Hadron spectroscopy in multibody B decays at LHCb

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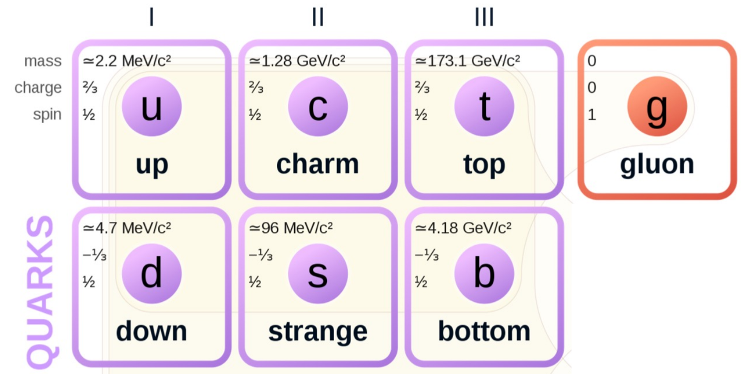
Outline

- Introduction
- LHCb experiment
- Physics analyses
  - Amplitude analysis of $B^0 \to D^- D^+ K^+ \pi^-$ Phys. Rev. Lett. 126 (2021) 122002
  - Amplitude analysis and branching fraction measurement of $B^+ \to D_s^+ D_s^- K^+$ arXiv:2210.15153,arXiv:2211.05034
- Summary and prospects

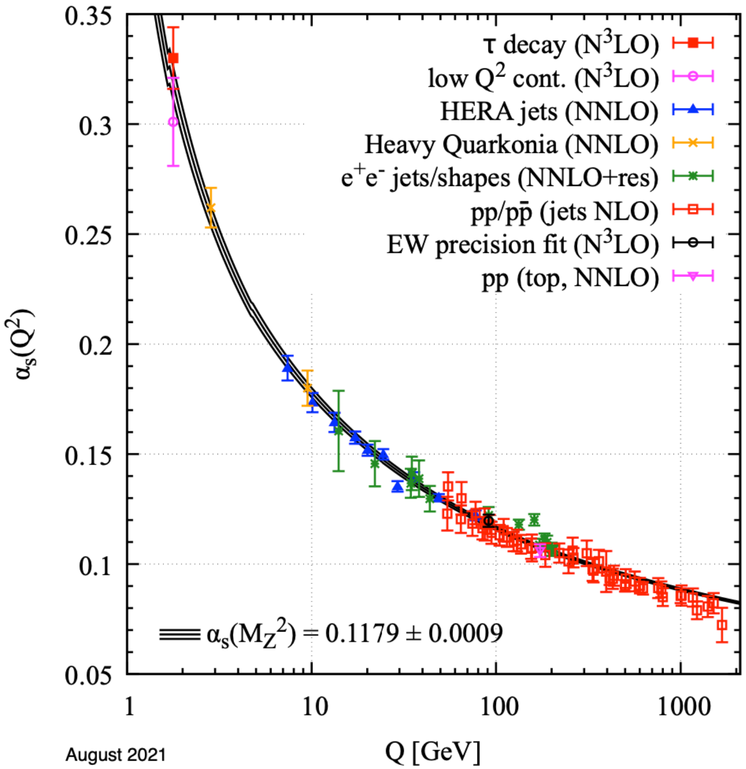
Introduction

Strong interaction

- Exists between quarks & gluons

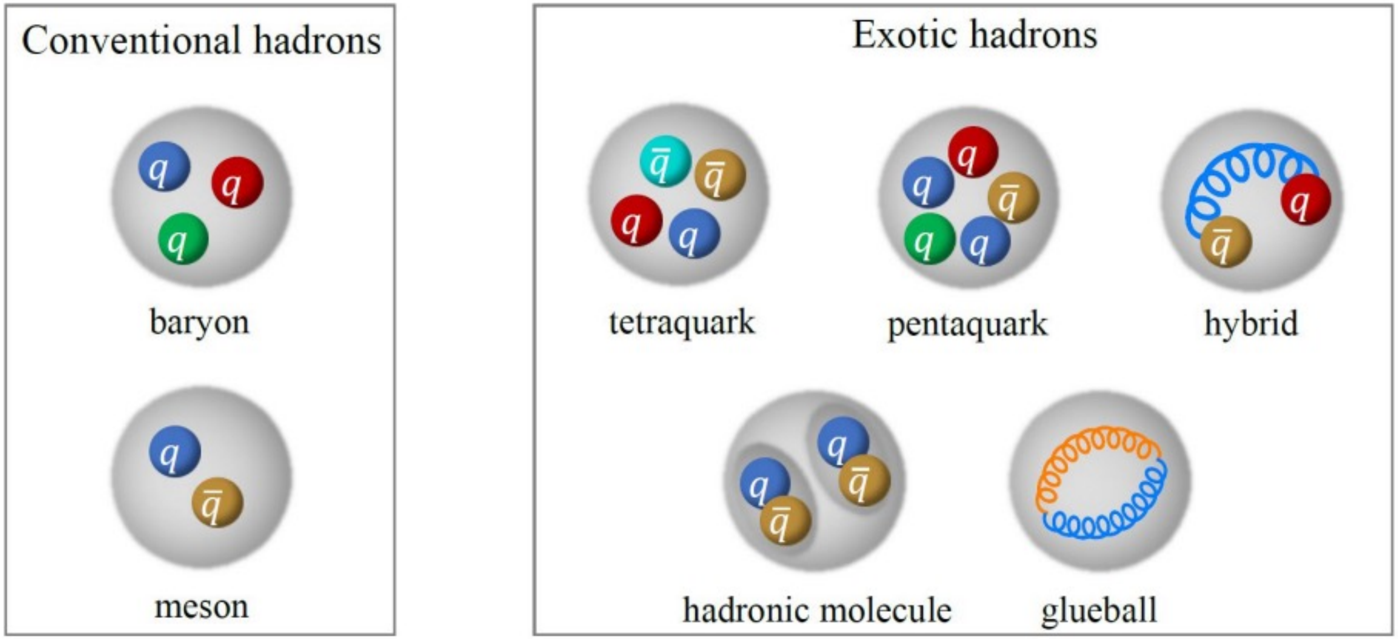


- Described by QCD
  - Asymptotic freedom
  - Perturbative in high energy
    - Precision calculation
  - Non-perturbative in low energy region
    - Precision calculation extremely difficult
    - Property of strong interaction not fully understood yet



Hadrons

- Composite particles composed of quarks and gluons via strong interaction
  - Binding energy is typically at low energy scale
  - Primary platform to study strong interaction and QCD in low energy region
- Phenomenal description is based on Quark Model but extended



- Abundant hadrons
  - Different contents
  - Different structures
  - Various excitation patterns (resonances)

-> Hadron spectroscopy

Hadrons: properties

- Quark content and structure
- Mass
- Width (1/lifetime)
- Spin-parity
- Decay:
  - A few decay weakly or even stable
  - Most decay strongly or electromagnetically
- …

How to determine these properties in theory?

How to measure these properties in experiment?

Hadrons: theoretical side

- Phenomenological theories --- usually based on quark model
  - e.g. Godfrey-Isgur (GI) potential model for conventional mesons

Phys. Rev. D 32, 189

$$H\,|\Psi\rangle = (H_0 + V)\,|\Psi\rangle = E\,|\Psi\rangle$$

$$H_0 = (p^2 + m_1^2)^{1/2} + (p^2 + m_2^2)^{1/2}$$

$$V_{ij}(\mathbf{p},\mathbf{r}) \to H_{ij}^{\text{conf}} + H_{ij}^{\text{hyp}} + H_{ij}^{\text{so}} + H_A$$

Similar to quantum mechanics in hydrogen system

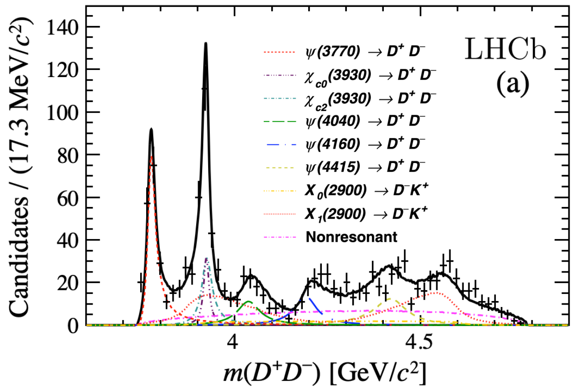
- Solving the equation will give:
  - States are classified according to $n^{2S+1}L_J$
  - $P = (-1)^{L+1}; C = (-1)^{L+S}$
  - Mass expressed as function of (n, L, S, J)
  - …

n: principle quantum number
S: spin sum
L: orbital angular momentum
J: total spin

- Lattice QCD --- first-principle method
  - Discretize time and space as lattices
  - Precision quite limited due to the huge amount of computation

Hadrons: experimental side

- A hadron usually appears as a peak in the invariant mass of the system of final-state particles

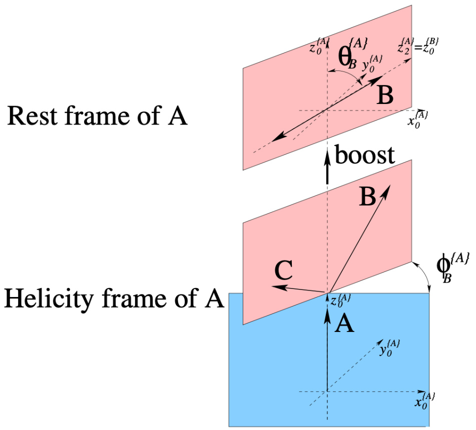


- Mass & width: mass lineshape
- Spin-parity: angular distribution
- Decay patterns: observation in different final states and measurement of the branching fraction
- Quark content: inferred from those of final-state particles
- Structure: inferred from other measured quantities and comparison with theoretical prediction

Hadrons: experimental side (cont.)

- Amplitude analysis: powerful tool to measure the properties of hadrons
  - e.g. for a multibody decay: $B^0 \to P_1P_2P_3P_4$, regard it as a cascade of two-body decays

$$B^0 \to R_{1j}R_{2n}, R_{1j} \to P_1P_2, R_{2n} \to P_3P_4$$



- Decay amplitude for two-body decay

$$\mathcal{M}_{A\to BC} = \mathcal{H}^{A\to BC}_{\lambda_B,\lambda_C} D^{J_A}_{m_A,\lambda_B-\lambda_C}(\phi_B,\theta_B,0)^*$$

This is the so-called helicity formalism

- B decay amplitude is the product of amplitudes of all cascade two-body decays
  - Including the propagators of intermediate resonant hadrons, e.g. Breit-Wigner function

$$\mathrm{BW}(m|m_0,\Gamma_0) = \frac{1}{m_0^2 - m^2 - i m_0\Gamma(m)},$$

- Total amplitude is the sum of the amplitudes involving different resonant hadrons
- Total amplitude contributes to PDF that fits to the phase-space distributions in data
  - Extract fitting parameters, like mass, width, spin-parity, branching fraction

Hadron spectroscopy in $B \to D\bar{D}K$ decays

$\bar{b} \to \bar{c}c\bar{s}$

- B: B^0, B^+
- D: D^0, $D^+_{(s)}$; D^{*0}, $D^{*+}_{(s)}$
- K: K^+, K^0; K^{*+}, K^{*0}

- Ideal platform to study hadrons containing charm quark(s)
  - Abundant final-state combinations
    - $D^{(*)}K^{(*)}$: D_s $[c\bar{s}]$
    - $D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$: (exotic) charmonium $[c\bar{c}(q\bar{q})]$, e.g. J/ψ
    - $\bar{D}^{(*)}K^{(*)}$: tetraquark containing $[csq\bar{q}']$
    - …

Experimental status of $B \to D\bar{D}K$ decays

- Many decay modes established

- Intermediate resonances
  - $D_{s1}(2536)^+, D_{s1}^*(2700)^+, \psi(3770)$, etc.

Prog. Theor. Exp. Phys. 2020 (2020) 083C01

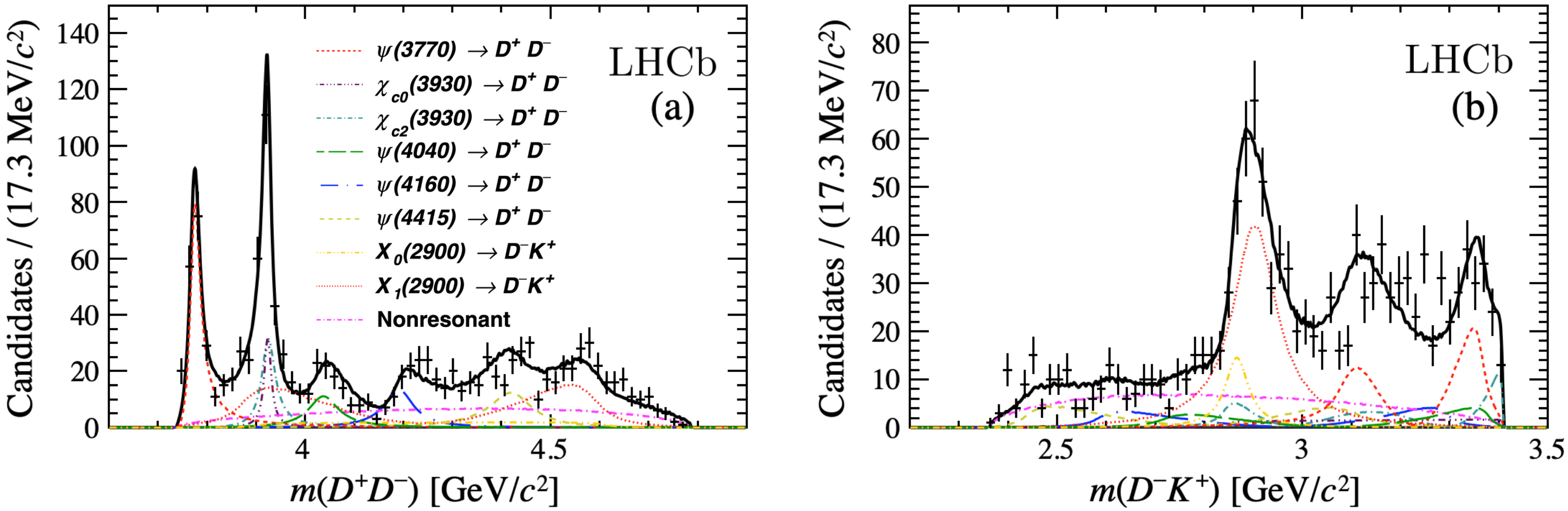
Neutral B mode	Charged B mode
$B^0 \to D^- D^0 K^+$	$B^+ \to \bar{D}^0 D^+ K^0$
$B^0 \to D^- D^{*0} K^+$	$B^+ \to \bar{D}^0 D^{*+} K^0$
$B^0 \to D^{*-} D^0 K^+$	$B^+ \to \bar{D}^{*0} D^+ K^0$
$B^0 \to D^{*-} D^{*0} K^+$	$B^+ \to \bar{D}^{*0} D^{*+} K^0$
$B^0 \to D^- D^+ K^0$	$B^+ \to \bar{D}^0 D^0 K^+$
$B^0 \to D^- D^{*+} K^0 + D^{*-} D^+ K^0$	$B^+ \to \bar{D}^0 D^{*0} K^+$
	$B^+ \to \bar{D}^{*0} D^0 K^+$
$B^0 \to D^{*-} D^{*+} K^0$	$B^+ \to \bar{D}^{*0} D^{*0} K^+$
$B^0 \to \bar{D}^0 D^0 K^0$	$B^+ \to D^- D^+ K^+$
$B^0 \to \bar{D}^0 D^{*0} K^0 + \bar{D}^{*0} D^0 K^0$	$B^+ \to D^- D^{*+} K^+$
	$B^+ \to D^{*-} D^+ K^+$
$B^0 \to \bar{D}^{*0} D^{*0} K^0$	$B^+ \to D^{*-} D^{*+} K^+$
$B^0 \to D^0 \bar{D}^0 K^+ \pi^-$	

- Amplitude analysis has rarely been touched due to low statistics
  - Small branching fraction: $\mathcal{B}(B \to D\bar{D}K) \times \mathcal{B}(D \to \mathrm{n}h)^2 \sim 10^{-7}$
  - Low efficiency: presence of many final-state tracks

Amplitude analysis of $B^+ \to D^+ D^- K^+$

- First $cs\bar{u}\bar{d}$ tetraquarks: $X_{0,1}(2900) \to D^- K^+$ Phys.Rev.D102(2020) 112003 Phys. Rev. Lett. 125 (2020) 242001

- $\chi_{c0}(3930)$
  - $J^{PC} = 0^{++}$, $M \sim 3924$ MeV
  - $M \sim m(D_s^+ D_s^-)$: a $c\bar{c}s\bar{s}$ tetraquark? ? JHEP, 2021, 06: 035 Sci. Bull., 2021, 66
    - search for it in $D_s^+ D_s^-$



Two extensions of $B^+ \to D^+D^-K^+$

- $B^0 \to D^+D^-K^+\pi^-$ undiscovered
  - Check for the resonances presented in $B^+ \to D^+D^-K^+$
  - Search for new D_s^+ states in $D^+K^+\pi^-$
    - Three-body system was rarely touched before
    - $K^*(892)^0 \to K^+\pi^-$: $m(D_s^+) > 2.76$ GeV; $J^P \neq 0^+$
    - $K^+\pi^-$ S-wave: $m(D_s^+) > 2.53$ GeV; $J^P = 0^-, 1^+, 2^-, \cdots$

    (while DK can only access $J^P = 0^+, 1^-, 2^+, \cdots$)

Derived from conservations of angular momentum and parity

- $B^+ \to D_s^+D_s^-K^+$ undiscovered
  - Search for conventional/exotic charminum in $D_s^+D_s^-$, e.g. $\chi_{c0}(3930)$
    - First time to study the $D_s^+D_s^-$ system in an exclusive B-meson decay

Yes!! Let's study these two decays at LHCb!

