New physics effects on quantum correlations in neutrino oscillation

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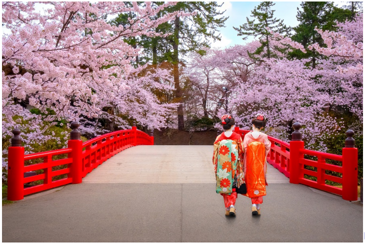

Neutrino Oscillation

- Neutrino oscillation is a quantum mechanical phenomenon where a neutrino created with a specific lepton flavour (electron, muon , or tau) can later be measured to have a different flavour.
- Due to non-zero mass, they oscillate from one flavor to another which has been confirmed by many experiments [Super Kamiokande (1998), Sudbury Neutrino Observatory (2002)], [Nobel Prize:2015].
- Flavor of neutrino determined by superposition of mass eigenstates.
- For neutrinos flavor eigenstates different from mass eigenstates
  $\nu_e = \nu_1 cos\theta + \nu_2 sin\theta$
  $\nu_\mu = -\nu_1 sin\theta + \nu_2 cos\theta.$
- Fundamentally neutrino oscillations are three flavor oscillations but in some cases, it can be reduced to effective two flavor oscillations.

Non Standard Interaction(NSI)

- Neutrino oscillation experiments have strong evidence that neutrino oscillations occur.
- Neutrino oscillation is leading effect for neutrino flavor transitions.
- NSI comprises the effect beyond standard model [Wolfensteinn, Phys. Rev. D 17 (1978)].
- From neutrino oscillation experiments, we have received precision measurements for some of the neutrino parameters, i.e. Δm^2_{21}, $\mid \Delta m^2_{31} \mid$, θ_{12}, θ_{23} [Ohlsson, Rept. Prog. Phys. 76 (2013)].
- Other parameters are still completely unknown such as sign(Δm^2_{31}), CP phases and the absolute neutrino mass scale.
- We are entering the precision era, such subleading effects can be estimated with more accuracy

Mode Entanglement in Neutrinos

Neutrino oscillation requires the flavour eigenstates ν_α to be represented as a linear combination of mass eigenstates ν_i as follows

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle ,$$

Time evolution of mass eigenstates is given by

$$|\nu_i(t)\rangle = e^{-\iota E_i t} |\nu_i\rangle ,$$

In the relativistic limit, neutrino flavour states are considered to be individual modes. In the two flavour neutrino system, it can be expressed as [Blasone et. al., Eur. Phys. Lett. 85 (2009)]

$$|\nu_\alpha\rangle \equiv |1\rangle_\alpha |0\rangle_\beta \equiv |10\rangle_{\alpha\beta} , \qquad |\nu\rangle_\beta \equiv |0\rangle_\alpha |1\rangle_\beta \equiv |01\rangle_{\alpha\beta} .$$

Mode Entanglement in Neutrinos

The time evolution of flavor eigenstate can then be written as

$$|\nu_\alpha(t)\rangle = \bar{U}_{\alpha\alpha}(t)\,|1\rangle_\alpha\,|0\rangle_\beta + \bar{U}_{\alpha\beta}(t)\,|0\rangle_\alpha\,|1\rangle_\beta\,,$$

The density matrix corresponding to the state for above eq. is expressed as

$$\rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \left|\bar{U}_{\alpha\alpha}(t)\right|^2 & \bar{U}_{\alpha\alpha}(t)\bar{U}^*_{\alpha\beta}(t) & 0 \\ 0 & \bar{U}^*_{\alpha\alpha}(t)\bar{U}_{\alpha\beta}(t) & \left|\bar{U}_{\alpha\beta}(t)\right|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

Various measures of quantum correlations can now be determined using the density matrix $\rho_\alpha(t) = |\nu_\alpha(t)\rangle\,\langle\nu_\alpha(t)|$ as the parameters of the density matrix, mixing angle and mass squared difference [Alok et. al., Nucl. Phys. B 909 (2016)].

Neutrino oscillation in Matter

- While travelling through the matter neutrinos undergo charged current (CC) and neutral current (NC) interactions with matter particles.
- Earth matter is composed of only nucleons and electrons.

