IRN Terascale

Quantum information in Higgs to tau tau at future lepton colliders

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Mohammad Mahdi Altakach

In collaboration with: P. Lamba, F. Maltoni, K. Mawatari and K. Sakurai

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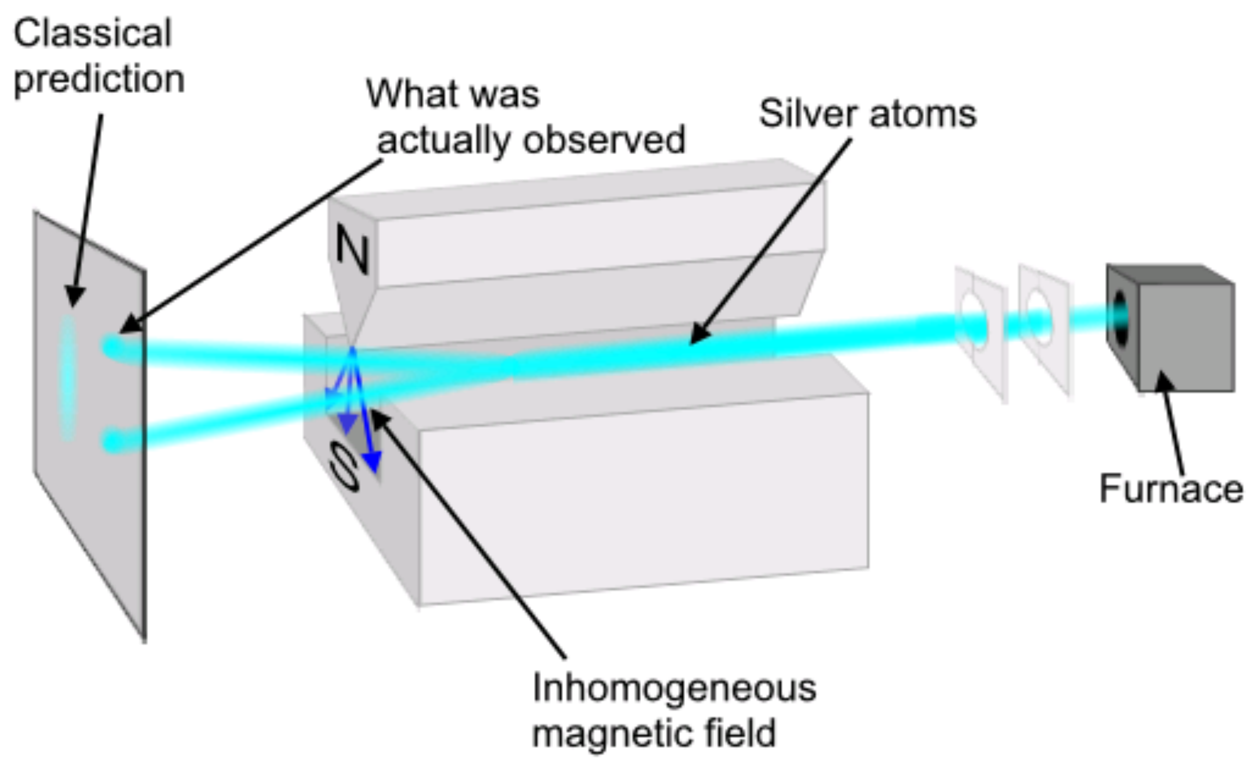
Part I

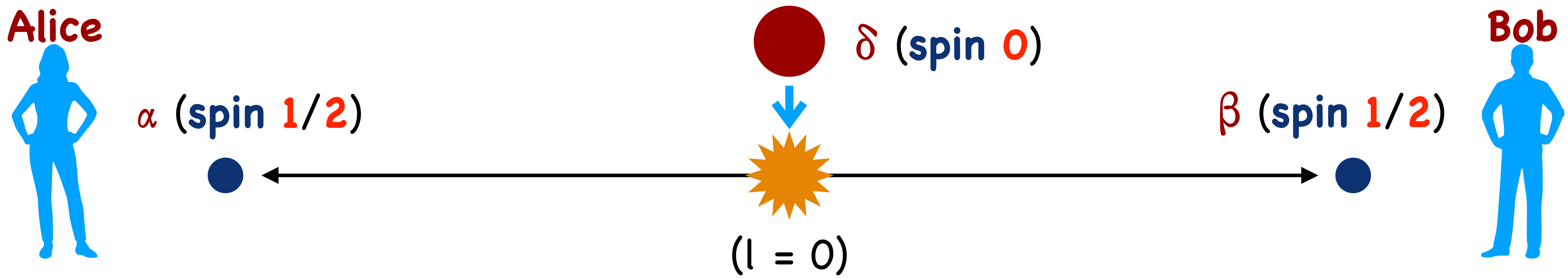
Introduction

Spin

- In **classical mechanics**, the components of **angular momentum** (l_x, l_y, l_z) take continuous real numbers.
- A striking fact, found in the **Stern-Gerlach** experiment, is that the measurement outcome of **spin** component is either **+1** or **-1** (in the $\hbar/2$ unit).