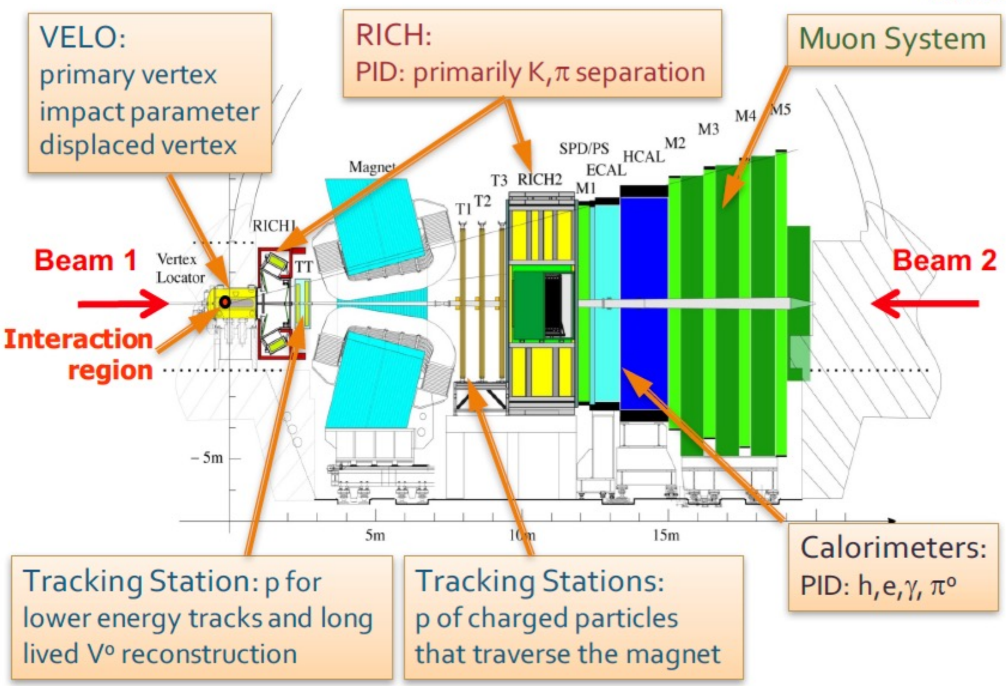
LHCb experiment

LHCb experiment

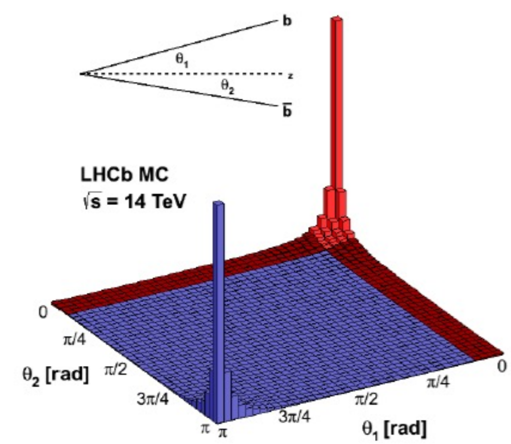
$$B^+ : B^0 : B_s^0 : \Lambda_b^0$$
$$(u\bar{b})\ (d\bar{b})\ (s\bar{b})\ (udb)$$
$$4 : 4 : 1 : 2$$

- LHC: beauty&charm factory
  - pp collision @ $\sqrt{s} = 13$ TeV : ~20000 $b\bar{b}$ /s
- LHCb detector: Dedicated for the precision reconstruction of heavy hadrons

The LHCb detector described in [JINST 3 (2008) S08005]



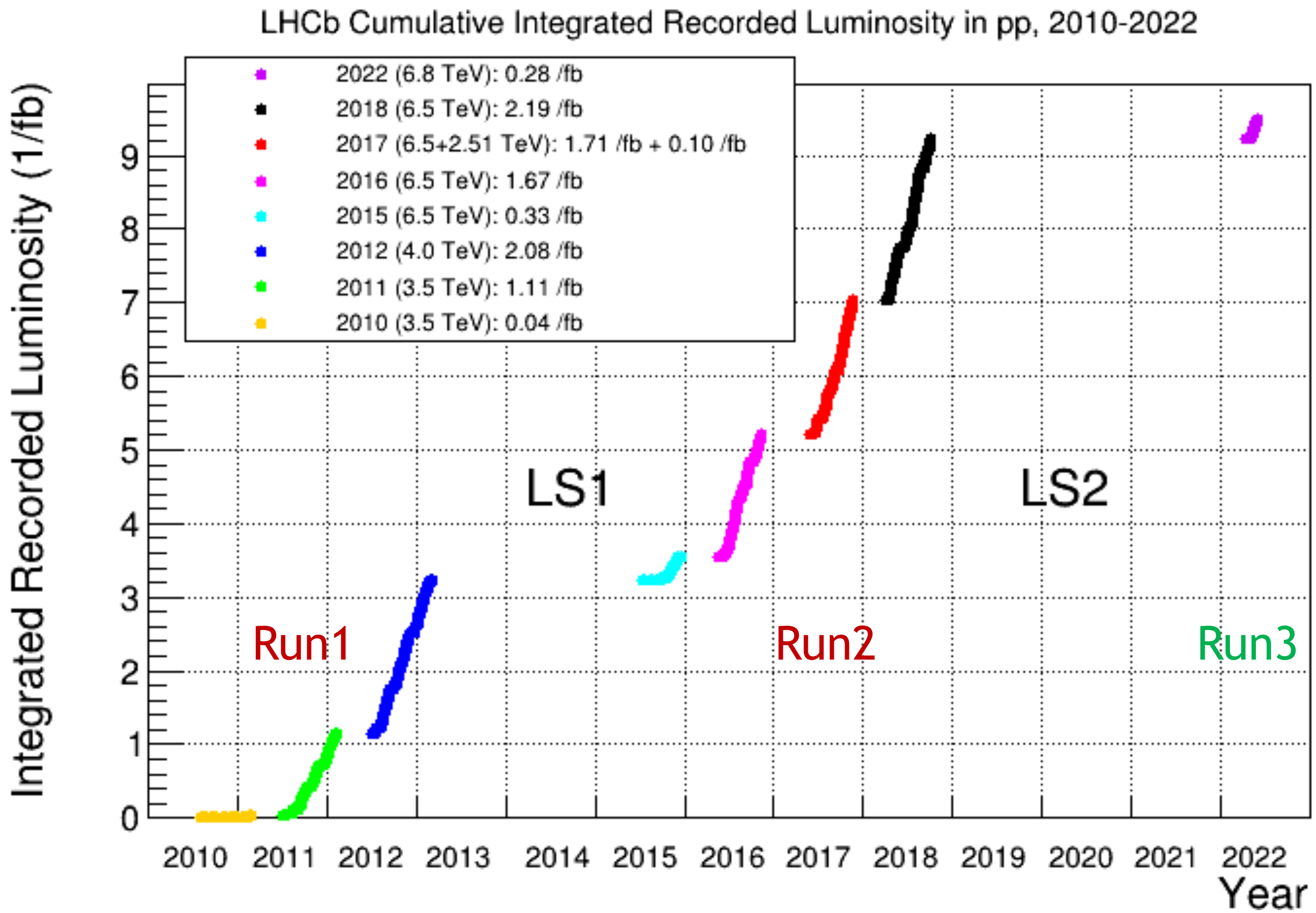
- $2 < \eta < 5$ range: $\sim 25\%$ of $b\bar{b}$ pairs inside LHCb acceptance



[Int. J. Mod. Phys. A 30 (2015) 1530022]

LHCb dataset

- Run1: 3 fb^{-1} pp collision @ 7, 8 TeV
- Run2: 6 fb^{-1} pp collision @ 13 TeV
- Run3: ongoing from 2022



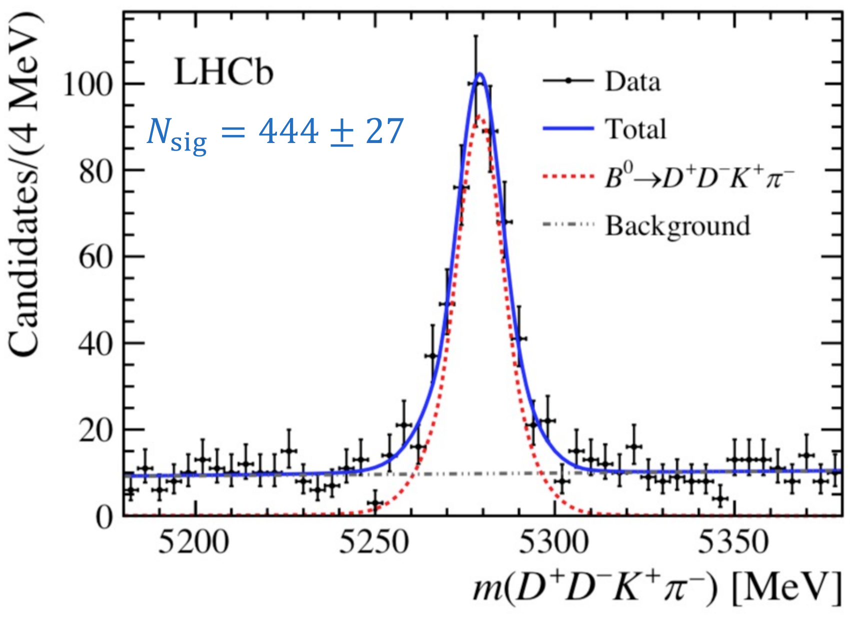
Amplitude analysis of $B^0 \to D^+ D^- K^+ \pi^-$

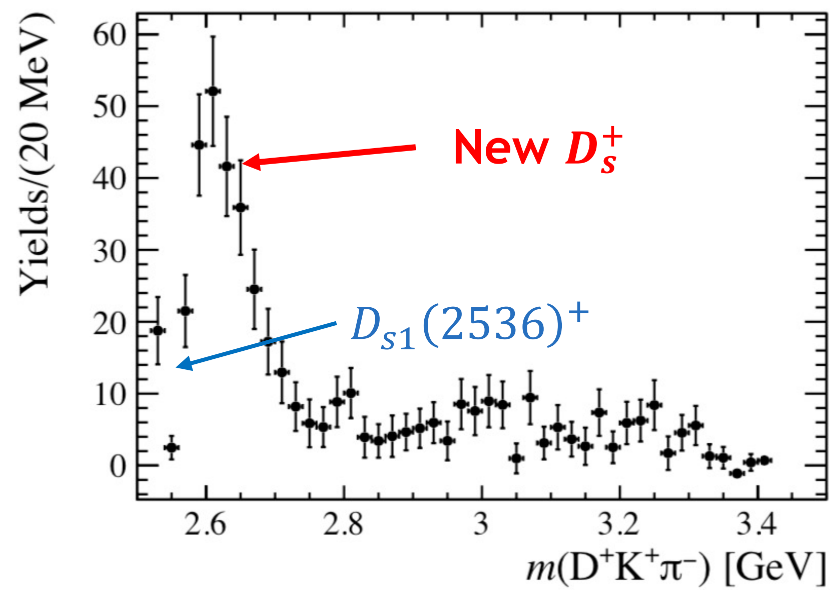
Phys. Rev. Lett. 126 (2021) 122002

$B^0 \to D^+D^-K^+\pi^-$ dataset

- Dataset: 16-18, $\mathcal{L} = 5.4\ \text{fb}^{-1}$
- Reconstruction: $B^0 \to D^+D^-K^+\pi^-,\ D^+ \to K^-\pi^+\pi^+$
- $m(K^+\pi^-) < 0.75\text{GeV}$

to focus on the low mass region at first due to the complexity of the four-body B^0 decay





Background subtracted using sPlot
Nucl.Instrum.Meth.A 555 (2005)

B^0 decay amplitude

- Using Helicity formalism
  - Decay chain: $B^0 \to D^- D_{sk}^+, D_{sk}^+ \to D^+ K^{*0}, K^{*0} \to K^+ \pi^-$
- Intermediate resonances:
  - $K^+\pi^-$: S-wave because $m(K^+\pi^-) < 0.75$ GeV
    - Modeled by $J^P = 0^+$ $K^*(700)^0$
  - $D^+K^+\pi^-$: $0^- + 0^+ \to 0^-, 1^+, 2^-, \cdots$
    - A non-resonant (NR) term with $J^P = 0^-$
    - $J^P = 1^+$ $D_{s1}(2536)^+$
    - A new D_{sJ}^+ state (three spin-parity tested: $J^P = 0^-, 1^+, 2^-$)
- Total amplitude

Helicity coupling	Wigner d-matrix	Momentum barrier factors for B^0 and D_{sk} decays

$$\mathcal{M} = \sum_k \mathcal{H}^{D_{sk}} d_{0,0}^{J_{D_{sk}}}(\theta_{D_s}) p^{L_{B^0}} F_{L_{B^0}}(pd) q^{L_{D_{sk}}} F_{L_{D_{sk}}}(qd)$$

$$\mathrm{BW}(m_{K^+\pi^-})\,\mathrm{BW}_{D_{sk}}(m_{D^+K^+\pi^-}),$$

Mass lineshapes

- θ_{D_s}: angle between D^+ and D^- momenta in the D_{sk}^+ rest frame
- p, q: center-of-mass momentum of $D^- D_{sk}^+$ and $D^+ K^{*0}$
- $d = 3$ GeV^{-1}~(0.6 fm): effective radius of the particle

Amplitude fit method

$$P_{\text{sig}}(\vec{x} \mid \vec{\omega}) = \frac{1}{I(\vec{\omega})} \left|\mathcal{M}(\vec{x} \mid \vec{\omega})\right|^2 \cdot \Phi(\vec{x})\epsilon(\vec{x})$$

$\vec{x}$: $(m_{D^+K^+\pi^-}, m_{K^+\pi^-}, \theta_{D_s})$
$\vec{\omega}$: fitting parameters

$\Phi(\vec{x})$: phase space factor
$\epsilon(\vec{x})$: efficiency
$I(\vec{\omega})$: normalisation factor

