Conference on Quantum Many-Body correlations in memory of Peter Schuck
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FFLO correlations in polarized ultracold Fermi gases

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Many-body correlations with Peter and his collaborators

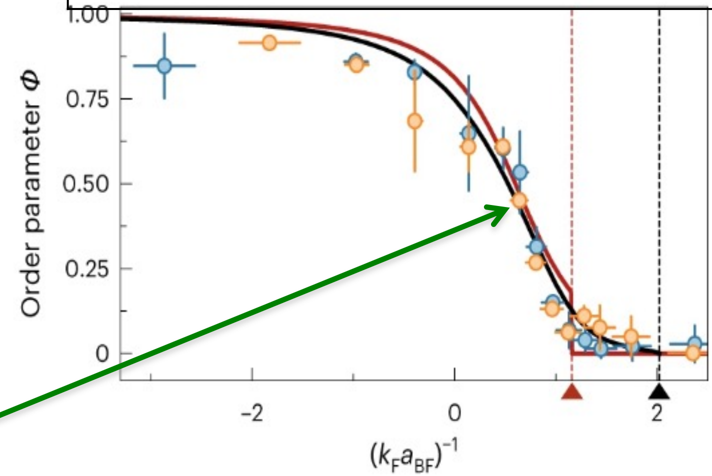
- Polarized Fermi systems ←---→ asymmetric nuclear matter have been object of several beautiful works by Peter and his collaborators and friends (Roepke, Baldo, Lombardo, Grasso, Urban)

- Many inspiring discussions with Peter (and Michael) over the years

G.C. Strinati, P. Pieri, G. Roepke, P. Schuck, M. Urban, The BCS-BEC crossover: From ultra-cold Fermi gases to nuclear systems, Phys. Reports **738**, 1 (2018)

- Multiple interactions with Peter also on boson-fermion pairing in Bose-Fermi mixtures, on which he worked after 2005 for a few years

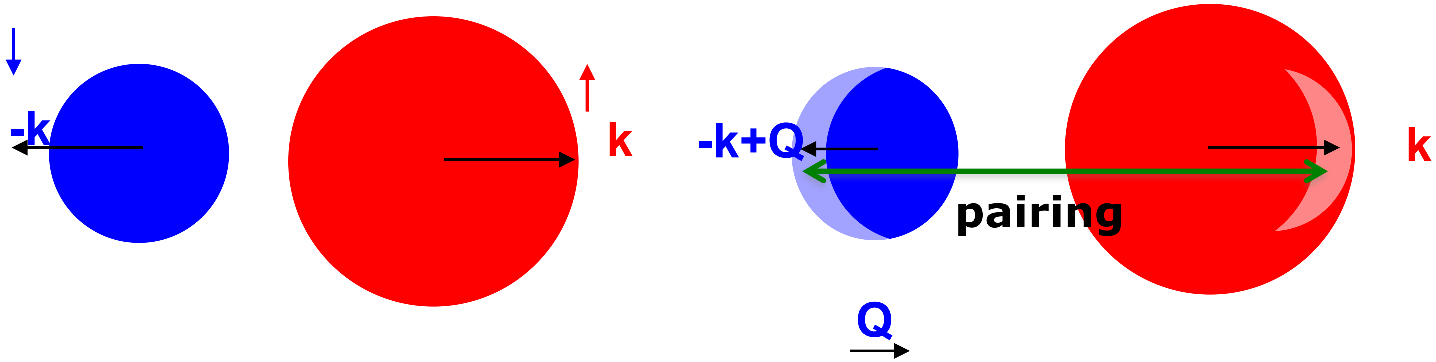
M. Duda et al. Nature Physics (2023)



Theoretical prediction: A. Guidini, G. Bertaina, D.E. Galli, P. Pieri, Phys. Rev. A (2015)

Polarized Fermi gases: search for FFLO phase

- Phase first proposed theoretically by Fulde & Ferrell and independently by Larkin & Ovchinnikov (1964)
- Search for FFLO phase has been one of the main motivations driving experiments with polarized Fermi gases
- Pairing between **k** and **−k+Q** to compensate mismatch of Fermi surfaces: pairs acquire a finite center of mass momentum **Q**



$$|\Psi_{FF}\rangle = \prod_{\mathbf{k}\in\mathcal{R}} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}}{2}\uparrow} \prod_{\mathbf{k}\notin\mathcal{R}} (u_\mathbf{k} + v_\mathbf{k} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}}{2}\uparrow} c^\dagger_{-\mathbf{k}+\frac{\mathbf{Q}}{2}\downarrow})|0\rangle$$

Search for FFLO fluctuating pairs in the normal phase

However:

- In fully isotropic systems, **at finite T,** thermal fluctuations destroy long-range FFLO ordering [Shimahara (1998), Ohashi (2002)] possibly turning it into algebraic order [Radzihovski (2011), Jakubczyk (2017,2021)]

- Phase diagram of polarized Fermi gas (mean field at T=0 & experiments at finite temperature in harmonic trap) is dominated by **phase separation**

Our idea:

- Search for FFLO fluctuating pairs in the normal phase

- Even if the system is the normal phase, non-condensed FFLO pairs might be present in certain regions of the phase diagram

- If this is the case, FFLO short-range correlations and fluctuations should reveal in corresponding pairing susceptibility

