# ON AXION COUPLINGS AND ANOMALIES

Based on :

P.Anastasopoulos, M.Bianchi, E.D., E.Kiritsis, [arXiv:hep-th/0605225 [hep-th]]
and on-going discussions with P. Lamba, S. Pokorski and K. Sakurai

confirming results in

J. Quevillon and C.Smith, [arXiv:1903.12559 [hep-ph]],
Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul and A.N.Rossia,
[arXiv:2011.10025 [hep-ph]].

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# Outline

1) Massive gauge fields, heavy fermions and triangle diagrams

- Stueckelberg-Chern-Simons couplings

2) Peccei-Quinn symmetry, axion couplings and anomalies

# 1) Heavy fermions, massive gauge fields and triangle diagrams

The decoupling theorem (Appelquist-Carrazzone) :

Heavy particles affect low-energy physics only through renormalization effects (running of couplings, etc)

Subtleties:

- Hierarchy problem (sensitivity of Higgs/scalar mass to heavy particles)

- Quantum anomalies (ex. top quark does not decouple in the Standard Model)

Obs: Other aspects of heavy particles (ex: heavy Higgs in SM) are not related to decoupling

Consider (abelian for simplicity) spontaneously broken gauge theory, chiral fermions:

- A set of light fermions
- A set of heavy chiral fermions, anomaly-free

$$Tr(X_L^i X_L^j X_L^k - X_R^i X_R^j X_R^k) = 0$$

Is there an effect of heavy fermions on low-energy physics (in addition to renormalization effects) ?

Some notations:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\psi}_{L,R}\gamma^\mu D_\mu \psi_{L,R} + |D_\mu \phi_I|^2 - (\lambda_I \phi_I \bar{\psi}_L \psi_R + h.c.) - V(\phi_I),$$

where

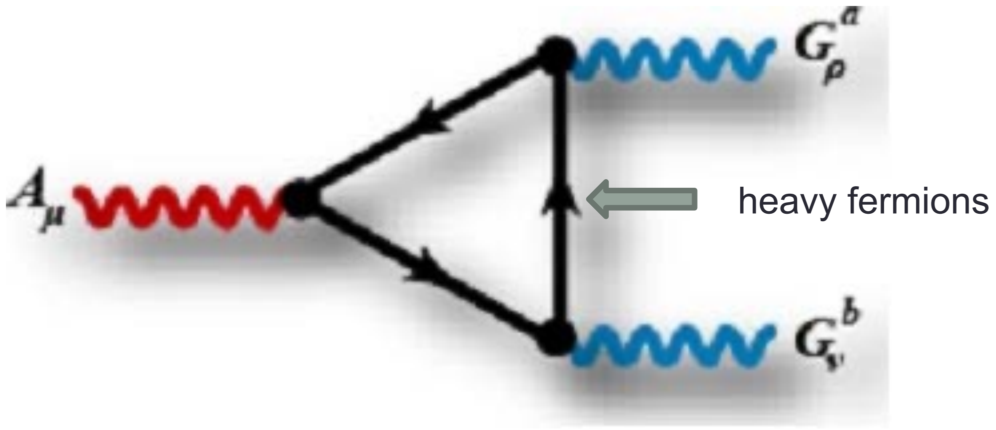
$$D_\mu \Psi_{L,R} = (\partial_\mu - i g_i X^i_{L,R} A^i_\mu)\Psi_{L,R}$$

$$\phi_I = \frac{1}{\sqrt{2}}(v_I + h_I)\, e^{i\frac{a_I}{v_I}}$$

. Gauge transf: $\delta A^i_\mu = \partial_\mu \alpha_i \ , \ \delta a_I = g_i X^i_I v_I \alpha_i$