- Maximum likelihood method

$$\begin{aligned} -\ln \mathcal{L}(\vec{\omega}) &= -s_W \sum_i W_i \ln P_{\text{sig}}(\vec{x}_i \mid \vec{\omega}) \\ &= -s_W \sum_i W_i \ln |\mathcal{M}(\vec{x}_i \mid \vec{\omega})|^2 + s_W \ln I(\vec{\omega}) \sum_i W_i \\ &\quad - s_W \sum_i W_i \ln[\Phi(\vec{x}_i)\epsilon(\vec{x}_i)]. \end{aligned}$$

$$s_W = \frac{\sum W_i}{\sum W_i^2}$$

- Background subtracted by introducing sWeights W_i

Amplitude fit result

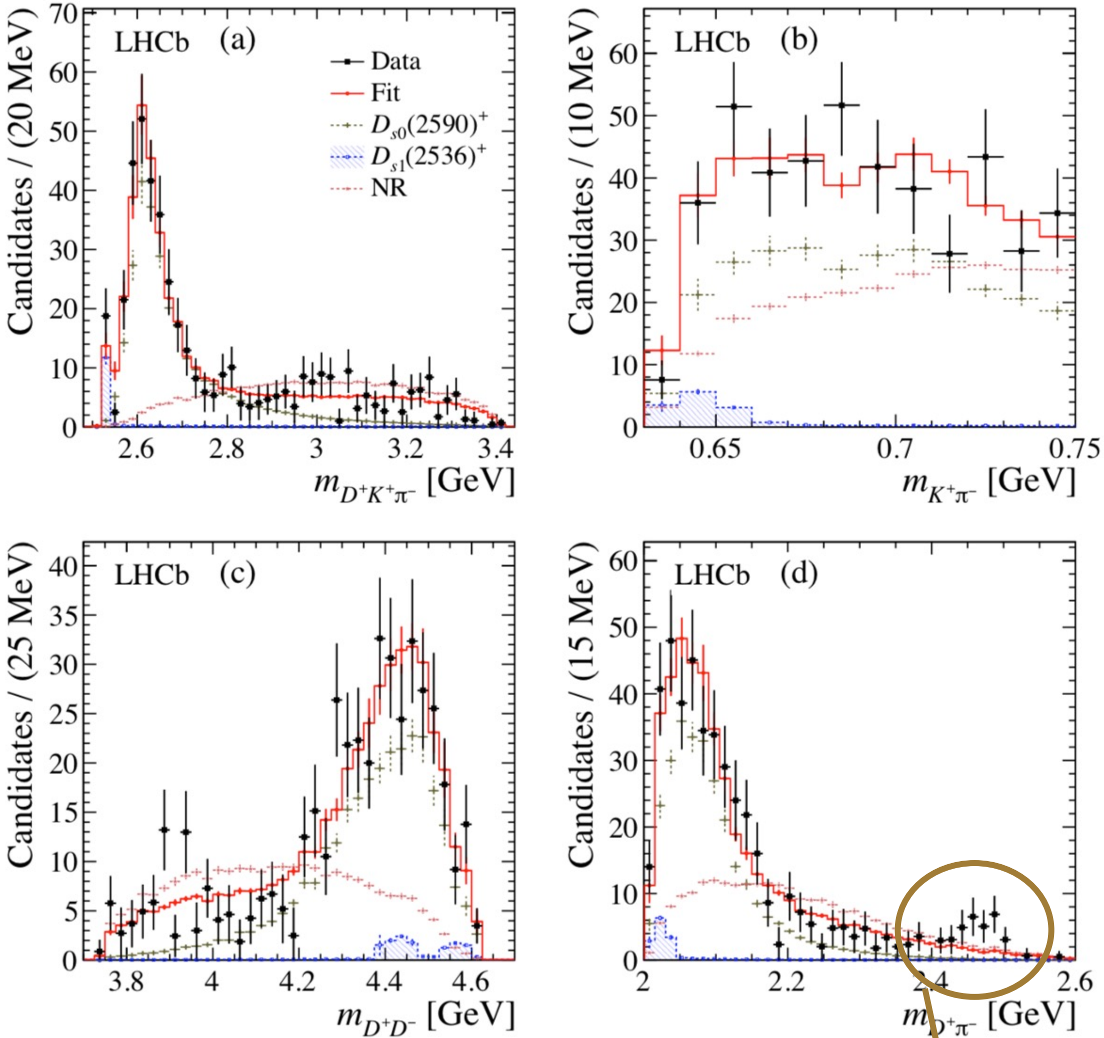
- 3D fit: $m(D^+K^+\pi^-), m(K^+\pi^-)$, $\cos\theta_{D_s}$
- Fit parameters
  - Helicity couplings of D_{sJ}^+ and $D_{s1}(2536)^+$
    - NR as reference
  - D_{sJ}^+ BW parameters
    - $D_{s1}(2536)^+$ and $K^*(700)^0$ BW parameters fixed to PDG
- $J^P = 0^-$ of D_{sJ}^+ leads to the best fit
  - $J^P = 1^+$ and 2^- are rejected by at least 15σ
  - Significance of D_{sJ}^+ : $> 20\sigma$

$$\boldsymbol{m_R = 2591 \pm 6 \pm 7\ \mathrm{MeV},}$$
$$\boldsymbol{\Gamma_R = 89 \pm 16 \pm 12\ \mathrm{MeV}}$$

$D_{s0}(2590)^+$

Mass projections

Background subtracted by sPlot

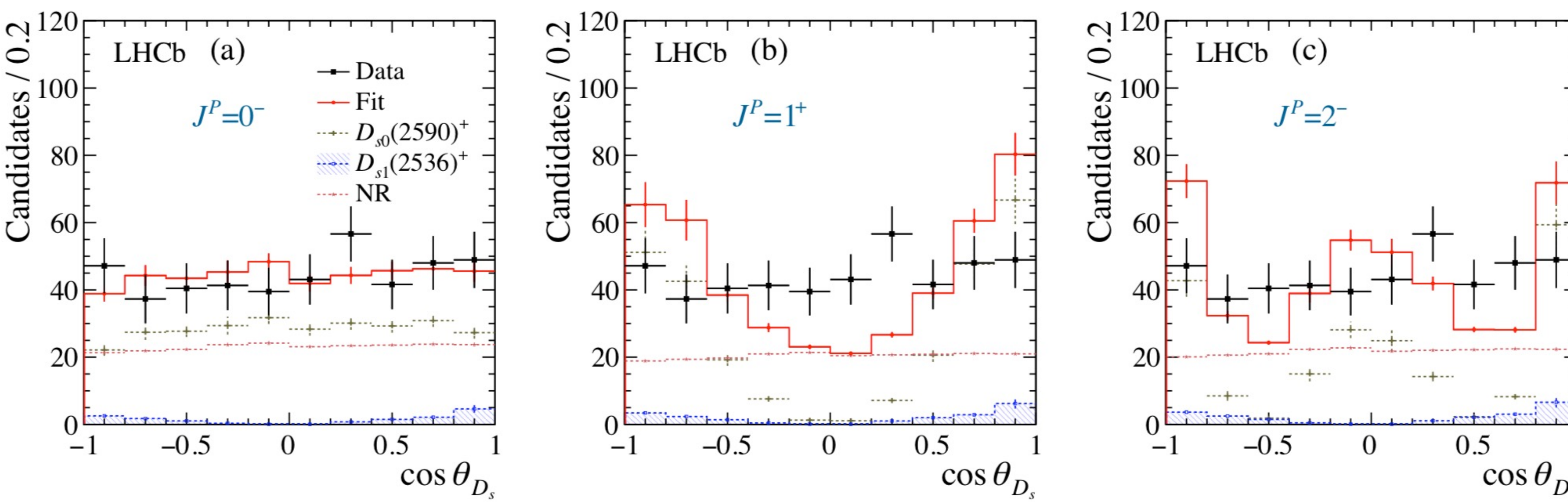


Fit well describes data

Small contribution of $D_2^*(2460)^+$ will be handled in systematic study

Angular projections

- $\cos\theta_{D_s}$ behavior described by $d^J_{0,0}(\cos\theta_{D_s})$ in the amplitude
  - $J^P = 0^-$: $|M|^2 \sim$ constant
  - $J^P = 1^+$: $|M|^2 \sim$ 2nd-order polynomial
  - $J^P = 2^-$: $|M|^2 \sim$ 4th-order polynomial
- $J^P = 0^-$ model is most consistent with data



Background subtracted by sPlot

Fit fractions

Fit fractions could be useful to obtain the partial decay width information of the states in the future

$$\mathcal{FF}^i = \frac{\int |\mathcal{M}^i(\vec{x} \mid \vec{\omega})|^2 \Phi(\vec{x}) d\vec{x}}{\int |\mathcal{M}(\vec{x} \mid \vec{\omega})|^2 \Phi(\vec{x}) d\vec{x}},$$

$$\mathcal{IF}^{ij} = \frac{2 \int \mathcal{R}e \left[\mathcal{M}^i(\vec{x} \mid \vec{\omega}) \cdot \mathcal{M}^{*j}(\vec{x} \mid \vec{\omega})\right] \Phi(\vec{x}) d\vec{x}}{\int |\mathcal{M}(\vec{x} \mid \vec{\omega})|^2 \Phi(\vec{x}) d\vec{x}}.$$

	Fit fraction $(\times 10^{-2})$
$D_{s0}(2590)^+$	63 ± 9 (stat) $\pm$ 9 (syst)
$D_{s1}(2536)^+$	3.9 ± 1.4 (stat) $\pm$ 0.8 (syst)
NR	51 ± 11 (stat) $\pm$ 19 (syst)
D_{s0}^+–NR	-18 ± 18 (stat) $\pm$ 24 (syst)
D_{s1}^+/D_{s0}^+	6.1 ± 2.4 (stat) $\pm$ 1.4 (syst)

Systematic uncertainties

The primary source is the $D_{s0}(2590)$ width model

Source	m_R [MeV]	Γ_R [MeV]	Fit fraction ($\times 10^{-2}$)				
			D_{s0}^+	D_{s1}^+	NR	D_{s0}^+–NR	D_{s1}^+/D_{s0}^+
$D_{s0}(2590)^+$ width model	6.1	8.0	4.7	0.0	15.0	19.6	0.5
$D_{s1}(2536)^+$ mass shape	0.3	4.3	2.3	0.6	3.5	5.3	1.1
$K^+\pi^-$ mass shape	2.7	2.6	3.0	0.2	1.2	4.4	0.1
Blatt–Weisskopf factor	0.7	3.4	2.8	0.3	1.3	3.0	0.2
Including $c\bar{c}$ resonances	1.1	5.4	2.7	0.1	6.3	10.0	0.4
$D^+\pi^-$ resonance veto	2.4	2.1	4.6	0.3	9.4	4.5	0.2
Simulation correction	0.2	1.1	0.3	0.1	0.7	0.8	0.2
Momentum calibration	0.5	0.4	1.2	0.0	1.4	2.5	0.2
Total	7.2	11.7	8.6	0.8	19.3	23.9	1.4

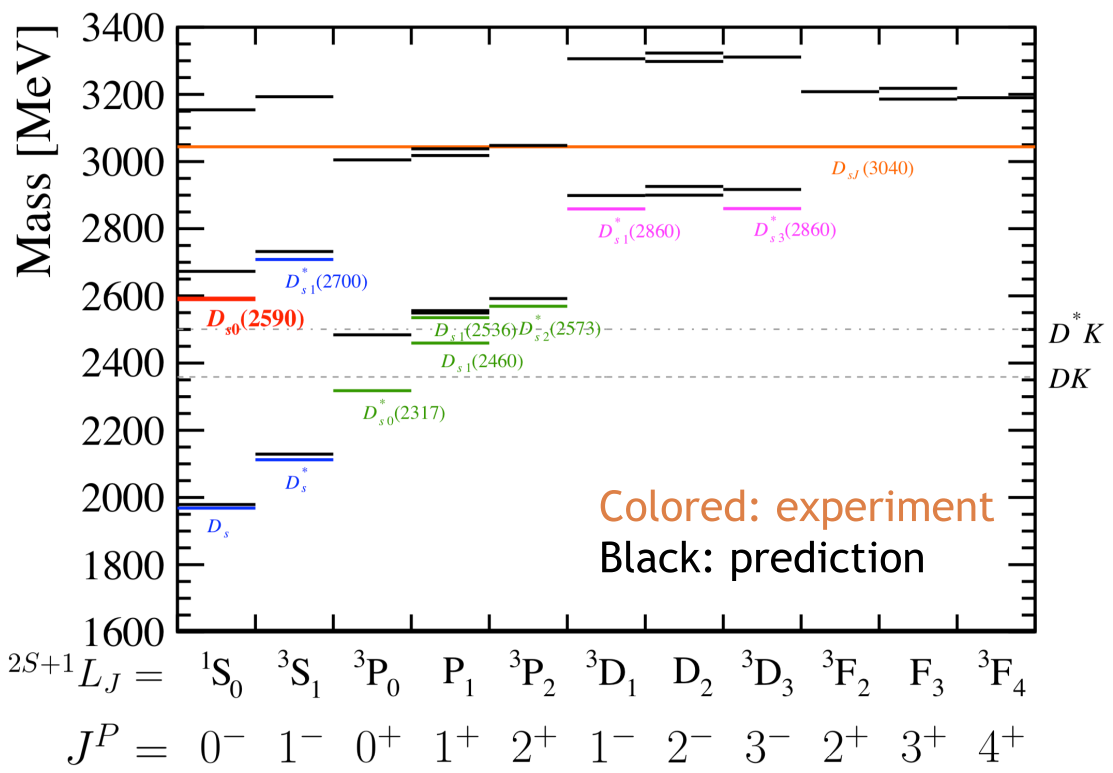
$D_{s0}(2590)^+$ in D_s^+ spectroscopy

A strong candidate for $D_s(2^1S_0)^+$, the radial excitation of the ground-state D_s^+ meson

Large discrepancy is seen in the $D_{s0}(2590)^+$ mass and the prediction
->
draw particular attention of theorists to interpret the nature of the $\boldsymbol{D_{s0}(2590)^+}$ state

- ✓ Coupled channel effect?
  - ✓ D^*K, $D_s^{(*)}\omega$, $D_s^{(*)}\eta$

arXiv:2204.02649



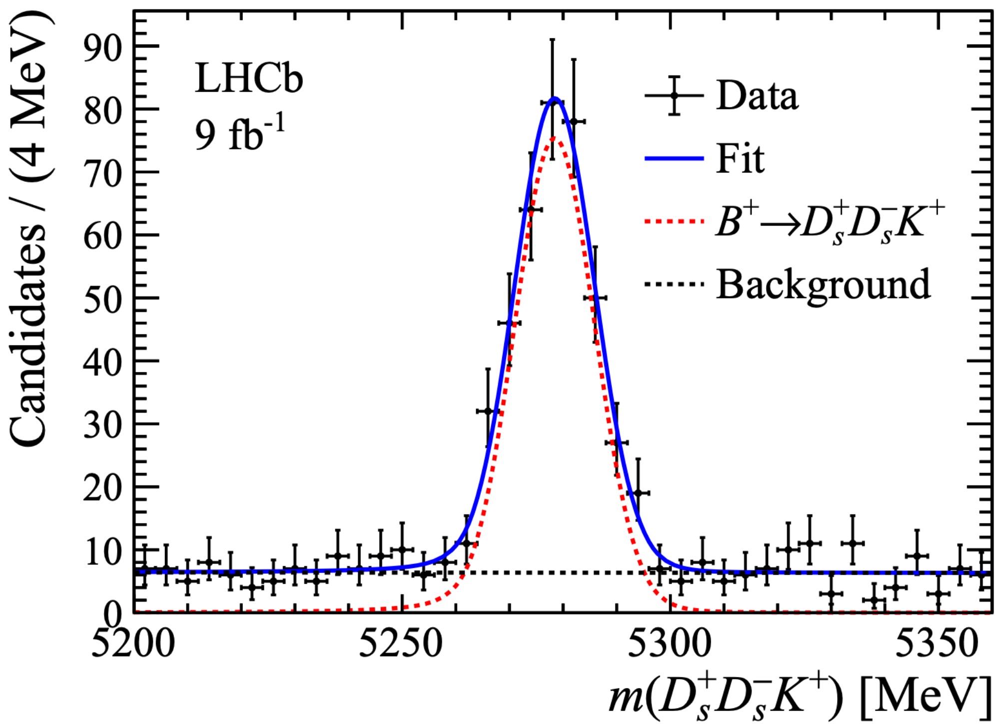
Phys. Rev. D93 (2016) 034035
Prog. Theor. Exp. Phys. 2020 (2020) 083C01

