Mapping out the **direct** connection between leptogenesis and CP violation at low energies

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$\varepsilon_{i\alpha}$

Heavy neutrino LNV decays

$\mathcal{J}_\nu$

Light neutrino oscillations

ZZX, Nucl. Phys. B (2023) 034, e-print: 2203.14185; paper in writing

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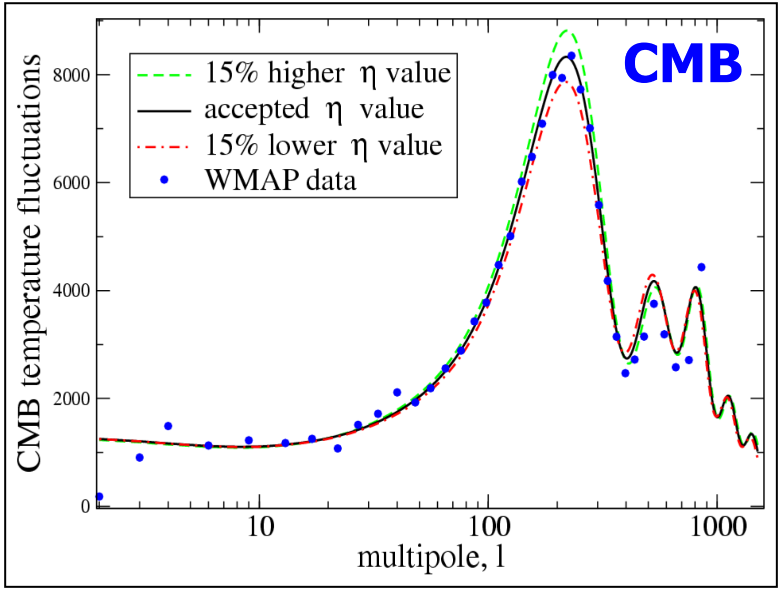
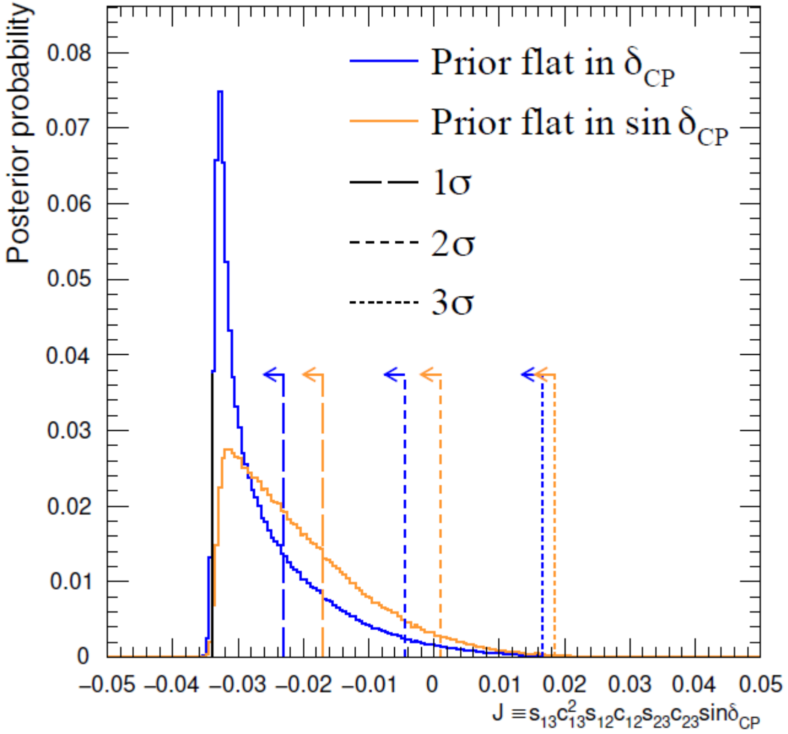
Motivation: experimental

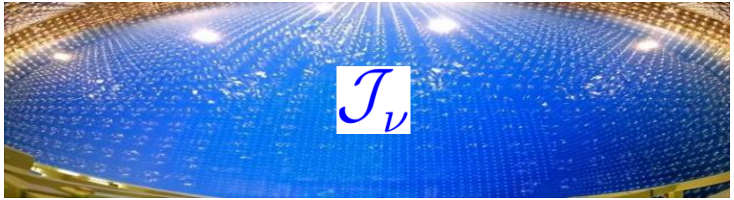
- ♦ **3-flavor neutrino oscillations firmly established; and 2σ evidence for CP violation recently achieved (T2K, 2303.03222).**