The gauge-invariant Stueckelberg combination is

$$\partial_\mu a_I - g_i X^i_I v_I A^i_\mu$$

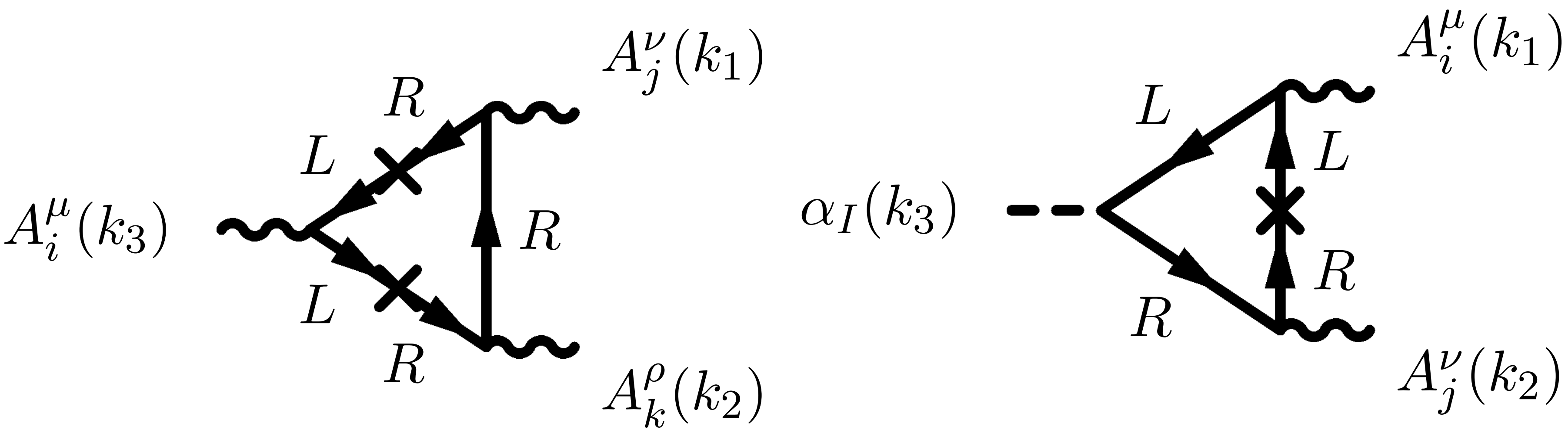



Since heavy fermions are anomaly-free, one could think that one only generate operators suppressed by the mass $M_\Psi = \lambda_I v_I$ of the heavy fermions

Not true ! One does generate gauge-invariant operators
(called Stueckelberg-Chern-Simons, SCS in what follows)

$$\mathcal{L}_{SCS} = E_{ij,k}\ \epsilon^{\mu\nu\rho\sigma}(A_{i,\mu} - P_i^I \partial_\mu a_I)(A_{j,\nu} - P_j^I \partial_\nu a_I) F_{k,\rho\sigma}$$

(ABDK,2006)



The gauge-invariant SCS terms can also be written as

$$\mathcal{L}_{SCS} = E_{ij,k}\ \epsilon^{\mu\nu\rho\sigma} A_{i,\mu} A_{j,\nu} F_{k,\rho\sigma} +$$
$$(E_{ij,k} - E_{ji,k}) P_i^I\ \epsilon^{\mu\nu\rho\sigma} a_I F_{j,\mu\nu} F_{k,\rho\sigma}$$

The first line above are called generalized Chern-Simons terms (GCS), the second line are axionic couplings

Remarks:

- Since heavy fermions are anomaly-free, there is no ambiguity in the triangle computation, no regularization needed. One finds (ABDK)

$$E_{ij,k} = \frac{1}{2} \sum (X_L^i X_R^j - X_R^i X_L^j)(X_R^k + X_L^k)$$

- Standard folklore: GCS terms are ambiguous and axionic couplings are related to anomalies. Both statements are wrong.

See also Quevillon and Smith, 2019 and Bonnefoy, Di Luzio, Grojean, Paul and Rossia, 2020

More general case : heavy + light chiral fermions.
Gauge anomalies cancel non-trivially between heavy and light fermions.

Considering heavy and light fermions separately, there are ambiguities in distributing anomaly among various currents. However, ambiguities are mass-independent and cancel in the full theory.

Integrating-out heavy fermions lead to the effective terms

$$\mathcal{L} = -\frac{1}{4g_i^2}(F^i_{\mu\nu})^2 + \frac{1}{2}(M_i^I A_{\mu,i} - \partial_i a_I)^2 +$$

$$\frac{E_{ij,k}}{24\pi^2}\ \epsilon^{\mu\nu\rho\sigma}(A_{i,\mu} - P_i^I \partial_\mu a_I)(A_{j,\nu} - P_j^I \partial_\nu a_I) F_{k,\rho\sigma} + \frac{D^I_{ij}}{24\pi^2} a_I \epsilon^{\mu\nu\rho\sigma} F_{i,\mu\nu} F_{j,\rho\sigma}$$

# 2) PQ symmetries, axions and anomalies

Until now I only talked about gauge symmetries and axions=goldstone bosons.

