Functional one-loop matching of effective field theories

MATCHETE

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Universität Zürich

In collaboration with:

Javier Fuentes-Martín, Matthias König, Julie Pagès, Anders Eller Thomsen

arXiv: [2212.04510], [2211.09144], [2012.08506]

IRN Terascale — LPSC Grenoble

25. April 2023

EFT matching

Scenario: theory $\mathscr{L}_{\rm UV}(\eta_H, \eta_L)$ with large scale separation $m_H \gg m_L$

- Analysis of low-energy phenomenology ⇒ construct effective theory (EFT)

$$\mathscr{L}_{\rm EFT}(\eta_L) = \mathscr{L}_{d=4}(\eta_L) + \sum_{d=5}^{\infty} \frac{1}{m_H^{d-4}} \sum_i C_i^{(d)} Q_i^{(d)}(\eta_L)$$

How to find:

Wilson coefficients $C_i^{(d)}$?

Effective operators $Q_i^{(d)}$?

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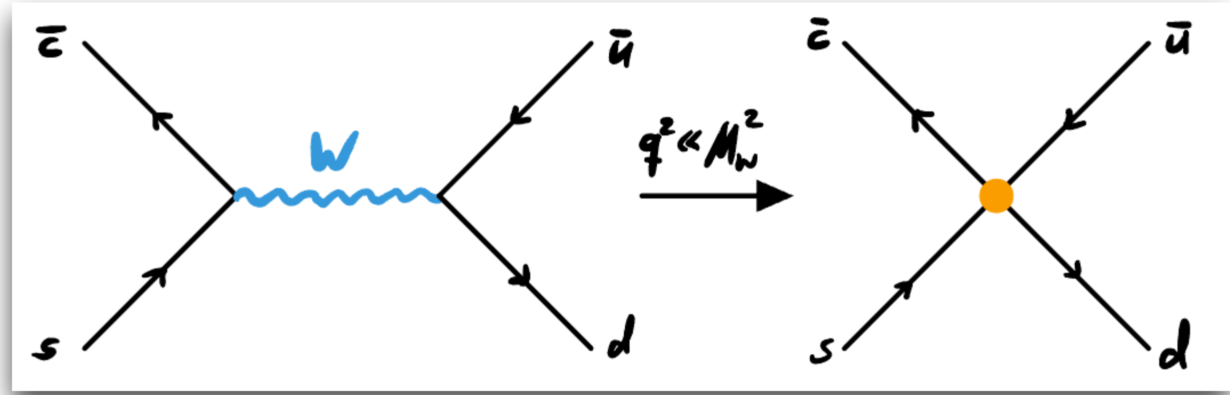
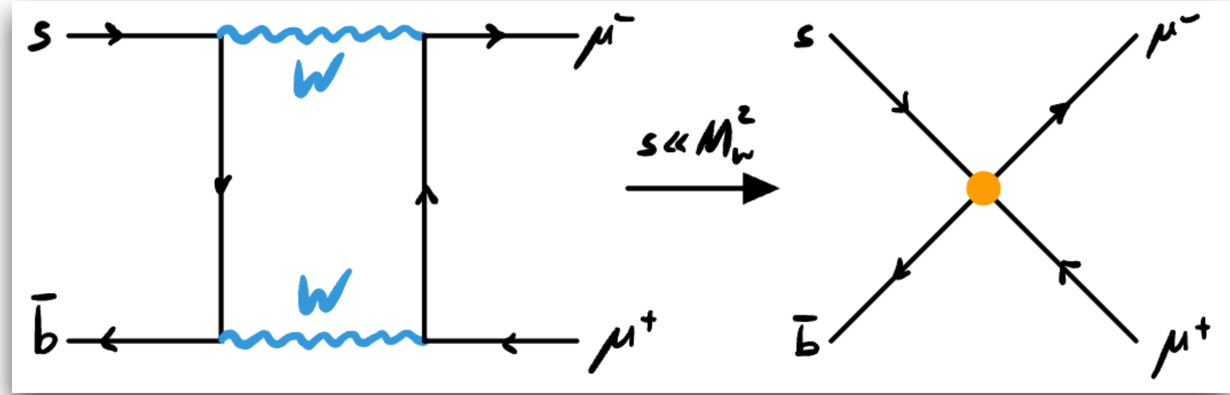
- Example: Fermi's theory — integrating out the W boson





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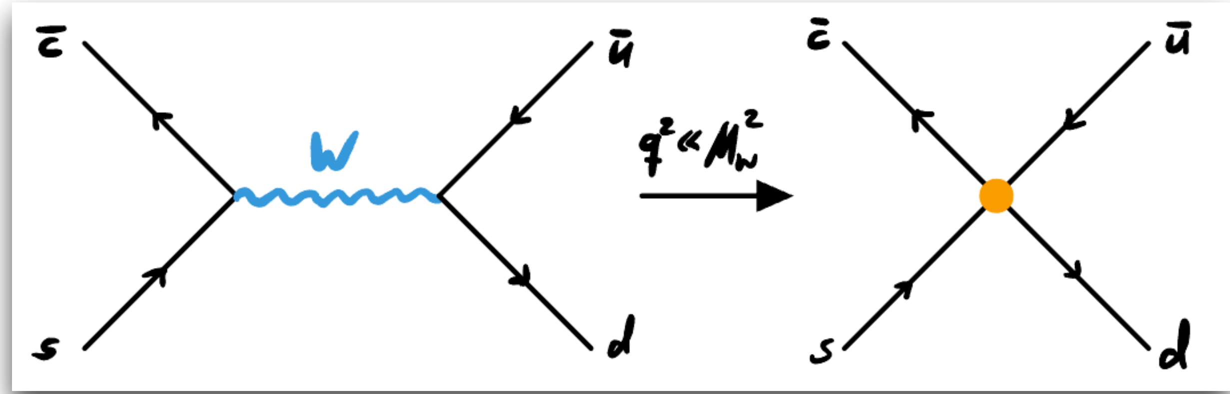
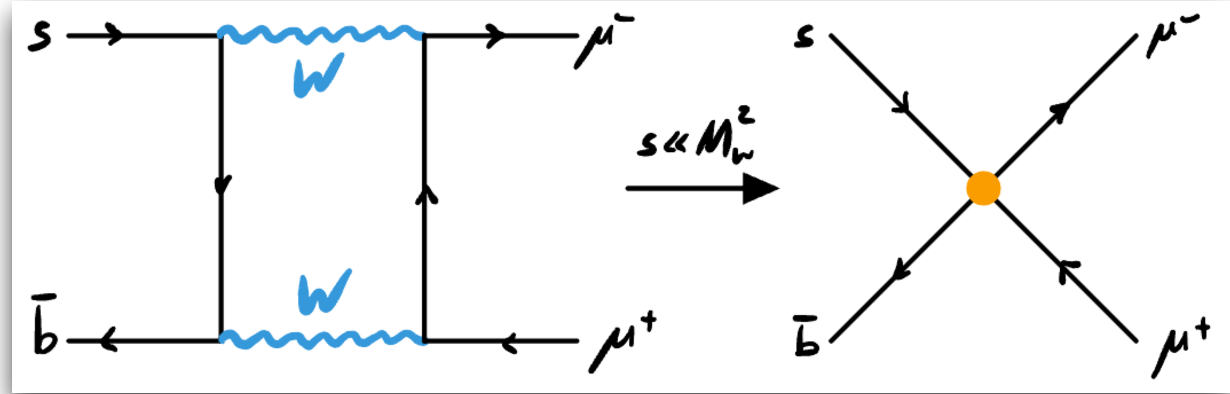
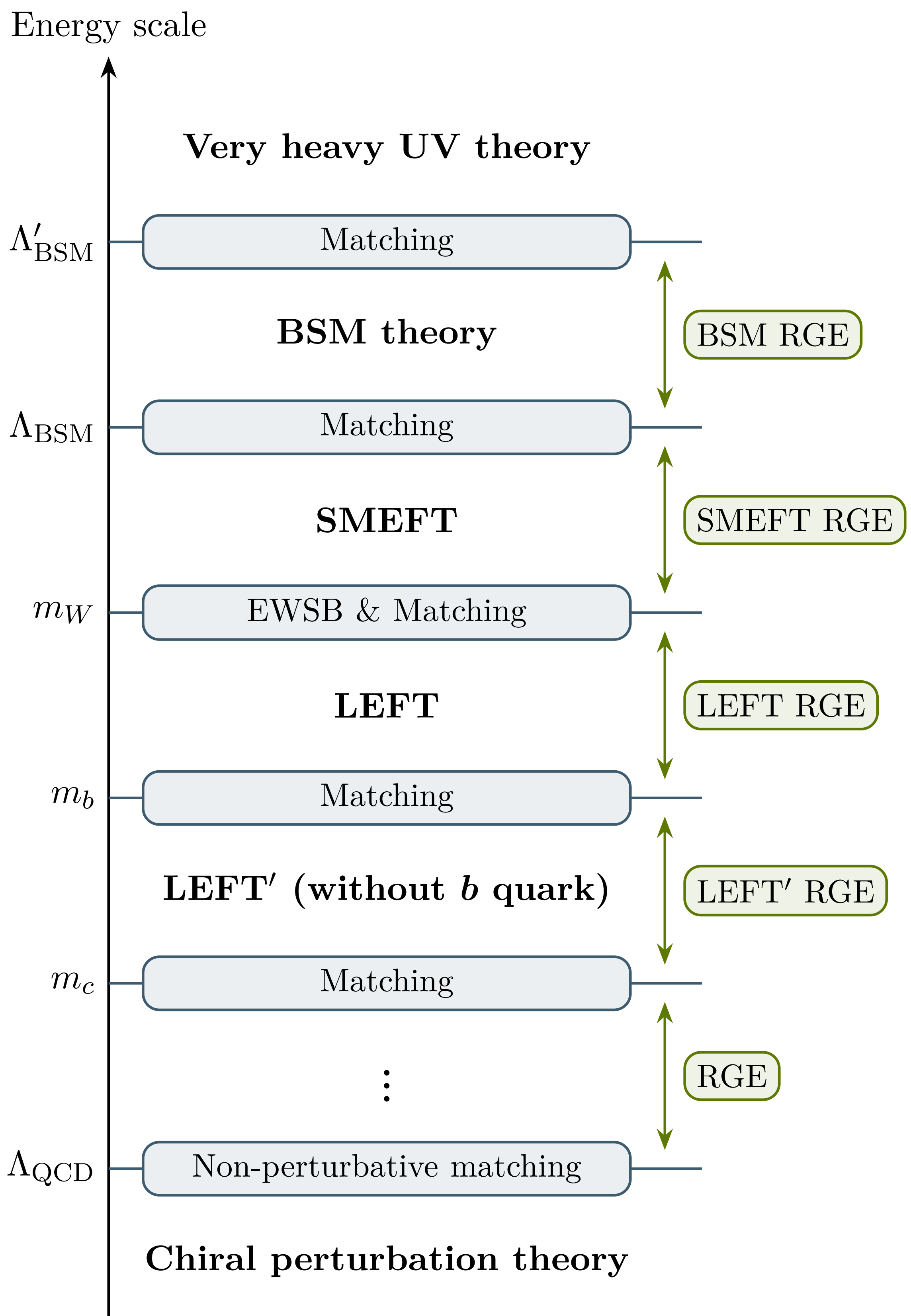




➡ Diagrammatic matching

- One-loop matching required for many interesting phenomena (e.g. FCNC)

Automating multiscale EFT analyses



Isidori, FW, Wyler [2303.16922]

