Alleviating the H_0 and σ_8 tensions via general conformal coupling between dark energy and dark matter

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Topic

1. H_0 and σ_8 tensions
2. General conformal transformation
3. Background universe
4. Perturbation
5. Conclusion

1. H_0 and σ_8 tensions

- H_0 is the Hubble expansion rate at present
- From CMB data (based on ΛCDM), $H_0 = 67.27$km s^{-1}Mpc^{-1} [1]
- From local measurments, $H_0 = 73$km s^{-1}Mpc^{-1} [2]

[1] P. A. R. Ade et al. (Planck Collaboration), Astron. Astrophys. 594, A13 (2016).
[2] A. G. Riess et al, arXiv: 2112.04510 [astro-ph.CO]

1. H_0 and σ_8 tensions

- σ_8 is the amplitude of matter density perturbation
- From CMB data (based on ΛCDM), $\sigma_8 = 0.83$ [3]
- From local measurments, $\sigma_8 = 0.76$ [4]

[3] H. Hildebrandt et al., Mon. Not. R. Astron. Soc. 465, 1454(2017)
[4] C. Heymans et al. A&A 646, A140 (2021)

2. General conformal transformation

- One of the possible model to alleviate H_0 and σ_8 tensions is conformal coupling model between dark energy and dark matter
- This work we use general conformal transformation [5]

$$\bar{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} \tag{1}$$

[5] V. Faraoni, E. Gunzig, P. Nardone, Fund. Cosmic Phys. 20:121, 1999

2. General conformal transformation

- To study the coupling between dark energy and dark matter, we write the action as

$$S = \int d^4x \Big[\sqrt{-g} \Big(\frac{R}{2} + P(\phi, X) + \mathcal{L}_{\mathcal{M}}(g_{\mu\nu}) \Big) + \sqrt{-\bar{g}} \mathcal{L}_c(\bar{g}_{\mu\nu}, \psi) \Big] \quad (2)$$

where we have set $1/\sqrt{8\pi G} = 1$,
R is the Ricci scalar,
$P(\phi, X) \equiv X - V(\phi)$,
$\mathcal{L}_{\mathcal{M}}$ is the Lagrangian of ordinary matter,
$\mathcal{L}_c$ is the Lagrangian of dark matter,
ψ is the matter field

2. General conformal transformation

- Varying eq.(2) with respect to the metric tensor $g_{\mu\nu}$, we obtain Einstein field equation as

$$\nabla_\mu G^{\mu\nu} = 8\pi G \nabla_\mu T_t^{\mu\nu} = 8\pi G \nabla_\mu (T_c^{\mu\nu} + T_\phi^{\mu\nu}) = 0. \tag{3}$$

where

$$T_\phi^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g} P(\phi, X)\right)}{\delta g_{\mu\nu}}, \tag{4}$$

$$T_c^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-\bar{g}} \bar{\mathcal{L}}_c)}{\delta g_{\mu\nu}} \tag{5}$$

2. General conformal transformation

- Varying the action eq. (2) with respect to scalar field ϕ, we obtain the EOM for scalar field ϕ as

$$\nabla_\alpha \nabla^\alpha \phi - V_{,\phi} = -\Gamma T_c - \nabla_\beta (\Xi \phi^{,\beta} T_c) \equiv -Q \tag{6}$$

where $\Gamma \equiv C_{,\phi}/[2(C + C_{,X} X)]$ and $\Xi \equiv C_{,X}/[2(C + C_{,X} X)]$

- Multiplying eq. (6) by $\phi_{,\beta}$, we obtain

$$\nabla_\alpha T^\alpha_{\beta(\phi)} = -\nabla_\alpha T^\alpha_{\beta(c)} = -Q\phi_{,\beta}. \tag{7}$$