Study of $B^+ \to D_s^+ D_s^- K^+$

arXiv:2210.15153
arXiv:2211.05034

$B^+ \to D_s^+ D_s^- K^+$ dataset

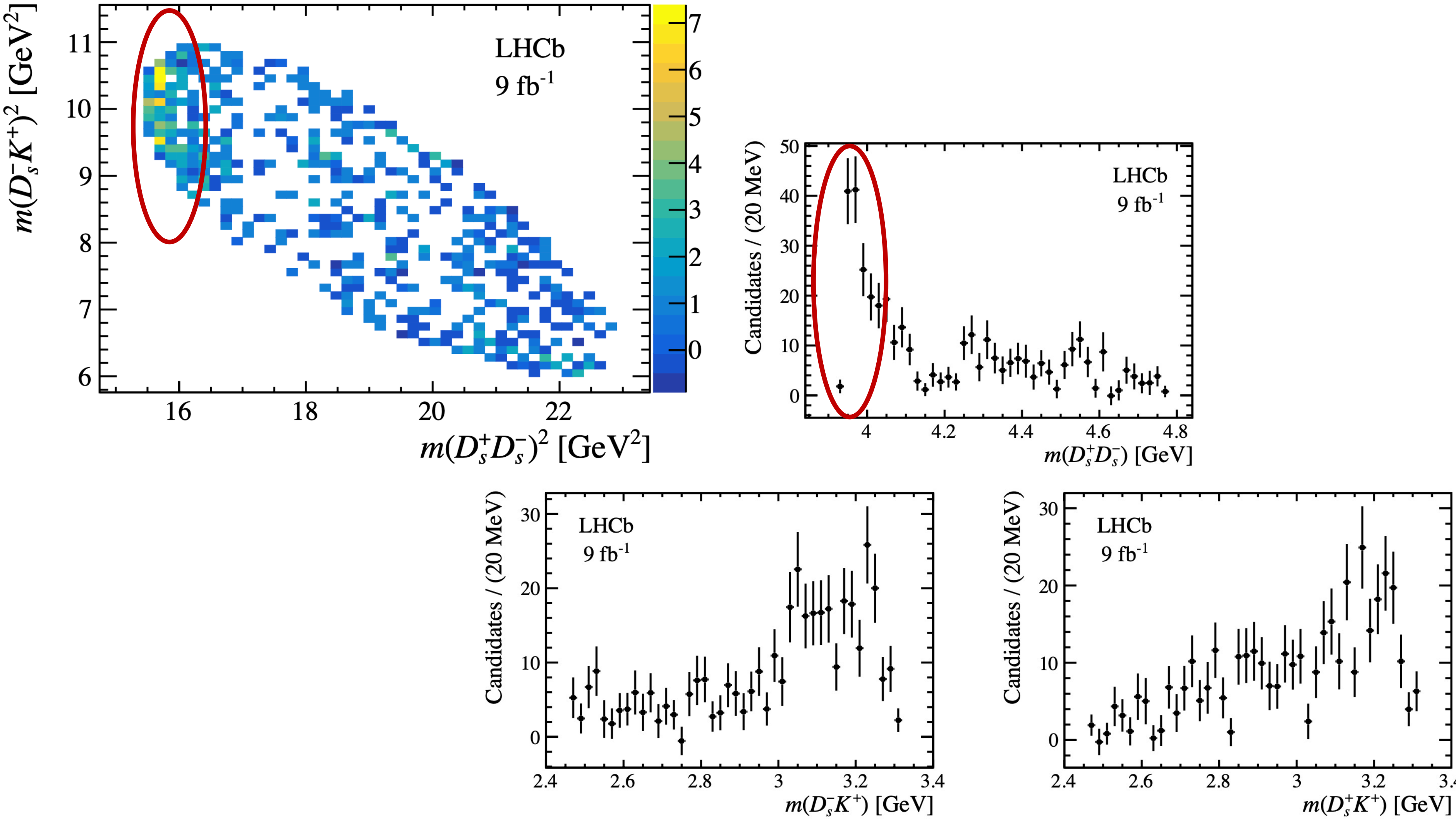
- Dataset: full Run1 + Run2 data, $\mathcal{L} = 9\ \text{fb}^{-1}$
- Reconstruction: $B^+ \to D_s^+ D_s^- K^+,\ D_s^\pm \to K^\mp K^\pm \pi^\pm$



$N_{\text{sig}} = 360 \pm 22$

Purity: 84%

Near-threshold structure in $D_s^+ D_s^-$

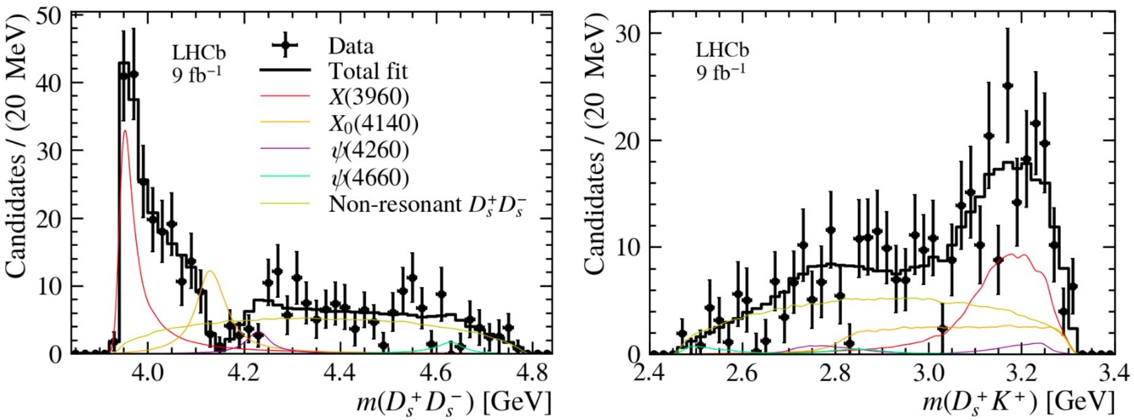


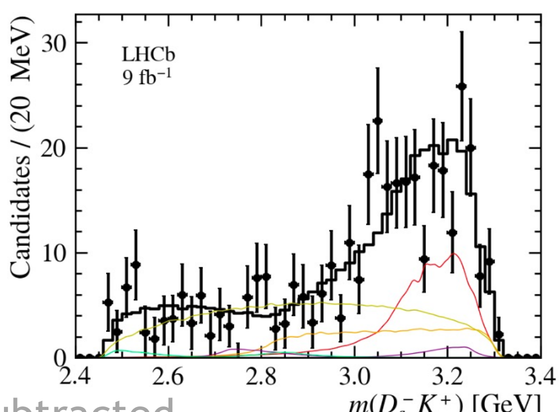
Background subtracted

Amplitude analysis

Observation of $X(3960)$ in $D_s^+ D_s^-$

- Amplitude analysis
  - Strategy is similar to the $B^0 \to D^+ D^- K^+ \pi^-$ analysis
- Default model
  - 0^{++}: $X(3960)$, $X_0(4140)$, non-resonant (NR) $\psi(4260)$ is $\psi(4230)$ in PDG2022
  - 1^{--}: $\psi(4260)$, $\psi\,(4660)$





✓ $X(3960)$ describes the near-threshold peak

✓ $X_0(4140)$ accounts for the dip at ~ 4.14 GeV via interference

Background subtracted

Amplitude fit result

Component	J^{PC}	M_0 (MeV)	Γ_0 (MeV)	$\mathcal{F}$ (%)	$\mathcal{S}$ (σ)
$X(3960)$	0^{++}	$3956 \pm 5 \pm 10$	$43 \pm 13 \pm 8$	$25.4 \pm 7.7 \pm 5.0$	12.6 (14.6)
$X_0(4140)$	0^{++}	$4133 \pm 6 \pm 6$	$67 \pm 17 \pm 7$	$16.7 \pm 4.7 \pm 3.9$	3.8 (4.1)
$\psi(4260)$	1^{--}	4230 (fixed)	55 (fixed)	$3.6 \pm 0.4 \pm 3.2$	3.2 (3.6)
$\psi(4660)$	1^{--}	4633 (fixed)	64 (fixed)	$2.2 \pm 0.2 \pm 0.8$	3.0 (3.2)
NR	0^{++}	-	-	$46.1 \pm 13.2 \pm 11.3$	3.1 (3.4)

- First uncertainty statistical, and second systematic
- Fixed parameters taken from PDG ($\psi(4260)$ is $\psi(4230)$ in PDG2022)

- $\mathcal{F}$: fit fraction
- $\mathcal{S}$: significance

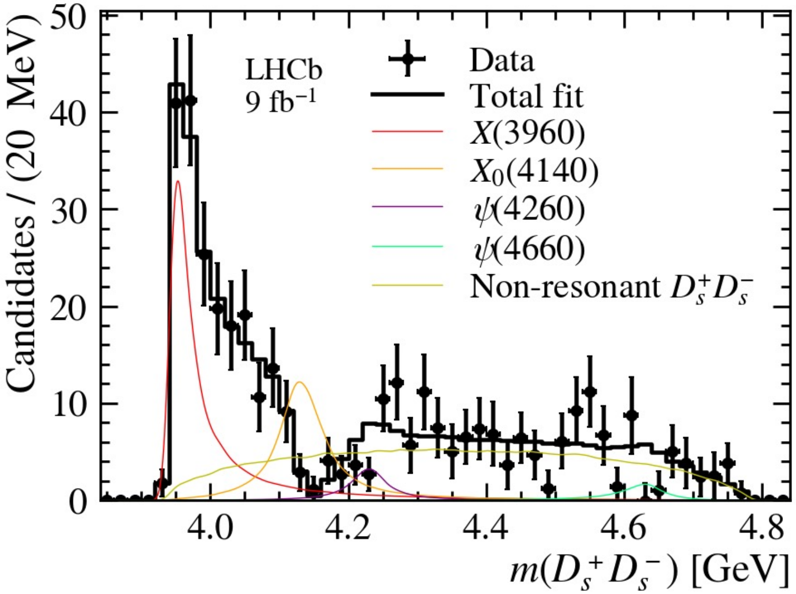
(numbers in brackets don not include systematic effect)

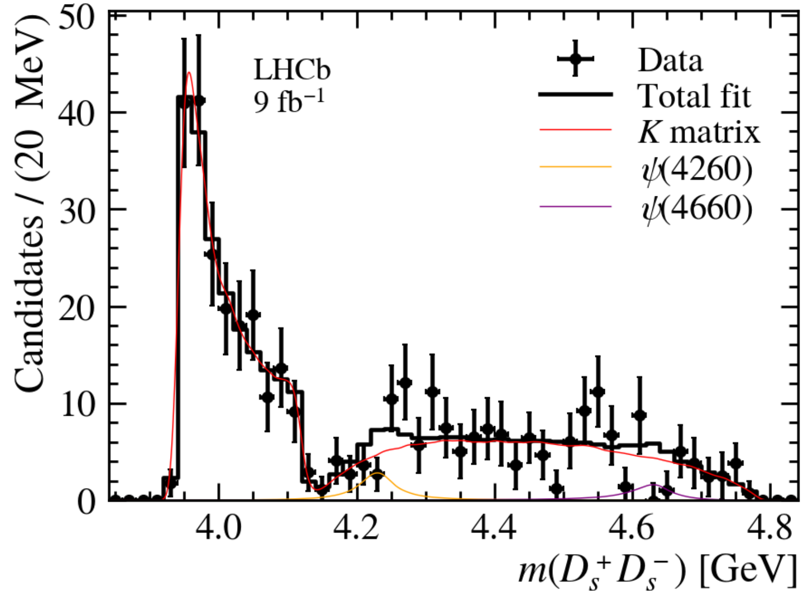
- Spin-parity tests:
  - $X(3960)$: 0^{++} favored; 1^{--} and 2^{++} rejected by at least 9σ
  - $X_0(4140)$: 0^{++} favored; 1^{--} and 2^{++} rejected by at least 3.5σ

Further investigation on $X_0(4140)$

- Dip around $4.14\ \mathrm{GeV}$ near the $J/\psi\phi$ threshold

Background subtracted





The default model: modelled by a new resonance, $X_0(4140)$

Can also be described by considering $J/\psi\phi \to D_s^+D_s^-$ rescattering in the K-matrix formula

No definitive conclusion on existence of $X_0(4140)$

$X(3960)$ and $\chi_{c0}(3930)$

- $X(3960)$: $M_0 = 3955 \pm 6 \pm 11$ MeV; $\Gamma_0 = 48 \pm 17 \pm 10$ MeV; $J^{PC} = 0^{++}$
- $\chi_{c0}(3930)$: $M_0 = 3924 \pm 2$ MeV; $\Gamma_0 = 17 \pm 5$ MeV; $J^{PC} = 0^{++}$

  Phys.Rev.D102(2020) 112003, Phys. Rev. Lett. 125 (2020) 242001

X denotes $X(3960)/\chi_{c0}(3930)$

- Are they the same particle? If yes
  - $\Gamma(X \to D^+D^-)$ v.s. $\Gamma(X \to D_s^+D_s^-)$ would imply the nature of the state, exotic or conventional?
    - Conventional charmonium predominantly decays into $D^{(*)}\overline{D}^{(*)}$
      - It is harder to excite an $s\bar{s}$ pair from vacuum compared with $u\bar{u}(d\bar{d})$

$\mathcal{FF}$: Fit fractions in the two B^+ decays

$$\frac{\Gamma(X \to D^+D^-)}{\Gamma(X \to D_s^+D_s^-)} = \frac{\mathcal{B}(B^+ \to D^+D^-K^+)\,\mathcal{FF}^X_{B^+ \to D^+D^-K^+}}{\mathcal{B}(B^+ \to D_s^+D_s^-K^+)\,\mathcal{FF}^X_{B^+ \to D_s^+D_s^-K^+}}$$

Unknown quantity yet.
Then measure it!

Branching fraction measurement

Strategy

- Relative measurement

$$\mathcal{R} \equiv \frac{\mathcal{B}(B^+ \to D_s^+ D_s^- K^+)}{\mathcal{B}(B^+ \to D^+ D^- K^+)} = \frac{N_{\rm sig}^{\rm corr}}{N_{\rm con}^{\rm corr}} \left[\frac{\mathcal{B}(D^+ \to K^- \pi^+ \pi^+)}{\mathcal{B}(D_s^+ \to K^- K^+ \pi^+)}\right]^2$$

Know quantities from PDG

- w_{sig}, w_{con}: sWeights determined from B^+ mass fits to extract the signal components

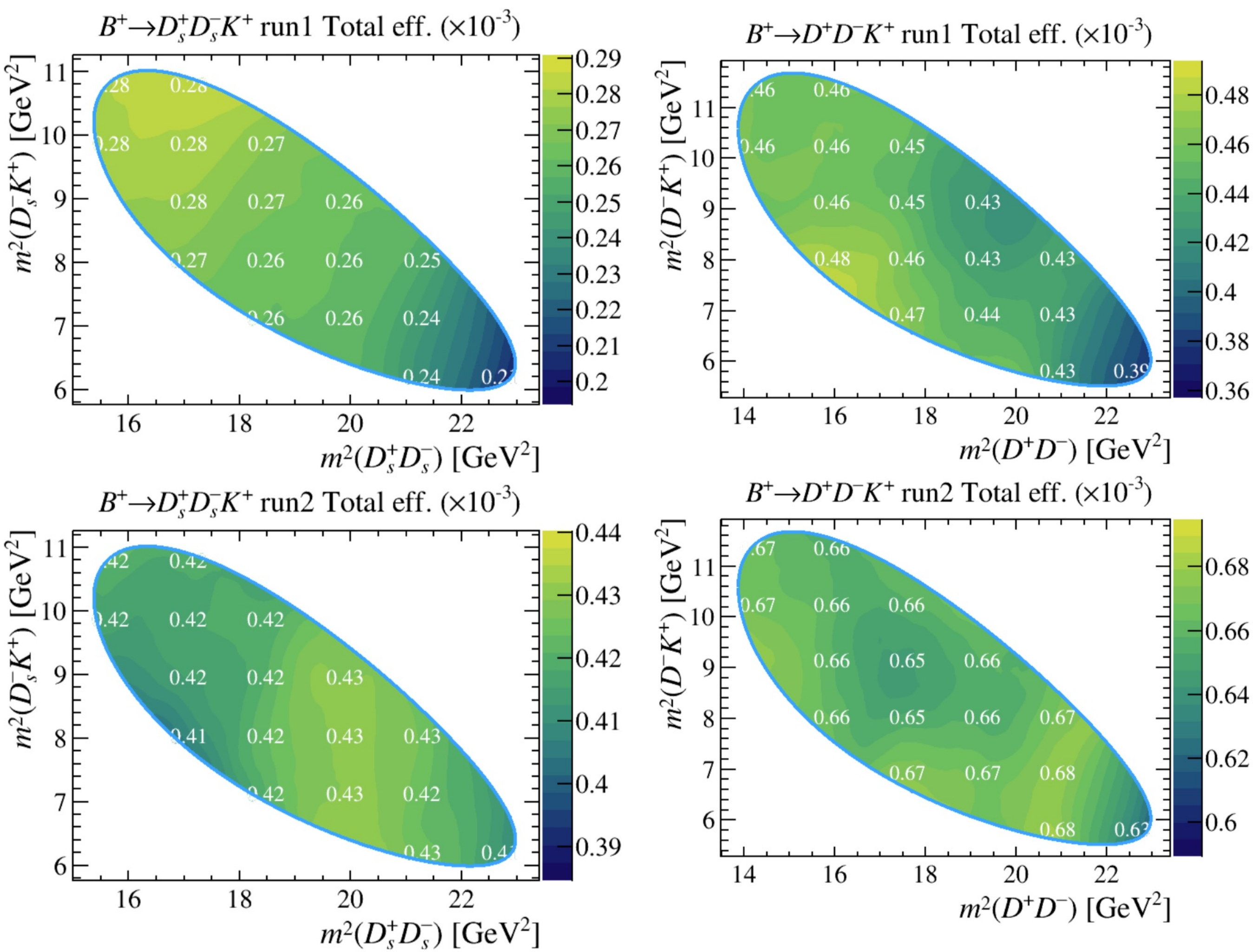
- ϵ_{sig}, ϵ_{con}: efficiency obtained from MC simulation

$$N_{\rm sig}^{\rm corr} = \sum_i \frac{w_{{\rm sig},\,i}}{\epsilon_{{\rm sig},\,i}(m^2(D_s^+ D_s^-), m^2(D_s^- K^+))}$$

$$N_{\rm con}^{\rm corr} = \sum_i \frac{w_{{\rm con},\,i}}{\epsilon_{{\rm con},\,i}(m^2(D^+ D^-), m^2(D^- K^+))}$$