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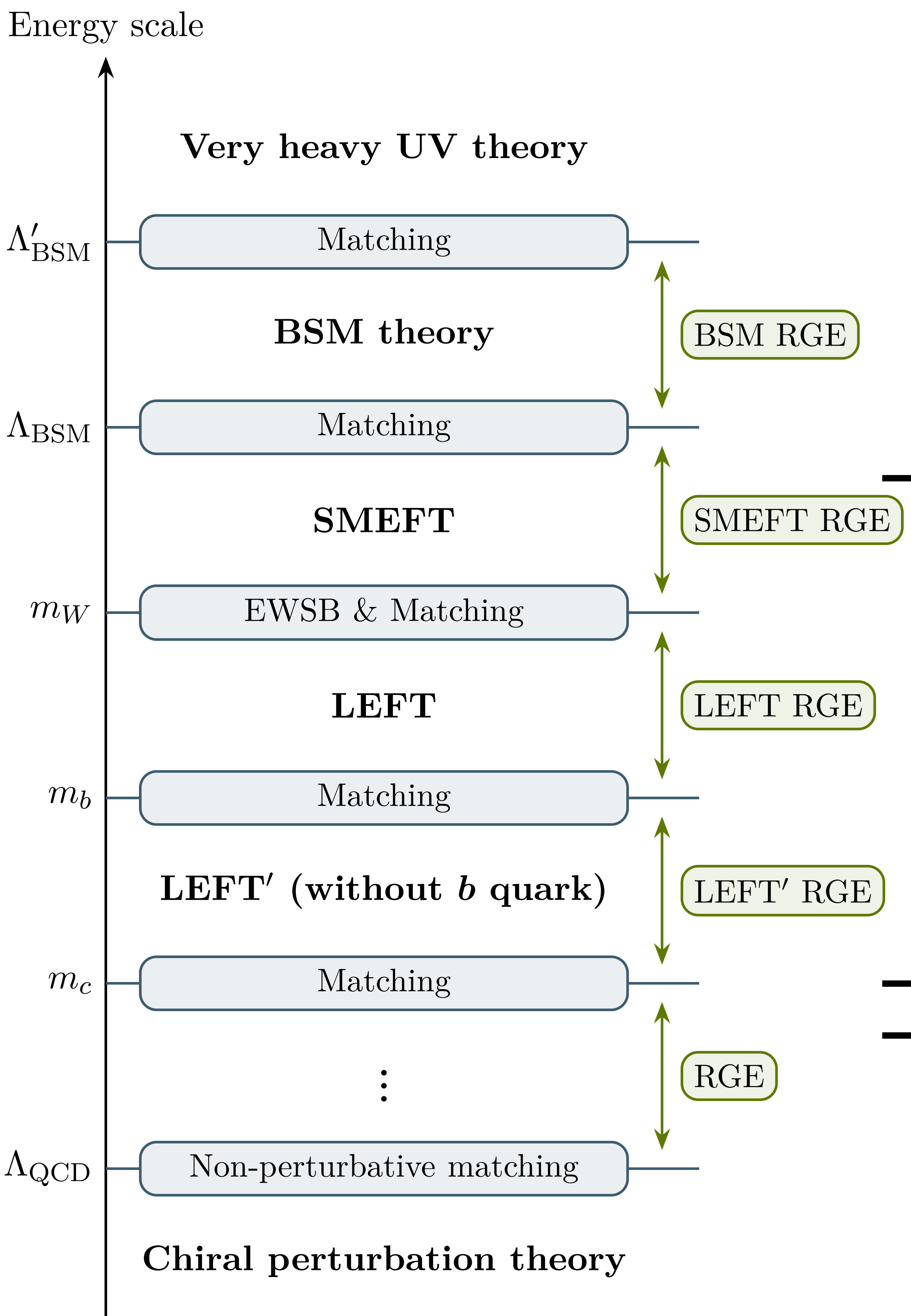

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Theory work:

Jenkins, Manohar, Trott [1308.2627], [1310.4838]
Alonso, Jenkins, Manohar, Trott [1312.2014]
Jenkins, Manohar, Stoffer [1709.04486]
Dekens, Stoffer [1908.05295]
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Tools:

`DsixTools`
Celis, Fuentes-Martin, Vicente, Virto [1704.04504]
Fuentes-Martin, Ruiz-Femenia, Vicente, Virto [2010.16341]
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→ Fit BSM theory parameters to low-energy experimental data

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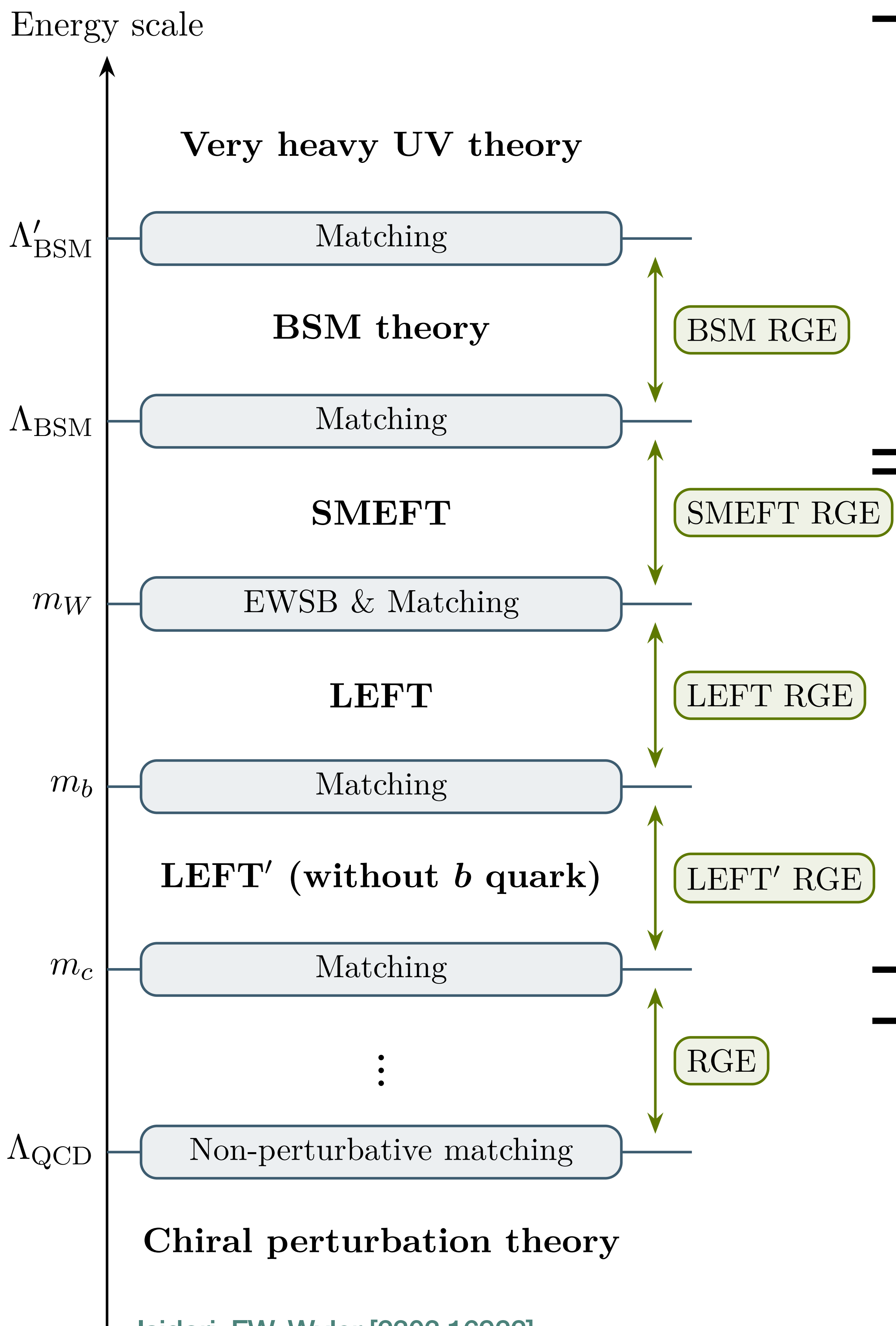

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Theory work: (mostly) model dependent …

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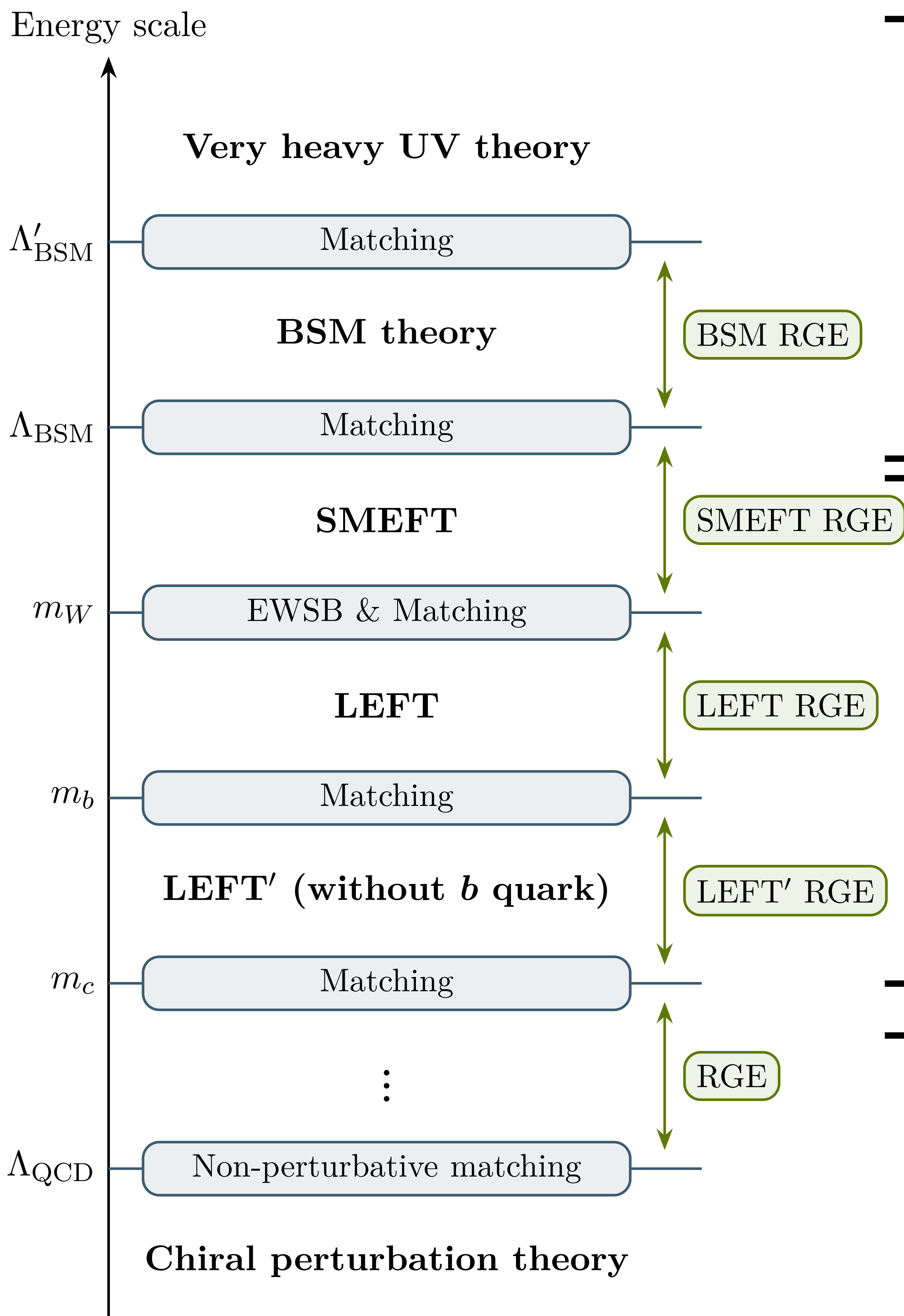

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→ Fit BSM theory parameters to low-energy experimental data

Plethora of BSM models requires automation of all steps (matching & running)

⇒ Goal: integration into single software suite

Functional matching

One-loop matching with path integral techniques

Functional Methods

- **Background field method:** shift all fields $\eta \to \hat{\eta} + \eta$

  $\hat{\eta}$: background fields (satisfy classical EOM)

  η: pure quantum fluctuation

- **Path integral representation of effective quantum action:**

$$\exp\left(i\Gamma_{\mathrm{UV}}(\hat{\eta})\right) = \int \mathcal{D}\eta \, \exp\left(i \int \mathrm{d}^d x \, \mathscr{L}_{\mathrm{UV}}(\eta + \hat{\eta})\right)$$

  - Perform path integral over η ("*integrating out*")
  - Expand in powers of m_H^{-1}

➡ Effective action of EFT containing all higher dimensional operators and coefficients

Dittmaier, Grosse-Knetter [hep-ph/9505266]
Henning, Lu, Murayama [1412.1837]
Drozd, Ellis, Quevillon, You [1512.03003]
del Aguila, Kunszt, Santiago [1602.00126]
Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]
Henning, Lu, Murayama [1604.01019]
Zhang [1610.00710]
Cohen, Lu, Zhang [2011.02484]
& many more

Tree-level and one-loop matching

- **Expanding the Lagrangian in η:**

$$\mathscr{L}_{\mathrm{UV}}(\eta) \to \mathscr{L}_{\mathrm{UV}}(\hat{\eta} + \eta) = \mathscr{L}_{\mathrm{UV}}(\hat{\eta}) + \frac{1}{2}\bar{\eta}_i \left. \frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta\bar{\eta}_i \, \delta\eta_j} \right|_{\eta=\hat{\eta}} \eta_j + \mathscr{O}(\eta^3)$$