3. Background universe

For the background universe, we use the line element,

$$ds^2 = -a^2 d\tau^2 + a^2 \delta_{ij} dx^i dx^i. \quad (8)$$

From eqs.(6) and (7), we obtain

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = Q_0, \quad (9)$$

$$\dot{\rho}_c + 3H\rho_c = -Q_0\dot{\phi}, \quad (10)$$

$$\dot{\rho}_b = -3H\rho_b, \quad (11)$$

$$\dot{\rho}_r = -4H\rho_r. \quad (12)$$

$$Q_0 = \frac{\Theta V_{,\phi} + 3H\Theta\dot{\phi} - 2X\Xi_{,\phi} + \Gamma}{\rho_c\Theta - 2X\Xi - 1}. \quad (13)$$

where $\Theta \equiv \Xi + 2X\Xi_{,X}$

3. Background universe: A. Autonomous equations

Let us compute the autonomous equations by defining the dimensionless dynamical variables as

$$x = \frac{\dot{\phi}}{\sqrt{6}H}, \qquad y = \frac{V}{3H^2}, \qquad \Omega_c = \frac{\rho_c}{3H^2}, \tag{14}$$

$$\Omega_b = \frac{\rho_b}{3H^2}, \qquad \Omega_r = \frac{\rho_r}{3H^2}. \tag{15}$$

We also define the dimensionless functions as

$$z = \frac{C_X}{C} H^2, \qquad \lambda = \frac{V_{,\phi}}{V}, \tag{16}$$

$$\gamma = \Gamma, \qquad \chi = \Xi H^2. \tag{17}$$

3. Background universe: A. Autonomous equations

To perform the dynamical analysis, we choose the potential of scalar field and conformal coefficient C as

$$V(\phi) = V_0 e^{\lambda\phi}, \qquad C(\phi, X) = C_0 e^{\lambda_1\phi}\Big[1 + e^{\lambda_2\phi}\Big(\frac{X}{V_0}\Big)^{\lambda_3}\Big], \tag{18}$$

where $C_0, \lambda_1, \lambda_2, \lambda_3$ are dimensionless constants.

3. Background universe: B. Fixed points

1. Field dominated point and scaling point
Setting $x' = y' = 0$, we obtain two fixed points as

$$x_f = \left\{-\frac{\lambda}{\sqrt{6}}, \frac{\sqrt{6}(2\lambda_3 + 1)}{B}\right\}, \tag{19}$$

$$y_f = \left\{1 - \frac{\lambda^2}{6}, 1 + \frac{6(2\lambda_3 + 1)^2}{B^2} + \frac{2\lambda(2\lambda_3 + 1)}{B}\right\}, \tag{20}$$

$$\Omega_c f = \left\{1, 1 + \frac{6(2\lambda_3 + 1)^2}{B^2} + \frac{2\lambda(2\lambda_3 + 1)}{B}\right\}, \tag{21}$$

$$\omega_{\phi f} = \left\{\frac{1}{3}(\lambda^2 - 3), -\frac{\lambda_1 + \lambda_2 + \lambda\lambda_3}{B\left(\frac{12(2\lambda_3+1)^2}{B^2} + \frac{2\lambda(2\lambda_3+1)}{B} + 1\right)}, \tag{22}$$

where $B = \lambda_1 + \lambda_2 - \lambda(3\lambda_3 + 2)$.

3. Background universe: B. Fixed points

We write λ, λ_1 and λ_2 in term of $\Omega_{\phi f}$ and $\omega_{\phi f}$:
Then, for scaling point, we obtain

$$x_f = \pm\sqrt{\frac{1}{2}\Omega_{\phi f}(1+\omega_{\phi f})} \quad \text{and} \quad y_f = \frac{1}{2}\Omega_{\phi f}(1+\omega_{\phi f}) \qquad (23)$$

3. Background universe: B. Fixed points

2. Kinetic dominated point and ϕMDE point

$$x_f^{(\text{kinetic})} = \pm 1, \tag{24}$$

$$x_f^{(\phi\text{MDE})} = -\frac{\lambda_1 + \lambda_2}{\sqrt{6}(3\lambda_3 + 2)} \mp \frac{\sqrt{\lambda_1^2 + 2\lambda_2\lambda_1 + \lambda_2^2 + 6\lambda_3(3\lambda_3 + 2)}}{\sqrt{6}(3\lambda_3 + 2)} \tag{25}$$

