Neutron star properties from semiclassical methods

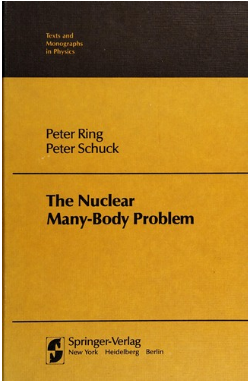
Constança Providência

Universidade de Coimbra, Portugal

QMBC, 21 March, 2023

Memory

In 1983 I was offered the book



The Nuclear Many-Body Problem

Peter Ring and Peter Schuck

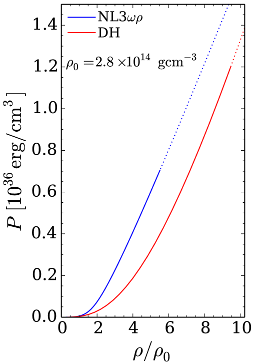
and without knowing this book was going to define my way in physics...

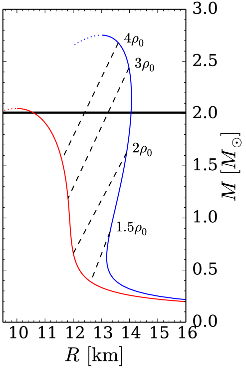
Semicalssical methods and neutron stars

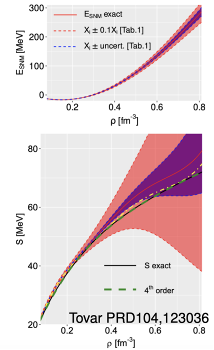
- What does a minimal set of nuclear matter constraints together with a $2M_\odot$ condition tell us about the neutron star EOS based on a microscopic model?
- Can we get the neutron star composition?
- Which are possible signatures of the presence of hyperons inside neutron stars?
- How can we describe the warm non-homogeneous stellar matter self-consistently?

Probing the interior of Neutron Stars

- Neutrons stars provide a laboratory for testing
  - nuclear physics: high density, highly asymmetric matter
  - QCD: deconfinement, quark matter, superconducting phases
- mass-radius $\rightarrow$ equation of state $\rightarrow$ composition?







(Mondal&Gulminelli PRD105 083016; Iman PRC105 015806 ; Essick PRL127 192701)

EOS: relativistic mean field description

RMF Lagrangian for stellar matter

- **Lagrangian density:** causal Lorentz-covariant Lagrangian (baryon densities and meson fields)

$$\mathscr{L}_{NLWM} = \sum_{B=baryons} \mathscr{L}_B + \mathscr{L}_{mesons} + \mathscr{L}_l + \mathscr{L}_\gamma,$$

- **Baryonic contribution:** $\mathscr{L}_B = \bar{\psi}_B \left[\gamma_\mu D_B^\mu - M_B^*\right] \psi_B,$
  $D_B^\mu = i\partial^\mu - g_{\omega B}\omega^\mu - \frac{g_{\rho B}}{2}\boldsymbol{\tau}\cdot\mathbf{b}^\mu - g_{\phi B}\phi^\mu$
  $M_B^* = M_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^*$

- **Meson contribution**

$$\mathscr{L}_{mesons} = \mathscr{L}_\sigma + \mathscr{L}_\omega + \mathscr{L}_\rho + \mathscr{L}_\phi + \mathscr{L}_{non-linear}$$

- **Lepton contribution:** $\mathscr{L}_l = \sum_l \bar{\psi}_l \left[\gamma_\mu i\partial^\mu - m_l\right] \psi_l$

EOS: relativistic mean field description

Density dependence of the EOS determined by introducing

- non-linear meson terms (Boguta&Bodmer 1977, Mueller&Serot 1996)

$$\mathscr{L}_{non-linear} = -\frac{1}{3} b g_\sigma^3 (\sigma)^3 - \frac{1}{4} c g_\sigma^4 (\sigma)^4 + \frac{\xi}{4!} (g_\omega \omega_\mu \omega^\mu)^4 + \Lambda_\omega g_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu g_\omega^2 \omega_\mu \omega^\mu ,$$

  - Parameters: $g_i (i = \sigma, \omega, \rho)$, b, c, ξ, Λ_ω (Malik arxiv:2301.08169)

- density dependent couplings: DD2, DDME2 (Typel NPA656 331, PRC81 015803; Lalazissis PRC71 024312)
  - $\Gamma_i(x) = \Gamma_{i0} \, h_i(x), x = \rho / rho_0$
  - $h_i(x) = exp[-(x^{a_i} - 1)], i = \sigma, \omega, \quad h_\rho(x) = exp[-a_\rho (x - 1)]$
  - Parameters: n_0, Γ_{i0}, a_i, $\quad i = \sigma, \omega, \rho$ (Malik ApJ930 17)