Alice & Bob



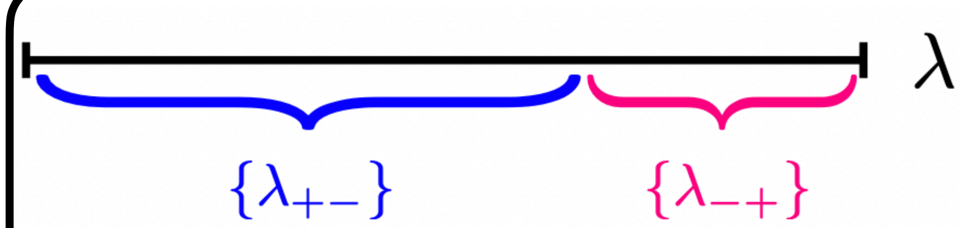
- **Alice** and **Bob** receive particles α and β, respectively, and measure the **spin z-component** of their particles. Repeat the process many times.
- **Alice** and **Bob** will find their results are completely random (**+1** and **-1**, **50**-**50**%).
- Nevertheless, their result is **100**% anti-correlated due to the **angular momentum** conservation. If **Alice's** result is **+1**, **Bob**'s result is always **-1** and vice versa.

Alice	+	+	-	+	-	-	+	+	+	-	+
Bob	-	-	+	-	+	+	-	-	-	+	-
$S_z^\alpha . S_z^\beta$	-	-	-	-	-	-	-	-	-	-	-

$$< S_z^\alpha . S_z^\beta > = -1$$

Hidden variable theory

- The most natural explanation would be as follows:
  - Since their result is sometimes **+1** and sometimes **-1**, it is natural to think that the **states** of α and β are different in each decay. The result look random, since we don't know in which sate the α and β particles are in each decay.
  - This means we can parametrise the **state** of α and β by a set of unknown (**hidden**) variables, λ.
  - For **i-th** decay, their states are: $\alpha(\lambda_i)$, $\beta(\lambda_i)$.



If $\lambda_i \in \{\lambda_{+-}\} \implies S_z[\alpha(\lambda_i)] = +1, \;\; S_z[\beta(\lambda_i)] = -1$

If $\lambda_i \in \{\lambda_{-+}\} \implies S_z[\alpha(\lambda_i)] = -1, \;\; S_z[\beta(\lambda_i)] = +1$

$$P(\lambda \in \{\lambda_{+-}\}) = P(\lambda \in \{\lambda_{-+}\}) = \frac{1}{2}$$

- In this explanation:
  - Particles have definite properties regardless of the measurement. (realism)
  - **Alice**'s measurement has no influence on **Bob**'s particle. (locality)

Quantum mechanics (QM)

- The explanation in **QM** is very different.
- Although their outcomes are different in each decay, **QM** says the **state** of the particles are exactly the same for all decays:

$$|\Psi^{(0,0)}\rangle \underset{\substack{\uparrow \\ \text{up to a phase } e^{i\theta}}}{\doteq} \frac{|\overset{\alpha}{+}\overset{\beta}{-}\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

(no realism)

- Before the measurements, particles have no definite spin. Outcomes are undetermined.
- At the moment when Alice makes her measurement, the state collapses into:

$$|\Psi\rangle \xrightarrow[\text{Alice's measurement}]{} \begin{cases} |+,-\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = +1 \\ |-,+\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = -1 \end{cases}$$

Bob's outcome is completely determined (before his measurement) and **100%** anti-correlated with **Alice**'s. (non-local)

Entanglement

- The origin of this bizarre feature is **entanglement**.

general

$$|\Psi\rangle \doteq c_{11}|++\rangle_z + c_{12}|+-\rangle_z + c_{21}|-+\rangle_z + c_{22}|--\rangle_z$$

separable

$$|\Psi_{\text{sep}}\rangle \doteq \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$$

entangled

$$|\Psi_{\text{ent}}\rangle \not\doteq \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$$

entangled

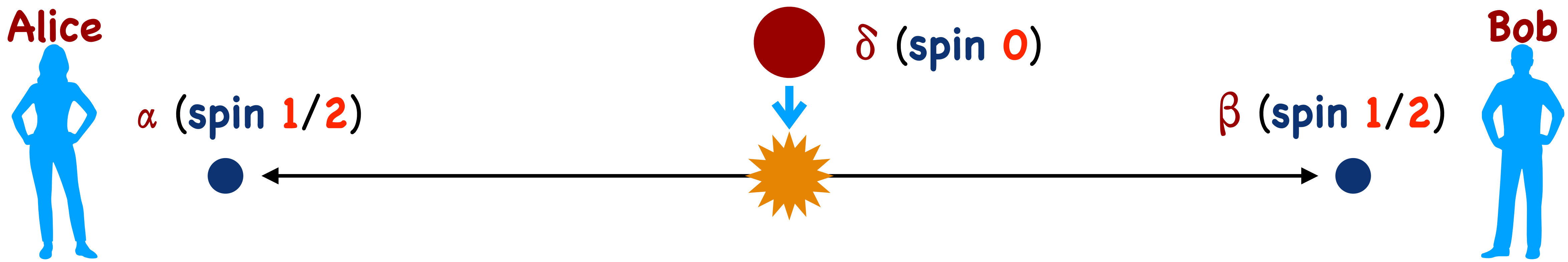
$$|\Psi^{(0,0)}\rangle \doteq \frac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

Bob's measurement collapses the **state** of β to $|+\rangle_z$ or $|-\rangle_z$ but does not influence the **state** of α.