We can define the usual Peccei-Quinn (PQ) symmetry in the following way:

- Gauge PQ and take the gauge coupling to zero at the end

  ⇒ global PQ symmetry

- We want a spontaneously broken anomalous symmetry, in order to solve the strong CP problem. In the gauged PQ case, one adds spectator ('t Hooft-like) massless fermions to cancel PQ anomalies, with no couplings to scalars/axions.

For N = N-1 + 1 PQ gauge fields and N complex scalars, in the unitary gauge

⇒ N-1 massive gauge fields + 1 PQ axion

The gauge fields + axion low-energy lagrangian is

$$\mathcal{L} = -\frac{1}{4}(F^i_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu a_{PQ})^2 + \frac{g_i g_j g_k E_{ij,k}}{24\pi^2}\ \epsilon^{\mu\nu\rho\sigma} A_{i,\mu} A_{j,\nu} F_{k,\rho\sigma} +$$

$$\frac{g_i g_j E_{iN,j}}{24\pi^2 f_{PQ}}\ \epsilon^{\mu\nu\rho\sigma} \partial_\mu a_{PQ} A_{i,\nu} F_{j,\rho\sigma} + \frac{g_i g_j D_{ij}}{24\pi^2 f_{PQ}} a_{PQ} \epsilon^{\mu\nu\rho\sigma} F_{i,\mu\nu} F_{j,\rho\sigma}$$

Notations: $X_{i=1\cdots N-1} \equiv Q^i \quad , \quad X_N \equiv X_{PQ}$

PQ breaking scale is $f_{PQ}^2 = \sum_{I=1}^N (X^I_{PQ})^2 v_I^2$

and

anomalous axion couplings

$$D_{ij} = \sum_H (Q_L^i Q_L^j X_L^{PQ} - Q_R^i Q_R^j X_R^{PQ}) \ ,$$

$$C_{ij} = \sum_H [Q_L^i Q_L^j + Q_R^i Q_R^j + \frac{1}{2}(Q_L^i Q_R^j + Q_R^i Q_L^j)](X_L^{PQ} - X_R^{PQ})$$

total axion couplings

These general formulae agree with the particular examples discussed in Quevillon,Smith and Bonnefoy et al

# Potential applications ?

Only brief comments, taken from

Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul and A.N.Rossia, [arXiv:2011.10025 [hep-ph]].

- Some couplings of the axion to SM gauge bosons are not related to anomalies:

$$a F_{\mu\nu} \tilde{Z}^{\mu\nu} \ , \ a Z_{\mu\nu} \tilde{Z}^{\mu\nu} \ , \ a W^+_{\mu\nu} \tilde{W}^{-\mu\nu}$$

(nonabelian extension: Brivio, Gavela et al, 2017; Bonnefoy et al.,2020)

Define

$$-\frac{16\pi^2}{e^2}\mathcal{L} \supset C_{\gamma\gamma}\frac{a}{f}F\tilde{F} + 2\frac{C_{Z\gamma}}{c_W s_W}\frac{a}{f}F\tilde{Z} + \frac{C_{ZZ}}{c_W^2 s_W^2}\frac{a}{f}Z\tilde{Z} + \frac{2C_{WW}}{s_W^2}W^+\tilde{W}^-$$

If these couplings would come entirely from anomalies, one would get the sum rules

$$C_{\gamma\gamma} + s_W^{-2}(1-t_W^2)C_{Z\gamma} - \frac{1}{s_W^2 c_W^2}C_{ZZ} = 0 \ , \quad C_{\gamma\gamma} + s_W^{-2}C_{Z\gamma} - (1+t_W^{-2})C_{WW} = 0$$

Heavy fermions chiral under SM and PQ do violate such sum rules.

- If kinematically allowed, new processes are also possible.

$$Z \to a\gamma \qquad Z' \to Za \quad , \quad Z' \to \gamma a$$

# Conclusions and Perspectives

- PQ axion couplings to massive gauge fields are not determined by anomalies (clearly stated by Quevillon-Smith for the PQ axion).

- Axion couplings to photons and gluons are still governed by anomalies

- Phenomenological consequences for ALP's, difficult to test for the PQ axion solving the strong CP problem.

# Thank You !