Universal One-Loop Effective Action in Gravity

Rémy Larue
(LPSC, Grenoble)

Based on "The Universal One-Loop Effective Action in Gravity"
J. Quevillon, R.L, [2303.10203]

Outline of the talk

- Introduction

Effective Field Theory (EFT) - Universal One-Loop Effective Action

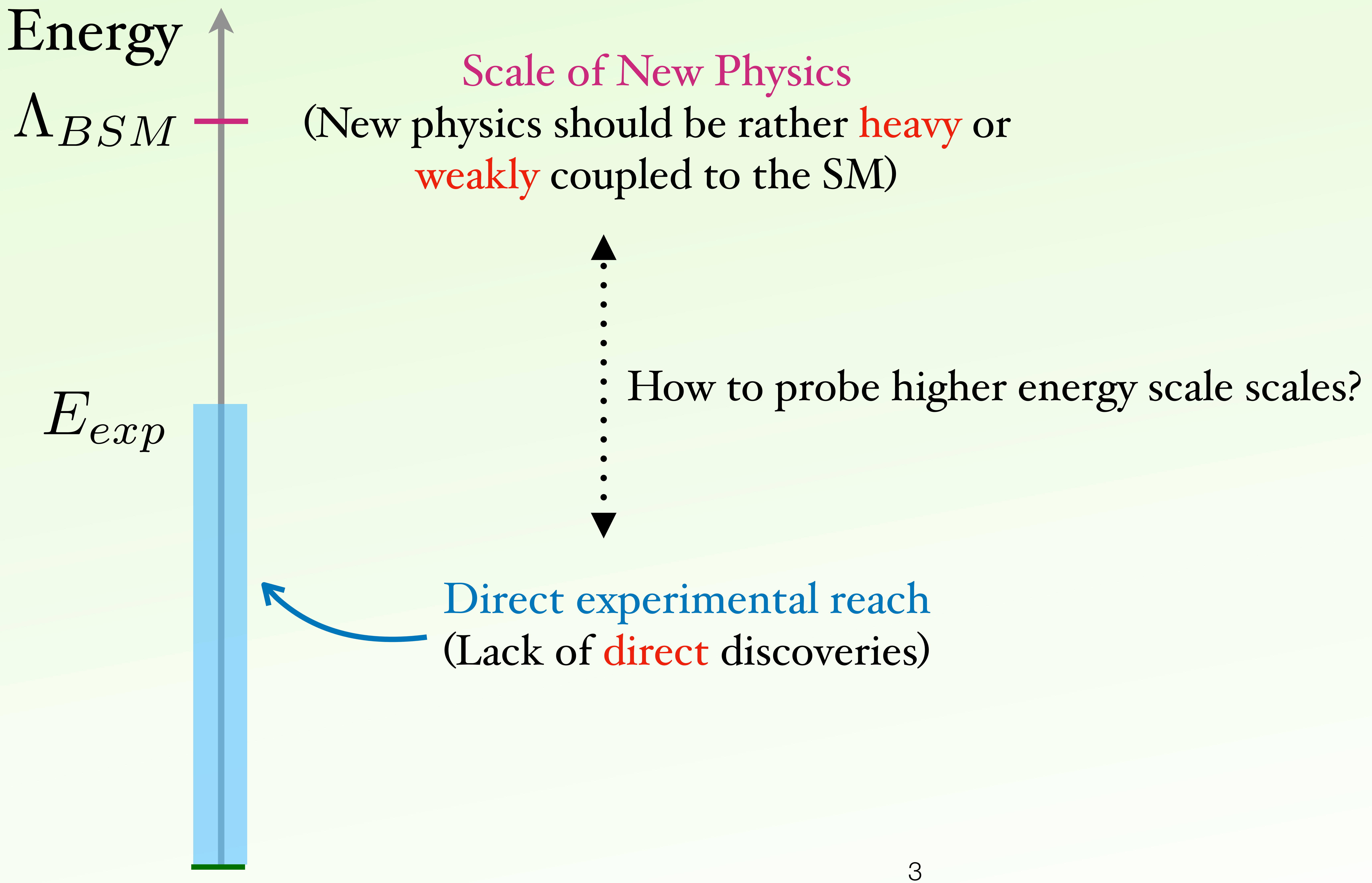
- Matching from the path integral

Functional methods - heat kernel vs CDE - limits of the bosonisation - curved spacetime methods

- Results

New effective operators - possible applications

EFT from the UV point of View



EFT from the UV point of View

Energy

Λ_{BSM} $S[\Phi_H, \phi_L]$

Matching (Compute Wilson coefficients) — Path integral methods → **UOLEA**

Matching done once and for all

$$S_{\text{eff}}[\phi_L] = S_{\text{SM}} + \sum_i c_i(\Lambda_{\text{BSM}})\mathcal{O}_i$$

