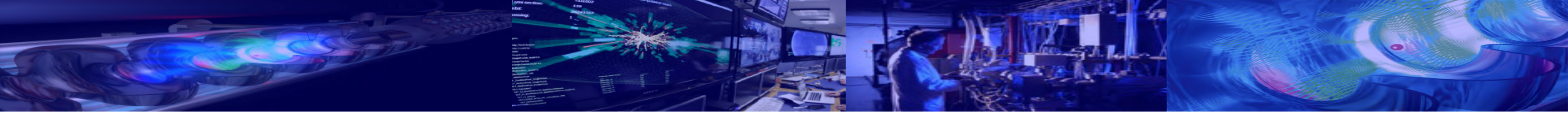


Peter Schuck
(1940-2022)

Relativistic Equation of Motion Framework for Nuclear Physics

Elena Litvinova

Western Michigan University

MICHIGAN STATE UNIVERSITY

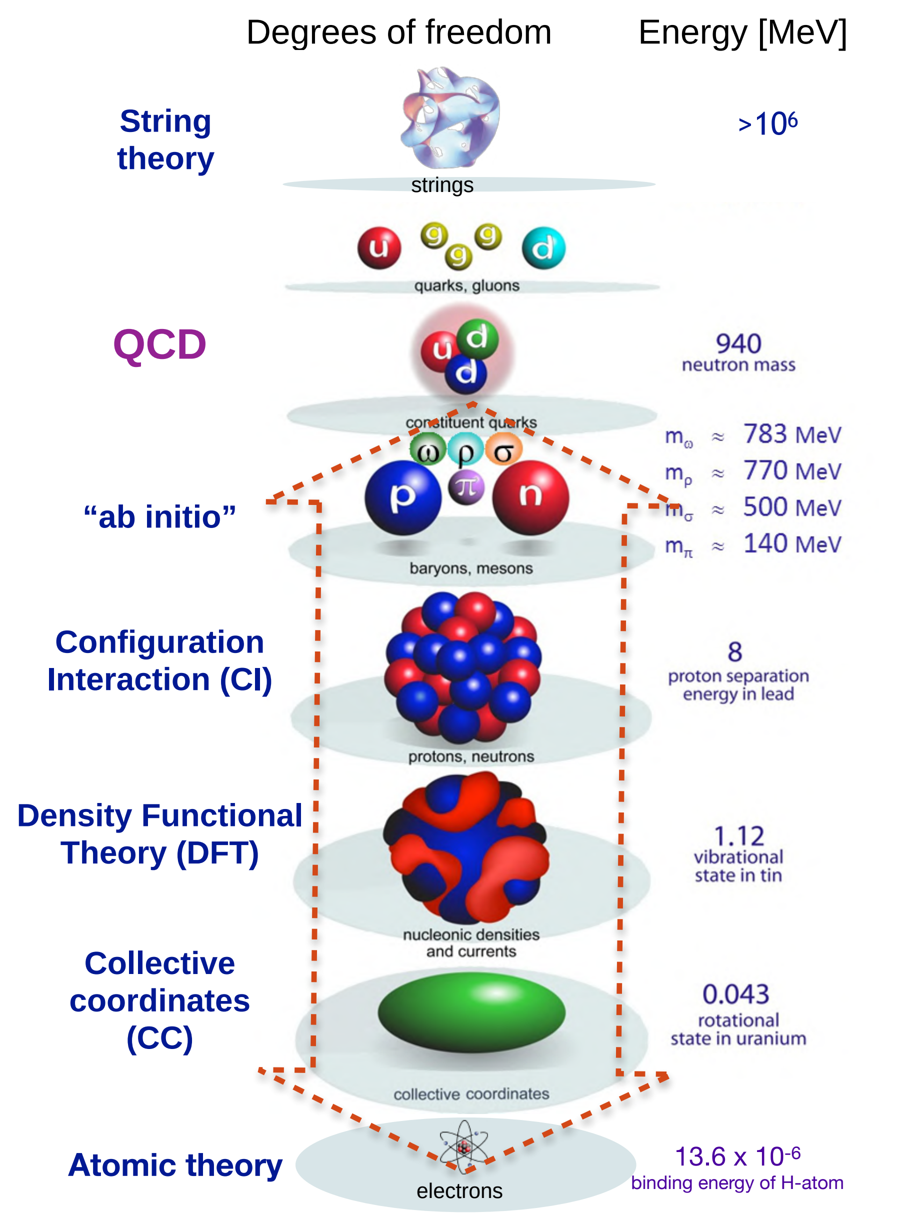
NSCL

NSF CAREER Award

Collaborators: Peter Schuck, Peter Ring, Manqoba Hlatshwayo,
Yinu Zhang, Caroline Robin, Herlik Wibowo

Conference on Quantum Many-Body Correlations in memory of Peter Schuck
IJCLab, Orsay, France, March 21-23, 2023

Hierarchy of energy scales and nuclear many-body problem



- ***The major conflict:***

Separation of energy scales => effective field theories

vs

The physics on a certain scale is governed by the next higher-energy scale

Hamiltonian:

$$H = K + V$$

center of mass

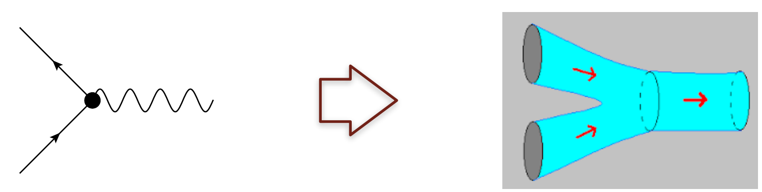
internal degrees of freedom: next energy scale

Standard Model:

free propagation and interaction (input)

String theory:

merging strings
NO "Interaction"

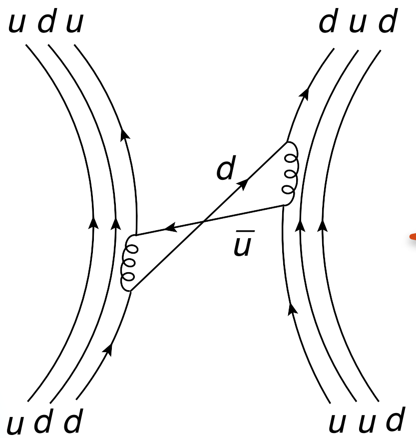


- ***Possible solution:***

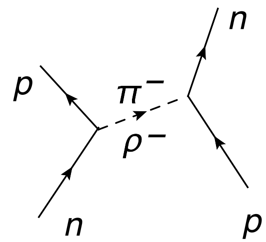
  - *Keep/establish connections between the scales via emergent phenomena*
  - *A universal approach to the strongly-coupled QMBP?*

The underlying mechanism of NN-interaction:

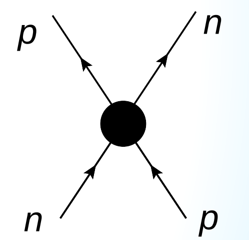
Quantum Chromodynamics (QCD, high energy)



Quantum Hadrodynamics (QHD, intermediate energy)



Nuclear Structure (NS, low energy)

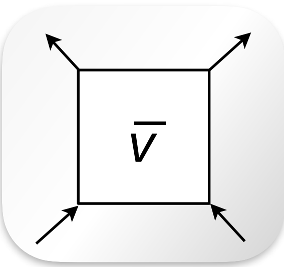


Relay of EFTs:

QCD → *QHD* → *NS*

Formalism:

- *Generic bare "interaction": model-independent, all channels included*
- *Higher-orders are treated via* ***in-medium propagators***
- *No perturbation theory*



In implementations:

- *Meson-exchange (ME) at leading order*
- *Effective coupling constants/ masses (adjusted on the mean-field (MF) level) + subtraction of beyond-MF double-counting (P. Ring et al.)*
- *Bare ME + subtraction of MF artifacts (in progress)*

A strongly-correlated many body system: single-fermion propagator, particle-hole propagator and related observables

$$H = \sum_{12} \bar{\psi}_1 (-i\gamma \cdot \nabla + M)_{12} \psi_2 + \frac{1}{4} \sum_{1234} \bar{\psi}_1 \bar{\psi}_2 \bar{v}_{1234} \psi_4 \psi_3 \; = T + V^{(2)}$$

Hamiltonian, *extendable to 3-body etc.*

$$G_{11'}(t-t') = -i\langle T\psi(1)\bar{\psi}(1')\rangle \qquad 1 = \{\xi_1, t\}$$

$$G_{11'}(\varepsilon) = \sum_n \frac{\eta_1^n \bar{\eta}_{1'}^{n*}}{\varepsilon - \varepsilon_n^+ + i\delta} + \sum_m \frac{\chi_1^m \bar{\chi}_{1'}^{m*}}{\varepsilon + \varepsilon_m^- - i\delta}$$

$$\eta_1^n = \langle 0^{(N)}|\psi_1|n^{(N+1)}\rangle \qquad \chi_1^m = \langle m^{(N-1)}|\psi_1|0^{(N)}\rangle$$

Ground state of N particles

(Excited) state of (N+1) particles

Single-particle propagator

Fourier transform: Spectral expansion

Residues *- spectroscopic (occupation) factors*

Poles *- single-particle energies*

$$R_{12,1'2'}(t-t') = -i\langle T(\bar{\psi}_1\psi_2)(t)(\bar{\psi}_{2'}\psi_{1'})(t')\rangle$$

$$R_{12,1'2'}(\omega) = \sum_{\nu>0} \Big[\frac{\rho_{21}^{\nu}\bar{\rho}_{2'1'}^{\nu*}}{\omega - \omega_\nu + i\delta} - \frac{\bar{\rho}_{12}^{\nu*}\rho_{1'2'}^{\nu}}{\omega + \omega_\nu - i\delta}\Big]$$

Excitation energies

$$\rho_{12}^{\nu} = \langle 0|\bar{\psi}_2\psi_1|\nu\rangle$$

Particle-hole response function

Fourier transform: Spectral expansion

Residues *- transition densities*

Poles *- excitation energies*

Exact equations of motion (EOM) for binary interactions: one-body problem

One-fermion propagator

$$G_{11'}(t-t') = -i\langle T\psi(1)\bar{\psi}(1')\rangle$$

EOM: Dyson Eq.

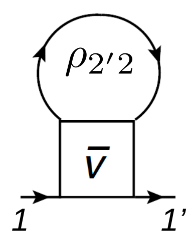
$$G(\omega) = G^{(0)}(\omega) + G^{(0)}(\omega)\Sigma(\omega)G(\omega) \qquad (*) \qquad \Sigma(\omega) = \Sigma^{(0)} + \Sigma^{(r)}(\omega)$$

Irreducible kernel (Self-energy, exact):

Instantaneous term (Hartree-Fock incl. "tadpole")
Short-range correlations

$$\Sigma^{(0)}_{11'} = -\langle\gamma^0\left\{[V,\psi_1],\bar{\psi}_{1'}\ \right\}\gamma^0\rangle$$

$$= \sum_{22'} \bar{v}_{121'2'}\langle\bar{\psi}_2\psi_{2'}\rangle =$$

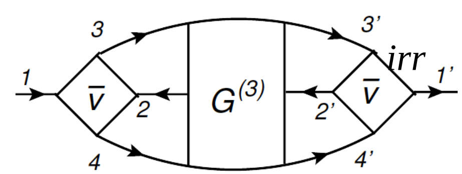


t-dependent (dynamical) term (symmetric version): **Long-range correlations**

$$\Sigma^{(r)}_{11'} = i\langle T\gamma^0[V,\psi_1](t)[V,\bar{\psi}_{1'}\ ](t')\gamma^0\rangle^{irr}$$

$$= -\frac{1}{4}\sum_{234}\sum_{2'3'4'} \bar{v}_{1234}G^{(3)irr}_{432',23'4'}(t-t')\bar{v}_{4'3'2'1'}$$

$$= -\frac{1}{4}$$



$$\rho_{11'} = -i\lim_{t=t'-0} G_{11'}(t-t')$$

is the full solution of (*):
includes the dynamical term!

Koltun-Migdal-Galitsky sum rule: **the binding energy**

"Ab-initio DFT":

$$E_0 = \frac{1}{2\pi}\int_{-\infty}^{\varepsilon_F^-} d\varepsilon \sum_{12}(T_{12}+\varepsilon\delta_{12})\mathrm{Im}G_{21}(\varepsilon)$$

Equation of motion (EOM) for the particle-hole response

Particle-hole propagator (response function):

$$R_{12,1'2'}(t-t') = -i\langle T(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle$$

spectra of excitations, masses, decays, …

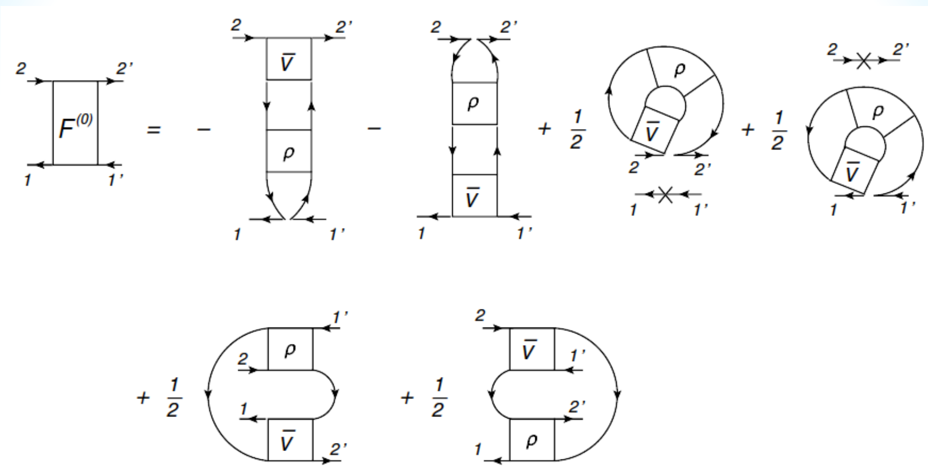
EOM: Bethe-Salpeter-Dyson Eq.