Efficiency

- **Denominator:** Generator-level simulated sample without any cut
- **Numerator:** Fully reconstructed simulated sample after all selections



Efficiency as function of Dalitz-plot variables

Branching fraction result

$$\mathcal{R} = \frac{\mathcal{B}(B^+ \to D_s^+ D_s^- K^+)}{\mathcal{B}(B^+ \to D^+ D^- K^+)} = 0.525 \pm 0.033 \pm 0.027 \pm 0.034$$

1. Stat.
2. Syst.
3. External

$$\frac{\Gamma(X \to D^+ D^-)}{\Gamma(X \to D_s^+ D_s^-)} = \frac{\mathcal{B}(B^+ \to D^+ D^- K^+)\, \mathcal{F}\mathcal{F}^X_{B^+ \to D^+ D^- K^+}}{\mathcal{B}(B^+ \to D_s^+ D_s^- K^+)\, \mathcal{F}\mathcal{F}^X_{B^+ \to D_s^+ D_s^- K^+}}$$
$$= 0.29 \pm 0.09\,(\text{stat}) \pm 0.10\,(\text{syst}) \pm 0.08\,(\text{ext})$$

X denotes $X(3960)/\chi_{c0}(3930)$

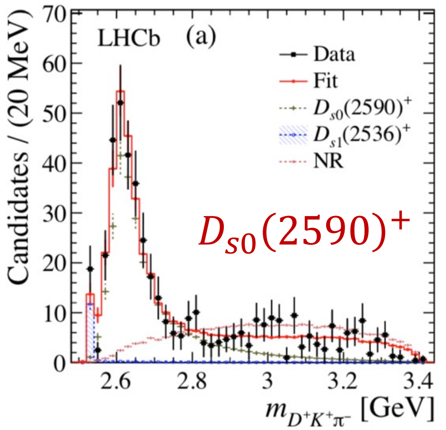
- If $X(3960)$ and $\chi_{c0}(3930)$ is the same state
  - $\Gamma(X \to D^+ D^-) < \Gamma(X \to D_s^+ D_s^-)$ disfavors the conventional interpretation
    - Conventional charmonium states predominantly decay into $D^{(*)}\overline{D}^{(*)}$

Summary and prospects

Summary

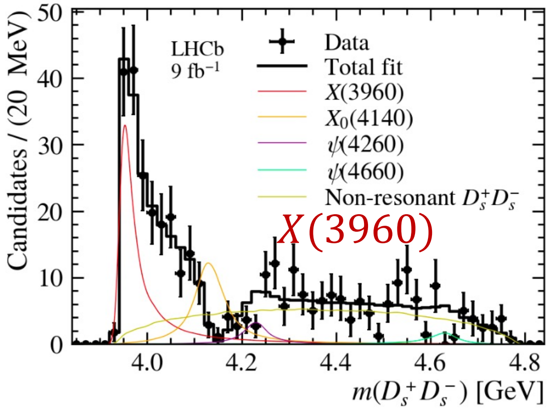
- Observations of two new excited mesons in multibody B decays

$B^0 \to D^+ D^- K^+ \pi^-$



Phys. Rev. Lett. 126 (2021) 122002

$B^+ \to D_s^+ D_s^- K^+$



arXiv:2210.15153, arXiv:2211.05034

- Properties measured using amplitude analysis

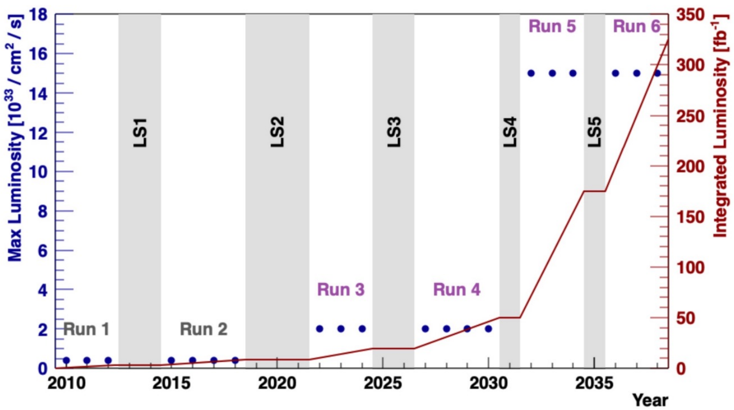
- $D_{s0}(2590)^+$: strong candidate for $D_s(2^1S_0)^+$
- $X(3960)$: charmonium(-like) state with $J^{PC} = 0^{++}$

Prospects on the $B \to D\bar{D}K$ analyses @ LHCb

- Excellent potential of $B \to D\bar{D}K$ decays for hadron spectroscopy studies
- Decays with purely charged final-state particles can be efficiently and precisely reconstructed @ LHCb
  - e.g. $B^+ \to D^{(*)+}\bar{D}^0K^+, B^+ \to D^0\bar{D}^0K^+$, etc.
  - Amplitude analyses of such decays are possible

Available yields ~$10^3 \sim 10^5$

- Decays involving $K^0/\pi^0/\gamma$ rarely touched
  - Low reconstruction efficiency and poor resolution
- Large LHCb data that will be collected in future runs
  - Allowing detailed investigations of the underlying resonances in some decays. e.g. $X_{0,1}(2900)$ in $B^0 \to D^+D^-K^+\pi^-$
  - Enabling the analyses of the decays involving $K^0/\pi^0/\gamma$



Possible future studies of $D_{s0}(2590)^+$

- $D_{s0}(2590)^+ \to D^{*0}K^+/D^{*+}K^0$ in principle possible
  - Investigated in $B \to \overline{D}D^{*0}K^+$ and $B \to \overline{D}D^{*+}K^0$ decays
  - $\Gamma(D_{s0}^+ \to D^*K)/\Gamma(D_{s0}^+ \to DK\pi)$ will be an additional input to help identify the $D_{s0}(2590)^+$ nature

- $DK\pi$ system can be investigated in other processes
  - $B_{(s)} \to DK\pi\pi/DK\pi K$
  - Prompt production
  - Measured results as cross checks for those in $B \to D\overline{D}K$ decays

Towards the nature of $X(3960)/\chi_{c0}(3930)$

- Precision measurements of $X(3960)/\chi_{c0}(3930)$ properties
  - $X(3960)$: $M_0 = 3955 \pm 6 \pm 11$ MeV; $\Gamma_0 = 48 \pm 17 \pm 10$ MeV
  - $\chi_{c0}(3930)$: $M_0 = 3924 \pm 2$ MeV; $\Gamma_0 = 17 \pm 5$ MeV
- To re-observe $X(3960) \to D_s^+ D_s^-$ in other decays
  - e.g. $B^0 \to D_s^+ D_s^- K^+ \pi^-$
- To re-observe $\chi_{c0}(3930)$ in the $D^0\overline{D}^0$ system
  - e.g. $B \to D^0 \overline{D}^0 K$
- If $X(3960)/\chi_{c0}(3930)$ is exotic, it could decay into $c\bar{c} + h$
  - $J^{PC} = 0^{++}$ $X(3915) \to J/\psi\omega$
    - Comparable properties with those of $X(3960)/\chi_{c0}(3930)$
  - Investigation of $B \to J/\psi\omega K$ will provide extra information
    - e.g. $\Gamma(X \to J/\psi\omega)/\Gamma(X \to D_s^+ D_s^-)$

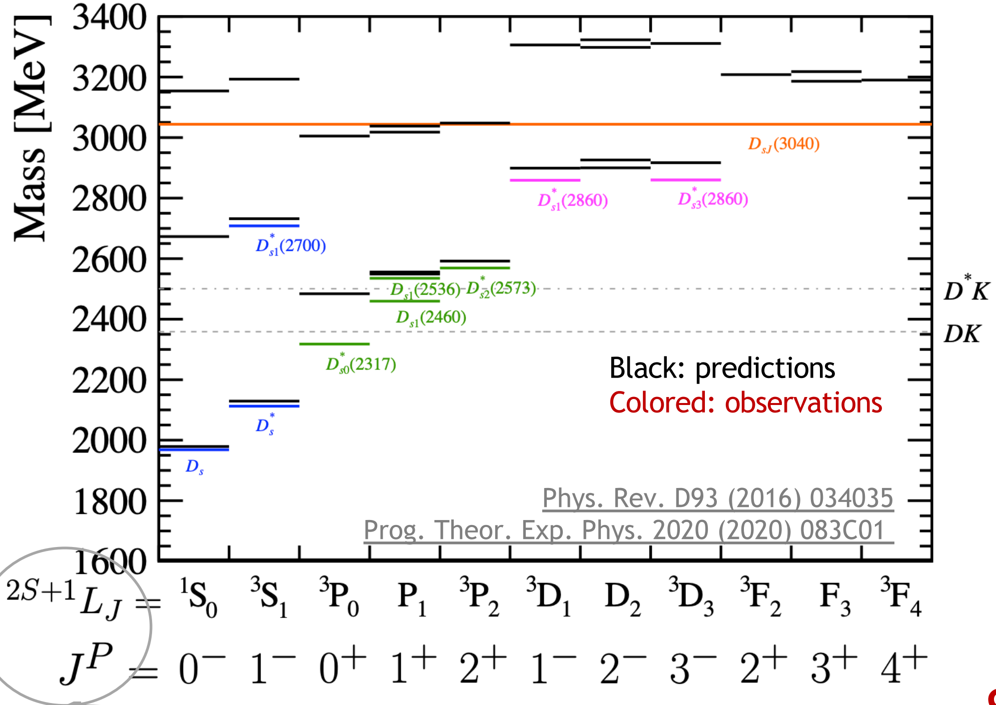
Thanks for listening

Backup

Introduction (Backup)

D_s^+ spectroscopy

Before the discovery of $D_{s0}(2590)$



- S: spin sum
- L: orbital angular momentum
- J: total spin; $P = (-1)^{L+1}$

- 10 mesons observed
  - $D_{sJ}(3040)$: J^P undetermined
  - $D_{s1}^*(2700)$: good candidate for 2^3S_1
  - $D_{s1,3}^*(2860)$: candidate for $1^3D_{1,3}$
  - $D_{s0}^*(2317)$ & $D_{s1}(2460)$:
    - Mass far below prediction
    - Still puzzles today
      - $cs\bar{u}\bar{d}$ tetraquark?
      - DK/D^*K molecular?

      Eur. Phys. J. C, 2017, 77(3)
  - Other states well established

- Six states unobserved below 3.1GeV
  - 2^1S_0: ~2.6GeV
  - $1^1D_2, 1^3D_2$:~2.86GeV
  - 2^3P_0, 2^1P_1, 2^3P_2: ~3GeV
  - Can be searched for in $D^{(*)}K^{(*)}$ system

Charm-strange mesons

State	J^P	Mass (MeV)	Width (MeV)	Observed decay modes
D_s^+	0^-	1968.35 ± 0.07	$\frac{1}{(5.04 \pm 0.04) \times 10^{-13}\ \mathrm{s}}$	$\eta\pi^+, K^+K^-\pi^+, etc.$
$D_{s1}^*(2112)^+$	1^-	2112.2 ± 0.4	< 1.9	$D_s^+\gamma, D_s^+e^+e^-, D_s^+\pi^0$
$D_{s0}^*(2317)^+$	0^+	2317.8 ± 0.5	< 3.8	$D_s^+\pi^0$
$D_{s1}(2460)^+$	1^+	2459.5 ± 0.6	< 3.5	$D_s^+\gamma, D_s^{*+}\pi^0, D_s^+\pi^+\pi^-$
$D_{s1}(2536)^+$	1^+	2535.11 ± 0.06	0.92 ± 0.05	$D_s^+\pi^+\pi^-, D^*K, DK\pi$
$D_{s2}^*(2573)^+$	2^+	2569.1 ± 0.8	16.9 ± 0.7	DK, D^*K
$D_{s1}^*(2700)^+$	1^-	2714 ± 5	122 ± 10	DK, D^*K
$D_{s1}^*(2860)^+$	1^-	2859 ± 27	159 ± 80	DK
$D_{s3}^*(2860)^+$	3^-	2860 ± 7	53 ± 10	DK, D^*K
$D_{sJ}(3040)^+$	$?^?$	3044^{+31}_{-9}	239 ± 60	D^*K

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Charm-strange mesons (cont.)