- ♦ **Almost an 100% baryon-antibaryon asymmetry of the Universe already established: the cosmic CP violation** ($\eta \sim 6.12 \times 10^{-10}$ **).**



+ BBN





Motivation: theoretical

- The most natural + economical way to have tiny neutrino masses —— seesaw (Minkowski 1977, Yanagida 1979, …):
  - Add R-handed neutrinos (N) IQ > 60
  - Allow ν-Yukawa interactions IQ > 90
  - Allow for N-self-interactions IQ > 130 as E. Majorana

- A bonus of seesaw: LNV + CPV decays of heavy Majorana neutrinos —— leptogenesis (Fukugita, Yanagida 1986).
  - One-loop CPV in heavy Majorana neutrino decays
  - A net lepton-antilepton asymmetry via CPV
  - The baryogenesis from leptogenesis

- One stone kills two birds: how is the CPV of leptogenesis connected to that in neutrino oscillations?
  - No direct connection in general (Buchmueller, Pluemacher 1998)
  - May have direct correlation in very special models (many papers)
  - My target: to establish the general + explicit links between them

OUTLINE

- **Seesaw + leptogenesis**
- **A full parameterization**
- **Jarlskog CPV invariant**
- **CPV in heavy N decays**

Formal seesaw: heavy neutrino decays

- The **canonical seesaw** mechanism formally works at an energy scale **far above the Fermi scale**:

(ZZX, 2203.14185)

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}} Y_l H l_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

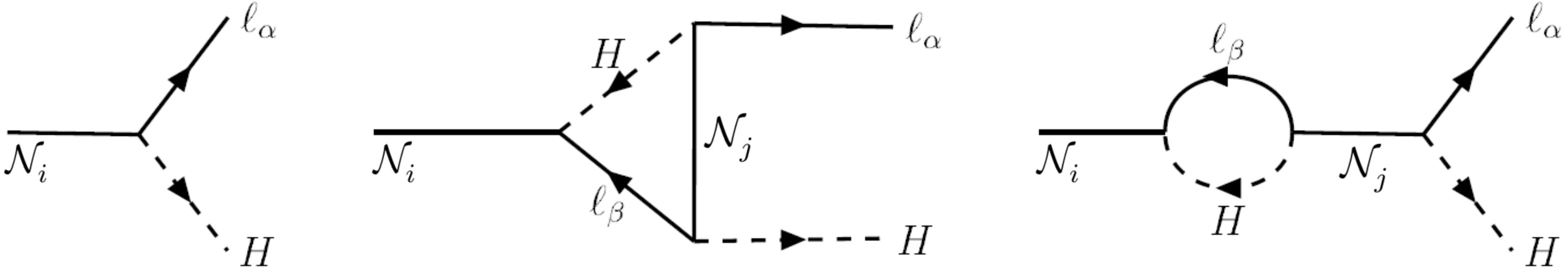
$$= \overline{l_{\rm L}} Y_l l_{\rm R} \phi^0 + \frac{1}{2}\overline{\begin{bmatrix}\nu_{\rm L} & (N_{\rm R})^c\end{bmatrix}} \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + \overline{\nu_{\rm L}} Y_l l_{\rm R} \phi^+ - \overline{l_{\rm L}} Y_\nu N_{\rm R} \phi^- + {\rm h.c.}$$

- The **basis transformation** related to LNV / CPV decays of heavy **Majorana** neutrinos **before SSB**:

$$U_0^{\prime\dagger} M_{\rm R} U_0^{\prime *} = \mathcal{D}_{\mathcal{N}} \; , \quad \mathcal{N}_{\rm R}^\prime = U_0^{\prime T} N_{\rm R} \; ; \qquad \mathcal{D}_{\mathcal{N}} \equiv {\rm Diag}\{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3\} \; , \quad \mathcal{N}^\prime = \begin{pmatrix}\mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3\end{pmatrix}^T$$

In the mass basis, heavy neutrinos decay via Yukawa interactions (all SM particles are massless):

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}} Y_l H l_{\rm R} + \overline{\ell_{\rm L}} \mathcal{Y}_\nu \widetilde{H} \mathcal{N}_{\rm R}^\prime + \frac{1}{2}\overline{(\mathcal{N}_{\rm R}^\prime)^c} \mathcal{D}_{\mathcal{N}} \mathcal{N}_{\rm R}^\prime + {\rm h.c.} \qquad \mathcal{Y}_\nu \equiv Y_\nu U_0^{\prime *}$$



Formal seesaw: light neutrino masses

- The canonical seesaw mechanism formally works at an energy scale far above the Fermi scale:

(ZZX, 2203.14185)

