Signature of nuclear matter properties on neutron star oscillations : equations of state representation and numerical simulations

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Action Incitative Pluriannuelle Ondes Gravitationnelles et Objets Compacts
November 14, 2022

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   - 3+1 formalism
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   - 2-parameter generalisation of pseudo-polytrope
   - 2D code (axisymmetric + symmetric around $z = 0$ plane)
6. Conclusion

Context

- Neutron stars : high mass star collapse
- Mass $\sim 1.4\,M_{\odot}$, radius $\sim 10\,\mathrm{km}$
- First detection : 1967 pulsar by Jocelyn Bell
- First formation detection : 1987 supernova SN1987a (12 neutrinos)

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- Post-coalescence life (~few 0.1s) : hypermassive neutron star ($M > 3 - 4M_\odot$) then collapse into black hole

Goals

- Write 3D hypermassive neutron star evolution code : gravitational waves modes.

[1] https://lorene.obspm.fr

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- Write 3D hypermassive neutron star evolution code : gravitational waves modes.
- Use different equations of state (EOS) in the code
- Code written using C++ and LORENE[1] (pseudospectral methods)
- Starting with spherical symmetry (1D) (no GW) extension to 2D presented at the end.

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Let $(\mathcal{M}, \mathbf{g})$ be a spacetime associated with $\mathbf{g}$ a metric of signature (-,+,+,+) ; it is assumed that there exists a foliation of $\mathcal{M}$ i.e. a scalar field $\hat{t}$ so that iso-$\hat{t}$ hypersurfaces

$$\Sigma_t = \{p \in \mathcal{M},\ \hat{t}(p) = t\} \tag{1}$$

verify

$$t \neq t' \Rightarrow \Sigma_t \cap \Sigma_{t'} = \emptyset \text{ et } \bigcup_{t \in \mathbb{R}} \Sigma_t = \mathcal{M} \tag{2}$$

Anywhere a timelike vector $\mathbf{n}$, orthogonal to Σ_t, is defined.

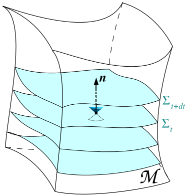

Figure: Illustration of a foliation

Let us define an induced metric γ (3-metric) which signature is $(+,+,+)$, its associated RIEMANN tensor R^i_{jkl}, RICCI, tensor $R_{ij} = R^k_{ijk}$, RICCI scalar $R = R^i_i$ (*intrinsic* curvature), the *extrinsic* curvature tensor K_{ij}, its trace $K = K^i_i$, the lapse function N, the shift vector β^i.

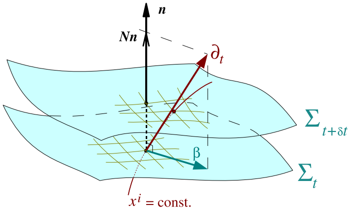

Figure: Illustration of the lapse and the shift

Knowing the lapse, the shift, and the 3-metric allows to fully determine $\mathcal{M}$'s geometric structure :

$$g_{\alpha\beta} = \begin{pmatrix} g_{00} & g_j \\ g_i & g_{ij} \end{pmatrix} = \begin{pmatrix} -N^2 + \beta_k\beta^k & \beta_j \\ \beta_i & \gamma_{ij} \end{pmatrix} \quad (3)$$

$\rightarrow$ Time and space are naturally separated

Model : cold, catalysed star

Thermo assumptions

- Fast cooling : T is a fraction of T_F in a few minutes - $T = 0$ neutron star
- β equilibrium : $p + e^- \leftrightarrow n + \nu_e$

We use the perfect fluid model :

$$\boxed{T_{\alpha\beta} = (e + p)u_\alpha u_\beta + p g_{\alpha\beta}} \qquad (4)$$

u unitary timelike vector, e total energy density, p pressure.

