Muon(ium) g-2

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with Ohayon (ETHZ→Technion) and Soreq (Technion)

Muon magnetic moment

Muons (even resting ones) possess a magnetic moment sourced by their spin angular momentum

electric charge

$$\vec{\mu}_\mu = g_\mu \left(\frac{Q_\mu}{2m_\mu} \right) \vec{S}_\mu$$

mass

Landé g-factor

For elementary particles Dirac equation predicts

$$g = 2$$

Yet vacuum fluctuations induce a (small) correction

$$g_\mu = 2(1 + a_\mu)$$

magnetic moment 'anomaly'

Measuring anomalous magnetic moments

Polarized muon from P-violating weak pion decay

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

Spin precession around momentum in B field [Thomas 1927]

$$\vec{\omega}_a = \frac{Q_\mu}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma_m^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

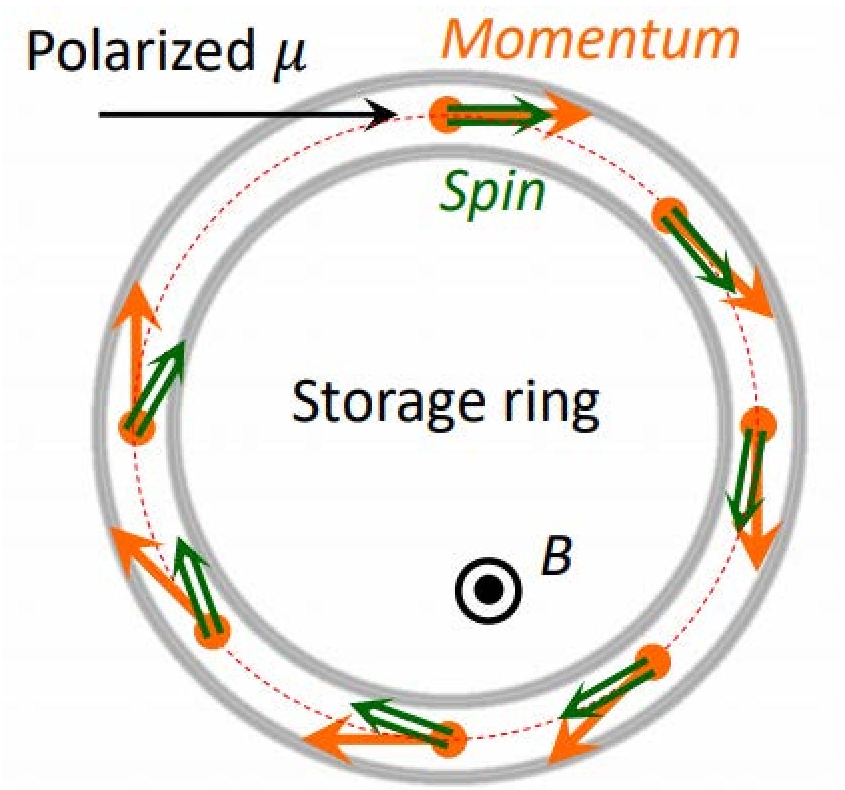
$$\simeq \frac{Q_\mu}{m_\mu} a_\mu \vec{B}$$

« magic » momentum $p_\mu \approx 3.09\,\mathrm{GeV}$

Electron from P-violating muon decay is a spin-analyzer

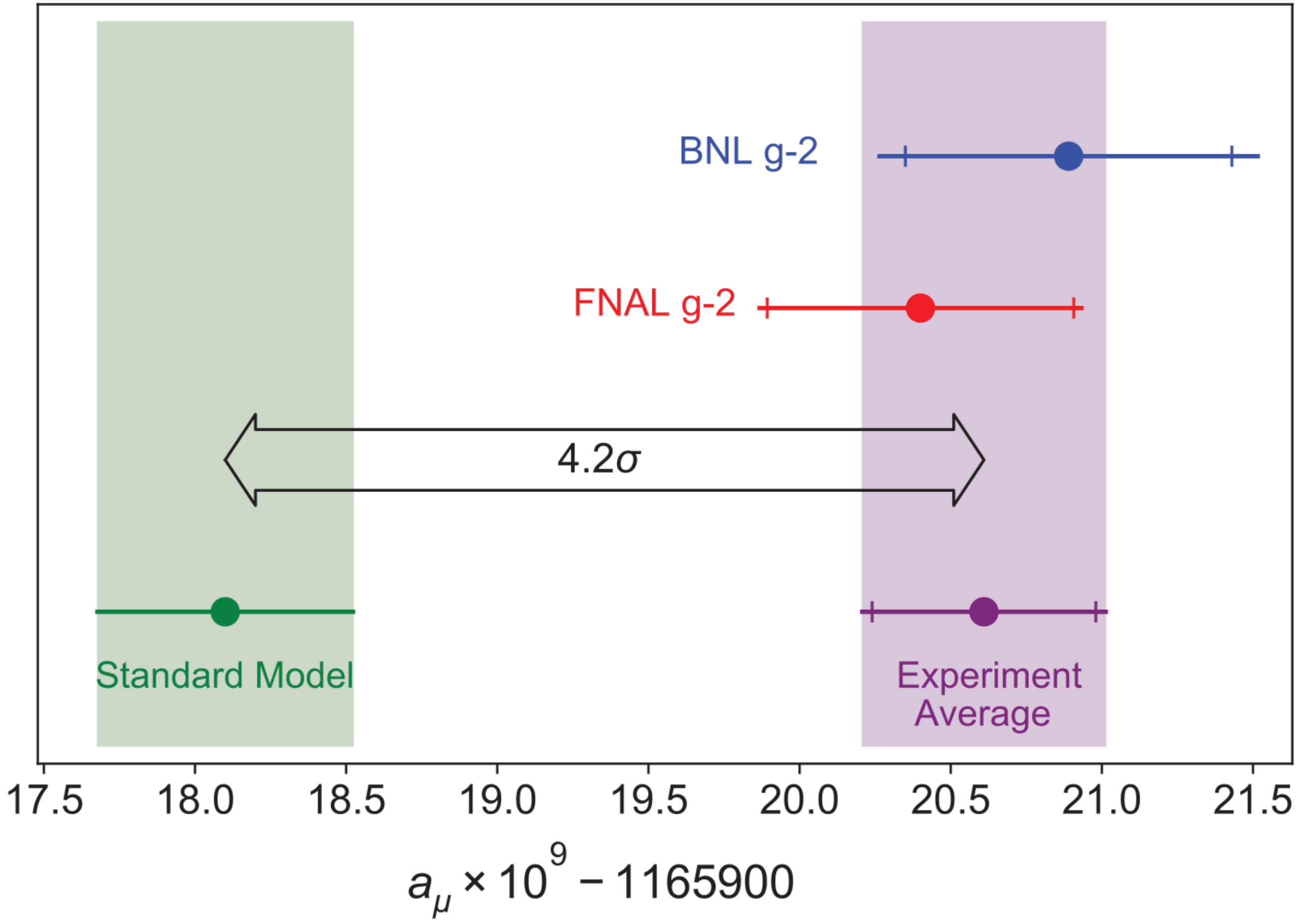
$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$$

