Pairing in systems with imbalance and BCS-BEC crossover

Armen Sedrakian

Frankfurt Institute for Advanced Studies, Frankfurt Main, Germany
Institute for Theoretical Physics, Wroclaw University

Conference on Qunatum-Many-Body Correlations in memory of Peter Schuck (QMBC 2023)

Collaboration with Peter Schuck (2000-2006)

1. arXiv:astro-ph/0611676 [pdf, ps, other] astro-ph nucl-th

doi 10.1103/PhysRevC.76.055805

Vertex renormalization in weak decays of Cooper pairs and cooling compact stars

Authors: Armen Sedrakian, Herbert Müther, Peter Schuck

Abstract: At temperatures below the critical temperature of superfluid phase transition baryonic matter emits neutrinos by breaking and recombination of Cooper pairs formed in the condensate. The strong interactions in the nuclear medium modify the weak interaction vertices and the associated neutrino loss rates. We study these modifications non-perturbatively by summing infinite series of particle-hole l... ▽ More

Submitted 29 May, 2007; **v1** submitted 21 November, 2006; **originally announced** November 2006.

Comments: 21 pages, 5 figures, uses RevTex; v2: Sections 4-6 revised

Journal ref: Phys.Rev.C76:055805,2007

2. arXiv:nucl-th/0407020 [pdf, ps, other] nucl-th astro-ph cond-mat.stat-mech

doi 10.1016/j.nuclphysa.2005.11.025

Alpha matter on a lattice

Authors: Armen Sedrakian, Herbert Müther, Peter Schuck

Abstract: We obtain the equation of state of interacting alpha matter and the critical temperature of Bose-Einstein condensation of alpha particles within an effective scalar field theory. We start from a non-relativistic model of uniform alpha matter interacting with attractive two-body and repulsive three-body potentials and reformulate this model as a O(2) symmetric scalar φ^6 field theory with negativ... ▽ More

Submitted 6 December, 2005; **v1** submitted 6 July, 2004; **originally announced** July 2004.

Comments: 14 pages, 2 figures; v2: improved presentation, results unchanged; to appear in Nucl. Phys. A

Journal ref: Nucl.Phys.A766:97-106,2006

3. arXiv:nucl-th/0308068 [pdf, ps, other] nucl-th doi 10.1016/j.physletb.2003.09.090

Pairing in nuclear systems with effective Gogny and V_low-k interactions

Authors: Armen Sedrakian, T. T. S. Kuo, Herbert Müther, Peter Schuck

Abstract: The pairing properties of nuclear systems are a sensitive probe of the effective nucleon-nucleon interactions. We compare the 1S0 pairing gaps in nuclear and neutron matter derived from the phenomenological Gogny interaction and a renormalization group motivated low-momentum V_low-k interaction extracted from realistic interactions. We find that the pairing gaps predicted by these interactions a... ▽ More

Submitted 23 April, 2004; **v1** submitted 25 August, 2003; **originally announced** August 2003.

Comments: 11 pages, 2 figures, uses elsart.sty; v2: references added, Brueckner calculations for the V_low-k spectrum; v3: author name correction

Journal ref: Phys.Lett. B576 (2003) 68-74

4. arXiv:nucl-th/0109024 [pdf, ps, other] nucl-th cond-mat.supr-con

doi 10.1103/PhysRevC.64.064314

Transition from BCS pairing to Bose-Einstein condensation in low-density asymmetric nuclear matter

Authors: U. Lombardo, P. Nozieres, P. Schuck, H. -J. Schulze, A. Sedrakian

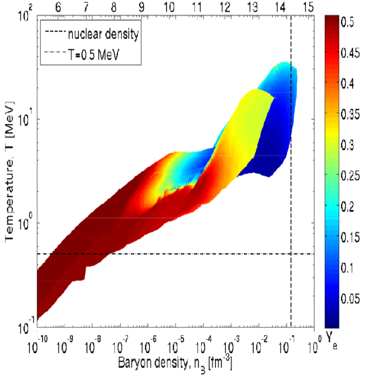
Abstract: We study the isospin-singlet neutron-proton pairing in bulk nuclear matter as a function of density and isospin asymmetry within the BCS formalism. In the high-density, weak-coupling regime the neutron-proton paired state is strongly suppressed by a minor neutron excess. As the system is diluted, the BCS state with large, overlapping Cooper pairs evolves smoothly into a Bose-Einstein condensate... ▽ More

