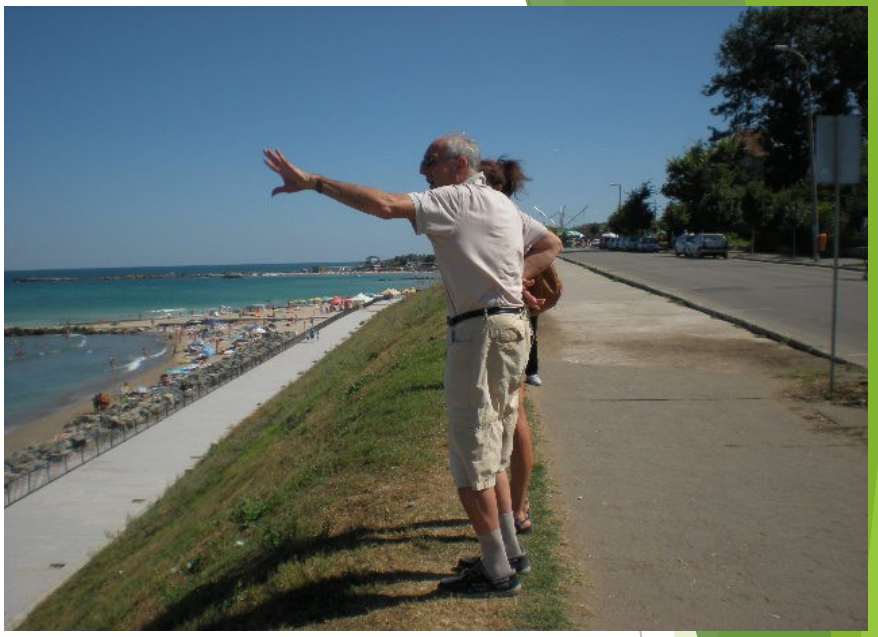
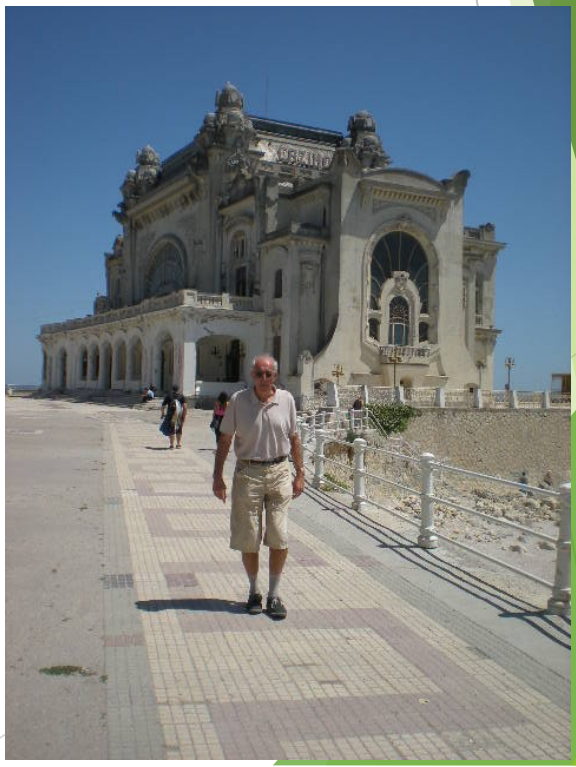
Cooperation with Peter Schuck on many-body correlations in nuclei
I. Self-consistent RPA
II.Four-body correlations

D.S. Delion (IFIN-HH, Bucharest)





Physics Reports 929 (2021) 1–84

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

Equation of Motion Method for strongly correlated Fermi systems and Extended RPA approaches

P. Schuck [a,b], D.S. Delion [c,d,*], J. Dukelsky [e], M. Jemai [f,g], E. Litvinova [h,i,j], G. Röpke [k], M. Tohyama [l]

[a] Université Paris-Saclay, CNRS-IN2P3, IJCLab, 91405 Orsay, France
[b] Université Grenoble Alpes, CNRS, LPMMC, 38000 Grenoble, France
[c] "Horia Hulubei" National Institute of Physics and Nuclear Engineering, 30 Reactorului, RO-077125, Bucharest-Magurele, Romania
[d] Academy of Romanian Scientists, 3 Ilfov RO-050044, Bucharest, Romania
[e] Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain
[f] Laboratory of Advanced Materials and Quantum Phenomena, Physics Department, Sciences Faculty of Tunis, El-Manar University, 2092 Tunis, Tunisia
[g] ISSATM, Carthage University, Avenue de la République, P.O. Box 77-1054 Amilcar, Tunis, Tunisia
[h] Department of Physics, Western Michigan University, Kalamazoo, MI 49008, USA
[i] National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA
[j] GANIL, CEA/DRF-CNRS/IN2P3, F-14076 Caen, France
[k] Universität Rostock, FB Physik, Universitätplatz, D-18051 Rostock, Germany
[l] Kyorin University School of Medicine, Mitaka, Tokyo 181-8611, Japan

I. Self-consistent RPA

PHYSICAL REVIEW C **72**, 064305 (2005)

Self-consistent random phase approximation and the restoration of symmetries within the three-level Lipkin model

D. S. Delion
National Institute of Physics and Nuclear Engineering, Bucharest Măgurele, POB MG-6, Romania

P. Schuck
Institut de Physique Nucléaire, F-91406 Orsay CEDEX, France

J. Dukelsky
Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain

(Received 7 April 2004; revised manuscript received 13 September 2005; published 13 December 2005)

We show that it is possible to restore broken symmetries associated with the Goldstone mode within the self-consistent random phase approximation (SCRPA) applied to the three-level Lipkin model. To do this, it is necessary to include the so-called scattering terms in the RPA operator. We determine one- and two-body densities as very convergent expansions in terms of the generators of the RPA basis. We show that SCRPA excitations correspond to the heads of some rotational bands in the exact spectrum.

DOI: 10.1103/PhysRevC.72.064305 PACS number(s): 21.60.Jz, 24.10.Cn

I. Self Consistent RPA

II. Three-level Lipkin model. Exact solution

III. Standard Hartree-Fock and RPA

IV. Goldstone mode

V. Rotational bands. Mass parameter

VI. Conclusions

I. Self consistent RPA (SCRPA)

A. Single particle mean field (MF) basis

B. Minimization procedure

C. SCRPA equations

D. Generalized MF equations

E. How to compute two-body densities ?

F. How to compute one-body densities ?

A. Single particle mean field (MF) basis

The aim of SCRPA is to describe in a consistent way collective excitations around a "deformed" vacuum. To this purpose we define the general "deformed" mean field single particle basis

$$a^{\dagger}_{k\mu} = \sum_{\alpha} C_{k\alpha} c^{\dagger}_{\alpha\mu} \ , \qquad (1)$$

as a superposition of "spherical" fermion creation operators on α-th level $c^{\dagger}_{\alpha\mu}$.

"Quadrupole-like" operators

$$K_{\alpha\beta} \equiv \sum_{\mu} c^{\dagger}_{\alpha\mu} c_{\beta\mu} \ , \tag{2}$$

become in the "deformed" mean field basis

$$A_{ij} = \sum_{\mu} a^{\dagger}_{i\mu} a_{j\mu} \ . \tag{3}$$

Hamiltonian

in the quadrupole-quadrupole scheme is given by

$$H = \sum_{\alpha} \epsilon_\alpha K_{\alpha\alpha} + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\delta\gamma} K_{\alpha\beta} K_{\gamma\delta} \ .$$

$$= \sum_{ij=0}^{n} F_{ij} A_{ij} + \frac{1}{2} \sum_{ijkl=0}^{n} G_{ijkl} A_{ij} A_{kl} \ , \qquad (4)$$

where the coefficients are given respectively by

$$F_{ij} \equiv \sum_{\alpha=0}^{n} \epsilon_\alpha C_{i\alpha} C_{j\alpha} \ ,$$

$$G_{ijkl} \equiv \sum_{\alpha\beta\gamma\delta=0}^{n} V_{\alpha\beta\gamma\delta} C_{i\alpha} C_{j\beta} C_{k\gamma} C_{l\delta} \ . \qquad (5)$$

SCRPA creation operators

$$Q^{\dagger}_{\nu} = \sum_{m>i} (X^{\nu}_{mi} \delta Q^{\dagger}_{mi} - Y^{\nu}_{mi} \delta Q_{mi}) \ , \qquad (6)$$

are written in terms of the normalized generators

$$\begin{aligned} \delta Q^{\dagger}_{mi} &= N^{-1/2}_{mi} A_{mi} \\ \delta Q_{mi} &= \delta Q^{\dagger}_{im} = N^{-1/2}_{mi} A_{im} \ . \end{aligned} \qquad (7)$$

Summation contains not only
particle (vacant) - hole (occupied) (ph) states,
but also the so-called scattering states, i.e.
pp and hh combinations.

Normalisation factor

is given by the mean value of the following commutator on the correlated vacuum

$$\begin{aligned}\langle 0|[A_{im}, A_{nj}]|0\rangle &= \delta_{mn}\delta_{ij}(\langle 0|A_{ii}|0\rangle - \langle 0|A_{mm}|0\rangle) \\ &\equiv \delta_{mn}\delta_{ij}N_{mi} \ , \end{aligned} \tag{8}$$

and takes effectively into account Pauli correlations.

Excited states

are defined by the SCRPA creation operators

$$|\nu\rangle = Q_\nu^\dagger |0\rangle \ . \tag{9}$$

In the derivation of SCRPA equations one supposes the existence of a correlated groundstate such that

$$Q_\nu |0\rangle = 0 \ . \tag{10}$$

We do not need to know its analytical form within the SCRPA procedure.

