Classifying CP-violating SMEFT operators

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1. Introduction

2. Method: Hilbert Series

3. Results of Classification

4. Summary & Discussion

CP violation in SM

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \theta\frac{g^2}{32\pi^2}F^{\mu\nu}\tilde{F}_{\mu\nu}$$ → $\theta_{\rm QCD}$

$$+ i\bar{\psi}\not{D}\psi + \mathrm{h.c.}$$

$$+\psi_i y_{ij}\psi_j\phi + \mathrm{h.c.}$$ → $\delta_{\rm CKM}$

$$+\left|D_\mu\phi\right|^2 - V(\phi)$$

- CP violations in SM are **too small** to describe the baryon asymmetry in the universe
  - Physics beyond the SM should have more CP violations

$\Rightarrow$ CPVs in SM Effective Field Theory (SMEFT)

- Measurement of CPV is **very sensitive**
  - Even high-dimensional operators in SMEFT can be probed experimentally

CP violation by a single SMEFT operator

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \frac{c}{\Lambda^{n-4}} \mathscr{O}^{(n)}_{\mathrm{CP-odd}}$$

If $\mathscr{O}^{(n)}_{\mathrm{CP-odd}}$ is invariant under $U(1)_B$, $U(1)_{L_i}$ $(i = 1,2,3)$,
$\mathscr{L}$ has a CP phase in c, in addition to δ_{CKM} and θ_{QCD} .

$\overset{\mathrm{def}}{\longrightarrow}$ $\mathscr{O}^{(n)}_{\mathrm{CP-odd}}$ is a CP-violating operator

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$\mathscr{L}$ has a CP phase in c, in addition to $\delta_{\rm CKM}$ and $\theta_{\rm QCD}$.

$\overset{\rm def}{\longrightarrow}$ $\mathscr{O}^{(n)}_{\rm CP-odd}$ is a CP-violating operator

How can we list these operators systematically?

Hilbert series

- A series of singlets under given group(s)
  - Suitable to construct EFT operators

[B. Henning, X. Lu, T. Melia & H. Murayama, 2017]

Fields Covariant derivative

$$H_0(\{\phi_i\}, D) = \int \mathrm{d}\mu_{SO(4)} \mathrm{d}\mu_{\text{gauge}} \frac{1}{P(D)} \prod_i \mathrm{PE}(\phi_i, D)$$

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$$H_0(\{\phi_i\}, D) = \int \mathrm{d}\mu_{SO(4)} \mathrm{d}\mu_{\text{gauge}} \frac{1}{P(D)} \prod_i \mathrm{PE}(\phi_i, D)$$

Pick up only singlets

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$$H_0(\{\phi_i\}, D) = \int \mathrm{d}\mu_{SO(4)} \mathrm{d}\mu_{\text{gauge}} \frac{1}{P(D)} \prod_i \mathrm{PE}(\phi_i, D)$$

Remove IBP redundancy

Hilbert series

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[B. Henning, X. Lu, T. Melia & H. Murayama, 2017]

$$H_0(\{\phi_i\}, D) = \int \mathrm{d}\mu_{SO(4)} \mathrm{d}\mu_{\text{gauge}} \frac{1}{P(D)} \prod_i \mathrm{PE}(\phi_i, D)$$

Remove EOM redundancy &
Implement bosonic/fermionic statistics

Hilbert series

- Coefficients of HS = # independent singlets
  - e.g.) HS for SMEFT

$$H_0(\text{SMEFT}) \supset 2HH^\dagger QQ^\dagger D$$

$$i[H^\dagger(D_\mu H) - (D_\mu H^\dagger)H]\bar{Q}\gamma^\mu Q,$$

$$i[H^\dagger\sigma^a(D_\mu H) - (D_\mu H^\dagger)\sigma^a H]\bar{Q}\gamma^\mu\sigma^a Q$$

CP transformation of operators

CP-odd operator

I. $\mathcal{O}_{\mathrm{I}} \xrightarrow{CP} \mathcal{O}_{\mathrm{I}}^{\dagger} \ (\neq \pm \mathcal{O}_{\mathrm{I}}) \quad \Rightarrow \quad \mathcal{O}_{\mathrm{I}} - \mathcal{O}_{\mathrm{I}}^{\dagger}$

II. $\mathcal{O}_{\mathrm{II}} \xrightarrow{CP} \mathcal{O}_{\mathrm{II}} \quad \Rightarrow$ X

III. $\mathcal{O}_{\mathrm{III}} \xrightarrow{CP} -\mathcal{O}_{\mathrm{III}} \quad \Rightarrow \quad \mathcal{O}_{\mathrm{III}}$