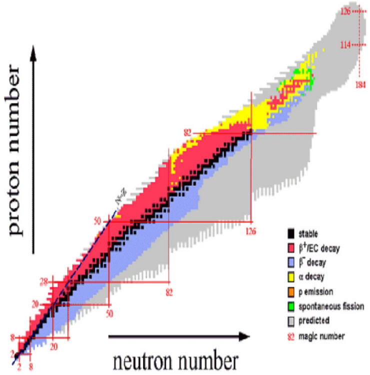
Submitted 10 September, 2001; **originally announced** September 2001.

Comments: 17 pages, uncluding 7 figures, PRC in press

Journal ref: Phys.Rev. C64 (2001) 064314

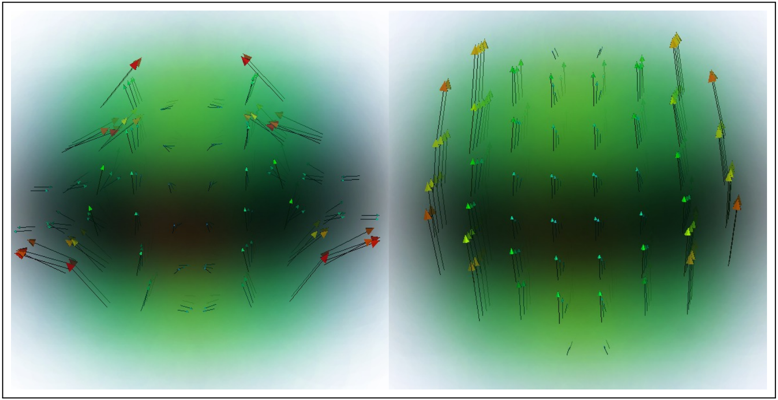
Isospin polarization suppresses $T = 0$ pairing





Isospin asymmetric the dominant $T = 1$ pairing is suppressed;
competition in $T = 0$ and $T = 1$ channels.

Spin polarization suppression of $T = 1$ pairing



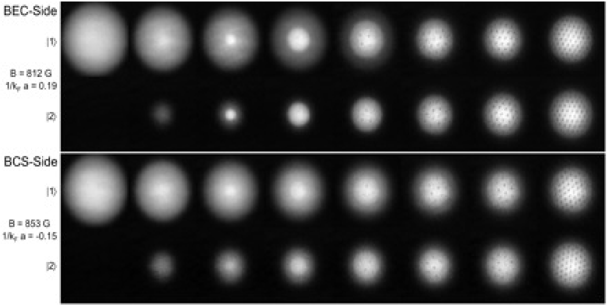
Ne nucleus in B field: left panel $B = 0$ right panel $B = 10^{17}$ G
Nuclear DFT calculations by Stein, Maruhn, Sedrakian, Reinhard, Phys. Rev. C 94, 035802 (2016).

Dilute alkali gases

- Critical temperature for condensation $T_c \sim$nK, $E_F \sim$ mK.
- Interactions can be manipulated in the range $[-\infty, \infty]$ and the unitary limit $|a| \to \infty$ is universal

$$a_{\text{eff}} = \frac{a_S}{B - B_0} \quad \text{(Feshbach resonance mechanism)}$$

- Several hyperfine states or atomic species can be trapped $n_\uparrow \neq n_\downarrow$ creating population imbalance (^{6}Li, ...)



Clouds of atomic gas evolved from highly polarized (left) to unpolarized limit (right). Vortices act as indicators of superfluidity, [from Zwierlein et al Science, 2006, vol. 311, p 492-496.]

Population imbalance

- Conventional BCS pairs particles on a Fermi surface with opposite momenta and spins: Cooper pair wave-function is invariant under time-reversal, i. e. simultaneous exchange of momentum and spin sign.
- In systems with population imbalance the pairing occurs between particles lying on different Fermi surfaces: The Cooper pair wave function is non-invariant under time reversal.