- Bayesian estimation of model parameters

Spanning the full range of NS properties with a microscopic model

Malik ApJ930 17, Malik arxiv: 2301.08169

Constraints				
Quantity		Value/Band	Ref	DDB
NMP (MeV)	ρ_0	0.153 ± 0.005	Typel & Wolter (1999)	✓
	ϵ_0	-16.1 ± 0.2	Dutra et al. (2014)	✓
	K_0	230 ± 40	Todd-Rutel & Piekarewicz (2005); Shlomo et al. (2006)	✓
	$J_{sym,0}$	32.5 ± 1.8	Essick et al. (2021a)	✓
PNM (MeV fm^{-3})	$P(\rho)$	$2 \times \mathrm{N^3LO}$	Hebeler et al. (2013)	✓
NS mass ($M_\odot$)	M_{max}	>2.0	Fonseca et al. (2021)	✓

Model parameters

Defining the priors: uniform distribution

Nuclear matter parameters DDB

(Malik ApJ930 17)

- $n_0, \Gamma_{i0}, a_i, i = \sigma, \omega, \rho$

hyperon parameters:

- vector-isoscalar mesons: SU(6)
- $g_{mi} = x_{mi}\, g_{mN},\ m = \sigma, \Xi$
  fitted to hypernuclei (BE Λ, Ξ)
- Σ not included

No	Parameters	P min	P max
1	$\Gamma_{\sigma,0}$	6.5	13.5
2	$\Gamma_{\omega,0}$	7.5	14.5
3	$\Gamma_{\rho,0}$	2.5	8.0
4	a_σ	0.0	0.30
5	a_ω	0.0	0.30
6	a_ρ	0.0	1.30
7	$x_{\sigma\Lambda}$	0.609	0.622
8	$x_{\sigma\Xi^-}$	0.309	0.322

Nuclear matter parameters NL

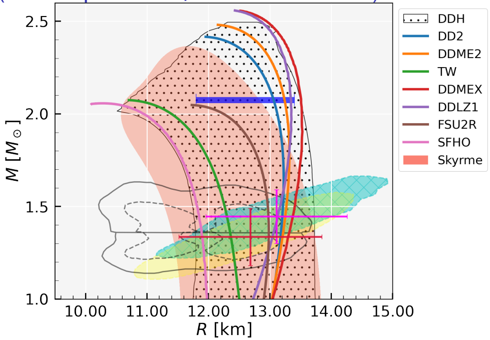
(Malik 2301.08169)

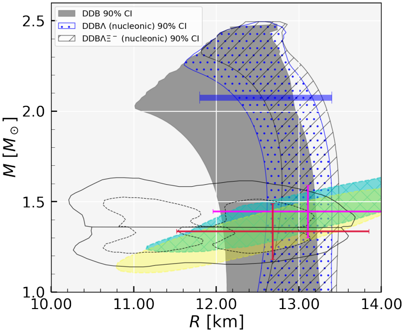
- $g_i (i = \sigma, \omega, \rho)$, b, c, ξ, Λ_ω
- *Set 0*: $\xi \in [0, 0.04]$
- *Set 1*: $\xi \in [0, 0.004]$
- *Set 2*: $\xi \in [0.004, 0.015]$
- *Set 3*: $\xi \in [0.015, 0.04]$

No	Parameters	*Set 0* min	*Set 0* max
1	g_σ	6.5	15.5
2	g_ω	6.5	15.5
3	g_ρ	6.5	16.5
4	$B = b \times 10^3$	0.5	9.0
5	$C = c \times 10^3$	-5.0	5.0
6	ξ	0.0	0.04
7	Λ_ω	0	0.12

Nucleonic RMF EOS (Bayesian Approach): how limitative is the method?

(Malik ApJ930 17; PRD106 063024)