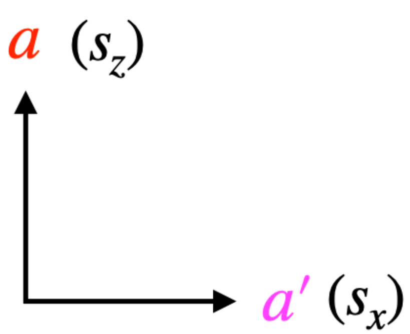
EPR paradox

- **Einstein**, **Podolsky** and **Rosen** (**EPR**) did not like the **QM** explanation.
- **EPR**'s **local**-**real** requirement: [Einstein, Podolsky, Rosen 1935].
  - Physical observables must be **real**: they have definite values irrespectively with the measurement.
  - Physical observables must be **local**: an action in one place cannot influence a physical observable in a space-like separated region.
- **QM** violates both **local** and **real** requirements.
- It seems difficult to experimentally discriminate **QM** and **general hidden variable theories**.
- **John Bell** (**1964**) derived simple inequalities that can discriminate **QM** from any **local**-**real** **hidden variable theories**: **Bell inequalities**.

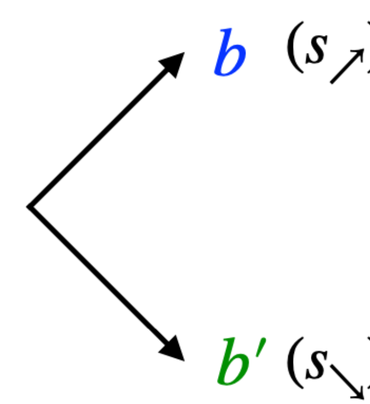
Bell inequalities



- The experiment consists of **4** sessions:



1. **Alice** and **Bob** measure $\mathbf{s_a}[\alpha]$ and $\mathbf{s_b}[\beta]$, respectively. Repeat the measurement many times and calculate $\langle \mathbf{s_a}.\mathbf{s_b} \rangle$.
2. Repeat (1) but for **a** and **b'**.
3. Repeat (1) but for **a'** and **b**.
4. Repeat (1) but for **a'** and **b'**.



- Finally we construct:

$$\boxed{R_{\mathrm{CHSH}} \equiv \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right|}$$

One can show in **hidden variable theories** that: $R_{CHSH} \leq 1$
[Clauser, Horne, Shimony, Holt, 1969].

Bell inequalities

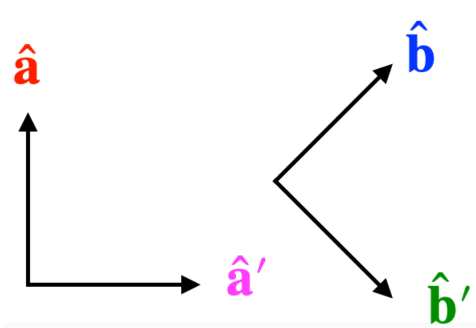
- In **QM**, for: $|\Psi^{(0,0)}\rangle \doteq \dfrac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$
- One can show: $\langle s_a s_b \rangle = \langle \Psi^{(0,0)} | s_a s_b | \Psi^{(0,0)} \rangle = (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})$
- Therefore:

$$R_{\mathrm{CHSH}} = \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

$$= \frac{1}{2} \left| \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} - \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}')}_{-\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}}')}_{\frac{1}{\sqrt{2}}} \right|$$

$$= \sqrt{2}$$

violates the upper bound of **hidden variable theories**!



Part II

Spin 1/2 biparticle system

Density operator

Probability of having $|\psi_1\rangle$

- For a statistical ensemble $\Big\{\{p_1 : |\psi_1\rangle\}, \{p_2 : |\psi_2\rangle\}, \{p_3 : |\psi_3\rangle\}, \ldots\Big\}$, we define the **density operator/matrix**:

$$\boxed{\hat{\rho} \equiv \sum_k p_k |\Psi_k\rangle\langle\Psi_k| \qquad \rho_{ab} \equiv \langle e_a|\hat{\rho}|e_b\rangle}$$

$$0 \le p_k \le 1$$

$$\sum_k p_k = 1$$

$$\langle e_a|e_b\rangle = \delta_{ab}$$

- **Density matrices** satisfy the conditions:

$$\hat{\rho}^\dagger = \hat{\rho}$$

$$\mathrm{Tr}\,\hat{\rho} = 1$$

$$\hat{\rho} \text{ is positive definite, that is } {}^\forall|\varphi\rangle;\ \langle\varphi|\hat{\rho}|\varphi\rangle \ge 0.$$