Running (Done once and for all)

E_{exp} $c_i(E_{\text{exp}})$ → Observables

Mapping (Done once and for all)

Matching

$$e^{iS_{\text{eff}}[\phi_L]} = \int \mathcal{D}\Phi_H e^{iS[\Phi_H,\phi_L]} \Rightarrow iS_{\text{eff}}[\phi_L] = \log\det\left[\frac{\delta^2 S}{\delta\Phi_H^2}\right]$$

Matching

$$e^{iS_{\text{eff}}[\phi_L]} = \int \mathcal{D}\Phi_H e^{iS[\Phi_H,\phi_L]} \Rightarrow iS_{\text{eff}}[\phi_L] = \log\det\left[\frac{\delta^2 S}{\delta\Phi_H^2}\right]$$

Assume a generic form for the action:

$$S[\Phi_H,\phi_L] = \int \mathrm{d}^d x\, \Phi_H \left(D^2 + m^2 + U[\Phi_H,\phi_L]\right)\Phi_H$$

Matching

$$e^{iS_{\text{eff}}[\phi_L]} = \int \mathcal{D}\Phi_H e^{iS[\Phi_H,\phi_L]} \Rightarrow iS_{\text{eff}}[\phi_L] = \log\det\left[\frac{\delta^2 S}{\delta\Phi_H^2}\right]$$

Assume a generic form for the action:

$$S[\Phi_H, \phi_L] = \int \mathrm{d}^d x\, \Phi_H \left(D^2 + m^2 + U[\Phi_H, \phi_L]\right) \Phi_H$$

$$\Longrightarrow \boxed{c_i(m,\mu)\mathcal{O}_i(D,U)}$$ UOLEA

Example: $\mathcal{L} = \Phi(D^2 + m^2 - \eta|H|^2)\Phi \;\Rightarrow \mathcal{O}_6 = |H|^6,\; c_6 = 24\eta^3$

Matching in gravity

$$e^{iS_{\text{eff}}[\phi_L]} = \int \mathcal{D}\Phi_H e^{iS[\Phi_H,\phi_L]} \Rightarrow iS_{\text{eff}}[\phi_L] = \log\det\left[\frac{\delta^2 S}{\delta\Phi_H^2}\right]$$

Assume a generic form for the action:

$$S[\Phi_H, \phi_L] = \int \mathrm{d}^d x\sqrt{-g}\,\Phi_H\left(D^2 + m^2 + U[\Phi_H, \phi_L, R]\right)\Phi_H$$

$$\Rightarrow \quad \boxed{c_i(m,\mu)\mathcal{O}_i(D, U, R)}$$

Functional methods

- Heat kernel

  Position space representation

$$\log\det\left[D^2+m^2+U\right]\simeq\sum_n\int \mathrm{d}\tau\,\tau^{n-3}a_n$$

  Iterative procedure, quite involved, lack of physical insight

Functional methods

- Heat kernel

  Position space representation

$$\log\det\left[D^2+m^2+U\right]\simeq\sum_n\int \mathrm{d}\tau\,\tau^{n-3}a_n$$

  Iterative procedure, quite involved, lack of physical insight

- Covariant Derivative Expansion (CDE)

  Momentum space, systematic, several codes exist

$$\log\det\left[D^2+m^2+U\right]\simeq\int \mathrm{d}^d q\,\mathrm{tr}\sum_{n\geq 0}\left[\Delta(D^2+2iq\cdot D+U)\right]^n\Delta$$

$$\Delta=(q^2-m^2)^{-1}\qquad\qquad \simeq\left(\int \mathrm{d}^d q\,\Delta^k q^{2j}\right)\mathrm{tr}\,U^l D^k$$

Limits of the bosonic action

$$S = \int \mathrm{d}^d x \, \bar{\psi}(i\not{D} - m - Q)\psi \xrightarrow{\text{Bosonisation}} \log\det\left[D^2 + m^2 + U[Q]\right]$$

Limits of the bosonic action

$$S = \int \mathrm{d}^d x\, \bar{\psi}(i\not{D} - m - Q)\psi \xrightarrow[\text{Bosonisation}]{} \log\det\left[D^2 + m^2 + U[Q]\right]$$

- However, it is not possible for a chiral fermion

  Cannot use the usual heat kernel coefficients

Limits of the bosonic action

$$S = \int \mathrm{d}^d x\, \bar{\psi}(i\not{D} - m - Q)\psi \xrightarrow{\text{Bosonisation}} \log\det\left[D^2 + m^2 + U[Q]\right]$$

