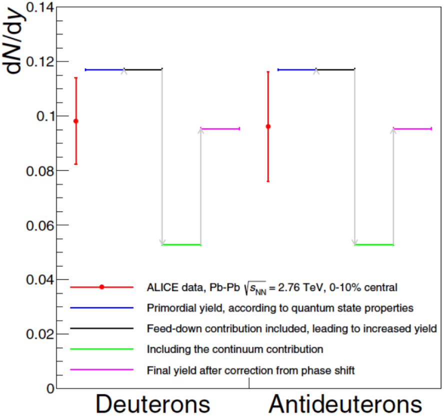


Figure: Integrand of the intrinsic partition function as function of the intrinsic energy in the deuteron channel. Mott dissociation and Levinson's theorem!

Application: Deuteron yields at LHC - ALICE



Production of deuterons at the chemical freeze-out temperature $T_{\rm fr} = 156$ MeV in the LHC-ALICE experiment for $\sqrt{s_{NN}} = 2.76$ TeV.

→ "snowballs in hell"

[Oliinychenko et al., PRC 99(2019)]

Important contributions from scattering state continuum in the deuteron channel! Cluster virial approach → Beth-Uhlenbeck EoS

B. Dönigus, G. Röpke, D.B., PRC 106, 044908 (2022)

Φ—Derivable Approach to the Cluster Virial Expansion

$$\Omega = \sum_{l=1}^{A} \Omega_l = \sum_{l=1}^{A} \left\{ c_l \left[\mathrm{Tr} \ln \left(-G_l^{-1} \right) + \mathrm{Tr} \left(\Sigma_l \;\; G_l \right) \right] + \sum_{\substack{i,j \\ i+j=l}} \Phi[G_i, G_j, G_{i+j}] \right\} ,$$

$$G_A^{-1} \quad = \quad G_A^{(0)^{-1}} - \Sigma_A \ , \ \Sigma_A(1 \dots A, 1' \dots A', z_A) = \frac{\delta \Phi}{\delta G_A(1 \dots A, 1' \dots A', z_A)}$$

Stationarity of the thermodynamical potential is implied

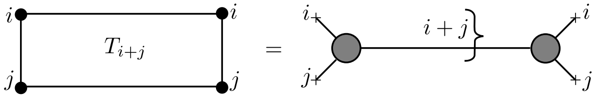
$$\frac{\delta \Omega}{\delta G_A(1 \dots A, 1' \dots A', z_A)} = 0 \ .$$

Cluster virial expansion follows for this Φ− functional

$\Phi =$ 

Figure: The Φ functional for $A-$particle correlations with bipartitions $A = i + j$.

Green's function and T-matrix: separable approximation



The T_A matrix fulfills the Bethe-Salpeter equation in ladder approximation

$$T_{i+j}(1,2,\ldots,A;1',2',\ldots A';z) = V_{i+j} + V_{i+j}G^{(0)}_{i+j}T_{i+j} \ ,$$

which in the separable approximation for the interaction potential,

$$V_{i+j} = \Gamma_{i+j}(1,2,\ldots,i;i+1,i+2,\ldots,i+j)\Gamma_{i+j}(1',2',\ldots,i';(i+1)',(i+2)',\ldots,(i+j)') \ ,$$

leads to the closed expression for the T_A matrix

$$T_{i+j}(1,2,\ldots,i+j;1',2',\ldots(i+j)';z) = V_{i+j}\left\{1-\Pi_{i+j}\right\}^{-1} \ ,$$

with the generalized polarization function

$$\Pi_{i+j} = \mathrm{Tr}\left\{\Gamma_{i+j}G^{(0)}_i\Gamma_{i+j}G^{(0)}_j\right\}$$

The one-frequency free $i-$particle Green's function is defined by the $(i-1)$-fold Matsubara sum

$$\begin{aligned} G^{(0)}_i(1,2,\ldots,i;\Omega_i) &= \sum_{\omega_1\ldots\omega_{i-1}} \frac{1}{\omega_1-E(1)}\frac{1}{\omega_2-E(2)}\cdots\frac{1}{\Omega_i-(\omega_1+\ldots\omega_{i-1})-E(i)} \\ &= \frac{(1-f_1)(1-f_2)\ldots(1-f_i)-(-)^i f_1 f_2\ldots f_i}{\Omega_i-E(1)-E(2)-\ldots E(i)} \ . \end{aligned}$$