- The expectation of an observable $\hat{O}$ is calculated by:

$$\boxed{\langle\hat{O}\rangle = \mathrm{Tr}\left[\hat{O}\hat{\rho}\right]}$$

Spin 1/2 biparticle system

- The spin system of α and β particles has **4** independent bases:

$$\boxed{\big(|e_1\rangle, |e_2\rangle, |e_3\rangle, |e_4\rangle\big) = \big(|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\big)}$$

3x**3**
matrix

- ==> ρ_{ab} is a **4**x**4** matrix (hermitian, Tr=**1**). It can be expanded as

$$\boxed{\rho = \frac{1}{4}\big(1 \otimes 1 + B_i \,.\, \sigma_i \otimes 1 + \bar{B}_i . 1 \otimes \sigma_i + C_{ij} \,.\, \sigma_i \otimes \sigma_j\big), \qquad B_i, \bar{B}_i, C_{ij} \in \mathbb{R}}$$

- For the **spin** operators $\hat{s}^{\alpha}$ and $\hat{s}^{\beta}$:

$$\boxed{\langle \hat{s}_i^{\alpha} \rangle = Tr[\hat{s}_i^{\alpha} \hat{\rho}] = B_i \qquad \langle \hat{s}_i^{\beta} \rangle = Tr[\hat{s}_i^{\beta} \hat{\rho}] = \bar{B}_i \qquad \langle \hat{s}_i^{\alpha} \hat{s}_j^{\beta} \rangle = Tr[\hat{s}_i^{\alpha} \hat{s}_j^{\beta} \hat{\rho}] = C_{ij}}$$

spin-spin
correlation

Entanglement

- If the **state** is **separable** (not **entangled**):

$$\boxed{\rho = \sum_k p_k \rho_k^{\alpha} \otimes \rho_k^{\beta}, \qquad 0 \le p_k \le 1 \ \text{ and } \sum_k p_k = 1}$$

- Then, a modified **matrix** by the partial transpose:

$$\boxed{\rho^{T_\beta} = \sum_k p_k \rho_k^{\alpha} \otimes [\rho_k^{\beta}]^T}$$

  is also a physical **density matrix**, i.e. **Tr=1** and non-negative.

- For biparticle systems, **entanglement** $\iff \rho^{T_\beta}$ to be non-positive. [Peres-Horodecki (1996,1997)].

- A simple sufficient condition for **entanglement** is:

$$\boxed{E \equiv C_{11} + C_{22} - C_{33} > 1}$$

Estimation of C_{ij}

- Let's suppose a **spin 1/2** particle α is **at rest** and spinning in the **s** direction.
- α decays into a measurable particle l_α and the rest X: $\alpha \rightarrow l_\alpha + (X)$
- The decay distribution is generally given by : $\dfrac{d\Gamma}{d\Omega} \propto 1 + x_\alpha(\hat{I}_\alpha . s)$

$x \in [-1, 1]$ is called spin-analysing power and depends on the decay
$x = 1$ for $\tau^\pm \rightarrow \pi^\pm \nu$

Unit direction vector of l_α measured at the rest frame of α

- One can show for $\alpha+\beta \rightarrow [l_\alpha + (X)] + [l_\beta + (X)]$:

$$\boxed{\langle \hat{s}_i^\alpha \hat{s}_j^\beta \rangle = -9.\langle \hat{I}_i^\alpha \hat{I}_j^\beta \rangle}$$