Classical example (1960's)

- Metallic superconductors with paramagnetic impurities. The effect of impurities is to induce an average slitting of Fermi-levels of spin-up and spin-down electrons. This can be described by adding a Pauli paramagnetic term to the spectrum:

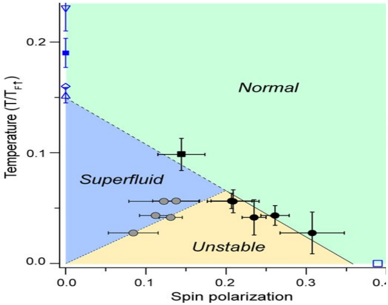
$$\epsilon_\uparrow = \frac{p^2}{2m} - \mu_\uparrow, \qquad \epsilon_\downarrow = \frac{p^2}{2m} - \mu_\downarrow,$$

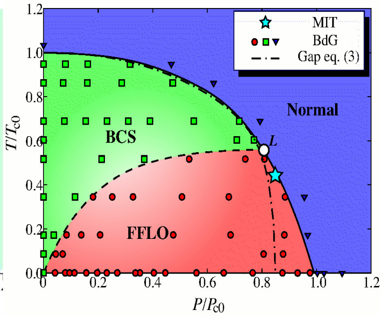
$$\mu_\uparrow = \mu + \delta\mu, \qquad \mu_\downarrow = \mu - \delta\mu, \qquad \delta\mu \propto \sigma B$$

-Concepts of "gapless superconductivity" (1961)
-Concepts of moving condensate - "Larkin-Ovchninnikov-Fulde-Ferrel phase" (1964)

Sharma and FFLO phases

Large spin polarizations destroy the superconductivity, but Fulde-Ferrell-Larkin-Ovchnnikov showed that a new phase can appear which is carrying current and is stable.





Pairing with population imbalance

Nuclear matter with imbalance - isospin asymmetry:

$$\hat{H} = \frac{1}{2m}\sum_{\alpha}\int d^3x \nabla\hat{\psi}^{\dagger}_{\alpha}(\boldsymbol{r})\nabla\hat{\psi}_{\alpha}(\boldsymbol{r}) - \sum_{\alpha\beta}\int d^3x \int d^3x' \hat{\psi}^{\dagger}_{\alpha}(\boldsymbol{r})\hat{\psi}^{\dagger}_{\beta}(\boldsymbol{r}) \underbrace{V(\boldsymbol{r},\boldsymbol{r}')}_{two-body-int.} \hat{\psi}_{\beta}(\boldsymbol{r}')\hat{\psi}_{\alpha}(\boldsymbol{r}).$$

Dyson-Schwinger equations (DSE) for species 1 and 2

$$\hat{G}^{-1}_{\alpha}(x_1)\hat{G}_{\alpha\beta}(x_1,x_2) = \hat{\mathbf{1}}\delta_{\alpha\beta}\delta(x_1-x_2) + i\sum_{\gamma}\int d^3x_3\, \hat{\Sigma}_{\alpha\gamma}(x_1,x_3)\hat{G}_{\gamma\beta}(x_3,x_2),$$

where $\hat{\mathbf{1}}$ is a unit matrix, $G^{-1}_{\alpha}(x) \equiv i\partial/\partial t + \boldsymbol{\nabla}^2/2m_{\alpha} + \mu_{\alpha}$ and GF of the superfluid state (Nambu-Gorkov space)

$$i\hat{G}_{\alpha\beta}(x_1,x_2) \equiv i\begin{pmatrix} G_{\alpha\beta}(x_1,x_2) & F_{\alpha\beta}(x_1,x_2) \\ F^{\dagger}_{\alpha\beta}(x_1,x_2) & G^{\dagger}_{\alpha\beta}(x_1,x_2)\end{pmatrix}, \quad \hat{G}^{-1}_{\alpha}(x) = \begin{pmatrix} G^{-1}_{\alpha}(x) & 0 \\ 0 & \left[G^{-1}_{\alpha}(x)\right]^{*}\end{pmatrix}.$$