Useful relationships for many-particle functions

$$G^{(0)}_{i+j} = G^{(0)}_{i+j}(1,2,\ldots,i+j;\Omega_{i+j}) \quad = \quad \sum_{\Omega_i} G^{(0)}_i(1,2,\ldots,i;\Omega_i)G^{(0)}_j(i+1,i+2,\ldots,i+j;\Omega_j) \ .$$

Another set of useful relationships follows from the fact that in the ladder approximation both, the full two-cluster ($i+j$ particle) T matrix and the corresponding Greens' function

$$G_{i+j} = G^{(0)}_{i+j}\left\{1-\Pi_{i+j}\right\}^{-1} \tag{1}$$

have similar analytic properties determined by the $i+j$ cluster polarization loop integral and are related by the identity

$$T_{i+j}G^{(0)}_{i+j} = V_{i+j}G_{i+j} \ . \tag{2}$$

which is straightforwardly proven by multiplying Equation for the $T_{i+j}-$ matrix with $G^{(0)}_{i+j}$ and using Equation (1). Since these two equivalent expressions in Equation (2) are at the same time equivalent to the two-cluster irreducible Φ functional these functional relations follow

$$T_{i+j} = \delta\Phi/\delta G^{(0)}_{i+j} \ ,$$
$$V_{i+j} = \delta\Phi/\delta G_{i+j} \ .$$

Next we prove the relationship to the Generalized Beth-Uhlenbeck approach!

GBU EoS from the $\Phi-$derivable approach

Consider the partial density of the $A-$particle state defined as

$$n_A(T,\mu) = -\frac{\partial \Omega_A}{\partial \mu} = -\frac{\partial}{\partial \mu} d_A \int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left[\ln\left(-G_A^{-1}\right) + \mathrm{Tr}\left(\Sigma_A\ G_A\right)\right] + \sum_{\substack{i,j \\ i+j=A}} \Phi[G_i, G_j, G_{i+j}] \ . \qquad (3)$$

Using spectral representation for $F(\omega)$ and Matsubara summation

$$F(iz_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\mathrm{Im} F(\omega)}{\omega - iz_n} \ , \quad \sum_{z_n} \frac{c_A}{\omega - iz_n} = f_A(\omega) = \frac{1}{\exp[(\omega-\mu)/T] - (-1)^A}$$

with the relation $\partial f_A(\omega)/\partial\mu = -\partial f_A(\omega)/\partial\omega$ we get for Equation (3) now

$$n_A(T,\mu) = -d_A \int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial\omega} \left[\mathrm{Im}\ln\left(-G_A^{-1}\right) + \mathrm{Im}\left(\Sigma_A\ G_A\right)\right] + \sum_{\substack{i,j \\ i+j=A}} \frac{\partial \Phi[G_i, G_j, G_A]}{\partial\mu} \ ,$$

where a partial integration over ω has been performed For two-loop diagrams of the sunset type holds a cancellation[3] which we generalize here for cluster states

$$d_A \int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial\omega} \left(\mathrm{Re}\Sigma_A\ \mathrm{Im} G_A\right) - \sum_{\substack{i,j \\ i+j=A}} \frac{\partial \Phi[G_i, G_j, G_A]}{\partial\mu} = 0 \ .$$

Using generalized optical theorems we can show that ($G_A = |G_A| \exp(i\delta_A)$)

$$\frac{\partial}{\partial\omega} \left[\mathrm{Im}\ln\left(-G_A^{-1}\right) + \mathrm{Im}\Sigma_A\ \mathrm{Re}G_A\right] = 2\mathrm{Im}\left[G_A\ \mathrm{Im}\Sigma_A \frac{\partial}{\partial\omega} G_A^*\ \mathrm{Im}\Sigma_A\right] = -2\sin^2\delta_A \frac{\partial\delta_A}{\partial\omega} \ .$$

The density in the form of a generalized Beth-Uhlenbeck EoS follows

$$n(T,\mu) = \sum_{i=1}^{A} n_i(T,\mu) = \sum_{i=1}^{A} d_i \int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_i(\omega) 2\sin^2\delta_i \frac{\partial\delta_i}{\partial\omega} \ .$$