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- **Tree-level matching:** $\mathscr{L}^{(0)}_{\mathrm{EFT}} = \mathscr{L}_{\mathrm{UV}}\left(\hat{\eta}_L, \hat{\eta}_H(\hat{\eta}_L)\right)$
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    - Gaussian path integral
    - Can be expressed in terms of superdeterminants SDet

generalization of Det to mixed spins

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higher loop orders

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Supertraces

Cohen, Lu, Zhang [2011.02484]

- **Supertraces:** $\Gamma^{(1)}_{\text{UV}} = \frac{i}{2}\text{STr}\left(\ln \mathcal{O}\right) = \pm\frac{i}{2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\langle k\,|\,\text{tr}(\ln \mathcal{O})\,|\,k\rangle$

- **Fluctuation operator:** $\mathcal{O}_{ij} \equiv \left.\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta\bar{\eta}_i\,\delta\eta_j}\right|_{\eta=\hat{\eta}} = \delta_{ij}\Delta_i^{-1} - X_{ij} = \Delta_i^{-1}(\delta_{ij} - \Delta_i X_{ij})$

$$\Delta_i^{-1} = \begin{cases} -(D^2 + M_i^2) \\ i\gamma^\mu D_\mu - M_i \\ g^{\mu\nu}(D^2 + M_i^2) \end{cases}$$

propagators — interaction terms

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propagators — interaction terms

- Expanding the logarithm, ΔX is at most $\mathcal{O}(m_H^{-1})$:

$$\Gamma^{(1)}_{\rm UV} = \frac{i}{2}\mathrm{STr}\left(\ln\ \Delta^{-1}\right) - \frac{i}{2}\sum_{n=1}^{\infty}\mathrm{STr}\left[(\Delta X)^n\right]$$

log-type supertrace — power-type supertrace

  - log-type: model independent, only depend on propagator type Δ
  - power-type: depend on the interaction terms X

Method of regions

- Expand loop integrands in *soft* $(k \sim m_L)$ and *hard* $(k \sim m_H)$ region before integration
  Beneke, Smirnov [hep-ph/9711391], Jantzen [1111.2589]
- Summing the results gives back the original integral expanded in m_L / m_H

$$\Gamma^{(1)}_{\mathrm{UV}} = \Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{hard}} + \Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{soft}}$$

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heavy particle loops **mixed particle loops** **light particle loops**

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- **One-loop EFT Lagrangian:**

Cohen, Lu, Zhang [2011.02484]

$$\int d^d x\, \mathscr{L}^{(1)}_{\rm EFT} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} = \frac{i}{2}{\rm STr}\left(\ln\, \Delta^{-1}\right)\Big|_{\rm hard} - \frac{i}{2}\sum_{n=1}^{\infty}{\rm STr}\left[(\Delta X)^n\right]\Big|_{\rm hard}$$

- Supertrace evaluation in manifest covariant form through CDE (see backup)

Operator reduction

- Supertrace output $\int \mathrm{d}^d x \, \mathscr{L}^{(1)}_{\mathrm{EFT}} = \Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{hard}}$ directly **provides EFT operators** (no a priori knowledge required), but $\mathscr{L}_{\mathrm{EFT}}$ is **not in a minimal basis**

➡ Many redundancies among the present operators

Operator reduction

- Supertrace output $\int \mathrm{d}^d x \, \mathscr{L}_{\mathrm{EFT}}^{(1)} = \Gamma_{\mathrm{UV}}^{(1)}\Big|_{\mathrm{hard}}$ directly **provides EFT operators** (no a priori knowledge required), but $\mathscr{L}_{\mathrm{EFT}}$ is **not in a minimal basis**

➡ Many redundancies among the present operators

- **Goal:** bring $\mathscr{L}_{\mathrm{EFT}}$ to minimal form by using:
    - Integration by parts identities
    - Diagonalize kinetic & mass mixing
    - Field redefinitions (*equations of motion*)
    - Reduction of Dirac algebra
    - Fierz identities
    - …

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  → **evanescent operators !!!** (Reduction of Dirac algebra and Fierz identities)

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Evanescent operators

Buras, Weisz [*Nucl.Phys.B* 333 (1990) 66-99]
Herrlich, Nierste [hep-ph/9412375]

$$\mathscr{L} \supset C_{lqde}^{prst}(\bar{\ell}^p \gamma^\mu q^t)(\bar{d}^s \gamma_\mu e^r) \xrightarrow[d=4]{\textbf{Fierz identity}} \mathscr{L}' \supset -2C_{lqde}^{prst}(\bar{\ell}^p e^r)(\bar{d}^s q^t)$$

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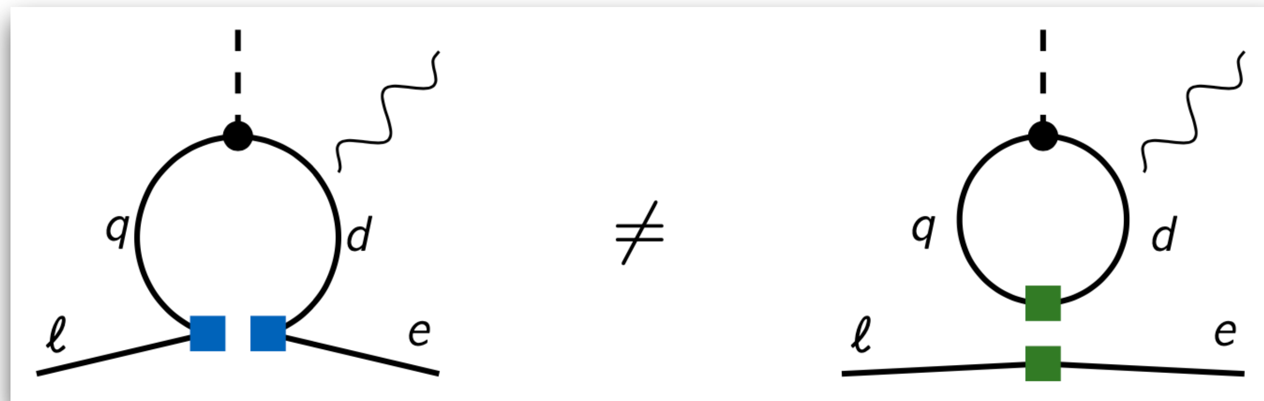

figure by A. Thomsen

The one-loop effective action built from $\mathscr{L}$ and $\mathscr{L}'$ do not agree:

$$\Gamma_{\rm EFT}^{(1)} \neq \Gamma_{\rm EFT}^{\prime(1)}$$

- In d dimensions we have: $C_{lqde}^{prst}(\bar{\ell}^p \gamma^\mu q^t)(\bar{d}^s \gamma_\mu e^r) = -2C_{lqde}^{prst}(\bar{\ell}^p e^r)(\bar{d}^s q^t) + C_{lqde}^{prst} E_{lqde}^{prst}$

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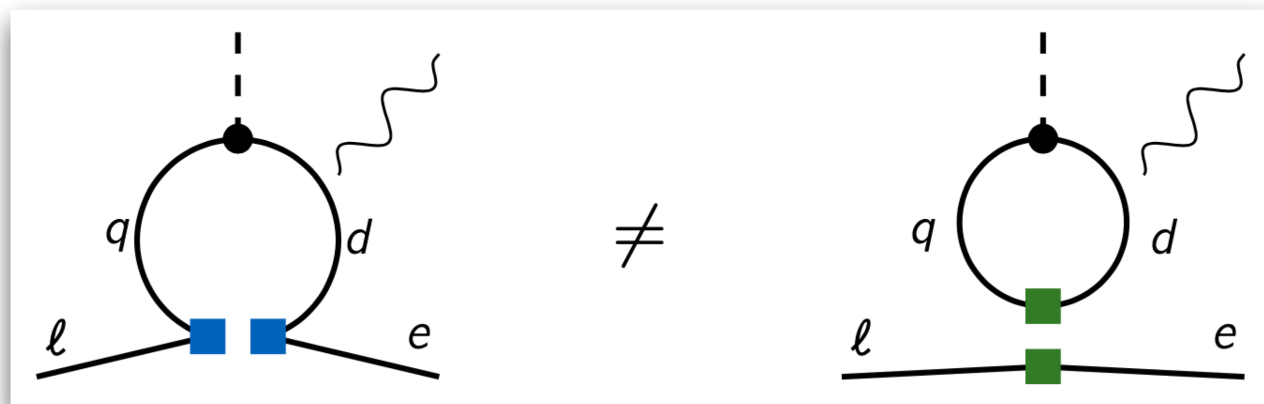

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- Effective one-loop action: $\Gamma^{(1)}_{\rm EFT} = \Gamma'^{(1)}_{\rm EFT} + \Delta S_E$
- Absorb physical effect of evanescent operators by finite one-loop shift of action ΔS_E (depends on all UV poles $\epsilon_{\rm UV}$ of SMEFT one-loop integrals)
- Computed for the SMEFT in **Fuentes-Martin, König, Pages, Thomsen, FW [2211.09144]**
- For LEFT see: **Aebischer, Buras, Kumar [2202.01225], Aebischer, Pesut [2208.10513]**

MATCHETE

MATCHing Effective Theories Efficiently

https://gitlab.com/matchete/matchete

Fuentes-Martin, König, Pages, Thomsen, FW [2212.04510]

MATCHETE

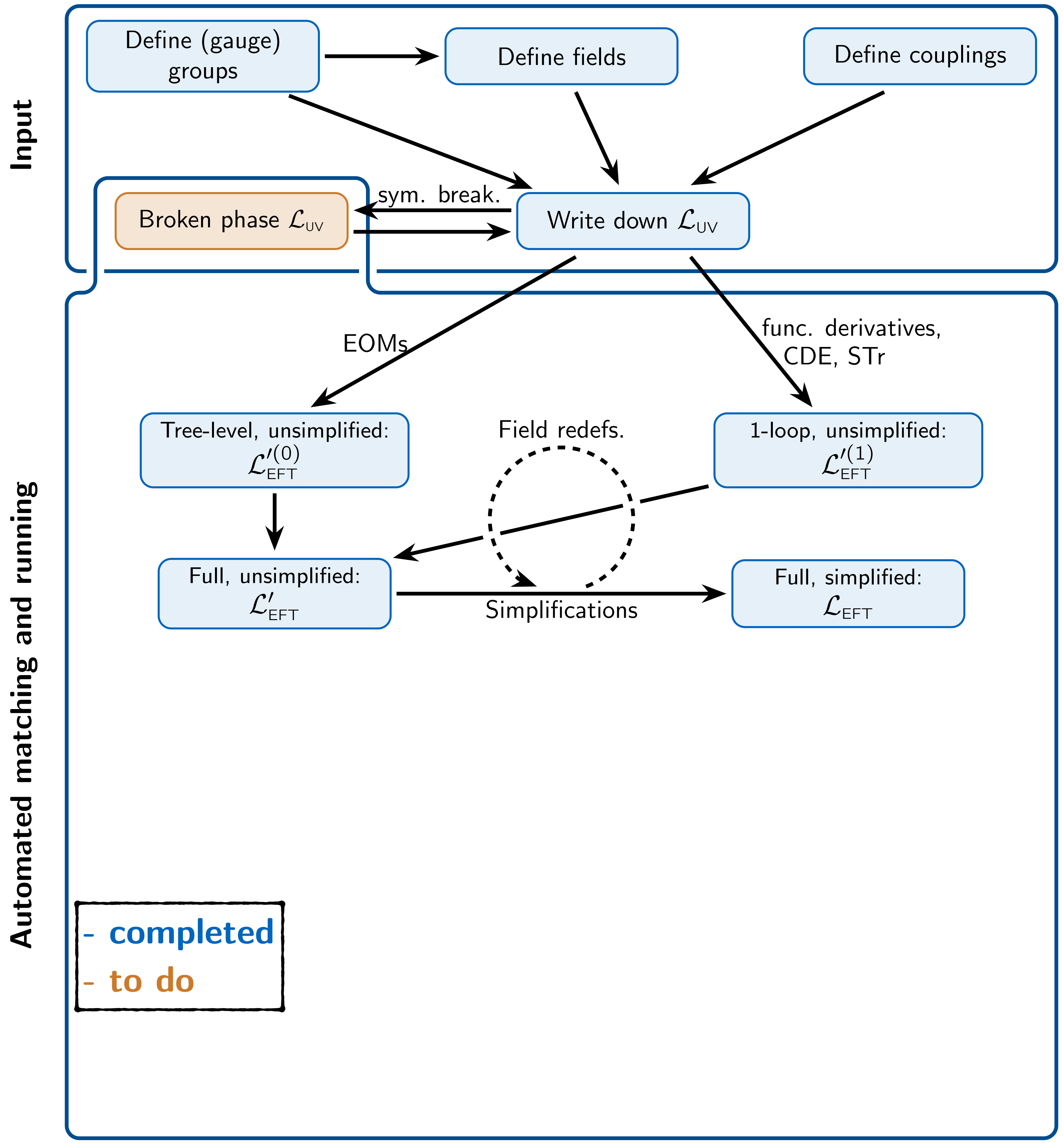
(*written in `Mathematica`)