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}} Y_l H l_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$= \overline{l_{\rm L}} Y_l l_{\rm R} \phi^0 + \frac{1}{2}\overline{\begin{bmatrix}\nu_{\rm L} & (N_{\rm R})^c\end{bmatrix}} \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + \overline{\nu_{\rm L}} Y_l l_{\rm R} \phi^+ - \overline{l_{\rm L}} Y_\nu N_{\rm R} \phi^- + {\rm h.c.}$$

- The basis transformation related to the origin of active Majorana neutrino masses before SSB:

$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

working masses:

$$\begin{cases} D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\} \\ D_N \equiv {\rm Diag}\{M_1, M_2, M_3\} \end{cases}$$

SSB ↓

$$\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}$$

6×6 mass matrix

$$\begin{bmatrix}\nu_{\rm L} \\ (N_{\rm R})^c\end{bmatrix} \longrightarrow \mathbb{U}^\dagger \begin{bmatrix}\nu_{\rm L} \\ (N_{\rm R})^c\end{bmatrix}, \quad \begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} \longrightarrow \mathbb{U}^T \begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix}$$

flavor states → mass states , flavor states → mass states

Observation (1): There is a mismatch between the bases of heavy neutrino masses and decays;

Observation (2): The formal seesaw mechanism works before SSB, and it keeps valid after SSB.

Weak charged-current interactions

- **A block diagonalization of the active-sterile neutrino flavor mixing in the seesaw mechanism:**

$$\mathbb{U} = \underbrace{\begin{pmatrix} I & \mathbf{0} \\ \mathbf{0} & U_0' \end{pmatrix}}_{\text{sterile}} \underbrace{\begin{pmatrix} A & R \\ S & B \end{pmatrix}}_{\text{interplay}} \underbrace{\begin{pmatrix} U_0 & \mathbf{0} \\ \mathbf{0} & I \end{pmatrix}}_{\text{active}}$$

$$\begin{cases} AA^\dagger + RR^\dagger = BB^\dagger + SS^\dagger = I \\ AS^\dagger + RB^\dagger = A^\dagger R + S^\dagger B = \mathbf{0} \\ A^\dagger A + S^\dagger S = B^\dagger B + R^\dagger R = I \end{cases}$$

- **Relation between flavor and mass states:**

$$\nu_{\rm L} = U\nu_{\rm L}' + R(N_{\rm R}')^c \; , \quad N_{\rm R} = S'^*(\nu_{\rm L}')^c + U'^* N_{\rm R}'$$

in which $U \equiv AU_0$, $U' \equiv U_0' B$ and $S' \equiv U_0' S U_0$

mass state $\begin{cases} \nu' = \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \end{pmatrix}^T \\ N' = \begin{pmatrix} N_1 & N_2 & N_3 \end{pmatrix}^T \end{cases}$

- **Weak charged-current interactions (in the basis of flavor = mass states of charged leptons):**

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W_\mu^- + {\rm h.c.}$$

The correlations:

$$UU^\dagger + RR^\dagger = I$$

$$UD_\nu U^T + RD_N R^T = \mathbf{0}$$

neutrino oscillations ← **light** (U) **heavy** (R) → **thermal leptogenesis**

A full Euler-like parametrization

- **A full Euler-like parametrization of the 6×6 unitary mixing matrix (ZZX, 0709.2220; 1110.0083):**

sterile — **active**

$$\mathbb{U} = \begin{pmatrix} I & \mathbf{0} \\ \mathbf{0} & U_0' \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} U_0 & \mathbf{0} \\ \mathbf{0} & I \end{pmatrix}$$

3 angles, 3 phases (sterile): $O_{56}O_{46}O_{45}$

interplay

3 angles, 3 phases (active): $O_{23}O_{13}O_{12}$

$$O_{36}O_{26}O_{16}O_{35}O_{25}O_{15}O_{34}O_{24}O_{14}$$

9 mixing angles, 9 CP-violating phases

$$O_{56} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{56} & \hat{s}^*_{56} \\ 0 & 0 & 0 & 0 & -\hat{s}_{56} & c_{56} \end{pmatrix} \quad \cdots\cdots \quad O_{12} = \begin{pmatrix} c_{12} & \hat{s}^*_{12} & 0 & 0 & 0 & 0 \\ -\hat{s}_{12} & c_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$O_{36} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{36} & 0 & 0 & \hat{s}^*_{36} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\hat{s}_{36} & 0 & 0 & c_{36} \end{pmatrix} \quad \cdots\cdots \quad O_{14} = \begin{pmatrix} c_{14} & 0 & 0 & \hat{s}^*_{14} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\hat{s}_{14} & 0 & 0 & c_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- **The 3×3 unitary flavor mixing matrix for three active neutrinos:**