[3] B. Vanderheyden & G. Baym, J. Stat. Phys. (1998), J.-P. Blaizot et al., PRD (2001)

Example: Deuterons in Nuclear Matter

The $\Phi-$derivable thermodynamical potential for the nucleon-deuteron system reads

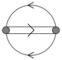
$$\Omega = -\mathrm{Tr}\,\{\ln(-G_1)\} - \mathrm{Tr}\{\Sigma_1 G_1\} + \mathrm{Tr}\,\{\ln(-G_2)\} + \mathrm{Tr}\{\Sigma_2 G_2\} + \Phi[G_1, G_2]\ ,$$

where the full propagators obey the Dyson-Schwinger equations

$$G_1^{-1}(1,z) = z - E_1(p_1) - \Sigma_1(1,z);\ \ G_2^{-1}(12,1'2',z) = z - E_1(p_1) - E_2(p_2) - \Sigma_2(12,1'2',z),$$

with selfenergies and Φ functional

$$\Sigma_1(1,1') \quad = \quad \frac{\delta\Phi}{\delta G_1(1,1')}\ ;\ \ \Sigma_2(12,1'2',z) = \frac{\delta\Phi}{\delta G_2(12,1'2',z)}\ , \Phi =$$



,

fulfilling stationarity of the thermodynamic potential $\partial\Omega/\partial G_1 = \partial\Omega/\partial G_2 = 0$.
For the density we obtain the cluster virial expansion

$$n = -\frac{1}{V}\frac{\partial\Omega}{\partial\mu} = n_{\mathrm{qu}}(\mu, T) + 2n_{\mathrm{corr}}(\mu, T)\ ,$$

with the correlation density in the generalized Beth-Uhlenbeck form

$$n_{\mathrm{corr}} = \int \frac{dE}{2\pi} g(E) 2\sin^2\delta(E)\frac{d\delta(E)}{dE}\ .$$

Cluster Virial Expansion for Quark-Hadron Matter within the GBU Approach

The cluster decomposition of the thermodynamic potential is given as

$$\Omega_{\rm total}(T,\mu,\phi,\bar\phi) = \Omega_{PNJL}(T,\mu,\phi,\bar\phi) + \Omega_{\rm pert}(T,\mu,\phi,\bar\phi) + \Omega_{MHRG}(T,\mu,\phi,\bar\phi), \qquad (4)$$

where the first two terms describe the quark and gluon degrees of freedom via the mean–field thermodynamic potential for quark matter in a gluon background field $\mathcal{U}$

$$\Omega_{PNJL}(T,\mu,\phi,\bar\phi) = \Omega_Q(T,\mu,\phi,\bar\phi) + \mathcal{U}(T,\phi,\bar\phi) \qquad (5)$$

with a perturbative correction $\Omega_{\rm pert}(T,\mu,\phi,\bar\phi)$.
The Mott-Hadron-Resonance-Gas (MHRG) part for the multi-quark clusters is

$$\Omega_{MHRG}(T,\mu,\phi,\bar\phi) = \sum_{i=M,B,\dots} \Omega_i(T,\mu,\phi,\bar\phi), \qquad (6)$$

where the multi–quark states, described by the GBU formula for color–singlet species:

$$\Omega_i(T,\mu,\phi,\bar\phi) = \pm d_i \int_0^\infty \frac{dp\, p^2}{2\pi^2} \int_0^\infty \frac{dM}{\pi} \frac{M}{E_p} \left\{ f_\phi^{(a),+} + f_\phi^{(a),-} \right\}\Big|_{\phi=1} \delta_i(M,T,\mu), \qquad (7)$$

color–triplet species (color antitriplet is analogous):

$$\Omega_i(T,\mu,\phi,\bar\phi) = \pm d_i \int_0^\infty \frac{dp\, p^2}{2\pi^2} \int_0^\infty \frac{dM}{\pi} \frac{M}{E_p} \left\{ f_\phi^{(a),+} + \left[f_\phi^{(a),-} \right]^* \right\} \delta_i(M,T,\mu), \qquad (8)$$

where d_i is the degeneracy factor, a is the net number of valence quarks in the cluster