For an incoming ν_e traversing through Earth, the corresponding Hamiltonian is given by [Guinty et. al., Oxford university press (2007)]

$$\mathcal{H}_m = \mathcal{H}_{vac} + \mathcal{H}_{mat} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} + U^\dagger \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} U,$$

The evolution operator in the mass eigen basis is represented as [Ohlsson et.al., J. Math. Phys. 41 (2000)]

$$U_m(L) = e^{-i\mathcal{H}_m L} = \phi \; e^{-iLT}$$

$$= \phi \sum_{a=1}^{2} e^{-iL\lambda_a} \frac{1}{2\lambda_a} \left(\lambda_a I + T\right).$$

NSI in Neutrino oscillation

In presence of NSI, the Hamiltonian is modified as follows [Ohlsson (2013)]

$$\mathcal{H}_{tot} = \mathcal{H}_{vac} + \mathcal{H}_{mat} + \mathcal{H}_{NSI} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$+U^{\dagger} A \begin{pmatrix} b + \epsilon_{\alpha\alpha}(x) & \epsilon_{\alpha\beta}(x) \\ \epsilon_{\beta\alpha}(x) & \epsilon_{\beta\beta}(x) \end{pmatrix} U.$$

$\epsilon_{\alpha\beta}(x)$ are the NSI parameters which are expressed as

$$\epsilon_{\alpha\beta}(x) = \sum_{f=e,u,d} \frac{N_f(x)}{N_e(x)} \epsilon^f_{\alpha\beta}.$$

The bounds on NSI parameters are extracted from global analysis of the data obtained from different oscillation and non-oscillation experiments [Esteban et.al., J. High Energy Phys., (2019), Coloma et. al., J. High Energy Phys. (2020)].

Local Realism

In 1964 John Bell formulated a mathematical statement in the form of inequalities which were based on following two assumptions [Bell, Physics 1 (1964)]

- Realism: A system has well defined values of an observable whether someone measures it or not.
- Locality: A measurement made on a system cannot influence other systems instantaneously.
- A system that can be described by a local realistic theory will satisfy this inequality.
- It turns out that nature experimentally invalidates that point of view and agreeing with quantum mechanics [Aspect et. al., Phys. Rev. Lett. 49 (1982)].

Measures of Quantum Correlations

Bell's Inequality

For a system consisting of two spin-1/2 particles A and B, the combined state with Hilbert space defined as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, is expressed in terms of the density matrix (ρ) as follows [Horodecki et. al., Phys.Lett. A 200 (1995)]

$$\rho = \frac{1}{4}\left[I \otimes I + (r.\sigma) \otimes I + I \otimes (s.\sigma) + \sum_{A,B=1}^{3} T_{AB}(\sigma_A \otimes \sigma_B)\right].$$

- T is correlation matrix and elements of this matrix are $T_{AB} = Tr[\rho(\sigma_A \otimes \sigma_B)]$. $T^\dagger T$ having eigenvalues u_i $(i = 1, 2, 3)$ out of which two largest positive eigenvalues are taken into account, denoted by u_i and u_j.Bell-CHSH inequality can be written as $M(\rho) = u_i + u_j \leq 1$.

Measures of Quantum Correlations

NAQC(non-local advantage of quantum coherence)

Coherence of a system represented by the state ρ can be quantified by l_1 norm which in the eigen basis of Pauli spin matrix σ_i $(i = x, y, z)$ is defined as [Mondal et.al., Phys. Rev. A 95 (2017)]

$$C^i_{l_1}(\rho) = \sum_{i_1, i_2} | \langle i_1 | \, \rho \, | i_2 \rangle |, (i_1 \neq i_2).$$

Here $|i_1\rangle$ and $|i_2\rangle$ are the eigen vectors of σ_i.