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega) \qquad (**) \qquad F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

Irreducible kernel (exact):

Instantaneous term ("bosonic" mean field):

Short-range correlations



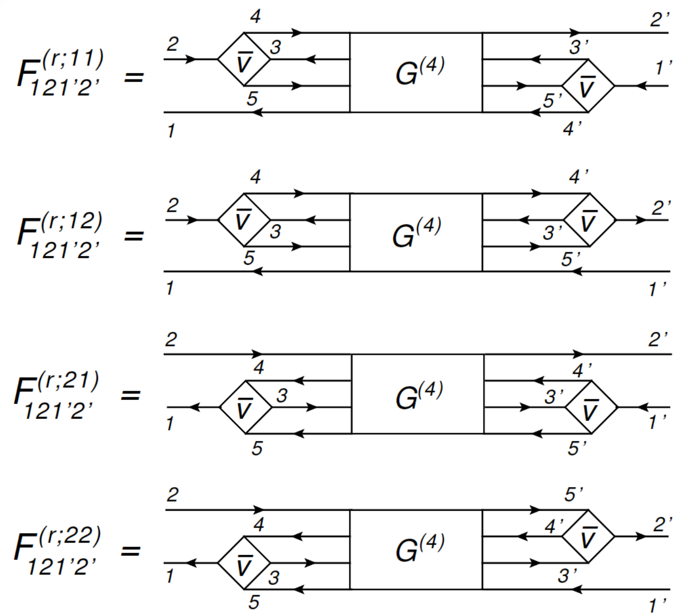
Self-consistent mean field $F^{(0)}$, where

$$\rho_{12,1'2'} = \delta_{22'}\rho_{11'} - i\lim_{t'\to t+0} R_{2'1,21'}(t-t')$$

contains the full solution of (**) **including the dynamical term!**

t-dependent (dynamical) term:

Long-range correlations



$$F^{(r)}_{12,1'2'}(t-t') = \sum_{ij} F^{(r;ij)}_{12,1'2'}(t-t')$$

Non-perturbative treatment of two-point $G^{(n)}$ in the dynamical kernels

- ***Quantum many-body problem in a nutshell:*** *Direct EOM for $G^{(n)}$ generates $G^{(n+2)}$ in the (symmetric) dynamical kernels and further high-rank correlation functions (CFs); an equivalent of the BBGKY hierarchy. $N_{Equations} = N_{Particles}$ & Coupled 🙈 !!!*

Truncation on two-body level

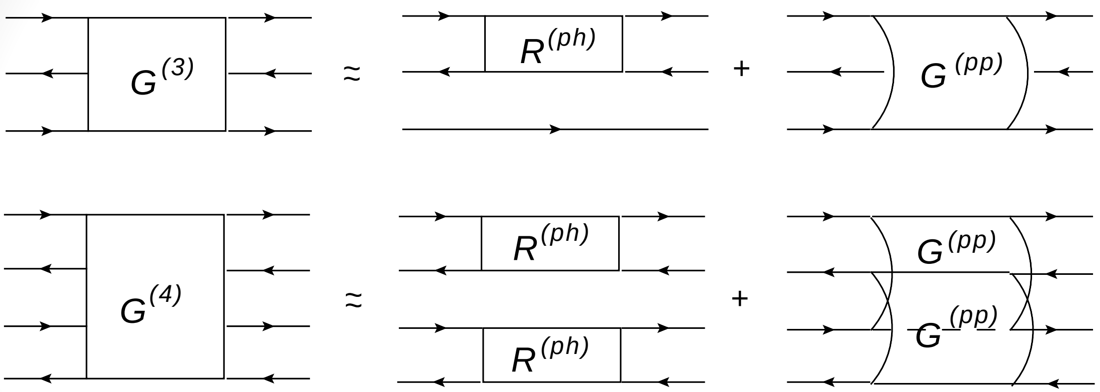
"Self-consistent GFs" *This work*

- ***Non-perturbative solution:***

Cluster decomposition

$$✦\, G^{(3)} = G^{(1)} G^{(1)} G^{(1)} + G^{(2)} G^{(1)} + \Xi^{(3)}$$

$$✦\, G^{(4)} = G^{(1)} G^{(1)} G^{(1)} G^{(1)} + G^{(2)} G^{(2)} + G^{(3)} G^{(1)} + \Xi^{(4)}$$

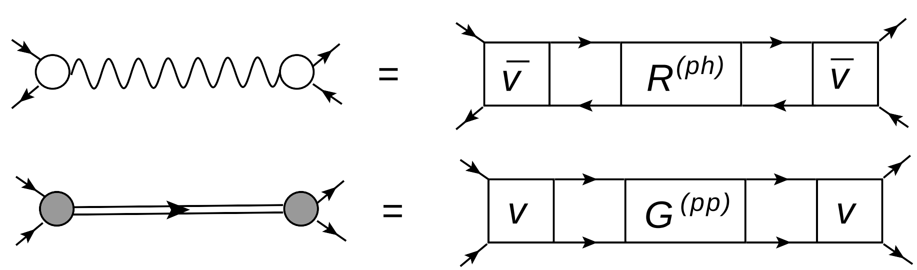


- *P. C. Martin and J. S. Schwinger, Phys. Rev.115, 1342 (1959).*
- *N. Vinh Mau, Trieste Lectures 1069, 931 (1970)*
- *P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994)*
- *…*

Exact mapping: particle-hole (2q) quasibound states

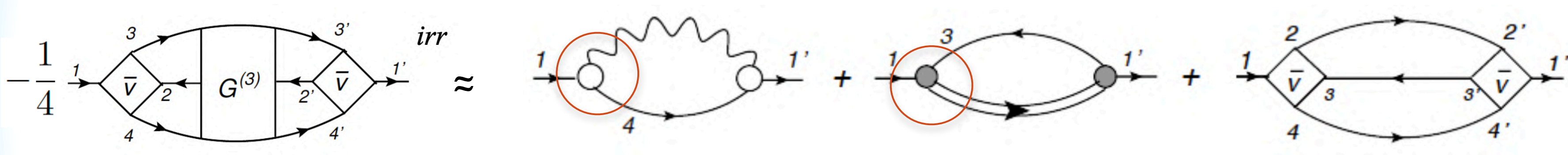
Emergence of effective "particles" (phonons, vibrations):

Emergence of superfluidity:



Emergence of effective degrees of freedom

Dynamical self-energy Σ^(r): — *"Radiative-correction"* — *Second-order*



Quasiparticle-vibration coupling (QVC)

Emergent phonon vertices and propagators: calculable from the underlying H, which does not contain phonon degrees of freedom

$$H = \sum_{12} h_{12}\psi_1^\dagger\psi_2 + \frac{1}{4}\sum_{1234}\bar{v}_{1234}\psi_1^\dagger\psi_2^\dagger\psi_4\psi_3$$

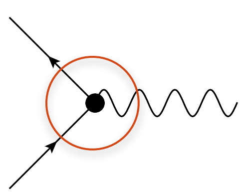
"Ab-initio"

$$H = \sum_{12} \tilde{h}_{12}\psi_1^\dagger\psi_2 + \sum_{\lambda\lambda'}\mathcal{W}_{\lambda\lambda'}Q_\lambda^\dagger Q_{\lambda'} + \sum_{12\lambda}\Big[\Theta_{12}^\lambda\psi_1^\dagger Q_\lambda^\dagger\psi_2 + h.c.\Big]$$

Effective QVC

Cf.: The Standard Model elementary interaction vertices: boson-exchange interaction is the input:

Possibly derivable?

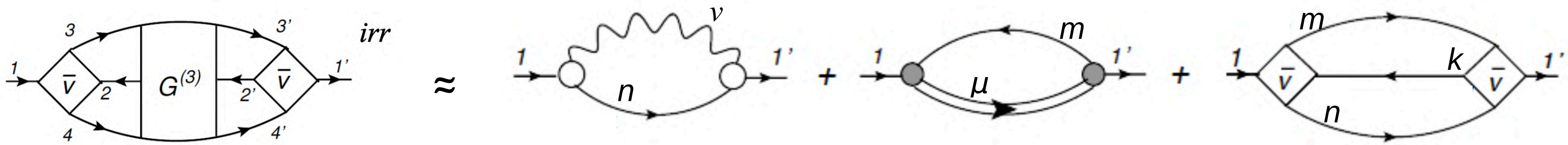


$$\gamma, g, W^\pm, Z^0$$

E.L., P. Schuck, PRC 100, 064320 (2019)
E.L., Y. Zhang, PRC 104, 044303 (2021)

Problems with approximate treatments: poles "mismatch", (non)-positivity and optical theorem

Dynamical self-energy:



Approximate:

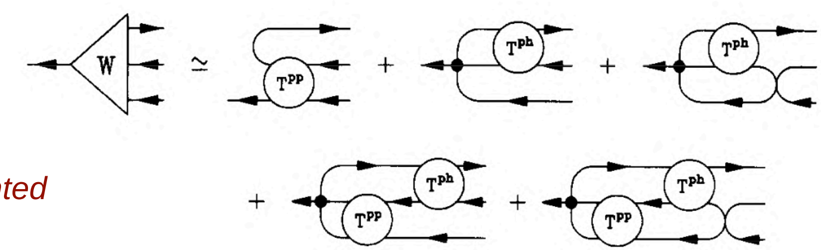
$$\Sigma^{(r)+}_{11'}(\omega) = \sum_{33'}\sum_{\nu n}\frac{\eta^n_3 g^{\nu}_{13} g^{\nu *}_{1'3'}\eta^{n*}_{3'}}{\omega - \omega_\nu - \varepsilon^{(+)}_n + i\delta} + \sum_{22'}\sum_{\mu m}\frac{\chi^{m*}_2 \gamma^{\mu(+)}_{12}\gamma^{\mu(+)*}_{1'2'}\chi^{m}_{2'}}{\omega - \omega^{(++)}_\mu - \varepsilon^{(-)}_m + i\delta} - \sum_{mnk}\frac{w^{mnk}_1 w^{mnk*}_{1'}}{\omega - \varepsilon^+_m - \varepsilon^+_n - \varepsilon^-_k + i\delta}$$

Exact:

$$\Sigma^{(r)+}_{11'}(\omega) \sim \langle v G^{(3)+}(\omega) v\rangle_{11'}$$

$$G^{(3)+}_{432',23'4'}(\omega) = \sum_{\varkappa}\frac{\langle 0|\bar\psi_2\psi_4\psi_3|\varkappa\rangle\langle\varkappa|\bar\psi_{3'}\bar\psi_{4'}\psi_{2'}|0\rangle}{\omega - \omega_\varkappa + i\delta}$$

- Watson-Faddeev series: P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994):

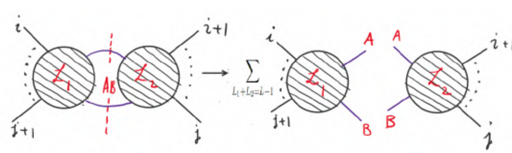


- 2p1h-RPA
P. Schuck, F. Villars and P. Ring
Nucl. Phys. A208, 302 (1973)

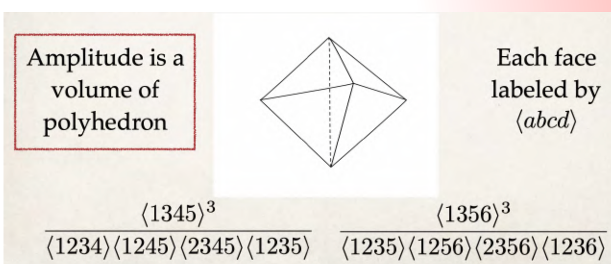
To be implemented for nuclei

Scattering amplitudes in particle physics (Yang-Mills theories etc.):

- Positivity preserved when eliminating all the virtual particles
- Amplitudes <=> Polyhedron "living" in the kinematic space
- Emergent unitarity



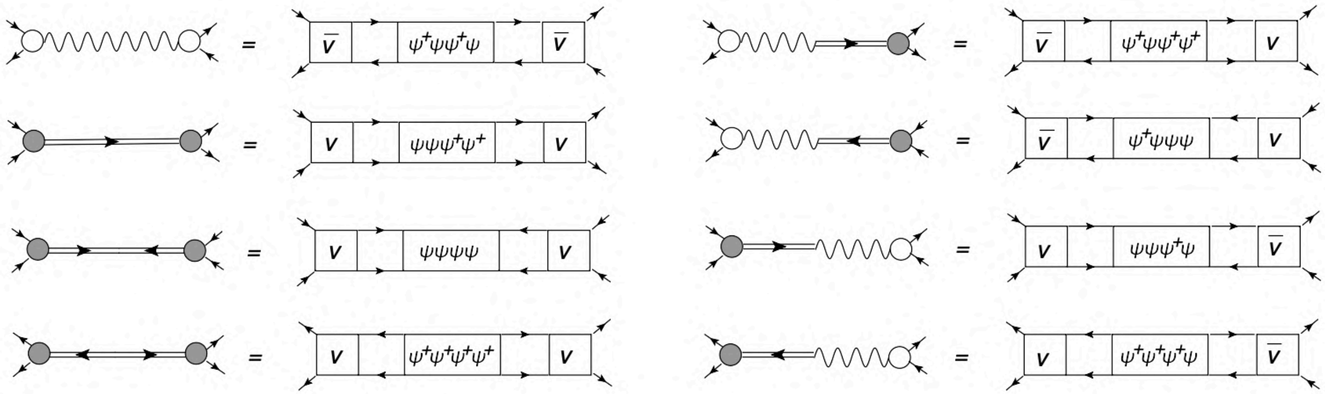
N. Arkani-Hamed, J. Trnka, A. Hodges et al.