Polyakov-loop modified distribution functions

For multiquark clusters with net number a of valence quarks holds

$$f_\phi^{(a),\pm} \stackrel{(a\ \text{even})}{=} \frac{(\phi - 2\bar{\phi} y_a^\pm) y_a^\pm + {y_a^\pm}^3}{1 - 3(\phi - \bar{\phi} y_a^\pm) y_a^\pm - {y_a^\pm}^3} \,, \quad (9)$$

$$f_\phi^{(a),\pm} \stackrel{(a\ \text{odd})}{=} \frac{(\bar{\phi} + 2\phi y_a^\pm) y_a^\pm + {y_a^\pm}^3}{1 + 3(\bar{\phi} + \phi y_a^\pm) y_a^\pm + {y_a^\pm}^3} \,, \quad (10)$$

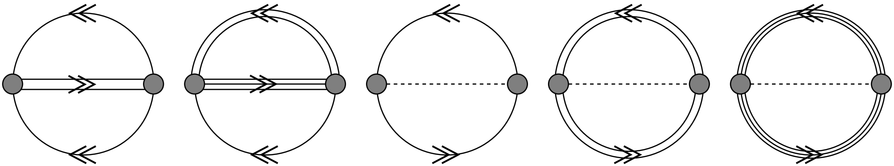
where $y_a^\pm = e^{-(E_p \mp a\mu)/T}$ and $E_p = \sqrt{\vec{p}^2 + M^2}$.
It is instructive to consider the two limits $\phi = \bar{\phi} = 1$ (deconfinement)

$$f_{\phi=1}^{(a=0,2,4,\dots),\pm} = \frac{y_a^\pm}{1 - y_a^\pm} \,, \quad f_{\phi=1}^{(a=1,3,5,\dots),\pm} = \frac{y_a^\pm}{1 + y_a^\pm} \,, \quad (11)$$

and $\phi = \bar{\phi} = 0$ (confinement),

$$f_{\phi=0}^{(a=0,2,4,\dots),\pm} = \frac{{y_a^\pm}^3}{1 - {y_a^\pm}^3} \,, \quad f_{\phi=0}^{(a=1,3,5,\dots),\pm} = \frac{{y_a^\pm}^3}{1 + {y_a^\pm}^3} \,. \quad (12)$$

$$\Omega = \sum_{i=Q,M,D,B} c_i \left[\mathrm{Tr} \ln \left(-G_i^{-1} \right) + \mathrm{Tr} \left(\Sigma_i \; G_i \right) \right] + \Phi \left[G_Q, G_M, G_D, G_B \right] \; ,$$



When Φ functional for the system is given by 2-loop diagrams holds

$$\begin{aligned} n &= -\frac{\partial \Omega}{\partial \mu} = \sum_a a \, n_a(T, \mu) \\ &= \sum_a a \, d_a \int \frac{d\omega}{\pi} \int \frac{d^3 q}{(2\pi)^3} \left\{ f_\phi^{(a),+} - \left[f_\phi^{(a),-} \right]^* \right\} 2 \sin^2 \delta_a(\omega, q) \frac{\partial \delta_a(\omega, q)}{\partial \omega} \; , \end{aligned}$$

Analogous for the entropy density $s = -\partial \Omega / \partial T$.