- **Define generic weakly coupled UV theory***
  (symmetries, fields, couplings)
  **with mass power-counting*

- **Automatic matching**
  computation of EoM & Q_{ij},
  STr enumeration & evaluation

- **Simplifications**
  Reduction of redundant operators:
  IbP, field redefinitions, (Fierz id.), …

➡ **Minimal basis** (e.g. Warsaw)



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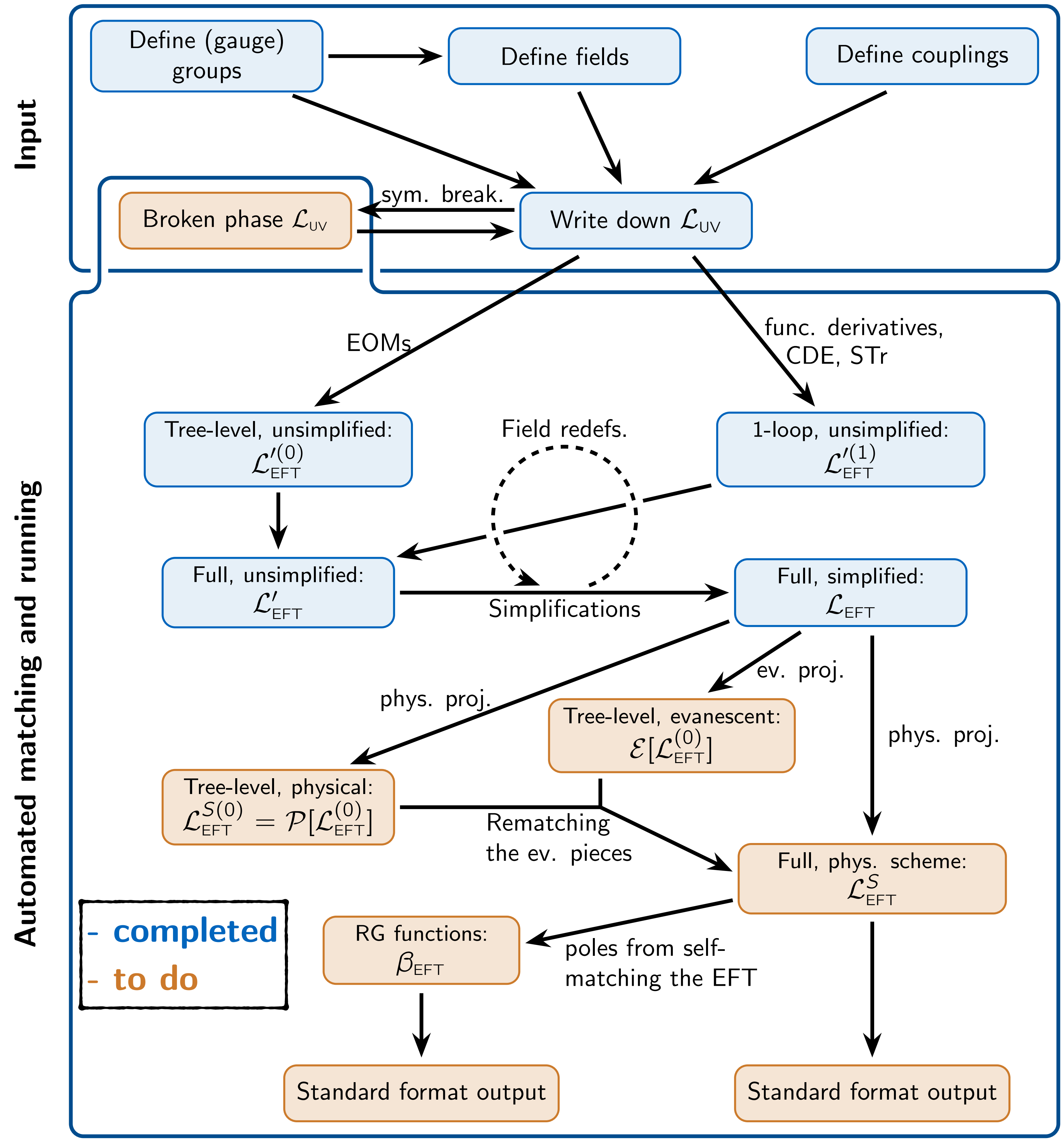
- **To do:**
  - Fully automatic treatment of Evanescent operators
  - β-functions
  - Heavy vector bosons



Real singlet scalar extension of SM

Loading the SM definitions »

The Higgs potential parameters are relabeled to mH and λH.

In[2]:=
```
ℒSM = LoadModel["SM", ModelParameters -> {"μ" -> mH, "λ" -> λh}];
ℒSM // NiceForm
```

Out[3]//NiceForm=

$$-\frac{1}{4}B^{\mu\nu 2} - \frac{1}{4}G^{\mu\nu A2} - \frac{1}{4}W^{\mu\nu I2} + D_\mu\bar{H}_i\, D_\mu H^i + mH^2\,\bar{H}_i\, H^i +$$
$$i\left(\bar{d}^p_a\cdot\gamma_\mu P_R\cdot D_\mu d^{ap}\right) + i\left(\bar{e}^p\cdot\gamma_\mu P_R\cdot D_\mu e^p\right) + i\left(\bar{l}^p_i\cdot\gamma_\mu P_L\cdot D_\mu l^{ip}\right) +$$
$$i\left(\bar{q}^p_{ai}\cdot\gamma_\mu P_L\cdot D_\mu q^{aip}\right) + i\left(\bar{u}^p_a\cdot\gamma_\mu P_R\cdot D_\mu u^{ap}\right) - \frac{1}{2}\lambda h\,\bar{H}_i\,\bar{H}_j\,H^i\,H^j -$$
$$\overline{Yd}^{pr}\,\bar{H}_i\left(\bar{d}^r_a\cdot P_L\cdot q^{aip}\right) - \overline{Ye}^{pr}\,\bar{H}_i\left(\bar{e}^r\cdot P_L\cdot l^{ip}\right) -$$
$$Ye^{pr}\,H^i\left(\bar{l}^p_i\cdot P_R\cdot e^r\right) - Yd^{pr}\,H^i\left(\bar{q}^p_{ai}\cdot P_R\cdot d^{ar}\right) -$$
$$Yu^{pr}\,\bar{H}_i\left(\bar{q}^p_{aj}\cdot P_R\cdot u^{ar}\right)\varepsilon^{ji} - \overline{Yu}^{pr}\,H^j\left(\bar{u}^r_a\cdot P_L\cdot q^{aip}\right)\bar{\varepsilon}_{ij}$$

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```

Out[3]//NiceForm=

$$-\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + D_\mu \bar{H}_i D_\mu H^i + mH^2 \bar{H}_i H^i + i\left(\bar{d}^p_a \cdot \gamma_\mu P_R \cdot D_\mu d^{ap}\right) + i\left(\bar{e}^p \cdot \gamma_\mu P_R \cdot D_\mu e^p\right) + i\left(\bar{l}^p_i \cdot \gamma_\mu P_L \cdot D_\mu l^{ip}\right) + i\left(\bar{q}^p_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{aip}\right) + i\left(\bar{u}^p_a \cdot \gamma_\mu P_R \cdot D_\mu u^{ap}\right) - \frac{1}{2} \lambda h \bar{H}_i \bar{H}_j H^i H^j - \overline{Yd}^{pr} \bar{H}_i \left(\bar{d}^r_a \cdot P_L \cdot q^{aip}\right) - \overline{Ye}^{pr} \bar{H}_i \left(\bar{e}^r \cdot P_L \cdot l^{ip}\right) - Ye^{pr} H^i \left(\bar{l}^p_i \cdot P_R \cdot e^r\right) - Yd^{pr} H^i \left(\bar{q}^p_{ai} \cdot P_R \cdot d^{ar}\right) - Yu^{pr} \bar{H}_i \left(\bar{q}^p_{aj} \cdot P_R \cdot u^{ar}\right) \varepsilon^{ji} - \overline{Yu}^{pr} H^j \left(\bar{u}^r_a \cdot P_L \cdot q^{aip}\right) \bar{\varepsilon}_{ij}$$

New field and couplings

Define new field

```
In[4]:= DefineField[ϕ, Scalar, SelfConjugate -> True, Mass -> {Heavy, M}]
```

Define new couplings

```
In[5]:= DefineCoupling[A, SelfConjugate -> True]
        DefineCoupling[κ, SelfConjugate -> True]
        DefineCoupling[μ, SelfConjugate -> True]
        DefineCoupling[λϕ, SelfConjugate -> True]
```

Real singlet scalar extension of SM

Loading the SM definitions

The Higgs potential parameters are relabeled to mH and λH.

```
In[2]:= ℒSM = LoadModel["SM", ModelParameters -> {"μ" -> mH, "λ" -> λh}];
        ℒSM // NiceForm
```

Out[3]//NiceForm=

$$-\frac{1}{4}B^{\mu\nu 2}-\frac{1}{4}G^{\mu\nu A2}-\frac{1}{4}W^{\mu\nu I2}+D_\mu\bar{H}_i D_\mu H^i+mH^2\bar{H}_i H^i+ i\left(\bar{d}^p_a\cdot\gamma_\mu P_R\cdot D_\mu d^{ap}\right)+i\left(\bar{e}^p\cdot\gamma_\mu P_R\cdot D_\mu e^p\right)+i\left(\bar{l}^p_i\cdot\gamma_\mu P_L\cdot D_\mu l^{ip}\right)+ i\left(\bar{q}^p_{ai}\cdot\gamma_\mu P_L\cdot D_\mu q^{aip}\right)+i\left(\bar{u}^p_a\cdot\gamma_\mu P_R\cdot D_\mu u^{ap}\right)-\frac{1}{2}\lambda h\,\bar{H}_i\bar{H}_j H^i H^j- \overline{Yd}^{pr}\bar{H}_i\left(\bar{d}^r_a\cdot P_L\cdot q^{aip}\right)-\overline{Ye}^{pr}\bar{H}_i\left(\bar{e}^r\cdot P_L\cdot l^{ip}\right)- Ye^{pr}H^i\left(\bar{l}^p_i\cdot P_R\cdot e^r\right)-Yd^{pr}H^i\left(\bar{q}^p_{ai}\cdot P_R\cdot d^{ar}\right)- Yu^{pr}\bar{H}_i\left(\bar{q}^p_{aj}\cdot P_R\cdot u^{ar}\right)\varepsilon^{ji}-\overline{Yu}^{pr}H^j\left(\bar{u}^r_a\cdot P_L\cdot q^{aip}\right)\bar{\varepsilon}_{ij}$$

New field and couplings

Define new field

```
In[4]:= DefineField[ϕ, Scalar, SelfConjugate -> True, Mass -> {Heavy, M}]
```

Define new couplings

```
In[5]:= DefineCoupling[A, SelfConjugate -> True]
        DefineCoupling[κ, SelfConjugate -> True]
        DefineCoupling[μ, SelfConjugate -> True]
        DefineCoupling[λϕ, SelfConjugate -> True]
```

Lagrangian

$$\mathscr{L}_{\text{int}} = -A\bar{H}H\phi - \frac{\kappa}{2}\bar{H}H\phi^2 - \frac{\mu}{3!}\phi^3 - \frac{\lambda_\phi}{4!}\phi^4$$

Write interaction terms

```
In[9]:= ℒint = (-A[] × Bar@H[i] × H[i] × ϕ[] - 1/2 κ[] × Bar@H[i] × H[i] × ϕ[] × ϕ[] - 1/3! μ[] ϕ[]^3 - 1/4! λϕ[] ϕ[]^4) // RelabelIndices;
        ℒNP = FreeLag[ϕ] + ℒint;
        ℒNP // NiceForm
```

Out[11]//NiceForm=

$$\frac{1}{2}\left(D_\mu\phi\right)^2-\frac{1}{2}M^2\phi^2-A\,\bar{H}_i H^i\phi-\frac{1}{2}\kappa\,\bar{H}_i H^i\phi^2-\frac{1}{6}\mu\,\phi^3-\frac{1}{24}\lambda\phi\,\phi^4$$

Define full UV Lagrangian

```
In[12]:= ℒUV = ℒSM + ℒNP;
```

Tree-level matching

Tree-level

Matching

```
In[16]:= ℒEFT0 = Match[ℒUV, LoopOrder -> 0, EFTOrder -> 6];
         ℒEFT0 - ℒSM // HcSimplify // NiceForm
```