Finite temperature

M. Pini, P. Pieri, G. C. Strinati, Phys. Rev. Res. **3**, 043068 (2021)

Method used in calculations: self-consistent T-matrix approach

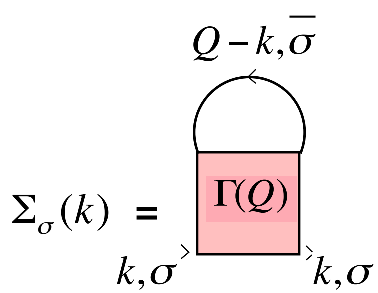
Modification of the original Nozieres-Schmitt-Rink approach for the BCS-BEC crossover in that: (i) Dyson's equation solved exactly; (ii) self-consistent G used everywhere instead of bare G_0; (iii) generalized to polarized systems with $\mu_\uparrow \neq \mu_\downarrow$

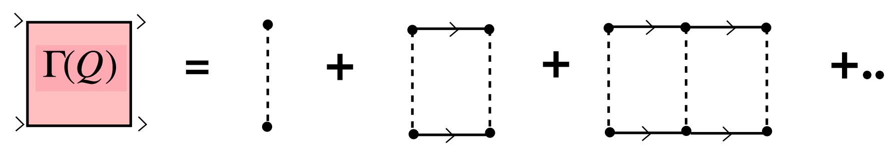
$$\Sigma_\sigma(k) = -\int \frac{d\mathbf{Q}}{(2\pi)^3} T \sum_\nu \Gamma(Q) G_{\bar{\sigma}}(k-Q)$$

$$\Gamma(Q) = -\left[\frac{m}{4\pi a} + \int \frac{d\mathbf{k}}{(2\pi)^3} \left(T \sum_\nu G_\uparrow(k) G_\downarrow(Q-k) - \frac{m}{k^2} \right) \right]^{-1}$$

$k = (\mathbf{k}, \omega_n)$

$\omega_n = 2\pi T(n + 1/2)$





$Q = (\mathbf{Q}, \Omega_\nu)$ $\Omega_\nu = 2\pi T \nu$

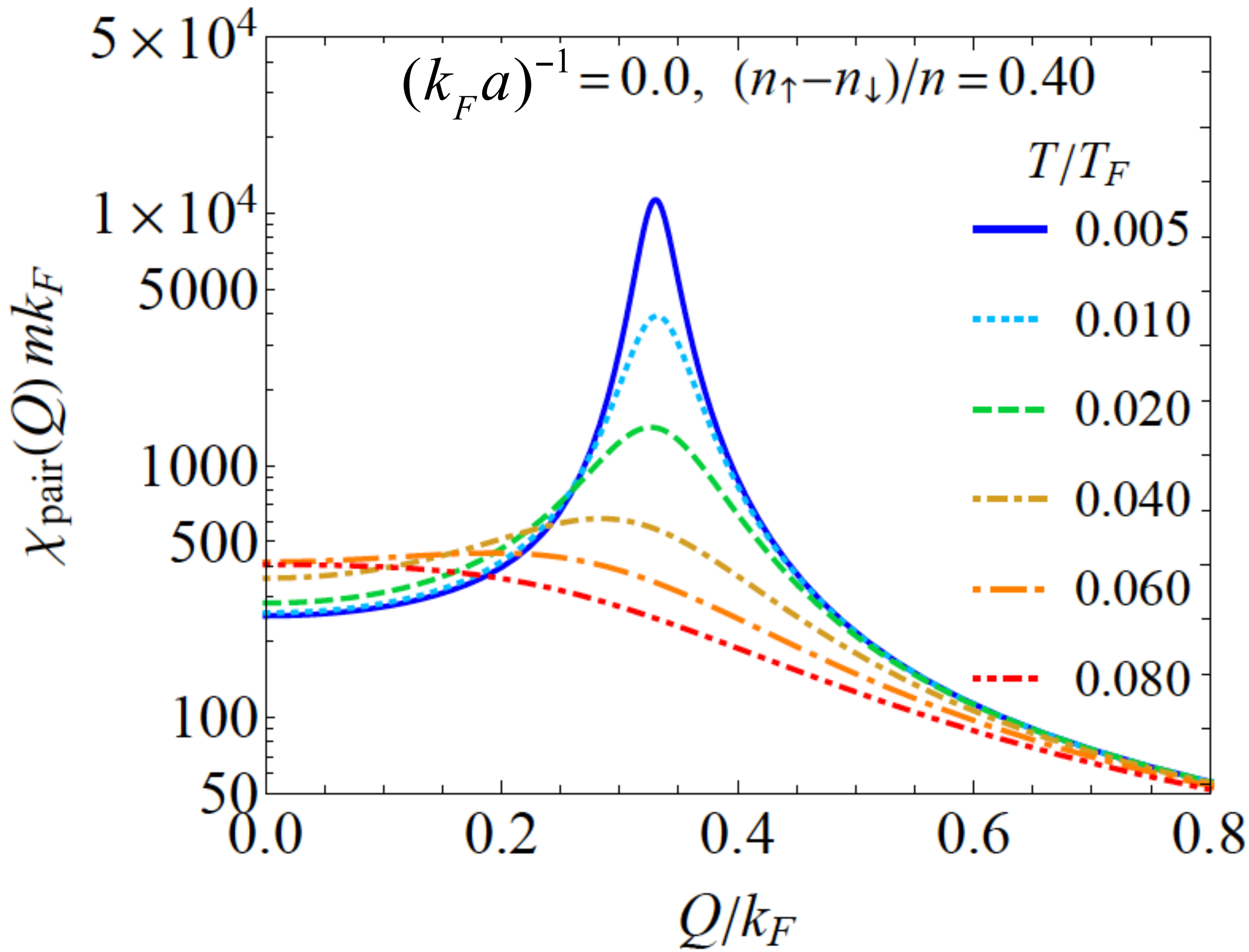
$$G_\sigma(\mathbf{k}, \omega_n) = [G_{0\sigma}(\mathbf{k}, \omega_n)^{-1} - \Sigma_\sigma(\mathbf{k}, \omega_n)]^{-1}$$

