IAA : Institut d'Astronomie et d'Astrophysique
Université Libre de Bruxelles

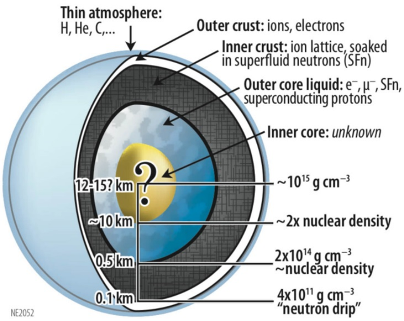
SUPERFLUID DYNAMICS IN NEUTRON STARS

ALLARD Valentin
March 23, 2023

QMBC 2023 : In memory of Peter Schuck

Nuclear superfluidity and neutron stars

Neutron stars contain **extremely dense matter** cold enough to be in various exotic quantum phases.



- **Neutron superfluidity** in the inner crust and **neutron-proton superfluid mixture** in the core.
- Complex **order parameter** $\tilde{\Delta} \Longrightarrow$ impact on thermal and transport properties.
- **Superfluid neutrons weakly coupled** to the rest of the star $\Longrightarrow$ **Finite currents** !

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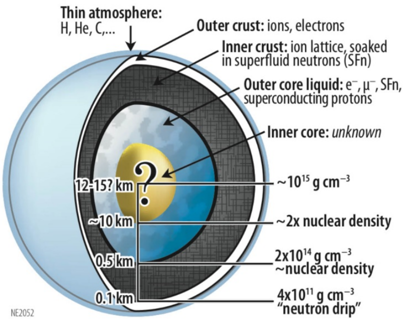

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- **Superfluid neutrons weakly coupled** to the rest of the star $\Longrightarrow$ Finite currents !

Most microscopic studies (see, e.g. A. Sedrakian and J. W. Clark, Eur. Phys. J. A **55** (2019)) work **assuming small currents** (which are relevant for observations e.g. pulsar glitches).

Superfluid hydrodynamics and entrainment effects

Superfluidity characterized by a complex **order parameter**

$$\tilde{\Delta}_q = \Delta_q \mathrm{e}^{i\phi_q} \qquad (q = n, p)$$

Superfluid = **Multifluid** hydrodynamics

- **Normal fluid** velocity $\boldsymbol{v_N}$
- **Superfluid** velocity (momentum) $\boldsymbol{V_q} = \hbar\boldsymbol{\nabla}\phi_q/(2m)$

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Entrainment effects (A. F. Andreev and E. P. Bashkin, Sov. Phys. JETP **42** (1975))

Interactions between superfluids $\Longrightarrow$ **Neutron flow** (of the mixture) **entraining** the **proton flow** and vice versa.

$$\boldsymbol{\rho_n} = mn_n\boldsymbol{v_N} + \rho_{nn}(\boldsymbol{V_n} - \boldsymbol{v_N}) + \rho_{np}(\boldsymbol{V_p} - \boldsymbol{v_N}) \neq mn_n\boldsymbol{V_n}$$

$$\boldsymbol{\rho_p} = mn_p\boldsymbol{v_N} + \rho_{pp}(\boldsymbol{V_p} - \boldsymbol{v_N}) + \rho_{pn}(\boldsymbol{V_n} - \boldsymbol{v_N}) \neq mn_p\boldsymbol{V_p}$$

- $\rho_{qq'}$ = **Entrainment matrix** = strength of $q - q'$ coupling $(q, q' = n, p)$
- $\boldsymbol{\rho_q}$ = **Mass current** of nucleon species q

Energy-density functional theory with currents

The dynamic of neutron-proton mixtures is governed by the **time-dependent Hartree-Fock Bogoliubov (TDHFB) equations**

$$
\begin{aligned}
i\hbar\partial_t n_q(\mathbf{r}\sigma,\mathbf{r}'\sigma',t) &= h_q(\mathbf{r},t)n_q(\mathbf{r}\sigma,\mathbf{r}'\sigma',t) - h_q^*(\mathbf{r}',t)n_q(\mathbf{r}\sigma,\mathbf{r}'\sigma',t) \\
&\quad + \sigma\sigma'\tilde{\Delta}_q(\boldsymbol{r},t)\tilde{n}_q(\boldsymbol{r}-\sigma,\boldsymbol{r'}-\sigma',t) - \tilde{n}_q(\boldsymbol{r}\sigma,\boldsymbol{r'}\sigma',t)\tilde{\Delta}_q^\star(\boldsymbol{r'},t)
\end{aligned}
$$

with single-particle hamiltonian

$$
h_q(\mathbf{r},t) = -\boldsymbol{\nabla}.\frac{\hbar^2}{2m_q^\oplus(\mathbf{r},t)}\boldsymbol{\nabla} + U_q(\mathbf{r},t) + \frac{1}{2i}\left[\mathbf{I}_q(\mathbf{r},t).\boldsymbol{\nabla} + \boldsymbol{\nabla}.\mathbf{I}_q(\mathbf{r},t)\right]
$$