- Code : 1D Chebyshev radial grid - 1 nucleus (star), 1 compactified domain.
- The boundary between the domains is comoving with the border of the star
- Hydrodynamical equations :

$$\nabla_\mu T^\mu_\nu = 0 \text{ and } \nabla_\mu (n_B u^\mu) = 0 \qquad (5)$$

+ Einstein equations :

$$^4R_{\alpha\beta} - \frac{1}{2}{}^4R g_{\alpha\beta} = 8\pi T_{\alpha\beta} \qquad (6)$$

written as PDE system + equation of state to close the system i.e. relation between thermo variables

With U_i the Eulerian velocity and

$$v^i = NU^i - \beta^i,\ H = \ln\left(\frac{e+p}{m_B n_B}\right),\ \Gamma^2 = (1 - U_i U^i)^{-1},\ c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}e} \tag{7}$$

The equations are

$$\begin{aligned}
\partial_t U_i = & -v^j D_j U_i - D_i N - \frac{N}{\Gamma^2}\left(D_i H - \frac{\Gamma^2(1-c_s^2)}{\Gamma^2 - c_s^2(\Gamma^2-1)} U_i U^j D_j H\right) \\
& + U_j D_i \beta^j + U_i U^j D_j N \\
& + \frac{N c_s^2}{\Gamma^2 - c_s^2(\Gamma^2-1)} U_i D_j U^j + \frac{N\Gamma^2(c_s^2-1)}{\Gamma^2 - c_s^2(\Gamma^2-1)} U_i U^j U^l K_{jl} \\
\partial_t H = & -v^i D_i H - c_s^2 N \frac{\Gamma^2}{\Gamma^2 - c_s^2(\Gamma^2-1)}\left[U^i U^j K_{ij} - \frac{U^i}{\Gamma^2} D_i H + D_i U^i\right]
\end{aligned}$$

[to be published]

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Polytrope

A polytrope is a 1-parameter equation of state. For neutron stars it describes cold star at β-equilibrium. The parameter is the density of baryons in the star :

$$p(n_B) = \kappa n_B^{\gamma} \tag{8}$$

- κ : pressure coefficient
- γ : adiabatic index

$\gamma = 2$: approximation for nuclear matter ; $\gamma = 5/3$: non-relativistic Fermi gas ; $\gamma = 4/3$: ultra relativistic Fermi gas
Easy to use, analytical and convenient for numerical tests but not realistic.

Realistic EOS : instabilities

Realistic EOSs induce instabilities in the code

- Phenomenological models
- Nuclear experimental and astrophysical data are extrapolated
- Represented as discrete tables (p, e, ..., are tabulated)
- Insufficient precision on sound speed (numerical derivatives)
- Thermodynamical consistency is not always possible (crust/core)

$\rightarrow$ Pseudo-polytropes fitting scheme (based on an idea by Jose PONS, Alicante, Spain [to be published])

Realistic EOS : representation

Drawbacks of tables :

- Lack of precision
- Lots of data for 2 or 3 parameters (thousands to millions of grid point)
- Longer computation time (interpolation)

Assets of a fitting scheme :

- Precision = machine accuracy
- No interpolation through tables
- Light storage (pseudo-polytropes : a few coefficients + a formula)

But :

- Crust ($n_B < 0.08\mathrm{fm}^{-3}$) difficult to handle : "removed"
- Losing physical features

Realistic EOS : pseudo-polytrope

Pseudo-polytropes (internal free energy parametrization) :

$$f(n_B) = m_B g(n_B) n_B^{\alpha} \tag{9}$$

+ thermo principles. In practice :

$$g(x = \ln n_B) = a + bx + cx^2 \tag{10}$$

Parameters a, b, c fitted on f/n_B, $\ln(p/n_B)$ and $\Gamma_1 = \frac{\mathrm{d}\ln p}{\mathrm{d}\ln n_B}$ on the interval $[n_1, n_{\max}]$.
Below n_1 a polytrope is chosen, with parameters adjusted for thermo consistency.
Polytrope recovered by : $g(x) = cst$ and $\gamma \equiv \alpha + 1$.

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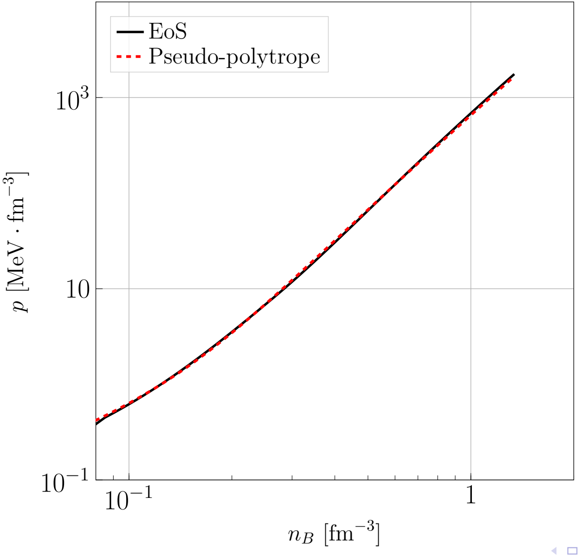
APR EoS [APR98] thermo