$$G_{0\sigma}(\mathbf{k}, \omega_n) = [i\omega_n - k^2/2m + \mu_\sigma]^{-1}$$

Within fully self-consistent t-matrix approach the pair susceptibility χ_{pair} is given by:

$$\chi_{\text{pair}}(\mathbf{Q}) = \Gamma(\mathbf{Q}, \Omega_\nu = 0)$$

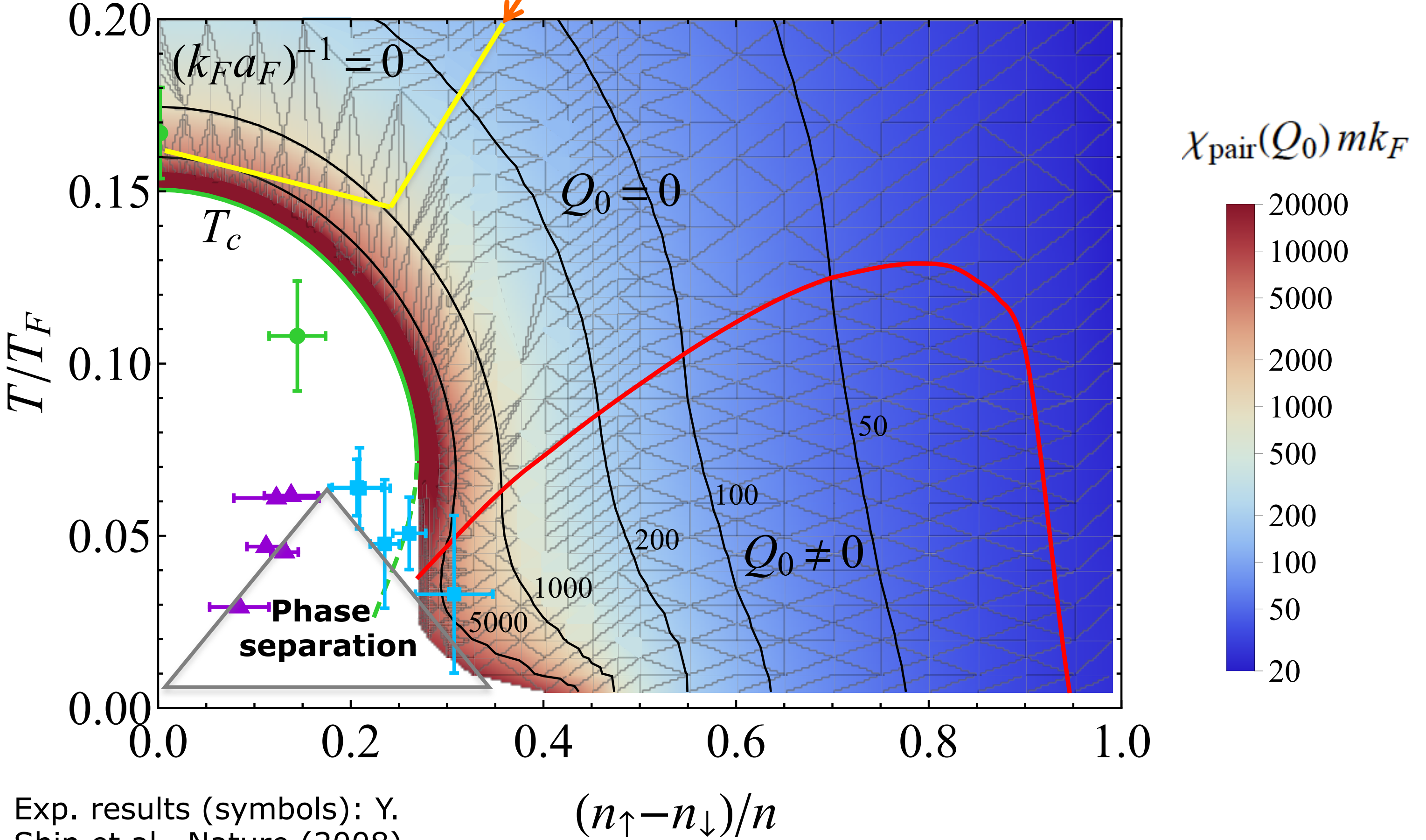
Polarized unitary Fermi gas: results for the pair susceptibility



Pronounced peak in the pair susceptibility at finite Q: **FFLO pairing fluctuations**.

Phase diagram of the polarized unitary Fermi gas

Recent complex Langevin QMC by F. Attanasio et al., PRA (2022) found $Q_0 = 0$ above this line

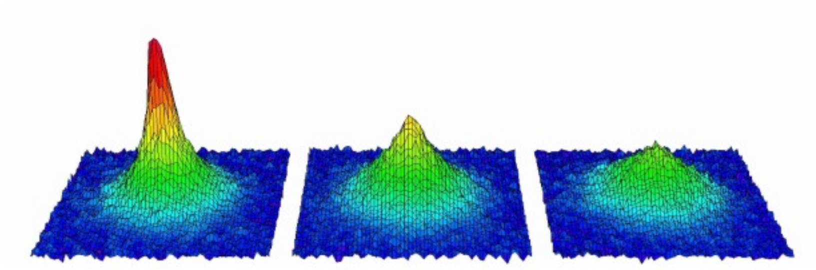


Exp. results (symbols): Y. Shin et al., Nature (2008)