The selfenergies $\Sigma_i = \delta\, \Phi\,[G_Q, G_M, G_D, G_B]/\delta\; G_i$

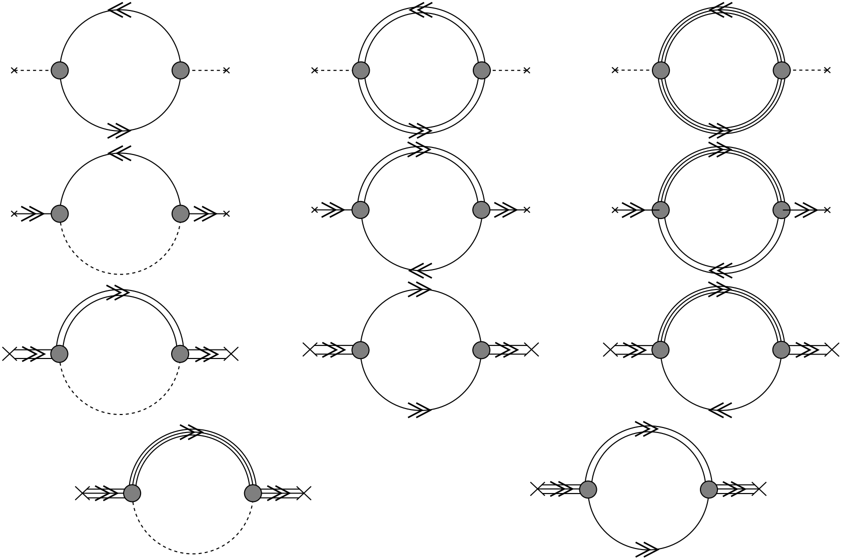


Figure: Selfenergies for Greens functions of Q-M-D-B system in 2-loop approx.

Lattice QCD based effective model: Multiquark-continuum and bound states

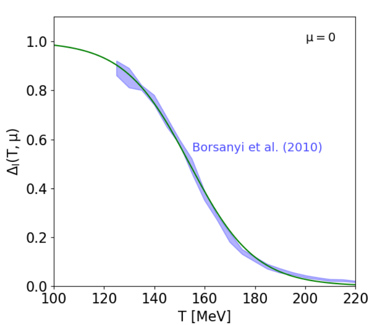
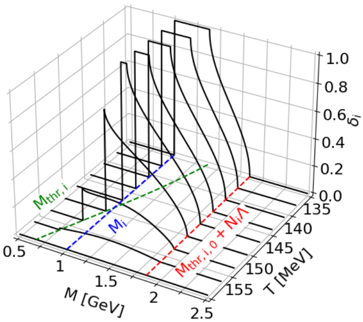
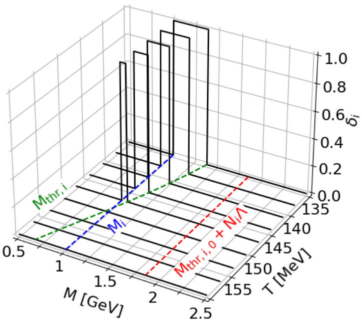


Figure: Left: Lattice result of the 2 flavor QCD chiral condensate from Borsanyi et al. (2010) and the fit to $\Delta_l(T,\mu)$; Right: Schematic model for the mass spectrum of multiquark states and their continuum thresholds at finite temperatures.

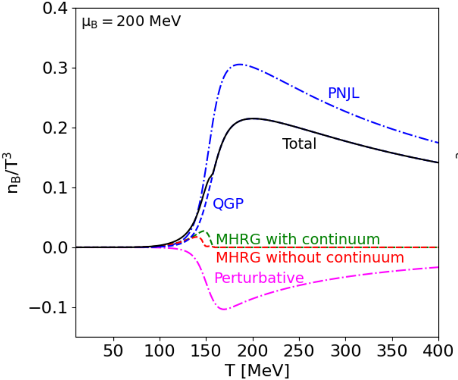
Models for multiquark phase shifts with Mott transition and Levinson theorem