measurable at colliders, but **needs to reconstruct the α (β) rest frames**

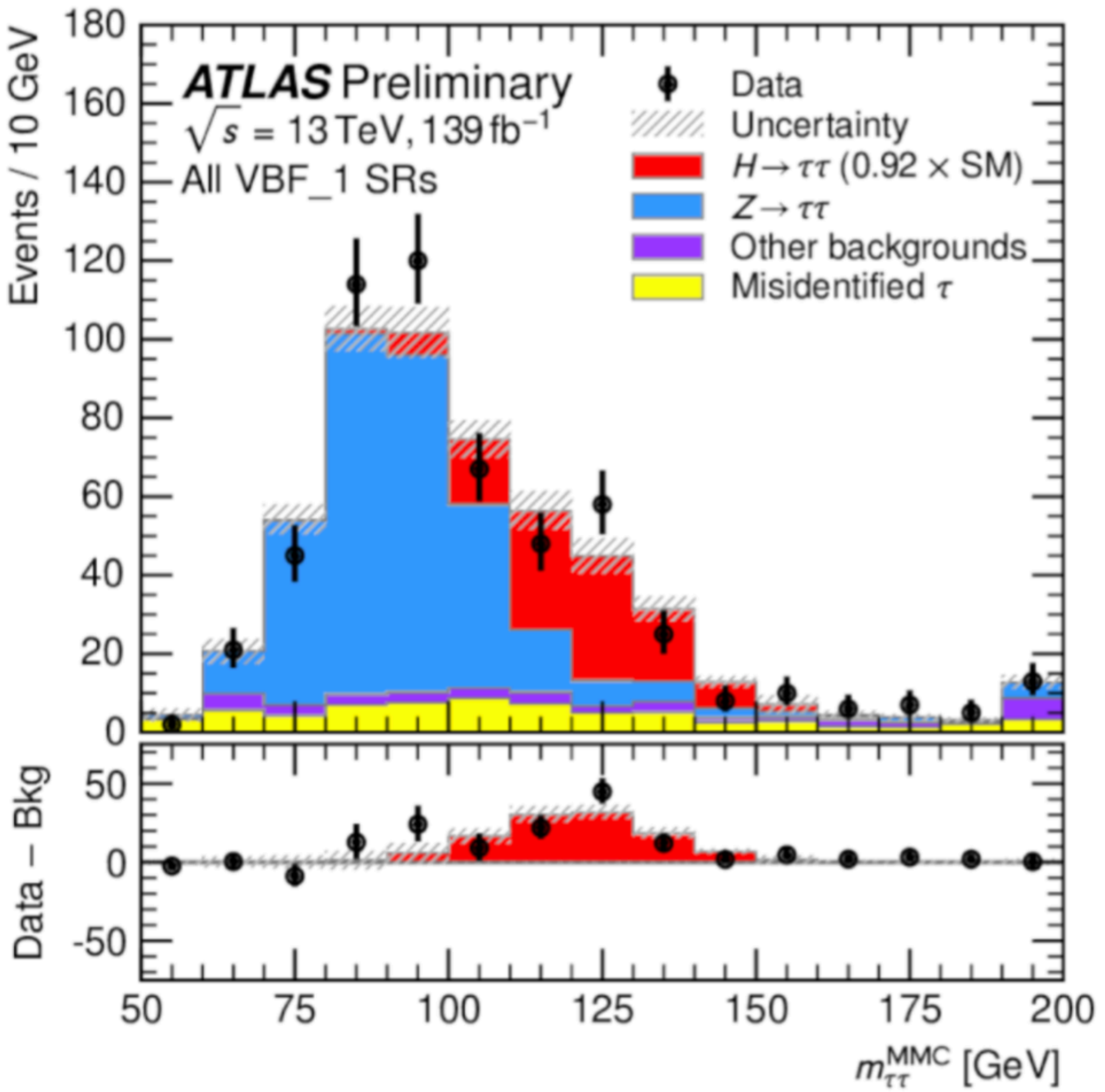
Part III

Higgs to tau tau @ lepton colliders

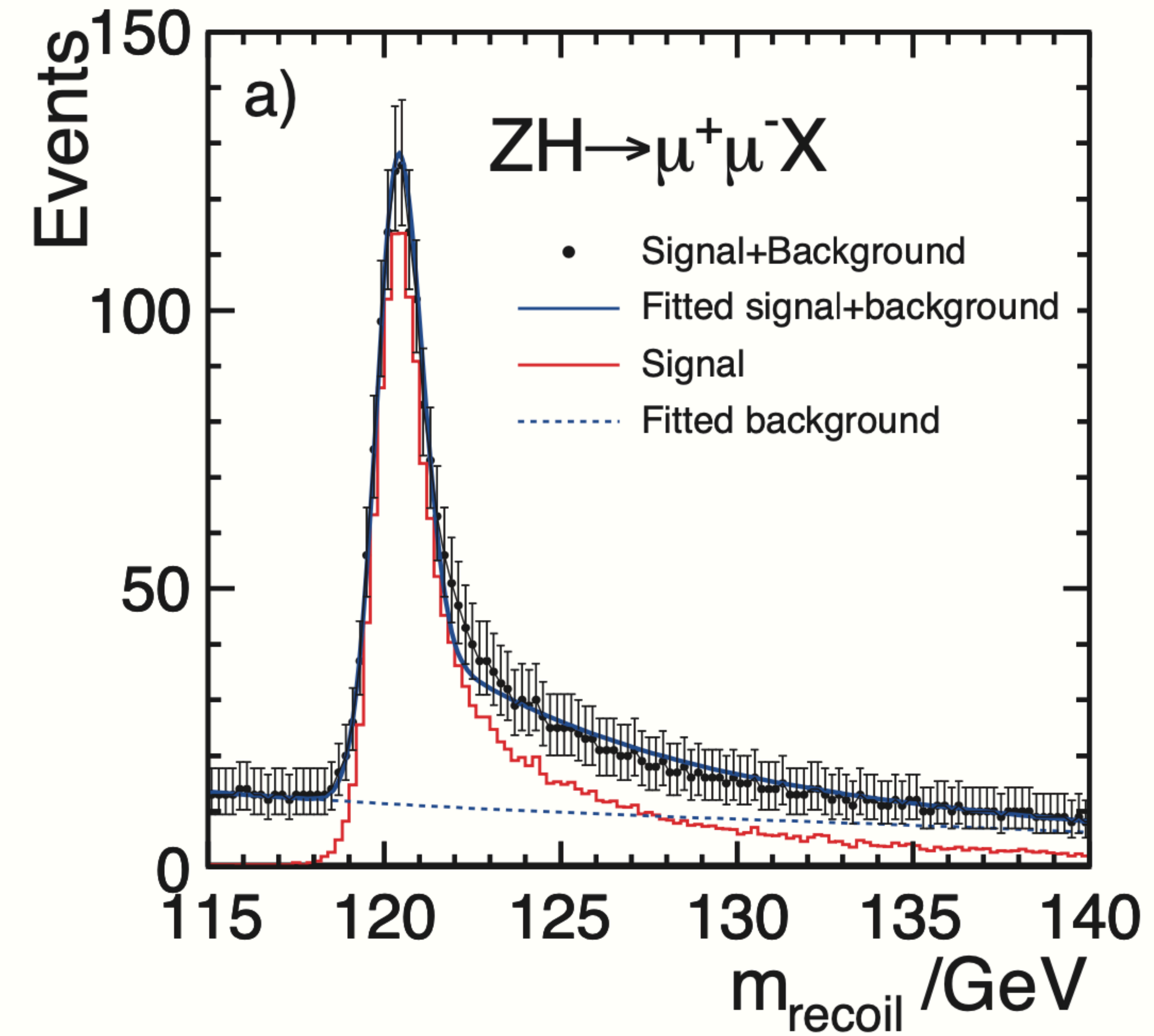
Why lepton colliders?