with potentials defined through particle density $n_q(\boldsymbol{r}\sigma,\boldsymbol{r'}\sigma',t)$ and pair density matrices $\tilde{n}_q(\boldsymbol{r}\sigma,\boldsymbol{r'}\sigma',t)$,

$$
\frac{\hbar^2}{2m_q^\oplus(\mathbf{r},t)} = \frac{\delta E}{\delta\tau_q(\mathbf{r},t)}, \qquad U_q(\mathbf{r},t) = \frac{\delta E}{\delta n_q(\mathbf{r},t)}, \qquad \mathbf{I}_q(\mathbf{r},t) = \frac{\delta E}{\delta \mathbf{j}_q(\mathbf{r},t)}
$$

$$
\tilde{\Delta}_q = 2\frac{\delta E}{\delta\tilde{n}_q^\star(\boldsymbol{r},t)} = \Delta_q(\boldsymbol{r},t)\mathrm{e}^{i\phi_q(\boldsymbol{r},t)}
$$

Mass current and Skyrme functionals

A **continuity equation** can be derived from the TDHFB equations

$$\partial_t \left(m n_q(\boldsymbol{r}, t) \right) + \boldsymbol{\nabla} . \boldsymbol{\rho_q}(\boldsymbol{r}, t) = 0$$

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Mass current $\boldsymbol{\rho_q}$ (N. Chamel and V. Allard, PRC100, 065801 (2019))

$$\boldsymbol{\rho_q}(\mathbf{r},t) = m n_q(\boldsymbol{r},t) \left(\frac{\hbar \boldsymbol{j_q}(\mathbf{r},t)}{m_q^{\oplus}(\mathbf{r},t) n_q(\boldsymbol{r},t)} + \frac{\boldsymbol{I_q}(\mathbf{r},t)}{\hbar} \right) = m n_q(\boldsymbol{r},t) \boldsymbol{v_q}(\boldsymbol{r},t)$$

Which allows to define the **true velocity** as $\boldsymbol{v_q}(\boldsymbol{r},t) = \boldsymbol{\rho_q}(\boldsymbol{r},t)/(m n_q(\boldsymbol{r},t))$

- Does not explicitly depend on the pairing field Δ_q.
- **General** case: valid for both uniform and non-uniform systems.

Homogeneous solutions : $T = 0$ K and small currents

For homogeneous neutron-proton superfluid mixture, **at low temperatures and small currents**, the normal component disappears ($\boldsymbol{v_N} = \boldsymbol{0}$)

$$\boldsymbol{\rho_n} = \rho_{nn}^{(\mathrm{TDHF})}\boldsymbol{V_n} + \rho_{np}^{(\mathrm{TDHF})}\boldsymbol{V_p}\,, \qquad \boldsymbol{\rho_p} = \rho_{pp}^{(\mathrm{TDHF})}\boldsymbol{V_p} + \rho_{pn}^{(\mathrm{TDHF})}\boldsymbol{V_n}$$

The entrainment matrix becomes **independent of pairing** $\Longrightarrow$ TDHF !
(N. Chamel and V. Allard, PRC100, 065801 (2019))

$$\rho_{np}^{(\mathrm{TDHF})} = \rho_{pn}^{(\mathrm{TDHF})} = \frac{1}{4}mn(1-\eta^2)\left(1 - \frac{m}{m_V^{\oplus}}\right)$$

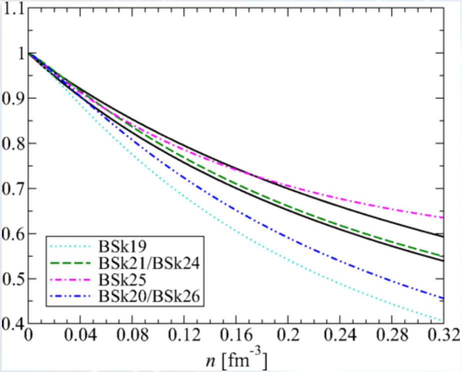
$$\rho_{nn}^{(\mathrm{TDHF})} = \frac{1}{2}mn(1+\eta) - \rho_{np}^{(\mathrm{TDHF})}\,, \qquad \rho_{pp}^{(\mathrm{TDHF})} = \frac{1}{2}mn(1-\eta) - \rho_{np}^{(\mathrm{TDHF})}\,.$$

with $n = (n_n + n_p)$ the total density and $\eta = (n_n - n_p)/n$ the isospin asymmetry.

The entrainment matrix is parametrized by the **isovector effective mass**!

Homogeneous solutions : $T = 0$ K and small currents

Isovector effective mass is also related to **giant resonances in atomic nuclei** !



Its density dependence is still uncertain !