Quite extended FFLO fluctuation region in the phase diagram

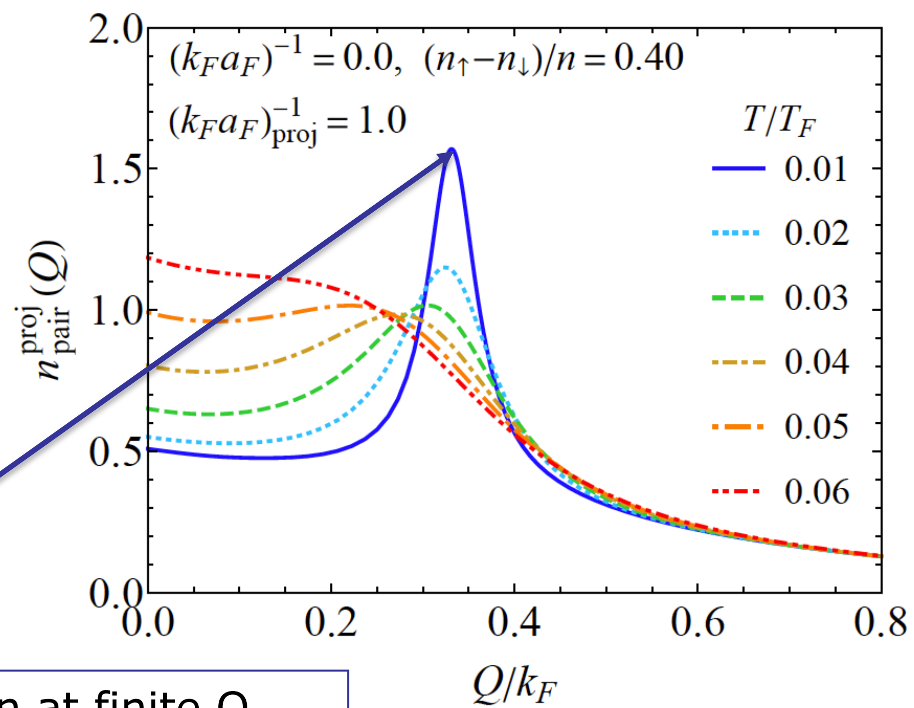
How to observe (strong) FFLO fluctuations?

- Use rapid magnetic-field sweep to the BEC side to measure projected pair-momentum distribution through time-of-flight imaging of Feshbach molecules.



C. Regal, M. Greiner, and D. S. Jin, PRL (2004)

Projected pair-momentum distribution



A peak in the pair momentum distribution at finite Q would be the smoking gun of strong FFLO correlations

Zero temperature

M. Pini, P. Pieri, G. C. Strinati, Phys. Rev. B **107**, 054505 (2023)

Self-consistent T-matrix approach exactly at T=0

Finite T

$$\Sigma_\sigma(k) = -\int \frac{d\mathbf{Q}}{(2\pi)^3} T \sum_\nu \Gamma(Q) G_{\bar{\sigma}}(k-Q)$$

$$k = (\mathbf{k}, \omega_n) \qquad \omega_n = 2\pi T(n+1/2)$$

$$Q = (\mathbf{Q}, \Omega_\nu) \qquad \Omega_\nu = 2\pi T \nu$$

$$G_\sigma(\mathbf{k}, \omega_n) = [G_{0\sigma}(\mathbf{k}, \omega_n)^{-1} - \Sigma_\sigma(\mathbf{k}, \omega_n)]^{-1}$$

$$G_{0\sigma}(\mathbf{k}, \omega_n) = [i\omega_n - k^2/2m + \mu_\sigma]^{-1}$$

$$\Gamma(Q) = -\left[\frac{m}{4\pi a} + \int \frac{d\mathbf{k}}{(2\pi)^3}\left(T\sum_\nu G_\uparrow(k) G_\downarrow(Q-k) - \frac{m}{k^2}\right)\right]^{-1}$$

T = 0

$$\Sigma_\sigma(k) = -\int \frac{d\mathbf{Q}}{(2\pi)^3} \int \frac{d\Omega}{2\pi} \Gamma(Q) G_{\bar{\sigma}}(k-Q)$$

$$k = (\mathbf{k}, \omega)$$

with ω and Ω continuous

$$Q = (\mathbf{Q}, \Omega)$$

$$G_\sigma(\mathbf{k}, \omega) = [G_{0\sigma}(\mathbf{k}, \omega)^{-1} - \Sigma_\sigma(\mathbf{k}, \omega)]^{-1}$$

$$G_{0\sigma}(\mathbf{k}, \omega) = [i\omega - k^2/2m + \mu_\sigma]^{-1}$$

$$\Gamma(Q) = -\left[\frac{m}{4\pi a} + \int \frac{d\mathbf{k}}{(2\pi)^3}\left(\int \frac{d\omega}{2\pi} G_\uparrow(k) G_\downarrow(Q-k) - \frac{m}{k^2}\right)\right]^{-1}$$

We continue to work with imaginary frequencies: $i\omega$ and $i\Omega$, continuous at zero T.