3. Background universe: B. Fixed points

Inserting $x_f^{(\phi\mathsf{MDE})}$ into the definition of Ω_ϕ, we get

$$\Omega_{\phi f}^{(\phi\mathsf{MDE})} = \left[1, 1, \frac{(\lambda_1 + \lambda_2 \pm \sqrt{\lambda_1^2 + 2\lambda_2\lambda_1 + \lambda_2^2 + \lambda_2^2 + 6\lambda_3(3\lambda_3 + 2)})^2}{6(3\lambda_3 + 2)^2}\right] \tag{26}$$

- Since $y = 0$ at these fixed point, then we get $\omega_{\phi f}^{(\phi\mathsf{MDE})} = 1$.
- This give the effective equation of state parameter $w_{\text{eff}} = \Omega_\phi \omega_\phi = \Omega_{\phi f}^{(\phi\mathsf{MDE})}$ is slightly positive during the ϕMDE epoch

3. Background universe: C. Stability

- Field dominated point

The eigenvalues for this case are

$$\mu_1 = 3\lambda_3(1+\omega_{\phi f}) + \lambda_2\sqrt{3(1+\omega_{\phi f})}, \tag{27}$$

$$\mu_2 = -\frac{3}{2}(1-\omega_{\phi f}), \tag{28}$$

$$\mu_3 = \frac{\lambda_3(9\omega_{\phi f} - 3) + +\omega_{\phi f} - (\lambda_1+\lambda_2)\sqrt{3(1+\omega_{\phi f})}}{4\lambda_3 + 2} \tag{29}$$

3. Background universe: C. Stability

The field dominated point is stable point when the following conditions are satisfied

$$\lambda_3 < -\frac{\lambda_2}{\sqrt{3\lambda_1}} \tag{30}$$

$$\lambda_1 \begin{cases} < -\frac{2\omega_{\phi f}(2\sqrt{3}\lambda_2 - 3\sqrt{1+\omega_{\phi f}})}{\sqrt{3}(1+\omega_{\phi f})} & \text{for} \quad \lambda_3 < -1/2 \\ > -\frac{2\omega_{\phi f}(2\sqrt{3}\lambda_2 - 3\sqrt{1+\omega_{\phi f}})}{\sqrt{3}(1+\omega_{\phi f})} & \text{for} \quad \lambda_3 > -1/2 \end{cases} \tag{31}$$

3. Background universe: C. Stability

- Scaling point

The eigenvalues for this case are

$$\mu_1 = 3\lambda_3(1+\omega_{\phi f}\Omega_{\phi f}) \mp \lambda_2\sqrt{3\Omega_{\phi f}(1+\omega_{\phi f}\Omega_{\phi f})}, \tag{32}$$

$$\mu_2 = -\frac{3}{4}(1-\omega_{\phi f}\Omega_{\phi f}) + 3\sqrt{\frac{r_a}{r_b}}, \tag{33}$$

$$\mu_3 = -\frac{3}{4}(1-\omega_{\phi f}\Omega_{\phi f}) - 3\sqrt{\frac{r_a}{r_b}}, \tag{34}$$

where

$$r_a = \lambda_3\Big[w_{\phi f}^2\left(2w_{\phi f}+1\right)\Omega_{\phi f}^3 + \left(-3w_{\phi f}^2 - 18w_{\phi f} + 16\right)\Omega_{\phi f}^2 + \left(16w_{\phi f} - 15\right)\Omega_{\phi f} + 1\Big] + \Omega_{\phi f}\Big[w_{\phi f}^2\left(w_{\phi f}+1\right)\Omega_{\phi f}^2 - 2\left(w_{\phi f}^2 + 5w_{\phi f} - 4\right)\Omega_{\phi f} + 9w_{\phi f} - 7\Big], \tag{35}$$

$$r_b = 16\left(\lambda_3\Omega_{\phi f} + 2\lambda_3 w_{\phi f}\Omega_{\phi f} + w_{\phi f}\Omega_{\phi f} + \Omega_{\phi f} + \lambda_3\right). \tag{36}$$