Hilbert series for CP-odd operators

$$H^{\mathrm{CP-odd}}(\{\phi_i\}, D) = \frac{1}{2}[H^{+}(\{\phi_i\}, D) - H^{-}(\{\phi_i\}, D)]$$

Singlets under

H^{+}	H^{-}
Lorentz & gauge	Lorentz & gauge & CP

Hilbert series for CP-odd operators

$$H^{\mathrm{CP-odd}}(\{\phi_i\}, D) = \frac{1}{2}[H^{+}(\{\phi_i\}, D) - H^{-}(\{\phi_i\}, D)]$$

$$H^{+}(\{\phi_i\}, D) \supset 2(\mathcal{O}_{\mathrm{I}} \oplus \mathcal{O}_{\mathrm{I}}^{\dagger}),\ \mathcal{O}_{\mathrm{II}},\ \mathcal{O}_{\mathrm{III}} \qquad H^{-}(\{\phi_i\}, D) \supset \mathcal{O}_{\mathrm{II}},\ -\mathcal{O}_{\mathrm{III}}$$

$$\supset \mathcal{O}_{\mathrm{I}} \oplus \mathcal{O}_{\mathrm{I}}^{\dagger},\ \mathcal{O}_{\mathrm{III}}$$

$$\wr$$

$$\mathcal{O}_{\mathrm{I}} - \mathcal{O}_{\mathrm{I}}^{\dagger}$$

Hilbert series for CP-violating operators

$$H^{\text{CP-violating}}(\{\phi_i\}, D) = \frac{1}{2}[H^{+}(\{\phi_i\}, D) - H^{-}(\{\phi_i\}, D)]$$

Singlets under

H^{+}	H^{-}
Lorentz & gauge & $U(1)_B$, $U(1)_{L_i}$	Lorentz & gauge & CP & $U(1)_B$, $U(1)_{L_i}$

CP-odd/violating operators

Mass dim.	5	6	7	8	9	10	11	12
CP-odd ops.	6	1422	771	22016	45228	1042942	1736133	37761366
CP-violating ops.	0	705	0	11777	0	437331	0	13891774
Time [s]	0.01	0.10	0.15	0.50	2.86	9.14	24.10	326.21

- The numbers for dim 6 are consistent with previous results.
  - Dim-6 CP-odd operators by R. Alonso et al. (2014)
  - Dim-6 CP-violating operators by Q. Bonnefoy et al. (2022)

CP-odd/violating operators

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- Dim 8: classification of $H^2W^2D^2$
  - Remmen and Rodd (2020) claim that there are 3 CP-odd operators.
  - Our Hilbert series tells that there are only 2.

↓

This is correct!

Dimension-6 CP-odd operators

```
HilbertCP =
     + G^3
     + W^3
     + 15*e^4
     + 36*l^2*e^2
     + 18*l^4
     + 36*d^2*e^2
     + 36*d^2*l^2
     + 18*d^4
     + 36*u^2*e^2
     + 36*u^2*l^2
     + 81*u^2*d*e
     + 72*u^2*d^2
     + 18*u^4
     + 81*q*d*l*e
     + 162*q*u*l*e
     + 81*q*u*d*l
     + 36*q^2*e^2
     + 72*q^2*l^2
     + 72*q^2*d^2
     + 54*q^2*u*e
     + 162*q^2*u*d
     + 72*q^2*u^2
     + 57*q^3*l
```

```
     + 36*q^4
     + 9*h*l*e*W
     + 9*h*l*e*B
     + 9*h*q*d*G
     + 9*h*q*d*W
     + 9*h*q*d*B
     + 9*h*q*u*G
     + 9*h*q*u*W
     + 9*h*q*u*B
     + h^2*G^2
     + h^2*W^2
     + h^2*B*W
     + h^2*B^2
     + 3*h^2*e^2*p
     + 6*h^2*l^2*p
     + 3*h^2*d^2*p
     + 9*h^2*u*d*p
     + 3*h^2*u^2*p
     + 6*h^2*q^2*p
     + 9*h^3*l*e
     + 9*h^3*q*d
     + 9*h^3*q*u
```