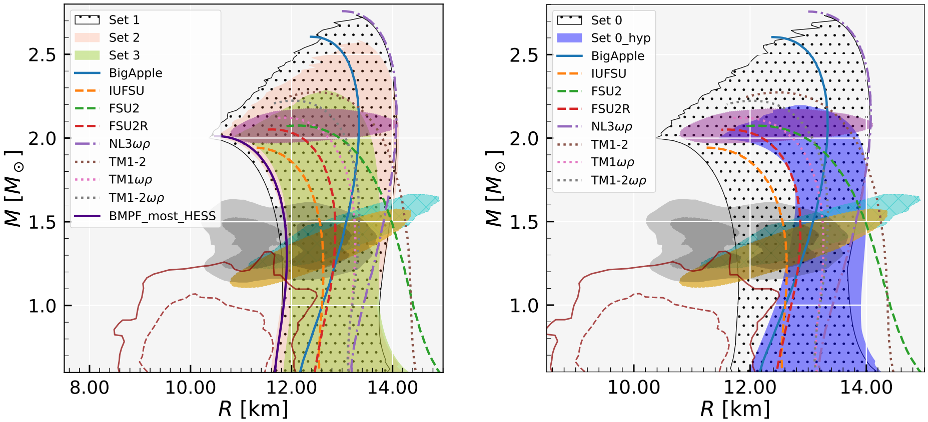


99% CI $P(R|M)$ joining DDB, DDBΛ, DDB$\Lambda\Xi$

90% CI $P(R|M)$

- Distributions for *pneμ* matter: no-hyperons, with Λ, with $\Lambda + \Xi$
- Constraints: $M \geq 2M_\odot$, χEFT, nuclear properties
- J0030+0451, J0740+6620 NICER data (Riley ApJL887L21, 918L27, MillerApJL887L24, 918L28)
- GW170817 LVC data (Abbott PRL121 161101)

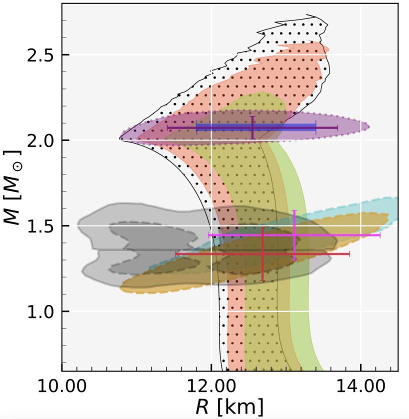
NS properties: full posterior NL

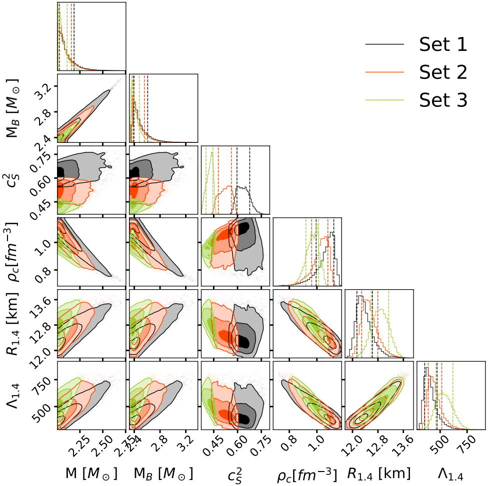


- **Observations:** GW170817, NICER J0740 and J0030, HESS
- **RMF models:** NL3$\omega\rho$,FSU2, FSU2R, IUFSU, BigApple, TM1-2($\omega\rho$)
- **Bayesian study Left**: Set 1 ($\xi < 0.004$), 2, 3 ($\xi > 0.015$)
- **Bayesian study Right**: Set 0 with and without hyperons

NS properties: RMF with non-linear meson terms

90% Conditional probability $P(R|M)$

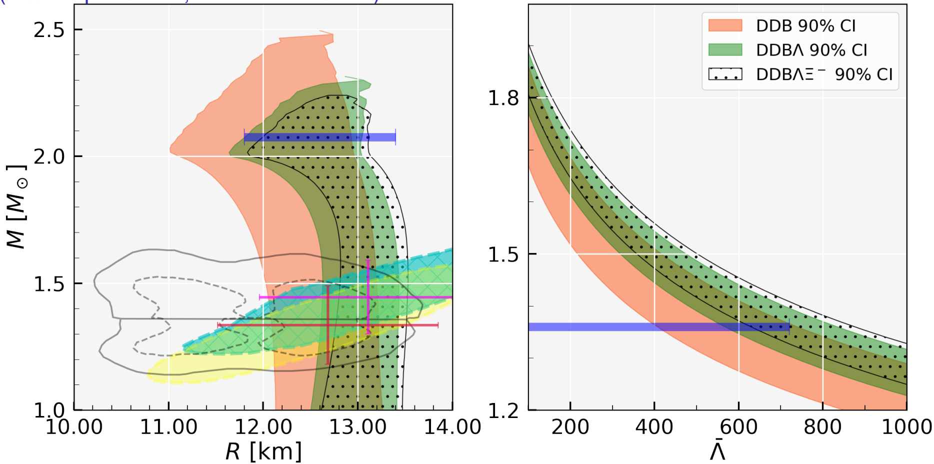




- High mass stars: the smaller ξ the larger M_{max}, R_{max}, c_s
- Canonical like mass stars: a larger ξ a larger radius ($R_{1.4}$)
- Current observations not constraining