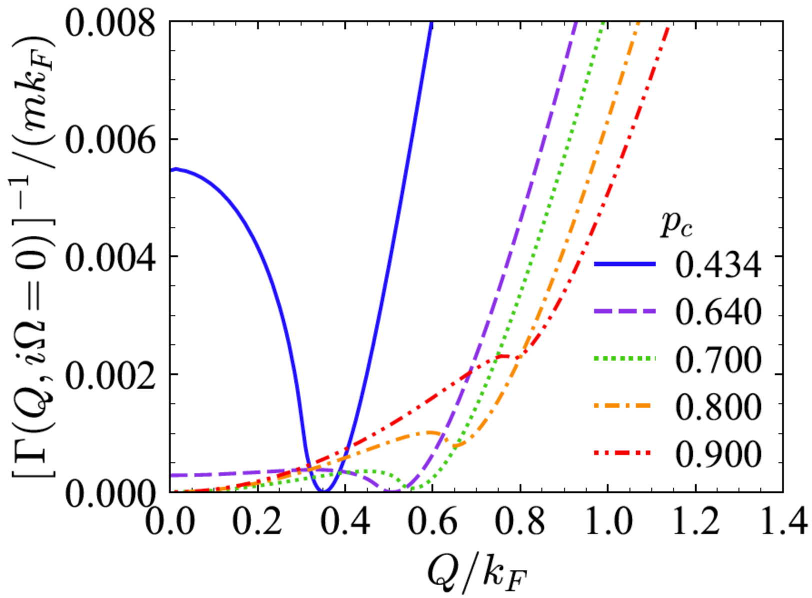
Phase diagram at T=0

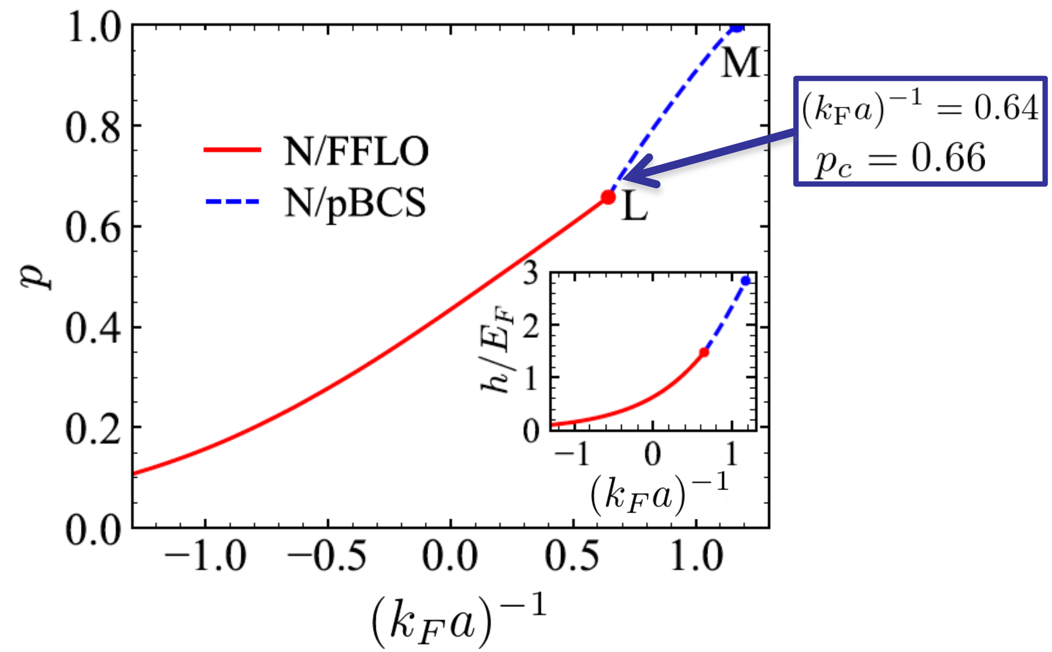
We vary dimensionless coupling $(k_F a)^{-1}$ (with $k_{\rm F} \equiv (3\pi^2 n)^{1/3}$, $n = n_\uparrow + n_\downarrow$) and determine critical polarization p_c [with $p = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$].

Second order phase transition determined by:

$$\left[\Gamma(|\mathbf{Q}| = Q_0, i\Omega = 0)|_{p=p_c}\right]^{-1} = 0 \longleftrightarrow \text{diverging pairing susceptibility } \chi_{\rm pair}(Q_0),$$

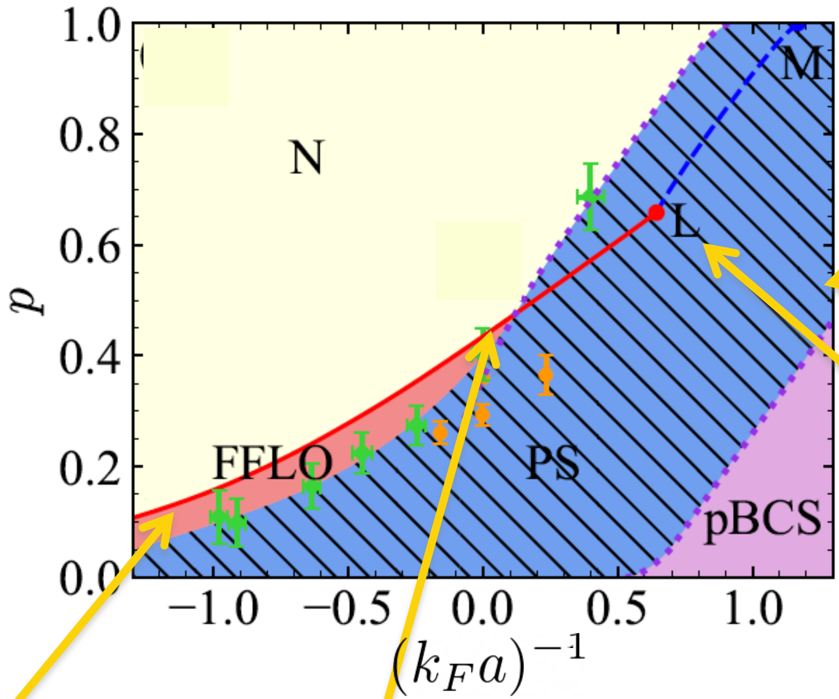
where Q_0 is the value of $|\mathbf{Q}|$ minimizing $\Gamma(|\mathbf{Q}|, i\Omega = 0)^{-1}$





At the Lifshitz point (L) the transition changes from N/FFLO to N/pBCS where pBCS is a polarized SF with standard BCS pairing $(Q_0 = 0)$.