- Background $Z/\gamma \to \tau^+\tau^-$ is much smaller at **lepton** colliders
- We need to reconstruct each τ rest frame to measure $\hat{I}$. This is challenging at **hadron** colliders since partonic CoM energy is unknown for each event

LHC



ILC



Simulation

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	0.83×10^{-4}
beam resolution e^- (%)	0.27	0.83×10^{-4}
$\sigma(e^+e^- \to HZ)$ (fb)	240.1	240.3
# of signal ($\sigma \cdot \mathrm{BR} \cdot L \cdot \epsilon$)	385	663
# of background ($\sigma \cdot \mathrm{BR} \cdot L \cdot \epsilon$)	20	36

- Events were generated with **Madgraph5**
- We incorporate the detector effect by smearing energies of visible particles

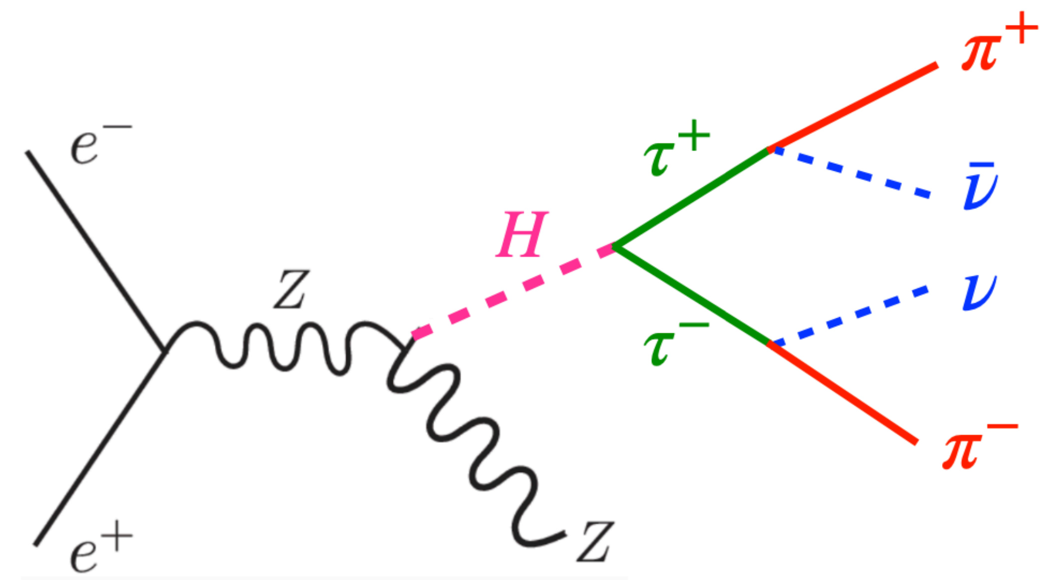
$$E^{true} \to E^{obs} = (1 + \sigma_E \,.\, \omega) \,.\, E^{true} \qquad \sigma_E = 0.03$$