APR EoS [APR98] : MR diagram = equilibrium sequence

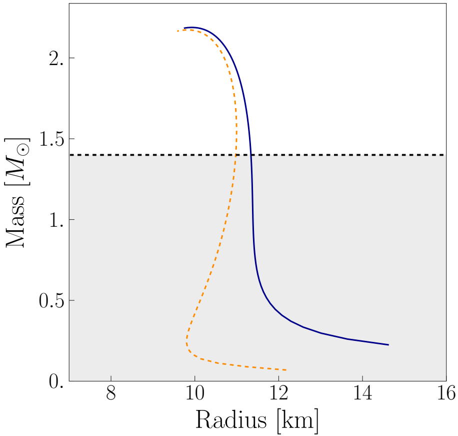


Figure: Mass-radius diagram for APR

Legend :

- Solid blue : 1-param version of APR in CompOSE
- Dashed orange : 1-param pseudo-polytropic fit

Features :

- Maximum mass
- Radii difficult to reproduce
- Different crust ($M < 1.4 M_\odot$)

Frequency extraction : principle for "$l = 0$"

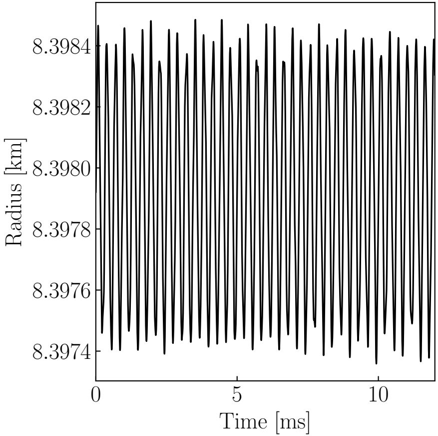


Figure: Radius vs time (EoS = APR)

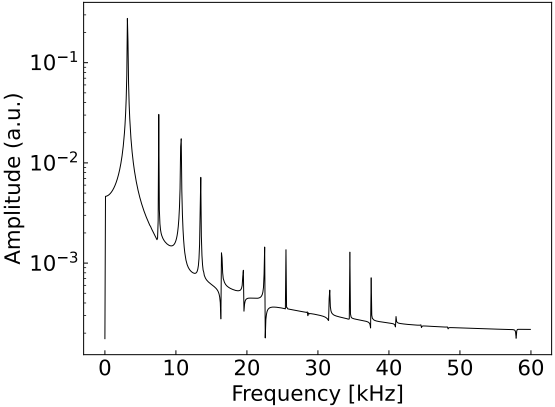


Figure: $\hat{R}(f)$ spectrum

EoS APR : Mf diagram.

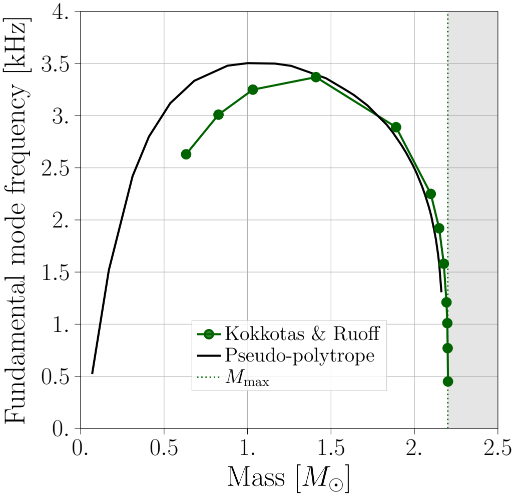


Figure: Mass-frequency diagram for APR. ref : Kokkotas & Ruoff [KR01]

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2-parameter pseudo-polytrope