boosted electron flies opposite
to the direction of muon spin



[Charpak+ 1962 → Bailey+ 1978] CERN
[Bennett+ 2006] BNL
[Abi+ 2021] FNAL

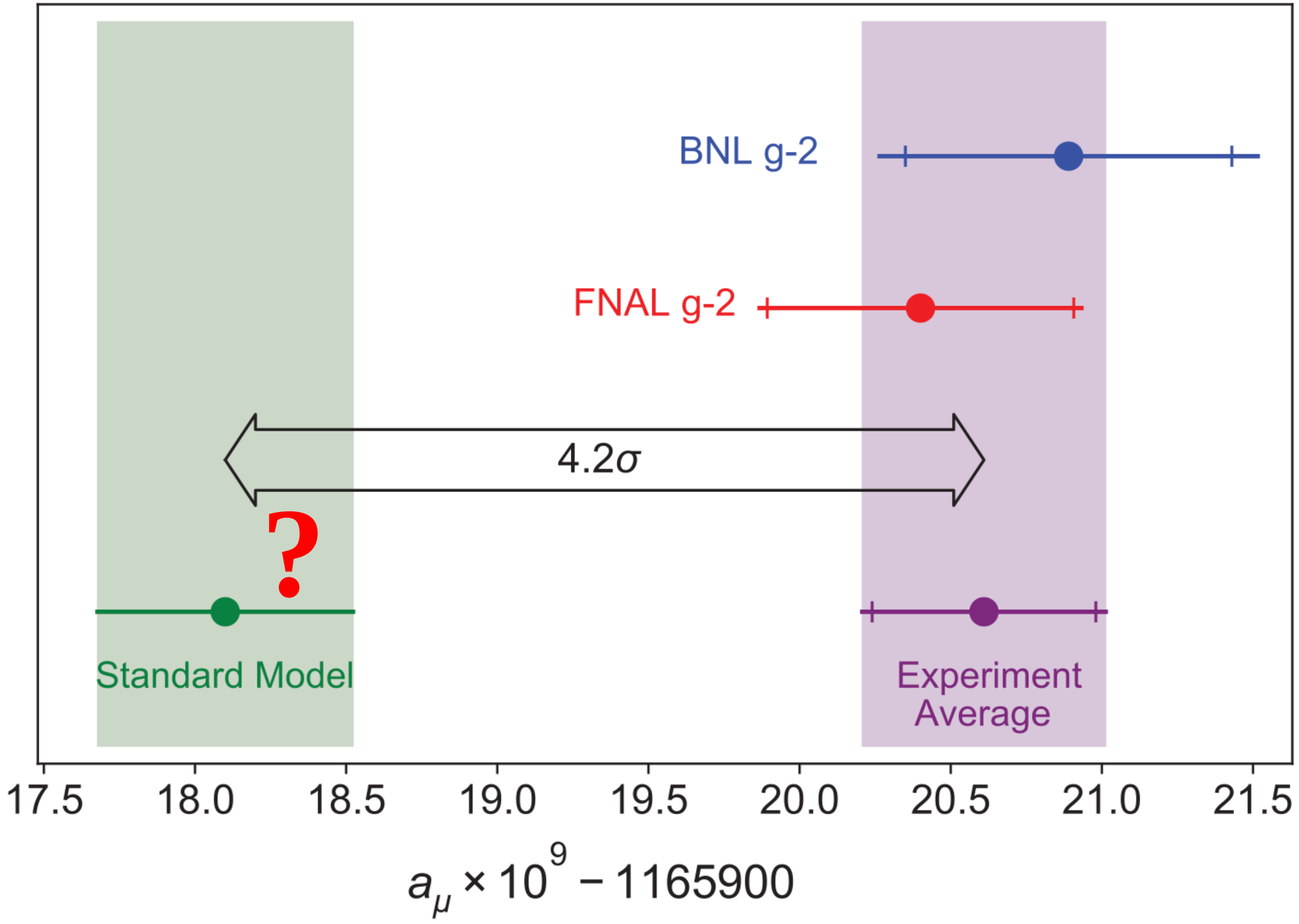
The Muon g-2 puzzle



Is this really an evidence of BSM Physics?

$$a_\mu^{\rm BSM} = 251(59) \times 10^{-11}$$

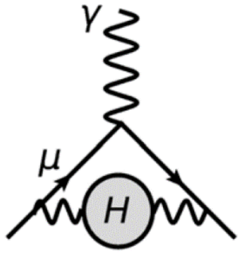
The Muon g-2 puzzle



Is this really an evidence of BSM Physics?

$$a_\mu^{\mathrm{BSM}} = 251(59) \times 10^{-11}$$

Do we really control the SM prediction?



R-ratio method:

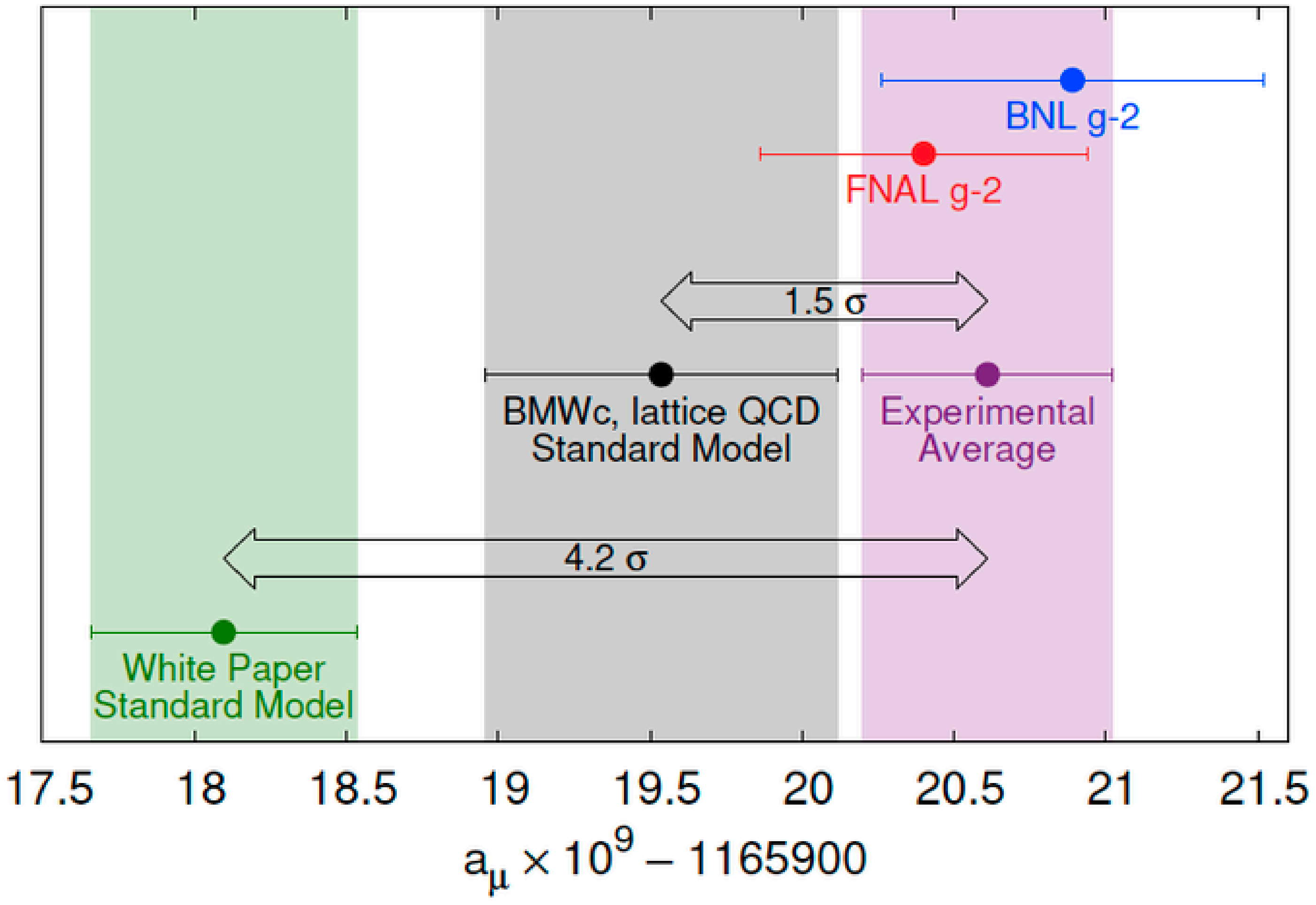
[Bouchiat-Michel 1961]