Dimension-8 CP-odd operators

```
HilbertCP =
    + 3*G^4
    + 3*W^2*G^2
    + 2*W^4
    + 2*B*G^3
    + 3*B^2*G^2
    + 3*B^2*W^2
    + B^4
    + 3*e^2*G^2*p
    + 3*e^2*W^2*p
    + 3*e^2*B^2*p
    + 18*e^4*p^2
    + 18*e^4*B
    + 3*l^2*G^2*p
    + 15*l^2*W^2*p
    + 18*l^2*B*W*p
    + 3*l^2*B^2*p
    + 72*l^2*e^2*p^2
    + 81*l^2*e^2*W
    + 81*l^2*e^2*B
    + 36*l^4*p^2
    + 63*l^4*W
    + 36*l^4*B
    + 18*d^2*G^2*p
    + 3*d^2*W^2*p
    + 18*d^2*B*G*p
    + 3*d^2*B^2*p
    + 72*d^2*e^2*p^2
    + 81*d^2*e^2*G
    + 81*d^2*e^2*B
    + 72*d^2*l^2*p^2
    + 81*d^2*l^2*G
    + 81*d^2*l^2*W
+ 81*d^2*l^2*B
+ 36*d^4*p^2
+ 81*d^4*G
+ 36*d^4*B
+ 18*u^2*G^2*p
+ 3*u^2*W^2*p
+ 18*u^2*B*G*p
+ 3*u^2*B^2*p
+ 72*u^2*e^2*p^2
+ 81*u^2*e^2*G
+ 81*u^2*e^2*B
+ 72*u^2*l^2*p^2
+ 81*u^2*l^2*G
+ 81*u^2*l^2*W
+ 81*u^2*l^2*B
+ 108*u^2*d*e*p^2
+ 243*u^2*d*e*G
+ 135*u^2*d*e*B
+ 144*u^2*d^2*p^2
+ 324*u^2*d^2*G
+ 162*u^2*d^2*B
+ 36*u^4*p^2
+ 81*u^4*G
+ 36*u^4*B
+ 162*q*d*l*e*p^2
+ 162*q*d*l*e*G
+ 162*q*d*l*e*W
+ 162*q*d*l*e*B
+ 243*q*u*l*e*p^2
+ 243*q*u*l*e*G
+ 243*q*u*l*e*W
+ 243*q*u*l*e*B
+ 162*q*u*d*l*p^2
+ 324*q*u*d*l*G
+ 162*q*u*d*l*W
+ 162*q*u*d*l*B
+ 18*q^2*G^2*p
+ 18*q^2*W*G*p
+ 15*q^2*W^2*p
+ 18*q^2*B*G*p
+ 18*q^2*B*W*p
+ 3*q^2*B^2*p
+ 72*q^2*e^2*p^2
+ 81*q^2*e^2*G
+ 81*q^2*e^2*W
+ 81*q^2*e^2*B
+ 144*q^2*l^2*p^2
+ 162*q^2*l^2*G
+ 243*q^2*l^2*W
+ 162*q^2*l^2*B
+ 144*q^2*d^2*p^2
+ 324*q^2*d^2*G
+ 162*q^2*d^2*W
+ 162*q^2*d^2*B
+ 81*q^2*u*e*p^2
+ 162*q^2*u*e*G
+ 81*q^2*u*e*W
+ 81*q^2*u*e*B
+ 243*q^2*u*d*p^2
+ 486*q^2*u*d*G
+ 243*q^2*u*d*W
+ 243*q^2*u*d*B
+ 144*q^2*u^2*p^2
+ 324*q^2*u^2*G
+ 162*q^2*u^2*W
+ 162*q^2*u^2*B
+ 81*q^3*l*p^2
+ 162*q^3*l*G
+ 162*q^2*u^2*W
+ 162*q^2*u^2*B
+ 81*q^3*l*p^2
+ 162*q^3*l*G
+ 135*q^3*l*W
+ 81*q^3*l*B
+ 72*q^4*p^2
+ 162*q^4*G
+ 126*q^4*W
+ 72*q^4*B
+ 18*h*l*e*G^2
+ 27*h*l*e*W*p^2
+ 27*h*l*e*W^2
+ 27*h*l*e*B*p^2
+ 27*h*l*e*B*W
+ 18*h*l*e*B^2
+ 108*h*l*e^3*p