Random number from a normal distribution

- We perform **100 pseudo-experiment** to estimate the statistical uncertainties

Solving kinematical constraints

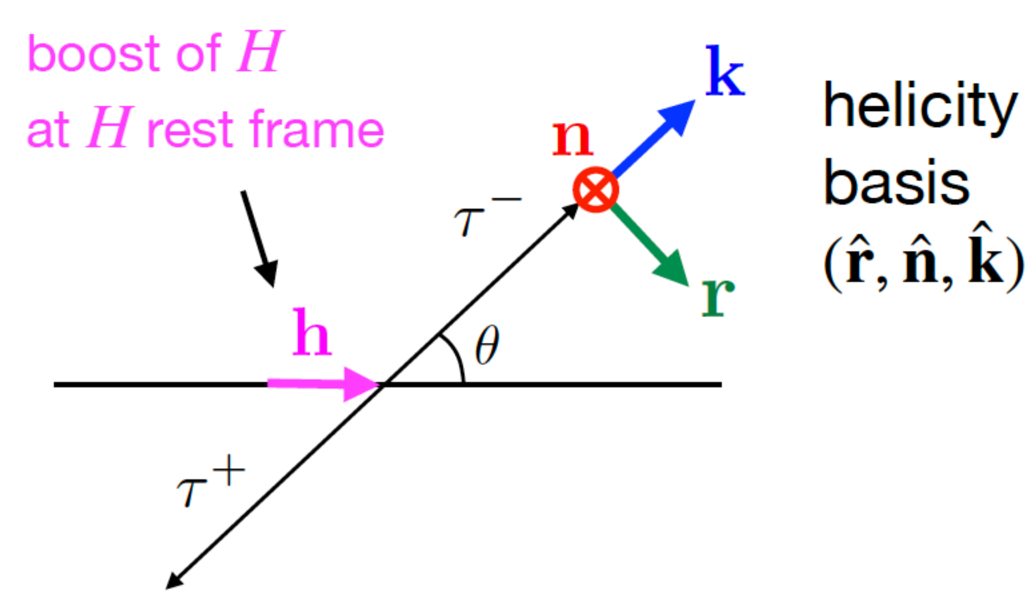
- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta: $(p_x^{\nu}, p_y^{\nu}, p_z^{\nu}), (p_x^{\bar\nu}, p_y^{\bar\nu}, p_z^{\bar\nu})$.
- **6** unknowns can be constrained by **2** mass-shell conditions and **4** energy-momentum conservation:



$$\boxed{\begin{aligned} m_\tau^2 &= (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar\nu}) \\ m_\tau^2 &= (p_{\tau^-})^2 = (p_{\pi^-} + p_{\nu}) \\ (p_{ee} - p_Z)^\mu &= p_H^\mu = \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar\nu})\right]^\mu \end{aligned}}$$

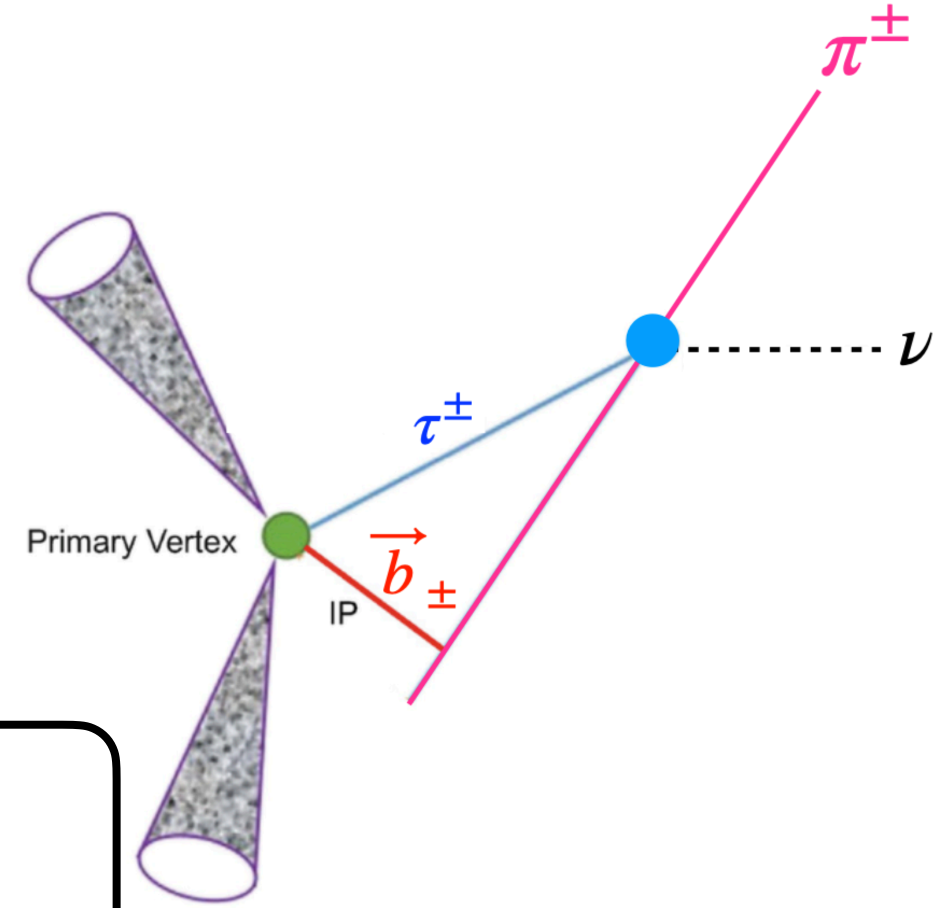
- With the reconstructed momenta, we define $(\hat{\boldsymbol{r}}, \hat{\boldsymbol{n}}, \hat{\boldsymbol{k}})$ basis at the Higgs rest frame.

$$\boxed{\begin{aligned} &C_{ij} = \langle \hat{s}_i^{(\tau^-)} \hat{s}_j^{(\tau^+)} \rangle = -9.\langle \hat{l}_i^- \hat{l}_j^+ \rangle \\ &(i, j = r, n, k) \end{aligned}}$$



Impact parameter (IP)

- We use the information of the **impact parameter** $\vec{b}_\pm$ measurement of $\pi^\pm$ to "correct" the observed energies of $\tau^\pm$ and Z decay products.
- We check whether the reconstructed τ momenta are consistent with the measured impact parameters.
- We construct the likelihood function and search for the most likely τ momenta.