$$a_\mu^{\mathrm{HVP-LO}} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

$$R(s) = \frac{3s}{4\pi\alpha^2} \sigma(e^+e^- \to \mathrm{hadrons})$$

$\pi\pi \;\sim 70\%$

The Muon g-2 puzzle

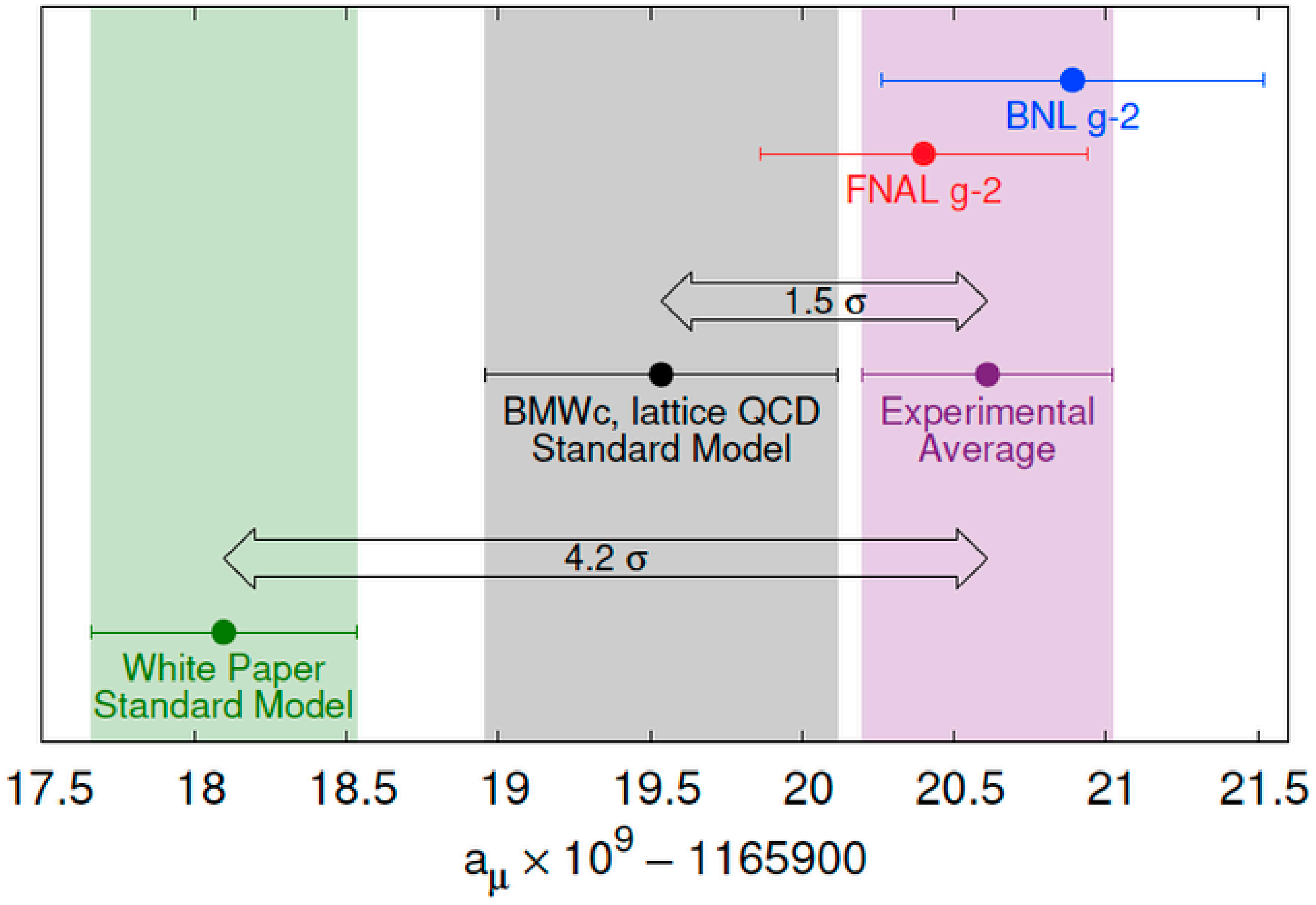


New lattice results cast doubts

[BMW coll. Nature 593 (2021) 7857]

$$a_\mu^{\mathrm{HVP-LO}} = 7075(55) \times 10^{-11}$$

The Muon g-2 puzzle

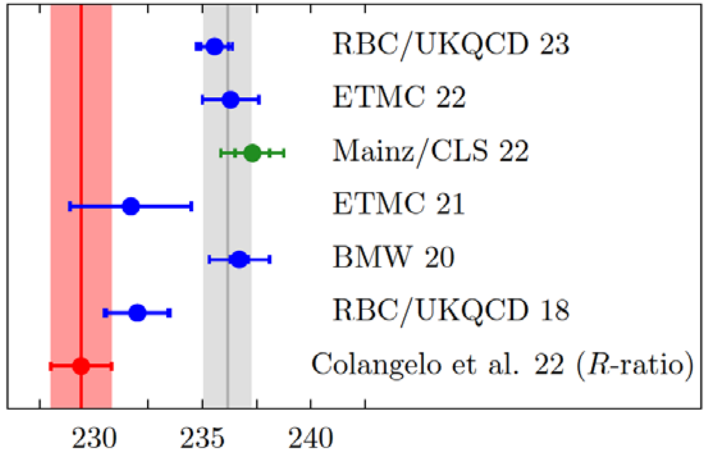


New lattice results cast doubts

[BMW coll. Nature 593 (2021) 7857]

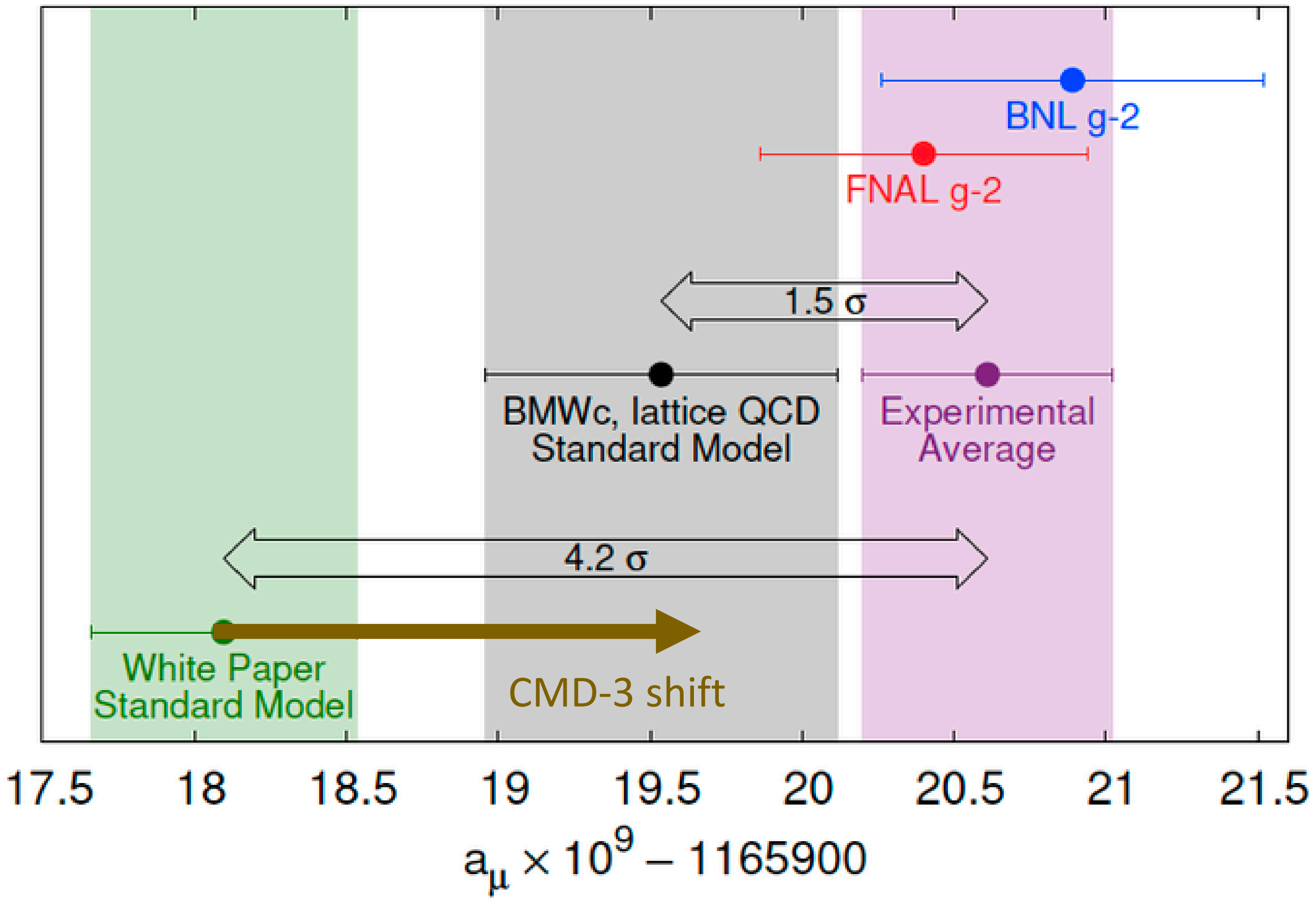
$$a_\mu^{\mathrm{HVP-LO}} = 7075(55) \times 10^{-11}$$

Corroborated by other lattice groups in the so-called *intermediate-window*: [see Witting's talk @MoriondEW2023]