+ 243*h*l^3*e*p
+ 243*h*d^2*l*e*p
+ 243*h*u*d*l*e*p
+ 135*h*u*d^2*l*p
+ 243*h*u^2*l*e*p
+ 135*h*u^2*d*l*p
+ 27*h*q*d*G*p^2
+ 45*h*q*d*G^2
+ 27*h*q*d*W*p^2
+ 27*h*q*d*W*G
+ 27*h*q*d*W^2
+ 27*h*q*d*B*p^2
+ 27*h*q*d*B*G
+ 27*h*q*d*B*W
+ 18*h*q*d*B^2
+ 243*h*q*d*e^2*p
+ 486*h*q*d*l^2*p
+ 243*h*q*d^3*p
+ 27*h*q*u*G*p^2
+ 45*h*q*u*G^2
+ 27*h*q*u*W*p^2
+ 27*h*q*u*W*G
+ 27*h*q*u*W^2
+ 27*h*q*u*B*p^2
+ 27*h*q*u*B*G
+ 27*h*q*u*B*W
+ 18*h*q*u*B^2
+ 243*h*q*u*e^2*p
+ 486*h*q*u*l^2*p
+ 243*h*q*u*d*e*p
+ 486*h*q*u*d^2*p
+ 135*h*q*u^2*e*p
+ 486*h*q*u^2*d*p
+ 243*h*q*u^3*p
+ 486*h*q^2*l*e*p
+ 243*h*q^2*d*l*p
+ 243*h*q^2*u*l*p
+ 81*h*q^3*e*p
+ 486*h*q^3*d*p
+ 486*h*q^3*u*p
+ h^2*G^2*p^2
+ h^2*G^3
+ 2*h^2*W^2*p^2
+ h^2*W^3
+ 3*h^2*B*W*p^2
+ h^2*B*W^2
+ h^2*B^2*p^2
+ 6*h^2*e^2*p^3
+ 18*h^2*e^2*W*p
+ 18*h^2*e^2*B*p
+ 15*h^2*e^4
+ 12*h^2*l^2*p^3
+ 54*h^2*l^2*W*p
+ 36*h^2*l^2*B*p
+ 117*h^2*l^2*e^2
+ 51*h^2*l^4
+ 6*h^2*d^2*p^3
+ 18*h^2*d^2*G*p
+ 18*h^2*d^2*W*p
+ 18*h^2*d^2*B*p
+ 36*h^2*d^2*e^2
+ 72*h^2*d^2*l^2
+ 18*h^2*d^4
+ 9*h^2*u*d*p^3
+ 18*h^2*u*d*G*p
+ 18*h^2*u*d*W*p
+ 18*h^2*u*d*B*p
+ 81*h^2*u*d*l^2
+ 6*h^2*u^2*p^3
+ 18*h^2*u^2*G*p
+ 18*h^2*u^2*W*p
+ 18*h^2*u^2*B*p
+ 36*h^2*u^2*e^2
+ 72*h^2*u^2*l^2
+ 81*h^2*u^2*d*e
+ 72*h^2*u^2*d^2
+ 18*h^2*u^4
+ 324*h^2*q*d*l*e
+ 27*h^2*q*d^2*l
+ 405*h^2*q*u*l*e
+ 162*h^2*q*u*d*l
+ 27*h^2*q*u^2*l
+ 12*h^2*q^2*p^3
+ 36*h^2*q^2*G*p
+ 54*h^2*q^2*W*p
+ 36*h^2*q^2*B*p
+ 72*h^2*q^2*e^2
+ 189*h^2*q^2*l^2
+ 27*h^2*q^2*d*e
+ 234*h^2*q^2*d^2
+ 81*h^2*q^2*u*e
+ 486*h^2*q^2*u*d
+ 234*h^2*q^2*u^2
+ 138*h^2*q^3*l
+ 90*h^2*q^4
+ 54*h^3*l*e*p^2
+ 18*h^3*l*e*W
+ 9*h^3*l*e*B
+ 54*h^3*q*d*p^2
+ 9*h^3*q*d*G
+ 18*h^3*q*d*W
+ 9*h^3*q*d*B
+ 54*h^3*q*u*p^2
+ 9*h^3*q*u*G
+ 18*h^3*q*u*W
+ 9*h^3*q*u*B
+ h^4*G^2
+ 2*h^4*W*p^2
+ 2*h^4*W^2
+ h^4*B*p^2
+ h^4*B*W
+ h^4*B^2
+ 3*h^4*e^2*p
+ 15*h^4*l^2*p
+ 3*h^4*d^2*p
+ 9*h^4*u*d*p
+ 3*h^4*u^2*p
+ 15*h^4*q^2*p
+ 9*h^5*l*e
+ 9*h^5*q*d
+ 9*h^5*q*u
```