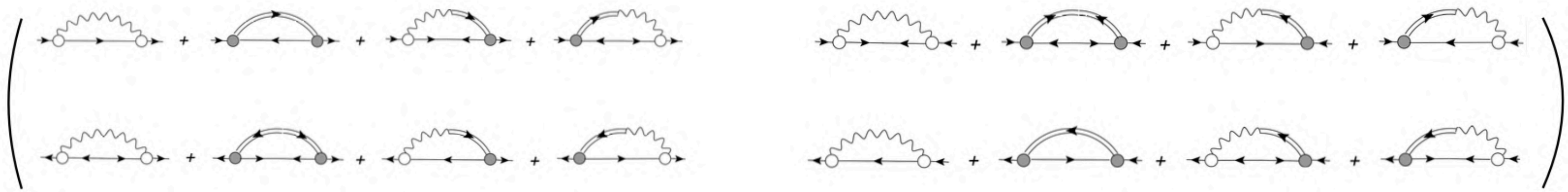
QVC in superfluid systems

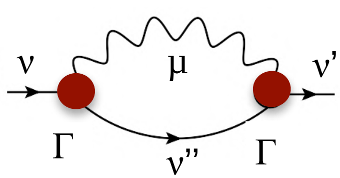
Superfluid dynamical kernel: adding particle-number violating contributions

Mapping on the QVC in the canonical basis



Quasiparticle dynamical self-energy (matrix):
normal and pairing phonons are unified

$\hat{\Sigma}^r =$ 



Bogoliubov transformation

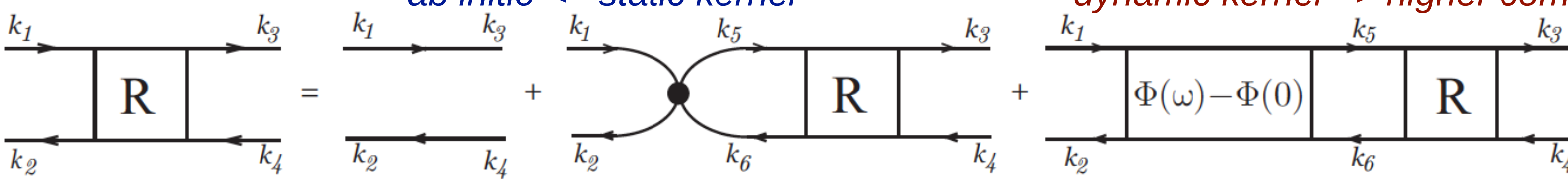
Cf.: Quasiparticle static self-energy (matrix) in HFB

$$\hat{\Sigma}^0 = \begin{pmatrix} \tilde{\Sigma}_{11'} & \Delta_{11'} \\ -\Delta^*_{11'} & -\tilde{\Sigma}^T_{11'} \end{pmatrix}$$

E.L., Y. Zhang, PRC 104, 044303 (2021)
Y. Zhang et al., PRC 105, 044326 (2022)

Nuclear response: toward a complete theory

ab initio <= static kernel — dynamic kernel => higher complexity

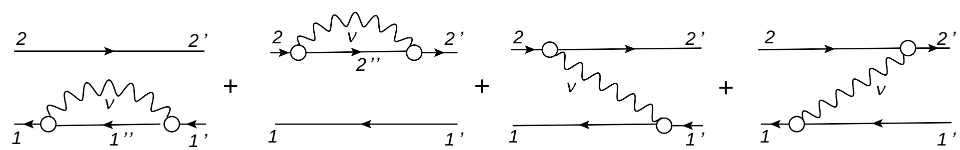


Dyson-Bethe-Salpeter Equation:

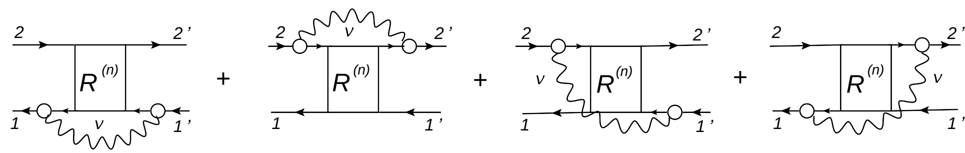
$$R(\omega) = R^0(\omega) + R^0(\omega)\,[V + \Phi(\omega) - \Phi(0)\,]\,R(\omega)$$

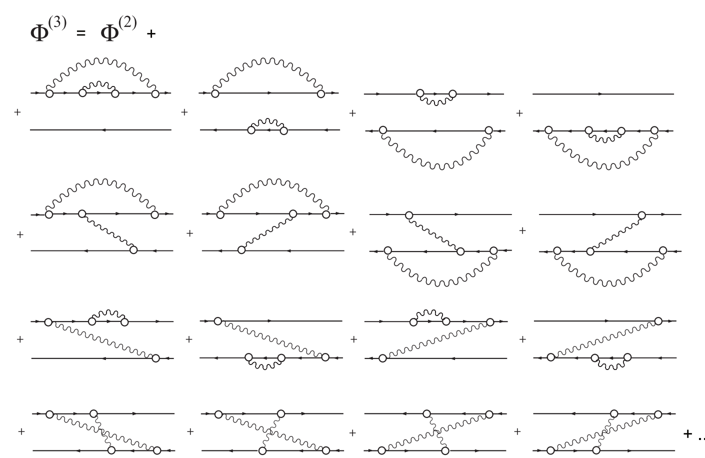
Subtraction for effective interactions (Tselyaev 2013)

Conventional NFT $\Phi_{21,2'1'} =$



Extended NFT: $\Phi^{(n+1)}_{212'1'} =$





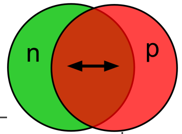
Generalized approach for the correlated propagators

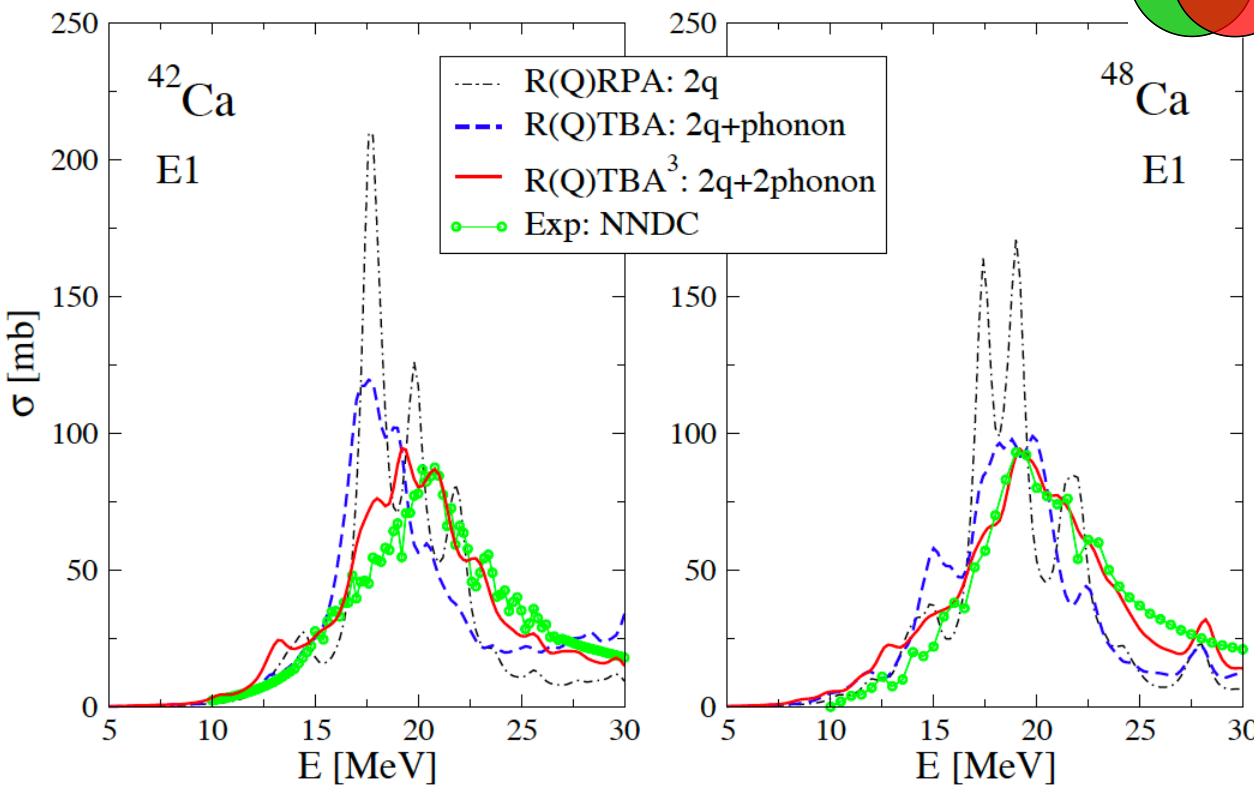
n-th order: E.L. PRC 91, 034332 (2015)

Ab-initio formulation,
$\Phi^{(3)}$ implementation; 2q+2phonon correlations:
E.L., P. Schuck, PRC 100, 064320 (2019)

RQTBA³ with correlated 3p3h configurations: 2q+2phonon

Giant Dipole Resonance in Ca isotopes



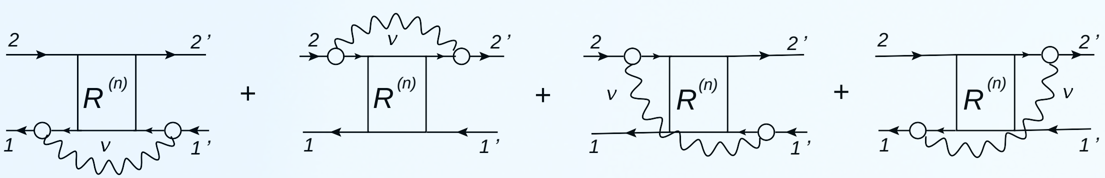


- *The new complex configurations 2q+2phonon included for the first time enforce fragmentation and spreading toward higher and lower energies, thus, modifying both giant and pygmy dipole resonances;*
- *Exp. Data: V.A. Erokhova et al., Bull. Rus. Acad. Phys. 67, 1636 (2003)*
- *RQTBA³ demonstrates an overall systematic improvement of the description of nuclear excited states heading toward spectroscopic accuracy without strong limitations on masses and excitation energies.*

E.L., P. Schuck,
PRC 100, 064320 (2019)

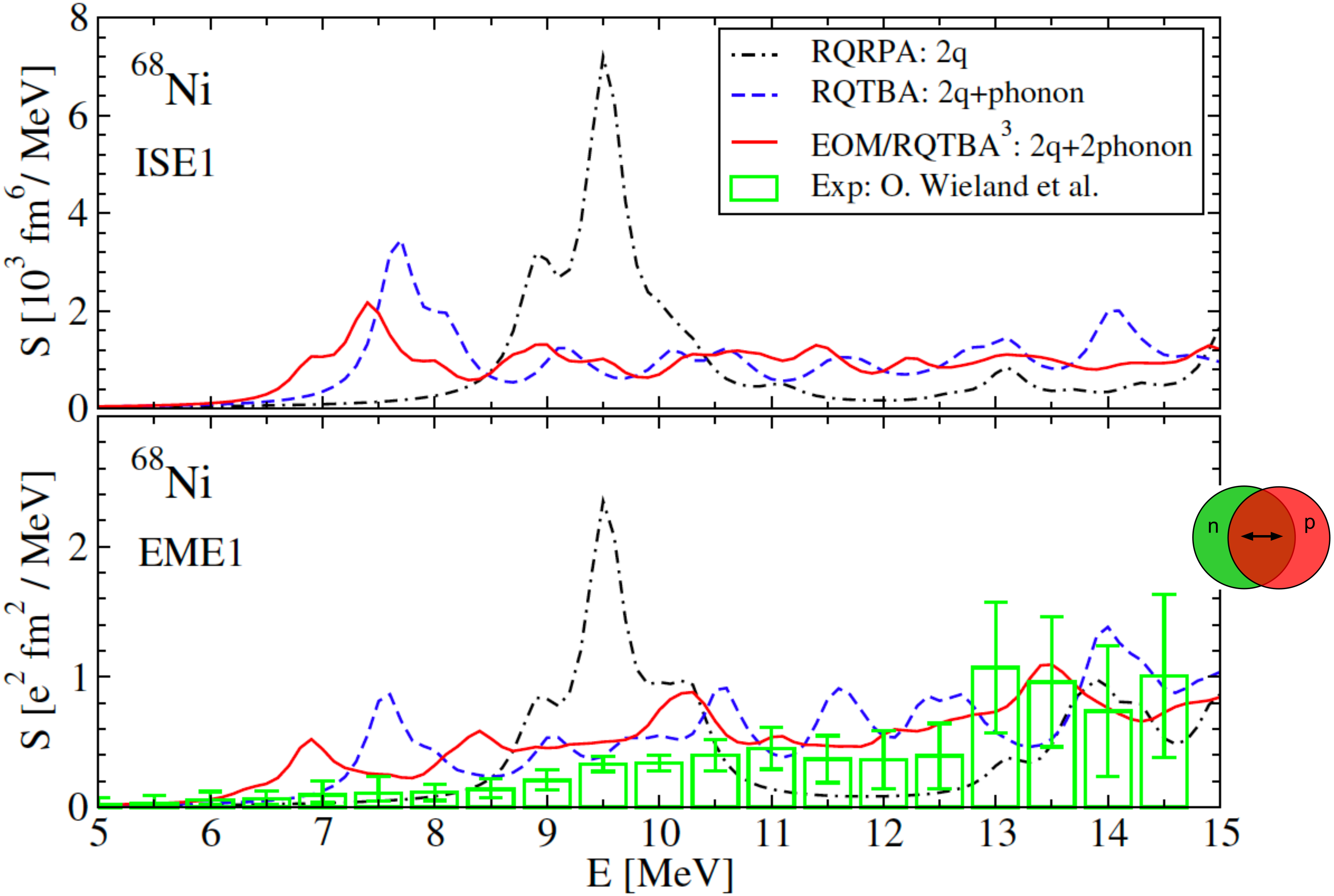
Interaction kernel:
(n = 2)

$$\Phi^{(n+1)}_{212'1'} =$$



RQTBA³ with correlated 3p3h configurations: 2q+2phonon

Low-Energy dipole strength distribution: IS vs EM

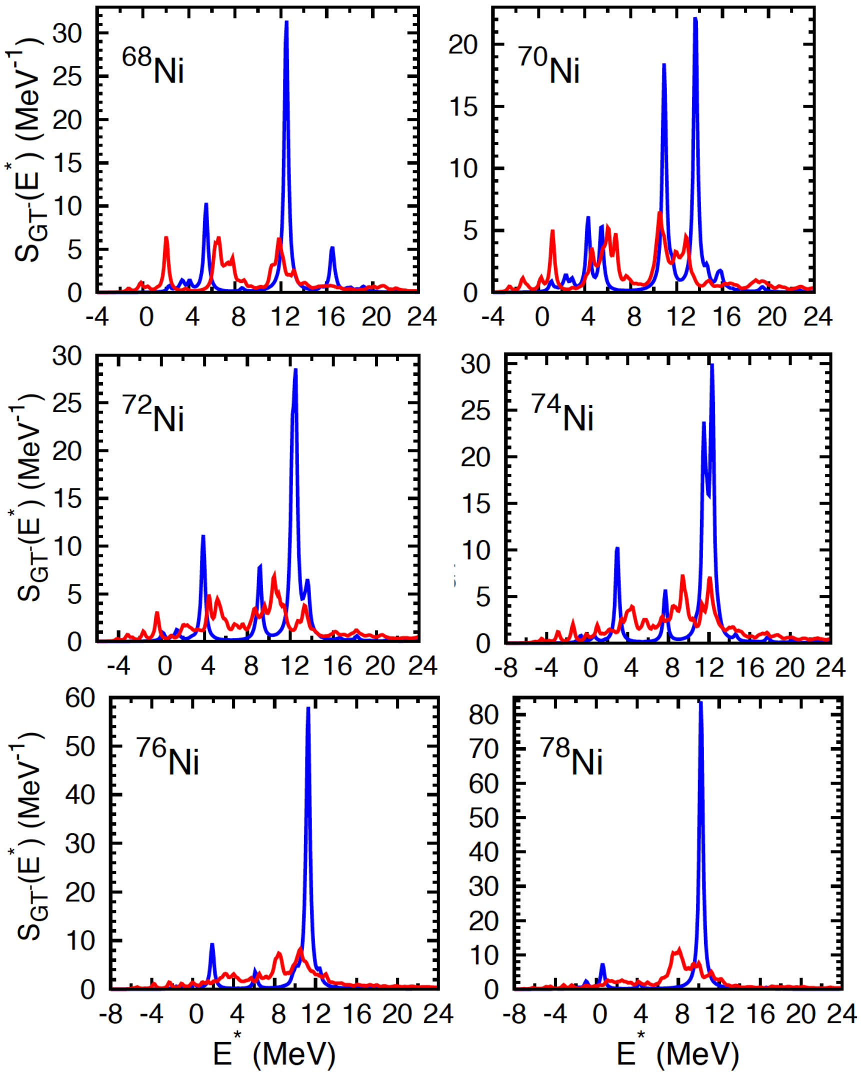


E.L., P. Schuck, PRC 100, 064320 (2019)