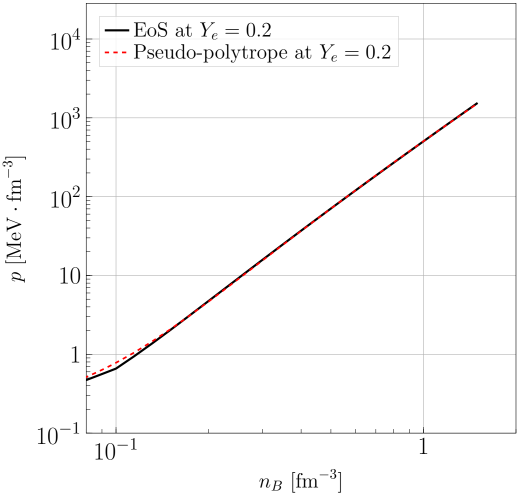
Generalisation to 2 parameters on $\varepsilon = e/m_n nB - 1$:

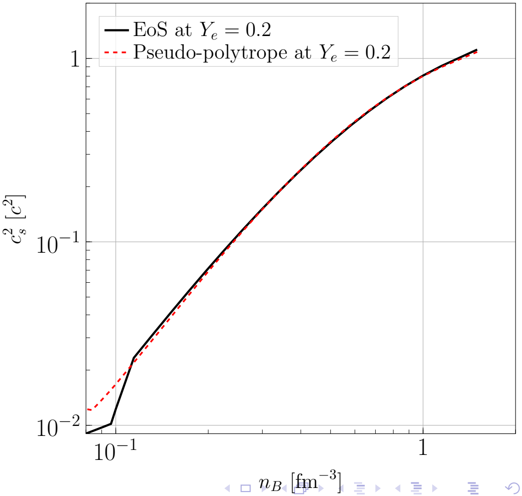
$$\varepsilon(x = \ln n_B, Y_e) = e^{\alpha x}(a(Y_e) + b(Y_e)x + c(Y_e)x^2) \tag{11}$$

+ simplified analytical crust, that meets several requirements :

- Continuity of $e(n_B)$ and $p(n_B)$ at crust-core transition
- Existence of β-equilibrium (i.e. ε must have an extremum in the Y_e direction for all n_B).

2-parameter pseudo-polytrope : thermodynamics for RG(SLy4) equation of state [GR15].





$l = 2, \ m = 0$ simulations

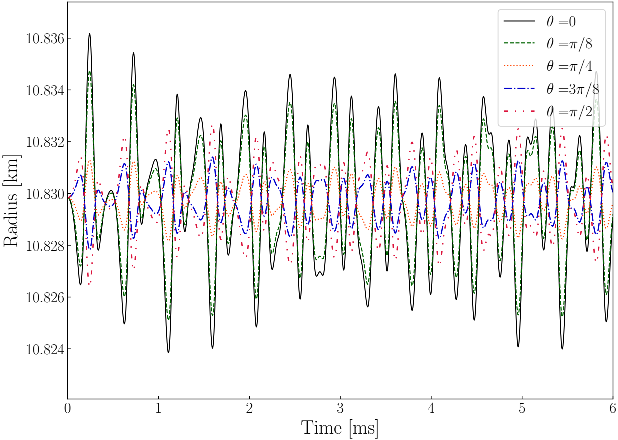


Figure: Radius vs time for NS with $l = 2, \ m = 0$ perturbation in axisymmetric configuration. EoS : $\gamma = 2$ polytrope.

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Conclusions, outlook

Recap :

- Pseudo-polytrope for 1- and 2-param EoS.
- Fitting scheme that produces close MR and Mf diagrams for higher masses ($M > 1.4M_\odot$).
- Asset : economical storage and reduced computing time.

Outlook :

- Papers on derivation of hydro equations and on representation of 1- and 2-param EoS to be submitted.
- EoS representation : 3-parameters i.e. hot NS.
- 3D code.
- Multi-domain inside the star.
- Quadrupole formula and its Post-Newtonian extensions to be implemented.

Thank you

Bibliography I

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[GR15] F. Gulminelli and Ad. R. Raduta, *Unified treatment of subsaturation stellar matter at zero and finite temperature*, Phys. Rev. C **92** (2015), no. 5, 055803 (en).

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Frequency extraction : principle

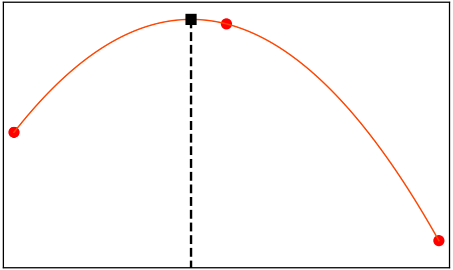

Figure: Frequency extraction principle