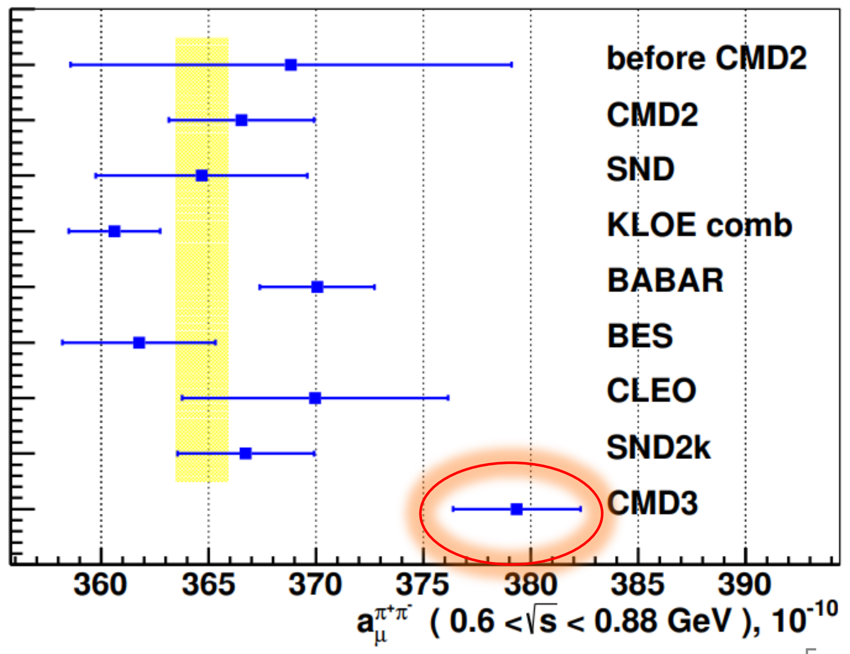
3.8σ

The Muon g-2 puzzle

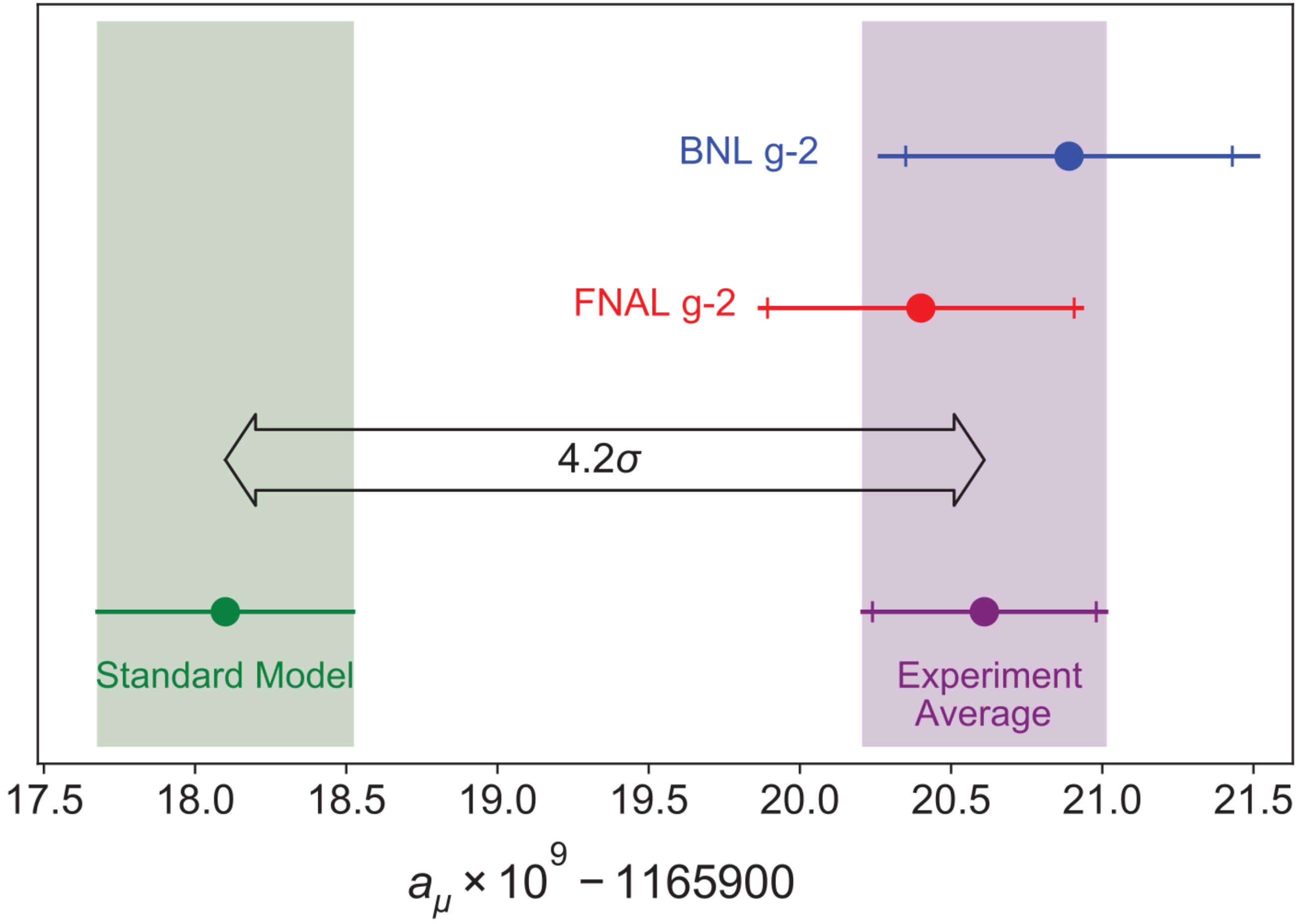


Recent $e^+e^- \to \pi\pi$ VEPP data also [CMD-3 coll. hep-ex/2302.08834]

$$a_\mu^{\mathrm{HVP-LO}}[\pi\pi] = 3793(30) \times 10^{-11}$$
$$(0.6 < \sqrt{s} < 0.9\,\mathrm{GeV})$$



Towards solving the puzzle



The jury is still out!

New experimental determinations of a_μ are more than welcome!

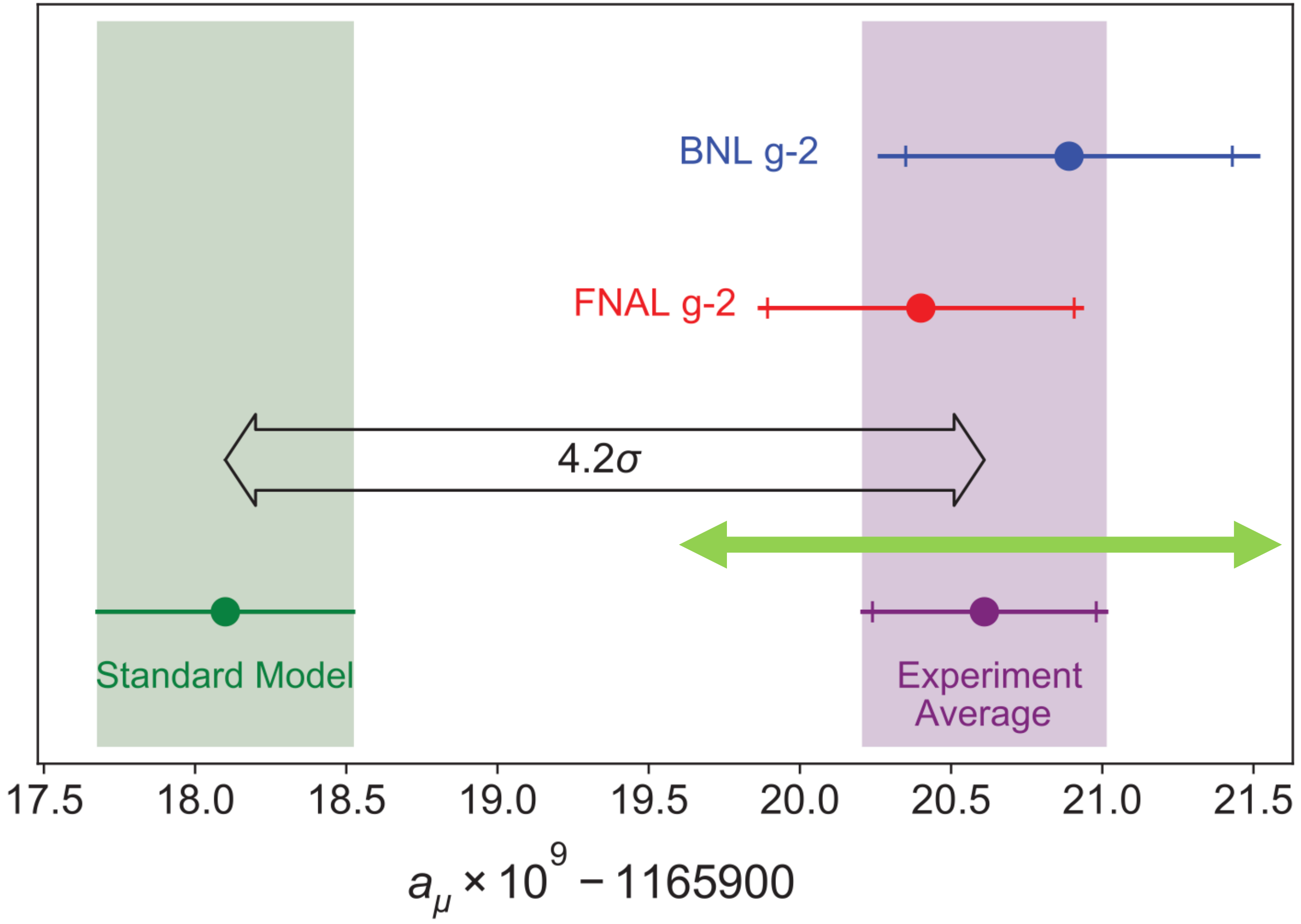
JPARC is coming up, but like BNL/FNAL it could be affected by « environmental » NP effects polluting the spin precession *e.g.*