Phase diagram at T=0: including phase separation



PS region combines exp. data data from Shin et al., 2008 and Olsen et al., 2015 (symbols with error bars) with QMC results from Pilati et al., 2008 (dotted lines)

Lifshitz point is hidden by phase separation region. However, the L point could be reached in principle following a metastable N phase (we found a positive definite compressibility matrix).

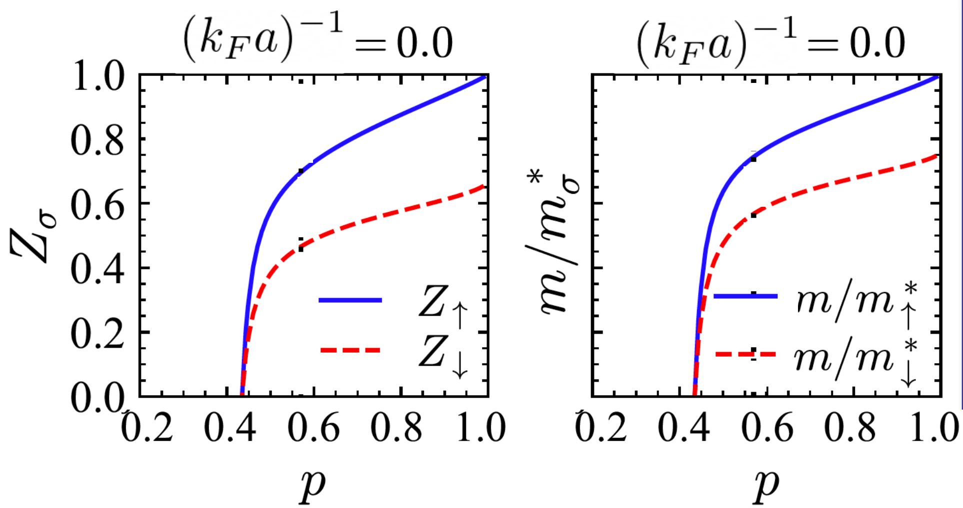
Relatively narrow region for observability of FFLO, still larger than within mean-field. It includes a polarization range at unitarity. Need boxlike trap for observability.

At unitarity, critical polarization to FFLO: ~ 0.435

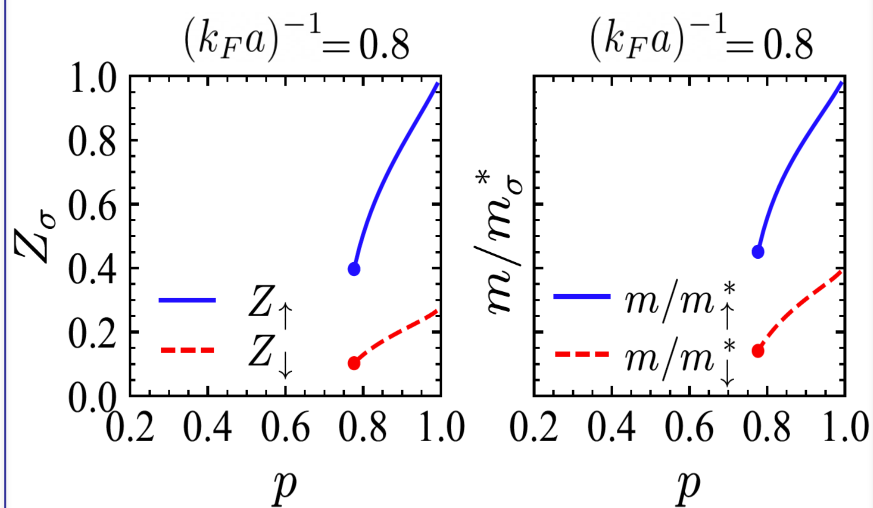
Quasi-particle residue and effective mass

Quasi-particle residue and effective mass at $|\mathbf{k}| = k_{F\sigma}$ can be calculated directly on the imaginary frequency axis:

$$Z_\sigma = \left[1 - \frac{\partial\, \mathrm{Im}\Sigma_\sigma(k_{F\sigma}, i\omega)}{\partial\omega}\bigg|_{\omega=0}\right]^{-1} \qquad \frac{m}{m^*_\sigma} = Z_\sigma\left[1 + \frac{m}{k_{F\sigma}} \frac{\partial\Sigma_\sigma(\mathbf{k}, i\omega=0)}{\partial|\mathbf{k}|}\bigg|_{|\mathbf{k}|=k_{F\sigma}}\right]$$