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Summary

- CPV is the key for BSM.
- We have developed an algorithm to classify SMEFT operators **based on their CP properties**.
    - **Hilbert series** technique
- Results
    - Our Hilbert series **reproduced and corrected** previous results.
    - High-dimensional operators can be obtained **in a few seconds**.
- Our method can be **easily** applied to other EFTs.
    - The FORM codes are now available!

Back up

Hilbert series

$$H_0 = \int \mathrm{d}\mu_{SO(4)}(x)\mathrm{d}\mu_{\text{gauge}}(y)\ \frac{1}{P(D,x)}\ \prod_i \mathrm{PE}(\phi_i, D, x, y)$$

$$P(D,x) = \frac{1}{(1-Dx_1)(1-Dx_1^{-1})(1-Dx_2)(1-Dx_2^{-1})}$$

$$\mathrm{PE}(\phi_i, D, x, y) = \exp\left[\sum_{n=1}^{\infty} (\pm 1)^{n+1}\frac{1}{n}\left(\frac{\phi_i}{D^{\Delta_i}}\right)^n \chi_i(D^n, x^n, y^n)\right]$$

± 1 for bosonic / fermionic ϕ_i

Dimension of ϕ_i

Character of ϕ_i
(trace of representation)

Hilbert series

$$H_0 = \int \mathrm{d}\mu_{SO(4)}(x)\mathrm{d}\mu_{\mathrm{gauge}}(y) \frac{1}{P(D,x)} \prod_i \mathrm{PE}(\phi_i, D, x, y)$$

Pick up only singlets by orthogonality of characters

$$\int \mathrm{d}\mu(x)\, \chi_R^*(x)\chi_{R'}(x) = \delta_{RR'}$$

$''\mathrm{gauge}'' = SU(3) \times SU(2) \times U(1)$ for SMEFT

Hilbert series for CP-odd operators

$$H^{\mathrm{CP-odd}}(\{\phi_i\}, D) = \frac{1}{2}[H^{+}(\{\phi_i\}, D) - H^{-}(\{\phi_i\}, D)]$$

$$\begin{cases} H^{+}(\{\phi_i\}, D) = \int \mathrm{d}\mu_{SO(4)}(x)\mathrm{d}\mu_{\text{gauge}}(y)\dfrac{1}{P(D,x)}\prod_i \mathrm{PE}(\phi_i, D, x, y) \\ H^{-}(\{\phi_i\}, D) = \int \mathrm{d}\mu_{Sp(2)}(\tilde{x})\mathrm{d}\mu_{\widetilde{\text{gauge}}}(\tilde{y})\dfrac{1}{P^{-}(D,\tilde{x})}\prod_i \mathrm{PE}^{-}(\phi_i, D, \tilde{x}, \tilde{y}) \end{cases}$$

$$P^{-}(D,\tilde{x}) = \frac{1-D^2}{(1-D\tilde{x})(1-D\tilde{x}^{-1})}$$

Hilbert series for CP-odd operators

$$H^-(\{\phi_i\}, D) = \int \mathrm{d}\mu_{Sp(2)}(\tilde{x}) \mathrm{d}\mu_{\widetilde{\mathrm{gauge}}}(\tilde{y}) \frac{1}{P^-(D, \tilde{x})} \prod_i \mathrm{PE}^-(\phi_i, D, \tilde{x}, \tilde{y})$$

The integrated groups are different from the original groups.

⇒ “folding” of Dynkin diagrams

$$SO(4) \xrightarrow{P} Sp(2)$$

$$\mathrm{gauge} \xrightarrow{C} \widetilde{\mathrm{gauge}} = Sp(2) \times SU(2)$$

The operator $H^2W^2D^2$

- According to Remmen and Rodd (2020),

$$i\epsilon^{IJK}(\mathscr{D}^\mu H^\dagger \sigma^I \mathscr{D}^\nu H)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$$

is CP-odd.

- However, this is CP-even:

$$i\epsilon^{IJK}[\mathscr{D}^\mu H^{\dagger i}(\sigma^I)_i^{\ j}\mathscr{D}^\nu H_j](W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$$

$$\xrightarrow{CP} i\epsilon^{IJK}[-\mathscr{D}^\nu H^{\dagger i}(\sigma^I)_i^{\ j}\mathscr{D}^\mu H_j][W^J_{\mu\rho}(-\widetilde{W}^{K\rho}_\nu) + (-\widetilde{W}^J_{\mu\rho})W^{K\rho}_\nu]$$

$$= +\, i\epsilon^{IJK}[\mathscr{D}^\mu H^{\dagger i}(\sigma^I)_i^{\ j}\mathscr{D}^\nu H_j](W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$$