SUPERFLUIDITY IN NUCLEAR SYSTEMS

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- Pairing interaction induced by the exchange of collective vibrations
- Two-particle transfer and the Josephson effect

PHYSICAL REVIEW C 72, 054314 (2005)

Pairing matrix elements and pairing gaps with bare, effective, and induced interactions

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https://www.sif.it/riviste/sif/sag/ricordo/broglia









Mean field calculation with low-momentum 2N and 3N interactions: 3-body force reduces the pairing gaps



T. Lesinski, K. Hebeler, T. Duguet, A.Schwenk, J. Phys. G 39 (2012) 015108

NFT has been mostly used in normal nuclei. Extension to superfluid, spherical nuclei within the Nambu-Gor' kov formalism (cf. Van der Sluys et al., NPA551(1993)210)

By extending the Dyson equation...

$$G_{\mu}^{-1} = (G_{\mu}^{o})^{-1} - \Sigma_{\mu}(\omega)$$

$$\Sigma_{\mu}(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \sum_{\mu'} \frac{1}{\hbar} G_{\mu'}(\omega') \sum_{\alpha} \frac{1}{\hbar} D_{\alpha}^{o}(\omega - \omega') * V_{\mu\mu',\alpha}^{2}$$

to the case of superfluid nuclei (Nambu-Gor' kov), it is possible to consider both



Renormalization of BCS quasiparticle energies and pairing gap

A. Idini et al., PRC 85 (2012) 014331

$$\begin{pmatrix} E_a + \Sigma_{11}(\tilde{E}_{a(n)}) & \Sigma_{12}(\tilde{E}_{a(n)}) \\ \Sigma_{12}(\tilde{E}_{a(n)}) & -E_a + \Sigma_{22}(\tilde{E}_{a(n)}) \end{pmatrix} \begin{pmatrix} x_{a(n)} \\ y_{a(n)} \end{pmatrix} = \tilde{E}_{a(n)} \begin{pmatrix} x_{a(n)} \\ y_{a(n)} \end{pmatrix}$$

 $\begin{array}{lll} V(ab(m)\lambda\nu) = & h(ab\lambda\nu)(u_{a}^{BCS}\tilde{u}_{b(m)} - v_{a}^{BCS}\tilde{v}_{b(m)}) & \tilde{u}_{a(n)} = x_{a(n)}u_{a}^{BCS} - y_{a(n)}v_{a}^{BCS} \\ W(ab(m)\lambda\nu) = & h(ab\lambda\nu)(u_{a}^{BCS}\tilde{v}_{b(m)} + v_{a}^{BCS}\tilde{u}_{b(m)}) & \tilde{v}_{a(n)} = x_{a(n)}v_{a}^{BCS} + y_{a(n)}u_{a}^{BCS}, \end{array}$

Generalized Gap Equation (schematic)



Renormalization of BCS pairing gap for states close to the Fermi energy (single node approximation, SLy4 mean field)



Experimental phonons were used for natural parity modes with λ =2,3,4,5



Many-Body Perturbation Theory Holt, Menendez, Schwenk, J. Phys. G 40 (2013) 075105



FIG. 3. (Color online) Three-point mass differences $\Delta_n^{(3)}$ in the calcium isotopes calculated to third order in MBPT with and without the leading chiral 3N forces, and in comparison with experiment [24, 67]. The legend is as in Fig. 1. Panel (a) shows the results of the third-order ladder contributions. Panels (b) and (c) include all MBPT diagrams to third order in the pf-shell and the extended $pfg_{9/2}$ valence space, respectively. The results in the pf-shell are with empirical SPEs. For the $pfg_{9/2}$ space, we show pairing gaps for both the MBPT and empirical SPEs.

When particle-hole contributions are included in a full ing in nuclei. Our results show that they can provide the This clearly demonstrates the importance of particle-hole Ref. [15] may be attributed to these effects. many-body processes, such as core-polarization, on pair-

third-order calculation, we find in Fig. 3 a clear improve- missing pairing strength required to reproduce experiment compared to including only ladder diagrams. In the ment on top of the direct NN+3N interactions. Analopf-shell, the three-point mass differences are increased, gously, the systematic differences between theoretical and leading to reasonable agreement with experimental data. experimental pairing gaps found in the EDF approach of

Many-body correlations in nuclear superfluidity

Elena Litvinova^{1,2,3} and Peter Schuck^{4,5}

$$\Delta_1 = -\sum_2 \mathcal{V}_{1\bar{1}2\bar{2}} \frac{\Delta_2}{2E_2},$$
(66)

where the bar denotes the conjugate or the time-reversed state [7] and the interaction matrix elements read

1 _

$$\mathcal{V}_{121'2'} = \frac{1}{4} \sum_{34} \delta_{1234} K_{341'2'}(2\lambda) = \frac{1}{2} \Big[K_{121'2'}^{(0)} + K_{121'2'}^{(r)}(2\lambda) \Big].$$
(67)

The integral part of the gap Eq. (66), thus, contains all the microscopic effects of the kernel K "on shell," regardless of the approximations made for its static $K^{(0)}$ and dynamical $K^{(r)}$ parts.