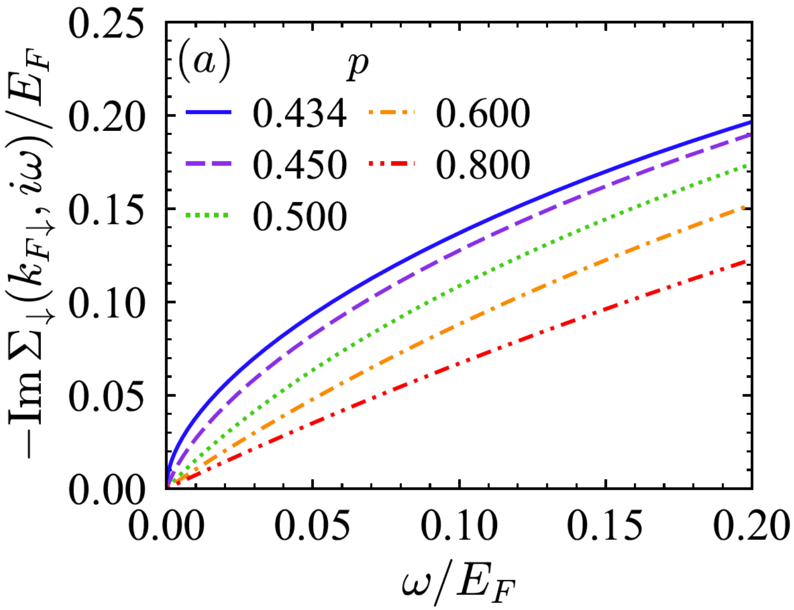


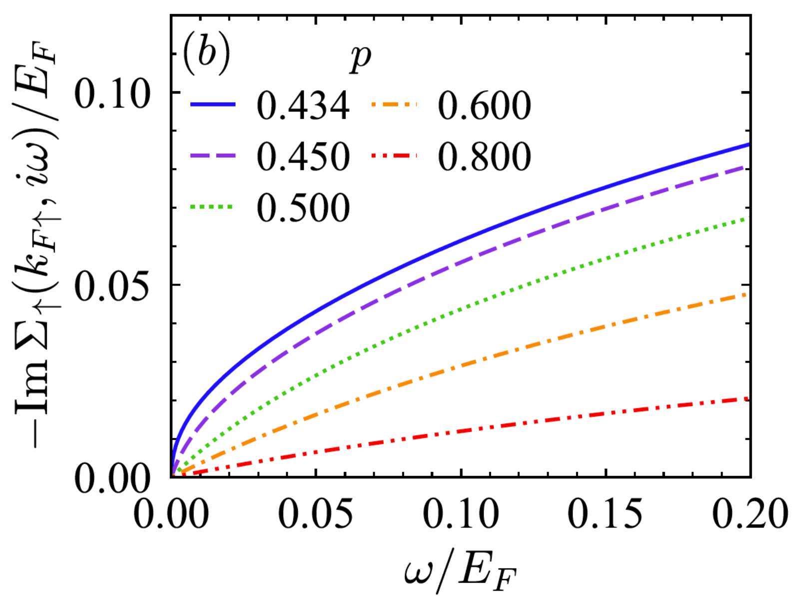
Vanishing quasi-particle residue and diverging effective mass at FFLO QCP: breakdown of FL properties analogous to what is found in heavy-fermions in the proximity of an AFM QCP.



When QCP is to standard BCS pairing, no breakdown of FL properties at QCP.

NFL behavior of self-energy (along imaginary axis)

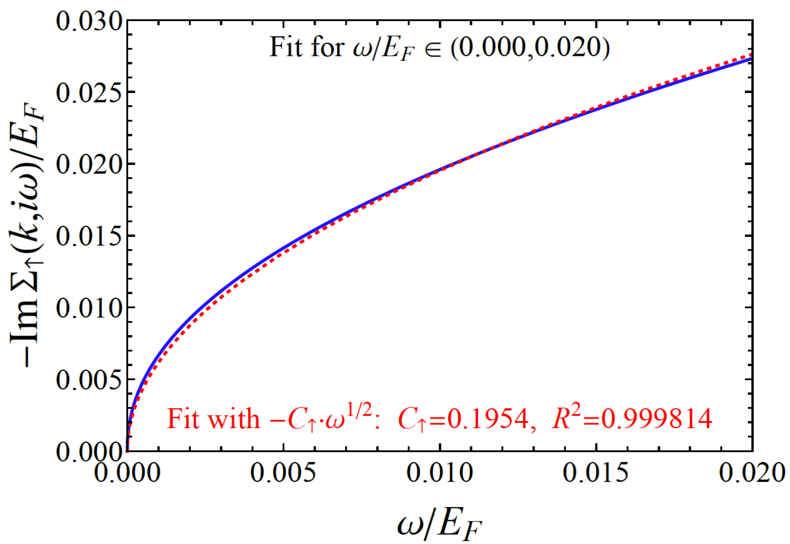




At FFLO QCP:

$$\mathrm{Im}\Sigma_\sigma(k_{F\sigma}, i\omega) = -C_\sigma \omega^{1/2}$$

$$\mathrm{Re}\Sigma_\sigma(k_{F\sigma}, i\omega) = \Sigma_\sigma(k_{F\sigma}, 0) + B_\sigma \omega^{1/2}$$



NFL behavior of self-energy (along real frequency axis) and dynamical critical exponent

Extending analytically small $i\omega$ behavior to small ζ in the upper complex plane:

$$\Sigma_\sigma(k_{F\sigma}, \zeta) = \Sigma_\sigma(k_{F\sigma}, 0) + (B_\sigma - iC_\sigma)\sqrt{-i\zeta}$$

yielding for $\zeta = \tilde{\omega} + i0^+$ the retarded self-energy:

$$\mathrm{Re}\Sigma^{\mathrm{R}}_\sigma(k_{F\sigma}, \tilde{\omega}) = \Sigma_\sigma(k_{F\sigma}, 0) + [B_\sigma - C_\sigma \mathrm{sgn}(\tilde{\omega})]\sqrt{|\tilde{\omega}|/2}$$

$$\mathrm{Im}\Sigma^{\mathrm{R}}_\sigma(k_{F\sigma}, \tilde{\omega}) = -[C_\sigma + B_\sigma \mathrm{sgn}(\tilde{\omega})]\sqrt{|\tilde{\omega}|/2}$$

manifestly NFL

Spectral-weight function at Fermi surface: $A_\sigma(k_{F\sigma}, \tilde{\omega}) = \dfrac{D_{\sigma\pm}}{|\tilde{\omega}|^{1/2}}$