- However, it is not possible for a chiral fermion

  Cannot use the usual heat kernel coefficients

- Chiral fermion: fermionic CDE (S.Ellis, J.Quevillon, H.Pham Ngoc Hoa, Z.Zhang, "The Fermionic Universal One-Loop Effective Action")

$$\log\det\left[i\not{D} - m - Q\right] \simeq \int \mathrm{d}^d q \sum_{n\geq 0} \left[\Delta(-i\not{D} + Q)\right]^n \Delta$$

$$\Delta = -(\not{q} + m)^{-1} \qquad \simeq \left(\int \mathrm{d}^d q \Delta^i q^{2j}\right) \mathrm{tr}\, U^k \not{D}^l \gamma' s$$

Curved spacetime functional methods

- Curved spacetime methods for computing bosonic determinants :

  Heat kernel (Avramidi 90’, De Witt 75’,…), CDE
  (Gaillard 89’, Alonso 20’)

- They remain quite involved

- Nothing available for a chiral UV theory

CDE in curved spacetime

- Momentum representation (Fourier transform) in curved spacetime

  ↳ Depends on the choice of coordinates

CDE in curved spacetime

- Momentum representation (Fourier transform) in curved spacetime

  ↳ Depends on the choice of coordinates

- Possible solution: rely on a specific choice of coordinate (often Riemann Normal Coordinates)

CDE in curved spacetime

- Momentum representation (Fourier transform) in curved spacetime

  ↳ Depends on the choice of coordinates

- Possible solution: rely on a specific choice of coordinate (often Riemann Normal Coordinates)
- Does not allow to obtain non-covariant results (anomalies, B.Filoche, R.L, J.Quevillon, H. Pham Ngoc Vuong, "Anomalies from an Effective Field Theory Perspective)

CDE in curved spacetime

- Momentum representation (Fourier transform) in curved spacetime

  ↳ Depends on the choice of coordinates

- Possible solution: rely on a specific choice of coordinate (often Riemann Normal Coordinates)
- Does not allow to obtain non-covariant results (anomalies, B.Filoche, R.L, J.Quevillon, H. Pham Ngoc Vuong, "Anomalies from an Effective Field Theory Perspective)
- We introduce a coordinate-independent Fourier transform

CDE in curved spacetime

- Previous CDE in curved spacetime were manifestly covariant, but computation much more involved
- Very systematic procedure, easy to automatise
- Still have the factorisation of momentum and operator part : universality
- Expansion greatly simplified in Riemann Normal Coordinates, plus it is manifestly covariant

Results

- First time computation of non-renormalisable operators with the CDE
  Recover the Euler-Heisenberg Effective Action

Results

- First time computation of non-renormalisable operators with the CDE
  Recover the Euler-Heisenberg Effective Action
- $$S = \int \mathrm{d}^d x \sqrt{-g}\, \bar{\psi}(i\not{D} - m - Q)\psi$$

with $Q \supset W_0 + iW_1\gamma_5 + V_\mu\gamma^\mu + iA_\mu\gamma^\mu\gamma_5$

Results

- First time computation of non-renormalisable operators with the CDE
  Recover the Euler-Heisenberg Effective Action
- $$S = \int \mathrm{d}^d x \sqrt{-g}\, \bar{\psi}(i\not{D} - m - Q)\psi$$

with $Q \supset W_0 + iW_1\gamma_5 + V_\mu\gamma^\mu + iA_\mu\gamma^\mu\gamma_5$

New effective renormalisable operators: $A^2R,\ W_1^2R,\ AW_1R$

and non-renormalisable operators: $W_0AAR,\ AVW_1R,\ W_0W_1^2R,\ \ldots$

$$S_{\rm eff}^{\rm bos} = \frac{c_s}{16\pi^2}\int \sqrt{-g}\,\mathrm{d}^4x\,\mathrm{tr}\Bigg\{ m^2\left(1-\log\left(\frac{\mathrm{m}^2}{\mu^2}\right)\right)\left(\frac{1}{6}R+U\right)$$

$$+\log\left(\frac{\mathrm{m}^2}{\mu^2}\right)\left[-\frac{1}{72}R^2+\frac{1}{180}R_{\mu\nu}R^{\mu\nu}-\frac{1}{180}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-\frac{1}{30}(\Box R)\right.$$
$$\left.-\frac{1}{6}RU-\frac{1}{6}(\Box U)-\frac{1}{2}U^2-\frac{1}{12}F^2\right]$$