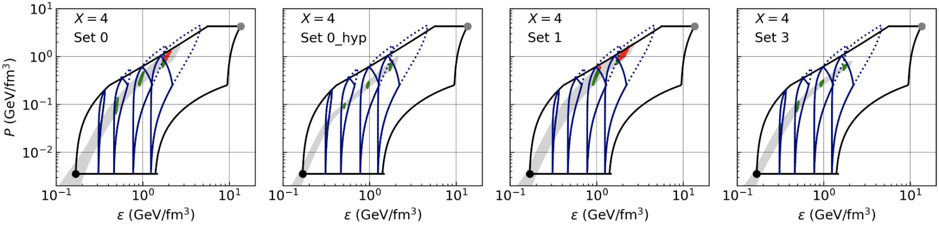
Nucleonic versus hyperonic EoS: density dependent

(Malik ApJ930 17; PRD106 063024)



- Hyperons couplings in DDBΛ snd DDB$\Lambda\Xi$: SU(6) for vector mesons, constrained by hypernuclei for σ-meson
- No hyperons: maximum mass $\approx 2.5 M_\odot$, $R_{1.4} \gtrsim$ 12km
- Hyperons: maximum mass $\approx 2.2 M_\odot$, $R_{1.4} >$ 12.5km

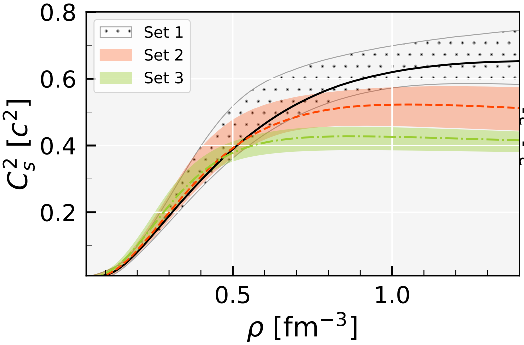
Does pQCD EoS impose extra constraints?

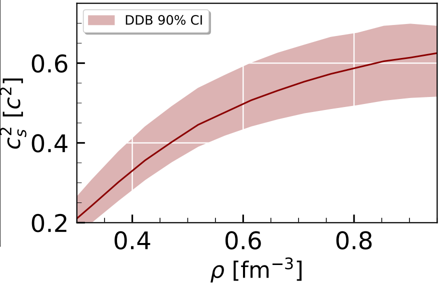


pQCD constraints of Komoltsev&Kurkela PRL128,202701

- stability, causality, and thermodynamic consistency.
- solid black lines: pQCD constraints in $\varepsilon - p$ domain
- solid blue lines: constraints at $n = 2, 3, 5, 8n_s$
- Excluded models: small ξ, large maximum mass, and large radii

Speed of sound





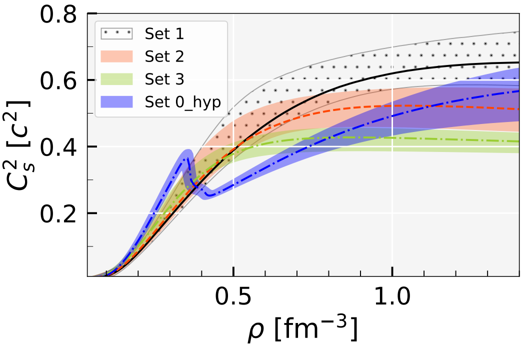
Set 1: $\xi < 0.004$

Set 2: $0.004 < \xi < 0.015$

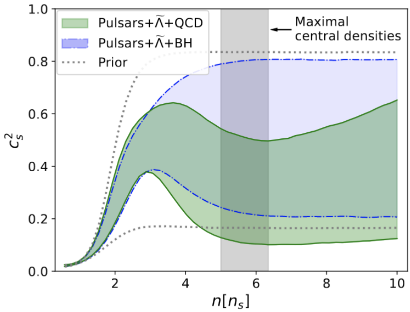
Set 3: $\xi > 0.015$,

- DDB:: c_s growing function of ρ
- NL: a large value of ξ moderates the c_s at large ρ

Speed of sound: hyperons



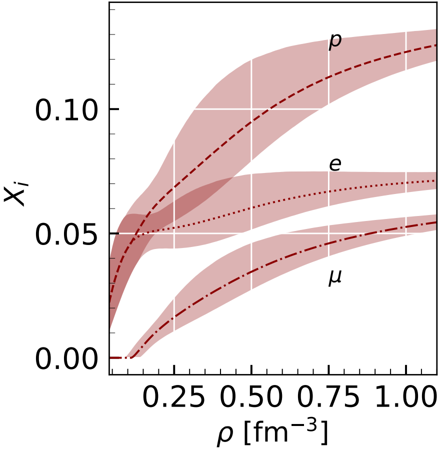
(Malik 2301.08169)