Out[17]//NiceForm=

$$\frac{1}{2} A^2 \frac{1}{M^2} \bar{H}_i \bar{H}_j H^i H^j + \frac{1}{6} A^2 \frac{1}{M^6} \left(-3 \kappa M^2 + A \mu\right) \bar{H}_i \bar{H}_j \bar{H}_k H^i H^j H^k + A^2 \frac{1}{M^4} \bar{H}_i D_\mu \bar{H}_j D_\mu H^i H^j + \left(\frac{1}{2} A^2 \frac{1}{M^4} \bar{H}_i \bar{H}_j D_\mu H^i D_\mu H^j + \text{H.c.}\right)$$

Tree-level matching

Tree-level

Matching

```
In[16]:= ℒEFT0 = Match[ℒUV, LoopOrder -> 0, EFTOrder -> 6];
         ℒEFT0 - ℒSM // HcSimplify // NiceForm
```

Out[17]//NiceForm=

$$\frac{1}{2} A^2 \frac{1}{M^2} \bar{H}_i \bar{H}_j H^i H^j + \frac{1}{6} A^2 \frac{1}{M^6} \left(-3\,\kappa\, M^2 + A\,\mu\right) \bar{H}_i \bar{H}_j \bar{H}_k H^i H^j H^k + A^2 \frac{1}{M^4} \bar{H}_i D_\mu \bar{H}_j D_\mu H^i H^j + \left(\frac{1}{2} A^2 \frac{1}{M^4} \bar{H}_i \bar{H}_j D_\mu H^i D_\mu H^j + \text{H.c.}\right)$$

Removing redundant operators on-shell

Simplify to the physical basis using field redefinitions.

```
In[18]:= ℒEFT0OnShell = ℒEFT0 // EOMSimplify;
         ℒEFT0OnShell - ℒSM // CollectOperators // HcSimplify // NiceForm
```

Out[19]//NiceForm=

$$\frac{1}{2} A^2 \frac{1}{M^4} \left(M^2 - 2\,mH^2\right) \bar{H}_i \bar{H}_j H^i H^j + \frac{1}{6} A^2 \frac{1}{M^6} \left(-6\,A^2 - 3\,M^2\,(\kappa - 2\,\lambda h) + A\,\mu\right) \bar{H}_i \bar{H}_j \bar{H}_k H^i H^j H^k - A^2 \frac{1}{M^4} \bar{H}_i D_\mu \bar{H}_j H^i D_\mu H^j + \left(\frac{1}{2} Ye^{rp} A^2 \frac{1}{M^4} \bar{H}_i H^i H^j \left(\bar{l}^r_j \cdot P_R \cdot e^p\right) + \frac{1}{2} Yd^{rp} A^2 \frac{1}{M^4} \bar{H}_i H^i H^j \left(\bar{q}^r_{aj} \cdot P_R \cdot d^{ap}\right) + \frac{1}{2} Yu^{rp} A^2 \frac{1}{M^4} \bar{H}_i \bar{H}_j H^j \left(\bar{q}^r_{ak} \cdot P_R \cdot u^{ap}\right) \varepsilon^{ki} + \text{H.c.}\right)$$

One-loop matching

One-loop

Matching

In[20]:=
```
ℒEFT = EchoTiming@EOMSimplify@Match[ℒUV, LoopOrder -> 1, EFTOrder -> 6] /. ϵ^-1 -> 0;
ℒEFT - ℒSM // CollectOperators // HcSimplify // NiceForm
```

» The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.

» Added new CG cg1 with indices {Bar[SU2L[fund]], SU2L[adj], Bar[SU2L[fund]]}

57.477

Out[21]//NiceForm=

$$\left(cHH - mH^2 + \frac{1}{6}\hbar\, cHH\, A^2 \frac{1}{M^4}\left(-3M^2 + 2\, cHH\left(8 + 3\, \mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right)\right)\bar H_i H^i +$$

$$\left(\frac{1}{2}A^2\frac{1}{M^4}\left(-2\, cHH + M^2\right) + \frac{1}{36}\hbar\frac{1}{M^6}\left(6 A\kappa\mu M^2\left(-5\, cHH + 3M^2\right) + 3M^4\kappa^2\left(-cHH + 3M^2\, \mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right) - 12\mu A^3\left(-3M^2 + cHH\left(14 + 3\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right)\right) +\right.$$

$$18A^4\left(-M^2\left(7 + 4\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right) + cHH\left(38 + 20\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right) - A^2\left(cHH\left(-348\kappa M^2 + 480\lambda h M^2 + 36\lambda\phi M^2 - 33\mu^2 - 252\kappa M^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right] +\right.\right.$$

$$288\lambda h M^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right] + 36\lambda\phi M^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right] + gL^2M^2\left(31 + 30\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right) + gY^2M^2\left(31 + 30\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\Bigg) +$$

$$9M^2\left(\mu^2 - M^2\right)\left(\lambda\phi\left(1 + \mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right) - 4\kappa\left(3 + 2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right) + 2\lambda h\left(7 + 6\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right)\Bigg)\Bigg)\Bigg)\bar H_i \bar H_j H^i H^j$$

+ 43 more terms

In[26]:=
```
PrintEffectiveCouplings[ℒEFT]
```

$$cHH = \hbar A^2 + \hbar\frac{1}{\epsilon}A^2 + mH^2 - \hbar\, mH^2 A^2\frac{1}{M^2} - \hbar\frac{1}{\epsilon}mH^2A^2\frac{1}{M^2} + \hbar\, mH^4A^2\frac{1}{M^4} + \hbar\frac{1}{\epsilon}mH^4A^2\frac{1}{M^4} + \frac{1}{2}\hbar\kappa M^2 + \frac{1}{2}\hbar\frac{1}{\epsilon}\kappa M^2 -$$

$$\frac{1}{2}\hbar A\mu - \frac{1}{2}\hbar\frac{1}{\epsilon}A\mu + \hbar A^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right] - \hbar\, mH^2A^2\frac{1}{M^2}\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right] + \hbar\, mH^4A^2\frac{1}{M^4}\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right] + \frac{1}{2}\hbar\kappa M^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right] - \frac{1}{2}\hbar A\mu\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]$$

Conclusions

- Functional methods well-suited for automation in MATCHETE
  - Currently supported UV states: Scalars, Fermions
  - For heavy vectors only tree-level matching is available
- Reduction of $\mathscr{L}_{\mathrm{EFT}}$ to a nearly minimal and *Warsaw like* basis
  - Fierz identities not yet automatically implemented due to evanescent operators
- Functional methods can be extended to computations of β-functions and evanescent operator contributions

Thank you for your attention!

Back up

Defining the SM — symmetries

Define gauge groups

```
In[2]:= DefineGaugeGroup[SU3c, SU[3], gs, G,
          FundAlphabet -> {"a","b","c","d","e","f"},
          AdjAlphabet -> {"A","B","C","D","E","F"}]
        DefineGaugeGroup[SU2L, SU[2], gL, W,
          FundAlphabet -> {"i","j","k","l","m","n"},
          AdjAlphabet -> {"I","J","K","L","M","N"}]
        DefineGaugeGroup[U1Y, U1, gY, B]
```

labels used for printing

label, gauge group, gauge coupling, gauge field

Define flavor indices

```
In[3]:= DefineFlavorIndex[Flavor,3,IndexAlphabet->{"p","r","s","t","u","v"}]
```

Defining the SM — fields

Fermions

```
In[4]:= DefineField[q, Fermion, Indices -> {SU3c[fund],SU2L[fund],Flavor},
          Charges -> {U1Y[1/6]}, Chiral -> LeftHanded, Mass -> 0]
        DefineField[u, Fermion, Indices -> {SU3c[fund], Flavor},
          Charges -> {U1Y[2/3]}, Chiral -> RightHanded, Mass -> 0]
        DefineField[d, Fermion, Indices -> {SU3c[fund], Flavor},
          Charges -> {U1Y[-1/3]}, Chiral -> RightHanded, Mass -> 0]
        DefineField[l, Fermion, Indices -> {SU2L[fund], Flavor},
          Charges -> {U1Y[-1/2]}, Chiral -> LeftHanded, Mass -> 0]
        DefineField[e, Fermion, Indices -> {Flavor},
          Charges -> {U1Y[-1]}, Chiral -> RightHanded, Mass -> 0]
```

Higgs

```
In[5]:= DefineField[H, Scalar, Indices -> {SU2L[fund]},
          Charges -> {U1Y[1/2]}, Mass -> 0];
```

Defining the SM — couplings

Yukawa couplings

```
In[6]:= DefineCoupling[Yu, Indices -> {Flavor, Flavor}]
        DefineCoupling[Yd, Indices -> {Flavor, Flavor}]
        DefineCoupling[Ye, Indices -> {Flavor, Flavor}]
```

Higgs mass and coupling

```
In[7]:= DefineCoupling[μ, SelfConjugate -> True, EFTOrder -> 1]
        DefineCoupling[λ, SelfConjugate -> True, EFTOrder -> 0]
```

Defining the SM — Lagrangian

Yukawa interactions

```
In[8]:= YukawaL = Ye[p,r] Bar[l[i,p]]**e[r] H[i]
          + Yd[p,r] Bar[q[a,i,p]]**d[a,r] H[i]
          + Yu[p,r] Bar[q[a,i,p]]**u[a,r] CG[eps[SU2L], {i,j}] Bar[H[j]];
```

Scalar potential

In[9]:= HiggsPotential = $-\mu[]^2$ Bar[H[i]]H[i] + $\frac{\lambda[]}{2}$ Bar[H[i]]H[i]Bar[H[j]]H[j];

Full SM Lagrangian

```
In[10]:= LSM = FreeLag[q, u, d, l, e, H, G, W, B]
           - PlusHc[YukawaL] - HiggsPotential //RelabelIndices;
         LSM //HcSimplify //NiceForm
```

Out[10]= $-\frac{1}{4}B^{\mu\nu 2} -\frac{1}{4}G^{\mu\nu A 2} -\frac{1}{4}W^{\mu\nu I 2} +D_\mu \bar{H}_i D^\mu H^i +\mu^2 \bar{H}_i H^i -\frac{1}{2}\lambda \bar{H}_i \bar{H}_j H^i H^j +i(\bar{d}^p_a \cdot \gamma_\mu P_R \cdot D_\mu d^{ap})$

$+i(\bar{e}^p \cdot \gamma_\mu P_R \cdot D_\mu e^p) +i(\bar{l}^p_i \cdot \gamma_\mu P_L \cdot D_\mu l^{ip}) +i(\bar{q}^p_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{aip}) +i(\bar{u}^p_a \cdot \gamma_\mu P_R \cdot D_\mu u^{ap})$

$+\left(-Ye^{rp} H^i (\bar{l}^r_i \cdot P_R \cdot e^p) -Yd^{rp} H^i (\bar{q}^r_{ai} \cdot P_R \cdot d^{ap}) -Yu^{rp} \bar{H}_i (\bar{q}^r_{aj} \cdot P_R \cdot u^{ap}) \varepsilon^{ji} + \mathrm{H.c.}\right)$

Covariant derivative expansion (CDE)

Henning, Lu, Murayama [1412.1837; 1604.01019]

- Operators $Q(iD_\mu, U_m)$ can depend on covariant derivatives D_μ and a set of momentum-independent functions U_m
- Supertraces not manifestly covariant (open covariant derivatives $D_\mu \mathbb{1}$)

$$\mathrm{STr}\left(Q(iD_\mu, U_m)\right) = \pm \int \frac{\mathrm{d}^d k}{(2\pi)^d} \langle k \,|\, \mathrm{tr}\left(Q(iD_\mu, U_m)\right) |\, k\rangle = \pm \int \mathrm{d}^d x \int \frac{\mathrm{d}^d k}{(2\pi)^d} \,\mathrm{tr}\left(Q(iD_\mu + k_\mu, U_m)\right) \mathbb{1}$$

- **Covariant derivative expansion (CDE)**

  Path integral transformation sandwiching the trace between $e^{-iD\cdot\partial_k}$ and $e^{iD\cdot\partial_k}$

  - These operators vanish when acting to the left / right
  - Pass $e^{-iD\cdot\partial_k}$ through Q to cancel against $e^{iD\cdot\partial_k}$
    $\Rightarrow$ putts all covariant derivatives D_μ into commutators/field-strengths $F_{\mu\nu}$

➡ Functional matching approach and supertraces are manifestly covariant

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➡ Functional matching approach and supertraces are manifestly covariant

CDE

Covariant Derivative Expansion of the supertrace:

$$\mathrm{STr}\left(Q(iD_\mu, U_k)\right) = \pm\int \mathrm{d}^d x \int \frac{\mathrm{d}^d p}{(2\pi)^d}\, e^{-iD\cdot\partial_p}\, \mathrm{tr}\left(Q(iD_\mu + p_\mu, U_k)\right) e^{iD\cdot\partial_p}$$