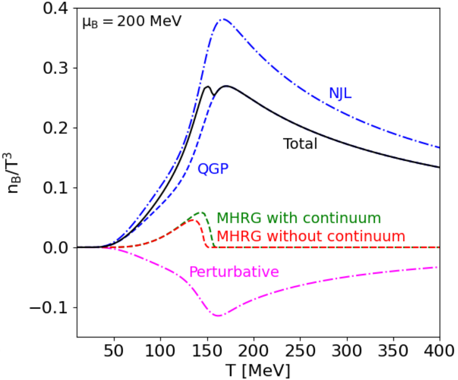




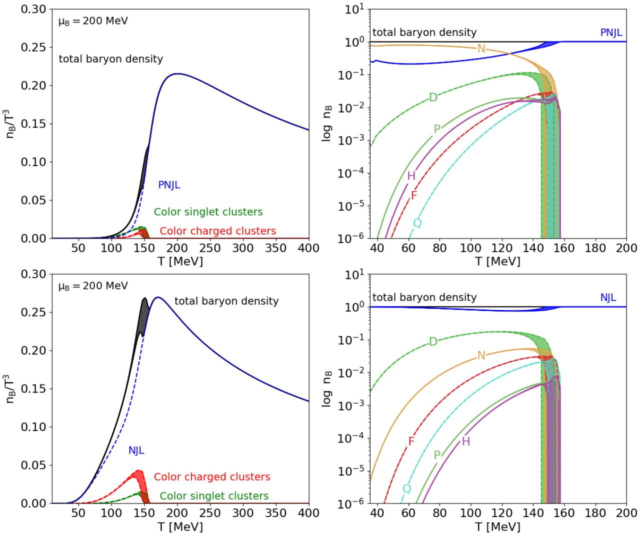
Left: Simple step-up-step-down model without continuum states;
Right: Phase shift model with continuum states.

Results for the baryon density at constant $\mu_B = 200$ MeV

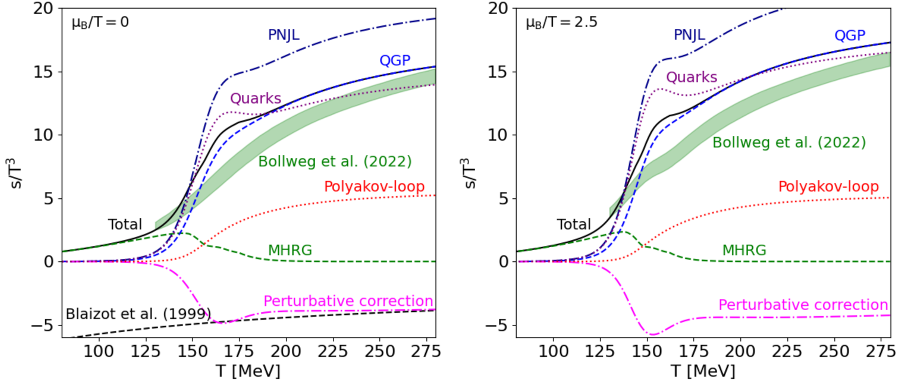




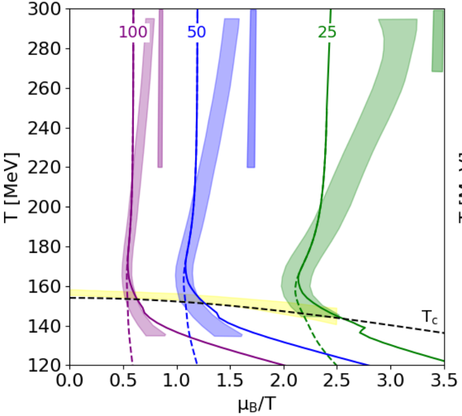
Results for multiquark cluster fractions at $\mu_B = 200$ MeV

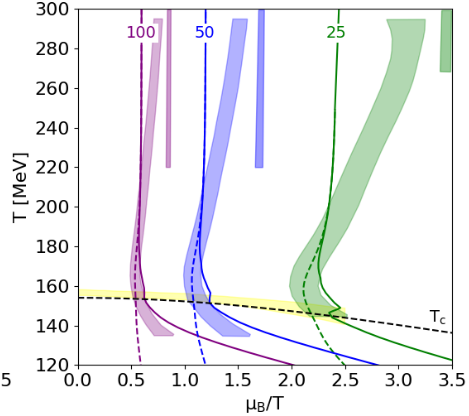


Results for the entropy density at constant s/n

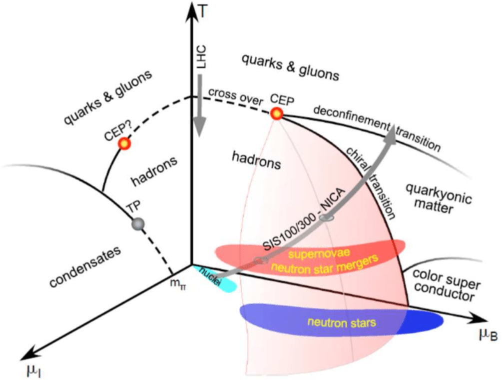


Results for the QCD phase diagram, lines of constant entropy per baryon $s/n = const$



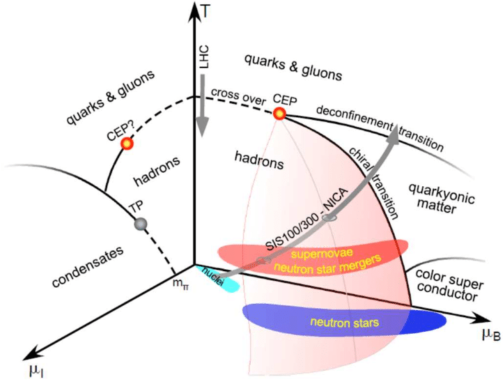


QCD Phase Diagram: Astrophysics vs. Heavy-Ion Coll.