Homogeneous solutions: finite T and finite currents

Focusing on homogeneous neutron-proton superfluid mixture with stationary flows in normal fluid rest frame ($\boldsymbol{v_N} = \mathbf{0}$), **TDHFB can be solved exactly** !

$$\mathcal{E}_{\boldsymbol{k}}^{(q)} = \hbar \boldsymbol{k} \cdot \mathbb{V}_{\boldsymbol{q}} + \sqrt{\varepsilon_{\boldsymbol{k}}^{(q)2} + \Delta_q^2}\,, \qquad \varepsilon_{\boldsymbol{k}}^{(q)} = \frac{\hbar^2 \boldsymbol{k}^2}{2m_q^\oplus} - \mu_q\,,$$

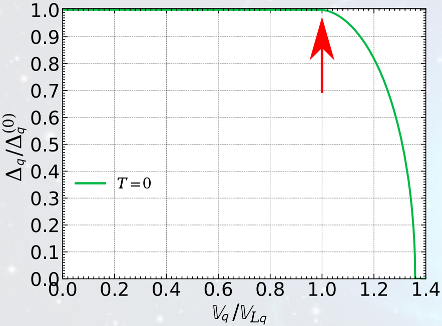
Effective superfluid velocity (V. Allard and N. Chamel, Universe 7(12) (2021))

$$\mathbb{V}_{\boldsymbol{q}} = \frac{m}{m_q^\oplus} \boldsymbol{V_q} + \frac{\boldsymbol{I_q}}{\hbar} \neq \boldsymbol{v_q}$$

- $\mathbb{V}_{\boldsymbol{q}}$ (q=n,p) contains the mutual contributions of $\boldsymbol{V_n}$ AND $\boldsymbol{V_p}$.
- **Dynamical decoupling** between quantities associated with protons or neutrons.

Order parameter : finite T and finite currents

The order parameter $\Delta_q(T, \mathbb{V}_q)$ is found to be **universal after proper rescaling** ! (V. Allard and N. Chamel, PRC103, 025804 (2021))



Velocity dependence entirely contained in the norm of $\boldsymbol{\mathbb{V}_q}$!

- $\Delta_q^{(0)}$ = Order parameter at $T = 0$ and in absence of velocity.

- $\mathbb{V}_{Lq} \approx \Delta_q^{(0)}/(\hbar k_{Fq})$ **(Landau's velocity)**.

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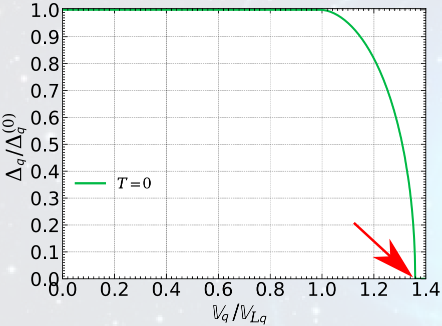

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- Transition to normal phase beyond $\mathbb{V}_{cq}^{(0)} \simeq e\mathbb{V}_{Lq}/2$

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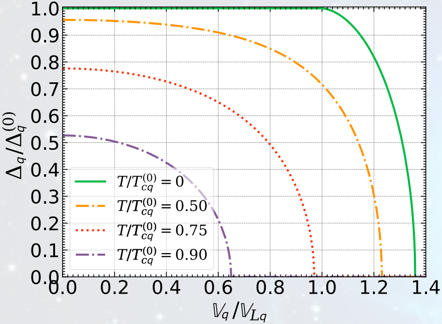

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- $\mathbb{V}_{Lq} \approx \Delta_q^{(0)}/(\hbar k_{Fq})$ **(Landau's velocity)**.
- $\mathbb{V}_{cq}^{(0)} \simeq \mathrm{e}\mathbb{V}_{Lq}/2$ (Critical velocity)
- $T_{cq}^{(0)} = \mathrm{e}^{\gamma}\Delta_q^{(0)}/\pi$ (BCS like)

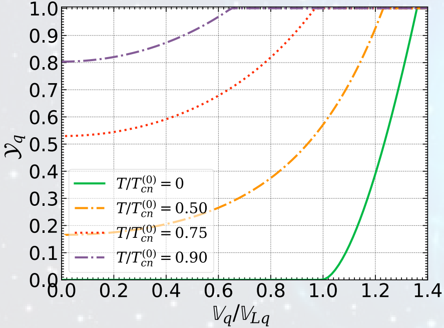
Interpolating functions available (V. Allard and N. Chamel, Universe **7(12)** (2021))!

Mass current and quasiparticle fractions

The mass current/true velocity (and entrainment matrix) take a simple form

$$\boldsymbol{\rho_q} = mn_q\left(1-\mathcal{Y}_q(T,\mathbb{V}_q)\right)\mathbb{V}_{\boldsymbol{q}}\,, \qquad \boldsymbol{v_q} = \left(1-\mathcal{Y}_q(T,\mathbb{V}_q)\right)\mathbb{V}_{\boldsymbol{q}}$$

With $\mathcal{Y}_q$, the **quasiparticle fractions** (universal after using $\mathbb{V}_q$ and rescaling)



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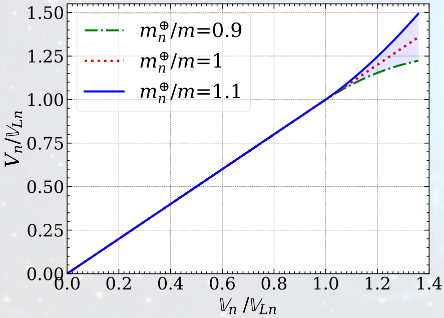
Velocities

Three kind of velocities

- Superfluid velocity $\boldsymbol{V_q}$: Rescaled **momentum**.
- Effective superfluid velocity $\mathbb{V}_q$: **Dynamical decoupling** between neutrons and protons.
- True velocity $\boldsymbol{v_q}$: Velocity of **mass-transport** of nucleons.