B. Minimisation procedure

of the mean excitation energy

$$\omega_\nu = \frac{\langle 0|[Q_\nu,[H,Q_\nu^\dagger]]|0\rangle}{\langle 0|[Q_\nu,Q_\nu^\dagger]|0\rangle}\ , \qquad (11)$$

with respect to X, Y amplitudes leads to the following

Equation of motion

$$\langle 0|[\delta Q_\nu,[H,Q_\nu^\dagger]]|0\rangle = \omega_\nu \langle 0|[\delta Q_\nu,Q_\nu^\dagger]|0\rangle\ . \qquad (12)$$

C. SCRPA equations

have formally the same structure as the linear RPA equations

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} \ . \tag{13}$$

SCRPA matrices

are given by the well-known relations

$$\begin{aligned} \mathcal{A}_{mi,nj} &= \langle 0 | \left[\delta Q_{mi}, \left[H, \delta Q^\dagger_{nj} \right] \right] | 0 \rangle \\ \mathcal{B}_{mi,nj} &= -\langle 0 | \left[\delta Q_{mi}, \left[H, \delta Q_{nj} \right] \right] | 0 \rangle = -\mathcal{A}_{mi,jn} \ . \end{aligned} \tag{14}$$

SCRPA matrix elements contain

One-body densities

$$\langle 0|A_{ij}|0\rangle = \delta_{ij}\langle 0|A_{ii}|0\rangle \ , \tag{15}$$

and

Two-body densities

$$\langle 0|A_{ij}A_{kl}|0\rangle \ . \tag{16}$$

The decoupled ansatz

$$\langle 0|A_{ij}A_{kl}|0\rangle \approx \langle 0|A_{ij}|0\rangle\langle 0|A_{kl}|0\rangle \ . \qquad (17)$$

leads to the so-called

Renormalized RPA (r-RPA).

D. Generalized MF equations

defining the "deformed" minimum are obtained as follows

$$\langle 0|[H, \delta Q^{\dagger}_{\nu}]|0\rangle = 0 \ . \tag{18}$$

They are of the form

$$\sum_{m} H_{nm} C_{m} = E_{\alpha} \langle 0|A_{\alpha\alpha}|0\rangle C_{n\alpha} \ , \tag{19}$$

where H_{nm} contains SCRPA one and two-body densities.

The SCRPA equations are solved together with generalized MF equations iteratively.

Self-consistent RPA

describes the motion
in an anharmonic potential
depending on the amplitude
of the oscillation:

the position of the minimum
depends on the amplitude

E. How to compute two-body densities ?

The inversion of the RPA phonon operator is given by

$$A_{mi} = N_{mi}^{1/2} \sum_{\nu} (X_{mi}^{\nu} Q_{\nu}^{\dagger} + Y_{mi}^{\nu} Q_{\nu})$$
$$A_{im} = N_{mi}^{1/2} \sum_{\nu} (X_{mi}^{\nu} Q_{\nu} + Y_{mi}^{\nu} Q_{\nu}^{\dagger}) \ . \qquad (20)$$

By using

$$Q|0\rangle = \langle 0|Q^{\dagger} = 0 \ , \qquad (21)$$

we obtain for the two-body densities $(m > i, n > j)$

$$\langle 0|A_{mi}A_{nj}|0\rangle = N_{mi}^{1/2} N_{nj}^{1/2} \sum_{\nu} Y_{mi}^{\nu} X_{nj}^{\nu}$$
$$\langle 0|A_{im}A_{jn}|0\rangle = N_{mi}^{1/2} N_{nj}^{1/2} \sum_{\nu} X_{mi}^{\nu} Y_{nj}^{\nu}$$
$$\langle 0|A_{mi}A_{jn}|0\rangle = N_{mi}^{1/2} N_{nj}^{1/2} \sum_{\nu} Y_{mi}^{\nu} Y_{nj}^{\nu}$$
$$\langle 0|A_{im}A_{nj}|0\rangle = N_{mi}^{1/2} N_{nj}^{1/2} \sum_{\nu} X_{mi}^{\nu} X_{nj}^{\nu} \ . \qquad (22)$$

F. How to compute one-body densities ?

In general one expands one-body density in terms of SCRPA operators $Q^{\dagger}_{\nu}$.

For the particular three-level Lipkin model we will use an exact procedure using the basic operators A_{mn}.

II. Three-level Lipkin model: SU(3) algebra

Three single particle levels α=**0,1,2**.
0 is a hole level, while **1,2** are particle levels.
Level degeneracy on projection μ is
$N = 2\Omega$ (number of particles).

"Quadrupole-like" operators

$$K_{\alpha\beta} \equiv \sum_{\mu=1}^{N} c^{\dagger}_{\alpha\mu} c_{\beta\mu} \ , \tag{23}$$

where $c^{\dagger}_{\alpha\mu}$ is a fermion creation operator on α-th level.

Commutation rules

$$[K_{\alpha\beta}, K_{\gamma\delta}] = \delta_{\beta\gamma} K_{\alpha\delta} - \delta_{\alpha\delta} K_{\gamma\beta} \ . \tag{24}$$

Hamiltonian

$$H = \sum_{\alpha=0}^{2} \epsilon_\alpha K_{\alpha\alpha} - \frac{V}{2} \sum_{\alpha=1}^{2} (K_{\alpha 0} K_{\alpha 0} + K_{0\alpha} K_{0\alpha}) \ . \quad (25)$$

Continuously broken symmetry

appears when $\epsilon_1 = \epsilon_2$.

The angular momentum projection operator

$$\hat{L}_0 = i(K_{21} - K_{12}) \ , \quad (26)$$

commutes with the Hamiltonian, i.e.

$$[H, \hat{L}_0] = 0 \ . \quad (27)$$

Exact solution

The eigenstates can be obtained from the diagonalisation procedure by the following expansion

$$|\nu\rangle = \sum_{n_1 \geq n_2} c^{(\nu)}_{n_1 n_2} |n_1 n_2\rangle \ , \tag{28}$$

where we introduced the Slater determinant

$$|n_1 n_2\rangle = \mathcal{N}_{n_1 n_2} K_{10}^{n_1} K_{20}^{n_2} |HF\rangle \ , \tag{29}$$

and the normalisation

$$\mathcal{N}_{n_1 n_2} \equiv \sqrt{\frac{(N - n_1 - n_2)!}{N! n_1! n_2!}} \ . \tag{30}$$

III. Standard Hartree-Fock (HF)

The expectation value of the quadupole operator is given by

$$\langle HF|A_{\alpha\beta}|HF\rangle = \delta_{\alpha\beta}\delta_{\alpha 0}N \ . \tag{31}$$

The mean field transformation matrix $C_{k\alpha}$ can be written as a product of two rotations

$$\begin{aligned} C_{k\alpha} &= \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix} \\ &= \begin{pmatrix} \cos\phi & \sin\phi\cos\psi & \sin\phi\sin\psi \\ -\sin\phi & \cos\phi\cos\psi & \cos\phi\sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix} \ . \end{aligned} \tag{32}$$

Hamiltonian mean value

$$\langle HF|H|HF\rangle = N\epsilon[e_0\cos^2\phi + e_1\sin^2\phi\cos^2\psi + e_2\sin^2\phi\sin^2\psi - \chi\sin^2\phi\cos^2\phi] \quad (33)$$

where we introduced the following dimensionless notations

$$e_k = \frac{\epsilon_k}{\epsilon}, \quad \chi = \frac{V(N-1)}{\epsilon} \; . \quad (34)$$

Hamiltonian minima

Spherical minimum

$$1) \quad \phi = 0, \quad \psi = 0 \ , \quad \chi < e_1 - e_0 \ , \tag{35}$$

Deformed minimum

$$2) \quad \cos 2\phi = \frac{e_1 - e_0}{\chi}, \quad \psi = 0, \quad \chi > e_1 - e_0 \ . \tag{36}$$

Moreover, our calculations have shown that for any MF minimum one obtains $\psi = 0$, independent of which kind of vacuum (correlated or not) we use to estimate the expectation values.

Standard RPA

Vacuum state is the Hartree-Fock ground state

$$|0\rangle = |HF\rangle \ . \tag{37}$$

We fix the origin of the particle spectrum with $e_0 = 0$. Then for a **spherical vacuum** $\phi = 0$ the RPA energies are given by

$$\omega_\nu = \epsilon_\nu \left[1 - \left(\frac{\chi}{e_\nu}\right)^2\right]^{1/2} \ , \quad \nu = 1, 2 \ , \tag{38}$$

If $\Delta\epsilon \equiv \epsilon_2 - \epsilon_1 = 0$, for the values of the strength $\chi > e_1$, in the **deformed region**, i.e. with $\phi \neq 0$ given by HF minimum, one obtains a **Goldstone mode**.

In this case by considering $e_1 = 1$ one obtains for the excitation energies

$$\omega_1 = \epsilon\sqrt{2(\chi^2 - 1)}\ ,$$
$$\omega_2 = 0\ . \qquad (39)$$

IV. Goldstone mode

The commutation relation

$$[H, L_0] = 0 \ , \tag{40}$$

can be seen as an

RPA equation with zero energy ω=0

$$[H, L_0] = \omega L_0 \ . \tag{41}$$

Thus, SCRPA will exhibit a Goldstone mode, as this is also the case with standard RPA.

That this property is conserved has already been announced by Rowe, but never has been explicitly verified.