$$\boxed{E_\alpha(\delta_\alpha) = (1 + \sigma_\alpha^E \cdot \delta_\alpha) \cdot E_\alpha^{\mathrm{obs}}}$$

$$\vec{b}_+ = |\vec{b}_+| \left(\sin^{-1}\Theta_+ \cdot \vec{e}_{\tau^+} - \tan^{-1}\Theta_+ \cdot \vec{e}_{\pi^+}\right)$$

$$\vec{\Delta}^i_{b_+}(\{\delta\}) \equiv \vec{b}_+ - |\vec{b}_+| \left(\sin^{-1}\Theta^i_+(\{\delta\}) \cdot \vec{e}^{\,i}_{\tau^+}(\{\delta\}) - \tan^{-1}\Theta^i_+(\{\delta\}) \cdot \vec{e}_{\pi^+}\right)$$

$$L^i_\pm(\{\delta\}) = \frac{[\Delta^i_{b_\pm}(\{\delta\})]^2_x + [\Delta^i_{b_\pm}(\{\delta\})]^2_y}{\sigma^2_{b_T}} + \frac{[\Delta^i_{b_\pm}(\{\delta\})]^2_z}{\sigma^2_{b_z}}$$

$$\boxed{L^i(\{\delta\}) = L^i_+(\{\delta\}) + L^i_-(\{\delta\})}$$

Results

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} 0.830 \pm 0.176 & 0.020 \pm 0.146 & -0.019 \pm 0.159 \\ -0.034 \pm 0.160 & 0.981 \pm 0.1527 & -0.029 \pm 0.156 \\ -0.001 \pm 0.158 & -0.021 \pm 0.155 & -0.729 \pm 0.140 \end{pmatrix}$	$\begin{pmatrix} 0.925 \pm 0.109 & -0.011 \pm 0.110 & 0.038 \pm 0.095 \\ -0.009 \pm 0.110 & 0.929 \pm 0.113 & 0.001 \pm 0.115 \\ -0.026 \pm 0.122 & -0.019 \pm 0.110 & -0.879 \pm 0.098 \end{pmatrix}$
E_k	2.567 ± 0.279 > 5σ	2.696 ± 0.215 > 5σ
R_{CHSH}	1.103 ± 0.163	1.276 ± 0.094 ~ 3σ

SM values:
$$C_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$E = 3$ **Entanglement** $\implies E > 1$

$R_{\mathrm{CHSH}} = \sqrt{2} \simeq 1.414$ **Bell-nonlocal** $\implies R_{\mathrm{CHSH}} > 1$

The superiority of **FCC-ee** over **ILC** is due to a better **beam resolution**

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	$0.83 \cdot 10^{-4}$
beam resolution e^- (%)	0.27	$0.83 \cdot 10^{-4}$

Part IV

Summary

Summary

- High energy tests of **entanglement** and **Bell inequality** has recently attracted an attention.

- We investigated feasibility of **quantum** property tests @ **ILC** and **FCC-ee**.

- **Quantum tests** require a precise reconstruction of the τ rest frames and **IP** information is crucial to achieve this.

	Entanglement	Bell-inquality
FCC-ee	$> 5\sigma$	$\sim 3\sigma$
ILC	$> 5\sigma$	

Backup slides

Hidden variable theory

One can show in hidden variable theories:

$$R_{\text{CHSH}} \equiv \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right| \leq 1$$

$$\begin{aligned}
|\langle ab\rangle - \langle ab'\rangle| &= \left|\int d\lambda\,(ab - ab')\,P\right| \\
&= \int d\lambda\,|ab(1 \pm a'b')P - ab'(1 \pm a'b)P| \\
&\leq \int d\lambda\Big(|ab||1 \pm a'b'|P + |ab'||1 \pm a'b|P\Big) \\
&= \int d\lambda\Big[(1 \pm a'b')P + (1 \pm a'b)P\Big] \\
&= 2 \pm (\langle a'b'\rangle + \langle a'b\rangle)
\end{aligned}$$

$$\pm aba'b'P - (\pm aba'b'P) = 0$$

$$|ab| = |ab'| = 1$$

$$|1 \pm a'b'|, |1 \pm a'b| \geq 0$$

$$a = s_a$$
$$b = s_b$$
$$\vdots$$

$$\rightarrow \quad \tilde{R}_{\text{CHSH}} = \frac{1}{2}\left(|\langle ab\rangle - \langle ab'\rangle| + |\langle a'b\rangle + \langle a'b'\rangle|\right) \leq 1$$

$$\max_{(\vec{a},\vec{b},\vec{a}',\vec{b}')}(R_{\text{CHSH}}) = \max_{(\vec{a},\vec{b},\vec{a}',\vec{b}')}\left(\tilde{R}_{\text{CHSH}}\right)$$

$$\boxed{\langle s_a s_b\rangle = \int s_a(\lambda)s_b(\lambda)P(\lambda)d\lambda}$$

$$\boxed{\int P(\lambda)d\lambda = 1}$$