$$+\frac{1}{m^2}\left[-\frac{1}{72}RF^2-\frac{1}{90}R_{\mu\nu}F^{\mu\lambda}F^{\nu}{}_{\lambda}-\frac{1}{180}R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\right.$$
$$+\left(\frac{1}{90}-\frac{a}{2}\right)(\mathcal{D}_\mu F^{\mu\nu})^2+aF_\mu{}^{\nu}F_\nu{}^{\rho}F_\rho{}^{\mu}+\left(\frac{1}{360}+\frac{a}{4}\right)(\mathcal{D}_\mu F_{\nu\rho})(\mathcal{D}^\mu F^{\nu\rho})$$
$$\left.-\frac{1}{12}U(\Box U)-\frac{1}{36}R(\Box U)-\frac{1}{12}RU^2-\frac{1}{6}U^3-\frac{1}{12}UF^2+\mathcal{O}(R^2)\right]$$
$$+\mathcal{O}(1/m^4)\Bigg\}.$$

$$
\begin{aligned}
S_{\text{eff}}^{\text{ferm}} \supset \frac{-1}{16\pi^2}\int \sqrt{-g}\,\mathrm{d}^4x\,\mathrm{tr}\Bigg\{ & -m^2\frac{1}{6}R\left(1-\log\left(\frac{\mathrm{m}^2}{\mu^2}\right)\right)+m\,\frac{1}{3}\log\left(\frac{\mathrm{m}^2}{\mu^2}\right)\;R\,W_0 \\
& +\log\left(\frac{\mathrm{m}^2}{\mu^2}\right)\left[-\frac{1}{144}R^2+\frac{1}{90}R_{\mu\nu}R^{\mu\nu}+\frac{7}{720}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}+\frac{1}{60}(\Box R)\right] \\
& \quad +\frac{1}{3}R\,W_0^2\left(1+\frac{1}{2}\log\left(\frac{\mathrm{m}^2}{\mu^2}\right)\right)+\frac{1}{3}R\,W_1^2\left(-1+\frac{1}{2}\log\left(\frac{\mathrm{m}^2}{\mu^2}\right)\right) \\
& +\frac{1}{m}\Bigg[W_1\left(-\frac{1}{48}\varepsilon_{\mu\nu\rho\sigma}R_{\alpha\beta}{}^{\mu\nu}R^{\alpha\beta\rho\sigma}\right) \\
& \quad +W_0\left(\frac{1}{45}R^{\mu\nu}R_{\mu\nu}-\frac{1}{72}R^2+\frac{7}{360}R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}+\frac{1}{3}(\Box R)\right) \\
& \quad +R^{\mu\nu}\left(-\frac{4}{3}W_0A_\mu A_\nu+i\frac{4}{3}A_\mu(\mathcal{D}_\nu W_1)+2iW_1(\mathcal{D}_\mu A_\nu))+\frac{2}{3}A_\mu[X_\nu,W_1]\right) \\
& \quad +R\left(-\frac{1}{3}A^\mu[X_\mu,W_1]+\frac{1}{9}W_0^3+\frac{1}{3}W_0W_1^2-i\frac{1}{3}W_1(\mathcal{D}_\mu A^\mu)\right)\Bigg]+\mathcal{O}(1/m^2)\Bigg\}\,.
\end{aligned}
$$

Applications

- Powerful tool to understand the behavior of quantum particles on a curved background (inflation, black holes, baryogenesis, ...), including chiral particles (axion models, neutrinos, ...)

Applications

- Powerful tool to understand the behavior of quantum particles on a curved background (inflation, black holes, baryogenesis, …), including chiral particles (axion models, neutrinos, …)
- Up to bottom approach to modified gravity theories, large scale structure EFTs

Applications

- Powerful tool to understand the behavior of quantum particles on a curved background (inflation, black holes, baryogenesis, ...), including chiral particles (axion models, neutrinos, ...)
- Up to bottom approach to modified gravity theories, large scale structure EFTs
- Quantisation of gravity (as an EFT):
  - Study interaction of a particle with the graviton on a curved background
  - Compute the saddle points of the gravitational path integral (integrate out the graviton)

Applications

- Powerful tool to understand the behavior of quantum particles on a curved background (inflation, black holes, baryogenesis, ...), including chiral particles (axion models, neutrinos, ...)
- Up to bottom approach to modified gravity theories, large scale structure EFTs
- Quantisation of gravity (as an EFT):
  - Study interaction of a particle with the graviton on a curved background
  - Compute the saddle points of the gravitational path integral (integrate out the graviton)
- Non-perturbative and coordinate independent computation of gravitational anomalies, crucial in model building when including gravity

Thanks for you attention

Momentum representation

- Define a contra-variant vector q_μ such that $\dfrac{\partial\, q_\mu}{\partial\, x^\nu} = 0$
- Can show that if $\mathcal{O}$ is covariant, then $\displaystyle\int \mathrm{d}^d x \mathrm{d}^d q\, e^{iq\cdot x} \mathcal{O} e^{-iq\cdot x}$ is coordinate independent, even though $q \cdot x$ is not
- Loose the commutativity of q^2 and ∂

  Generates the gravity operators
- Can define a coordinate independent functional trace