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

the unitary sterile flavor mixing U_0' **can be gotten by replacements:** $12 \leftrightarrow 45,\ 13 \leftrightarrow 46 \text{ and } 23 \leftrightarrow 56$

in which $c_{ij} \equiv \cos\theta_{ij}$ **and** $\hat{s}_{ij} \equiv e^{i\delta_{ij}}\sin\theta_{ij}$

- **These two matrices enter the weak charged-current interactions:**

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ -\hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} & c_{24}c_{25}c_{26} & 0 \\ -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} & -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} & \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} & -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} & c_{34}c_{35}c_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} & & \end{pmatrix}$$

$$R = \begin{pmatrix} \hat{s}^*_{14}c_{15}c_{16} & \hat{s}^*_{15}c_{16} & \hat{s}^*_{16} \\ -\hat{s}^*_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} & -\hat{s}^*_{15}\hat{s}_{16}\hat{s}^*_{26} + c_{15}\hat{s}^*_{25}c_{26} & c_{16}\hat{s}^*_{26} \\ +c_{14}\hat{s}^*_{24}c_{25}c_{26} & & \\ -\hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} & -\hat{s}^*_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} & \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} - c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} & +c_{15}c_{25}\hat{s}^*_{35}c_{36} & c_{16}c_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} + c_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} & & \end{pmatrix}$$

The active-sterile interplay matrices (2)

- **These two matrices have nothing to do with currently known observables:**

$$B = \begin{pmatrix} c_{14}c_{24}c_{34} & 0 & 0 \\ \begin{array}{l} -c_{14}c_{24}\hat{s}^*_{34}\hat{s}_{35} - c_{14}\hat{s}^*_{24}\hat{s}_{25}c_{35} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}c_{35} \end{array} & c_{15}c_{25}c_{35} & 0 \\ \begin{array}{l} -c_{14}c_{24}\hat{s}^*_{34}c_{35}\hat{s}_{36} + c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}\hat{s}_{36} \\ -c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}c_{36} + \hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}\hat{s}_{36} \\ +\hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}c_{36} - \hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}c_{36} \end{array} & \begin{array}{l} -c_{15}c_{25}\hat{s}^*_{35}\hat{s}_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}c_{36} \\ -\hat{s}^*_{15}\hat{s}_{16}c_{26}c_{36} \end{array} & c_{16}c_{26}c_{36} \end{pmatrix}$$

$$S = \begin{pmatrix} -\hat{s}_{14}c_{24}c_{34} & -\hat{s}_{24}c_{34} & -\hat{s}_{34} \\ \begin{array}{l} \hat{s}_{14}c_{24}\hat{s}^*_{34}\hat{s}_{35} + \hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}c_{35} \\ -c_{14}\hat{s}_{15}c_{25}c_{35} \end{array} & \hat{s}_{24}\hat{s}^*_{34}\hat{s}_{35} - c_{24}\hat{s}_{25}c_{35} & -c_{34}\hat{s}_{35} \\ \begin{array}{l} \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}\hat{s}_{36} - \hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}\hat{s}_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}c_{36} + c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}\hat{s}_{36} \\ +c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}c_{36} - c_{14}c_{15}\hat{s}_{16}c_{26}c_{36} \end{array} & \begin{array}{l} \hat{s}_{24}\hat{s}^*_{34}c_{35}\hat{s}_{36} + c_{24}\hat{s}_{25}\hat{s}^*_{35}\hat{s}_{36} \\ -c_{24}c_{25}\hat{s}_{26}c_{36} \end{array} & -c_{34}c_{35}\hat{s}_{36} \end{pmatrix}$$

A mismatch between mass and decay states

- With the block diagonalization of the seesaw flavor structure, we obtain
- $Y_\nu \phi^{0*} = R D_N \left[I - \left(B^{-1} S A^{-1} R \right)^T \right] U'^T$ **SSB** → $M_{\rm D} = R D_N \left[I - \left(B^{-1} S A^{-1} R \right)^T \right] U'^T$
- $M_{\rm R} = U' \left[D_N - \left(B^{-1} S A^{-1} R \right) D_N \left(B^{-1} S A^{-1} R \right)^T \right] U'^T$ ← **texture reconstruction**
- $N'_{\rm R} = (U'^*)^{-1} \left[N_{\rm R} - S'^* (\nu'_{\rm L})^c \right] = (B^*)^{-1} \left[\mathcal{N}'_{\rm R} - U_0'^T S'^* (\nu'_{\rm L})^c \right]$ ← **a basis mismatch**

mass states for seesaw to work (↑ $N'_{\rm R}$)

mass states for heavy neutrino decays (↑ $\mathcal{N}'_{\rm R}$)

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = (B^*)^{-1} \left[\begin{pmatrix} \mathcal{N}_1 \\ \mathcal{N}_2 \\ \mathcal{N}_3 \end{pmatrix} - U_0'^T S'^* \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \right]$$

Electroweak precision measurements and neutrino oscillation data put a stringent constraint on the non-unitarity of the PMNS matrix *U* —— it is below or far below 1%, implying *A* ≈ *B* ≈ *I* .