- Transformation properties:
  - $e^{-iD\cdot\partial_p}(p_\mu + iD_\mu)e^{-iD\cdot\partial_p} = p_\mu + i\tilde{G}_{\mu\nu}\partial_p^\nu$
  - $\tilde{G}_{\mu\nu} \equiv \sum_{n=0}^{\infty} \frac{(-i)^n}{(n+2)n!}(D_{\{\alpha_1,\ldots,\alpha_n\}}G_{\mu\nu})\partial_p^{\alpha_1}\cdots\partial_p^{\alpha_n}$
  - $\tilde{U}_k \equiv e^{-iD\cdot\partial_p}U_k e^{-iD\cdot\partial_p} = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!}(D_{\{\alpha_1,\ldots,\alpha_n\}}U_k)\partial_p^{\alpha_1}\cdots\partial_p^{\alpha_n}$
- The covariant supertrace

$$\mathrm{STr}\left(Q(iD_\mu, U_k)\right) = \pm\int \mathrm{d}^d x \int \frac{\mathrm{d}^d p}{(2\pi)^d}\, \mathrm{tr}\left(Q(p_\mu + i\tilde{G}_{\mu\nu}\partial_p^\nu, \tilde{U}_k(x))\right)$$

Example: scalar toy-model

Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

Two real scalars with mass hierarchy $M \gg m$

$$\mathscr{L}_{\rm UV}(\varphi, \Phi) = \frac{1}{2}\left(\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2\right) + \frac{1}{2}\left(\partial_\mu \Phi \partial^\mu \Phi - M^2 \Phi^2\right) - \frac{\kappa}{4!}\varphi^4 - \frac{\lambda}{3!}\varphi^3 \Phi$$

- Integrate out Φ applying the functional method up to $d = 6$
- **Tree-level matching:**
  - Equation of motion: $M^2 \hat{\Phi} = -D^2 \hat{\Phi} - \frac{\lambda}{3!}\hat{\varphi}^3$
  - Solution: $\hat{\Phi} = -\frac{\lambda}{6M^2}\hat{\varphi}^3 + \mathcal{O}(M^{-4})$
  - Tree-level EFT Lagrangian:

$$\mathscr{L}^{(0)}_{\rm EFT} = \frac{1}{2}\left(\partial_\mu \hat{\varphi} \partial^\mu \hat{\varphi} - m^2 \hat{\varphi}^2\right) - \frac{\kappa}{4!}\hat{\varphi}^4 + \frac{10\lambda^2}{6!M^2}\hat{\varphi}^6$$

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Example: scalar toy-model

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$$\mathscr{L}_{\rm UV}(\varphi,\Phi) = \frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2\right) + \frac{1}{2}\left(\partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2\right) - \frac{\kappa}{4!}\varphi^4 - \frac{\lambda}{3!}\varphi^3\Phi$$

- Integrate out Φ applying the functional method up to $d = 6$
- **Tree-level matching:**
  - Equation of motion: $M^2\hat{\Phi} = -D^2\hat{\Phi} - \frac{\lambda}{3!}\hat{\varphi}^3$
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  - Tree-level EFT Lagrangian:

$$\mathscr{L}^{(0)}_{\rm EFT} = \frac{1}{2}\left(\partial_\mu\hat{\varphi}\partial^\mu\hat{\varphi} - m^2\hat{\varphi}^2\right) - \frac{\kappa}{4!}\hat{\varphi}^4 + \frac{10\lambda^2}{6!M^2}\hat{\varphi}^6$$

Scalar toy-model @ one-loop

- The fluctuation operator $\mathcal{O}_{ij}$

$$\Delta_\Phi^{-1} = -\partial^2 - M^2, \qquad X_{\Phi\Phi} = 0, \qquad X_{\varphi\Phi}^{[2]} = (X_{\varphi\Phi}^{[2]})^\dagger = -\frac{\lambda}{2}\hat{\varphi}^2,$$

$$\Delta_\varphi^{-1} = -\partial^2, \qquad X_{\varphi\varphi}^{[2]} = -m^2 - \frac{\kappa}{2}\hat{\varphi}^2 - \lambda\hat{\varphi}\hat{\Phi} = -m^2 - \frac{\kappa}{2}\hat{\varphi}^2 - \frac{\lambda^2}{6M^2}\hat{\varphi}^4$$

- Supertraces to compute with the CDE:
  - Log-type: $\mathrm{STr}\left(\ln \Delta_\Phi^{-1}\right)\Big|_{\mathrm{hard}}$
  - Power-type: $\mathrm{STr}\left(\Delta_\Phi X_{\Phi\varphi}^{[2]} \Delta_\varphi X_{\varphi\Phi}^{[2]}\right)\Big|_{\mathrm{hard}}$, $\mathrm{STr}\left(\Delta_\Phi X_{\Phi\varphi}^{[2]} \Delta_\varphi X_{\varphi\varphi}^{[2]} \Delta_\varphi X_{\varphi\Phi}^{[2]}\right)\Big|_{\mathrm{hard}}$

- One-loop EFT Lagrangian from supertrace evaluation:

$$\mathscr{L}_{\mathrm{EFT}}^{(1)} = \frac{1}{16\pi^2}\frac{\lambda^2}{16}\left[2\left(1 + \frac{m^2}{M^2}\right)\hat{\varphi}^4 - \frac{1}{M^2}\hat{\varphi}^2\partial^2\hat{\varphi}^2 + \frac{\kappa}{M^2}\hat{\varphi}^6\right]$$

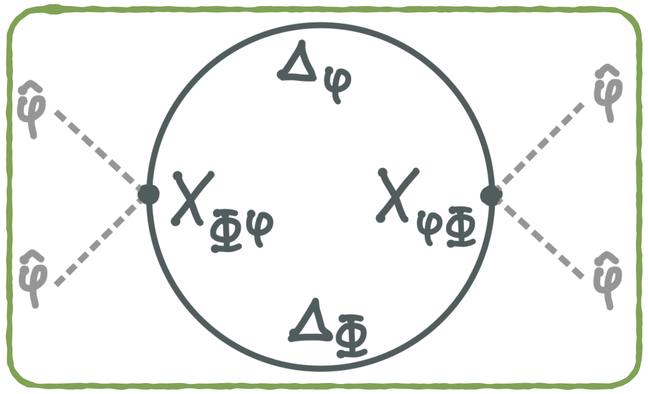
Scalar toy-model @ one-loop

- The fluctuation operator $\mathcal{O}_{ij}$

$$\Delta_\Phi^{-1} = -\partial^2 - M^2, \qquad X_{\Phi\Phi} = 0, \qquad X_{\varphi\Phi}^{[2]} = (X_{\varphi\Phi}^{[2]})^\dagger = -\frac{\lambda}{2}\hat{\varphi}^2,$$

$$\Delta_\varphi^{-1} = -\partial^2, \qquad X_{\varphi\varphi}^{[2]} = -m^2 - \frac{\kappa}{2}\hat{\varphi}^2 - \lambda\hat{\varphi}\hat{\Phi} = -m^2 - \frac{\kappa}{2}\hat{\varphi}^2 - \frac{\lambda^2}{6M^2}\hat{\varphi}^4$$

- Supertraces to compute with the CDE:
  - Log-type: $\mathrm{STr}\left(\ln \Delta_\Phi^{-1}\right)\Big|_{\mathrm{hard}}$
  - Power-type: $\mathrm{STr}\left(\Delta_\Phi X_{\Phi\varphi}^{[2]}\Delta_\varphi X_{\varphi\Phi}^{[2]}\right)\Big|_{\mathrm{hard}}$, $\mathrm{STr}\left(\Delta_\Phi X_{\Phi\varphi}^{[2]}\Delta_\varphi X_{\varphi\varphi}^{[2]}\Delta_\varphi X_{\varphi\Phi}^{[2]}\right)\Big|_{\mathrm{hard}}$



diagrammatic representation of supertraces

- One-loop EFT Lagrangian from supertrace evaluation:

$$\mathscr{L}_{\mathrm{EFT}}^{(1)} = \frac{1}{16\pi^2}\frac{\lambda^2}{16}\left[2\left(1 + \frac{m^2}{M^2}\right)\hat{\varphi}^4 - \frac{1}{M^2}\hat{\varphi}^2\partial^2\hat{\varphi}^2 + \frac{\kappa}{M^2}\hat{\varphi}^6\right]$$

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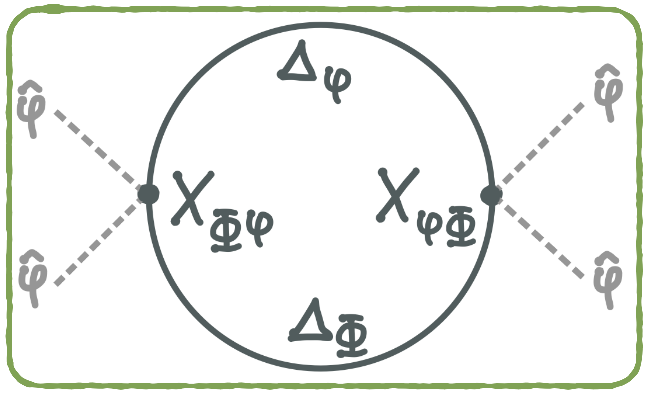

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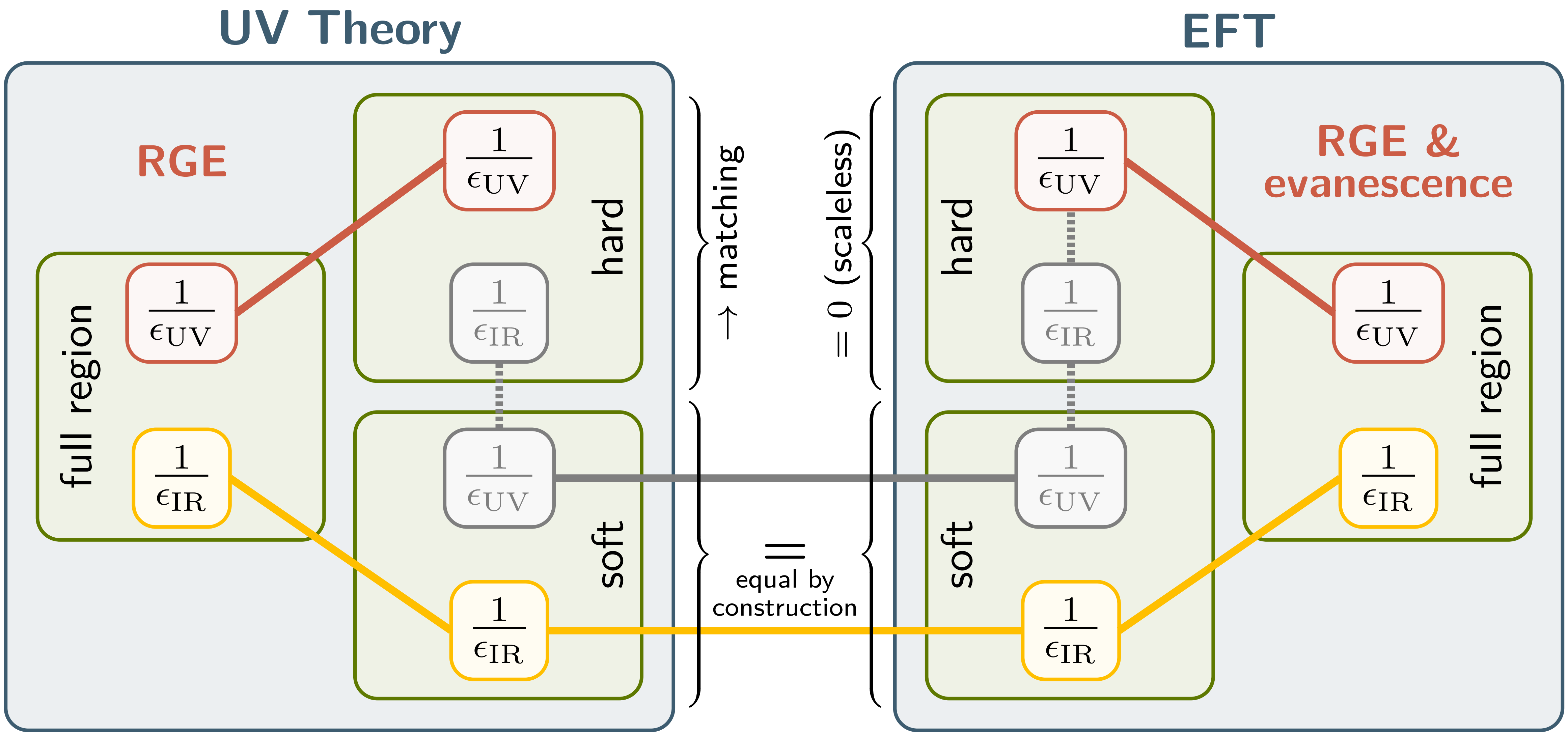
Method of regions



Isidori, FW, Wyler [2303.16922]

- The artificial IR poles of the hard region of the UV theory integrals provide the counterterms for the full EFT Lagrangian.