[Davoudiasl-Szafron hep-ph/2210.14959]
[Agrawal et al. hep-ph/2210.17547]

MUonE will measure HVP directly, should be clean fromNP, see *e.g.*

[Masiero-Paradisi-Passera PRD 2020]

Towards solving the puzzle



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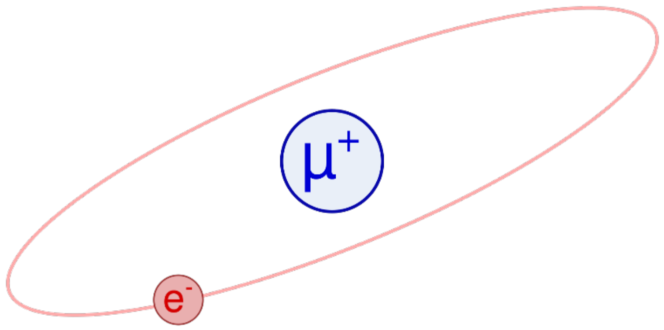
New experimental determinations of a_μ are more than welcome!

Muonium spectroscopy will provide another independent determination at 1ppm!

An alternative approach

extract a_μ from high-precision spectroscopy of muonic bound states



Muonium (Mu) = antimuon-electron bound state
(like an exotic hydrogen isotope)

pure leptonic atom controled by QED
(strong-interaction enters only through HVP, suppressed by m_e^2/m_μ^2)

unstable since muon decays $\rightarrow$ $\tau_{\mathrm{Mu}} \simeq 2.2\,\mu\mathrm{s}$

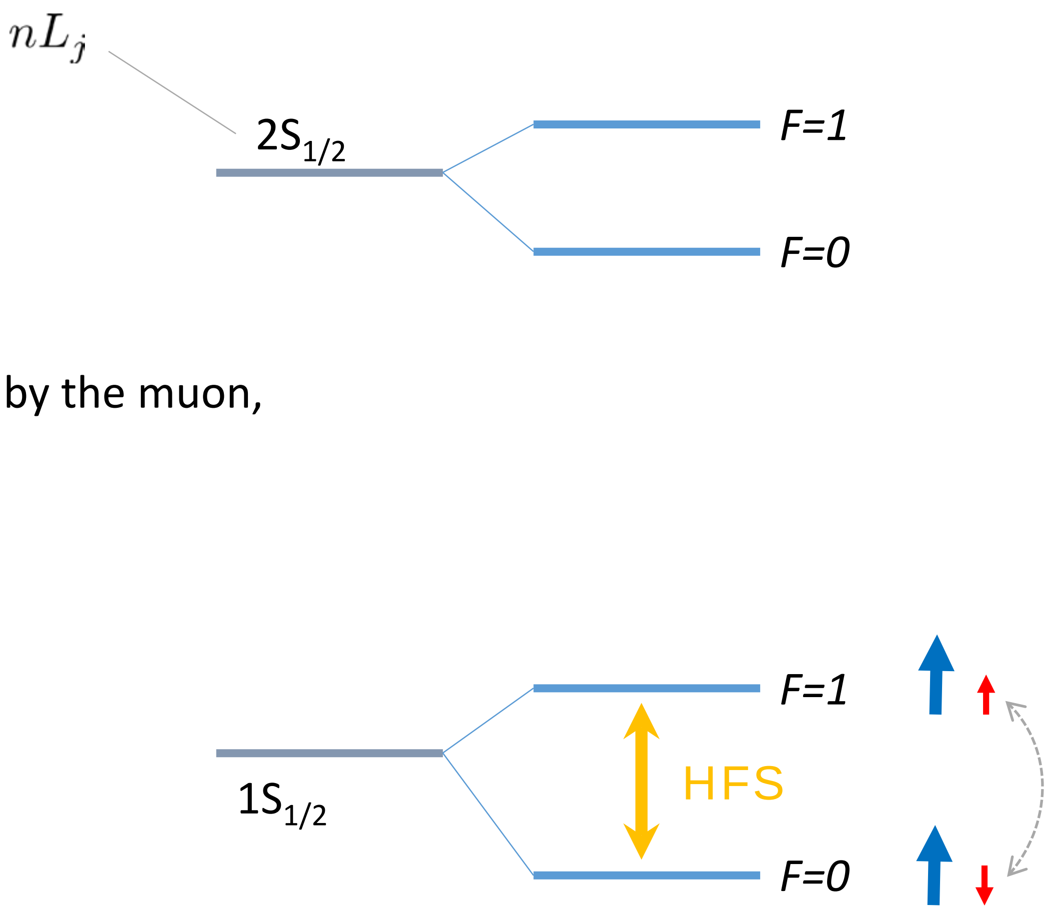
Other muonic bound states exists:
like muonic hydrogen/deuterium = muon-proton/deuteron bound states
but only Lamb shift 2S-2P is measured with precision (not very sensitive to a_μ)
and the theory prediction is plagued with large uncertainty from finite nuclear size corrections.

Muonium energy levels

full angular momentum:
$$\vec{F} = \vec{L} + \vec{S}_e + \vec{S}_\mu$$

Hyperfine splitting (HFS) for S levels
arises from magnetic dipole-dipole interaction

$$H_{\mathrm{HFS}} = -\frac{2\mu_0}{3}\,\vec{\mu}_e \cdot \vec{\mu}_\mu\,\delta^3(r)$$



The electron spin flips in the (static) magnetic field sourced by the muon, which lifts the degeneracy of the S state.

The 1S-HFS in muonium is very precisely measured:

$$\nu_{\mathrm{HFS}}^{\mathrm{exp}} = 4\,463\,302\,765(53)\,\mathrm{Hz}\ \ (12\mathrm{ppb})$$

[Liu et al. PRL 1999]

Ground-state HFS theory

Rydberg constant
$R_\infty \equiv \alpha^2 m_e c/(2h)$

fine-structure constant

nonrelativistic Fermi energy from $H_{\rm HFS}$

$$\nu_{\rm HFS} = \frac{16}{3}(1 + a_\mu)\frac{m_e}{m_\mu}\frac{R_\infty c\alpha^2}{(1+m_e/m_\mu)^3}[1 + \delta_{\rm HFS}]$$

electron-muon mass ratio

$\mathcal{O}(\alpha)$ correction

[CODATA 2018 + refs therein]

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electron-muon
mass ratio

$\mathcal{O}(\alpha)$ correction
[CODATA 2018 + refs therein]