Then the upper limit of the following quantity is given by

$$\sum_{i=x,y,z} C^i_{l_1}(\rho) \leq \sqrt{6} \approx 2.45.$$

Measures of Quantum Correlations

NAQC(non-local advantage of quantum coherence)

To understand NAQC, let us consider an entangled state, consisting of two subsystems A and B, expressed by the density matrix ρ . The violation of $C^i_{l_1}(\rho)$ infers the fact that the single system description of the coherence of the subsystem B is not feasible. Therefore NAQC of the state B is achieved by the condition [Ming et. al., Phys. Rev. A 98 (2018)]

$$N_{l_1}(\rho) = \frac{1}{2} \sum_{i,j,a} p(\rho_{B|\Pi^a_i}) C^i_{l_1}(\rho_{B|\Pi^a_i}) > \sqrt{6}.$$

NAQC is a stronger measure of non-local correlation than Bell's inequality.

Quantum correaltion in Neutrino Oscillation

- The coherent time evolution of neutrino flavor eigenstates implies that there is a linear superposition between the mass eigenstates which make up a flavour state.
- Thus neutrino oscillations are related to the multi-mode entanglement of single-particle states which can be expressed in terms of flavor transition probabilities [Alok et. al., Nucl. Phys. B 909 (2016)].
- Hence neutrino is an interesting candidate for study of quantum correlations.

We are interested in studying measures of quantum correlations in neutrinos. In particular, we intend to study the measures of quantum correlations such as NAQC and Bell-Inequality in neutrino oscillation system.

Experimental Setups

On the basis of sensitivity to NO in terms of different oscillation channels, we can classify experimental set-ups in three categories as follows

- DUNE, MINOS and T2K are the long baseline (LBL) accelerator experiments.These experiments are mainly sensitive to θ_{23} and Δ_{32} parameters, (for appropriate approximations viz. $\{\theta_{12}, \Delta m^2_{21}\} \to 0$), driving the oscillation channel $\nu_\mu \to \nu_\tau$.
- KamLAND and JUNO are the LBL reactor experiments operate with $\bar{\nu}_e$ beam and look for the $\bar{\nu_e}$ appearance channel. In the limit $\theta_{13} \to 0$, the effective two flavour oscillation formula consists mainly the parameters Δm^2_{21} and θ_{12}.
- Daya Bay is the short baseline (SBL) reactor experiment. Daya-Bay is almost unable to observe oscillations for small effective mass square difference, hence for $\Delta m^2_{21} \to 0$, it has sensitivity for Δm^2_{31} and θ_{13} oscillation parameters [Alok et. al., Nucl. Phys. B 909 (2016)].

Results and Discussions

The analytical expressions of the two parameters, $M(\rho)$ and $N_{l_1}(\rho)$, are obtained as [Yadav et. al., EPJC 82, no. 5, 1-10 (2022)]

$$M(\rho) = f_a(x, y, r) + f_b(x, y, z, r) + f_c(x, z, r),$$

$$N_{l_1}(\rho) = 2 + \sqrt{\frac{2f_b(x, y, z, r)}{3}},$$

where the form of quantities f_a, f_b and f_c are given as

$$f_a(x, y, r) = \frac{e^{\mathrm{Im}(4r)}[x^2 + y^2 + (x^2 - y^2)\cos(2r)]^2}{4x^4},$$

$$f_b(x, y, z, r) = \frac{3e^{\mathrm{Im}(4r)} z^2 \sin^2 r (x^2 + y^2 + (x^2 - y^2)\cos(2r))}{x^4},$$

$$f_c(x, z, r) = \frac{e^{\mathrm{Im}(4r)} z^4 \sin^4 r}{x^4},$$

with $r = \frac{Lx}{4E}$.

Results and Discussions

The quantities x, y and z are functions of NSI parameters and are given by

$$x = \sqrt{x_1 - x_2 + x_3},$$

$$y = \frac{-x_2}{2\,\Delta m^2\,\cos(2\theta)} + \Delta m^2\,\cos(2\theta),$$

$$z = \frac{x_3 - (\Delta m^2)^2\cos(4\theta)}{2\,\Delta m^2\,\sin(2\theta)},$$

with

$$x_1 = 4A^2E^2(4\epsilon_{\alpha\beta}^2 + (\epsilon_{\alpha\alpha} + b - \epsilon_{\beta\beta})^2),$$

$$x_2 = 4\,A\,E\,(\epsilon_{\alpha\alpha} + b - \epsilon_{\beta\beta})\,\Delta m^2\cos(2\theta),$$

$$x_3 = 8\,A\,E\,\epsilon_{\alpha\beta}\,\Delta m^2\,\sin(2\theta) + (\Delta m^2)^2.$$