Example: Results obtained from neutron matter ($n_p = 0$) $\Longrightarrow$ **Non-linear universal relations**!

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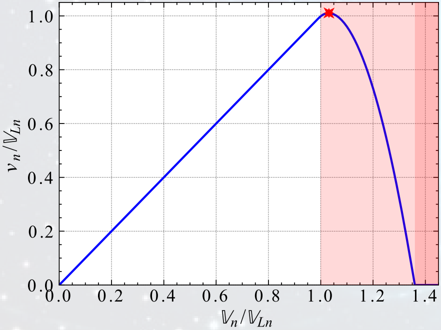

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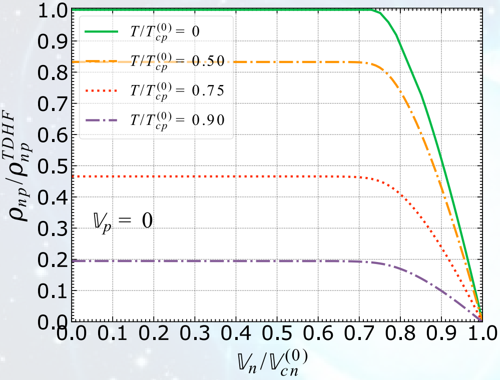
Entrainment matrix : finite T and finite currents

Entrainment matrix numerically computed in *V. Allard and N. Chamel, Universe 7(12) (2021)* !



Universal expressions and interpolating functions !

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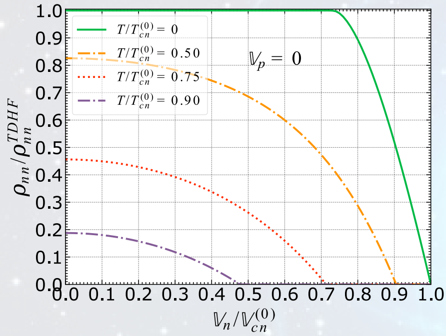
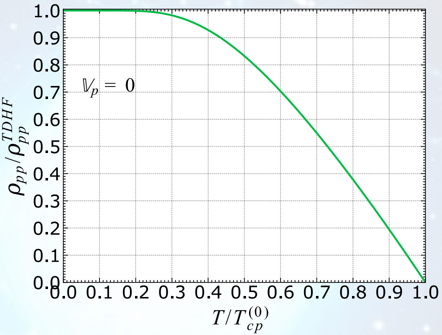
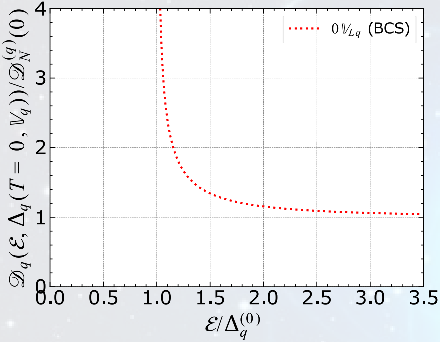


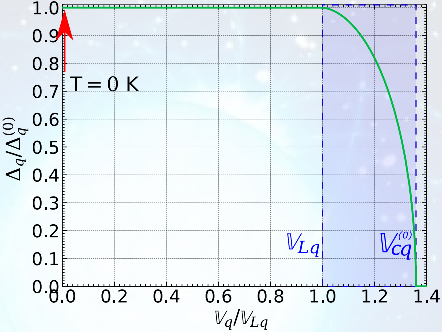


Universal expressions and interpolating functions !

Quasiparticle density of states and gapless regime

Finite currents influence the quasiparticle density of states (DoS).



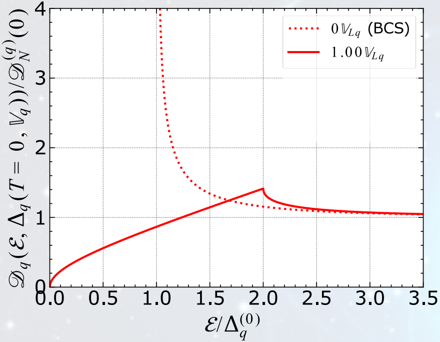


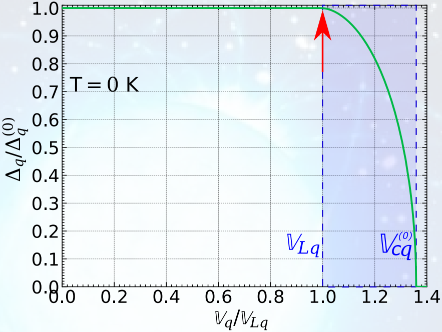
BCS regime : $\mathbb{V}_q < \mathbb{V}_{Lq}$

No quasiparticle excitation for $\mathcal{E} < \Delta_q^{(0)}$ (**energy gap**).