➡ The EFT is automatically renormalized.

Method of regions

Beneke, Smirnov [hep-ph/9711391]
Jantzen [1111.2589]

How to evaluate loop integrals in supertraces ?

- **Method of regions in dimensional regularization:**
  - The loop-integrals contain light m_L and heavy m_H masses $(m_H \gg m_L)$
  - Separate and expand in momentum regions:
    soft-region: $p \sim m_L$ $\leftrightarrow$ *hard-region:* $p \sim m_H$
  - Integrate each region over the full d-dimensional space
  - Summing both integrals gives the full integral without expansion

$$I = \int \mathrm{d}^d p \, \frac{N}{(p^2 - m_L^2)(p^2 - m_H^2)} = I_{\mathrm{soft}} + I_{\mathrm{hard}}$$

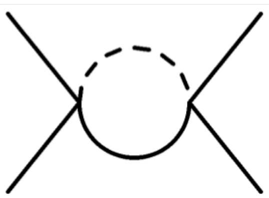
$$I_{\mathrm{soft}} = \int \mathrm{d}^d p \, \frac{N}{(p^2 - m_L^2)(-m_H^2)} \left[1 + \frac{p^2}{m_H^2} + \frac{p^4}{m_H^4} + \cdots \right], \quad I_{\mathrm{hard}} = \int \mathrm{d}^d p \, \frac{N}{p^2 (p^2 - m_H^2)} \left[1 + \frac{m_L^2}{p^2} + \frac{m_L^4}{p^4} + \cdots \right]$$

- All the short distance effects we are interested in are encoded in hard region

Method of regions: toy model

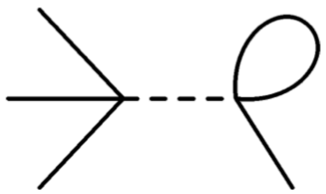
$$\mathscr{L}_{\rm UV}(\varphi,\Phi) = \frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2\right) + \frac{1}{2}\left(\partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2\right) - \frac{\kappa}{4!}\varphi^4 - \frac{\lambda}{3!}\varphi^3\Phi$$

UV theory (soft and hard contributions)



$$= \frac{i}{16\pi^2}\lambda^2\left[3 + 3\frac{m^2}{M^2} + \frac{s+t+u}{2M^2}\right]\Bigg|_{\rm hard}$$

$$+ \frac{i}{16\pi^2}\lambda^2\left[-3\frac{m^2}{M^2} + 3\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right]\Bigg|_{\rm soft} + \mathcal{O}(M^{-4}),$$



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$\Gamma^{(1)}_{\rm UV}\Big|_{\rm hard}$

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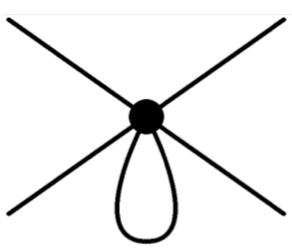
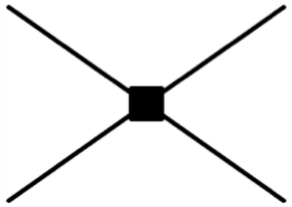
EFT (only soft contributions)



$$= \frac{i}{16\pi^2}\lambda^2\left[-5\frac{m^2}{M^2} + 5\frac{m^2}{M^2}\log\left(\frac{m^2}{M^2}\right)\right] + \mathcal{O}(M^{-4}),$$



$$= iC_{\varphi^4} - i\frac{C_{\varphi^4\partial^2}}{M^2}(s+t+u)$$

$\mathscr{L}^{(0)}_{\rm EFT}$

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Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

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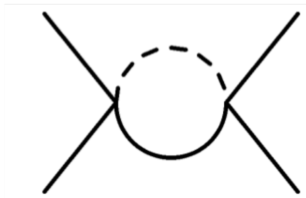

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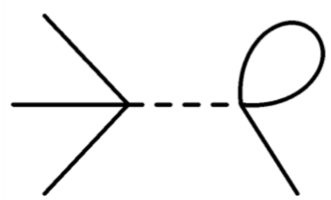

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$\Gamma^{(1)}_{\rm UV}\big|_{\rm hard}$

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cancel exactly

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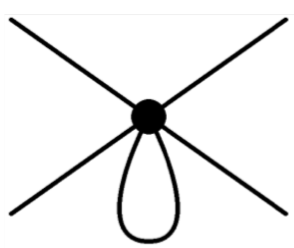

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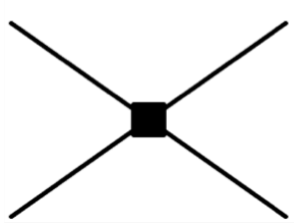

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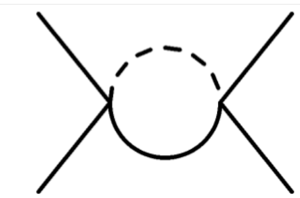

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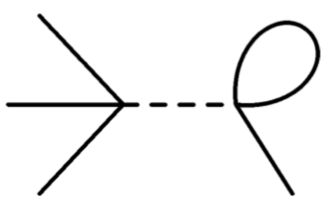

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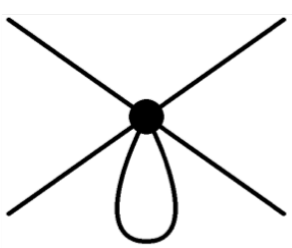

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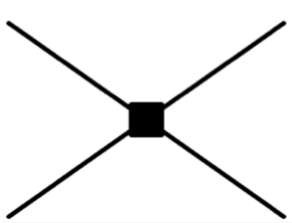

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matching condition

Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

γ_5 prescription in d dimensions

Fuentes-Martin, König, Pages, Thomsen, FW [2012.08506]

- Continuation of γ_5 to d dimensions: semi-naive dimensional regularization
- $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad \{\gamma^\mu, \gamma_5\} = 0, \quad \gamma_5^2 = 0 \quad$ for $\mu, \nu = 1, \ldots, d$
- Abandon cyclicity of tr with odd # of $\gamma_5 \rightarrow$ require: $\mathrm{tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5] = -4i\varepsilon^{\mu\nu\rho\sigma}$

➡ Reading point ambiguity of $\mathcal{O}(\epsilon)$ — which is the left-most γ matrix?

  - Only relevant in divergent diagrams: ambiguity $\mathcal{O}(\epsilon)$ can combine with pole $\mathcal{O}(\epsilon^{-1})$

- UV theory does not contain UV poles (anomaly cancellation)
- Method of regions can introduce spurious IR divergences in the UV theory due to expansion in hard momentum region
- These poles cancel exactly agains UV divergences introduced in the EFT

➡ $\mathcal{O}(\epsilon)$ ambiguities also cancel if same reading point is chosen in EFT and UV

RGE computation

- **Deriving Renormalization Group Equations (RGE) using supertraces:**
  - In the EFT there are no heavy particles
  - Do not expand loop integrals in heavy masses
  - Compute the UV poles of all supertraces in the EFT
  - ➡ β functions of the EFT
- **Possible issues:**
  - The matching might not generate the full EFT basis
  - RGE derivation yields new operators
  - ➡ Repeated application of RGE derivation

Field redefinitions — EOM

Criado, Perez-Victoria [1811.09413]

- LSZ formula: *S*-matrix invariant under field redefinitions
- Perturbative field redefinition: $$\eta \to \tilde{\eta} = \eta + \frac{1}{\Lambda}\delta\eta$$
- Shifting the EFT Lagrangian:

$$\mathscr{L}[\eta] \to \mathscr{L}[\tilde{\eta}] = \mathscr{L}[\eta] + \frac{1}{\Lambda}\frac{\delta \mathscr{L}[\tilde{\eta}]}{\delta\tilde{\eta}}\bigg|_{\tilde{\eta}=\eta}\delta\eta$$

EOM

Toy model:

$$\partial^2\hat{\varphi} = -m^2\hat{\varphi} - \left(\frac{\kappa}{3!} - \frac{\lambda^2}{32\pi^2}\right)\hat{\varphi}^3$$

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EOM

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Example: vector-like fermions

Defining models:

Preliminary

In[3]:=
```
<< Matchete`
```

MATCHETE

Gauge groups

In[5]:=
```
(* U(1) gauge group with coupling e and field-strength A *)
DefineGaugeGroup[U1e, U[1], e, A]
```

Field content (except for gauge fields)

In[6]:=
```
(* heavy vector-like fermion with charge 1 *)
DefineField[Ψ, Fermion, Charges → {U1e[1]}, Mass → {Heavy, M}]
```

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```

In[8]:=
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ψ light fermion with charge 1
ϕ massless real scalar

3) Define couplings
y Yukawa coupling order $\mathcal{O}(m_L^0)$

Lagrangian

Writing the Lagrangian

Preliminary

Free Lagrangian

In[10]:=
```
ℒfree = FreeLag[];
ℒfree // NiceForm
```

Out[11]//NiceForm=

$$-\frac{1}{4} A^{\mu\nu\,2} + \frac{1}{2} \left(D_\mu\phi\right)^2 + i \left(\bar{\psi} \cdot \gamma_\mu \cdot D_\mu\psi\right) - m \left(\bar{\psi} \cdot \psi\right) + i \left(\bar{\Psi} \cdot \gamma_\mu \cdot D_\mu\Psi\right) - M \left(\bar{\Psi} \cdot \Psi\right)$$

Interactions

In[12]:=
```
ℒint = -y[] × Bar[ψ[]] ** PR ** Ψ[] ϕ[] // PlusHc;
ℒint // NiceForm
```

Out[13]//NiceForm=

$$-y\,\phi \left(\bar{\psi} \cdot P_R \cdot \Psi\right) - \bar{y}\,\phi \left(\bar{\Psi} \cdot P_L \cdot \psi\right)$$

Full Lagrangian

In[14]:=
```
ℒ = ℒfree + ℒint;
ℒ // NiceForm
```

Out[15]//NiceForm=

$$-\frac{1}{4} A^{\mu\nu\,2} + \frac{1}{2} \left(D_\mu\phi\right)^2 + i \left(\bar{\psi} \cdot \gamma_\mu \cdot D_\mu\psi\right) - m \left(\bar{\psi} \cdot \psi\right) + i \left(\bar{\Psi} \cdot \gamma_\mu \cdot D_\mu\Psi\right) - M \left(\bar{\Psi} \cdot \Psi\right) - y\,\phi \left(\bar{\psi} \cdot P_R \cdot \Psi\right) - \bar{y}\,\phi \left(\bar{\Psi} \cdot P_L \cdot \psi\right)$$

Tree-level matching

Tree-level matching

Preliminary

In[16]:=
```
ℒEFT0 = Match[ℒ, LoopOrder → 0, EFTorder → 6];
ℒEFT0 // NiceForm
```

Out[17]//NiceForm=

$$-\frac{1}{4}A^{\mu\nu 2}+\frac{1}{2}(D_\mu\phi)^2+i(\bar\psi\cdot\gamma_\mu\cdot D_\mu\psi)-m(\bar\psi\cdot\psi)+i\,\bar y\,y\,\frac{1}{M^2}\,\phi\,D_\mu\phi\,(\bar\psi\cdot\gamma_\mu P_L\cdot\psi)+i\,\bar y\,y\,\frac{1}{M^2}\,\phi^2\,(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi)$$

Simplifications

IBP simplification

In[18]:=
```
ℒEFT0IBP = ℒEFT0 // IBPSimplify // RelabelIndices;
ℒEFT0IBP // HcSimplify // NiceForm
```

Out[19]//NiceForm=

$$-\frac{1}{4}A^{\mu\nu 2}+\frac{1}{2}(D_\mu\phi)^2+i(\bar\psi\cdot\gamma_\mu\cdot D_\mu\psi)-m(\bar\psi\cdot\psi)+\left(-\frac{i}{2}\,\bar y\,y\,\frac{1}{M^2}\,\phi^2\,(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi)+\text{h.c.}\right)$$