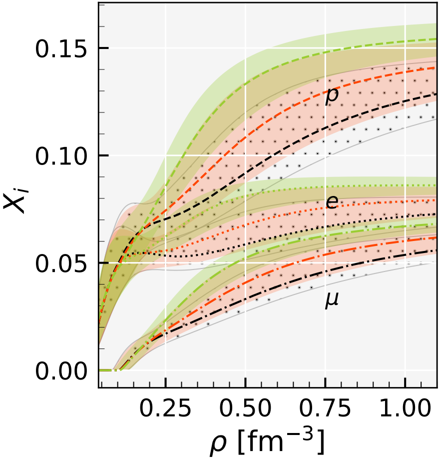
(Gorda arxiv:2204.11877)

- Hyperons: $2M_\odot$ requires a harder EOS, smaller ξ
- peak structure: onset of hyperons or large ξ
- Agnostic EoS with astro and pCD constraints: compatible but different behavior

Proton fraction



(Malik ApJ930 17)



(Malik 2301.08169)

- DDB: exponential decrease of $g_\rho \rightarrow$ no direct Urca
- NL: the larger ξ, the stiffer is the symmetry energy, favoring larger proton fractions.

Effect non-linear terms on meson fields

$$\sigma = \frac{g_\sigma}{m^2_{\sigma,\text{eff}}} \sum_i \rho_i^s \qquad m^2_{\sigma,\text{eff}} = m^2_\sigma + b g^3_\sigma \sigma + c g^4_\sigma \sigma^2$$

$$\omega = \frac{g_\omega}{m^2_{\omega,\text{eff}}} \sum_i \rho_i \qquad m^2_{\omega,\text{eff}} = m^2_\omega + \frac{\xi}{3!} g^4_\omega \omega^2 + 2\Lambda_\omega g^2_\rho g^2_\omega \rho^2$$

$$\rho = \frac{g_\rho}{m^2_{\rho,\text{eff}}} \sum_i I_3 \rho_i, \qquad m^2_{\rho,\text{eff}} = m^2_\rho + 2\Lambda_\omega g^2_\omega g^2_\rho \omega^2,$$

- $m_{\omega,\text{eff}}$ increases with $\omega \rightarrow$ at high densities $\omega \propto \rho^\alpha$, $\alpha < 1$, softening of the EOS.
- $m_{\rho,\text{eff}}$, increases with density $\rightarrow$ weaker ρ (softer symmetry energy).
  - but if $\xi \neq 0$, softening is smaller since the ω field does not grow so fast with the baryonic density.

Neutron Star EOS: Future

- Use future observations (radio, x-ray, gravitational waves) to constrain the EoS obtained within a microscopic model
- How can we get the NS composition? Include observations sensitive to composition
- Extend present approach to hybrid stars
- Learn from agnostic approaches: understand possible excluded regions due to nuclear matter constraints

Light clusters in neutron stars

- Clusters in Supernova and NS margers :
  - quantum statistical approach (QS):
    - quantum correlations with the medium, take into account the excited states and temperature effect.
    - Mass shifts available only for a few nuclear species and a limited density domains (RopkePRC92 054001)
    - Can be implemented with approximations (Typel et al 2010)
  - RMF approach with light clusters
    - considered as new degrees of freedom.
    - characterized by a density, and possibly temperature, dependent effective mass,
    - interact with the medium via meson couplings.
    - In-medium effects are incorporated via the meson couplings, the effective mass shift, or both.
  - Constraints (*ab-initio* calculations, experimental) are needed to fix the couplings!

Constraining light clusters

- Cluster formation has been measured in heavy ion collisions (Qin et al PRL 108 (2012), Hagel et al PRL108,062702): equilibrium constants, Mott points and medium cluster binding energies
- Cluster formation in Supernova EOS constrained with equilibrium constants from HIC was studied in (Hempel et al PRC91, 045805 (2015))
  - the SN EoS should incorporate: mean-field interactions of nucleons, inclusion of all relevant light clusters, and a suppression mechanism of clusters at high densities

EOS: including light clusters

(Pais PRC99 055806)

- Lagrangian density: $\mathscr{L} = \mathscr{L}_N + \mathscr{L}_c + \mathscr{L}_m + \mathscr{L}_e$
  - nucleons, tritium, helion

$$\mathscr{L}_j = \bar{\psi}_j \left[\gamma_\mu i D_j^\mu - M_j^* \right] \psi_j \qquad i = p, n, t, {}^3\mathrm{He}$$