Quasiparticle density of states and gapless regime

The gap disappears at Landau's velocity !





Gapless regime (V. Allard and N. Chamel, Phys. Sci. Forum 7(1), (2023))

- **Gapless superfluidity** ($\Delta_q \neq 0$) **for** $\mathbb{V}_{Lq} \leq \mathbb{V}_q \leq e\mathbb{V}_{Lq}/2$.
- **Impact on transport properties** (e.g. the specific heat) !

Gapless superfluidity and specific heat

The presence of an energy gap influences the specific heat.

Normal phase : $\Delta_q = 0$

Specific heat proportionnal to the temperature :

$$c_N^{(q)}(T) \approx \frac{\pi^2}{3}\mathcal{D}_N^{(q)}(0)k_{\mathrm{B}}^2 T$$

(with $\mathcal{D}_N^{(q)}(0)$ = Density of quasiparticle states in normal phase)

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BCS superfluidity : $\Delta_q \neq 0$ and $\mathbb{V}_q = 0$

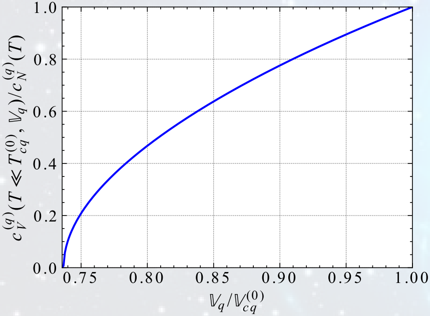
Exponential suppression at low temperatures:

$$c_V^{(q)}(T \ll T_{cq}^{(0)}) \propto \mathrm{e}^{-\Delta_q^{(0)}/(k_\mathrm{B}T)} c_N^{(q)}(T)$$

Gapless superfluidity and specific heat

Gapless superfluidity : $\mathbb{V}_{Lq} < \mathbb{V}_q \leq e\mathbb{V}_{Lq}/2$

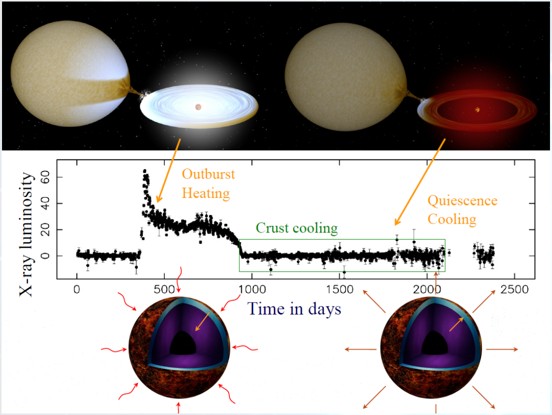
The specific heat $c_V^{(q)}$ becomes comparable to $c_N^{(q)}(T)$.



Universal expression for $c_V^{(q)}(T,\mathbb{V}_q)/c_N^{(q)}(T)$ as a function of $\mathbb{V}_q/\mathbb{V}_{Lq}$ (V. Allard and N. Chamel, PRC. (2022), *submitted*)

Application : Quasipersistent soft X-ray transients

Neutron star crust heated during **accretion regime** (for ~ 1-10 years) before **cooling phase** (see R. Wijnands et al, J. Astrophys. Astr. **38** (2017), for a review).



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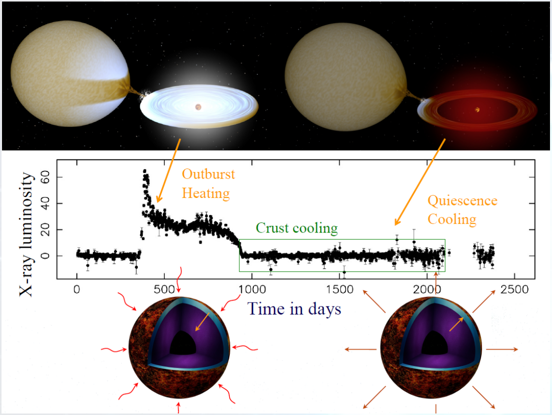
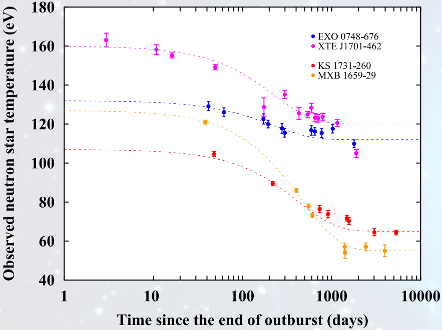


Thermal relaxation observed for several sources up to 10^4 days.

Cooling curve

The **cooling curve** $T_{\text{eff}}^{\infty}(t)$ allows to **probe the NS interior**.