We have checked that

for an RPA operator restricted to ph and hp configurations the Goldstone mode does NOT come at zero energy.

The reason for this is simple: usually a symmetry operator contains also (hh) and (pp) configurations, and without them, it is atrophiated and SCRPA fails to produce a zero mode.

In standard RPA this does not matter because hh and pp configurations decouple. Beyond standard RPA it matters and, as we will show,

the inclusion of scattering terms will produce the Goldstone mode.

This is the reason why we think that the above Hamiltonian is adequate since it can be studied in the limit $\delta\epsilon = \epsilon_2 - \epsilon_1 \to 0$ where the spontaneously broken symmetry shows up.

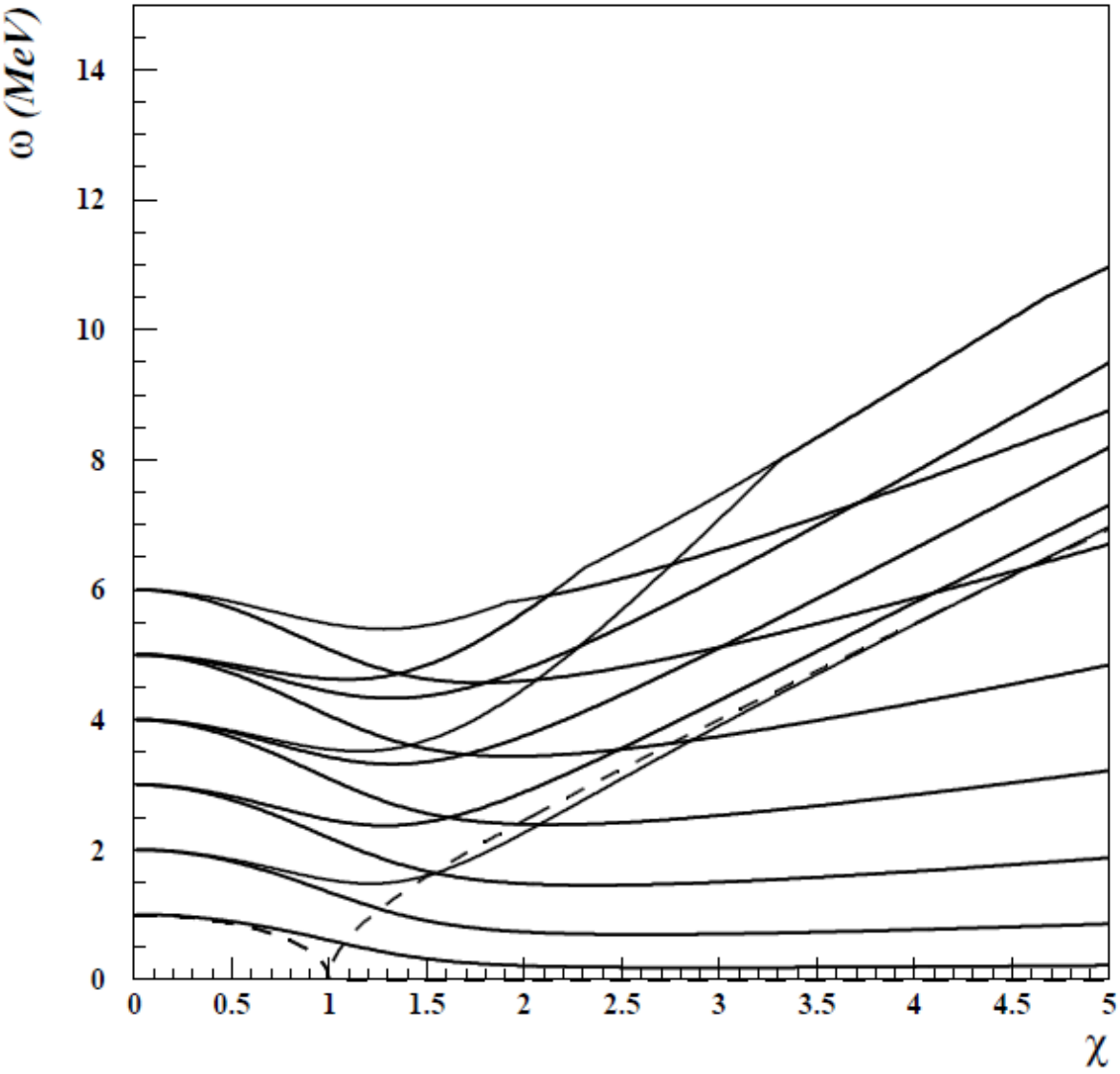


FIG. 1: **EXACT SOLUTION**
Exact energies versus the strength parameter χ.
$N = 20$, $\epsilon_1 = 0, \epsilon_2 = \epsilon_3 = 1MeV$. By dashes are given standard RPA values.
Notice the appearance of a zero mode solution (Goldstone mode) beyond χ=1.

SCRPA creation operators

$$Q^{\dagger}_{\nu} = \sum_{m>i=0}^{2} \left(X^{\nu}_{mi} \delta Q^{\dagger}_{mi} - Y^{\nu}_{mi} \delta Q_{mi} \right) , \qquad (13)$$

are written in terms of the normalized generators

$$\begin{aligned} \delta Q^{\dagger}_{mi} &= N^{-1/2}_{mi} A_{mi} \\ \delta Q_{mi} &= \delta Q^{\dagger}_{im} = N^{-1/2}_{mi} A_{im} \ . \end{aligned} \qquad (14)$$

1,0 and **2,0** are particle-hole (ph) terms, while **2,1** is the particle-particle (pp) scattering term.

How to compute one-body densities ?

Let us consider the expansion

$$A_{mm} = \sum_{n_1 n_1 = 0}^{N} c_{n_1 n_2}(m) A_{10}^{n_1} A_{20}^{n_2} A_{02}^{n_2} A_{01}^{n_1} \ , \qquad (42)$$

To find $c_{n_1 n_2}(m)$ we compute the expectation values on the correlated vacuum $|0\rangle$ and use the inversion of the RPA operator.

One finally obtains from (42) a nonlinear system of equations, determining the normalisation factors N_{10}, N_{20}.

One directly obtains one-body densities

$$\begin{aligned}
\langle 0|A_{mm}|0\rangle &= \left[y_{mm} + \frac{y_{11}y_{22}}{N}\right] \\
&\times \left[1 + \frac{2}{N}(y_{11} + y_{22}) + \frac{3}{N^2}y_{11}y_{22}\right]^{-1} \\
\langle 0|A_{00}|0\rangle &= N - \langle 0|A_{11}|0\rangle - \langle 0|A_{22}|0\rangle \ , \qquad (43)
\end{aligned}$$

where

$$y_{mn} = \sum_{\nu} Y^{\nu}_{m0} Y^{\nu}_{n0} \ . \qquad (44)$$

SPHERICAL REGION

The eigenvalues for the two-dimensional $\omega_k^{(2)},\ k=1,2$ and three dimensional $\omega_k^{(3)},\ k=1,2,3$ versions of the SCRPA versus the strength χ (first column) in the spherical region. The particle number is $N=20$ and $\epsilon_n = n\ MeV,\ n=0,1,2$. In the columns 2-4 are given the exact solutions $\omega_k\ , k=1,2,5$. In the last columns are given groundstate energies for two-dimensional and three-dimensional SCRPA and exact values, respectively.

χ	ω_1	ω_2	ω_5	$\omega_1^{(2)}$	$\omega_2^{(2)}$	$\omega_1^{(3)}$	$\omega_2^{(3)}$	$\omega_3^{(3)}$	$E_0^{(2)}$	$E_0^{(3)}$	$E_0^{(exact)}$
0.900	0.641	1.472	2.567	0.657	1.854	0.661	1.855	2.742	-0.397	-0.399	-0.361
0.920	0.625	1.452	2.549	0.644	1.848	0.648	1.849	2.732	-0.418	-0.420	-0.379
0.940	0.609	1.432	2.531	0.631	1.842	0.636	1.844	2.723	-0.441	-0.443	-0.398
0.960	0.593	1.412	2.513	0.618	1.837	0.624	1.838	2.714	-0.464	-0.467	-0.418
0.980	0.576	1.393	2.495	0.605	1.831	0.612	1.832	2.707	-0.489	-0.491	-0.439
1.000	0.559	1.374	2.476	0.593	1.825	0.600	1.827	2.700	-0.514	-0.517	-0.460
1.020	0.542	1.355	2.458	0.581	1.819	0.588	1.821	2.694	-0.540	-0.543	-0.483
1.040	0.525	1.336	2.439	0.570	1.814	0.577	1.815	2.689	-0.568	-0.571	-0.506
1.060	0.508	1.318	2.421	0.559	1.808	0.567	1.810	2.685	-0.596	-0.599	-0.529
1.080	0.491	1.300	2.402	0.548	1.802	0.556	1.804	2.681	-0.625	-0.628	-0.554
1.100	0.474	1.283	2.384	0.537	1.796	0.546	1.798	2.679	-0.655	-0.659	-0.580