Field redefinitions

In[20]:=
```
ℒEFT0IBP // EoMSimplify // HcSimplify // NiceForm
```

Out[20]//NiceForm=

$$-\frac{1}{4}A^{\mu\nu 2}+\frac{1}{2}(D_\mu\phi)^2+i(\bar\psi\cdot\gamma_\mu\cdot D_\mu\psi)+\left(-m(\bar\psi\cdot P_R\cdot\psi)+\frac{1}{2}\,m\,\bar y\,y\,\frac{1}{M^2}\,\phi^2\,(\bar\psi\cdot P_R\cdot\psi)+\text{h.c.}\right)$$

One-loop matching

One-loop matching

Preliminary

In[21]:=
```
ℒEFT1 = Match[ℒ, LoopOrder → 1, EFTorder → 6];
ℒEFT1 // NiceForm
```

Out[22]//NiceForm=

$$-\frac{1}{4} A^{\mu\nu\,2} + \frac{7}{270}\hbar e^2 \frac{1}{M^2}\left(D_\rho A^{\mu\nu}\right)^2 + \frac{1}{20}\hbar e^2\frac{1}{M^2} A^{\mu\nu} D^2 A^{\mu\nu} + \frac{1}{240}\hbar e^2 \frac{1}{M^2} D_\nu D_\rho A^{\mu\nu} A^{\mu\rho} + \frac{1}{240}\hbar e^2\frac{1}{M^2} D_\rho D_\nu A^{\mu\nu}A^{\mu\rho} + \frac{1}{90}\hbar e^2\frac{1}{M^2} D_\rho A^{\mu\nu} D_\nu A^{\mu\rho} +$$

$$\frac{7}{270}\hbar e^2 \frac{1}{M^2} D_\nu A^{\mu\nu} D_\rho A^{\mu\rho} + \frac{7}{240}\hbar e^2\frac{1}{M^2} A^{\mu\nu} D_\nu D_\rho A^{\mu\rho} + \frac{7}{240}\hbar e^2 \frac{1}{M^2} A^{\mu\nu} D_\rho D_\nu A^{\mu\rho} + \frac{1}{120}\hbar e^2\frac{1}{M^2} D_\mu D_\rho A^{\mu\nu} A^{\nu\rho} + \frac{1}{120}\hbar e^2 \frac{1}{M^2} D_\rho D_\mu A^{\mu\nu} A^{\nu\rho} -$$

$$\frac{2}{135}\hbar e^2\frac{1}{M^2} D_\rho A^{\mu\nu} D_\mu A^{\nu\rho} + \frac{1}{27}\hbar e^2\frac{1}{M^2} D_\mu A^{\mu\nu} D_\rho A^{\nu\rho} - \frac{1}{40}\hbar e^2 \frac{1}{M^2} A^{\mu\nu} D_\mu D_\rho A^{\nu\rho} - \frac{1}{40}\hbar e^2\frac{1}{M^2} A^{\mu\nu} D_\rho D_\mu A^{\nu\rho} - \frac{1}{3} \hbar e^2 A^{\mu\nu\,2}\,\mathrm{Log}\left[\bar\mu^2 \frac{1}{M^2}\right] +$$

$$\frac{1}{2} \left(D_\mu\phi\right)^2 - 2\hbar\bar y y m^2\phi^2 - 2\hbar \bar y y m^4 \frac{1}{M^2}\phi^2 - 2\hbar\bar y y M^2\phi^2 + \frac{1}{9}\hbar\bar y y\frac{1}{M^2}\phi D^2D^2\phi + \frac{1}{9}\hbar\bar y y \frac{1}{M^2}\phi D_\mu D_\nu D_\mu D_\nu\phi + \frac{1}{9} \hbar\bar y y\frac{1}{M^2}\phi D_\mu D^2 D_\mu \phi -$$

$$2\hbar\bar y y m^2\phi^2\, \mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right] - 2\hbar\bar y y m^4\frac{1}{M^2}\phi^2\, \mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right] - 2\hbar\bar y y M^2\phi^2\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right] - \frac{1}{2}\hbar\bar y y\phi D^2\phi + 2\hbar\bar y y m^2\frac{1}{M^2}\phi D^2\phi -$$

$$\hbar\bar y y\phi D^2\phi\, \mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right] + i\left(\bar\psi\cdot\gamma_\mu\cdot D_\mu\psi\right) - m\left(\bar\psi\cdot\psi\right) - \frac{i}{18}\hbar\bar y y\frac{1}{M^2}\left(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu D^2\psi\right) - \frac{i}{18}\hbar\bar y y \frac{1}{M^2}\left(\bar\psi\cdot\gamma_\mu P_L\cdot D_\nu D_\mu D_\nu\psi\right) -$$

$$\frac{i}{18}\hbar\bar y y\frac{1}{M^2}\left(\bar\psi\cdot\gamma_\mu P_L\cdot D^2 D_\mu\psi\right) + \frac{i}{18}\hbar\bar y y\frac{1}{M^2}\left(D^2D_\nu\bar\psi\cdot\gamma_\nu P_L\cdot\psi\right) + \frac{i}{18}\hbar\bar y y\frac{1}{M^2}\left(D_\mu D_\nu D_\mu\bar\psi\cdot\gamma_\nu P_L\cdot\psi\right) + \frac{i}{18}\hbar\bar y y\frac{1}{M^2}\left(D_\mu D^2\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right) +$$

$$\frac{3i}{8}\hbar\bar y y\left(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi\right) + \frac{i}{4}\hbar\bar y y\left(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi\right)\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right] - \frac{3i}{8}\hbar\bar y y\left(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right) - \frac{i}{4}\hbar\bar y y\left(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right)\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right] -$$

$$\hbar\bar y^2 y^2\phi^4\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right] + 2\hbar m^2\bar y^2 y^2\frac{1}{M^2}\phi^4 + \frac{1}{3}\hbar\bar y^3 y^3\frac{1}{M^2}\phi^6 + \frac{13}{12}\hbar\bar y^2y^2\frac{1}{M^2}\phi^2\left(D_\mu\phi\right)^2 + \frac{13}{12}\hbar\bar y^2 y^2\frac{1}{M^2}D^2\phi\,\phi^3 + \frac{1}{3}\hbar\bar y y e^2\frac{1}{M^2}\phi^2 A^{\mu\nu\,2} +$$

$$\frac{5}{12}\hbar e\bar y y\frac{1}{M^2}D_\nu A^{\mu\nu}\left(\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right) - \frac{1}{12}\hbar e\bar y y\frac{1}{M^2}D_\mu A^{\mu\nu}\left(\bar\psi\cdot\gamma_\nu P_L\cdot\psi\right) - \frac{1}{8}\hbar e\bar y y\frac{1}{M^2}A^{\mu\nu}\left(\bar\psi\cdot\Gamma_{\mu\nu\rho}P_L\cdot D_\rho\psi\right) +$$

$$\frac{1}{8}\hbar e\bar y y\frac{1}{M^2}A^{\mu\nu}\left(D_\rho\bar\psi\cdot\Gamma_{\mu\nu\rho}P_L\cdot\psi\right) + i\bar y y\frac{1}{M^2}\phi D_\mu\phi\left(\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right) + i\bar y y\frac{1}{M^2}\phi^2\left(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi\right) - \frac{5i}{4}\hbar\bar y^2y^2\frac{1}{M^2}\phi^2\left(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi\right) -$$

$$i\hbar\bar y^2y^2\frac{1}{M^2}\phi^2\left(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi\right)\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right] + \frac{5i}{4}\hbar\bar y^2 y^2\frac{1}{M^2}\phi^2\left(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right) + i\hbar\bar y^2y^2\frac{1}{M^2}\phi^2\left(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right)\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]$$

Operator simplifications

Simplification

Preliminary

In[23]:=
```
ℒEFT1IBP = ℒEFT1 // IBPSimplify // RelabelIndices // CollectTerms;
ℒEFT1IBP // HcSimplify // NiceForm
```

Out[24]//NiceForm=

$$-\frac{2}{15}\hbar e^2 \frac{1}{M^2} D_\nu A^{\mu\nu} D_\rho A^{\mu\rho} + i\left(\bar\psi\cdot\gamma_\mu\cdot D_\mu\psi\right) - m\left(\bar\psi\cdot\psi\right) + \frac{1}{3}\hbar\bar y y\frac{1}{M^2}D^2\phi D^2\phi + \left(-\frac{1}{4}-\frac{1}{3}\hbar e^2 \mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)A^{\mu\nu\,2} +$$
$$\hbar\left(2m^2\bar y^2 y^2\frac{1}{M^2} - \bar y^2 y^2\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)\phi^4 + \hbar\left(y m^2\left(-2\bar y - 2\bar y\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right) + y m^4\frac{1}{M^2}\left(-2\bar y - 2\bar y\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right) + y M^2\left(-2\bar y - 2\bar y\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)\right)\phi^2 +$$
$$\left(\frac{1}{2} + \hbar\left(-2\bar y y m^2\frac{1}{M^2} + y\left(\frac{1}{2}\bar y + \bar y\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)\right)\right)\left(D_\mu\phi\right)^2 + \hbar y\left(\frac{3i}{4}\bar y + \frac{i}{2}\bar y\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)\left(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi\right) + \frac{1}{3}\hbar\bar y^3 y^3\frac{1}{M^2}\phi^6 +$$
$$\frac{13}{18}\hbar\bar y^2 y^2\frac{1}{M^2}D^2\phi\,\phi^3 + \frac{1}{3}\hbar\bar y y e^2\frac{1}{M^2}\phi^2 A^{\mu\nu\,2} + \frac{4}{9}\hbar e\bar y y\frac{1}{M^2}D_\nu A^{\mu\nu}\left(\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right) - \frac{1}{4}\hbar e\bar y y\frac{1}{M^2}A^{\mu\nu}\left(\bar\psi\cdot\Gamma_{\mu\nu\rho}P_L\cdot D_\rho\psi\right) +$$
$$\left(-\frac{i}{6}\hbar\bar y y\frac{1}{M^2}\left(D^2\bar\psi\cdot\gamma_\nu P_L\cdot D_\nu\psi\right) + \left(-\frac{1}{2}\bar y y\frac{1}{M^2} + \hbar y^2\frac{1}{M^2}\left(\frac{5i}{4}\bar y^2 + i\bar y^2\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)\right)\phi^2\left(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right) + \mathrm{h.c.}\right)$$

In[25]:=
```
ℒEFT1IBP // EoMSimplify // CollectTerms // HcSimplify // Quiet // NiceForm
```

Out[25]//NiceForm=

$$-\frac{2}{15}\hbar e^2\frac{1}{M^2}D_\nu A^{\mu\nu}D_\rho A^{\mu\rho} + \frac{1}{2}\left(D_\mu\phi\right)^2 + i\left(\bar\psi\cdot\gamma_\mu\cdot D_\mu\psi\right) - \frac{1}{6}\hbar m\bar y y\frac{1}{M^2}\left(\bar\psi\cdot P_L\cdot D^2\psi\right) - \frac{1}{6}\hbar m\bar y y\frac{1}{M^2}\left(\bar\psi\cdot P_R\cdot D^2\psi\right) + \left(-\frac{1}{4}-\frac{1}{3}\hbar e^2\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)A^{\mu\nu\,2} +$$
$$\hbar\left(2m^2\bar y^2 y^2\frac{1}{M^2} - \bar y^2 y^2\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)\phi^4 + \hbar\left(y m^2\left(-2\bar y - 2\bar y\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right) + y m^4\frac{1}{M^2}\left(-2\bar y - 2\bar y\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right) + y M^2\left(-2\bar y - 2\bar y\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)\right)\phi^2 +$$
$$\frac{1}{3}\hbar\bar y^3 y^3\frac{1}{M^2}\phi^6 + \frac{1}{3}\hbar\bar y y e^2\frac{1}{M^2}\phi^2 A^{\mu\nu\,2} + \frac{4}{9}\hbar e\bar y y\frac{1}{M^2}D_\nu A^{\mu\nu}\left(\bar\psi\cdot\gamma_\mu P_L\cdot\psi\right) - \frac{1}{4}\hbar e\bar y y\frac{1}{M^2}A^{\mu\nu}\left(\bar\psi\cdot\Gamma_{\mu\nu\rho}P_L\cdot D_\rho\psi\right) +$$
$$\left(\left(-m + \hbar m y\left(\frac{3}{8}\bar y + \frac{1}{4}\bar y\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)\right)\left(\bar\psi\cdot P_R\cdot\psi\right) + \left(\frac{1}{2}m\bar y y\frac{1}{M^2} + \hbar m y^2\frac{1}{M^2}\left(-\frac{37}{16}\bar y^2 - \frac{19}{8}\bar y^2\,\mathrm{Log}\left[\bar\mu^2\frac{1}{M^2}\right]\right)\right)\phi^2\left(\bar\psi\cdot P_R\cdot\psi\right) + \mathrm{h.c.}\right)$$