State	$n^{2S+1}L_J$	Mass (MeV)			Width (MeV)		
		Exp.[1]	GI[5]	GI-S[6]	Exp.[1]	GI[5]	GI-S[6]
D_s^+	1^1S_0	1968.35 ± 0.07	1979	1967	$\frac{1}{(5.04\pm0.04)\times10^{-13}\ \text{s}}$	-	-
$D_{s1}^*(2112)^+$	1^3S_1	2112.2 ± 0.4	2129	2115	< 1.9	1.03×10^{-3}	-
$D_{s0}^*(2317)^+$	1^3P_0	2317.8 ± 0.5	2484	2463	< 3.8	221	-
$D_{s1}(2460)^+$	$1P_1$	2459.5 ± 0.6	2549	2529	< 3.5	0.135	-
$D_{s1}(2536)^+$	$1P_1'$	2535.11 ± 0.06	2556	2534	0.92 ± 0.05	140	-
$D_{s2}^*(2573)^+$	1^3P_2	2569.1 ± 0.8	2592	2571	16.9 ± 0.7	10.07	-
$D_{s1}^*(2860)^+$	1^3D_1	2859 ± 27	2899	2865	159 ± 80	197.2	-
-	$1D_2$	-	2900	-	-	115.1	-
-	$1D_2'$	-	2926	-	-	195	-
$D_{s3}^*(2860)^+$	1^3D_3	2860 ± 7	2917	2883	53 ± 10	46	14
-	1^3F_2	-	3208	3159	-	292.5	416
-	$1F_3$	-	3186	-	-	182.6	372
-	$1F_3'$	-	3218	-	-	323	193
-	1^3F_4	-	3190	3143	-	182	151
-	2^1S_0	-	2673	2646	-	73.6	76.6
$D_{s1}^*(2700)^+$	2^3S_1	2714 ± 5	2732	2704	122 ± 10	123.4	-
-	2^3P_0	-	3005	2960	-	145.6	166.6
$D_{sJ}(3040)^+$	$2P_1$	3044^{+31}_{-9}	3018	-	239 ± 60	143	286
	$2P_1'$		3038	2992		147.6	131.3
-	2^3P_2	-	3048	3004	-	131.5	86.3

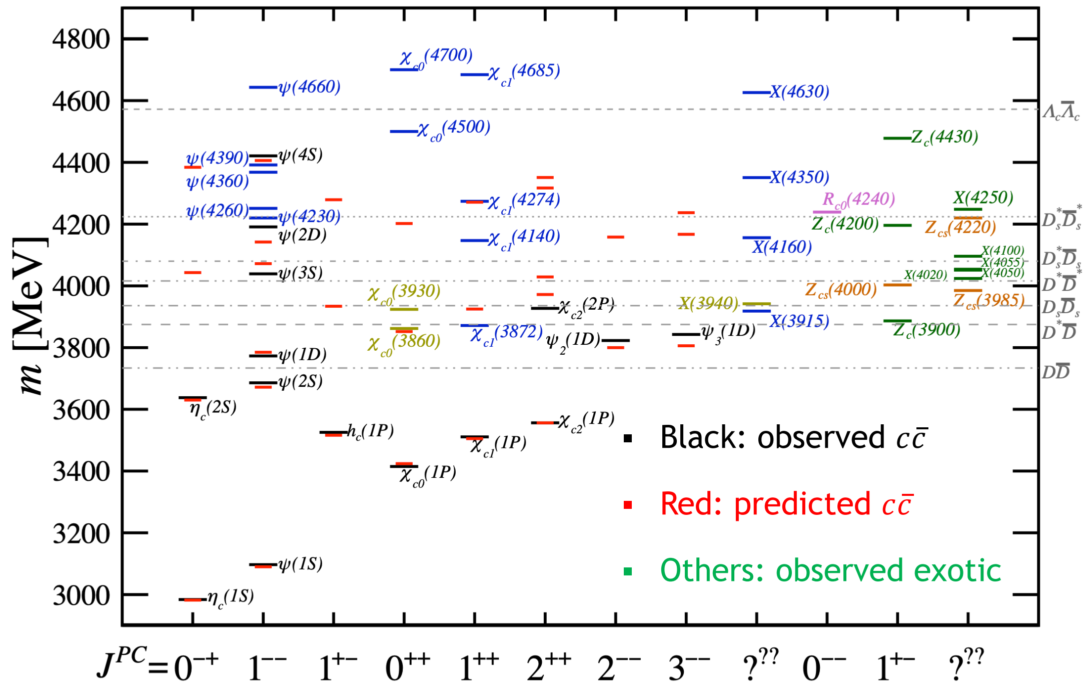
[1] Prog. Theor. Exp. Phys., 2020, 2020(8)
[5] Phys. Rev. D, 2016, 93(3): 034035
[6] Phys. Rev. D, 2015, 91: 054031

$$\begin{pmatrix} |nL_L\rangle \\ |nL_L'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{13} & \sin\theta_{13} \\ -\sin\theta_{13} & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} |n^1L_L\rangle \\ |n^3L_L\rangle \end{pmatrix}$$

$$\begin{pmatrix} |D_{s1}^*(2700)\rangle \\ |D_{s1}^*(2860)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{SD} & \sin\theta_{SD} \\ -\sin\theta_{SD} & \cos\theta_{SD} \end{pmatrix} \begin{pmatrix} |2^3S_1\rangle \\ |1^3D_1\rangle \end{pmatrix}$$

Charmonium

- Rich structures
- Conventional charmonium
  - Predominantly decay into $D^{(*)}\overline{D}^{(*)}$ if mass above $D^{(*)}\overline{D}^{(*)}$
    - OZI allowed
- Exotic charmonium
  - Have $c\bar{c} + h/\gamma$ decay process
    - OZI suppressed for conventional states
  - Inner structure unclear



Prog. Theor. Exp. Phys. 2020 (2020) 083C01 Phys. Rev. D, 2005, 72: 054026

- Experimental information to help identify charmonium states
  - Precise measurements of the mass, width
  - Investigations of different decay modes
- More states are expected in experiment
  - Open charm: $D_{(s)}^{(*)}\overline{D}_{(s)}^{(*)}$, $\Lambda_c^+\Lambda_c^-$
  - $c\bar{c} + h/\gamma$

Conventional charmonium

State	$n^{2S+1}L_J$	Mass (MeV)				Width (MeV)	
		Exp.	NR	GI	GI-S	Exp.	NR
$\eta_c(1S)$	$\eta_c(1^1S_0)$	2983.9 ± 0.4	2982	2975	2979	32.0 ± 0.7	-
J/ψ	$\psi(1^3S_1)$	3096.900 ± 0.006	3090	3098	3097	0.0926 ± 0.0017	-
$\eta_c(2S)$	$\psi(2^1S_0)$	3637.5 ± 1.1	3630	3623	3623	$11.3^{+3.2}_{-2.9}$	-
$\psi(2S)$	$\psi(2^3S_1)$	3686.10 ± 0.06	3672	3676	3673	0.294 ± 0.008	-
-	$\eta_c(3^1S_0)$	-	4043	4064	3991	-	80
$\psi(4040)$	$\psi(3^3S_1)$	4039 ± 1	4072	4100	4022	80 ± 10	74
-	$\eta_c(4^1S_0)$	-	4384	4425	4250	-	61
$\psi(4415)$	$\psi(4^3S_1)$	4421 ± 4	4406	4450	4463	62 ± 20	78
$\chi_{c0}(1P)$	$\chi_c(1^3P_0)$	3414.71 ± 0.30	3424	3445	3433	10.8 ± 0.6	-
$\chi_{c1}(1P)$	$\chi_c(1^3P_1)$	3510.67 ± 0.05	3505	3510	3510	0.84 ± 0.04	-
$\chi_{c2}(1P)$	$\chi_c(1^3P_2)$	3556.17 ± 0.07	3556	3550	3554	1.97 ± 0.09	-
$h_c(1P)$	$h_c(1^1P_1)$	3525.38 ± 0.11	3516	3517	3519	0.7 ± 0.4	-
$\{\chi_{c0}(3860)\}$	$\chi_c(2^3P_0)$	3862^{+50}_{-35}	3852	3916	3842	201^{+180}_{-110}	30
$\{\chi_{c0}(3930)\}$		3923.8 ± 1.6				17.4 ± 5.1	
$\{X(3940)\}$	$\chi_c(2^3P_1)$	3942 ± 9	3925	3953	3901	37^{+27}_{-17}	165
$\chi_{c2}(3930)$	$\chi_c(2^3P_2)$	3922.5 ± 1.0	3972	3979	3937	35.2 ± 2.2	80
-	$h_c(2^1P_1)$	-	3934	3956	3908	-	87
$\psi(3770)$	$\psi(1^3D_1)$	3773.7 ± 0.4	3785	3819	3787	27.2 ± 1.0	43
$\psi(3823)$	$\psi(1^3D_2)$	3823.7 ± 0.5	3800	3838	3798	< 5.2	-
$\psi(3842)$	$\psi(1^3D_3)$	3842.71 ± 0.20	3806	3849	3799	2.8 ± 0.6	0.5
-	$\eta_c(1^1D_2)$	-	3799	3837	3796	-	-
$\psi(4160)$	$\psi(2^3D_1)$	4191 ± 5	4142	4194	4089	70 ± 10	74
-	$\psi(2^3D_2)$	-	4158	4208	4100	-	92
-	$\psi(2^3D_3)$	-	4167	4217	4103	-	148
-	$\eta_c(2^1D_2)$	-	4158	4208	4099	-	111

Prog. Theor. Exp. Phys., 2020, 2020(8)
Phys. Rev. D, 2005, 72: 054026
Phys. Rev. D, 2009, 79: 094004

Exotic charmonium

Prog. Theor. Exp. Phys., 2020, 2020(8)

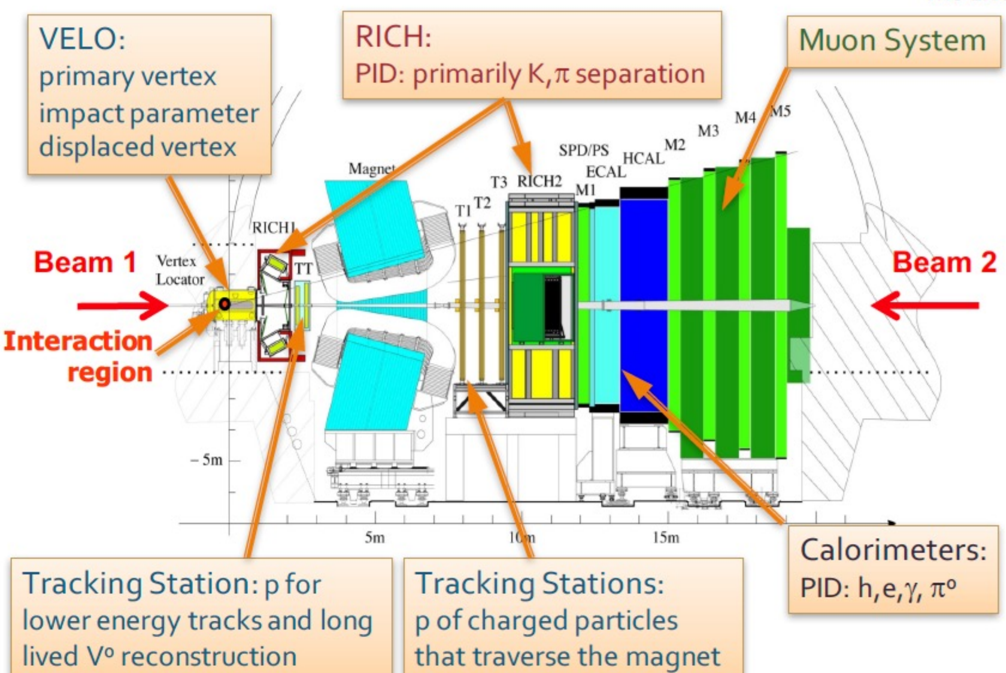
State	J^{PC}	Decay(s)	State	J^{PC}	Decay(s)
$\chi_{c1}(3872)$	1^{++}	$D^0\bar{D}^0\pi^0, \bar{D}^{*0}D^0,$	$X(4630)$	$?^{?+}$	$J/\psi\phi$
		$J/\psi\pi\pi, J/\psi\omega, J/\psi\rho,$	$\psi(4660)$	1^{--}	$\psi(2S)\pi\pi, \Lambda_c^+\Lambda_c^-,$
		$J/\psi\gamma, \chi_{c1}\pi^0$			$D_s^+D_{s1}(2536)^-$
$X(3915)$	$(0,2)^{++}$	$J/\psi\omega, \gamma\gamma$	$\chi_{c1}(4685)$	1^{++}	$J/\psi\phi$
$\chi_{c1}(4140)$	1^{++}	$J/\psi\phi$	$\chi_{c0}(4700)$	0^{++}	$J/\psi\phi$
$\psi(4230)$	1^{--}	$\chi_{c0}\omega, h_c\pi\pi,$	$Z_c(3900)$	1^{+-}	$J/\psi\phi, D\bar{D}^*$
		$\eta_c 3\pi, J/\psi\eta,$	$X(4020)^\pm$	$?^{?-}$	$h_c\pi, D^*\bar{D}^*$
		$J/\psi\pi\pi, \psi(2S)\pi\pi,$	$X(4050)^\pm$	$?^{?+}$	$\chi_{c1}\pi$
		$\gamma\chi_{c1}(3872),$	$X(4055)^\pm$	$?^{?-}$	$\psi(2S)\pi$
		$D_s^+D_{s1}(2536)^-, l\bar{l}$	$X(4100)^\pm$	$?^{??}$	$\eta_c\pi$
$\psi(4260)$	1^{--}	$e^+e^-, J/\psi\pi\pi,$	$Z_c(4200)^\pm$	1^{+-}	$J/\psi\pi, \psi(2S)\pi$
		$J/\psi KK$	$R_{c0}(4240)^\pm$	0^{--}	$\psi(2S)\pi$
$\chi_{c1}(4274)$	1^{++}	$J/\psi\phi$	$X(4250)^\pm$	$?^{?+}$	$\chi_{c1}\pi$
$X(4350)$	$?^{?+}$	$J/\psi\phi, \gamma\gamma$	$Z_c(4430)$	1^{+-}	$J/\psi\pi, \psi(2S)\pi$
$\psi(4360)$	1^{--}	$e^+e^-, J/\psi\pi\pi, \psi(2S)\pi\pi$	$Z_{cs}(3985)^\pm$	1^+	$D^{*0}D_s^-, D^0D_s^{*-}$
$\psi(4390)$	1^{--}	$h_c\pi\pi, J/\psi\eta$	$Z_{cs}(4000)^\pm$	1^+	$J/\psi K^\pm$
$\chi_{c0}(4500)$	0^{++}	$J/\psi\phi$	$Z_{cs}(4220)^\pm$	1^+	$J/\psi K^\pm$

LHCb experiment

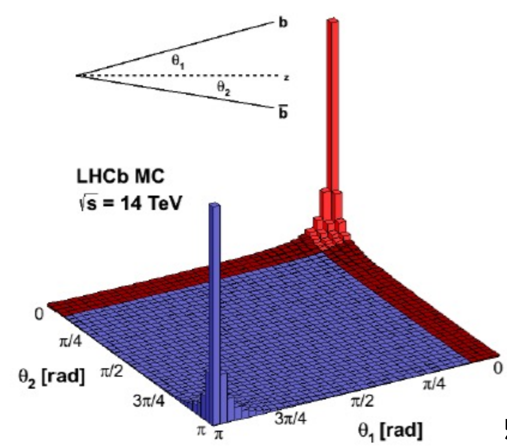
$$\begin{array}{ccccccc} B^+ & : & B^0 & : & B_s^0 & : & \Lambda_b^0 \\ (u\bar{b}) & & (d\bar{b}) & & (s\bar{b}) & & (udb) \\ 4 & : & 4 & : & 1 & : & 2 \end{array}$$

- LHC: beauty&charm factory
  - pp collision @ $\sqrt{s} = 13$ TeV : ~20000 $b\bar{b}$ /s
- LHCb detector: Dedicated for the precision reconstruction of heavy hadrons
  - Powerful particle-ID — $\epsilon(K \to K) \sim 95\%$ mis-ID $\epsilon(\pi \to K) \sim 5\%$; $\epsilon(\mu \to \mu) \sim 97\%$ mis-ID $\epsilon(\pi \to \mu) \sim 1 - 3\%$
  - High momentum and mass resolution — $\Delta p/p = 0.4 \sim 0.6\%$ $(5 - 100\ \mathrm{GeV}/c)$; $\sigma_m = 8\ \mathrm{MeV}/c^2$ for $B \to J/\psi X$ (constrained $m_{J/\psi}$)
  - Precise vertex reconstruction — $\sigma_{IP} = 20\mu\mathrm{m}$ to select long-lived beauty & charm candidates

The LHCb detector described in [JINST 3 (2008) S08005]



- $2 < \eta < 5$ range: $\sim 25\%$ of $b\bar{b}$ pairs inside LHCb acceptance



[Int. J. Mod. Phys. A 30 (2015) 1530022]

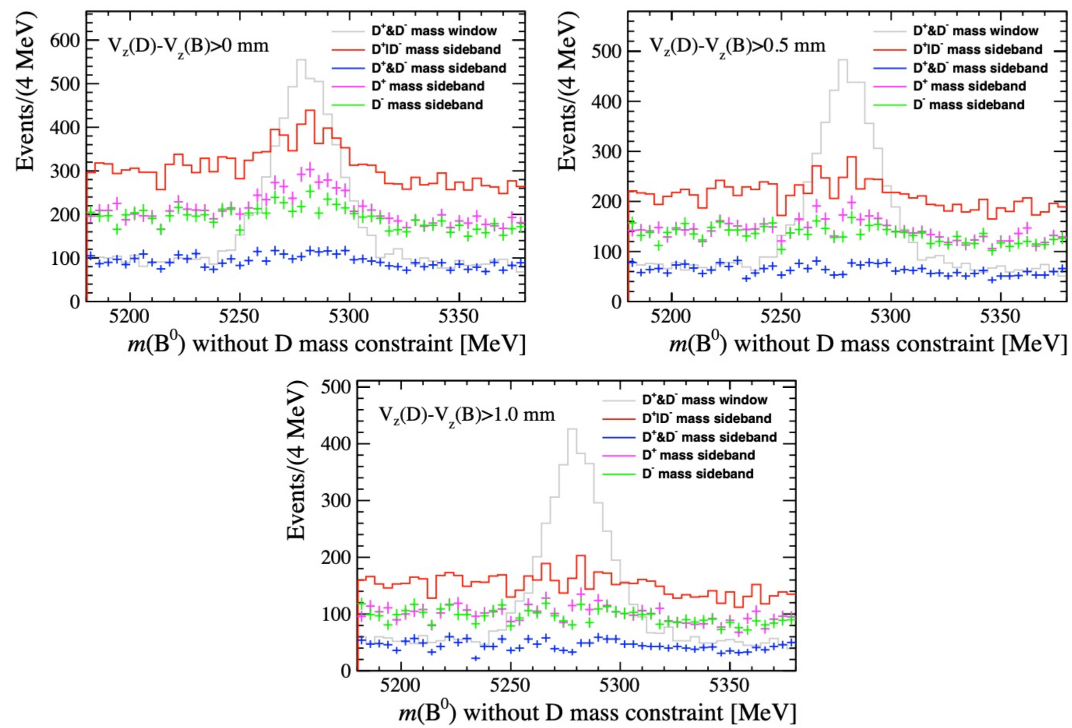
$B^0 \to D^- D^+ K^+ \pi^-$ analysis

(Backup)