Z-exchange
$-65\,{\rm Hz}$

$$\delta_{\rm HFS} = \delta_{\rm Dirac} + \delta_{\rm rad} + \delta_{\rm rec} + \delta_{\rm rad-rec} + \delta_{\rm weak} + \delta_{\rm had}$$

hadronic vacuum pol.
$= 237.7(1.5)\,{\rm Hz}$

relativistic
(exact)

radiative
known up to $\mathcal{O}(Z\alpha^4)$
including a_e

recoil
known up to
$\mathcal{O}[(m_e/m_\mu)(Z\alpha)^3]$
$\sim 60\,{\rm Hz}$ uncertainty

radiative-recoil
known up to $\mathcal{O}[(m_e/m_\mu)\alpha^3]$
$\sim 10\,{\rm Hz}$ uncertainty

Total TH uncertainty $\sim 70\,{\rm Hz}\,(16{\rm ppb})$
dominanted by (yet) uncalculated QED corrections at three-loop order

[Eides-Shelyuto IJMPA 2016]

antimuon charge
$Z = 1$

Ground-state HFS theory

$$\nu_{\mathrm{HFS}} = \frac{16}{3}(1+a_\mu)\frac{m_e}{m_\mu}\frac{R_\infty c\alpha^2}{(1+m_e/m_\mu)^3}[1+\delta_{\mathrm{HFS}}]$$

Rydberg constant $R_\infty \equiv \alpha^2 m_e c/(2h)$

fine-structure constant

nonrelativistic Fermi energy from H_{HFS}

Need to extract from another observable → electron-muon mass ratio

$\mathcal{O}(\alpha)$ correction

[CODATA 2018 + refs therein]

$$\delta_{\mathrm{HFS}} = \delta_{\mathrm{Dirac}} + \delta_{\mathrm{rad}} + \delta_{\mathrm{rec}} + \delta_{\mathrm{rad-rec}} + \delta_{\mathrm{weak}} + \delta_{\mathrm{had}}$$

Z-exchange $-65\,\mathrm{Hz}$

hadronic vacuum pol. $= 237.7(1.5)\,\mathrm{Hz}$

relativistic (exact)

antimuon charge $Z=1$

radiative known up to $\mathcal{O}(Z\alpha^4)$ including a_e

recoil known up to $\mathcal{O}[(m_e/m_\mu)(Z\alpha)^3]$

$\sim 60\,\mathrm{Hz}$ uncertainty

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[Eides-Shelyuto IJMPA 2016]

Alternative muon mass determination

To extract a_μ from muonium HFS, another observable is needed to fix the muon mass

The second best determination of m_e/m_μ
is provided by the muonium 1S-2S transition

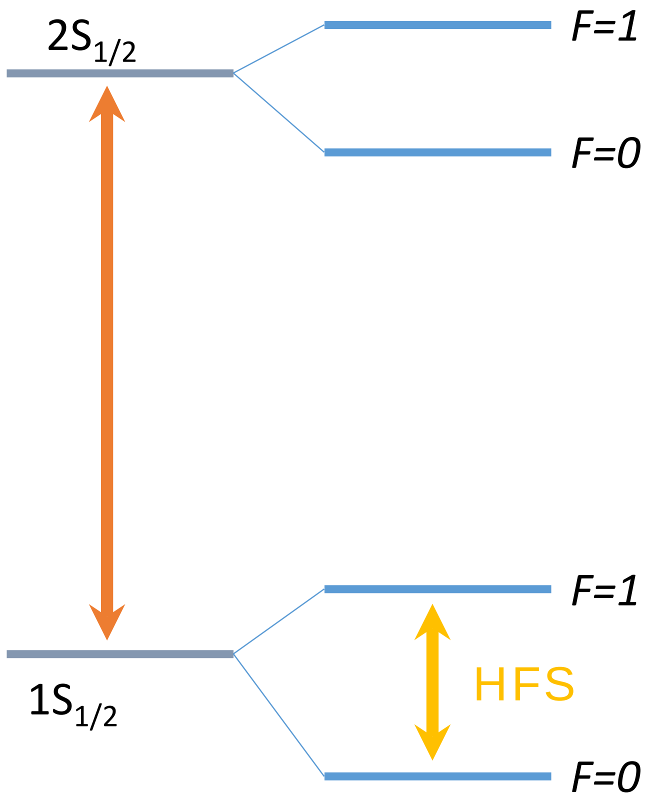
The muon mass enters as a recoil correction to all Mu energies through the reduced mass $m_r \equiv m_e m_\mu/(m_e + m_\mu)$

$$E_n^{\mathrm{Mu}} \simeq -\alpha^2 \frac{m_r c^2}{2n^2} = -\frac{R_\infty ch}{1+m_e/m_\mu}\frac{1}{n^2}$$

The 1S-2S is the best measured Mu transition between different n's:

$$\nu_{\mathrm{1S-2S}}^{\mathrm{exp}} = 2\,455\,528\,941.0(9.8)\ \mathrm{MHz}\ \ (4\mathrm{ppb})$$

[Meyer et al. PRL 2000]



1S-2S theory

nonrelativistic energy
(including recoil)

$\mathcal{O}(\alpha^2)$ correction

$$\nu_{1S-2S} = \frac{3}{4}\frac{R_\infty c}{(1+m_e/m_\mu)}[1+\delta_{1S-2S}]$$

[CODATA 2018 + refs therein]
rescaling hydrogen formulae
with the muon mass and removing
nuclear finite size and pol. effects

1S-2S theory

nonrelativistic energy (including recoil)

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$$\nu_{1S-2S} = \frac{3}{4}\frac{R_\infty c}{(1+m_e/m_\mu)}[1+\delta_{1S-2S}]$$

[CODATA 2018 + refs therein]

rescaling hydrogen formulae with the muon mass and removing nuclear finite size and pol. effects

vacuum pol. known up to $\mathcal{O}[\alpha(Z\alpha)^4]$

2+3 photon exchange known up to $\mathcal{O}[\alpha^3(Z\alpha)^4]$

muon self-E

$$\delta_{1S-2S} = \delta_{\rm Dirac} + \delta_{\rm rel-rec} + \delta_{e\rm SE} + \delta_{\rm VP} + \delta_{2\gamma} + \delta_{3\gamma} + \delta_{\rm rad-rec} + \delta_{\mu\rm SE}$$

relativistic (exact)

relativistic-recoil known up to $\mathcal{O}[(m_e/m_\mu)(Z\alpha)^4]$

electron self-E $\mathcal{O}[\alpha(Z\alpha)^4]$ known

radiative-recoil known up to $\mathcal{O}[(m_e/m_\mu)\alpha(Z\alpha)^3]$

Total TH uncertainty $\sim 20\,\mathrm{kHz}\,(8\mathrm{ppt})$
from (yet) uncalculated QED (rad-rec) corrections at three-loop order

[Karshenboim et al. PLB 2019]

Least-square adjustment of muonium data

Following the CODATA procedure [see CODATA 1998] we construct a least-square fit of the Mu HFS and 1S-2S transitions to extract *both* m_e/m_μ and a_μ from spectroscopy

input datum	value	relative uncertainty	identification	reference
ν_{1S-2S}	2 455 528 941.0(9.8) MHz	4.0×10^{-9}	RAL-99	[40]
ν_{HFS}	4 463 302 776(51) Hz	1.2×10^{-8}	LAMPF-99	[38]
ν_{HFS}	4 463 302.88(16)kHz	3.6×10^{-8}	LAMPF-82	[55]
$\delta E(1S)/h$	0.000(14) MHz	4.3×10^{-12}	theory	[43]
$\delta E(2S)/h$	0.0(1.8) kHz	2.2×10^{-12}	theory	[43]
$\delta E(HFS)/h$	0.000(70) kHz	1.6×10^{-8}	theory	[52]

input datum	observational equation
ν_{1S-2S}	$\nu_{1S-2S} \doteq [E_M(2S; m_e/m_\mu) + \delta^{th}_{2S} - E_M(1S; m_e/m_\mu) - \delta^{th}_{1S}]/h$
ν_{HFS}	$\nu_{HFS} \doteq \nu^{th}_{HFS}(m_e/m_\mu, a_\mu) + \delta^{th}_{HFS}/h$
$\delta E(1S)/h$	$\delta E(1S) \doteq \delta^{th}_{1S}$
$\delta E(2S)/h$	$\delta E(2S) \doteq \delta^{th}_{2S}$
$\delta E(HFS)/h$	$\delta E(HFS) \doteq \delta^{th}_{HFS}$

Using CODATA 2018 recommended values for R_∞ and α, current Mu data yield:

$$m_e/m_\mu = 4\,836\,329(4) \times 10^{-9}$$

$$a_\mu^{Mu} = 116\,637(82) \times 10^{-8} (700\text{ppm})$$

← very large uncertainty (Muon g-2 coll. result is ~ 0.35ppm) dominated by the 1S-2S measurement uncertainty

larger value than Muon g-2 coll. result

$a_\mu^{Mu} - a_\mu^{exp} \simeq 4.5 \times 10^{-7}$ but consistent w/in uncertainties

However, significant improvements in muonium spectroscopy expected!