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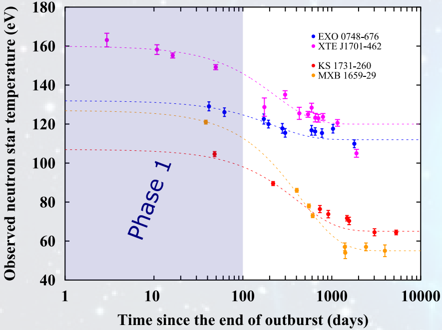

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- **Phase 1** : T^{∞}_{eff} sensitive to the **outer crust**.

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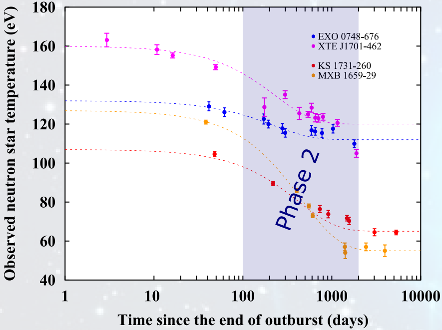

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- **Phase 1** : $T_{\rm eff}^{\infty}$ sensitive to the **outer crust**.
- **Phase 2** : $T_{\rm eff}^{\infty}$ sensitive to the **inner crust**.

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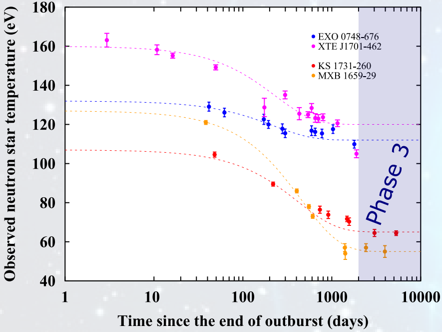

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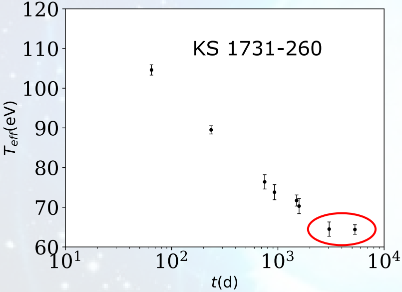
- **Phase 2** : $T_{\mathrm{eff}}^{\infty}$ sensitive to the **inner crust**.
- **Phase 3** : $T_{\mathrm{eff}}^{\infty}$ sensitive to the **outer core** $\Longrightarrow$ Thermal equilibrium.

Problematic systems

Two systems exhibit **challenge our current understanding** :

KS 1731-260

- Colder than expected (E. M. Cackett et al., ApJL **722** (2010)).



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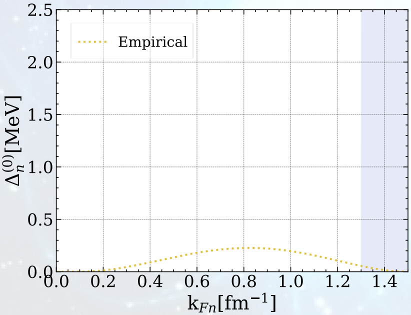

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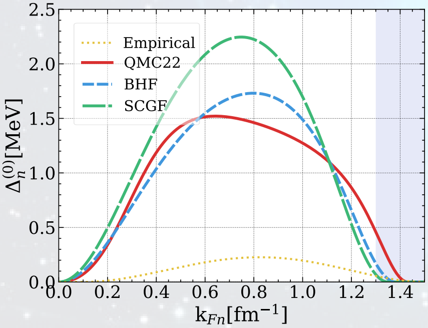

Microscopic neutron pairing

- **QMC22:** S. Gandolfi et al, Condens. Matter, **7(1)** (2022).
- **BHF:** L. G. Cao et al, Phys. Rev. C **74** (2006).
- **SCGF:** M. Drissi and A. Rios, Eur. Phys. J. A **58** (2022).

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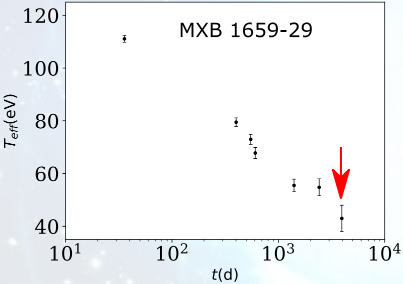
MXB 1659-29

- **Unexpected** late-time temperature drop (Cackett et al.,ApJ **774** (2013)).