Data: O. Wieland et al., Phys. Rev. C 98, 064313 (2018)

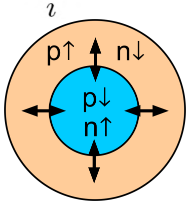
Spin-isospin excitations (Gamow-Teller resonance)
2q+phonon configurations in the dynamical kernel (pn-RQTBA)

Overall strength

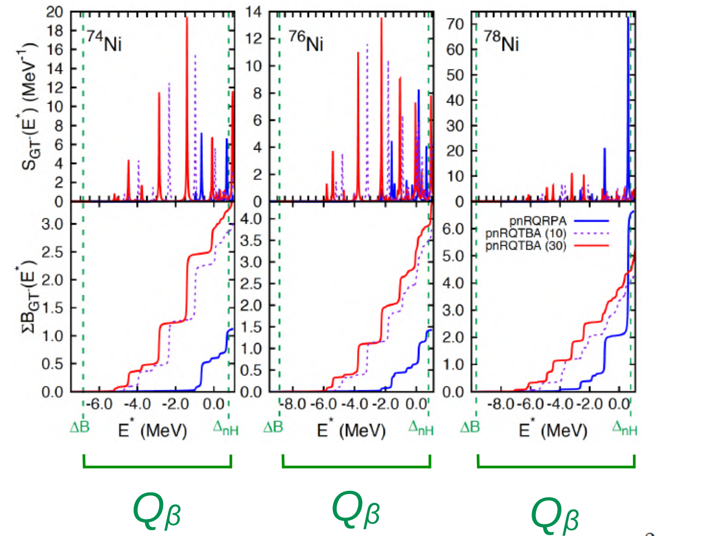


- pn-RQRPA (blue)
- pn-RQTBA (red)

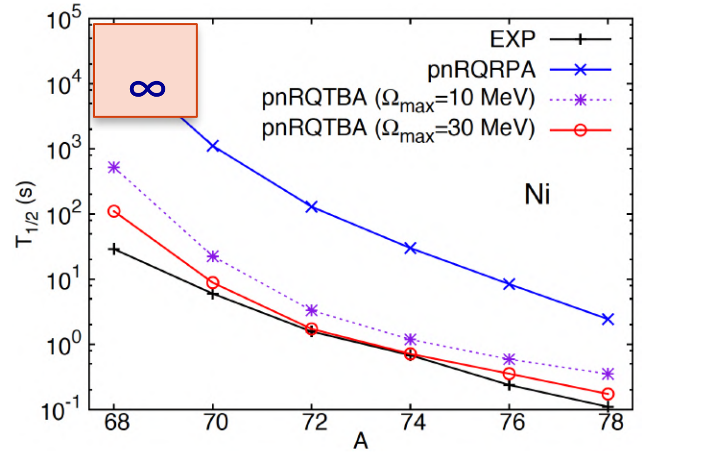
$$P = \sum_i \sigma^{(i)} \tau_{\pm}^{(i)}$$



Low-energy part: Q_β window



$$\frac{1}{T_{1/2}} = \sum_m \lambda_{if}^m = D^{-1} g_A^2 \sum_m \int dE_e \left| \sum_{pn} < 1_\lambda^+ || \boldsymbol{\sigma}\tau_- || 0^+ > \right|^2 \frac{dn_m}{dE_e}$$

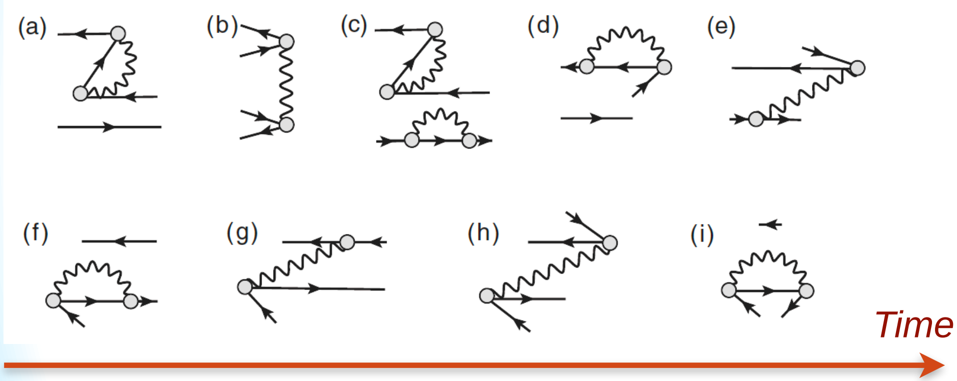


C. Robin, E.L.,
Eur. Phys. J. A 52, 205 (2016)

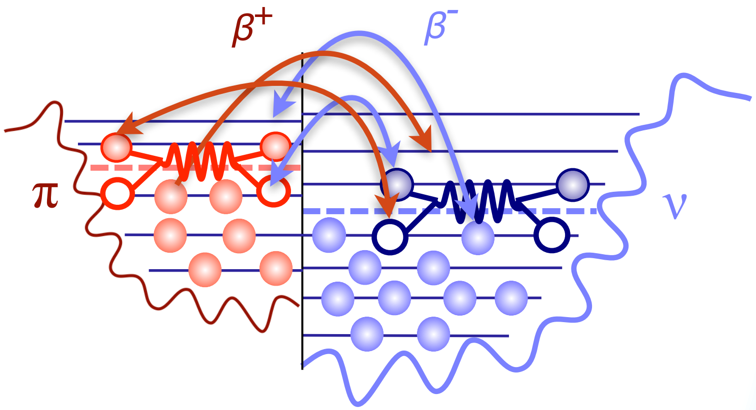
No fits, no artificial quenching,
no adjustable proton-neutron pairing

More correlations: Emergent "time machine"

Ground state correlations induced by QVC:
backward-going diagrams (V. Tselyaev, 1989)

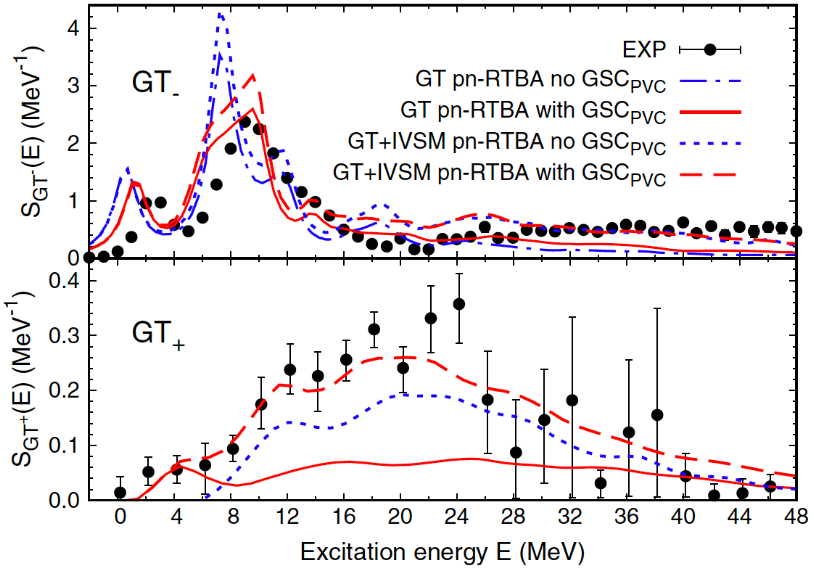


New unblocking mechanism:



The backward-going diagrams are solely responsible for the β+ strength in neutron-rich nuclei

Gamow-Teller strength in 90-Zr:



C. Robin, E.L., Phys. Rev. Lett. 123, 202501 (2019)

Finite-temperature response: the ph+phonon dynamical kernel

$$R_{12,1'2'}(t-t') = -i\langle \mathcal{T}(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle \to -i\langle \mathcal{T}(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle_T$$

$$< ... > \equiv < 0|...|0 > \quad \to \quad < ... >_T \equiv \sum_n exp\Big(\frac{\Omega - E_n - \mu N}{T}\Big) < n|...|n >$$

averages → *thermal averages*

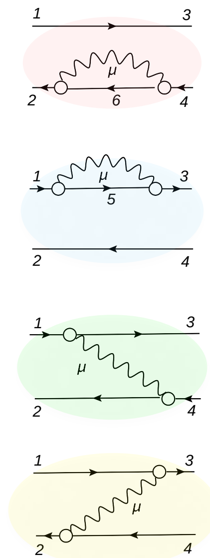
Method: EOM for Matsubara Green's functions

$$\mathcal{R}_{14,23}(\omega,T) = \tilde{\mathcal{R}}^0_{14,23}(\omega,T) + \\ + \sum_{1'2'3'4'} \tilde{\mathcal{R}}^0_{12',21'}(\omega,T)\big[\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega,T)\big]\mathcal{R}_{3'4,4'3}(\omega,T)$$

$$\delta\Phi_{1'4',2'3'}(\omega,T) = \Phi_{1'4',2'3'}(\omega,T) - \Phi_{1'4',2'3'}(0,T)$$

T > 0: *1p1h+phonon dynamical kernel:* *T = 0:*

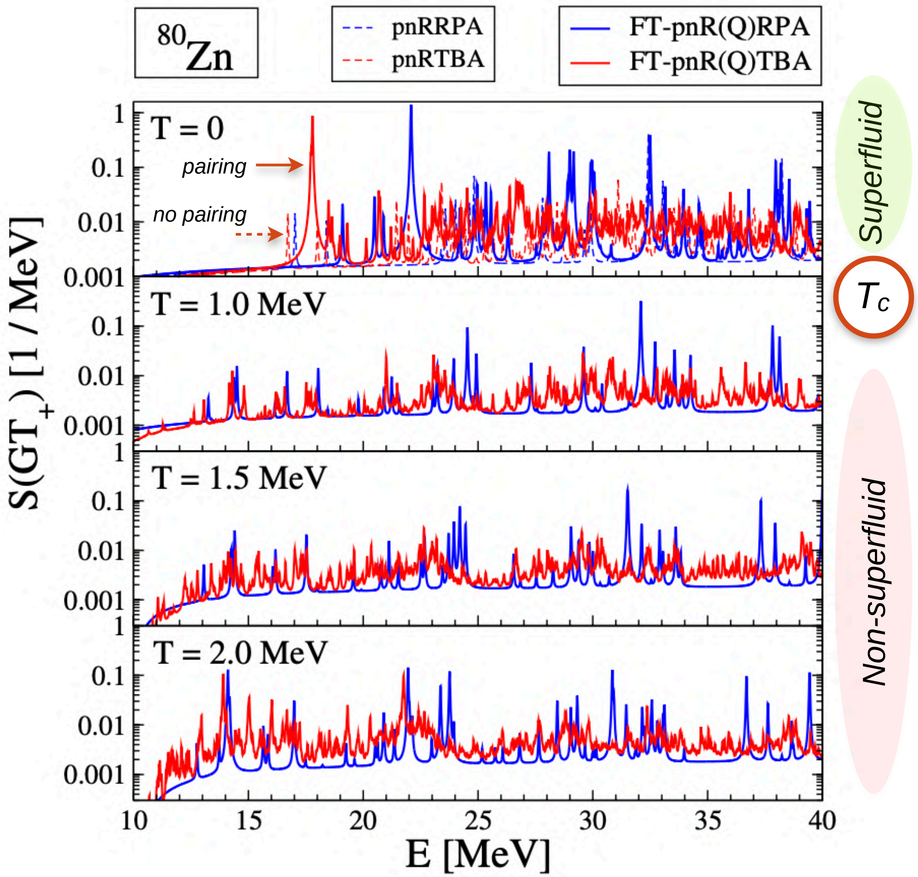
$$\Phi^{(ph)}_{14,23}(\omega,T) = \frac{1}{n_{43}(T)}\sum_{\mu;\eta_\mu=\pm1}\eta_\mu\Big[\delta_{13}\sum_6 \gamma^{\eta_\mu}_{\mu;62}\gamma^{\eta_\mu *}_{\mu;64} \times \\ \times \frac{\big(N(\eta_\mu\Omega_\mu)+n_6(T)\big)\big(n(\varepsilon_6-\eta_\mu\Omega_\mu,T)-n_1(T)\big)}{\omega-\varepsilon_1+\varepsilon_6-\eta_\mu\Omega_\mu} + \\ + \delta_{24}\sum_5 \gamma^{\eta_\mu}_{\mu;15}\gamma^{\eta_\mu *}_{\mu;35} \times \\ \times \frac{\big(N(\eta_\mu\Omega_\mu)+n_2(T)\big)\big(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_5(T)\big)}{\omega-\varepsilon_5+\varepsilon_2-\eta_\mu\Omega_\mu} - \\ - \gamma^{\eta_\mu}_{\mu;13}\gamma^{\eta_\mu *}_{\mu;24} \times \\ \times \frac{\big(N(\eta_\mu\Omega_\mu)+n_2(T)\big)\big(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_3(T)\big)}{\omega-\varepsilon_3+\varepsilon_2-\eta_\mu\Omega_\mu} - \\ - \gamma^{\eta_\mu *}_{\mu;31}\gamma^{\eta_\mu}_{\mu;42} \times \\ \times \frac{\big(N(\eta_\mu\Omega_\mu)+n_4(T)\big)\big(n(\varepsilon_4-\eta_\mu\Omega_\mu,T)-n_1(T)\big)}{\omega-\varepsilon_1+\varepsilon_4-\eta_\mu\Omega_\mu}\Big],$$