Results and Discussions

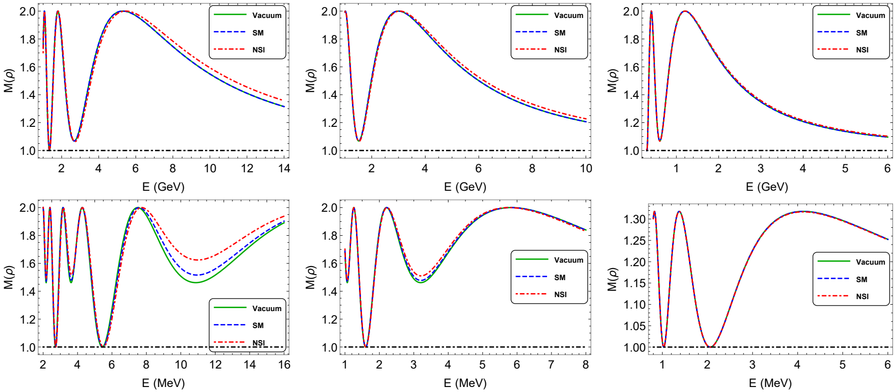

Figure: Variation of M(ρ) with energy (E) for the accelerator and reactor experiments. (a) Upper left: DUNE; (b) upper middle: MINOS; (c) upper right: T2K; (d) lower left: KamLAND; (e) lower middle: JUNO; and (f) lower right: Daya Bay Dotted (black) line represents the classical bound of M(ρ).

Results and Discussions

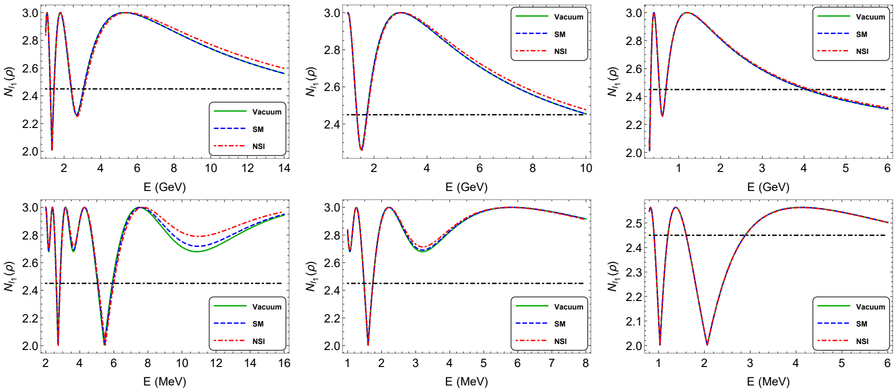

Figure: Variation of NAQC parameter with energy (E) for the accelerator and reactor experiments: DUNE, $L = 1300$ km, $E \approx 1 - 14$ GeV; MINOS, $L = 735$ km, $E \approx 1 - 10$ GeV; T2K, $L = 295$ km, $E \approx 0 - 6$ GeV; KamLAND, $L = 180$ km, $E \approx 1 - 16$ MeV; JUNO, $L = 53$ km, $E \approx 1 - 8$ MeV; and Daya Bay, $L = 2$ km, $E \approx 0.8 - 6$ MeV.

Results and Discussions

Expts.	Measure	% inc. w.r.t. vac	% inc. w.r.t. SM int.
DUNE	$M(\rho)$	4.3	4.3
	$N_{l_1}(\rho)$	4	4
MINOS	$M(\rho)$	2.4	2.4
	$N_{l_1}(\rho)$	2.3	2.3
T2K	$M(\rho)$	0.7	0.7
	$N_{l_1}(\rho)$	0.6	0.6
KamLAND	$M(\rho)$	16.5	11
	$N_{l_1}(\rho)$	11	7
JUNO	$M(\rho)$	5	3.3
	$N_{l_1}(\rho)$	3.5	2.4
Daya Bay	$M(\rho)$	≈ 0	≈ 0
	$N_{l_1}(\rho)$	≈ 0	≈ 0

Table: Percentage (%) increase in Bell's inequality parameter $M(\rho)$ and NAQC parameter $N_{l_1}(\rho)$ in presence of NSI in comparison to vacuum and SM interaction for six different experimental set-ups.