The DSE equations are closed via the approx. for the self-energy matrix (anomalous part reads)

$$\Delta_{\alpha\beta}(x_1,x_2) = \sum_{\gamma\kappa}\int \Gamma_{\alpha\beta\gamma\kappa}(x_1,x_2;x_3,x_4)F_{\gamma\kappa}(x_3,x_4)dx_3dx_4.$$

Quasiclassical approximation

Inhomogeneous systems - separation of CM and relative motions

$$\hat{G}(x,X) \to \hat{G}(\omega,\boldsymbol{p},\mathbf{R},T), \qquad x = x_1 - x_2, \qquad X = (x_1+x_2)/2$$

The DSE now is written as

$$\sum_{\gamma} \begin{pmatrix} \omega - \epsilon^{+}_{\alpha\gamma} & -\Delta_{\alpha\gamma} \\ -\Delta^{\dagger}_{\alpha\gamma} & \omega + \epsilon^{-}_{\alpha\gamma} \end{pmatrix} \begin{pmatrix} G_{\gamma\beta} & F_{\gamma\beta} \\ F^{\dagger}_{\gamma\beta} & G^{\dagger}_{\gamma\beta} \end{pmatrix} = \delta_{\alpha\beta}\hat{\mathbf{1}}, \tag{1}$$

where

$$\boxed{\epsilon^{\pm}_{\alpha\beta} = \left(\mathbf{P}/2 \pm \boldsymbol{p}\right)^2/2m_\alpha - \mu_\alpha \pm \mathrm{Re}\,\Sigma_{\alpha\beta} - \mathrm{Im}\,\Sigma_{\alpha\beta},} \tag{2}$$

The quasiparticle excitation spectrum is determined in the standard fashion by finding the poles of the propagators

$$\omega_{\pm\pm} = \epsilon_A \pm \sqrt{\epsilon_S + \frac{1}{2}\mathrm{Tr}\left(\Delta\Delta^{\dagger}\right) \pm \frac{1}{2}\sqrt{[\mathrm{Tr}\left(\Delta\Delta^{\dagger}\right)]^2 - 4\mathrm{Det}\left(\Delta\Delta^{\dagger}\right)}}.$$

$$\Delta \equiv \Delta_{\alpha\beta} \qquad \epsilon_S = (\epsilon^{+} + \epsilon^{-})/2, \qquad \epsilon_A = (\epsilon^{+} - \epsilon^{-})/2$$

$$\omega_{\pm\pm} = \epsilon_A \pm \sqrt{\epsilon_S + \frac{1}{2}\mathrm{Tr}\left(\Delta\Delta^\dagger\right) \pm \frac{1}{2}\sqrt{[\mathrm{Tr}\left(\Delta\Delta^\dagger\right)]^2 - 4\mathrm{Det}\left(\Delta\Delta^\dagger\right)}}.$$

Four-fold split spectrum: - isospin asymmetry and finite momentum
- competition between spin-1 and spin-0 pairing

Solve coupled equations for densities

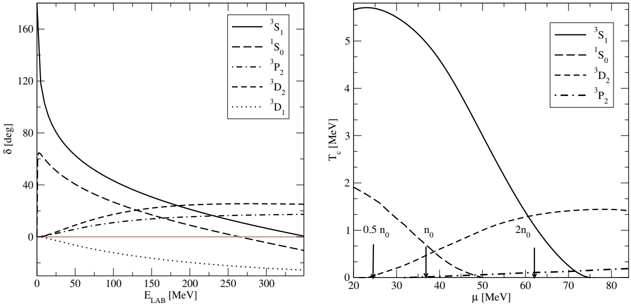
$$\rho_{n/p}(\vec{Q}) = -2\int \frac{d^4k}{(2\pi)^4}\mathrm{Im}[G^+_{n/p}(k,\vec{Q}) - G^-_{n/p}(k,\vec{Q})]f(\omega), \tag{3}$$

and pairing gap

$$\Delta(Q) = \frac{1}{2}\sum_{a,r}\int \frac{d^3k'}{(2\pi)^3} V_{l,l'}(k,k') \frac{\Delta_{l'}(k',Q)}{2\sqrt{E_S(k')^2 + \Delta_{l'}(k',Q)}}[1 - 2f(E^r_a)], \tag{4}$$

where $V_{l,l'}(k,k')$ is the interaction in the $^3S_1$-$^3D_1$ partial wave.