  - alphas, deuterons

$$\begin{aligned}
\mathscr{L}_\alpha &= \frac{1}{2}(iD_\alpha^\mu \phi_\alpha)^*(iD_{\mu\alpha}\phi_\alpha) - \frac{1}{2}\phi_\alpha^* M_\alpha^2 \phi_\alpha, \\
\mathscr{L}_d &= \frac{1}{4}(iD_d^\mu \phi_d^\nu - iD_d^\nu \phi_d^\mu)^*(iD_{d\mu}\phi_{d\nu} - iD_{d\nu}\phi_{d\mu}) - \frac{1}{2}\phi_d^{\mu *} M_d^2 \phi_{d\mu},
\end{aligned}$$

$$\begin{aligned}
iD_j^\mu &= i\partial^\mu - g_{vj}\omega^\mu - \frac{g_{\rho j}}{2}\boldsymbol{\tau}\cdot\mathbf{b}^\mu, \\
M_j^* &= m^* = m - g_s\phi_0, \quad j = p, n \\
M_j^* &= M_j - g_{sj}\phi_0 - \mathbf{B}_j, \quad j = t, h, d, \alpha
\end{aligned}$$

- couplings: constrained by HIC data ou first principle calculations

Mass shift in clusters - g_{sj}

(Pais PRC99 055806)

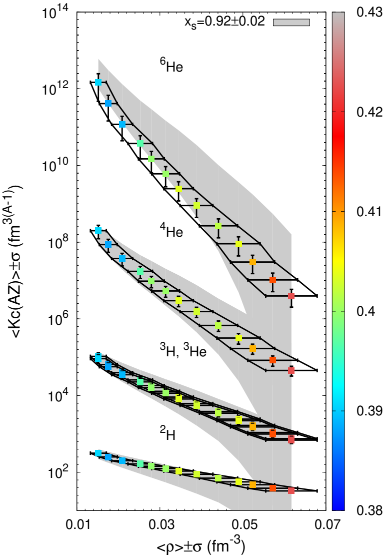
- Binding energy for each cluster: $B_j = A_j m^* - M_j^*$
- $m^* = m - g_s \phi_0$, nucleon effective mass
- $M_j^* = A_j m - g_{sj} \phi_0 - (B_j^0 + \delta B_j)$, cluster effective mass
- $g_{sj} = x_{sj} A_j g_s$, the cluster- scalar meson coupling
  - needs to be determined from experiments
- δB_j: the energy states occupied by the gas are excluded (double counting avoided!)

Equilibrium constants: model versus experiment

System ^{136}Xe+^{124}Sn (INDRA - GANIL), Bougault *et al* JPG 47 (2020) 025103

Chemical equilibrium constants :

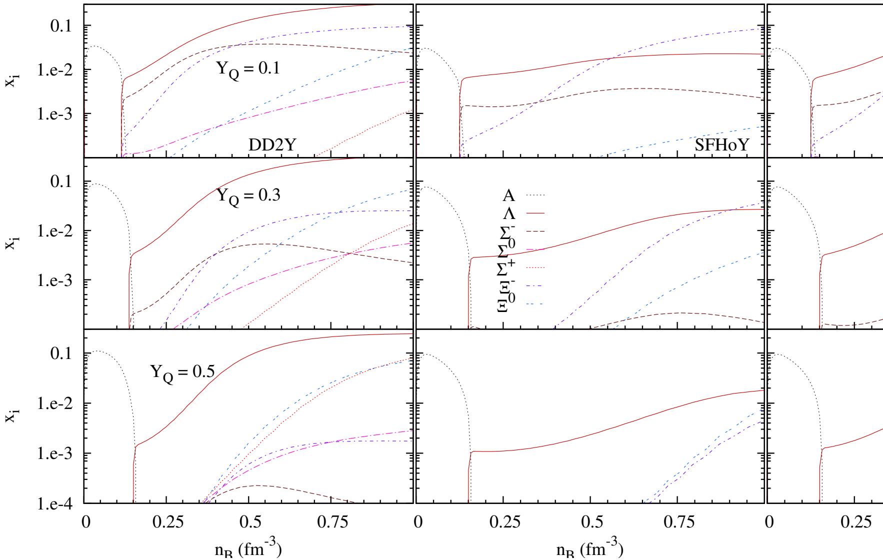
- $K_c[i] = \rho_i/(\rho_p^{Z_i}\rho_n^{N_i})$
- chemical equilibrium constants for homogeneous matter with five light clusters
- calculated at the average value of (T, $\rho_{\rm exp}$, $y_{pg,\rm exp}$)
- cluster-meson scalar coupling constants $g_{s_i} = x_{s_i} A_i g_s$, with $x_{s_i} = 0.92 \pm 0.02$
- global proton fraction: color code



(Pais PRL125 012701)