Gorkov algebraic diagrammatic construction formalism at third order

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Local approximation

The pairing gap associated with the bare interaction is surface peaked; the induced interaction reinforces this feature



Microscopic justification of surface peaked, density-dependent pairing force

A. Pastore et al., Phys. Rev. C78 (2008) 024315

Interaction	α	η
$v_{ m Arg} onumber v_{ m Arg+ind}$	0.66 2.0	0.84 1.32

$$v^{\delta}(\vec{r}_1, \vec{r}_2) = v_0 \left[1 - \eta \left(\frac{\rho \left(\frac{\vec{r}_1 + \vec{r}_2}{2} \right)}{\rho_0} \right)^{\alpha} \right] \delta(\vec{r}_1 - \vec{r}_2),$$

A. Pastore et al., Phys. Rev. C78 (2008) 024315

One of the basic questions about nuclear pairing is the role of induced interactions in the effective pairing interaction [5,54–57]. Indirect information about this can in principle be obtained by exhibiting the density dependence and the isospin dependence of the effective interaction. It is therefore of interest to examine interactions including a density dependence to see the sensitivity. The rms residual for the neutron OES with volume, mixed, and surface pairing in HF+BCS theory are shown in Table IV. There is a slight favoring of the surface interaction, but we deem that the difference in the residuals (10%) is too slight to be significant. The weak sensitivity to the density dependence confirms the results of other studies [10,58]. *G.F. Bertsch et al., PRC 79 (2009) 034306*

Binding energies do not provide a clean measure of pairing correlations since they have contributions which are not directly related to them. These include the impact of (quasi)particle-vibrational coupling on the binding energies of odd-mass nuclei. The inclusion of particle-vibrational coupling increases the accuracy of the description of the single-particle configurations in odd-*A* nuclei but such studies are limited to spherical nuclei (see Refs. [76,77) It is interesting that such features have already been mentioned in seminal article of Decharge and Gogny [78] where they indicated that treating explicitly the residual interaction through configuration mixing in odd and even nuclei is expected to lower the OES by approximately 300 keV in the Sn isotopes. *S. Teeti and A.V. Afanasjev, Phys. Rev. C* 103 (2021) 034310

Dear Enrico,

Umberto Lombardo is in Orsay and we are discussing screening of the pairing interaction in the different channels (in infinite matter). In this respect I have a question: in finite nuclei, did you ever consider screening in the p-n S=1, T=0, ie deuteron, channel ? I mean vibration renormalisation in this channel ? Would be interesting and would specifically interest me.

Hope you are fine.

Best regards,

Peter.

- The Josephson effect and two-nucleon transfer reactions

Josephson Junction in Condensed Matter

Because of the phase coherence, each superconductor behaves as a singlelevel quantum-mechanical system

$$i\hbarrac{\partial}{\partial t}egin{pmatrix} \sqrt{n_A}e^{i\phi_A}\ \sqrt{n_B}e^{i\phi_B} \end{pmatrix} = egin{pmatrix} eV & K\ K & -eV \end{pmatrix}egin{pmatrix} \sqrt{n_A}e^{i\phi_A}\ \sqrt{n_B}e^{i\phi_B} \end{pmatrix}$$

$$\dot{n}_A = rac{2K\sqrt{n_A n_B}}{\hbar} \sin arphi.
onumber \ rac{\partial arphi}{\partial t} = rac{2eV(t)}{\hbar}$$

$$rac{\partial arphi}{\partial t} = rac{2eV(t)}{\hbar}$$

 $I(t) = I_c \sin(\varphi(t))$

$$\varphi = \phi_B - \phi_A$$

Josephson effect

- Josephson junction: 2 superconductors separated by an insulating barrier of width d.
- When a constant potential (battery) V is applied, an alternating current (ac) of frequency ν_J = 2eV/h is induced, and the corresponding radiation is emitted.
- The charge carriers are Cooper pairs tunneling through the insulating junction.

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Josephson effect

- If $d \ll \xi$, the Josephson current is $I_J \approx I_N$, where I_N is the normal (single-electron) current.
- The correlation length can be estimated to be $\xi = \hbar v_F / (\Delta \pi) \approx 10^4 \text{\AA}, v_F \equiv$ Fermi velocity.

If $d > \xi$

The supercurrent vanishes and only a direct (dc) normal current I_N of single electron carriers flows.