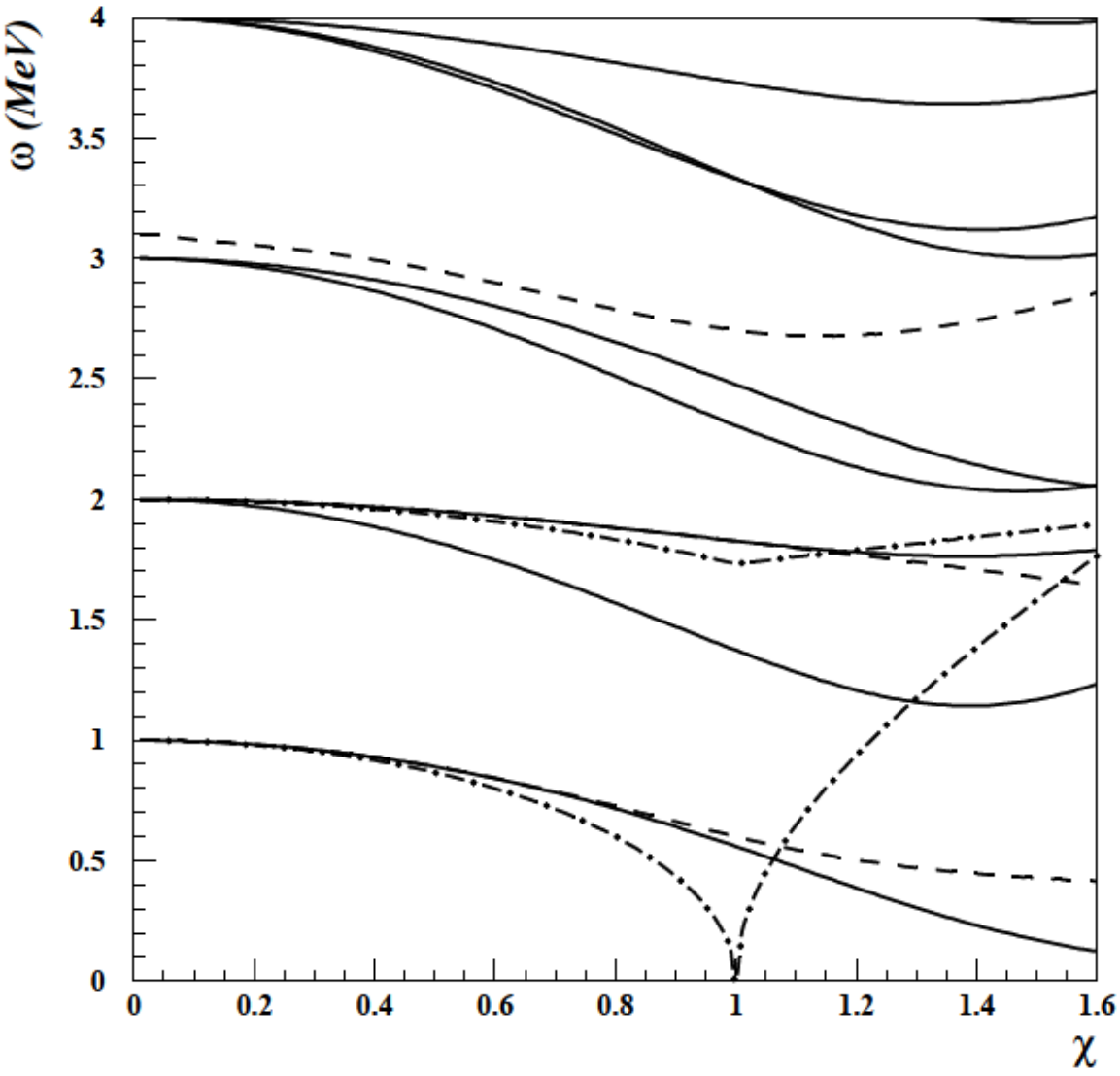


FIG. 2: **SCRPA IN THE SPHERICAL REGION**
Excitation energies versus the strength parameter χ.
$N = 20$ and $e_0 = 0,\ e_1 = 1,\ e_2 = 2$ (dashed lines).
By solid lines are given the lowest exact eigenvalues and by dot-dashes the standard RPA energies.

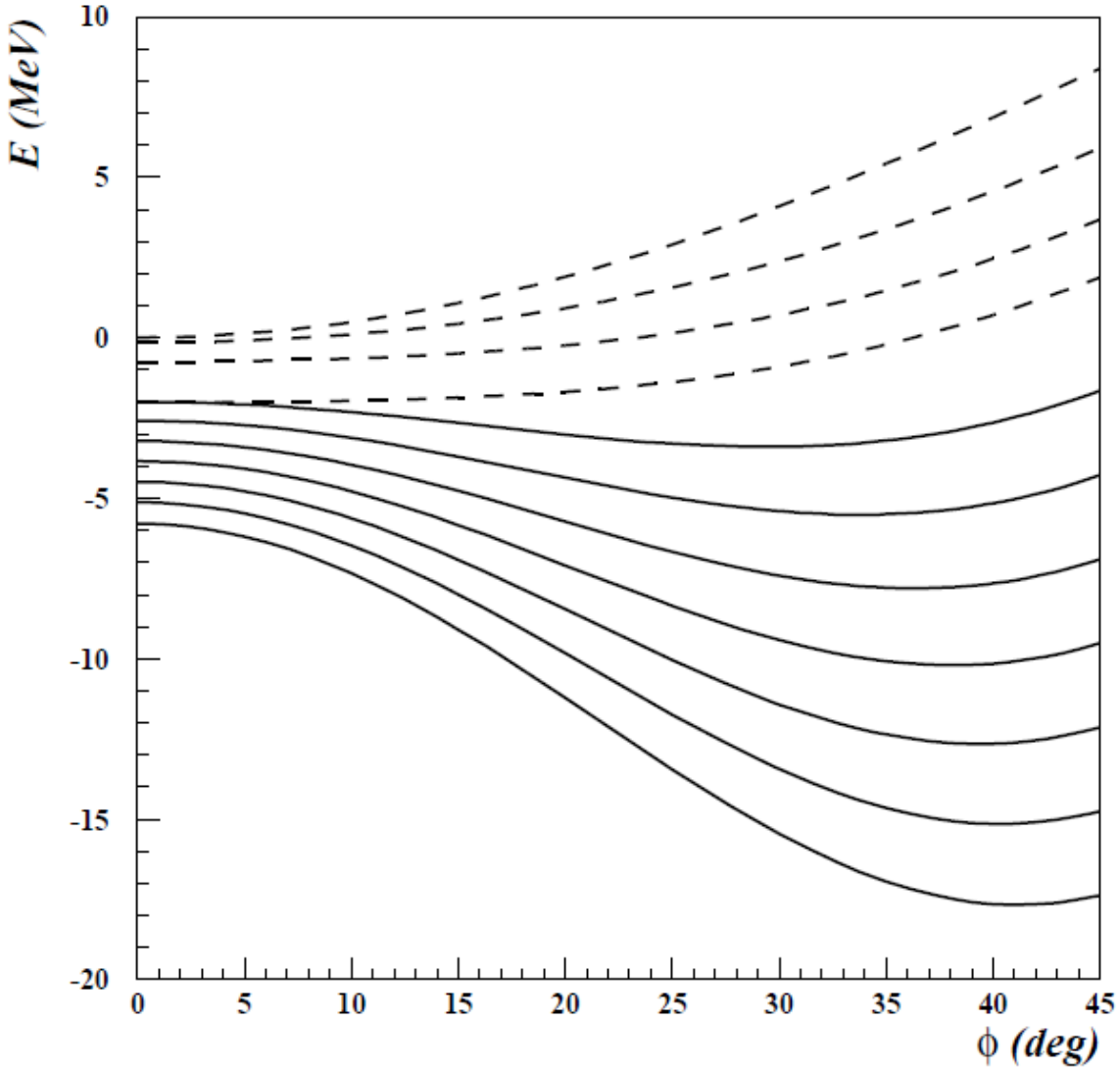


FIG. 3: **TRANSITION FROM SPHERICAL TO THE DEFORMED REGION**

The SCRPA expectation value of the Hamiltonian versus the angle ϕ, for $N = 20$ and different values of the strength parameter χ (from the top of the figure, $\chi = 0, 0.5, ..., 5$). By dashes are given the values for the spherical region and by solid lines for the deformed region.