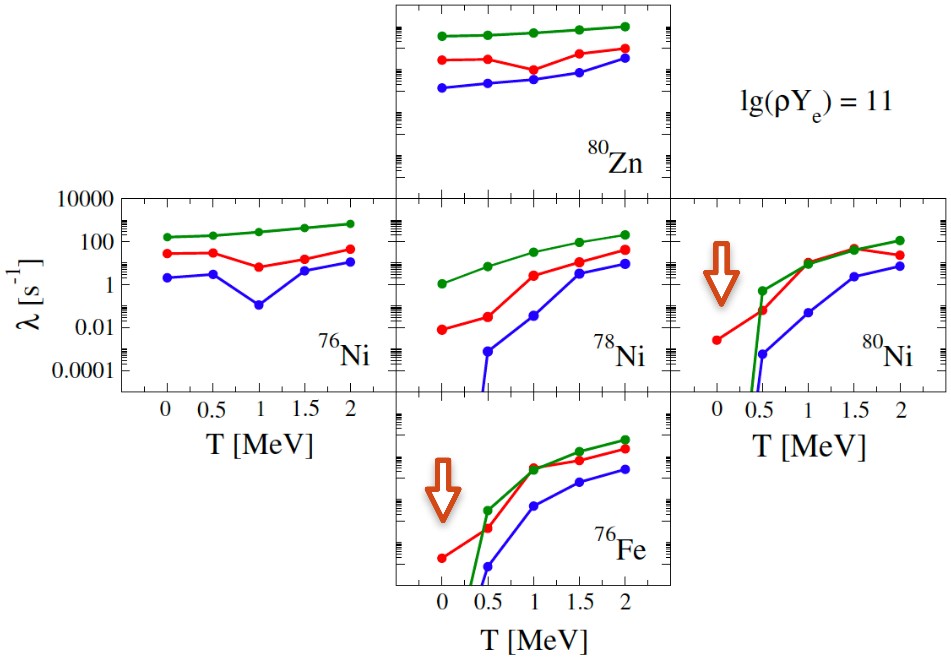
$$\Phi^{(ph,ph)}_{14,23}(\omega) = \sum_\mu \times \\ \times\Big[\delta_{13}\sum_6 \frac{\gamma^{\mu}_{62}\gamma^{\mu *}_{64}}{\omega-\varepsilon_1+\varepsilon_6-\Omega_\mu} + \\ +\delta_{24}\sum_5 \frac{\gamma^{\mu}_{15}\gamma^{\mu *}_{35}}{\omega-\varepsilon_5+\varepsilon_2-\Omega_\mu} - \\ - \frac{\gamma^{\mu}_{13}\gamma^{\mu *}_{24}}{\omega-\varepsilon_3+\varepsilon_2-\Omega_\mu} - \\ - \frac{\gamma^{\mu *}_{31}\gamma^{\mu}_{42}}{\omega-\varepsilon_1+\varepsilon_4-\Omega_\mu}\Big]$$

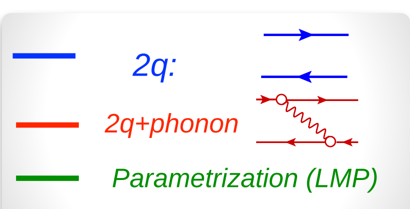
GT+ response and electron capture (EC) rates at T>0: the neighborhood of ^{78}Ni

GT+ response



Electron capture rates around ^{78}Ni





E.L., C. Robin, H. Wibowo, PLB 800, 135134 (2020)
E.L., P. Schuck, PRC 104, 044330 (2021)
E.L., C. Robin, PRC 103, 024326 (2021)

Interplay *of superfluidity and collective effects in core-collapse supernovae:*

- *Amplifies the EC rates and, consequently,*
- *Reduces the electron-to-baryon ratio leading to lower pressure*
- *Promotes the gravitational collapse*
- *Increases the neutrino flux and effective cooling*
- *Allows heavy nuclei to survive the collapse*

Formalism at T>0: the pairing channel

Averages redefined:

$$G_{12,1'2'}(t-t') = -i\langle\mathcal{T}(\psi_1\psi_2)(t)(\bar\psi_{2'}\bar\psi_{1'})(t')\rangle \rightarrow -i\langle\mathcal{T}(\psi_1\psi_2)(t)(\bar\psi_{2'}\bar\psi_{1'})(t')\rangle_T$$

Grand Canonical average: $< ... > \equiv < 0|...|0 > \quad \rightarrow \quad < ... >_T \equiv \sum_n exp\Big(\frac{\Omega - E_n - \mu N}{T}\Big) < n|...|n >$

Matsubara imaginary-time formalism: temperature-dependent dynamical kernel

Direct:

$$\mathcal{K}^{(r;11)}_{121'2'}(\omega_n) = -\sum_{\nu'\nu''} w_{\nu'}w_{\nu''}$$
$$\times\Big[\sum_{\nu\mu}\frac{\Theta^{\mu\nu;\nu'\nu''(+)}_{121'2'}}{i\omega_n - \omega_{\nu\nu'} - \omega^{(++)}_{\mu\nu''}}\big(e^{-(\omega_{\nu\nu'}+\omega^{(++)}_{\mu\nu''})/T} - 1\big)$$
$$-\sum_{\nu\varkappa}\frac{\Theta^{\varkappa\nu;\nu'\nu''(-)}_{121'2'}}{i\omega_n + \omega_{\nu\nu'} + \omega^{(--)}_{\varkappa\nu''}}\big(e^{-(\omega_{\nu\nu'}+\omega^{(--)}_{\varkappa\nu''})/T} - 1\big)\Big]$$

Exchange:

$$\mathcal{K}^{(r;12)}_{121'2'}(\omega_n) = \sum_{\nu'\nu''} w_{\nu'}w_{\nu''}$$
$$\times\Big[\sum_{\nu\mu}\frac{\Sigma^{\mu\nu;\nu'\nu''(+)}_{121'2'}}{i\omega_n - \omega_{\nu\nu'} - \omega^{(++)}_{\mu\nu''}}\big(e^{-(\omega_{\nu\nu'}+\omega^{(++)}_{\mu\nu''})/T} - 1\big)$$
$$-\sum_{\nu\varkappa}\frac{\Sigma^{\varkappa\nu;\nu'\nu''(-)}_{121'2'}}{i\omega_n + \omega_{\nu\nu'} + \omega^{(--)}_{\varkappa\nu''}}\big(e^{-(\omega_{\nu\nu'}+\omega^{(--)}_{\varkappa\nu''})/T} - 1\big)\Big],$$

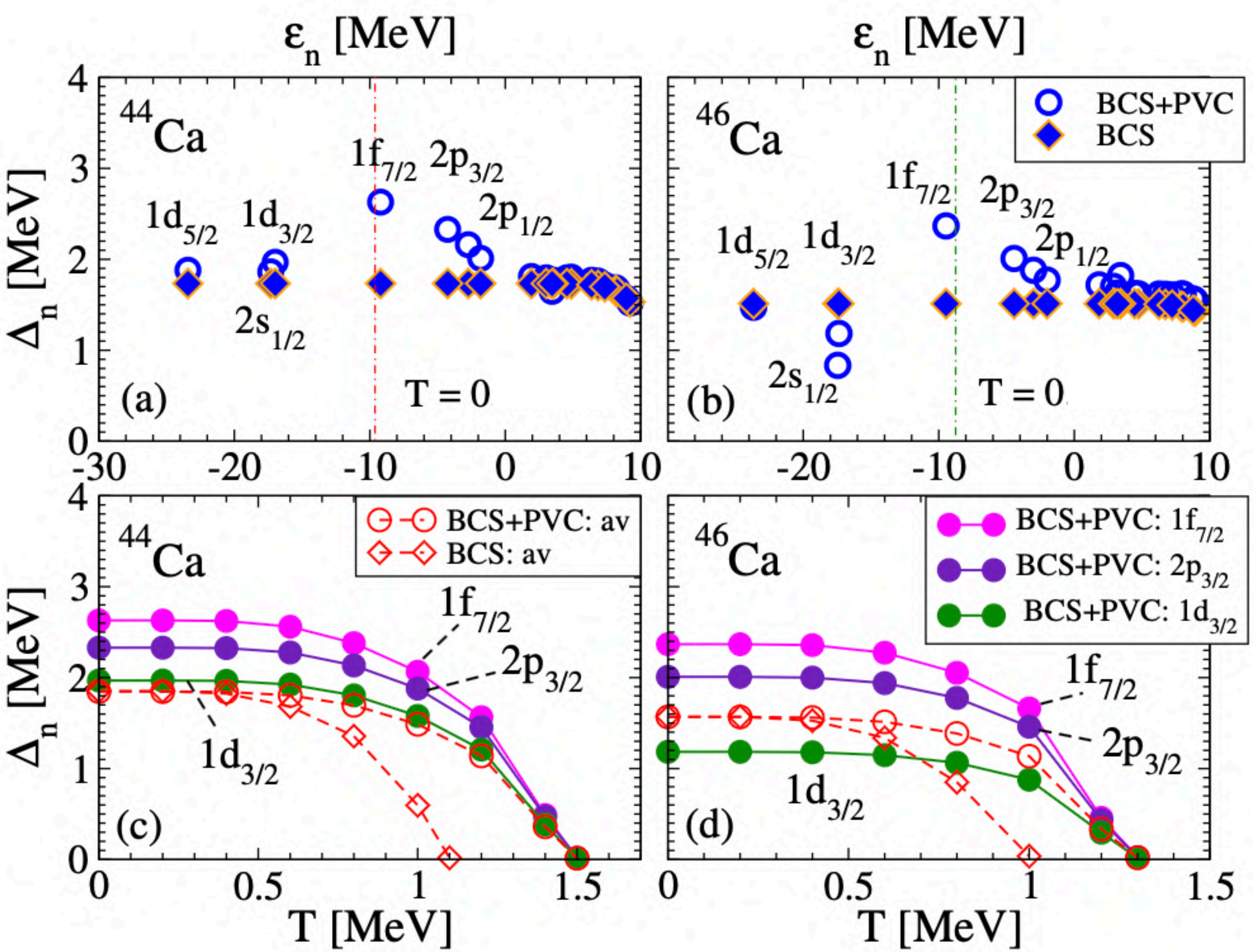
E.L., P.Schuck, Phys. Rev. C 104, 044330 (2021)

BCS-like gap Eq., but with non-trivial T-dependence in $K^{(r)}$:

$$f_1(T) = \frac{1}{\exp(E_1/T) + 1}$$

$$\Delta_1(T) = -\sum_2 \mathcal{V}_{1\bar{1}2\bar{2}}\frac{\Delta_2(T)(1-2f_2(T))}{2E_2}$$

$$\mathcal{V}_{121'2'} = \frac{1}{2}\Big(K^{(0)}_{121'2'} + K^{(r)}_{121'2'}(2\lambda)\Big)$$



E.L., P.Schuck, Phys. Rev. C 104, 044330 (2021)

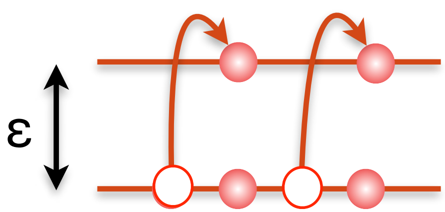
Lipkin Hamiltonian on quantum computer

The algorithm: *Variational Quantum Eigensolver (VQE) + quantum EOM (qEOM)*

- *VQE: a minimal encoding scheme is found ("J-scheme") and implemented, based on the symmetry of the LMG Hamiltonian. Yields an accurate ground state |0>.*
- *qEOM generates efficiently the EOM matrix:*

Two-level
Lipkin Hamiltonian:
exactly solvable

$$\hat{H} = \epsilon \hat{J}_z - \frac{v}{2}\left(\hat{J}_+^2 + \hat{J}_-^2\right)$$



Generalized eigenvalue equation:

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B}^* & \mathcal{A}^* \end{bmatrix} \begin{bmatrix} X^n \\ Y^n \end{bmatrix} = E_{0n} \begin{bmatrix} \mathcal{C} & \mathcal{D} \\ -\mathcal{D}^* & -\mathcal{C}^* \end{bmatrix} \begin{bmatrix} X^n \\ Y^n \end{bmatrix}$$

$$\mathcal{A}_{\mu_\alpha \nu_\beta} = \langle 0 | \left[\left(\hat{K}^\alpha_{\mu_\alpha} \right)^\dagger, \left[\hat{H}, \hat{K}^\beta_{\nu_\beta} \right] \right] | 0 \rangle$$

$$\mathcal{B}_{\mu_\alpha \nu_\beta} = - \langle 0 | \left[\left(\hat{K}^\alpha_{\mu_\alpha} \right)^\dagger, \left[\hat{H}, \left(\hat{K}^\beta_{\nu_\beta} \right)^\dagger \right] \right] | 0 \rangle$$

$$\mathcal{C}_{\mu_\alpha \nu_\beta} = \langle 0 | \left[\left(\hat{K}^\alpha_{\mu_\alpha} \right)^\dagger, \hat{K}^\beta_{\nu_\beta} \right] | 0 \rangle$$

$$\mathcal{D}_{\mu_\alpha \nu_\beta} = - \langle 0 | \left[\left(\hat{K}^\alpha_{\mu_\alpha} \right)^\dagger, \left(\hat{K}^\beta_{\nu_\beta} \right)^\dagger \right] | 0 \rangle \, .$$