- $\mathcal{D}_N = B \left[D_N - \left(B^{-1} S A^{-1} R \right) D_N \left(B^{-1} S A^{-1} R \right)^T \right] B^T$
- **Although the mismatch is negligibly small in the canonical case, it is conceptually interesting!**

A bridge between light and heavy

- **The exact seesaw formula —— a bridge between the original and derivational flavor parameters:**

$$U D_\nu U^T + R D_N R^T = \mathbf{0} \longrightarrow M_\nu \equiv U_0 D_\nu U_0^T = \left(\mathrm{i} A^{-1} R\right) D_N \left(\mathrm{i} A^{-1} R\right)^T$$

Degrees of freedom (mass + mixing angle + CPV phase): 3 + 3 + 3 (derivational) ⟵ 3 + 9 + 6 (original)

- **To calculate the Jarlskog invariant of CP violation in the active neutrino oscillations, which is defined by**

$$\mathcal{J}_\nu \sum_\gamma \varepsilon_{\alpha\beta\gamma} \sum_k \varepsilon_{ijk} = \mathrm{Im}\left[(U_0)_{\alpha i} (U_0)_{\beta j} (U_0)^*_{\alpha j} (U_0)^*_{\beta i}\right]$$

$$\begin{cases} D_\nu \equiv \mathrm{Diag}\{m_1, m_2, m_3\} \\ D_N \equiv \mathrm{Diag}\{M_1, M_2, M_3\} \\ \Delta m^2_{ij} \equiv m_i^2 - m_j^2 \end{cases}$$

- **On the one hand, we use the light degrees of freedom to obtain the relation:**

$$\mathrm{Im}\left[\left(M_\nu M_\nu^\dagger\right)_{e\mu} \left(M_\nu M_\nu^\dagger\right)_{\mu\tau} \left(M_\nu M_\nu^\dagger\right)_{\tau e}\right] = \mathcal{J}_\nu \Delta m^2_{21} \Delta m^2_{31} \Delta m^2_{32}$$

- **On the other hand, we use the original seesaw-related parameters to calculate the same quantity *in the leading order approximation* of $A^{-1}R$.**

$$A^{-1} R \simeq \begin{pmatrix} \hat{s}^*_{14} & \hat{s}^*_{15} & \hat{s}^*_{16} \\ \hat{s}^*_{24} & \hat{s}^*_{25} & \hat{s}^*_{26} \\ \hat{s}^*_{34} & \hat{s}^*_{35} & \hat{s}^*_{36} \end{pmatrix}$$

But here, we switch off one heavy neutrino for simplicity.

The result in the minimal seesaw

- **The minimal seesaw framework with only 2 heavy Majorana neutrinos —— a benchmark scenario**
- **Consequences: (1) only 2 massive light (active) neutrinos; (2) only 2 CPV phases in the unitary PMNS matrix.**
- **Parameter counting: Original —— 2 masses + 6 mixing angles + 3 CPV phases;**
  Derivational —— 2 masses + 3 mixing angles + 2 CPV phases.

$$\alpha \equiv \delta_{14} - \delta_{15}\ ,\ \beta \equiv \delta_{24} - \delta_{25}\ ,\ \gamma \equiv \delta_{34} - \delta_{35}$$

$$\begin{aligned}\mathrm{Im}\left[\left(M_\nu M_\nu^\dagger\right)_{e\mu}\left(M_\nu M_\nu^\dagger\right)_{\mu\tau}\left(M_\nu M_\nu^\dagger\right)_{\tau e}\right] = C_0 \big[& C_\alpha \sin 2\alpha + C_\beta \sin 2\beta + C_\gamma \sin 2\gamma \\ & + C_{\alpha+\beta}\sin(\alpha+\beta) + C_{\beta+\gamma}\sin(\beta+\gamma) + C_{\gamma+\alpha}\sin(\gamma+\alpha) \\ & + C_{\alpha-\beta}\sin(\alpha-\beta) + C_{\beta-\gamma}\sin(\beta-\gamma) + C_{\gamma-\alpha}\sin(\gamma-\alpha)\big] = \mathcal{J}_\nu \Delta m^2_{21}\Delta m^2_{31}\Delta m^2_{32}\end{aligned}$$