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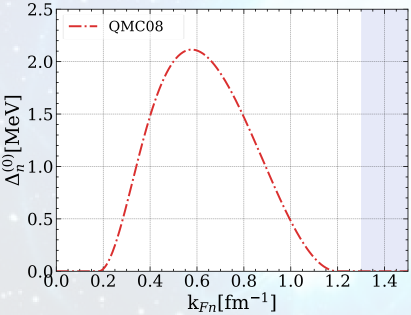
- Neutrons in normal phase at high densities (Deibel et al., ApJ **839** (2017))



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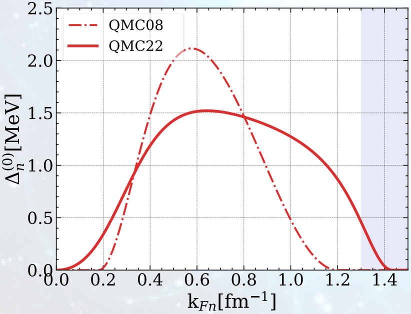
Two systems exhibit **challenge our current understanding** :

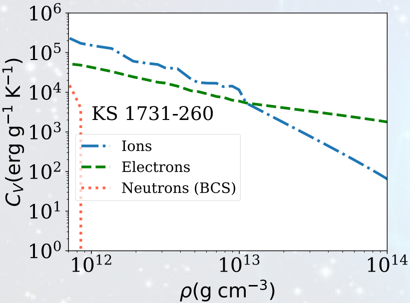
MXB 1659-29

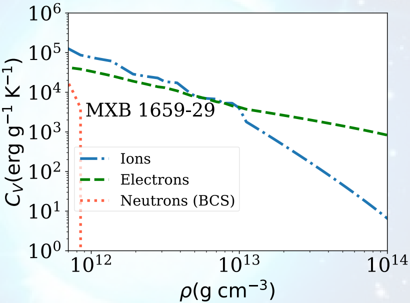
- **Unexpected** late-time temperature drop (Cackett et al.,ApJ **774** (2013)).
- Neutrons in normal phase at high densities (Deibel et al., ApJ **839** (2017)) **BUT contradicted by more recent calculations** !



Cooling of Neutron Star crust

Gapless regime allows to **explain observations using recent/realistic nuclear pairing (e.g., QMC22, BHF of SCGF)**! (V. Allard and N. Chamel, Phys. Rev. L. (2022), *submitted*)





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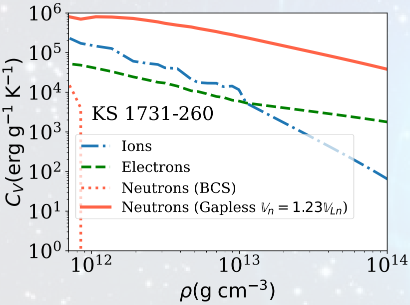

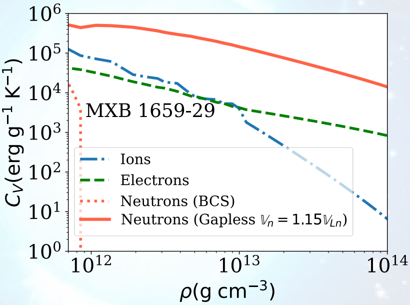

Gapless neutrons give the **major contribution** to the specific heat.

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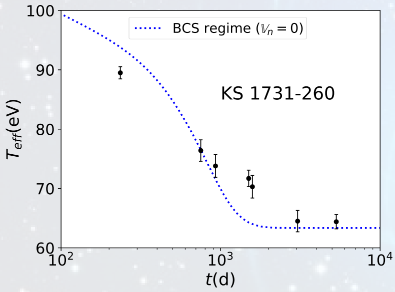

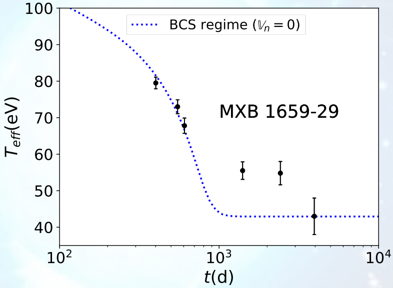

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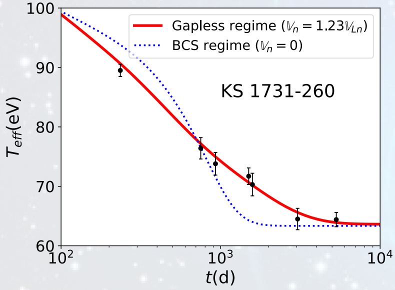

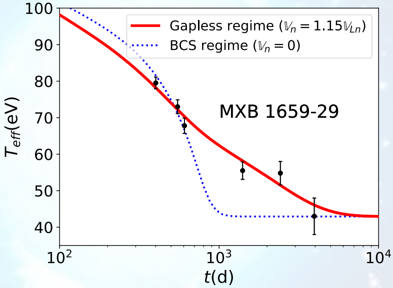

Delayed thermal relaxation in the crust.