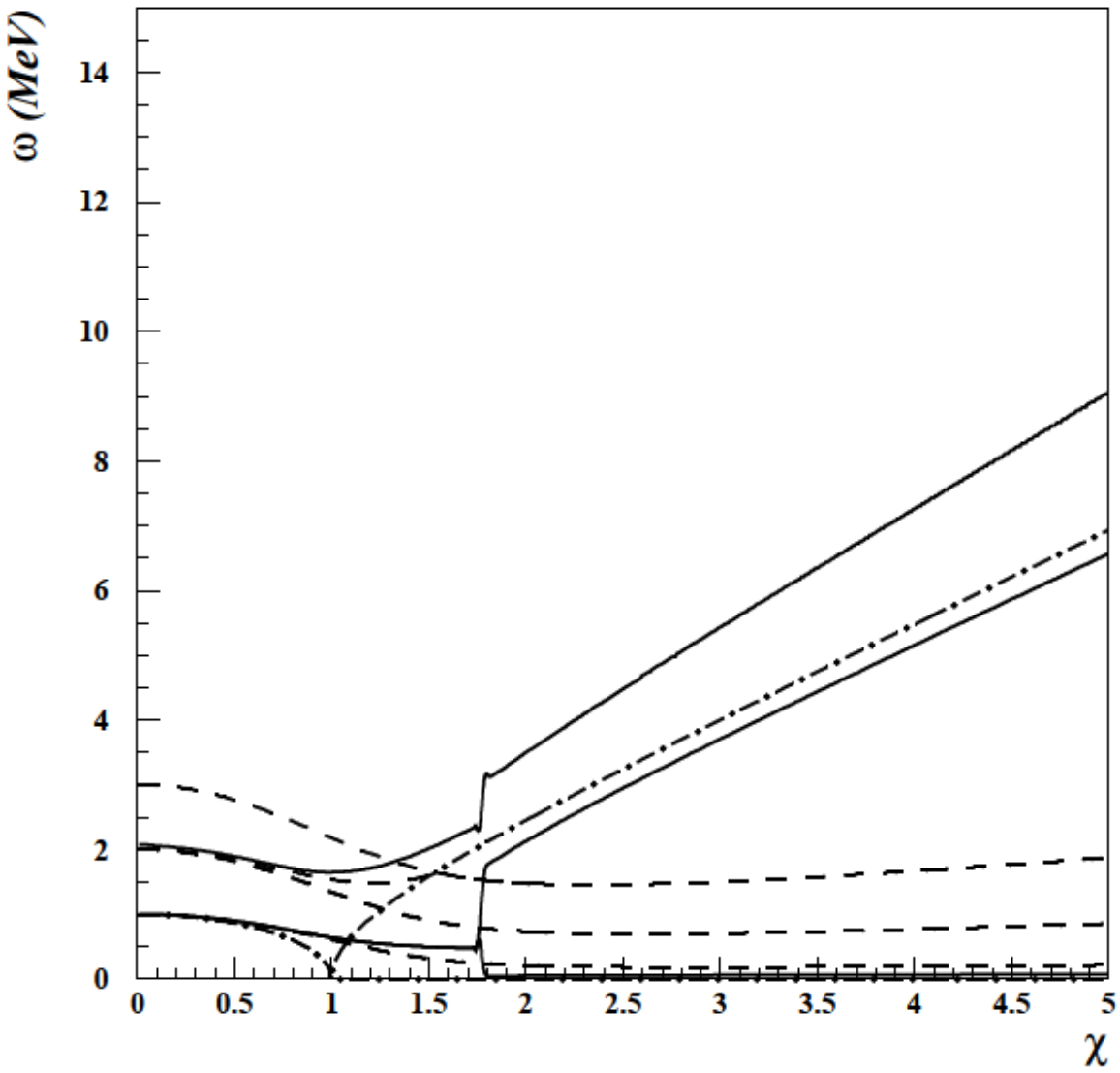


FIG. 4: **GOLDSTONE MODE**

SCRPA excitation energies versus the strength parameter χ, for $N = 20$, $\Delta\epsilon = 0.001$ MeV (full line). By dashes are given the lowest exact eigenvalues and by dot-dashes the standard RPA energies. After the phase transition point $\chi = 1$ (standard RPA) and $\chi \approx 2.1$ (SCRPA) a Goldstone mode at zero energy appears.

In a model with a continuously broken symmetry a clear rotational band structure is revealed.

The exact solution, found by a diagonalisation procedure, has a definite angular momentum projection L_0. The expectation value of the L_0^2 operator has **integer values**

$$\sqrt{\langle L_0^2 \rangle} = J = 0, 1, 2, ... \tag{45}$$

The "rotational bands" with $J = 0, 1, 2, ...$ are built on top of the RPA excitations

The exact energies can be written as follows

$$\omega_J = \frac{J(J+1)}{2M} , \tag{46}$$

where M is the "exact" mass parameter.

SCRPA mass parameter

of the rotational band can be written as follows

$$M = 2L_0^* \left(\mathcal{A} - \mathcal{B}\right)^{-1} L_0 \ , \tag{47}$$

where $\mathcal{A}, \mathcal{B}$ are the SCRPA matrices and L_0 denotes the part with $m > i$ components of the momentum operator

$$\begin{aligned} \hat{L}_0 &= i(K_{21} - K_{12}) \\ &= i\left[A_{20} - A_{02})\sin\phi + (A_{21} - A_{12})\cos\phi\right] \ , \end{aligned} \tag{48}$$

which should be written in terms of normalised generators $\delta Q^\dagger$, i.e.

$$L_0 = i\left(0,\ N_{20}^{1/2} sin\phi,\ N_{21}^{1/2} cos\phi\right) \ . \tag{49}$$

For the standard RPA case, by using the matrix elements given by and $N_{20} = N$, one obtains an analytical solution, namely

$$M = \frac{N(\chi - 1)}{\epsilon\chi(\chi + 1)} \ . \tag{50}$$

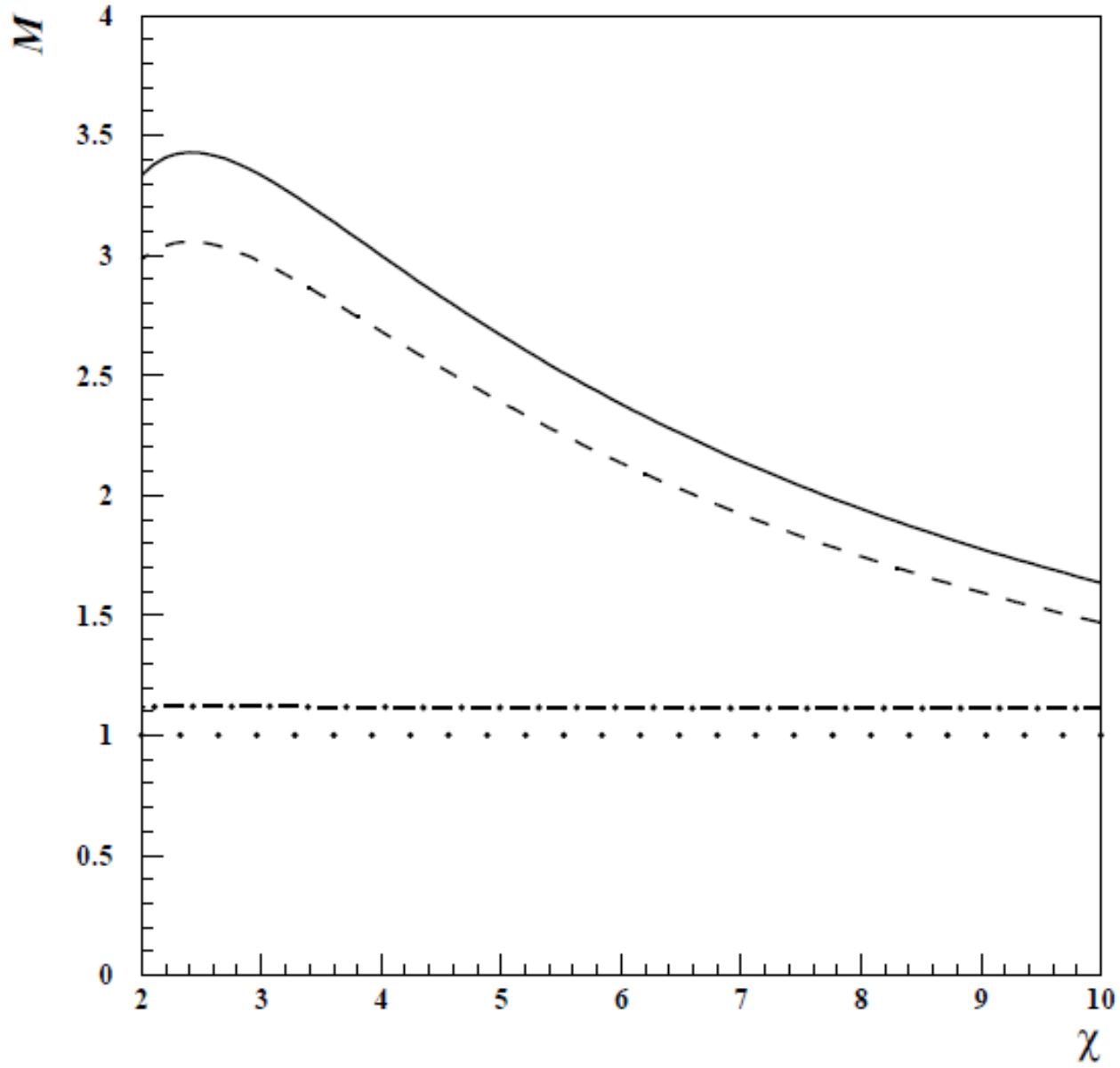


FIG. 5: **STANDARD RPA INERTIAL PARAMETER** versus χ (solid line) the inertial parameter from the exact energy spectrum (dashed line) and their ratio (dot-dashed line) for $N = 20$.