- **Coefficients**

$$\begin{aligned}C_0 = {} & M_1^2 M_2^2 \left[s_{14}^2\left(s_{25}^2+s_{35}^2\right) + s_{24}^2\left(s_{15}^2+s_{35}^2\right) + s_{34}^2\left(s_{15}^2+s_{25}^2\right)\right. \\ & \left. - 2s_{14}s_{15}s_{24}s_{25}\cos(\alpha-\beta) - 2s_{24}s_{25}s_{34}s_{35}\cos(\beta-\gamma) - 2s_{14}s_{15}s_{34}s_{35}\cos(\gamma-\alpha)\right]\end{aligned}$$

$$C_\alpha = M_1 M_2 s_{14}^2 s_{15}^2\left(s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2\right)$$

$$C_\beta = M_1 M_2 s_{24}^2 s_{25}^2\left(s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2\right)$$

$$C_\gamma = M_1 M_2 s_{34}^2 s_{35}^2\left(s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2\right)$$

$$C_{\alpha+\beta} = M_1 M_2 s_{14}s_{15}s_{24}s_{25}\left(s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2 + s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2\right)$$

$$C_{\beta+\gamma} = M_1 M_2 s_{24}s_{25}s_{34}s_{35}\left(s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2 + s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2\right)$$

$$C_{\gamma+\alpha} = M_1 M_2 s_{14}s_{15}s_{34}s_{35}\left(s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2 + s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2\right)$$

$$C_{\alpha-\beta} = \left[M_1^2\left(s_{14}^2+s_{24}^2+s_{34}^2\right)s_{34}^2 - M_2^2\left(s_{15}^2+s_{25}^2+s_{35}^2\right)s_{35}^2\right]s_{14}s_{15}s_{24}s_{25}$$

$$C_{\beta-\gamma} = \left[M_1^2\left(s_{14}^2+s_{24}^2+s_{34}^2\right)s_{14}^2 - M_2^2\left(s_{15}^2+s_{25}^2+s_{35}^2\right)s_{15}^2\right]s_{24}s_{25}s_{34}s_{35}$$

$$C_{\gamma-\alpha} = \left[M_1^2\left(s_{14}^2+s_{24}^2+s_{34}^2\right)s_{24}^2 - M_2^2\left(s_{15}^2+s_{25}^2+s_{35}^2\right)s_{25}^2\right]s_{14}s_{15}s_{34}s_{35}$$

- **Conclusion:**

low $\mathcal{J}_\nu$ ← **9 ways** — **seesaw scale** $\alpha\ ,\ \beta\ ,\ \gamma$

CPV in heavy Majorana neutrino decays

- **The flavor-dependent CP-violating asymmetries in LNV decays of heavy Majorana neutrinos:**

$$\varepsilon_{i\alpha} \equiv \frac{\Gamma(\mathcal{N}_i \to \ell_\alpha + H) - \Gamma(\mathcal{N}_i \to \overline{\ell_\alpha} + \overline{H})}{\sum_\alpha \left[\Gamma(\mathcal{N}_i \to \ell_\alpha + H) + \Gamma(\mathcal{N}_i \to \overline{\ell_\alpha} + \overline{H})\right]}$$

$$x_{ji} \equiv \mathcal{M}_j^2/\mathcal{M}_i^2$$

$$= \frac{1}{8\pi\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)_{ii}} \sum_{j\neq i} \left\{ \mathrm{Im}\left[\left(\mathcal{Y}_\nu^*\right)_{\alpha i} \left(\mathcal{Y}_\nu\right)_{\alpha j} \left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)_{ij} \xi(x_{ji}) + \left(\mathcal{Y}_\nu^*\right)_{\alpha i} \left(\mathcal{Y}_\nu\right)_{\alpha j} \left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^*_{ij} \zeta(x_{ji}) \right] \right\}$$

where $\xi(x_{ji}) = \sqrt{x_{ji}}\left\{1 + 1/\left(1 - x_{ji}\right) + \left(1 + x_{ji}\right)\ln\left[x_{ji}/\left(1 + x_{ji}\right)\right]\right\}$ **,** $\zeta(x_{ji}) = 1/\left(1 - x_{ji}\right)$

- **Baryogenesis via leptogenesis in the early Universe**

- **A net lepton number asymmetry:** $\Gamma(\mathcal{N}_i \to \ell_\alpha + H) < H(T = \mathcal{M}_i)$