According to general consideration (Senthil, 2008) at QCP where QP residue vanishes:

$$A_\sigma(k_{F\sigma}, \tilde{\omega}) \sim \frac{1}{|\tilde{\omega}|^{1/z}}$$

where z is the **dynamical critical exponent**

We found for the dynamical pairing susceptibility: $\chi_{\mathrm{pair}}(\mathbf{Q}, i\Omega) \simeq \dfrac{(mk_{\mathrm{F}})^{-1}}{\epsilon + b(|\mathbf{Q}| - Q_0)^2 - (d_1 + id_2)i\Omega}$

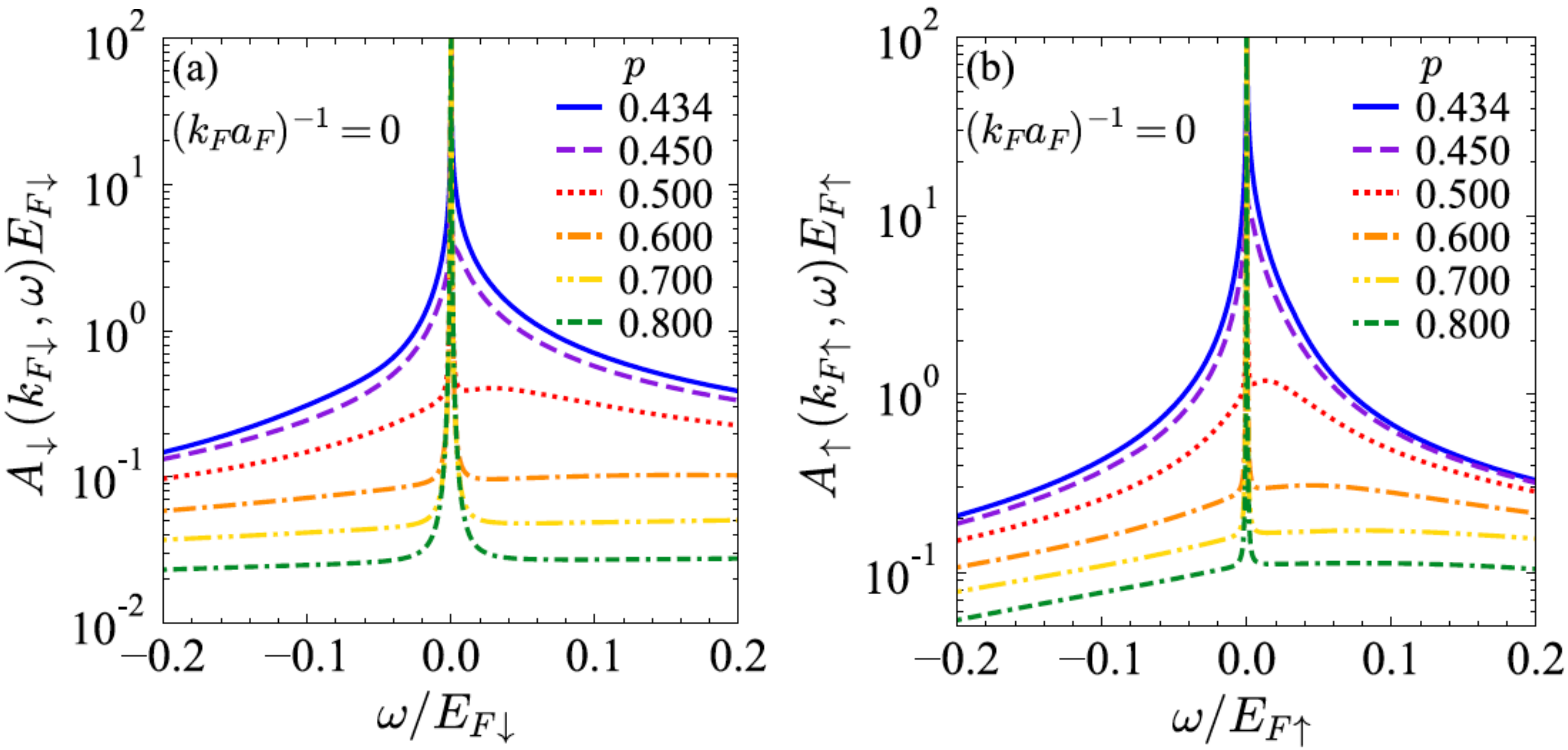
Yielding, on the real frequency axis, after setting $\xi = \sqrt{b/\epsilon}$, the scaling form:

$$\chi_{\mathrm{pair}}(\mathbf{Q}, \tilde{\Omega}; \xi) = \frac{\xi^2}{mk_{\mathrm{F}}b}\Phi_0[(|\mathbf{Q}| - Q_0)\xi, m_1\tilde{\Omega}\xi^2]$$

$$\boxed{z = 2}$$

$$m_1 = (d_1 + id_2)/b$$

Spectral weight function at Fermi surface, from large polarizations to QCP for the unitary Fermi gas



Analytical continuation performed numerically with Padé approximants. Away from the QCP, delta-like peak over incoherent background. At QCP, power law behavior.

Conclusions

- At finite (low) temperature FFLO correlation in the unitary Fermi gas should be sufficiently strong to be observable in the pair momentum distribution.

- At T=0 region of stability for FFLO significantly larger than mean-field prediction. It includes a polarization range at unitarity

- At FFLO QCP: vanishing quasi-particle residue and diverging effective mass: breakdown of FL properties analogous to what is found in heavy-fermions at AFM QCP.

- Power law behaviors of retarded self-energy and spectral weight function at FFLO QCP are determined by dynamical critical exponent as argued by Senthil, 2008.

Thank you!