$B^0 \to D^+D^-K^+\pi^-$ background

- Physical background are negligible with 5280 ± 100 MeV
  - Mis-ID bkg：Cabibbo suppressed； f_s/f_d suppressed
  - Partially reconstructed bkg： $D^{*+} \to D^+\pi^0/\gamma$
    - $D^{*+} \to D^+\pi^0$： excluded out of the mass window 5280 ± 100 MeV
    - $D^{*+} \to D^+\gamma$: $\mathcal{B}(D^{*+} \to D^+\gamma) = (1.6 \pm 0.4)\%$ is very small

- Non-double-charm background
  - $B^0 \to [K^-\pi^+\pi^+]D^-K^+\pi^-$, $B^0 \to [K^-\pi^+\pi^+][K^-\pi^+\pi^+]K^+\pi^-$



Amplitude construction

- Using Helicity formalism
  - Decay chain: $B^0 \to D^- D_{sk}^+, D_{sk}^+ \to D^+ K^{*0}, K^{*0} \to K^+\pi^-$
- Intermediate resonances:
  - $K^+\pi^-$: S-wave because $m(K^+\pi^-) < 0.75$ GeV
    - Modeled by $J^P = 0^+ \; K^*(700)^0$
  - $D^+K^+\pi^-$: $0^- + 0^+ \;\to\; 0^-, 1^+, 2^-, \cdots$
    - A non-resonant (NR) term with $J^P = 0^-$
    - $J^P = 1^+ \; D_{s1}(2536)^+$
    - A new D_{sJ}^+ state (three spin-parity tested: $J^P = 0^-, 1^+, 2^-$)
- Matrix element

Helicity coupling	Wigner d-matrix	Momentum barrier factors for B^0 and D_{sk} decays

$$\mathcal{M} = \sum_k \mathcal{H}^{D_{sk}} d_{0,0}^{J_{D_{sk}}}(\theta_{D_s}) p^{L_{B^0}} F_{L_{B^0}}(pd) q^{L_{D_{sk}}} F_{L_{D_{sk}}}(qd)$$

$$\mathrm{BW}(m_{K^+\pi^-})\,\mathrm{BW}_{D_{sk}}(m_{D^+K^+\pi^-}),$$

Mass lineshapes

- θ_{D_s}: angle between D^+ and D^- momenta in the D_{sk}^+ rest frame
- p, q: center-of-mass momentum of $D^-D_{sk}^+$ and D^+K^{*0}
- $d = 3\ \mathrm{GeV}^{-1} \sim (0.6\ \mathrm{fm})$: effective radius of the particle

Mass lineshapes

- Relativistic Breit-Wigner function

$$\mathrm{BW}(m|m_0,\Gamma_0) = \frac{1}{m_0^2 - m^2 - i m_0 \Gamma(m)}$$

  - m_0: BW mass
  - $\Gamma_0 \equiv \Gamma(m_0)$: BW width
  - $\Gamma(m)$: mass-dependent width (total width)

Sum over all open channels

$$\Gamma(m) = \sum_c \Gamma^c(m) \equiv \sum_c g_c^2 \rho'_c(m) \qquad \rho'_c(m) \propto \int \mathrm{d}\Phi_N^c \, |\mathcal{M}^c|^2$$

- Width formula:
  - $K_0^*(700)^0$:

$$\Gamma^{K^*\to K\pi}(m_{K\pi}) = \Gamma_0^{K^*\to K\pi} \frac{q^{K\pi}}{q_0^{K\pi}} \frac{m_0^{K^*}}{m_{K\pi}}$$

  - $D_{s1}(2536)^+$: set to constant because it is very narrow (0.9MeV)
  - New D_{sJ}^+:

$$\Gamma^{D_{sJ}}(m_{D^+K^+\pi^-}) = \Gamma^{D_{sJ}\to D^*K}(m_{D^+K^+\pi^-}) + \Gamma^{D_{sJ}\to DK\pi}(m_{D^+K^+\pi^-})$$

Inferred $D_{sJ} \to D^*K$ decay width	$D_{sJ} \to DK\pi$ decay width

D_{sJ} decay width

$$\Gamma^{D_{sJ}}(m_{D^+K^+\pi^-}) = \Gamma^{D_{sJ}\to D^*K}(m_{D^+K^+\pi^-}) + \Gamma^{D_{sJ}\to DK\pi}(m_{D^+K^+\pi^-})$$

- $\Gamma^{D_{sJ}\to D^*K}(m_{DK\pi})$: two-body decay width

$$\Gamma^{D_{sJ}\to D^*K}(m_{DK\pi}) = \Gamma_0^{D_{sJ}\to D^*K} \frac{m_{DK\pi}}{m_0}\left(\frac{q^{D^*K}}{q_0^{D^*K}}\right)^{2L^{D^*K}+1} \frac{F^2_{L^{D^*K}}(q^{D^*K}d)}{F^2_{L^{D^*K}}(q_0^{D^*K}d)}$$

- $\Gamma^{D_{sJ}\to DK\pi}(m_{DK\pi})$: three-body decay width

$$\Gamma^{D_{sJ}\to DK\pi}(m_{DK\pi}) \propto \int d\Phi_{DK\pi} |\mathcal{M}^{D_{sJ}\to DK\pi}(m_{DK\pi})|^2$$

  - D_{sJ} decay amplitude depends on the $K^+\pi^-$ mass lineshape
    - No prior knowledge about the $K^+\pi^-$ mass lineshape
      ($K^*(700)^0$ BW may not be suitable because here $m_{DK\pi}$ could be very large, and more possible channels could open)
  - Four choices of $\Gamma^{D_{sJ}\to DK\pi}(m_{DK\pi})$ are tested in the amplitude fit

Four D_{sJ}^+ width models:

a. Constant
b. 3-body width with $K^+\pi^-$ LASS model Nucl. Phys., 1988, B296
c. 3-body width with unity $K^+\pi^-$ amplitude
d. 3-body width with $K^*(700)^0$ BW amplitude

MC integration

$$I(\vec{\omega}) \equiv \int \left|M(\vec{x} \mid \vec{\omega})\right|^2 \Phi(\vec{x})\epsilon(\vec{x})\, d\vec{x}$$

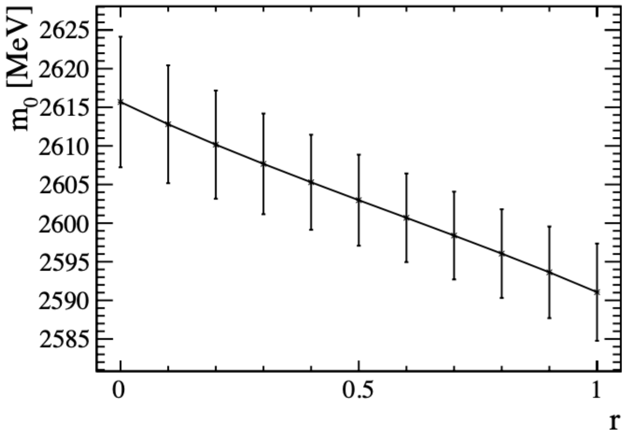
$$\approx \frac{1}{\sum_j^{N_{\mathrm{MC}}} w_j^{\mathrm{MC}}} \sum_j^{N_{\mathrm{MC}}} w_j^{\mathrm{MC}} \left|\mathcal{M}(\vec{x}_j \mid \vec{\omega})\right|^2$$

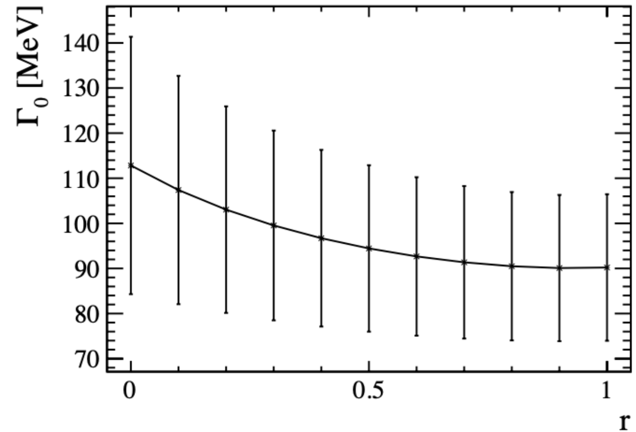
PDF normalization using MC integration by summing over all MC events after the selection (w_j^{MC} for MC correction)

Mass&width

- BW parameters vary a lot with the change of r

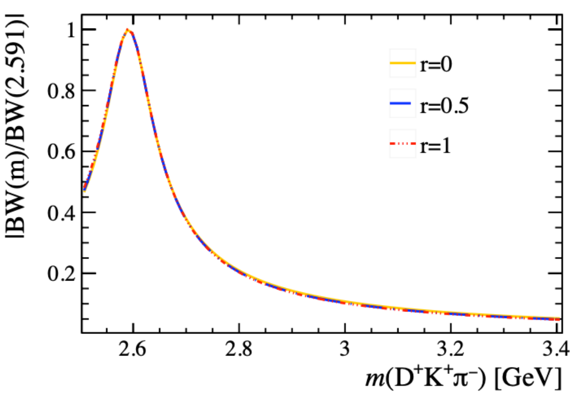
$\Gamma^{D_{sJ}\to DK\pi}\sim\text{constant}$

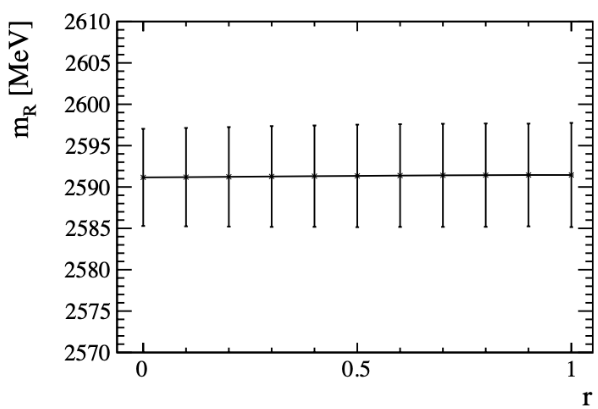


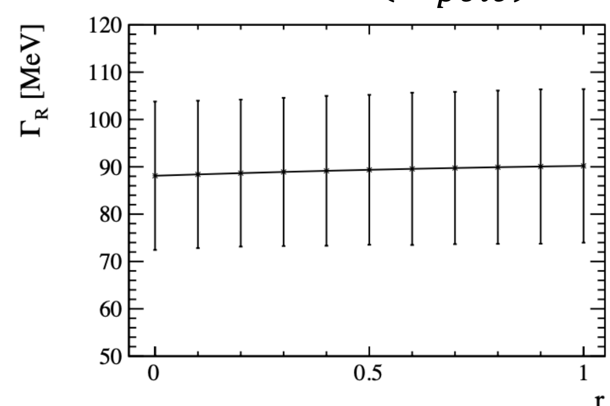


- But similar mass lineshapes and pole mass&width

$$\frac{1}{\mathrm{BW}(m_{pole})}=0$$





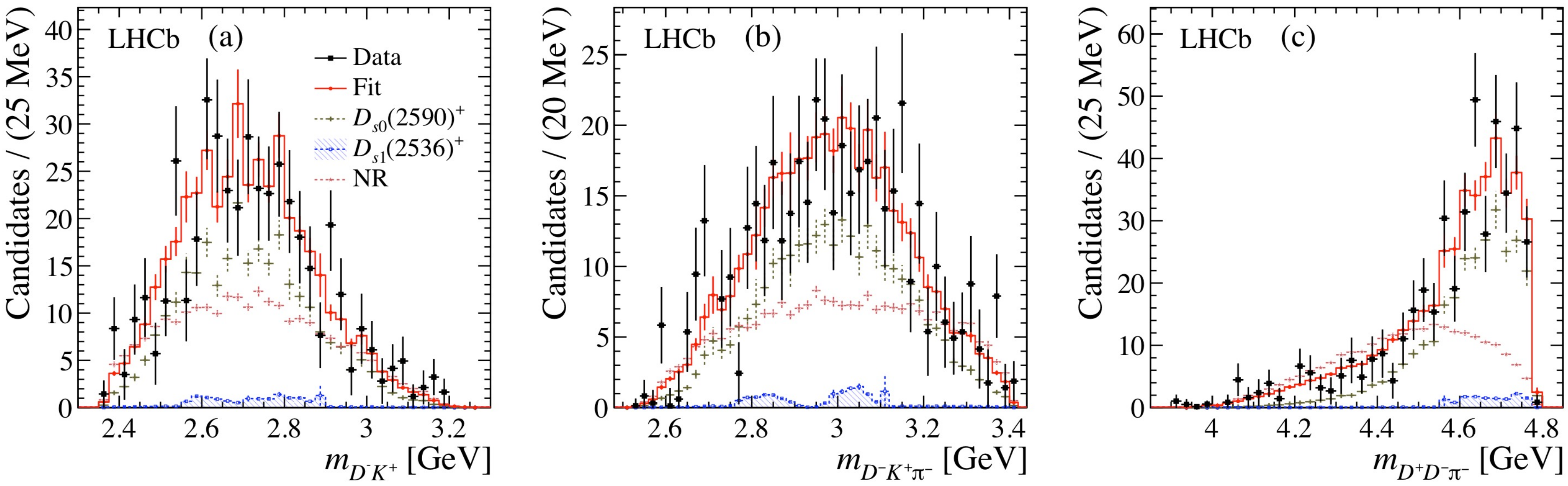


- BW parameters generally do not have strict physical meaning
  - Depending on decay processes and the lineshape parameterizations
- Pole mass and width are physical quantities
  - Independent of decay processes and parameterizations

Peak position and FWHM

More mass projections in fit

$B^0 \to D^- D^+ K^+ \pi^-$



Significance

- Using an empirical formula

$$p = \text{TMath::Prob}(-2\Delta \ln \mathcal{L},\ \nu \cdot \Delta\text{ndof})$$

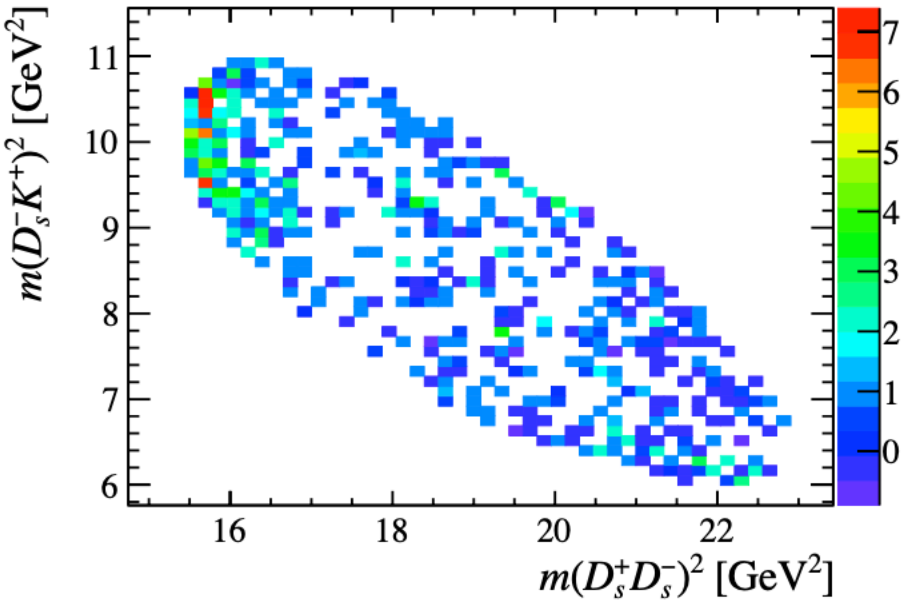
$$\sigma = \sqrt{2} \cdot \text{TMath::ErfcInverse}(p)$$