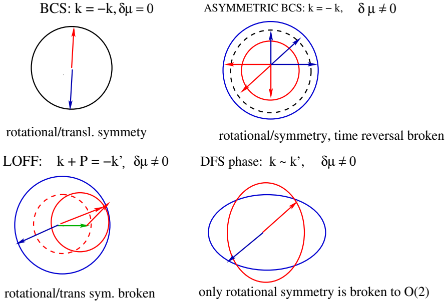
Phase shifts and pairing channels in nuclear matter



The low-density limit in nuclear systems corresponds to the deuteron Bose-Einstein condensate. Therefore, we have a density driven BCS-BEC crossover!

regime	$\log\left(\frac{\rho}{\rho_0}\right)$	T [MeV]	d [fm]	ξ_{rms} [fm]	ξ_a [fm]
WCR	-0.5	0.5	1.68	3.17	1.41
ICR	-1.5	0.5	3.61	0.94	1.25
SCR	-2.5	0.2	7.79	0.57	1.79

Studying the phase diagram of dilute nuclear matter (Stein et al, 2012, 2014, 2016)

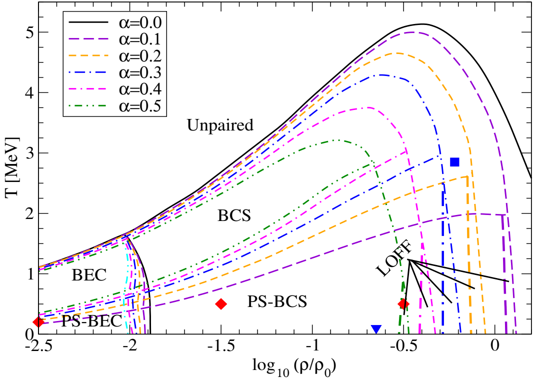


Possible phases, including "spatial mixing" for s and n phases by a factor $0 \leq x \leq 1$

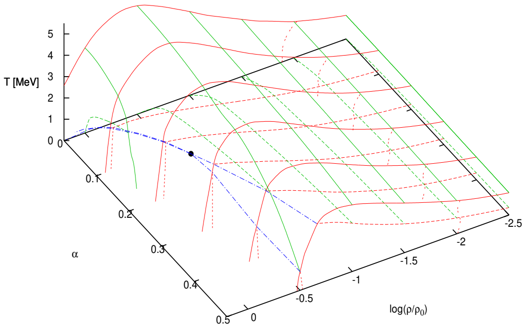
$$\begin{cases} Q = 0, & \Delta \neq 0, & x = 0, & \text{BCS phase,} \\ Q \neq 0, & \Delta \neq 0, & x = 0, & \text{LOFF phase,} \\ Q = 0, & \Delta \neq 0, & x \neq 0, & \text{PS phase,} \\ Q = 0, & \Delta = 0, & x = 1, & \text{unpaired phase,} \end{cases}$$

The phase diagram of SD-paired nuclear matter

Temperature-density phase diagram for varying asymmetry



- Competing phases: BCS, LOFF, PS, Unpaired
- BCS - BEC crossover, with LOFF disappearing in the low density limit
- tetra-critical points (Lifshitz point), i.e., an inhomogeneous phase terminates at the point
- triangle: LOFF quenched by BCS-BEC crossover, quadrangle: quatro-critical-point