From: NuPECC Long Range Plan 2017

QCD Phase Diagram: Astrophysics vs. Heavy-Ion Coll.



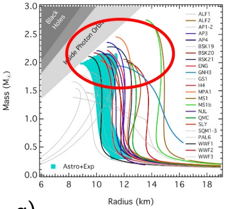
From: NuPECC Long Range Plan 2017

Prominent contributions to deconfinement in modern multimessenger Astrophysics:

- Quark deconfinement transition triggers the supernova explosion of a very massive ($M = 50M_\odot$) blue supergiant progenitor star
  T. Fischer et al., Nature Astron. 2 (2018) 960
- Unambiguous signal of a strong phase transition in the postmerger GW from a binary NS merger predicted
  A. Bauswein et al., Phys. Rev. Lett. 122 (2019) 061102
- Strong deconfinement phase transition in NS can be detected by observing the mass twin star phenomenon
  D. B. et al., Universe 6 (2020) 81

Confining relativistic density functional w. color supercond.

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - \hat{m})q + \mathcal{L}_V + \mathcal{L}_D - \mathcal{U}$$



- **Vector repulsion**

$$\mathcal{L}_V = -G_V(\overline{q}\gamma_\mu q)^2$$

  - needed to reach $2M_\odot$
  - motivated by non-perturbative gluon exchange

  S. Yong et al., Phys. Rev. D 100, 034018 (2019)

- **Diquark pairing**

$$\mathcal{L}_D = G_D \sum_{A=2,5,7} (\overline{q} i\gamma_5\tau_2\lambda_A q^c)(\overline{q}^c i\gamma_5\tau_2\lambda_A q)$$

  - motivated by Cooper theorem
  - color superconductivity

- **Density functional** (G_0 – coupling, $\alpha \geq 0$, $\langle\overline{q}q\rangle_0$ – χ-condensate in vacuum)

$$\mathcal{U} = G_0\left[(1+\alpha)\langle\overline{q}q\rangle_0^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]^{\frac{1}{3}}$$

  - motivated by String Flip Model
  - χ-symmetric interaction

O. Ivanytskyi and D.B., Phys. Rev. D 105 (2022) 114042

Expansion around $\langle \bar{q}q \rangle$ and $\langle \bar{q}i\vec{\tau}\gamma_5 q \rangle = 0$

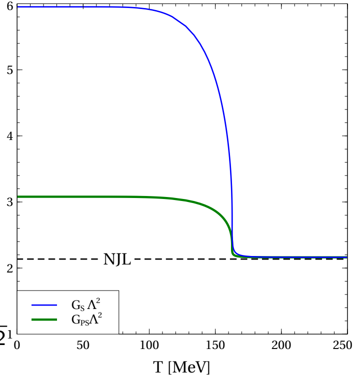
$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{\textbf{0}^{\textbf{th}}\ \textbf{order}} + \underbrace{\left(\bar{q}q - \langle \bar{q}q \rangle\right)\Sigma_{MF}}_{\textbf{1}^{\textbf{st}}\ \textbf{order}} - \underbrace{G_S\left(\bar{q}q - \langle \bar{q}q \rangle\right)^2 - G_{PS}\left(\bar{q}i\vec{\tau}\gamma_5 q\right)^2}_{\textbf{2}^{\textbf{nd}}\ \textbf{order}} + \ldots$$

- **Mean-field self-energy**

$$\Sigma_{MF} = \frac{\partial \mathcal{U}_{MF}}{\partial \langle \bar{q}q \rangle}$$