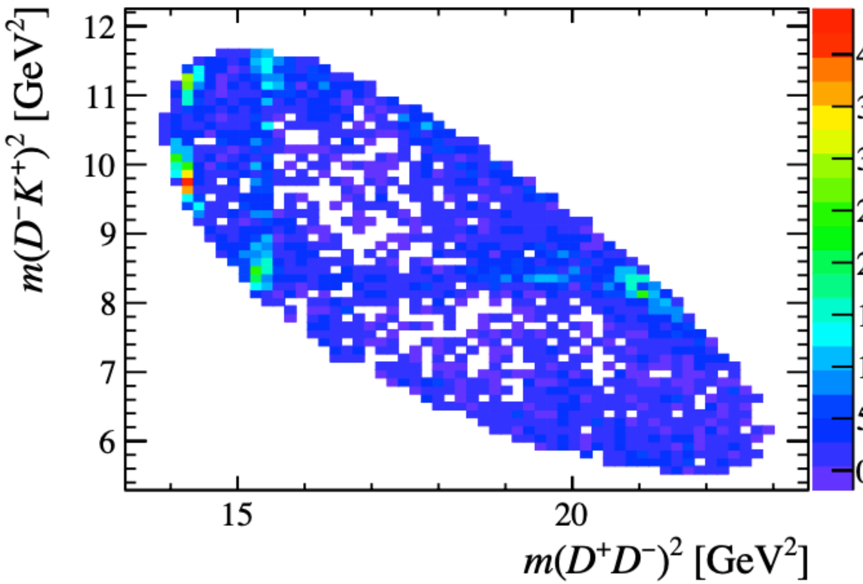
  - Null hypothesis - $J^P = 0^-$ hypothesis
  - $\nu = 2$ is an empirical value

$B^0 \rightarrow D_s^+ D_s^- K^+$ analysis

(Backup)

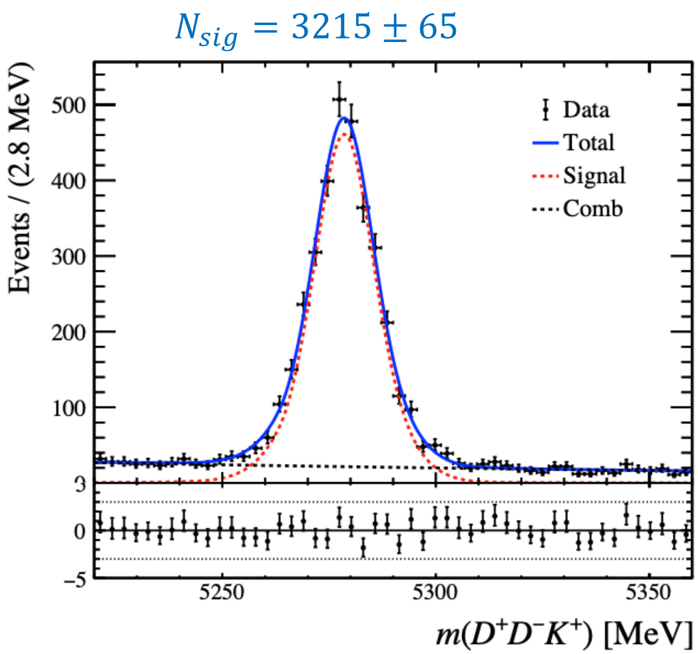
Dalitz plots

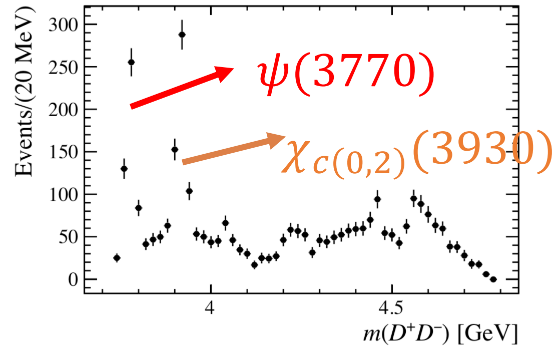


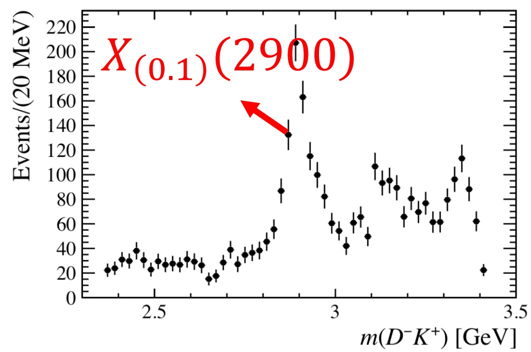


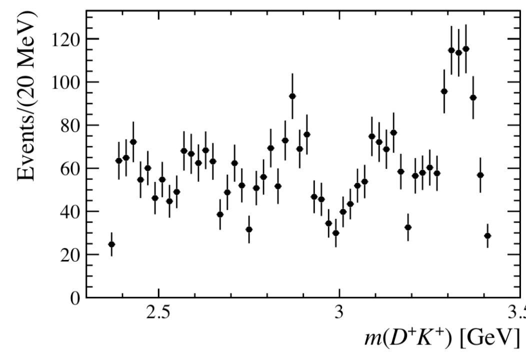
$B^+ \to D^+ D^- K^+$ data sample

- Reconstruction: $B^+ \to D^+ D^- K^+, D^+ \to K^- \pi^+ \pi^+$







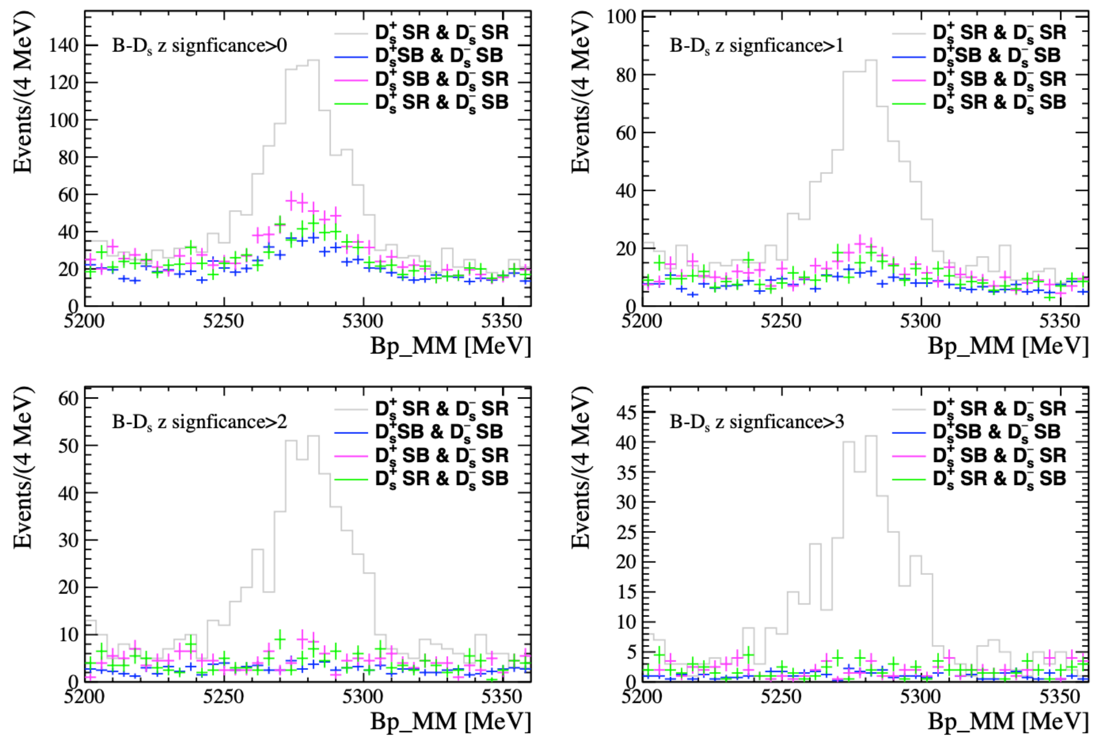


These structures have already been analyzed by
Phys.Rev.D102(2020) 112003
Phys. Rev. Lett. 125 (2020) 242001

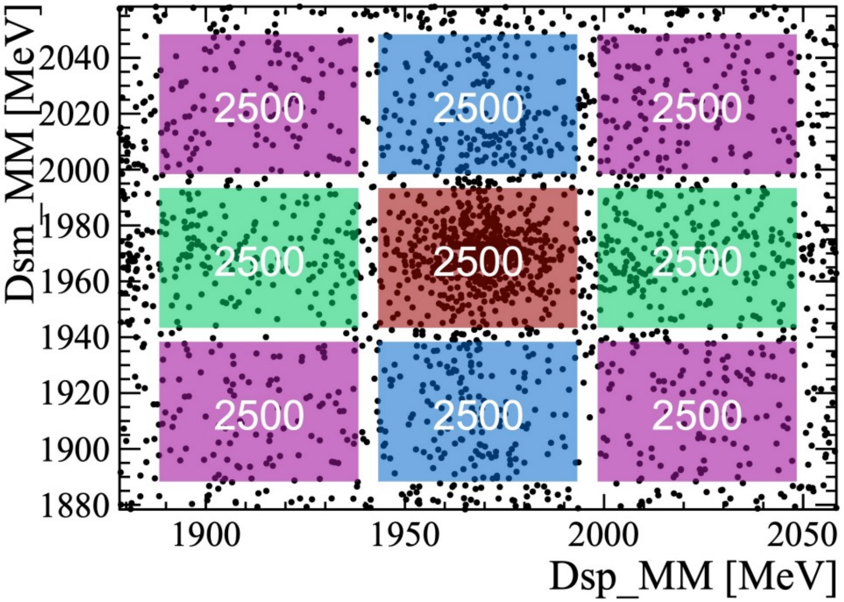
$B^+ \to D_s^+ D_s^- K^+$ physical background

- Partially reconstructed background
  - $B^+ \to D_s^+ D_s^- K^{*+},\ K^{*+} \to K^+ \pi^0$: Ouside the mass window(5280 ± 80 MeV)
  - $B^+ \to D_s^{(*)+} D_s^{(*)-} K^+, D_s^{*\pm} \to D_s^\pm \gamma$: Ouside the mass window(5280 ± 80 MeV)

- Non-double-charm background
  - $B^+ \to [K^- K^+ \pi^+]\ D_s^-\ K^+$
  - $B^+ \to [K^- K^+ \pi^+][K^+ K^- \pi^-] K^+$



$B^+ \to D_s^+ D_s^- K^+$ NDC fraction



Case	n_{sig}	n_{bkg}
Region a	57.0 ± 17.7	618.0 ± 29.4
Region b	36.9 ± 14.5	395.1 ± 23.7
Signal	355.7 ± 22.7	276.2 ± 20.9
n_{NDC}	19.3 ± 9.5	
f_{NDC} (%)	5.4 ± 2.7	

- Region a: only one D sideband (Blue&Green)
- Region b: two D sideband (Pink)
- Signal region: two D mass window (Red)

- B^+ signals estimated using a simple fit
  - Signal shape: Gaussian with mean set to PDG mass and width to 13 MeV (Typical resolution in MC)
  - Background shape: exponential

$$n_{\text{NDC}} = n_{\text{sig}}^{\text{green}} \cdot \frac{S_{\text{sig}}}{S_{\text{green}}} + n_{\text{sig}}^{\text{blue}} \cdot \frac{S_{\text{sig}}}{S_{\text{blue}}} - n_{\text{sig}}^{\text{pink}} \cdot \frac{S_{\text{sig}}}{S_{\text{pink}}}$$
$$= n_{\text{sig}}^{\text{a}} \cdot \frac{S_{\text{sig}}}{S_{\text{a}}/2} - n_{\text{sig}}^{\text{b}} \cdot \frac{S_{\text{sig}}}{S_{\text{b}}},$$

(The residual NDC fraction will be subtracted in branching fraction calculation)

$B^+ \to D^+ D^- K^+$ physical background

- Peaking background
  - Such background is thoroughly surveyed in the previous analysis (LHCb-PAPER-2020-024, LHCb-PAPER-2020-025)
  - Can be excluded if choosing B^+ mass > 5220 MeV

- NDC background
  - $\frac{dz}{\sigma_{dz}} > 2$ to suppress the background
  - Similar method to estimate NDC fraction

Case	n_{sig}	n_{bkg}
Region a	204.2 ± 36.4	2601.8 ± 61.0
Region b	14.2 ± 22.2	1159.0 ± 39.9
Signal	3084.7 ± 63.7	1399.6 ± 48.8
n_{NDC}	98.6 ± 19.0	
f_{NDC} (%)	3.2 ± 0.6	

(The residual NDC fraction will be subtracted in branching fraction calculation)

Branching fraction

$$N_{\text{sig}}^{\text{corr}} = 950406.31 \pm 56534.18\,(\text{stat}),$$
$$N_{\text{con}}^{\text{corr}} = 5329569.64 \pm 103700.12\,(\text{stat}).$$

$$\sigma(N_{\text{sig}}^{\text{corr}}) = \sqrt{\sum_i \left(\frac{w_{\text{sig},\,i}}{\epsilon_{\text{sig},\,i}(m^2(D_s^+ D_s^-), m^2(D_s^- K^+))} \right)^2}$$

$$\sigma(N_{\text{con}}^{\text{corr}}) = \sqrt{\sum_i \left(\frac{w_{\text{con},\,i}}{\epsilon_{\text{con},\,i}(m^2(D^+ D^-), m^2(D^- K^+))} \right)^2}$$

- Multiplying $(1 - f_{\text{NDC}}^{\text{sig}})/(1 - f_{\text{NDC}}^{\text{con}})$ for NDC background subtraction
- Multiplying $1 - \frac{\sigma_{N_{\text{sig}}}}{N_{\text{sig}}} \cdot (\text{bias of } \text{N}_{\text{sig}}\ \text{pull})$ for bias correction

$$\mathcal{R} = \frac{\mathcal{B}(B^+ \to D_s^+ D_s^- K^+)}{\mathcal{B}(B^+ \to D^+ D^- K^+)} = 0.525 \pm 0.033\,(\text{stat}) \pm 0.027\,(\text{syst}) \pm 0.034\,\,(\text{ext})$$

Systematic uncertainties

Systematic source	Relative uncertainty (%)
L0 trigger correction	2.3
Signal model variation	0.3
Background model variation	0.1
B^+ mass fit bias	0.1
Limited size of MC samples	0.5
KDE parameters	0.4
Charmless and single-charm background	2.9
PID resampling	2.8
BDT working point	1.6
Tracking efficiency	1.0
Multiple candidate removal	0.7
MC truth match efficiency	0.6
Total syst. (stat.)	5.1 (6.3)

Systematic uncertainties in amplitude analysis

Source		L0	MC	PID	Comp.	Bl-W	$M_0\&\Gamma_0$	Model	Tot.
$X(3960)$	M_0 (MeV)	0	2	0	2	0	1	11	11
	Γ_0 (MeV)	0	1	0	3	1	2	9	10
	FF (%)	0.6	0.7	0.5	7.1	0.0	2.8	1.0	7.8
$X_0(4140)$	M_0 (MeV)	0	1	0	10	1	4	1	11
	Γ_0 (MeV)	0	1	2	5	1	4	1	7
	FF (%)	0.1	0.5	0.0	6.9	0.1	2.9	1.9	7.5
$\psi(4260)$	FF (%)	0.0	0.0	0.0	3.0	0.0	0.1	0.1	3.0
$\psi(4660)$	FF (%)	0.0	0.0	0.0	0.4	0.0	0.1	0.2	0.4
NR	FF (%)	0.7	1.7	0.7	9.8	0.1	3.7	3.2	10.7