3. Background universe: C. Stability

The first eigenvalue can be negative when

$$\lambda_3 < \pm\frac{\lambda_2\sqrt{3\Omega_{\phi f}(1+\omega_{\phi f})}}{3(1+\omega_{\phi f}\Omega_{\phi f})}. \tag{37}$$

The real part of both μ_2 and μ_3 can be negative when $r_a = 0$, these give

$$\lambda_{3a} = -\frac{\Omega_{\phi f}[\omega_{\phi f}^2\Omega_{\phi f}^2(1+\omega_{\phi f}) - 2(\omega_{\phi f}^2+5\omega_{\phi f}-4)\Omega_{\phi f}+9\omega_{\phi f}-7]}{\omega_{\phi f}^2\Omega_{\phi f}^3(1+2\omega_{\phi f})+\Omega_{\phi f}^2 K+(16\omega_{\phi f}-15)\Omega_{\phi f}+1} \tag{38}$$

$$\lambda_{3b} = -\frac{\Omega_{\phi f}(1+\omega_{\phi f})}{2\omega_{\phi f}\Omega_{\phi f}+\Omega_{\phi f}+1} \tag{39}$$

$$K = -3\omega_{\phi f}^2 - 18\omega_{\phi f} + 16$$

The scaling point can be stable when $\lambda_3 < \lambda_{3a}$ or $\lambda_3 > \lambda_{3b}$

3. Background universe: C. Stability

- Kinetic dominated point

For this point $x_f = \pm 1$, the eigenvalues are

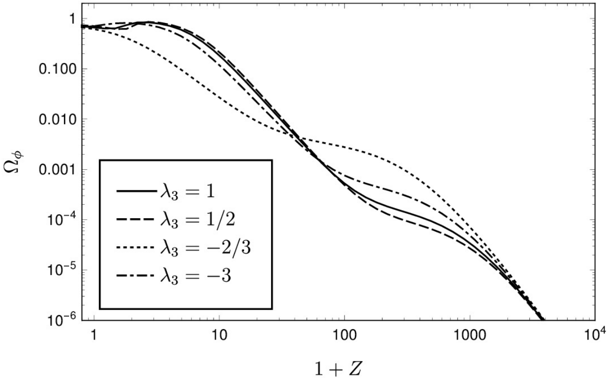
$$\mu_1 = \frac{3(\lambda_3 + 1)}{2\lambda_3 + 1} \pm \frac{\sqrt{6}(\lambda_1 + \lambda_2)}{4\lambda_3 + 2}, \tag{40}$$

$$\mu_2 = 6\lambda_3 \mp \sqrt{6}\lambda_2, \qquad \text{and} \qquad \mu_3 = 6 \pm \sqrt{6}\lambda. \tag{41}$$

3. Background universe: C. Stability

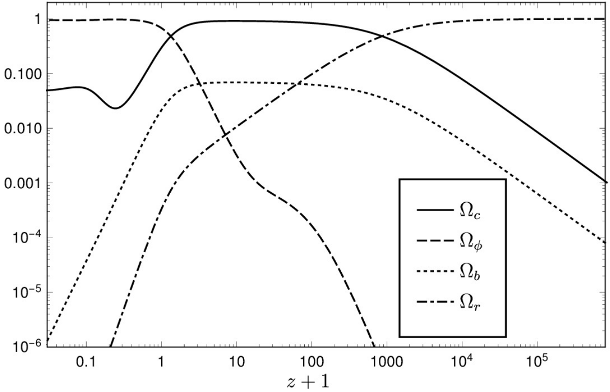
- For ϕMDE point, the eigenvalues of this point are complicated
- We are interested in the case where ϕMDE is followed by accelerating epoch described by scaling point

3. Background universe: D. Evolution from the ϕMDE to scaling point



We set $\lambda_2 = 1, \omega_{\phi f} = -0.99$ and $\Omega_{\phi f} = 0.7$.