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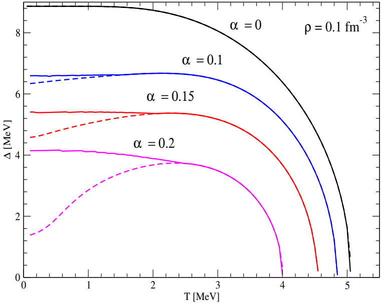
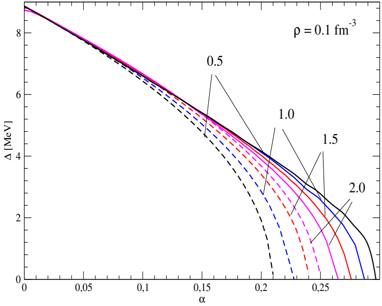
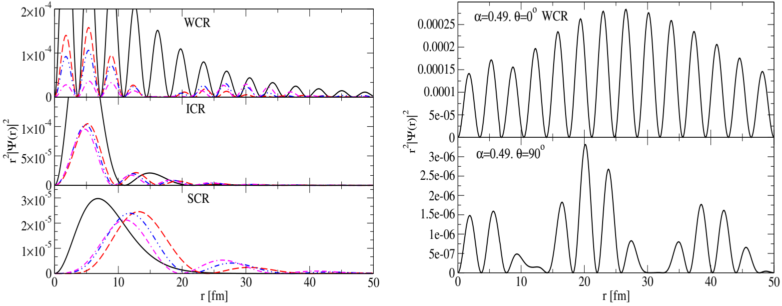
Density and polarization dependence of the gap





- the anomalies in the T-dependence are lifted by the LOFF phase
- the LOFF allows for the paired state for larger polarizations

Density probabilities of Cooper pairs



regime	$\log\left(\frac{\rho}{\rho_0}\right)$	T [MeV]	d [fm]	$\xi_{\rm rms}$ [fm]	ξ_a [fm]
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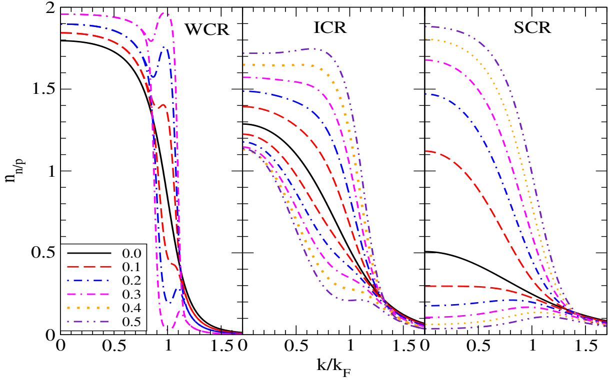
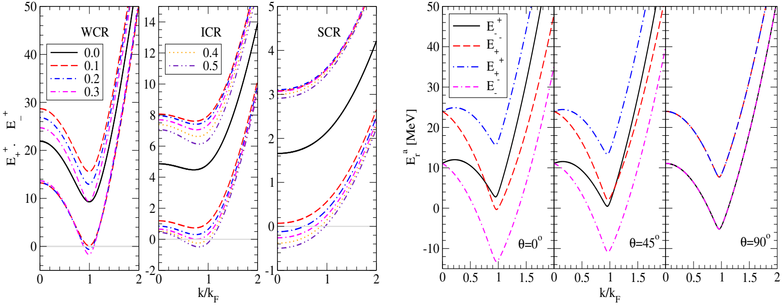
Occupation numbers of majority and minority components



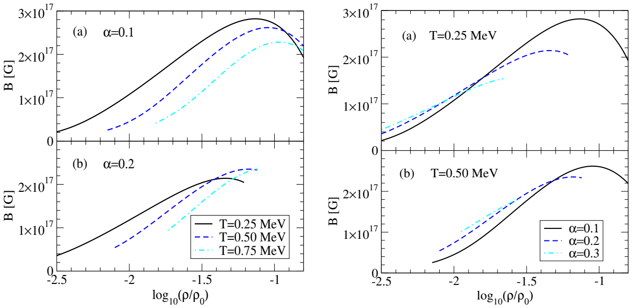
Dependence of the particle occupation numbers on momentum *k* in units of Fermi-momentum for the three coupling regimes and various asymmetries indicated in the plot.