Cooling of Neutron Star crust

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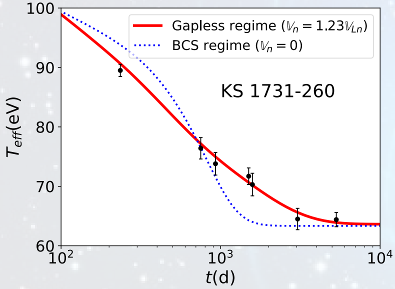
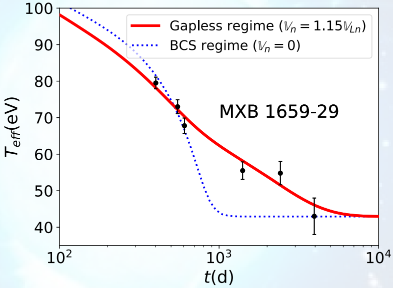


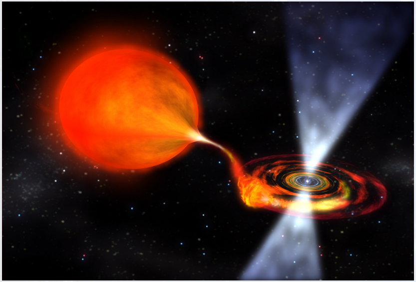


Question: How do we obtain finite value of $\mathbb{V}_n$?

Origin of finite superflows in neutron stars

Recycling scenario

Crust (ions + electrons + muons in normal phase) spun up during accretion (transfer of angular momentum from infalling material) $\Longrightarrow \mathbb{V}_n$ increasing.



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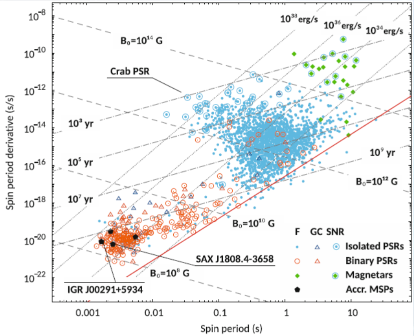


Accreting millisecond X-ray pulsars ?

Burst oscillations likely related to neutron star spin. (see, e.g; Wijnands et al, Astrophys. Space. Sci. **363** (2002); Astrophys. J. Lett. **606** (2004) and Smith et al, Astrophys. J. **479** (1997))

- MXB : $P_s \sim 1.91$ ms
- KS : $P_s \sim 1.76$ ms

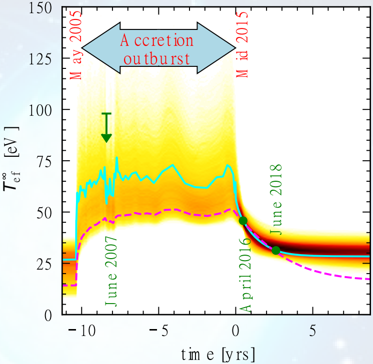
Other potential systems ?

Some systems exhibit particular features

HETE J 1900.1-2455

- **High specific heat** required
- But only 2 observations...

> *"a significant fraction of the dense core is not superfluid/superconductor."* (Degenaar et al., MNRAS 508 (2021))



To clarify

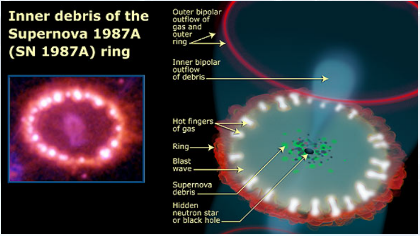
Further observations are expected (*Chandra*) !

Other potential systems ?

Gapless regime in isolated neutron stars ?

SN 1987A remnant

- Neutron star ?
- Suppression of neutron superfluidity also advanced (Page et al., ApJ 898 (2020)).



Caveat

Further observations are needed to make sure the central object is a neutron star...

Conclusions and perspectives

Conclusions

- **Multifluid hydrodynamics** : three types of velocities $\boldsymbol{\mathbb{V}_q} \neq \boldsymbol{V_q} \neq \boldsymbol{v_q}$ (for a given nucleon flow)!
- Effects of superflow appearing through an “effective“ superfluid velocity $\boldsymbol{\mathbb{V}_q}$.
- Order parameter Δ_q, quasiparticle fractions $\mathcal{Y}_q$ and entrainment matrix $\rho_{qq'}$ are found to be **universal** after introducing $\mathbb{V}_q$ and after proper rescaling.
- **Gapless** regime for $\mathbb{V}_{Lq} \leq \mathbb{V}_q \leq \mathrm{e}\mathbb{V}_{Lq}/2$: **impact on neutron star cooling**.

Perspectives : bridging gap between cooling and hydrodynamics

- Combine the neutron star cooling with hydrodynamical models.
- Study the cooling of promising systems.

References

Formalism: TDHFB with currents

- N. Chamel and V. Allard, PRC 100, 065801: `https://doi.org/10.1103/PhysRevC.100.065801`
- V. Allard and N. Chamel, PRC 103, 025804: `https://doi.org/10.1103/PhysRevC.103.025804`

Full numerical results and interpolations

- V. Allard and N. Chamel, Universe 2021, 7(12) (2021): `https://doi.org/10.3390/universe7120470`

Submitted papers: Gapless superfluidity in neutron stars

- V. Allard and N. Chamel, *Gapless superfluidity in neutron stars- I. Thermal properties*, PRC (2022)
- V. Allard and N. Chamel, *Evidence of gapless neutron superfluidity from the late time cooling of transiently acceting neutron stars*, PRL (2022)

Thanks for your attention !