Big improvements coming up!

The Mu-MASS experiment at PSI plans to reduce the 1S-2S uncertainty to

[Crivelli Hyperfine Interact. 2018] $\sim 10\,\mathrm{kHz}(4\mathrm{ppt})$

$\sim 10^3$ improvement!

This could be further reduced to $\sim \mathrm{few\,kHz}$ after the High-Intensity Muon Beam upgrade at PSI

[Kiselev et al. J-PARC symbosium 2019]

The MuSEUM experiment using a high-intensity pulsed muon beam at J-PARC will reduce the HFS uncertainty to

[Tanaka et al. 2021] $\sim 10\,\mathrm{Hz}(2.2\mathrm{ppb})$

~ 10 improvement!

The linewidth can be reduced by selecting the « old muonium » tail (if statistics is high enough) which could bring down the HFS uncertainty to

$\sim 4\,\mathrm{Hz}(1\mathrm{ppb})$

Theory is expected to also improve with a complete calculation of the 3-loop contribution in bound-state QED [Eides 2018]

Projected a_μ uncertainty from muonium

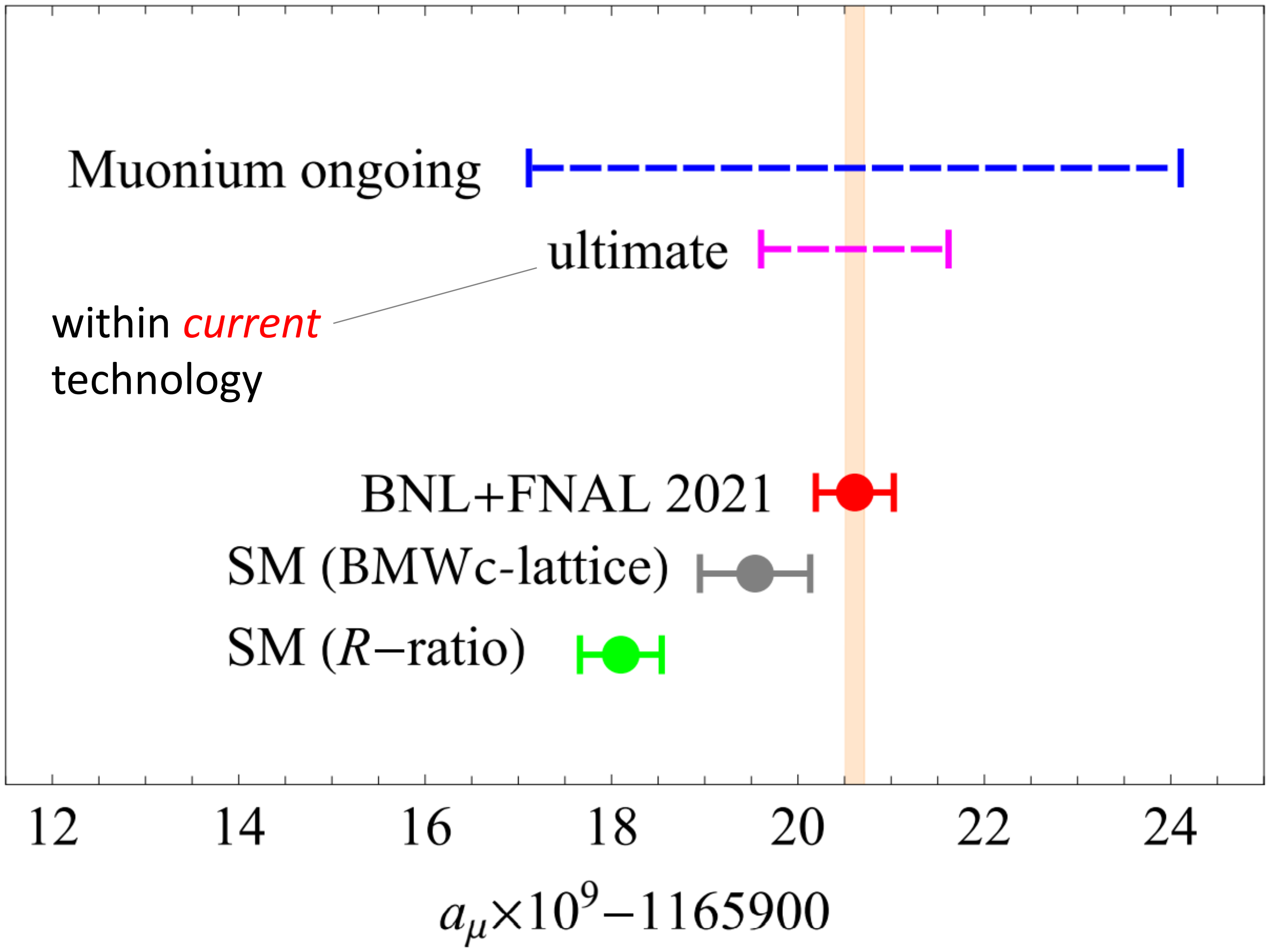
parameter (unit)	quantity	u_r current	u_r ongoing	u_r ultimate
m_e/m_μ (ppb)	ν_{1S-2S}(exp)	825	0.84	0.34
	QED(1S-2S)	1.7	1.2	0.1
	R_∞	0.40	0.13	
	total	825	1.5	0.37
a_μ (ppm)	ν_{1S-2S}(exp)	708	0.73	0.29
	ν_{HFS}(exp)	10	1.9	0.77
	QED(1S-2S)	1.4	1.0	0.07
	QED(HFS)	14	1.9	0.2
	HVP(HFS)	0.29	0.16	
	R_∞	0.35	0.13	
	α	0.26	0.14	
	total	708	3.0	0.88

rather conservative
based on planned experiments

$\mathcal{O}(1\text{ppm})$
assuming plausible
future improvements

with official goals
of Mu-MASS/MuSEUM

Shedding light on Muon g-2 puzzle



A value of a_μ^{Mu} at $\mathcal{O}(1\mathrm{ppm})$ is not competitive to current spin-precession measurements

However, it may help to understand the origin of the $\sim 2\mathrm{ppm}$ difference between (R-ratio) SM and experiment

New physics contamination

The extraction of a_μ^{Mu} from spectroscopy is indirect
since it assumes that muonium theory follows QED.

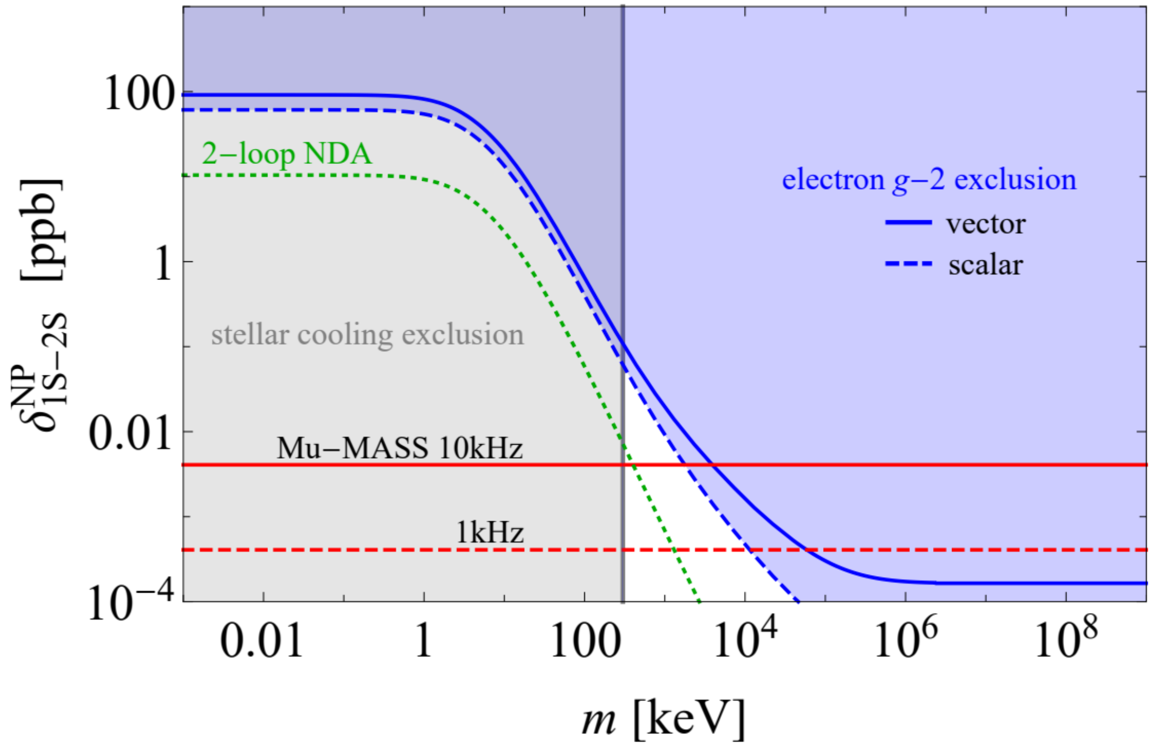
As the current puzzle may be caused by the existence of new physics,