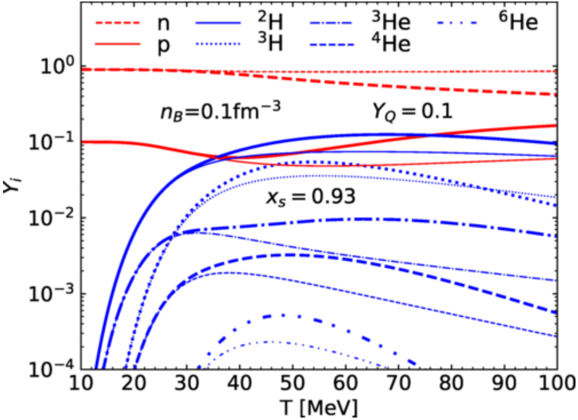
Hyperon fractions

T = 30 MeV, Y_Q = 0.1, 0.3, 0.5 (Fortin 1711.09427)



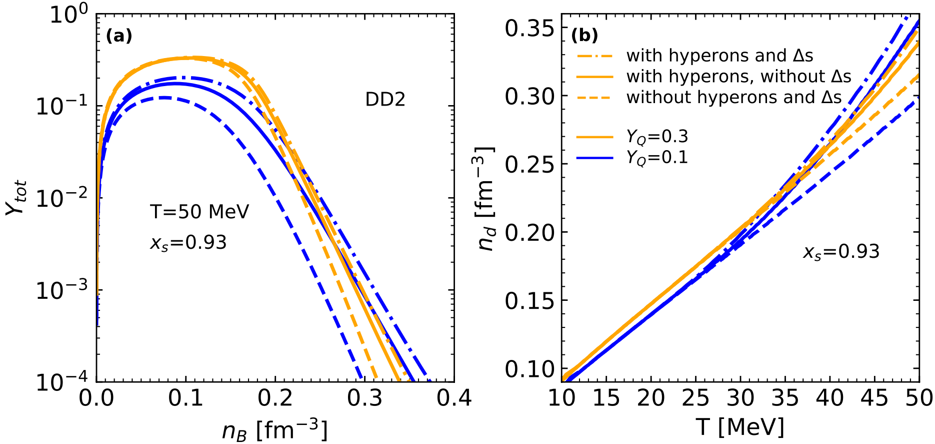
Hyperon effect on light cluster and unbound nucleons

Custodio PRC104 035801, PRC105 065803



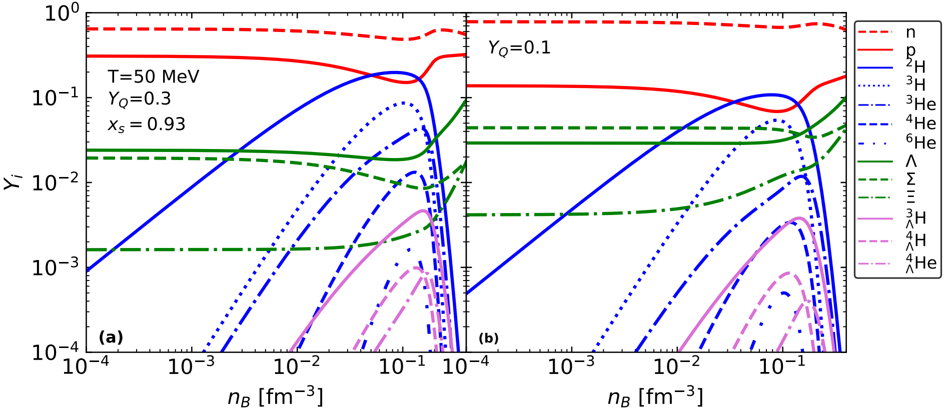
- Unbound nucleon and light cluster fractions with hyperons (thick lines) and without hyperons (thin lines)
- Hyperons: smaller fraction of unbound nucleons and larger fraction of light clusters
- Hyperons: the cluster dissolution density increases

Hyperon/deltas effect of light cluster abundances



- Total mass fraction of the light clusters versus density (T = 50 MeV): larger fractions in the presence of heavy baryons
- dissolution density of the clusters versus temperature: heavy baryons shift dissolution to larger densities

Light hyperclusters



- Mass fractions of the unbound protons and neutrons Λ, Σ and Ξ, light clusters and light hypernuclei
- Hypernuclei may be more abundant than α-particles or other heavier clusters, for small Y_Q

Hyperons/deltas and hypernuclei in NS

Custodio PRC104 035801, PRC105 065803

The presence of hyperons/deltas

- shifts the dissolution of clusters to larger densities
- increases the amount of clusters
- smaller charge fractions $Y_Q \to$ larger effects
- the dissolution of the less-abundant clusters occurs at larger densities due to smaller Pauli-blocking effects.
- hypernuclei set in at $T > 25$ MeV.
  - If Y_Q is small, hypernuclei may be more abundant than α-particles or other heavier clusters.