Results and Discussions

Experiments	$N_{l_1}(\rho)$	$M(\rho)$
DUNE (GeV)	1-1.25 ,1.5-2.45 ,3-14	1-14
MINOS (GeV)	1-1.3 ,1.75-10	1-10
T2K (GeV)	0.4-0.5 , 0.7-4	0-6
KamLAND (MeV)	1-2.5 , 3-5 ,6-16	1-16
JUNO (MeV)	1-1.45 , 1.8-8	1-8
Daya Bay (MeV)	0.8-0.9 ,1.2-1.6 ,2.9-6	0.8-6

Table: Energy regions showing the violation of $M\rho$ and NAQC for six different experimental set-ups. For accelerator experiments the energy range lies in GeV region, while for reactor experiments they are in MeV.

Entanglement

- The study of correlations has extensive literature on quantum systems such as quantum teleportation and so on, and now various investigations into particle physics-related systems.
- The most basic idea among them that can be accurately measured and described using the methods of quantum resource theories is entanglement.
- There are numerous ways to quantify entanglement in three-entanglement measures, i.e. Entanglement of formation, Concurrence, and Negativity.

Entanglement Measure

- Entanglement of formation quantifies the minimum amount of entanglement required to prepare a given quantum state, defined as

$$EOF(\rho_{ABC}(t)) = \frac{1}{2}[S(\rho_A) + S(\rho_B) + S(\rho_C)]$$

where ρ_A, ρ_B and ρ_C is reduced density matrices which is partial trace of density matrix. $S_(\rho_A)$, $S_(\rho_B)$ and $S_(\rho_C)$ is von Neumann entropy defined as $S_(\rho_A) = -Tr(\rho_A log \rho_A)$.
EOF in terms of survival and oscillation probabilities for vacuum can presented as follows [Ming et. al, (2021)]:

$$EOF^{\alpha} = -\frac{1}{2}[P_{\alpha e} log_2 P_{\alpha e} + P_{\alpha\mu} log_2 P_{\alpha\mu} + P_{\alpha\tau} log_2 P_{\alpha\tau} \\ +(P_{\alpha\mu} + P_{\alpha\tau}) log_2 (P_{\alpha\mu} + P_{\alpha\tau}) + (P_{\alpha e} + P_{\alpha\tau}) log_2 (P_{\alpha e} + P_{\alpha\tau}) \\ +(P_{\alpha\mu} + P_{\alpha e}) log_2 (P_{\alpha\mu} + P_{\alpha e})]$$

Entanglement Measure

- Concurrence measures the degree of entanglement between three subsystems of a larger quantum system

$$C(\rho_{ABC}) = [3 - Tr(\rho_A)^2 - Tr(\rho_B)^2 - Tr(\rho_C)^2]^{\frac{1}{2}}$$

where $\rho_A = Tr_{BC}(\rho_{ABC}(t))$, $\rho_B = Tr_{AC}(\rho_{ABC}(t))$ and $\rho_C = Tr_{AB}(\rho_{ABC}(t))$.
Following is a presentation of Concurrence for vacuum in terms of oscillation and survival probabilities [Ming et. al, (2021)]:

$$C^{\alpha} = \sqrt{3 - 3(P_{\alpha e}^2 + P_{\alpha\mu}^2 + P_{\alpha\tau}^2) - 2P_{\alpha\mu}P_{\alpha\tau} - 2P_{\alpha e}(P_{\alpha\mu} + P_{\alpha\tau})}$$