- **Effective medium dependent couplings**

$$G_S = -\frac{1}{2}\frac{\partial^2 \mathcal{U}_{MF}}{\partial \langle \bar{q}q \rangle^2}, \qquad G_{PS} = -\frac{1}{6}\frac{\partial^2 \mathcal{U}_{MF}}{\partial \langle \bar{q}i\vec{\tau}\gamma_5 q \rangle^2}$$



Mass radius diagram

- **Hydrostatic equilibrium**

$$\frac{dp}{dr} = -\frac{Gm\epsilon}{r^2}\frac{\left(1+\frac{p}{\epsilon}\right)\left(1+\frac{4\pi r^3 p}{m}\right)}{\left(1-\frac{2Gm}{r}\right)}$$

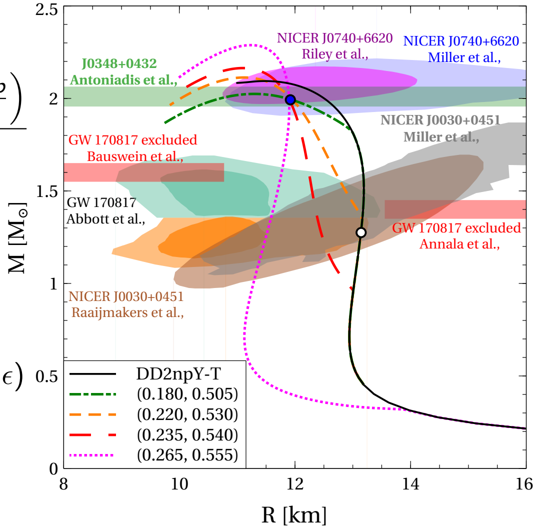
$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

- **Cold matter EoS**

$$\begin{cases} p = p(\mu_B) \\ \epsilon = \mu_B \frac{\partial p}{\partial \mu_B} - p \end{cases} \Rightarrow p = p(\epsilon)$$

- **Total radius R and mass M**

$$r = R \quad \Rightarrow \begin{cases} p = 0 \\ m = M \end{cases}$$



O. Ivanytskyi and D. Blaschke, Phys. Rev. D 105, 114042 (2022)

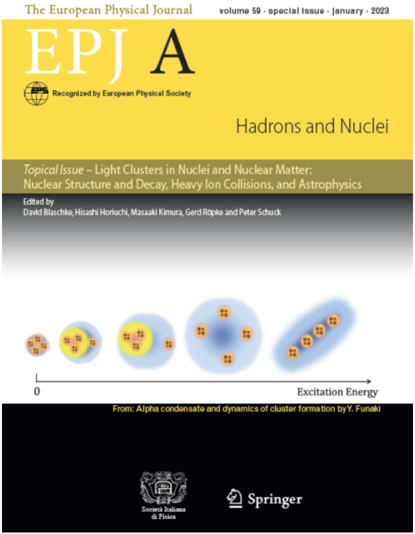
Summary

- cluster virial expansion developed for sunset-type Φ functionals made of cluster Green's functions and a cluster T-matrix
- cluster Φ functional approach to quark-meson-diquark-baryon system developed and example for meson dissociation outlined
- quark Pauli blocking in hadronic matter is contained in the approach
- selfconsistent density-functional approach to quark matter with confinement and chiral symmetry breaking obtained as limiting case
- applications to nuclear clustering and quark deconfinement in the astrophysics of supernovae and compact stars as well as in heavy-ion collisions

Outlook

- cluster virial expansion for quark-hadron matter as a relativistic density functional with bound state formation and dissociation
- Ginzburg-Landau-type density functional for the QCD phase diagram besides the one for the liquid-gas phase transition in nuclear matter.

EPJ A Topical Collections (TC)



From ECT* Trento Workshop in 2019

New TC:

"The Nuclear Many-Body Problem"

Devoted to the legacy of Peter Schuck

Topics:

- The interacting boson model and collective phenomena in nuclear systems
- Nuclear energy density functionals
- Equation of motion method and extended RPA
- Quantum condensates and pairing
- Alpha-particle clustering
- Pions and related experiments, astrophysics
- Applications in solid state physics, quartetting in semiconductor layers, etc.

More details on the symposium website and

https://epja.epj.org/epja-open-calls-for-papers

New Deadline: 30. June 2023