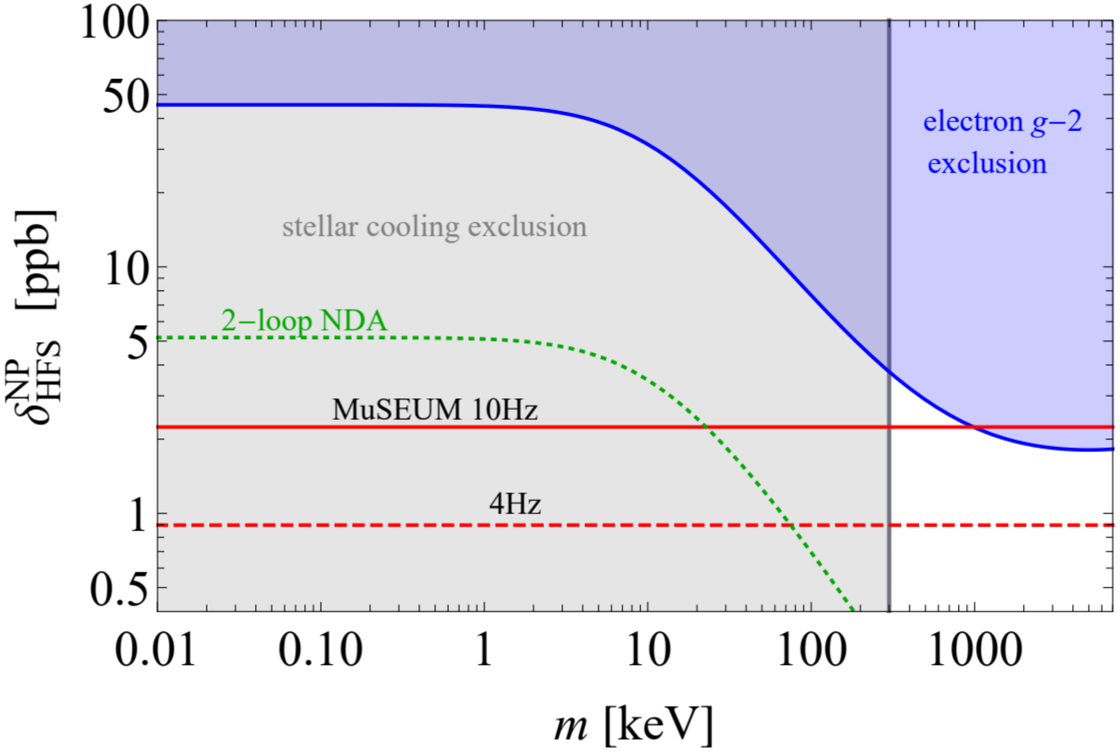
could it *contaminate* the muonium lines used to extract a_μ ?

We addressed the question by assuming
the existence of the new boson (scalar or vector)
with a muon-coupling that resolves the muon g-2 puzzle
and a free coupling to electrons

If NP *only* to muons, muonium theory is *unchanged*.
An additional coupling to electrons is *constrained* by
the th/exp agreement for electron g-2,
and astrophysics from stellar cooling.

Maximal NP effects in muonium





Except in a small range of NP mass around $\sim 1\text{MeV}$
NP effects are sufficiently constrainted
to be below the expected Mu-MASS/MuSEUM uncertainty.

Conclusions

Muonium will provide an independent value of a_μ at $\sim 1\,\mathrm{ppm}$ within few years, thanks to

- improved measurements coming up (Mu-MASS@PSI|MuSEUM@JPARC)
- completing the 3-loop QED calculation in Mu (underway)
- mildly reducing uncertainty of the Rydberg constant R_∞ (already available)

Can it be pushed *further* and surpass precession determination?
(what if we have a new intense source of muons?)

backups

Improving 1S-2S measurement

The 1S-2S is a two-photon transition ($\Delta L = 0$) with low excitation efficiency.

To increase the transition probability, a high-power pulsed laser was used in previous experiments. The price to pay was a broadening of the linewidth from $\simeq 145\,\mathrm{kHz}$ (muon lifetime) to $\sim 20\,\mathrm{MHz}$ and an extra $\sim 10\,\mathrm{MHz}$ systematic uncertainty from « chirping »

The Mu-MASS experiment at PSI proposed to circumvent this limitation by using cavity-enhanced continuous-wave excitation, together with an intense low-energy muon beam, thus planning to reduce the 1S-2S uncertainty to $\sim 10\,\mathrm{kHz}(4\mathrm{ppt})$ [Crivelli Hyperfine Interact. 2018]

This could be further reduced to $\sim \mathrm{few\ kHz}$ after the High-Intensity Muon Beam upgrade at PSI

[Kiselev et al. J-PARC symbosium 2019]

Improving HFS measurement

The a_μ^{Mu} uncertainty can be further reduced by improving the HFS measurement

Previous measurements at LAMPF were statistics limited.
The MuSEUM experiment using a high-intensity pulsed muon beam at J-PARC
is expected to bring down the statistics uncertainty to $\sim 10\,\mathrm{Hz}(2.2\mathrm{ppb})$ [Tanaka et al. 2021]

A reduction of systematics is also needed at this level of uncertainty.
The dominant one is due to pressure shift from the finite gas density in the experiment. [Kanda et al. 2021]
which could be reduced by measuring the HFS in vacuum or in a gas admixture with opposite shifts.

Further improvements are *very* challenging.
A $10\,\mathrm{Hz}$ uncertainty already requires resolving the line to 10^{-4} of the linewidth (from muon decay),
only done once in spectroscopy: the 2S-4P transition in hydrogen [Beyer et al. Science 2017]

The linewidth can be reduced by selecting the « old muonium » tail (if statistics is high enough)
which could bring down the HFS uncertainty to $\sim 4\,\mathrm{Hz}(1\mathrm{ppb})$

Improving 1S-2S theory

Once experimental uncertainty is down to $\sim \mathrm{few\,kHz}$, the theory must be improved by a factor ~ 10

The main theory uncertainty comes from the uncalculated radiative-recoil terms at three-loop QED of $\mathcal{O}[(m_e/m_\mu)\alpha(Z\alpha)^6]$

There is extra insentive to calculate them:
Once the proton radius puzzle is fully resolved,
such terms will become the limiting factor
to further improvements of R_∞ in hydrogen.

Subleading uncertainty from uncalculated recoil terms of $\mathcal{O}[(m_e/m_\mu)^2(Z\alpha)^6]$ at three-loop QED should also be reduced.

R_∞ should also improve by a factor few.
The QED uncertainty in hydrogen was recently reduced to $\sim 1\,\mathrm{kHz}$ [Karshenboim et al. PLB 2019]
meaning that a three-fold improvement
is already possible relative to CODATA 2018.

All of the above would then allow to determine the electron-muon mass ratio to $\sim 0.37\mathrm{ppb}$
thus making it a subleading source of uncertainty for a_μ^{Mu}

Improving HFS theory

The HFS theory should improve in the meantime by a factor ~ 20.

To this level the uncertainty is only limited by uncalculated terms in QED.
(The HVP uncertainty is $\sim 1\,\mathrm{Hz}$, still subdominant.)

The required QED calculation is currently being done, with a goal of $\sim \mathrm{few\,Hz}$. [Eides 2018]
This is motivated by the upcoming MuSEUM measurement,
aiming at a reduced uncertainty of m_e/m_μ and thus of a_μ in future Fermilab/J-PARC runs.