Entanglement Measure

- Negativity is defined as the absolute value of the sum of the negative eigenvalues of the partially transposed density matrix and it can be used for both pure and mixed states, defined as

$$N = (N_{A-BC} N_{B-CA} N_{C-AB})^{\frac{1}{3}}$$

where $N_{A-BC} = -\sum_i \lambda_i^A$, $N_{B-CA} = -\sum_j \lambda_j^B$ and $N_{C-AB} = -\sum_k \lambda_k^C$. Here λ_i^A, λ_j^B and λ_k^C are negative eigenvalues of $\rho_{ABC}^{T_\alpha}(t)$ which is partial transpose of matrix $\rho_{ABC}(t)$.
Here following is a presentation of the Negativity in terms of survival and oscillation probabilities for vacuum [Ming et. al., (2021)]:

$$N^\alpha = [\sqrt{P_{\alpha e}}\sqrt{P_{\alpha\mu} + P_{\alpha\tau}}\sqrt{P_{\alpha e}}\sqrt{P_{\alpha\mu}}\sqrt{P_{\alpha e} + P_{\alpha\mu}}\sqrt{P_{\alpha\tau}}]^{\frac{1}{3}}$$

Results and Discussions

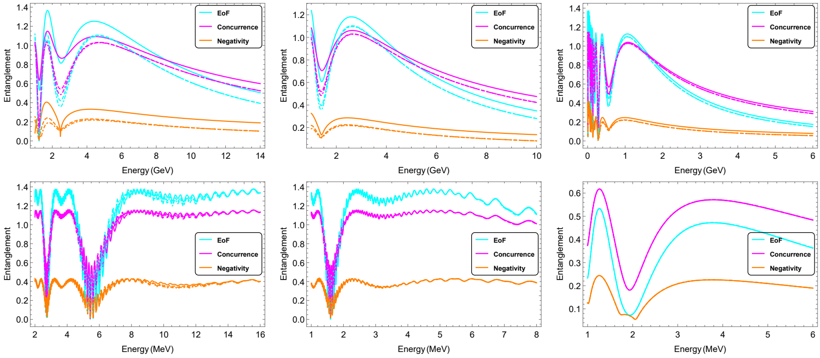

Figure: Tripartite entanglement measures as a function of the energy of neutrino (E) for different experiments: DUNE, $L = 1300$ km, $E \approx 1 - 14$ GeV; MINOS, $L = 735$ km, $E \approx 1 - 10$ GeV; T2K, $L = 295$ km, $E \approx 0 - 6$ GeV; KamLAND, $L = 180$ km, $E \approx 1 - 16$ MeV; JUNO, $L = 53$ km, $E \approx 1 - 8$ MeV; and Daya Bay, $L = 2$ km, $E \approx 0.8 - 6$ MeV.

Results and Discussions

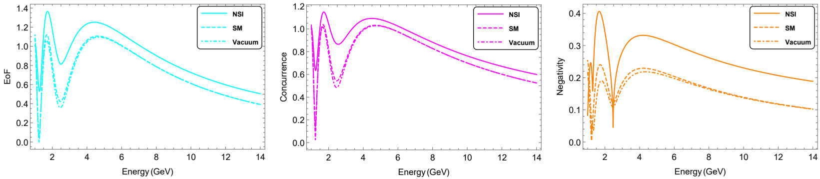

Figure: Tripartite entanglement measures: (a) Left: EOF, (b) Middle: Concurrence, and (c) Right: Negativity, as a function of the energy of neutrino (E) for DUNE experiment. The dot-dashed line represents oscillation in Vacuum, whereas the dashed lines for SM and solid lines for NSI.

Accord

- In the past decade, the community has shown a growing interest in studying quantum correlations beyond entanglement, and significant progress has been made.
- Recently, accord, a new measure of quantum correlations, has been defined. It is the minimization of the maximum over unitary matrices.
- Accord is directly defined in terms of a simple experimental procedure and has a clear, intuitive meaning, and this is the main advantage of the accord over current measures of quantum correlations
- Accord is equivalent to concurrence for pure states, a lower bound on discord for mixed states.