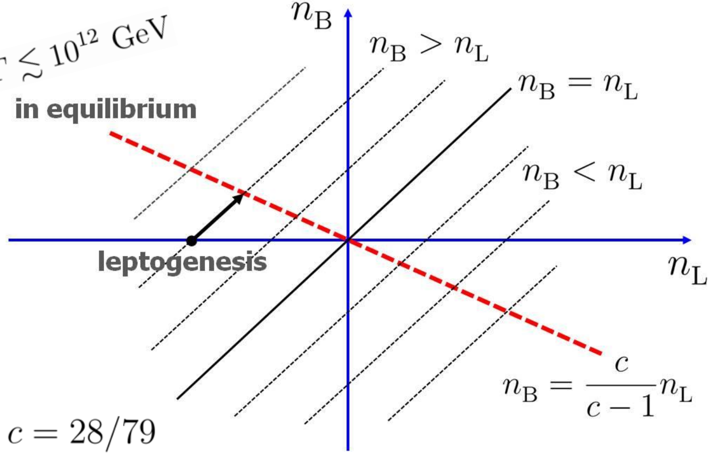
$$Y_\mathrm{L} \equiv \frac{n_\mathrm{L} - n_{\overline{\mathrm{L}}}}{s} = \frac{1}{g_*}\sum_{i,\alpha} \kappa_{i\alpha}\varepsilon_{i\alpha}$$

sphaleron-induced (B-L)-conserving process in thermal equilibrium

$$Y_\mathrm{B} \equiv \frac{n_\mathrm{B} - n_{\overline{\mathrm{B}}}}{s} = -cY_\mathrm{L}$$

A net baryon number asymmetry

10^2 GeV $\lesssim T \lesssim 10^{12}$ GeV



CPV in 1st heavy Majorana neutrino decays

- **The CP-violating asymmetries in LNV decays of the first heavy Majorana neutrino:**

$$\varepsilon_{1e} = \frac{M_1^2 s_{14} s_{15}}{8\pi\langle\phi^0\rangle^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)} \Big[x_{21}\xi(x_{21}) \left[s_{14}s_{15}\sin 2\alpha + s_{24}s_{25}\sin(\alpha+\beta) + s_{34}s_{35}\sin(\alpha+\gamma)\right] + x_{21}\zeta(x_{21}) \left[s_{24}s_{25}\sin(\alpha-\beta) + s_{34}s_{35}\sin(\alpha-\gamma)\right]\Big]$$

$$\varepsilon_{1\mu} = \frac{M_1^2 s_{24} s_{25}}{8\pi\langle\phi^0\rangle^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)} \Big[x_{21}\xi(x_{21}) \left[s_{14}s_{15}\sin(\alpha+\beta) + s_{24}s_{25}\sin 2\beta + s_{34}s_{35}\sin(\beta+\gamma)\right] + x_{21}\zeta(x_{21}) \left[s_{14}s_{15}\sin(\beta-\alpha) + s_{34}s_{35}\sin(\beta-\gamma)\right]\Big]$$

$$\varepsilon_{1\tau} = \frac{M_1^2 s_{34} s_{35}}{8\pi\langle\phi^0\rangle^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)} \Big[x_{21}\xi(x_{21}) \left[s_{14}s_{15}\sin(\alpha+\gamma) + s_{24}s_{25}\sin(\beta+\gamma) + s_{34}s_{35}\sin 2\gamma\right] + x_{21}\zeta(x_{21}) \left[s_{14}s_{15}\sin(\gamma-\alpha) + s_{24}s_{25}\sin(\gamma-\beta)\right]\Big]$$

$$\alpha \equiv \delta_{14} - \delta_{15}\,, \quad \beta \equiv \delta_{24} - \delta_{25}\,, \quad \gamma \equiv \delta_{34} - \delta_{35}$$

$$\varepsilon_1 = +\frac{M_1^2 x_{21}\xi(x_{21})}{8\pi\langle\phi^0\rangle^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)} \Big[s_{14}^2 s_{15}^2 \sin 2\alpha + s_{24}^2 s_{25}^2 \sin 2\beta + s_{34}^2 s_{35}^2 \sin 2\gamma + 2s_{14}s_{15}s_{24}s_{25}\sin(\alpha+\beta) + 2s_{14}s_{15}s_{34}s_{35}\sin(\alpha+\gamma) + 2s_{24}s_{25}s_{34}s_{35}\sin(\beta+\gamma)\Big]$$