Pasta phases and light clusters

Pais et al PRC 91, 055801 2015

- Pasta phase in the Compressible Liquid Drop (CLD) approximation:
  - Minimization of total free energy density
  - pasta phases (f volume fraction) versus low density background nucleon gas $(1-f)$.
  - Minimization with respect to r_d, ρ_B^I, y_p^I, f
  - The Gibbs equilibrium conditions ($T = T^I = T^{II}$)

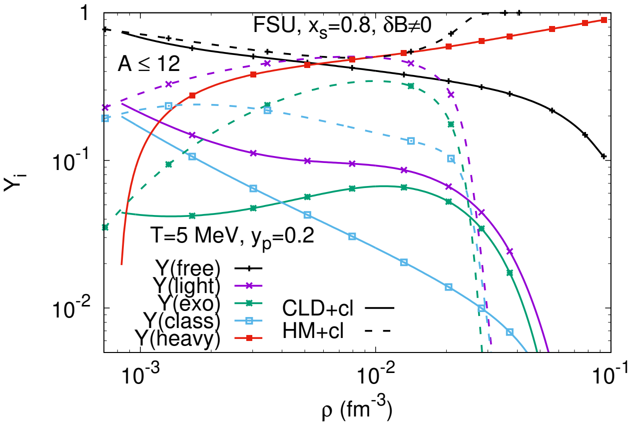
$$\mu_n^I = \mu_n^{II},$$
$$\mu_p^I = \mu_p^{II} - \frac{\varepsilon_{surf}}{f(1-f)(\rho_p^I - \rho_p^{II})},$$
$$P^I = P^{II} - \varepsilon_{surf}\left(\frac{1}{2\alpha} + \frac{1}{2\Phi}\frac{\partial\Phi}{\partial f} - \frac{\rho_p^{II}}{f(1-f)(\rho_p^I - \rho_p^{II})}\right).$$

- Total free energy density $\mathscr{F}$ and total ρ_p of the system:

$$\mathscr{F} = f\mathscr{F}^I + (1-f)\mathscr{F}^{II} + F_e + \varepsilon_{surf} + \varepsilon_{Coul},$$
$$\rho_p = \rho_e = y_p\rho = \rho_p^I + (1-f)\rho_p^{II},$$
$$\varepsilon_{surf} = 2\varepsilon_{Coul}$$

Cluster fractions: including pasta phases

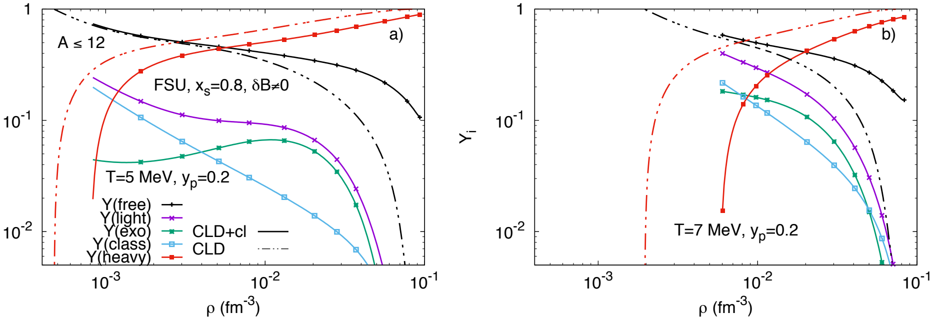
$T = 5$ MeV, $y_p = 0.2$ (Pais PRC99 055806)



- The heavy cluster (CLD+cl calculation): light clusters less abundant but increases their melting density
- Increasing T → onset of both heavy and light clusters moves to larger densities.

Single pasta versus Cluster fractions with pasta

$T = 5$ and 10 MeV, $y_p = 0.2$ (Pais PRC99 055806)



- inclusion of light clusters
  - moves the onset of the heavy cluster to larger densities
  - reduces the mass fraction of nucleons in the heavy clusters
  - increases the fraction of free nucleons in the background
  - if a too restrictive scenario concerning competing degrees of freedom is used: overestimation of the role of the heavy cluster

Future

- Inclusion of light clusters
  - Bayesian analysis that allows a better constraining of the cluster couplings taking into account new INDRA data (collaboration with Caen)
  - cluster dissolution: undertand until which temperatures the clusters and hyperclusters survive as individual structures
  - include the heavy clusters and heavy baryons self-consistently
  - build a EoS to be used in simulations

Thank you !