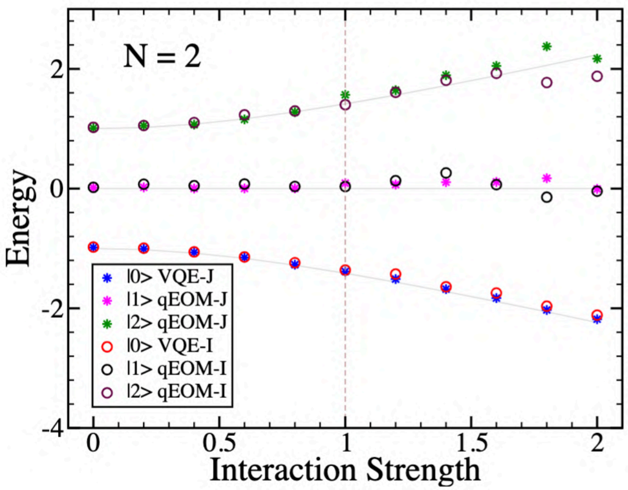
Excitation operator: complexity α, K^α

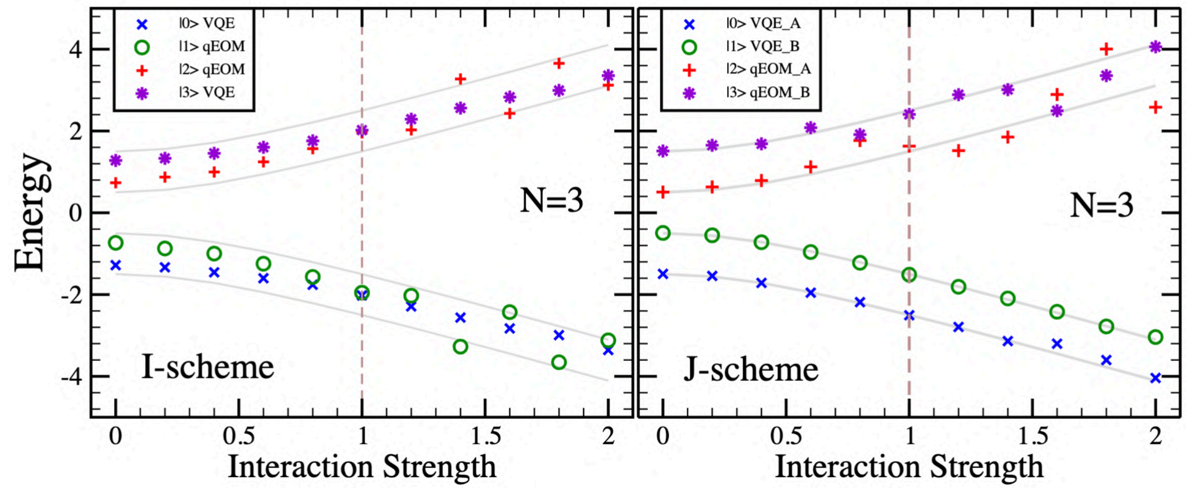
$$\hat{K}^1_{\mu_1} = a_i^\dagger a_{j'} \qquad \hat{K}^2_{\mu_2} = a_i^\dagger a_j^\dagger a_{j'} a_{i'}$$

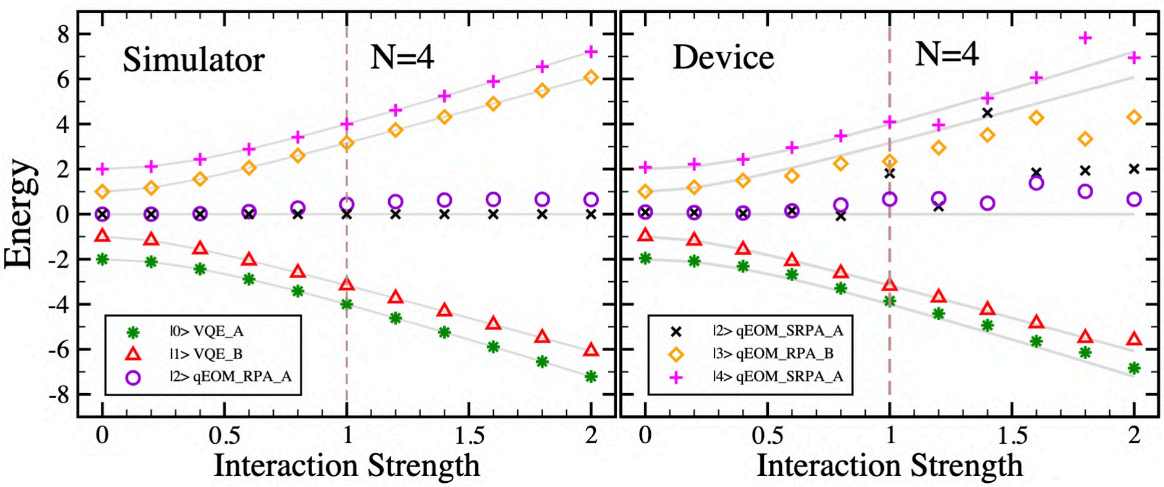
…

M. Hlatshwayo, et al., Phys. Rev. C 106, 024319 (2022)

Lipkin Hamiltonian on quantum computer: hardware results







Conventions:

- n_q = *number of states*
- *N = number of particles*
- *ν = v /ε effective interaction strength*

- ***I-scheme:*** *individual spin basis,* $n_q = 2^N$

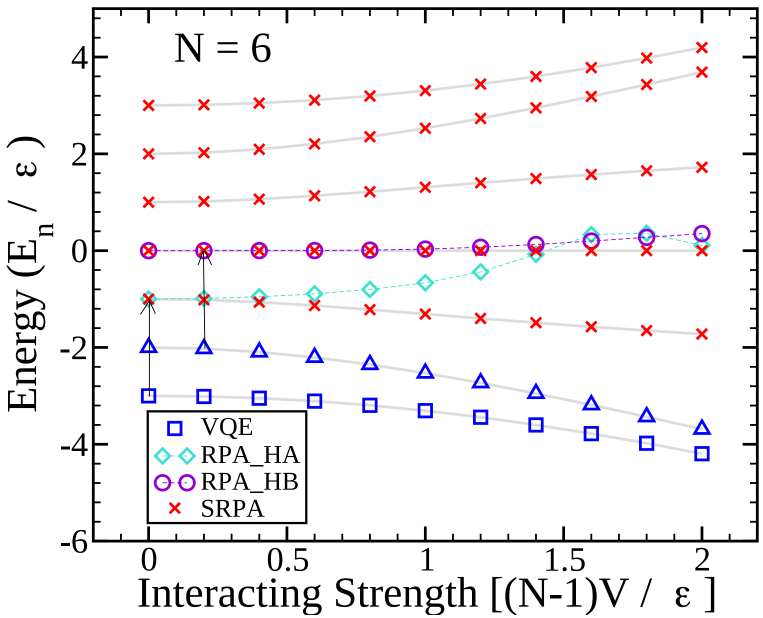
- ***J-scheme:*** *total spin basis (coupled form), symmetry:* $n_q = N/2 + 1$

Observations:

- *Higher-rank excitation ~ higher accuracy*
- *Stronger coupling ~ lower accuracy*
- *More particles ~ lower accuracy*
- *Less qubits ~ higher accuracy*

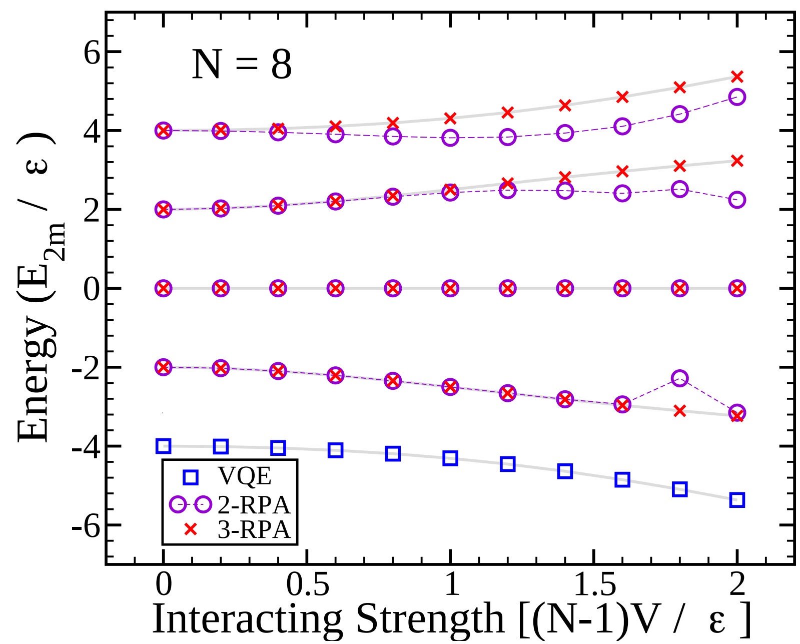
M. Hlatshwayo, et al., Phys. Rev. C 106, 024319 (2022)

More particles, higher complexity (preliminary; simulator)



Second RPA vs RPA:
RPA breaks down early
in the strong-coupling regime

“Third” RPA and higher:
More accurate solutions,
tbd on a device



Quantum advantage:
The same Pauli strings
<XYZ> are to be measured
for α = 1,2,3

Outlook

Summary:

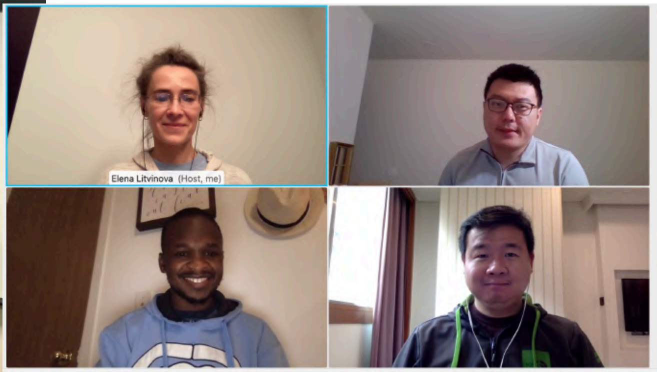
- The relativistic nuclear field theory (RNFT) is formulated and advanced in the Equation of Motion (EOM) framework, with the emphasis on emergent collectivity.
- The emergent collective effects renormalize interactions in correlated media, underly the spectral fragmentation mechanisms, affect superfluidity and weak decay rates.
- Relativistic NFT is generalized to finite temperature and applied to nuclear superfluidity.
- Weak rates at astrophysical conditions are extracted: the correlations beyond mean field are found significant.

Current and future developments:

- Deformed nuclei: correlations vs shapes; first results just released (Yinu Zhang et al.);
- Efficient algorithms; quantum computing (Manqoba Hlatshwayo et al.);
- Implementation of the EC rates into the core-collapse supernovae simulations;
- Toward an “ab initio” description: implementations with bare NN-interactions;
- Superfluid pairing at T>0 to extend the application range (r-process);
- Relativistic EOM’s, bosonic EOM’s, beyond Standard Model, …

Many thanks

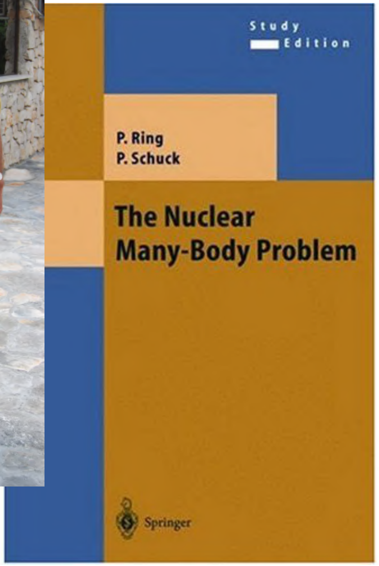
Yinu Zhang (WMU)
Manqoba Hlatshwayo (WMU)
Herlik Wibowo (AS Taipei)
Caroline Robin (U. Bielefeld & GSI)
Peter Schuck (IPN Orsay)
Peter Ring (TU München)
Tamara Niksic (U Zagreb)

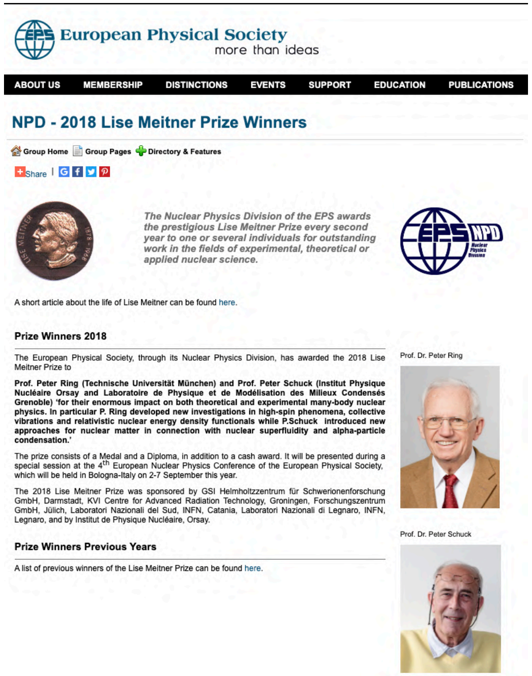


US-NSF CAREER PHY-1654379 (2017-2023)

US-NSF PHY-2209376 (2022-2025)

CAREER Award





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NPD - 2018 Lise Meitner Prize Winners

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The Nuclear Physics Division of the EPS awards the prestigious Lise Meitner Prize every second year to one or several individuals for outstanding work in the fields of experimental, theoretical or applied nuclear science.

A short article about the life of Lise Meitner can be found here.

Prize Winners 2018

The European Physical Society, through its Nuclear Physics Division, has awarded the 2018 Lise Meitner Prize to

Prof. Peter Ring (Technische Universität München) and Prof. Peter Schuck (Institut Physique Nucléaire Orsay and Laboratoire de Physique et de Modélisation des Milieux Condensés Grenoble) 'for their enormous impact on both theoretical and experimental many-body nuclear physics. In particular P. Ring developed new investigations in high-spin phenomena, collective vibrations and relativistic nuclear energy density functionals while P.Schuck introduced new approaches for nuclear matter in connection with nuclear superfluidity and alpha-particle condensation.'

The prize consists of a Medal and a Diploma, in addition to a cash award. It will be presented during a special session at the 4th European Nuclear Physics Conference of the European Physical Society, which will be held in Bologna-Italy on 2-7 September this year.

The 2018 Lise Meitner Prize was sponsored by GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, KVI Centre for Advanced Radiation Technology, Groningen, Forschungszentrum GmbH, Jülich, Laboratori Nazionali del Sud, INFN, Catania, Laboratori Nazionali di Legnaro, INFN, Legnaro, and by Institut de Physique Nucléaire, Orsay.