CPV in 2nd heavy Majorana neutrino decays

♦ **The CP-violating asymmetries in LNV decays of the second heavy Majorana neutrino:**

$$\varepsilon_{2e} = -\frac{M_2^2 s_{14} s_{15}}{8\pi\langle\phi^0\rangle^2 \left(s_{15}^2 + s_{25}^2 + s_{35}^2\right)} \Big[x_{12}\xi(x_{12}) \left[s_{14}s_{15}\sin 2\alpha + s_{24}s_{25}\sin(\alpha+\beta) + s_{34}s_{35}\sin(\alpha+\gamma)\right] + x_{12}\zeta(x_{12}) \left[s_{24}s_{25}\sin(\alpha-\beta) + s_{34}s_{35}\sin(\alpha-\gamma)\right]\Big]$$

$$\varepsilon_{2\mu} = -\frac{M_2^2 s_{24} s_{25}}{8\pi\langle\phi^0\rangle^2 \left(s_{15}^2 + s_{25}^2 + s_{35}^2\right)} \Big[x_{12}\xi(x_{12}) \left[s_{14}s_{15}\sin(\alpha+\beta) + s_{24}s_{25}\sin 2\beta + s_{34}s_{35}\sin(\beta+\gamma)\right] + x_{12}\zeta(x_{12}) \left[s_{14}s_{15}\sin(\beta-\alpha) + s_{34}s_{35}\sin(\beta-\gamma)\right]\Big]$$

$$\varepsilon_{2\tau} = -\frac{M_2^2 s_{34} s_{35}}{8\pi\langle\phi^0\rangle^2 \left(s_{15}^2 + s_{25}^2 + s_{35}^2\right)} \Big[x_{12}\xi(x_{12}) \left[s_{14}s_{15}\sin(\alpha+\gamma) + s_{24}s_{25}\sin(\beta+\gamma) + s_{34}s_{35}\sin 2\gamma\right] + x_{12}\zeta(x_{12}) \left[s_{14}s_{15}\sin(\gamma-\alpha) + s_{24}s_{25}\sin(\gamma-\beta)\right]\Big]$$

$$\alpha \equiv \delta_{14} - \delta_{15}\,, \quad \beta \equiv \delta_{24} - \delta_{25}\,, \quad \gamma \equiv \delta_{34} - \delta_{35}$$

$$\varepsilon_{2} = -\frac{M_2^2 x_{12}\xi(x_{12})}{8\pi\langle\phi^0\rangle^2 \left(s_{15}^2 + s_{25}^2 + s_{35}^2\right)} \Big[s_{14}^2 s_{15}^2 \sin 2\alpha + s_{24}^2 s_{25}^2 \sin 2\beta + s_{34}^2 s_{35}^2 \sin 2\gamma + 2 s_{14}s_{15}s_{24}s_{25}\sin(\alpha+\beta) + 2 s_{14}s_{15}s_{34}s_{35}\sin(\alpha+\gamma) + 2 s_{24}s_{25}s_{34}s_{35}\sin(\beta+\gamma)\Big]$$

A summary of the phase dependence

- The **Jarlskog** invariant and the CPV asymmetries depend on three independent CPV phases:

	2α	2β	2γ	$\alpha+\beta$	$\beta+\gamma$	$\alpha+\gamma$	$\alpha-\beta$	$\beta-\gamma$	$\gamma-\alpha$
$\mathcal{J}_\nu$	√	√	√	√	√	√	√	√	√
ε_{1e}	√			√		√	√		√
$\varepsilon_{1\mu}$		√		√	√		√	√	
$\varepsilon_{1\tau}$			√		√	√		√	√
ε_1	√	√	√	√	√	√			
ε_{2e}	√			√		√	√		√
$\varepsilon_{2\mu}$		√		√	√		√	√	
$\varepsilon_{2\tau}$			√		√	√		√	√
ε_2	√	√	√	√	√	√			

- A more straightforward correlation between the two CPV observables needs some assumptions.

Concluding remarks

- The canonical seesaw mechanism is the most natural and economical mechanism to produce tiny Majorana neutrino masses and interpret the cosmological matter-antimatter asymmetry by thermal leptogenesis mechanism, but all these only work qualitatively.

- *For the first time*, we have derived the generic + explicit expressions of the Jarlskog invariant for light neutrino oscillations in terms of the original seesaw flavor parameters, and of CP asymmetries for heavy neutrino decays. Then their connections become transparent.

- A full numerical exploration of the seesaw parameter space can be done, using a good computer. And some constraints will be available once CPV in neutrino oscillations is measured.

$\varepsilon_{i\alpha}$

Heavy neutrino LNV decays

$\mathcal{J}_\nu$

Light neutrino oscillations