Dispersion relations for majority and minority components



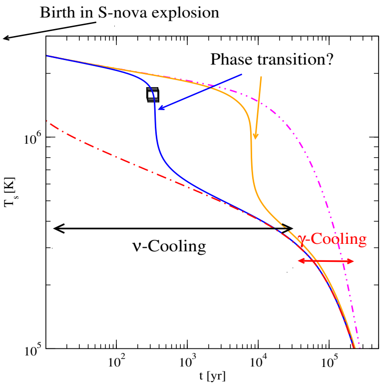
Dispersion relations for quasiparticle spectra in the case of BCS condensate as a function of momentum in units of Fermi-momentum. For each asymmetry the upper branch correspond to E_+^+ and the lower one to E_-^+ solution.

Spin-polarized matter



(a) Magnetic field required to create a specified spin polarization as a function of the density for two polarization values.
(b) Same but for two temperatures T = 0.25 MeV and 0.5 MeV.

Cooling of compact stars



$$\left(\int_0^{R_c} n c_\nu(r,T)\, dV_p\right) \frac{dT'}{dt} = -\int_0^{R_c} n\epsilon_\nu(r,T) e^{2\Phi} dV_p + 4\pi\sigma R^2 T_S^4 e^{2\Phi_c}$$

Neutrino and photon radiation processes

- Modified Urca/brems process

$$n + n \to n + p + e + \bar{\nu},$$
$$n + n \to n + p + \nu + \bar{\nu},$$

- Crustal bremsstrahlung

$$e + (A, Z) \to e + (A, Z) + \nu + \bar{\nu},$$

- Cooper pair-breaking-formation

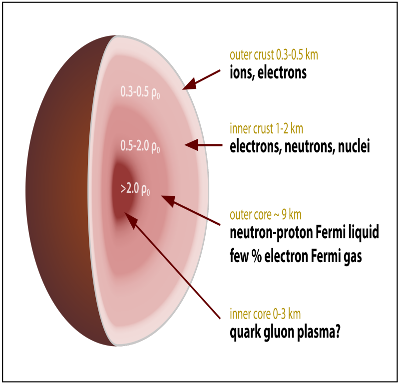
$$[NN] \to [NN] + \nu + \bar{\nu},$$

- Surface photo-emission

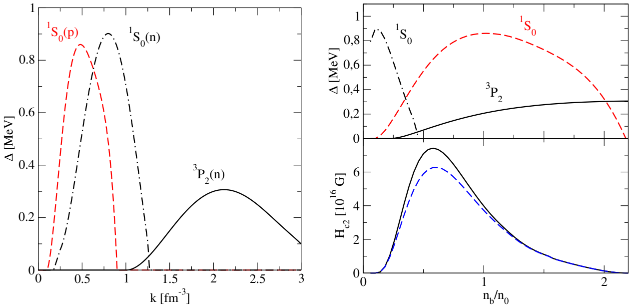
$$L_\gamma = 4\pi\sigma R^2 T^4$$

- Urca process

$$n \to p + e + \bar{\nu}, \quad p + e \to n + \nu.$$



Spin-polarized matter



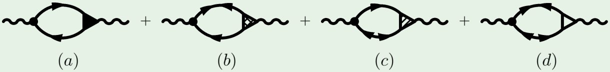
(a) The suppression of proton pairing with increasing field; (b) the suppression of the neutrino emissivity from proton condensate as pairing disappears.

Computing the PB emissivity

The neutrino emissivity is expressed in terms of the polarization tensor of baryonic matter

$$\varepsilon_{\nu\bar{\nu}} = -2\left(\frac{G_F}{2\sqrt{2}}\right)^2 \int d^4q g(\omega)\omega \sum_{i=1,2} \int \frac{d^3q_i}{(2\pi)^3 2\omega_i} \Im[L^{\mu\lambda}(q_i)\,\Pi_{\mu\lambda}(q)]\delta^{(4)}(q - \sum_i q_i),$$

Four polarization tensors in Nambu-Gorkov space:



To describe a superfluid we need the propagators

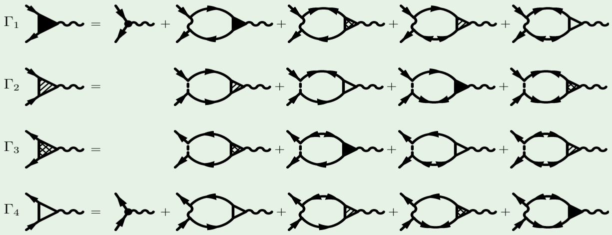
$$\mathcal{G}_{\sigma,\sigma'}(i\omega_n,\boldsymbol{p}) = \begin{pmatrix} \hat{G}_{\sigma\sigma'}(i\omega_n,\boldsymbol{p}) & \hat{F}_{\sigma\sigma'}(i\omega_n,\boldsymbol{p}) \\ \hat{F}^{+}_{\sigma\sigma'}(i\omega_n,\boldsymbol{p}) & \hat{G}^{+}_{\sigma\sigma'}(i\omega_n,\boldsymbol{p}) \end{pmatrix}.$$

which in the momentum space is given by

$$\hat{G}_{\sigma\sigma'}(i\omega_n,\boldsymbol{p}) = \delta_{\sigma\sigma'}\left(\frac{u_p^2}{i\omega_n - \varepsilon_p} + \frac{v_p^2}{i\omega_n + \varepsilon_p}\right),$$

$$\hat{F}_{\sigma\sigma'}(i\omega_n,\boldsymbol{p}) = -i\sigma_y u_p v_p\left(\frac{1}{i\omega_n - \varepsilon_p} - \frac{1}{i\omega_n + \varepsilon_p}\right),$$

Vertex functions



To leading order in the small q/ω expansion, the temporal (00) and spatial (jj) components of density (D) and spin (S) response functions may be written as

$$\Pi_D^{00} = -\frac{4q^4 v_F^4}{45\omega^4}\,\mathcal{G}, \quad \Pi_D^{jj} = -\frac{2q^2 v_F^4}{9\omega^2}\,\mathcal{G}, \quad \Pi_S^{00} = -v_F^2\,\mathcal{G}, \quad \Pi_S^{jj} = -\frac{q^2 v_F^2}{\omega^2}\,\mathcal{G},$$

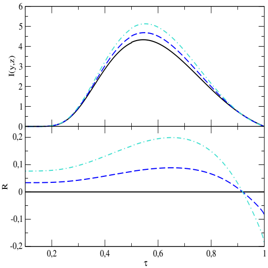
$$\mathcal{G}(\boldsymbol{v},\omega,\boldsymbol{q}) = \Delta^2 \int_{-\infty}^{+\infty} d\xi_p \left[\frac{\epsilon_+ - \epsilon_-}{\epsilon_+ \epsilon_-} \frac{f(\epsilon_-) - f(\epsilon_+)}{\omega^2 - (\epsilon_+ - \epsilon_-)^2 + i\eta} - \frac{\epsilon_+ + \epsilon_-}{\epsilon_+ \epsilon_-} \frac{1 - f(\epsilon_-) - f(\epsilon_+)}{\omega^2 - (\epsilon_+ + \epsilon_-)^2 + i\eta} \right].$$

Vector and axial vector current ν-$\bar{\nu}$ emissivity

The emissivities are given by

$$\epsilon_\nu^V = \frac{16 G^2 c_V^2}{1215\pi^3} \nu(0) v_F^4 I\left(\frac{\Delta}{T}\right) T^7,$$

$$\epsilon_\nu^A = \frac{4 G^2 g_A^2}{15\pi^3} \zeta_A \nu(0) {v_F}^2 I\left(\frac{\Delta}{T}\right) T^7,$$



$$I\left(\frac{\Delta}{T}\right) = \left(\frac{\Delta}{T}\right)^7 \int_1^\infty \frac{dy\, y^5}{\sqrt{y^2-1}} f\left[\left(\frac{\Delta}{T}\right) y\right]^2 \left[1 + \left(\frac{7}{33} + \frac{41}{77}\gamma\right) v_F^2\right].$$

- Systematic expansion in v_F^2, axial current emission is dominant
- The emissivity has a maximum at $\tau = T/T_c = 0.6$, i.e., affects cooling close to the phase transition

Conclusions

Peter Schuck was an inspiration as a scientist and as a human being for many of us, definitely for me. His memory and legacy will stay with us for many decades to come.