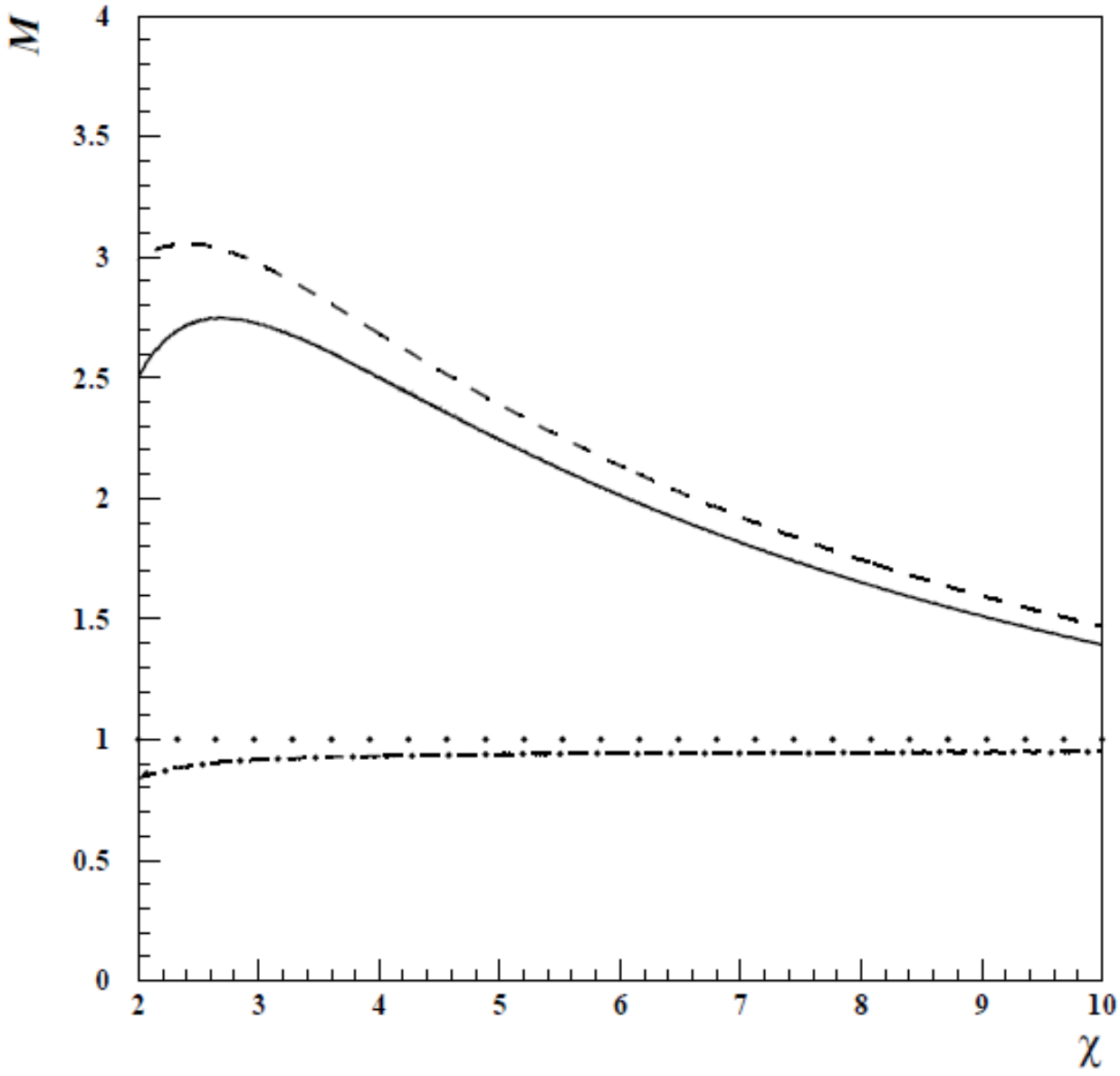


FIG. 6: **SCRPA INERTIAL PARAMETER** versus χ (solid line) the inertial parameter from the exact energy spectrum (dashed line) and their ratio (dot-dashed line) for $N = 20$.

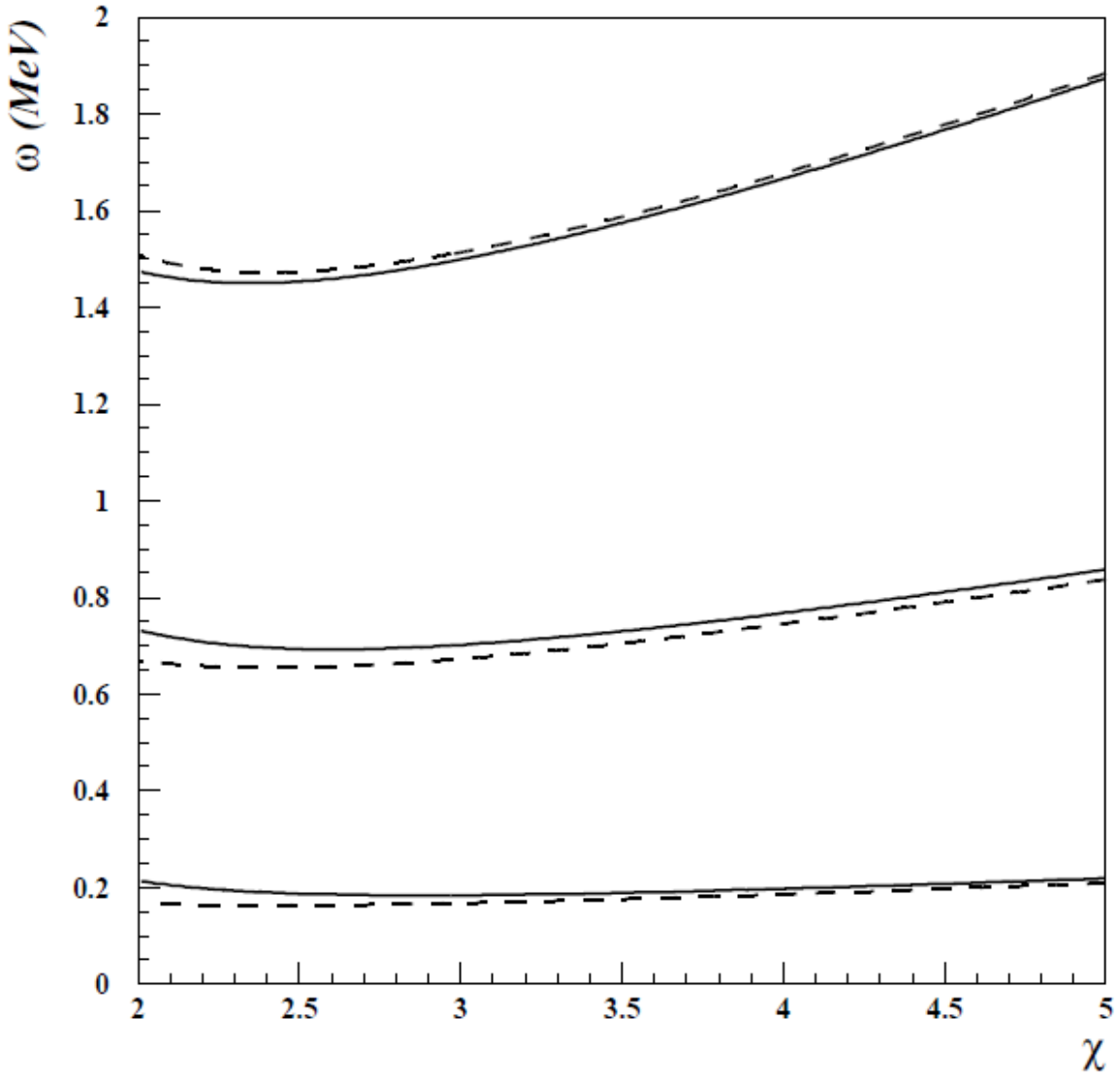


FIG. 7: **SCRPA ROTATIONAL SPECTRUM** (dashed lines) and the exact energies (solid lines) for $N = 20$. The three levels correspond to $J = 1, 2, 3$.

VI. Conclusions

1. The three-level Lipkin model has the advantage of allowing for a **continuously broken symmetry** on the mean field level with the appearance of a **Goldstone mode**.

2. The **RPA operator** should contain, in addition to the usual ph components $a_k^\dagger a_0$, also the so-called anomalous or **scattering terms** $a_2^\dagger a_1$.

3. We also calculated the **moment of inertia** of the rotational band which works in SCRPA (with scattering terms) in a way very analogous to standard RPA. Very good agreement with the exact solution is found.

4. Therefore the present formulation of SCRPA allows to mentain all the **formal and desirable properties of standard RPA**.

II. Four-body correlations

PHYSICAL REVIEW C **85**, 064306 (2012)

Shell model plus cluster description of negative parity states in ^{212}Po

D. S. Delion,[1,2,3] R. J. Liotta,[4] P. Schuck,[5,6] A. Astier,[7] and M.-G. Porquet[7]

[1] *"Horia Hulubei" National Institute of Physics and Nuclear Engineering, 407 Atomiştilor, Bucharest-Măgurele, RO-077125, România*

[2] *Academy of Romanian Scientists, 54 Splaiul Independenţei, Bucharest, RO-050085, România*

[3] *Bioterra University, 81 Gârlei, Bucharest, RO-013724, România*

[4] *KTH, Alba Nova University Center, SE-10691 Stockholm, Sweden*

[5] *IPN, IN2P3/CNRS and Université Paris-Sud, F-91406 Orsay, France*

[6] *LPMMC, CNRS and Université Joseph Fourier, F-38042 Grenoble Cedex 9, France*

[7] *CSNSM, IN2P3/CNRS and Université Paris-Sud, F-91405 Orsay, France*

(Received 16 February 2012; revised manuscript received 15 May 2012; published 7 June 2012)

The intraband electromagnetic transitions in ^{210}Po and ^{210}Pb are well described within the shell model approach. In contrast, similar transitions in ^{212}Po are one order of magnitude smaller than the experimental values, suggesting the existence of an α-cluster component in the structure of this nucleus. To probe this assumption we introduced Gaussian-like components in the single-particle orbitals. We thus obtained an enhancement of intraband transitions, as well as a proper description of the absolute α-decay width in ^{212}Po. We analyzed the recently measured unnatural parity states I^- in ^{212}Po in terms of the collective octupole excitation in ^{208}Pb coupled to positive parity states in ^{210}Pb. They are connected by relatively large dipole transitions to yrast positive natural parity states. We described $E1$ transitions by using the same α-cluster component and an effective neutron dipole charge $e_\nu = -eZ/A$. $B(E2)$ values and absolute α-decay width in ^{212}Po are simultaneously described within the shell model plus a cluster component depending upon one free strength parameter.