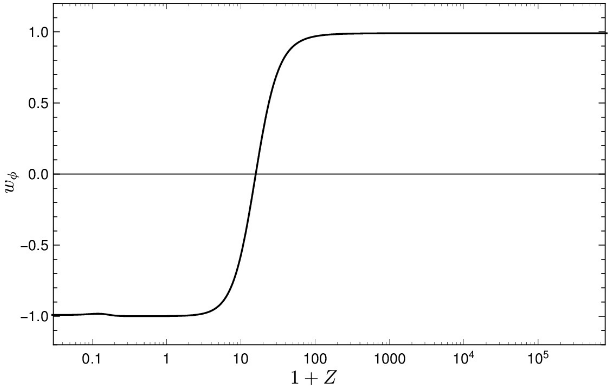
3. Background universe: D. Evolution from the ϕMDE to scaling point



The evolution of Ω_r, Ω_{dm} and Ω_ϕ for $\lambda_3 = -3/2$

3. Background universe: D. Evolution from the ϕMDE to scaling point

- The evolution of ω_ϕ



4. Perturbation

In order to compute the evolution of perturbation, we use Newtonian gauge

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Psi)\delta_{ij}dx^i dx^j \,. \quad (42)$$

We obtain the evolution of density and velocity perturbation as

$$\dot{\delta}_c - 3\dot{\Psi} - (\frac{k^2}{a})v_c = \dot{\phi}\tilde{Q}_0\delta_c - \dot{\phi}\frac{\delta Q}{\rho_c} - \tilde{Q}_0\delta\dot{\phi}, \quad (43)$$

$$\dot{v}_c + (H - \dot{\phi}\tilde{Q}_0)v_c + \frac{1}{a}\Psi = \frac{\tilde{Q}_0}{a}\delta\phi \quad (44)$$

4. Perturbation

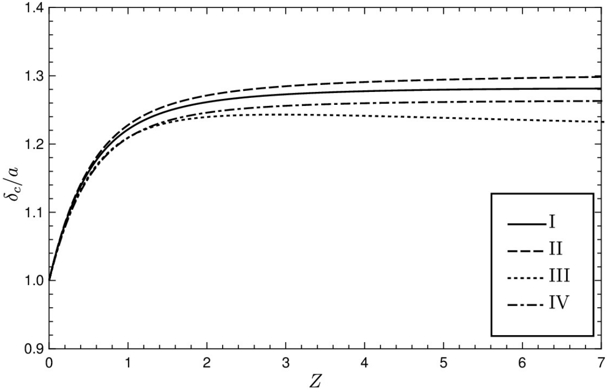
Differentiating eq.(43) wrt. time, we obtain

$$\delta_c'' + C_1\delta_c' - \frac{3}{2}(G_{cc}\Omega_c\delta_c + G_{cb}\Omega_b\delta_b) = 0, \quad (45)$$

where C_1, G_{cc} and G_{cb} are the functions of $x, y, z, \Omega_b, \Omega_c$ and parameters of model.

- The effective gravitational coupling depends on G_{cc}.

4. Perturbation



- The evolution of δ_c/a. For lines I and II, $(\lambda_3, \Omega_{\phi f}, \omega_{\phi f}) = (1/2, 0.96, -0.9)$ and $(-3/2, 0.99, -0.99)$.
- The line III is the usual conformal coupling case $(z = 0)$ with $(\lambda, \lambda_1) = (-1/10, -2/10)$.
- The line IV represents ΛCDM model.

5. Conclusion

- The background universe can evolve from the radiation era to ϕMDE and toward the accelerated expansion era at late time
- The existence of ϕMDE modifies the evolution of the universe, then the H_0 tension can be solved
- The growth of the linear matter perturbation is weaker than ΛCDM, such that the σ_8 tension can be alleviated

Thank you!