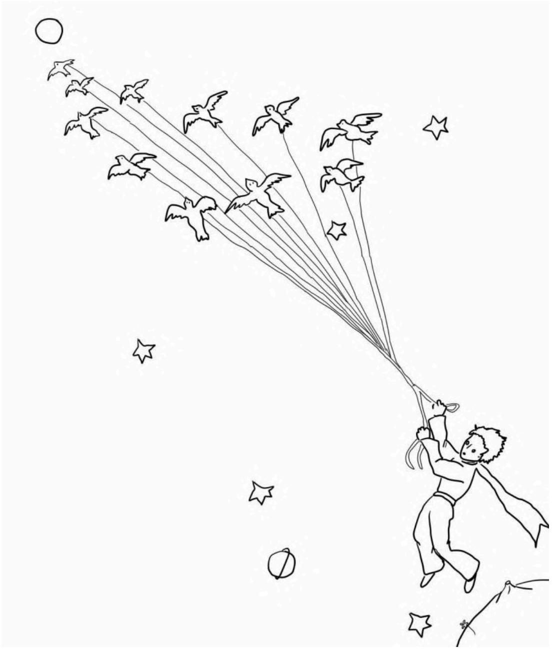
Prof. Dr. Peter Ring

Prof. Dr. Peter Schuck

Prize Winners Previous Years

A list of previous winners of the Lise Meitner Prize can be found here.

Thank you!



Finite amplitude method extended beyond QRPA (preliminary):

Generalized FAM (FAM-QVC)

QVC

$$\delta\mathcal{R}^{(20)}_{\mu\nu}(\omega) = \frac{\delta\mathcal{H}^{20}_{\mu\nu}(\omega) + \sum_{\mu'\nu'}\Phi^{(+)}_{\mu\nu'\nu\mu'}(\omega)\delta\mathcal{R}^{(20)}_{\mu'\nu'}(\omega) + F^{20}_{\mu\nu}}{\omega - E_\mu - E_\nu}$$

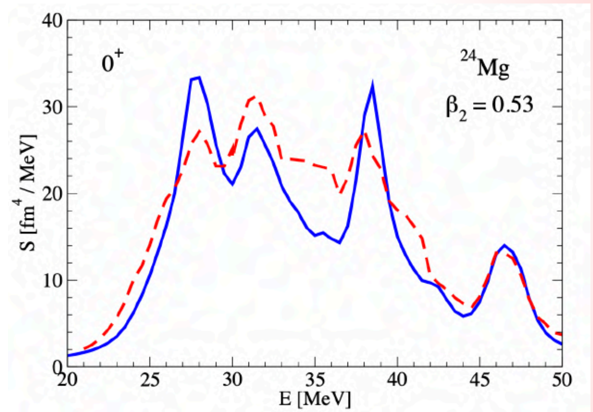
$$\delta\mathcal{R}^{(02)}_{\mu\nu}(\omega) = \frac{\delta\mathcal{H}^{02}_{\mu\nu}(\omega) + \sum_{\mu'\nu'}\Phi^{(-)}_{\mu\nu'\nu\mu'}(\omega)\delta\mathcal{R}^{(02)}_{\mu'\nu'}(\omega) + F^{02}_{\mu\nu}}{-\omega - E_\mu - E_\nu}.$$

QVC amplitude (leading approximation):

$$\Phi^{(+)}_{\mu\mu'\nu\nu'}(\omega) = \Big[\delta_{\mu'\nu'}\sum_{\gamma n}\frac{\Gamma^{(11)n}_{\mu\gamma}\Gamma^{(11)n*}_{\nu\gamma}}{\omega - \omega_n - E_{\mu'\gamma}} + \delta_{\mu\nu}\sum_{\gamma n}\frac{\Gamma^{(11)n}_{\mu'\gamma}\Gamma^{(11)n*}_{\nu'\gamma}}{\omega - \omega_n - E_{\mu\gamma}} + \sum_{n}\frac{\Gamma^{(11)n}_{\mu\nu}\Gamma^{(11)n*}_{\nu'\mu'}}{\omega - \omega_n - E_{\mu'\nu}} + \sum_{n}\frac{\Gamma^{(11)n}_{\mu'\nu'}\Gamma^{(11)n*}_{\nu\mu}}{\omega - \omega_n - E_{\mu\nu'}}\Big]$$

E.L., Y. Zhang, Phys. Rev. C 106, 064316 (2022)

Proof of principle:
IVGMR in 24-Mg
in a restricted model space



Ongoing:

- *Convergence improvement*
- *Optimization*
- *Cross-check routines*

Are there theoretical limits on accuracy?

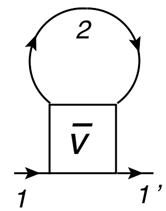
- Higher-rank configurations = higher accuracy? Can we quantify this? How accurately we can describe the observed spectra, in principle?
- Spectroscopic accuracy in nuclear structure: experiment (laser spectroscopy [eV], nuclear resonance fluorescence [keV]) … no standards for theory. ~100 keV?
- Chemical accuracy 1 kcal/mol = 0.043 eV is possible with the gold standard for quantum chemistry calculations, namely the canonical coupled cluster (CC) expansion truncated at the second order in the electronic excitation operator and including an approximate treatment of the triple excitations (CCSD(T), where S stands for single, D for double, and (T) for non-iterative triple) [P.J. Ollitrault et al, Phys. Rev. Res. 2, 043140, 2020]
- CCSD(T) includes up to (correlated) 3p3h configurations and scales as $O(N^7)$ with the number of degrees of freedom N of the model Hamiltonian.
- In nuclear structure, there are relatively rare calculations with (correlated) 3p3h configurations for medium-heavy nuclei (QPM, EOM/RQTBA^3, CC). The results are still not ideal.
- Is the problem in the underlying strong "forces", which are not weak and known with limited accuracy? Or the many-body methods? Likely both.
- Working with model (solvable) Hamiltonians allows one to solely focus on the many-body problem. Can be studied with quantum and hybrid algorithms on NISQ devices.

Mean field approximation and beyond

Exact "ab-initio" self-energy :

$$\Sigma_{11'}(\omega) = \Sigma^{(0)}_{11'} + \Sigma^{(r)}_{11'}(\omega)$$

=



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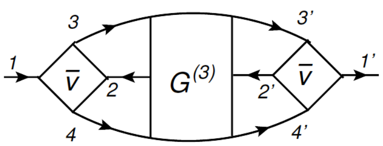
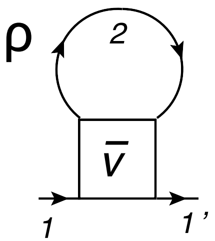


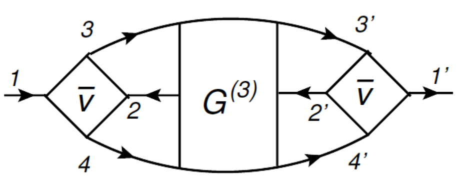
Mean field approximation (density functional theory, DFT)

$$\Sigma_{11'}(\omega) =$$



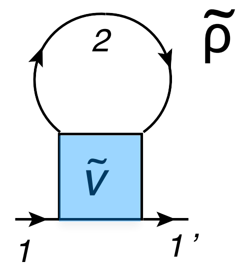
static

+



ω-dependent

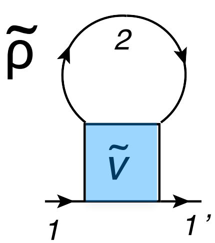
≈



DFT: static $[\omega = \tilde{\varepsilon}_1\ (?)]$

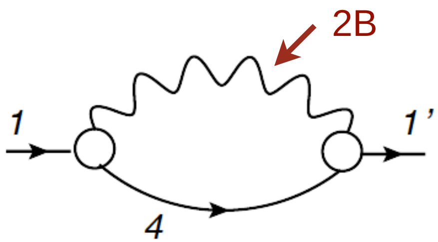
Beyond mean field: particle-vibration coupling (PVC), leading approximation:

$$\Sigma_{11'}(\omega) =$$



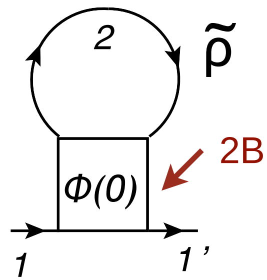
DFT: static, basis

+



PVC ω-dependent

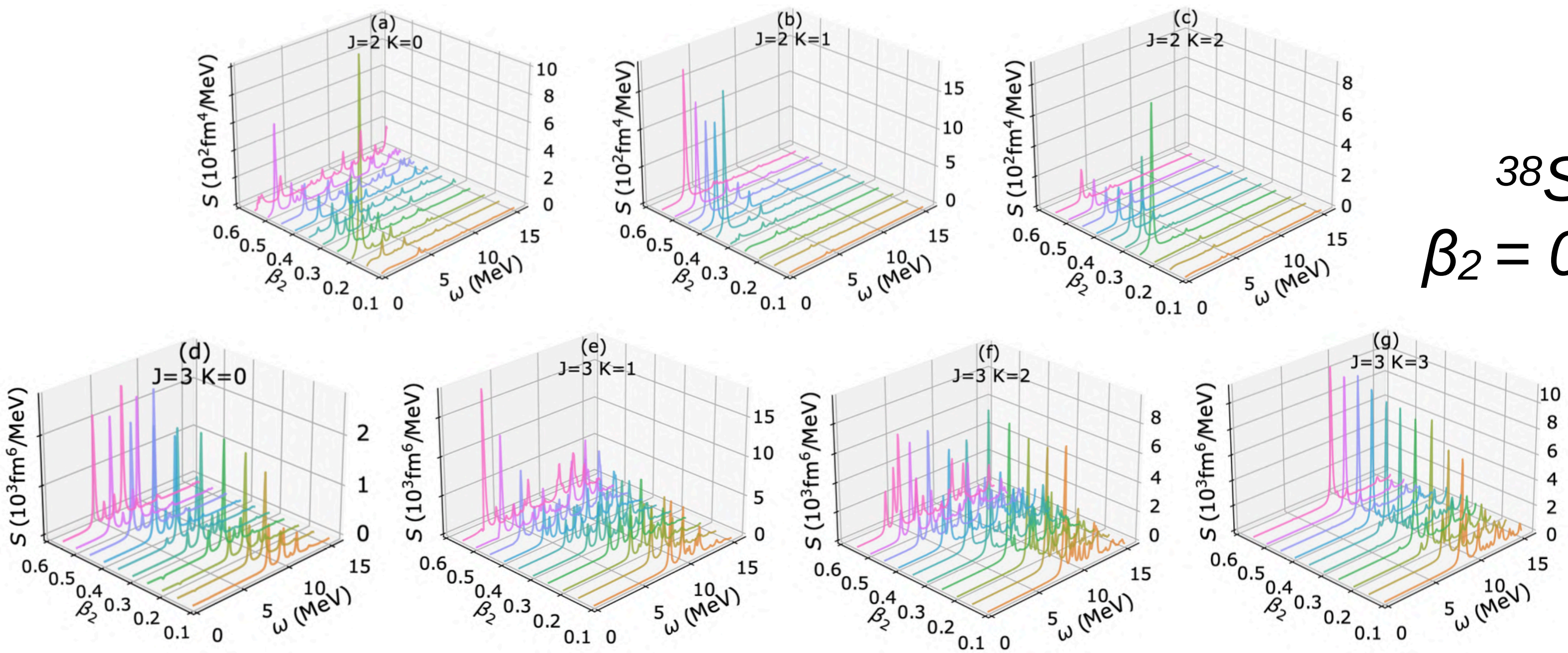
−



PVC double counting removal

Single-(quasi)particle states. New implementation: FAM-QRPA+QVC

(i) Relativistic meson-nucleon Lagrangian + (ii) Relativistic Hartree-Bogoliubov (RHB) + (iii) Quasiparticle random phase approximation (QRPA): J = 2^+ - 5^-, K = [0,J]. Finite amplitude method (FAM): A. Bjelčić et al., CPC 253, 107184 (2020). Relativistic DD-PC1 interaction.



^{38}Si

$\beta_2 = 0.31$

(iv) QVC vertex extraction:

$$\Gamma^{(ij)n}_{\mu\mu'} = \frac{1}{\langle n|F^\dagger|0\rangle}\oint_{\gamma_n} \delta H^{ij}_{\mu\mu'}(\omega)\frac{d\omega}{2\pi i}$$

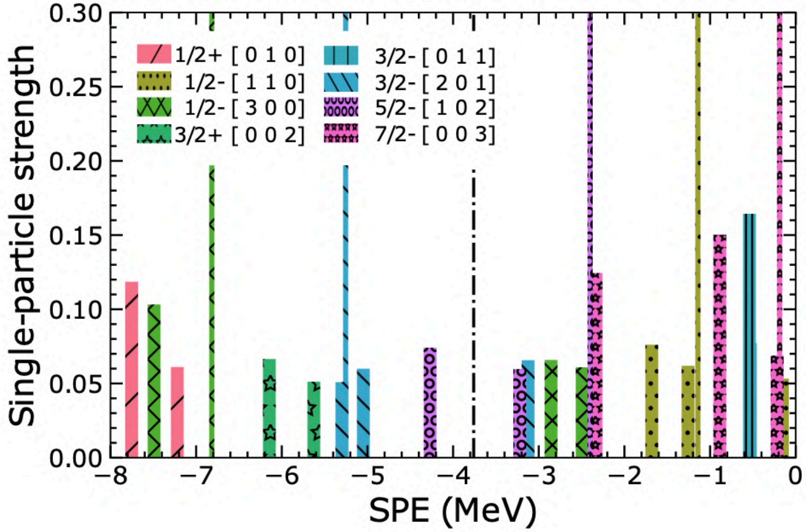
Variation of the HFB Hamiltonian at the QRPA pole

(v) Dyson Eq. solution

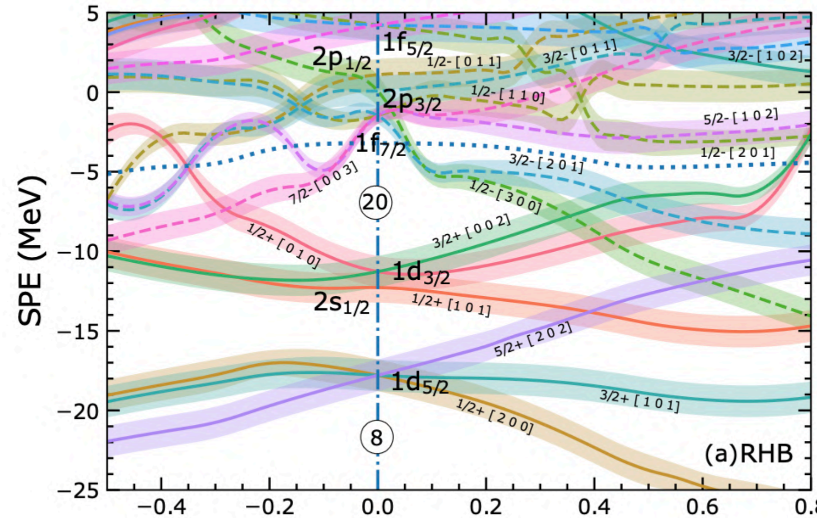
E.L., Y. Zhang, PRC 104, 044303 (2021)