DOI: 10.1103/PhysRevC.85.064306 PACS number(s): 21.60.Jz, 23.20.Js, 23.60.+e, 27.80.+w

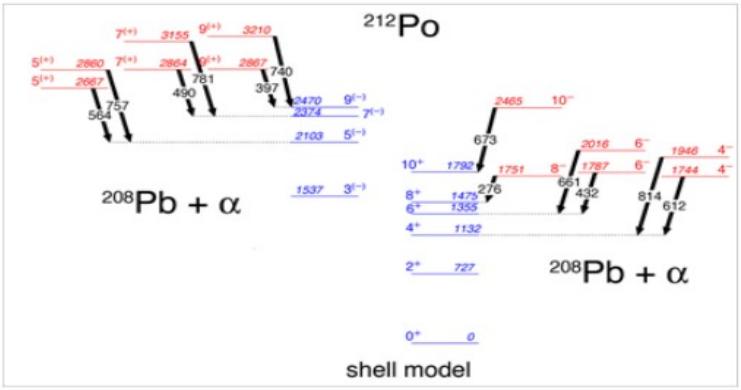
APS physics | Physics

Viewpoint: Do alpha particles cluster inside heavy nuclei?

Michael P. Carpenter, Argonne National Laboratory, Argonne, IL 60439, USA

January 25, 2010 • *Physics* 3, 8

New excited states have been observed in ^{212}Po that are associated with a configuration in which an alpha particle is combined with a doubly-magic ^{208}Pb core.



Selected for a Viewpoint in *Physics*

PRL **104,** 042701 (2010) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2010

Novel Manifestation of α-Clustering Structures: New "$\alpha + {}^{208}$Pb" States in ^{212}Po Revealed by Their Enhanced $E1$ Decays

A. Astier,[1] P. Petkov,[1,2] M.-G. Porquet,[1] D. S. Delion,[3,4] and P. Schuck[5]

Nuclear structure in ^{212}Po

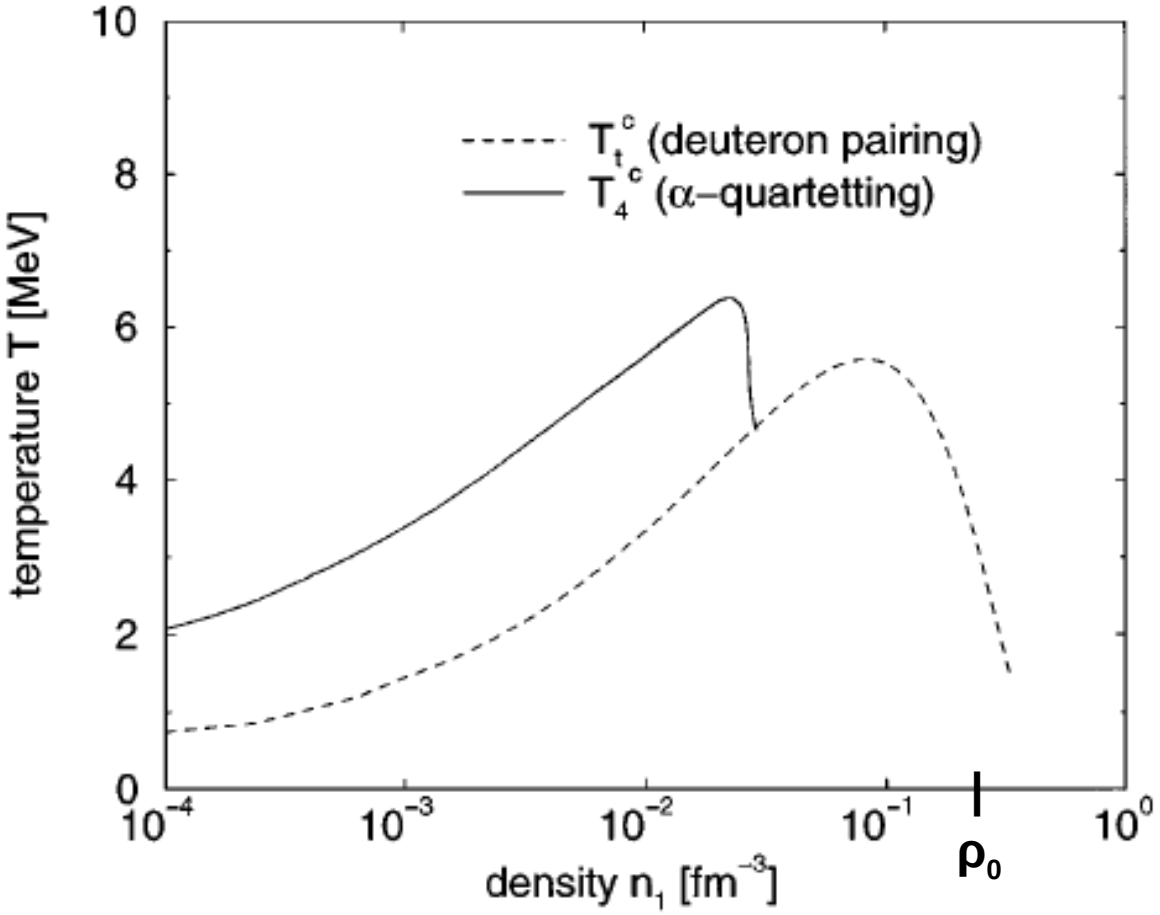
Positive parity states 2^+, 4^+, 6^+, 8^+ are given by neutron broken pairs

$$|^{212}\mathrm{Po}(J^+)\rangle = |^{210}\mathrm{Pb}(J^+) \otimes {}^{210}\mathrm{Po}(\mathrm{g.s.})\rangle$$

Negative parity states 4^-, 6^-, 8^- are given by neutron broken pairs coupled to an octupole state

$$|^{212}\mathrm{Po}(I^-)\rangle = |[^{210}\mathrm{Pb}(J^+) \otimes {}^{210}\mathrm{Pb}(3^-)]_{I^-} \otimes {}^{210}\mathrm{Po}(\mathrm{g.s.})\rangle$$

Phase diagram for deuteron and α-particle



G. Ropke, A. Schnell, P. Schuck, P. Nozieres
Four-particle condensate in strongly coupled fermion systems
Phys. Rev. Lett. 80, 3177 (1998).

Pairing survives at the equilibrium density ρ_0 and α-quarteting collapses at about 10% ρ_0 , i.e.
an α-particle can exist only on the nuclear surface

Single particle wave function

We add to the sp radial wave function a cluster component for each angular momentum

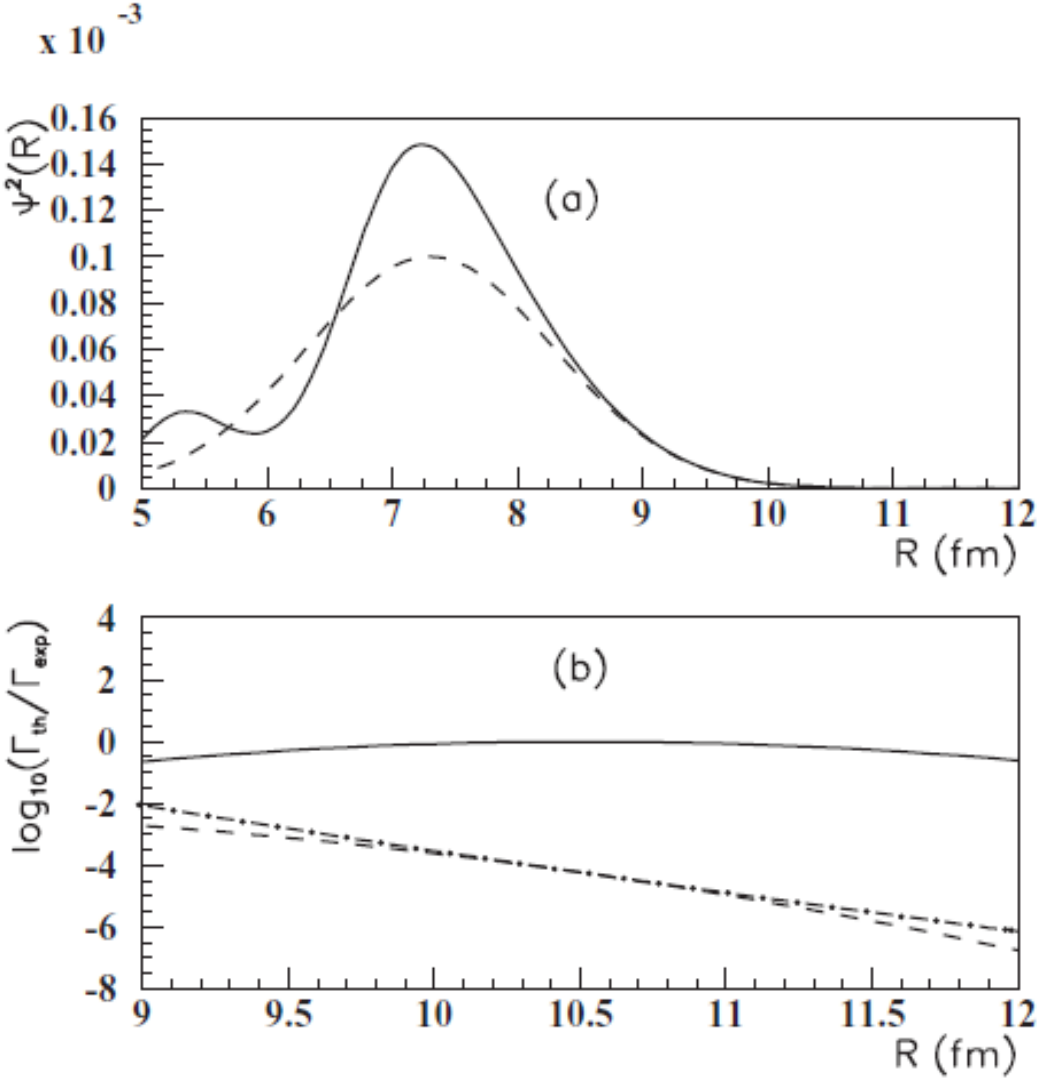
$$\psi_l(r) = \psi_l^{(SM)}(r) + \psi_l^{(clus)}(r) \ .$$