Accord

- According to the theory of decoherence, the interaction of a system with its environment can result in the loss of coherence with respect to preferred subspaces, depending on the nature of the interaction.
- For example, if an environment couples to the spatial degree of freedom of a system, it will reduce the system's spatial coherence.
- In accounts of the emergence of classicality, such incoherence plays an important role.
- The coupling to the environment will produce mixed quantum mechanical states while initially we started with pure quantum mechanical state.
- In this work, we analyze the recent measure accord for mixed states in quantum decoherence system with Majorana phase for two flavour neutrino system

Formalism

To investigate the effects of a non-diagonal form of the decoherence matrix, for convenience, we take into account $\gamma_1 = \gamma_2 = \gamma$, $D_{\lambda\mu}$ given by

$$D_{\lambda\mu} = - \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \gamma_1 & \alpha & 0 \\ 0 & \alpha & \gamma_2 & 0 \\ 0 & 0 & 0 & \gamma_3 \end{pmatrix}$$

The density matrix at any time, t represent as

$$\rho(t) = \begin{pmatrix} \rho_0(t) + \rho_3(t) & \rho_1(t) - i\rho_2(t) \\ \rho_1(t) + i\rho_2(t) & \rho_0(t) - \rho_3(t) \end{pmatrix}$$

Results and Discussions

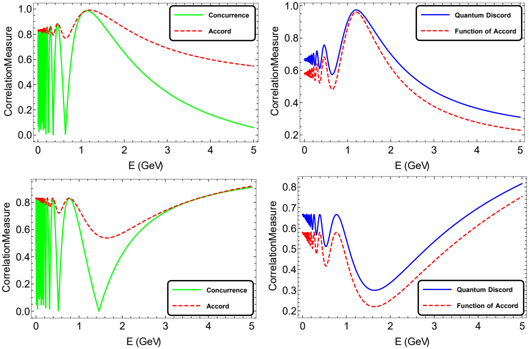

Figure: In upper pannel (Left).Accord vs. Concurrence. (Right) Accord vs. quantum discord for non-zero Majorana phase and In lower pannel (Left).Accord vs. Concurrence. (Right) Accord vs. quantum discord for zero Majorana phase with neutrino energy (E), Parameters are $\phi = \frac{\pi}{4}$, $x = 1.3 \times 10^4 km$, $\sin^2\theta_{12} = 0.861$, $\Delta m^2_{12} = 7.5 \times 10^{-5} eV^2$. Moreover, we set $\gamma = 1.2 \times 10^{-23} GeV$, $\gamma_3 = 2.3 \times 10^{-23} GeV$, $\alpha = 1 \times 10^{-23} GeV$. $E \approx 0 - 5$ GeV and $\Delta m^2 = 7.5 * 10^{-5} eV^2$. In (a) green line refers to concurrence while the dashed red line for an accord. In (b) blue line refers to quantum discord while the dashed red line is for the function of the accord.

Results and Discussions

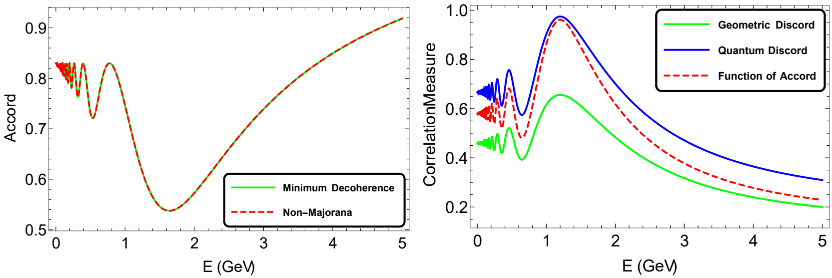

Figure: **Left)** Variation of accord with neutrino energy for Minimum Decoherence model and non-Majorana model, Parameters are $\phi = 0$. **Right)** Variation of correlation measures i.e. Quantum Discord, the function of accord and geometric discord with energy (E) with $\phi = \frac{\pi}{4}$. Other parameters for both plots are $x = 1.3 \times 10^4 km$, $\sin^2\theta_{12} = 0.861$, $\Delta m^2_{12} = 7.5 \times 10^{-5} eV^2$. Moreover, we set $\gamma = 1.2 \times 10^{-23} GeV$, $\gamma_3 = 2.3 \times 10^{-23} GeV$, $\alpha = 1 \times 10^{-23} GeV$., $E \approx 0 - 5$ GeV.

Thank You