Single-(quasi)particle states in ^{38}Si

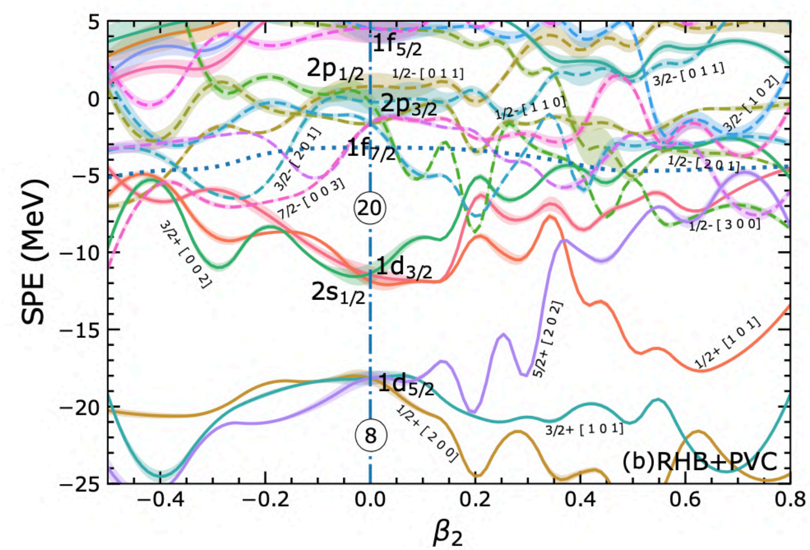
Fragmentation of quasiparticle states: RHB vs RHB+QVC



Nilsson diagram: RHB

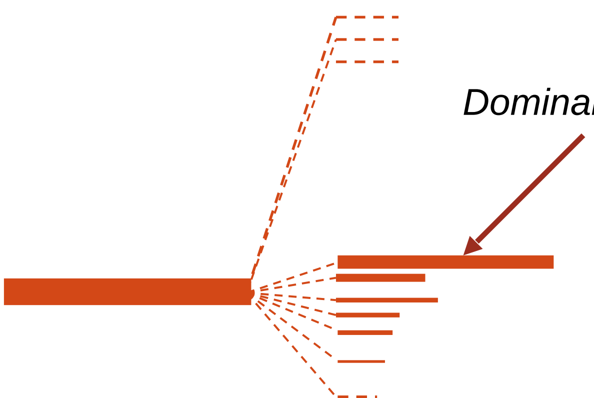


Nilsson diagram: RHB+QVC (dominant only)



Fragmentation mechanism: schematic

Dominant level



Sum rules:

$$E_k = \sum E_k^{(\nu)} S_k^{(\nu)}$$

$$\sum_\nu S_k^{(\nu)} = 1$$

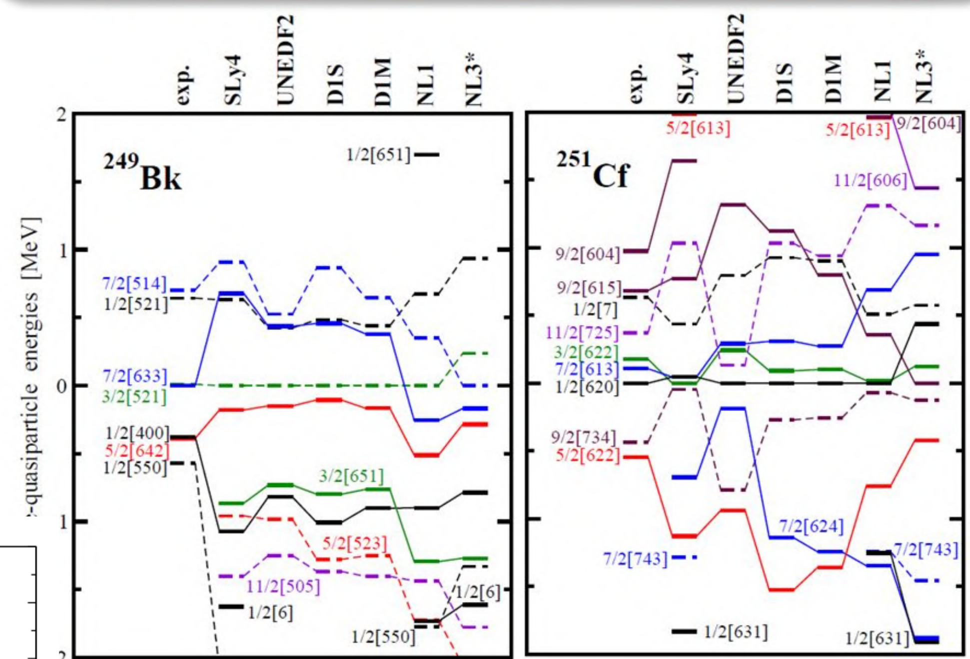
Y. Zhang et al., PRC 105, 044326 (2022)

Single-(quasi)particle states in $^{249,251}Cf$

A. Afanasjev et al.: Long-standing problem of the description of single-particle states in deformed nuclei.

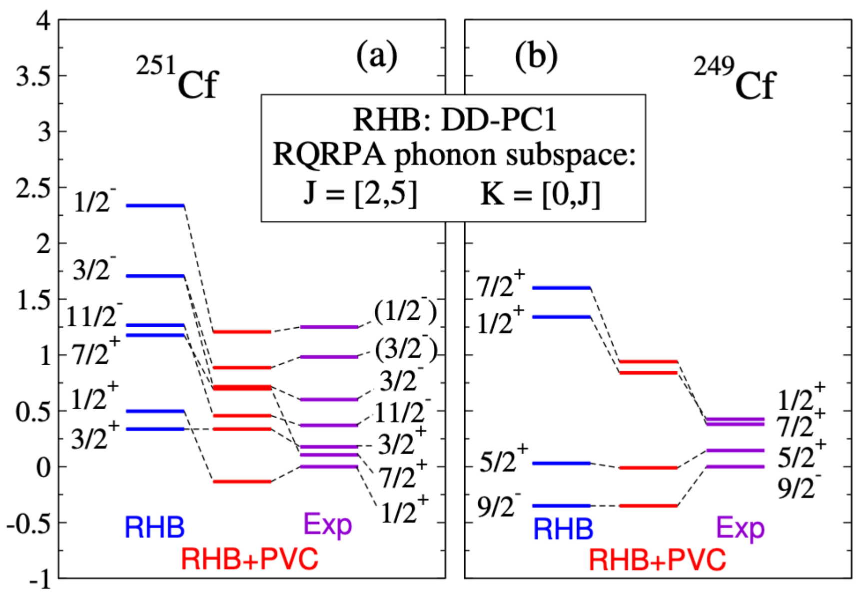
Systematic studies for ^{249}Bk and ^{251}Cf in the mean-field approximation:

Deformed one-quasiparticle states: covariant and non-relativistic mean-field calculations vs experiment:



^{250}Cf

$\beta_2 = 0.29$



Beyond mean field: RHB+QVC calculations. Dominant fragments in ^{251}Cf and ^{249}Cf.

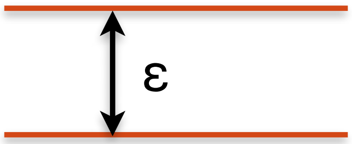
The spectroscopic factors are quenched even stronger than in spherical nuclei. Can this be measured?

Y. Zhang et al., PRC 105, 044326 (2022)

Lipkin Hamiltonian on quantum computer

Two-level Lipkin (Meshkov-Glick), LMG, Hamiltonian:

$$\hat{H} = \epsilon \hat{J}_z - \frac{v}{2}\left(\hat{J}_+^2 + \hat{J}_-^2\right) - \frac{w}{2}\left(\hat{J}_+\hat{J}_- + \hat{J}_-\hat{J}_+\right)$$



Quasispin operators:

$$\hat{J}_z = \frac{1}{2}\sum_{p=1}^{N}\left(\hat{a}^\dagger_{p,+}\hat{a}_{p,+} - \hat{a}^\dagger_{p,-}\hat{a}_{p,-}\right),$$

N = 2j + 1

$$\hat{J}_+ = \sum_{p=1}^{N}\hat{a}^\dagger_{p,+}\hat{a}_{p,-} \text{ and } \hat{J}_- = \left(\hat{J}_+\right)^\dagger$$

Excitation operator:

Configuration complexity:

$$\hat{O}^\dagger_n = \sum_{\alpha}\sum_{\mu_\alpha}\left[X^\alpha_{\mu_\alpha}(n)\hat{K}^\alpha_{\mu_\alpha} - Y^\alpha_{\mu_\alpha}(n)\left(\hat{K}^\alpha_{\mu_\alpha}\right)^\dagger\right]$$

$$\hat{K}^1_{\mu_1} = a^\dagger_i a_{j'} \qquad \hat{K}^2_{\mu_2} = a^\dagger_i a^\dagger_j a_{j'} a_{i'} \qquad \ldots$$

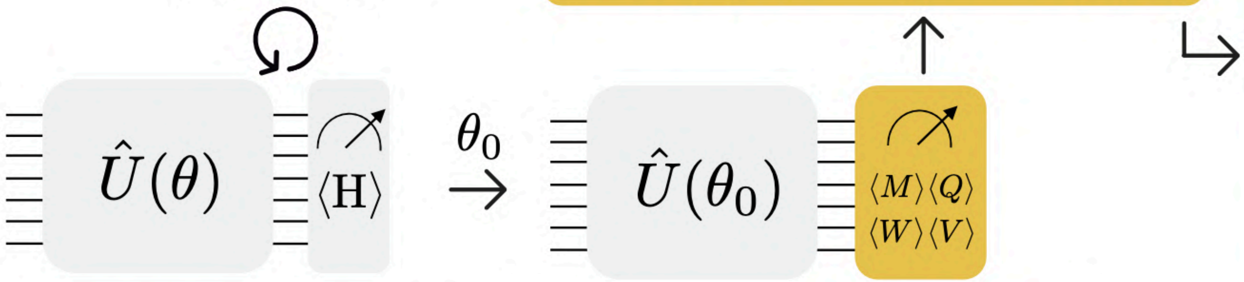
M. Hlatshwayo, et al., Phys. Rev. C 106, 024319 (2022)

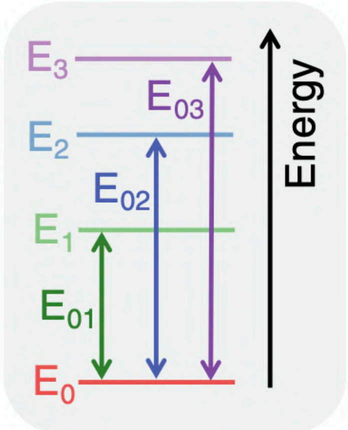
Atomic nuclei on quantum computer: accessing emergence via entanglement

Variational Quantum Eigensolver (VQE) + Quantum Equation of Motion (qEOM):

$$\min_{\theta}\left(\frac{\langle\Psi(\theta)|\,\hat{\mathrm{H}}\,|\Psi(\theta)\rangle}{\langle\Psi(\theta)|\Psi(\theta)\rangle}\right)$$

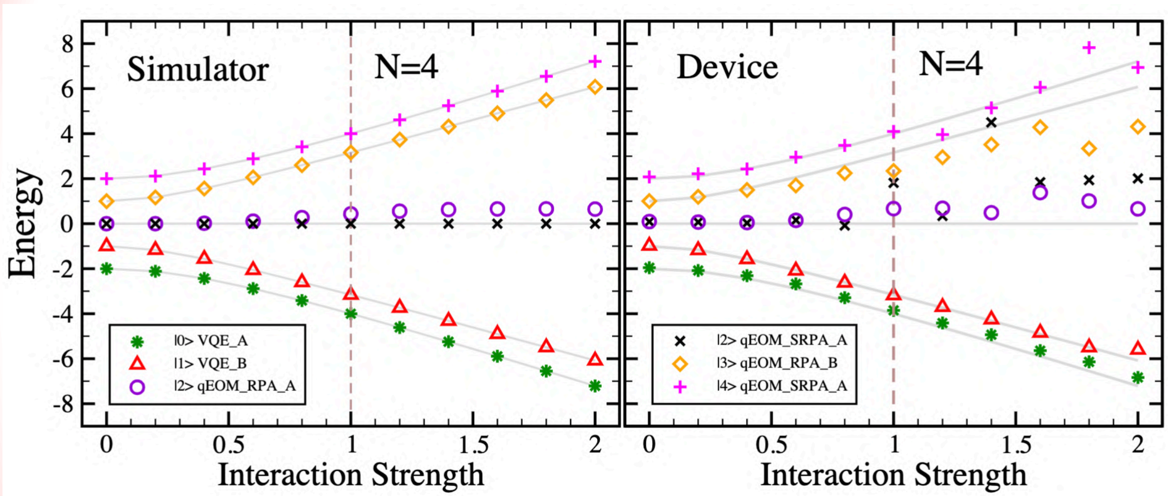
$$\begin{pmatrix}\mathbf{M} & \mathbf{Q}\\ \mathbf{Q^*} & \mathbf{M^*}\end{pmatrix}\begin{pmatrix}\mathbf{X}_n\\ \mathbf{Y}_n\end{pmatrix} = E_{0n}\begin{pmatrix}\mathbf{V} & \mathbf{W}\\ -\mathbf{W^*} & -\mathbf{V^*}\end{pmatrix}\begin{pmatrix}\mathbf{X}_n\\ \mathbf{Y}_n\end{pmatrix}$$





P. Ollitrault et al., Phys. Rev. Res. 2, 043140 (2020)

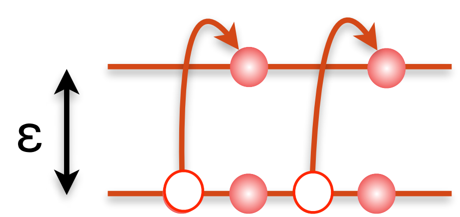
Implementation for N = 4 (IBM-Q): RPA vs SRPA vs exact



Beyond SRPA, toward quantum advantage [in progress]

*Two-level Lipkin Hamiltonian: **exactly solvable***

$$\hat{H} = \epsilon \hat{J}_z - \frac{v}{2}\left(\hat{J}_+^2 + \hat{J}_-^2\right)$$



M. Hlatshwayo et al., Phys. Rev. C 106, 024319 (2022)