Cluster component

is given by Gaussian-like sp wave function peaked on the nuclear surface r_0

$$\psi_l^{(clus)}(r) = \mathcal{N}_l^{(clus)} e^{-\beta_c (r-r_0)^2/2}$$

Mean field with surface α-clustering in ^{212}Po explains decay width between ground states



Formation probability versus cm radius
total: solid line
cluster comp: dashes

Log (width / exp.) versus cm radius

The same cluster amplitude ≈ 0.3 explains B(Eλ) values and absolute α-decay width

Surface α-clustering term with the amplitude ≈ 0.3 explains large electromagnetic E1 transitions in ^{212}Po

B(E2:J+2→J)-values

$J' \to J$	^{210}Po $B(E2)_{exp}$	$B(E2)_{th}$	^{210}Pb $B(E2)_{exp}$	$B(E2)_{th}$	^{212}Po $B(E2)_{exp}$	$B(E2)_{th}$
$2 \to 0$	0.56(12)	6.7	1.4(4)	3.9		9.2
$4 \to 2$	4.6(2)	12.9	3.2(7)	3.5		20.8
$6 \to 4$	3.0(1)	8.9	2.2(3)	2.4	13.5(36)	14.4
$8 \to 6$	1.18(3)	3.9	0.62(5)	1.0	4.60(9)	5.8

B(E1:I⁻→J⁺)-values

I^-	J^+	E_{MSM} (MeV)	$E(^{212}\mathrm{Po}(I^-))$ (MeV)	$E_{exp}(^{212}\mathrm{Po}(I^-))$ (MeV)	$B(E1)^{(1)}_{th}$ (10^4 W.u.)	$B(E1)^{(2)}_{th}$ (10^4 W.u.)	$B(E1)_{exp}$ (10^4 W.u.)
2^-	2^+	−0.407	1.236		5	1	
	4^+	−0.204	1.907		15	63	
4^-	4^+	−0.303	1.808	1.744	9	11	25
	6^+	−0.107	2.201	1.946	2	4	11
6^-	6^+	−0.213	1.886	1.787	37	122	66
	8^+	−0.490	2.197	2.016	3	8	19
8^-	6^+	−0.489	1.816	1.751	43	148	200
	8^+	−0.215	2.240	1.986	8	24	
10^-	8^+	−0.360	2.135	2.465	2	1	18

Microscopic description

PHYSICAL REVIEW C **107**, 024302 (2023)

Cluster mean-field description of α emission

A. Dumitrescu[1,2,3] and D. S. Delion[1,2,4]

[1] *"Horia Hulubei" National Institute of Physics and Nuclear Engineering, 30 Reactorului, POB MG-6, RO-077125 Bucharest-Măgurele, România*

[2] *Academy of Romanian Scientists, 3 Ilfov RO-050044 Bucharest, România*

[3] *Department of Physics, University of Bucharest, 405 Atomistilor, POB MG-11, RO-077125 Bucharest-Măgurele, România*

[4] *Bioterra University, 81 Gârlei, RO-013724 Bucharest, România*

(Received 18 November 2022; accepted 25 January 2023; published 6 February 2023)

We show that the Hartree-Fock-Bogoliubov (HFB) method is able to describe experimental values of α decay widths by including a residual nucleon-nucleon surface Gaussian interaction (SGI) within the standard procedure used to calculate the nuclear mean field. We call this method the cluster HFB (CHFB) approach. In this way we correct the deficient asymptotic behavior of the corresponding single-particle wave functions generated by the standard mean field. The corrected mean field becomes a sum between the standard mean Woods-Saxon–like field and a cluster Gaussian component centered at the same radius as the SGI. Thus, we give a confirmation of the mean field plus cluster potential structure, which was assumed in our previous work on α-decay widths. Systematic calculations evidence the linear correlation between the SGI strength and fragmentation potential, allowing for reliable predictions concerning the half-lives of superheavy emitters.

DOI: 10.1103/PhysRevC.107.024302

Surface Gaussian Interaction (SGI)

$$v(r_\tau, R_\tau) = -v_0 \exp\left(-\frac{r_\tau^2}{b_{\rm rel}^2}\right) \times \left[1 + x_c \exp\left(-\frac{(R_\tau - R_0)^2}{b_{\rm c.m.}^2}\right)\right],$$

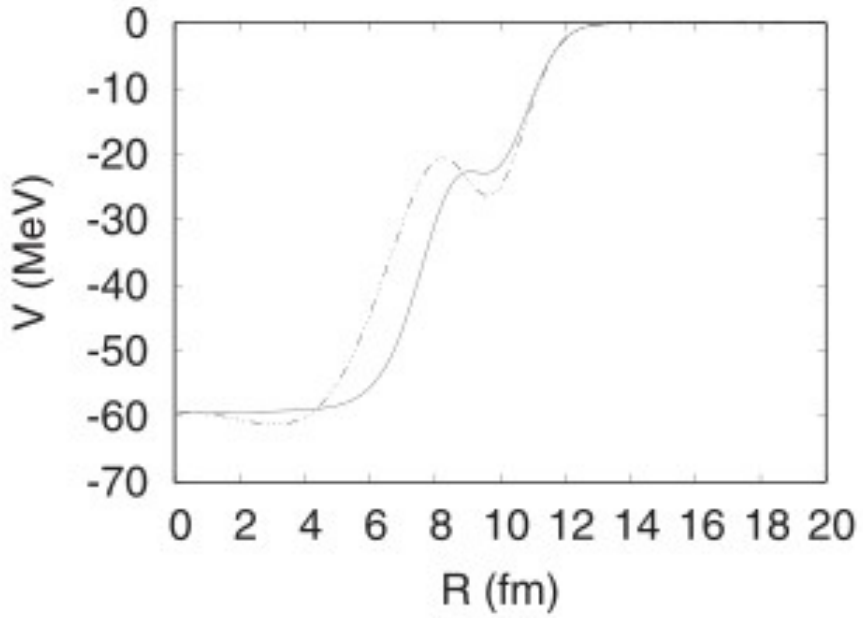


FIG. 1. Proton HFB mean field plus SGI interaction (dashed line) and WS plus SGI potential (solid line) in the case of ^{242}Pu.

Decay systematics

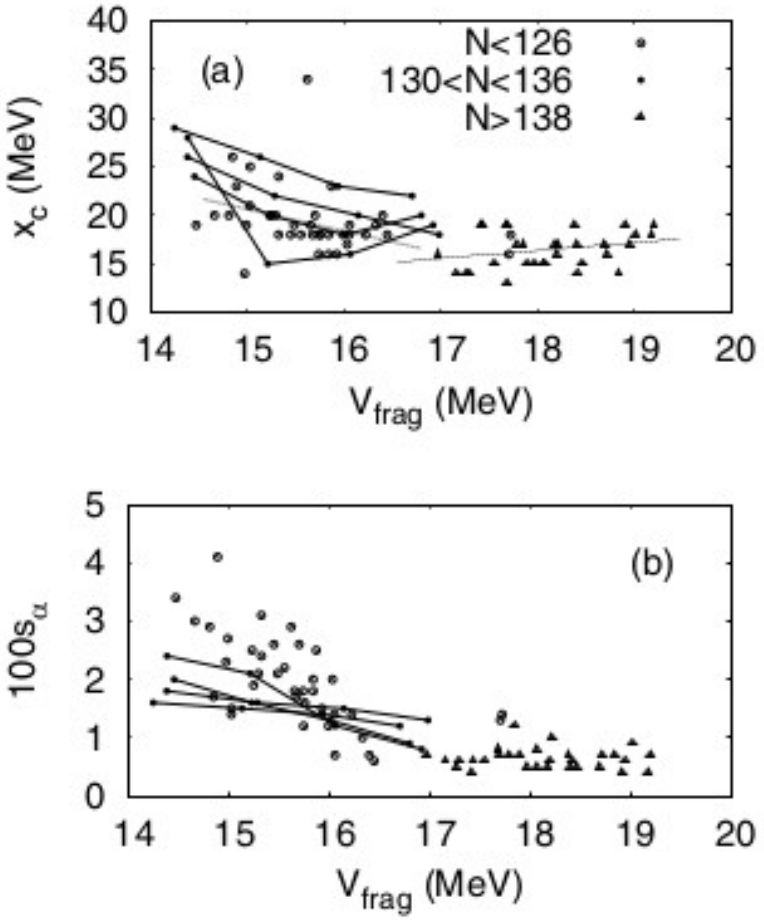


FIG. 6. Residual interaction strength (a) and α-decay spectroscopic factor (b) versus the fragmentation potential.

Conclusions

Absolute decay widths and electromagnetic transitions can be described microscopically by using a mixed single particle basis, containing additional clustering components