Can we reproduce the cross sections?

 σ_{1n} and σ_{2n} are calculated for the reaction between the two superfluid nuclei ¹¹⁶Sn and ⁶⁰Ni.

How do we do the calculations?

Cross sections and dipoles

- 2-n cross section: $\sigma_{2n} \to T_{2n} = \sum_{\gamma} \langle \phi_f | \mathbf{v}(\mathbf{r}_1) | \phi_{\gamma} \rangle \langle \phi_{\gamma} | \mathbf{v}(\mathbf{r}_2) | \phi_i \rangle$
- dipole vector: $\vec{d} \rightarrow T_d = e_{eff} \sum_{\gamma} \langle \phi_f | v(r_1) | \phi_{\gamma} \rangle (\vec{r_1} + \vec{r_2}) \langle \phi_{\gamma} | v(r_2) | \phi_i \rangle$
- 3D projections of \vec{d} : $d_i \rightarrow T_{d_i} = e_{eff} \sum_{\gamma} \langle \phi_f | \mathbf{v}(\mathbf{r}_1) | \phi_{\gamma} \rangle (\mathbf{r}_{1i} + \mathbf{r}_{2i}) \langle \phi_{\gamma} | \mathbf{v}(\mathbf{r}_2) | \phi_i \rangle$

 γ emission strength function

• amplitude for emission of photon of polarization q in direction θ_{γ} :

$$\mathcal{T}^{q}(heta_{\gamma}) = \sum_{i} \mathcal{D}^{1}_{i,q}(heta_{\gamma}) T_{d_{i}},$$

• differential cross section for γ emission:

$$\frac{d^2\sigma_{\gamma}}{d\Omega_{\gamma}dE_{\gamma}} = \rho(E_f)\left(\frac{E_{\gamma}^2}{(\hbar c)^3}\right)\sum_q |\mathcal{T}^q(\theta_{\gamma})|^2\,\delta(E-E_{\gamma}-E_f+Q)$$

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Is there a nuclear Josephson effect? A prediction

• The finite collision time, as well as recoil effects, provides a width $(\Delta E \sim \hbar / \tau_{coll})$.

Nuclear superconductivity?

Physics

VIEWPOINT

The Tiniest Superfluid Circuit in Nature

A new analysis of heavy-ion collision experiments uncovers evidence that two colliding nuclei behave like a Josephson junction—a device in which Cooper pairs tunnel through a barrier between two superfluids.

By Piotr Magierski

Velocity of the transferred (nuclear) Cooper pair: Depairing velocity.

$$v_c = \frac{\hbar s_c}{m} = \frac{2\Delta}{mv_F} = \frac{\hbar}{m\xi}$$

$$\xi = \hbar v_F / 2 \Delta$$
,
coherence length
of the superfluid

FIG. 5: Supercurrent I_Q (in units of I_Q^0) vs. superfluid velocity v (in unit of v_L) for various temperatures. From top to bottom: $T = (0.1, 0.25, 0.4, 0.5, 0.556, 0.75, 0.9) T_c^0$. The curves terminate at the critical velocities $v_c(T)$ appropriate to these temperatures. The maximum supercurrent for a particular curve determines the value of the critical current at that temperature.

Revisiting the critical velocity of a clean one-dimensional

superconductor

Tzu-Chieh Wei Institute for Quantum Computing and Department of Physics and Astronomy, University of Waterloo, Waterloo, ON N2L 3G1, Canada*

Paul M. Goldbart

Department of Physics, Institute for Condensed Matter Theory, and Federick Seitz Materials Research Laboratory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, U.S.A. (Dated: April 15, 2009) Extension of the Cooper pair in an isolated nucleus : almost the same for different forces.

It is constrained by the nuclear mean and field is much smaller than the nuclear matter estimate h v_F / $\pi \Delta$ (~ 20 fm)

M. Matsuo, Phys. Rev. C56 (2007) 3054
N. Pillet, N. Sandulescu, P. Schuck, Phys. Rev. C76 (2007)24310
K. Hagino, H. Sagawa, J. Carbonell, P. Schuck, PRL 99 (2007)22506

The nucleon pair is transferred due to the mean field acting twice. The amplitude due to the action of the pairing force is about one order of magnitude weaker

D.R. Bes and O. Civitarese, Nucl. Phys. A 983 (2019) 53

Coherence length during the reaction: possibly, a somewhat different concept

The two nucleons remain correlated even when they are separated by the distance of closest approach

PIAVE-ALPI ACCELERATOR

Search for a Josephson-like effect in the ¹¹⁶Sn+⁶⁰Ni system

PRISMA + AGATA experiment

Spokesperson(s): L. Corradi, S. Szilner

(GLIMOS: L. Corradi)

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