Three-body contact of the resonant Fermi gas

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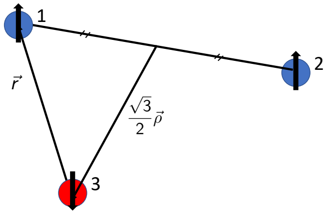
F. Werner and XL, arXiv:2211.09765
(submitted to Comptes rendus de l'Académie des sciences).

Outline

Introduction: three-body contact

System: Ultracold spin 1/2 interacting Fermi gases (BEC-BCS crossover)

- Jacobi coordinates:



$\mathbf{R} = (\mathbf{r}, \rho)$ is hyperradius (6D)

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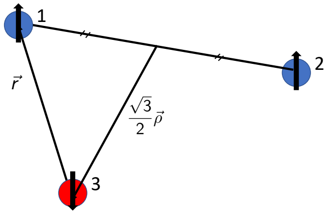

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Definition

$$N_3(\epsilon) \underset{\epsilon\to 0}{\sim} C_3\, \epsilon^{2s+2}$$

C_3 is *three-body contact*.
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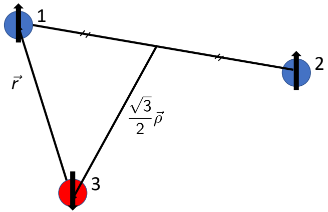

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Analogous to number of pairs $\uparrow\downarrow$ (●●) in $r < \epsilon$: $N_2(\epsilon) \underset{\epsilon \to 0}{\sim} C_2\, \frac{\epsilon}{4\pi}$
C_2 is Tan's (two-body) contact.

Introduction: three-body contact

Like Tan's contact C_2, C_3 is due to *interactions*.

No interaction: $N_2^{(0)}(\epsilon) \propto \epsilon^3$, $N_3^{(0)}(\epsilon) \propto \epsilon^8$.

N_2 and N_3 are enhanced $\Rightarrow$Bunching effect.

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Zero range attractive interactions (scattering length a):
short range behavior of many-body wavefunction

2 or 3 particles close ($r_{12}, R \ll |a| \,\mathrm{or}\, (m|E|)^{-1/2}$):

V. Efimov 1970; D. Petrov, C. Salomon, G. Shlyapnikov 2004; S. Tan 2004.

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$$\Psi(1,2,3\cdots) \underset{R\to 0}{\propto} \boxed{R^{s-2}} \times \left(\sum_{m=-1}^{1} \Phi_m(\mathbf{\Omega}) B_m(\cdots) \right) \Rightarrow N_3(\epsilon) \underset{\epsilon\to 0}{\sim} \epsilon^{2\,s+2}$$

Introduction: three-body contact

Momentum distribution: $N_\sigma(\mathbf{k}) \underset{k\to+\infty}{\sim} \frac{C_2}{k^4}$

S. Tan 2005, 1D: M. Olchanyi, V. Dunjko 2003

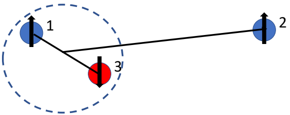
Number of pairs of opposite spin particles at a distance $< \epsilon$ of c.o.m. momentum $\mathbf{K}$:

Property

$N_2(\epsilon, \mathbf{K}) \underset{\epsilon\to 0}{\sim} \frac{\epsilon}{4\pi} N_P(\mathbf{K})$

$N_P(\mathbf{K}) \underset{K\to+\infty}{\sim} \frac{C_3}{K^{2s+4}} \times (constant)$.

$\Psi \sim \frac{1}{r} R^{s-1}$



Non degenerate regime

$n_\sigma \Lambda_T^3 \ll 1, \quad \Lambda_T = \sqrt{\frac{2\pi\hbar^2}{m\,k\,T}}$ thermal wavelength: virial-like expansion

Virial expansion of three-body contact

$$\mathcal{C}_3 \underset{n_\sigma \Lambda_T^3 \to 0}{\sim} n^3 \left(\frac{\hbar^2}{m k_B T}\right)^{2-s} f\left(\frac{\lambda_T}{a}\right)$$

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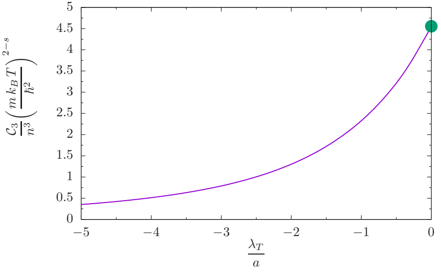
Results :



Unitary limit:
exact result
(3-body problem)
$f(0) = \frac{9\sqrt{3}\ \pi^3}{2^{2s+1}\,\Gamma(s+2)}$

Non degenerate regime, two approaches:

Wave-function:
(Unitary Limit)

UL : separability

$\psi(\mathbf{R}) = \frac{F(R)}{R^2}\, \phi(\boldsymbol{\Omega})$

exact results for F and ϕ.

$F(R) \underset{R\to 0}{\sim} const\, R^{s}$

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Green's functions:
3-body correlation function

$g_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) =$
$\langle \psi^\dagger_\uparrow(\mathbf{r}_1)\psi^\dagger_\uparrow(\mathbf{r}_2)\psi^\dagger_\downarrow(\mathbf{r}_3)\psi_\downarrow(\mathbf{r}_3)\psi_\uparrow(\mathbf{r}_2)\psi_\uparrow(\mathbf{r}_1)\rangle$

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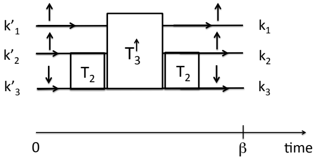
(virial expansion with diagrams: XL 2011)

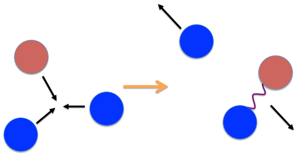


$\Im(t_3^{l=1}(k,k')) \underset{k,\,k'\to+\infty}{\sim} \frac{B(E, a^{-1})}{(k\,k')^{s+1}}$

Three-body contact and three-body losses

System:

spin 1/2 (● ●) fermions
@ ultralow T (*e. g.* ^{6}Li).



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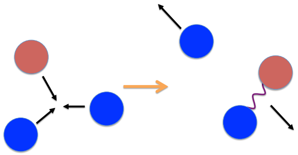

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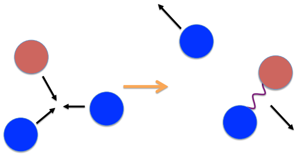

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Property

$$\Gamma_3 = -\frac{\hbar}{m} 4\, s(s+1) Im[a_3]\, C_3 \propto\ C_3$$

a_3: "three-body parameter" for short-range physics

$$\Psi(\mathbf{R},\cdots) \underset{R\to 0}{\sim} \sum_{m=-1}^{1} \left(R^s - a_3\, R^{-s}\right) \frac{1}{R^2} \Phi_m(\mathbf{\Omega}) B(\cdots)$$

Conclusion-projects

- three-body contact: new observable, anologous to two-body Tan's contact, but for three-body correlations.
- analogous relations for short-range correlations $N_3(\epsilon)$, high-momentum tails of close pairs has $K^{-7.5454..}$ behavior.
- lowest order virial expansion of C_3. This could be used has a calibration to measure C_3, through three-body losses, at any T.
- j-particle contact ?

Derivative of the energy with respect to the three-body parameter a_3

Property

$$\left.\frac{\partial E}{\partial a_3}\right|_a = \frac{\hbar^2}{m} 2\, s(s+1)\, C_3.$$