



# Looking beyond Dark Matter in Axion Haloscopes

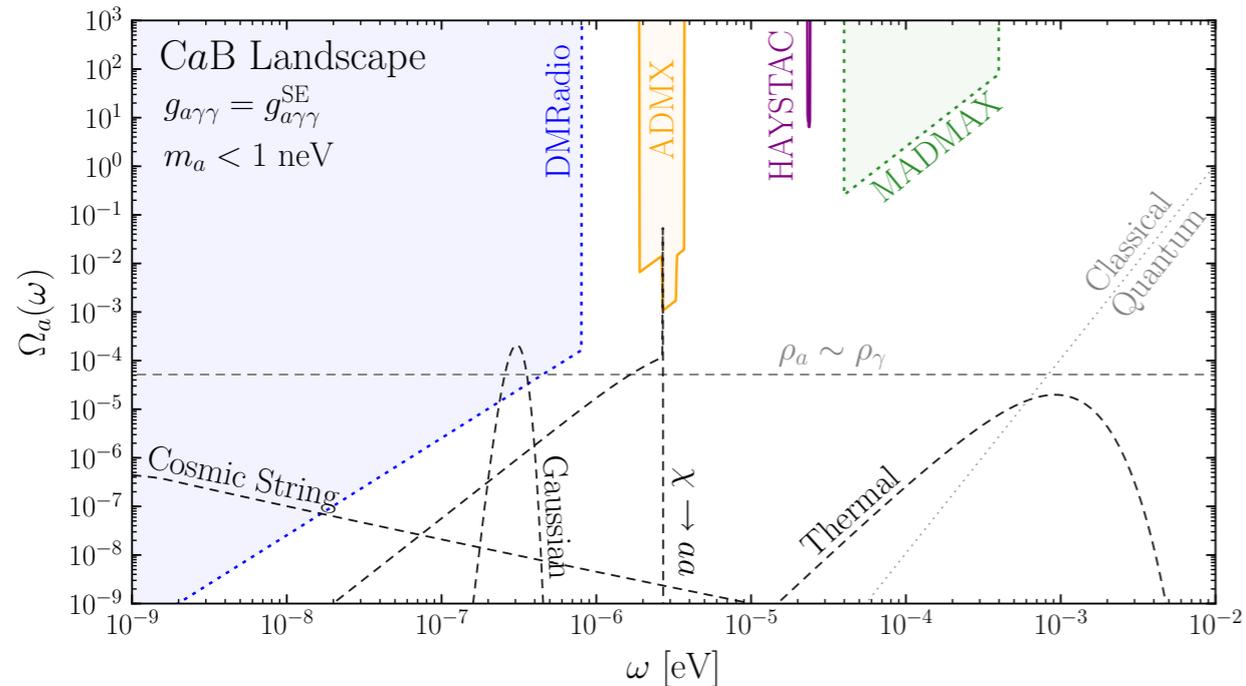
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# The Cosmic Axion Background

PRD 2021 (Editors' Suggestion)

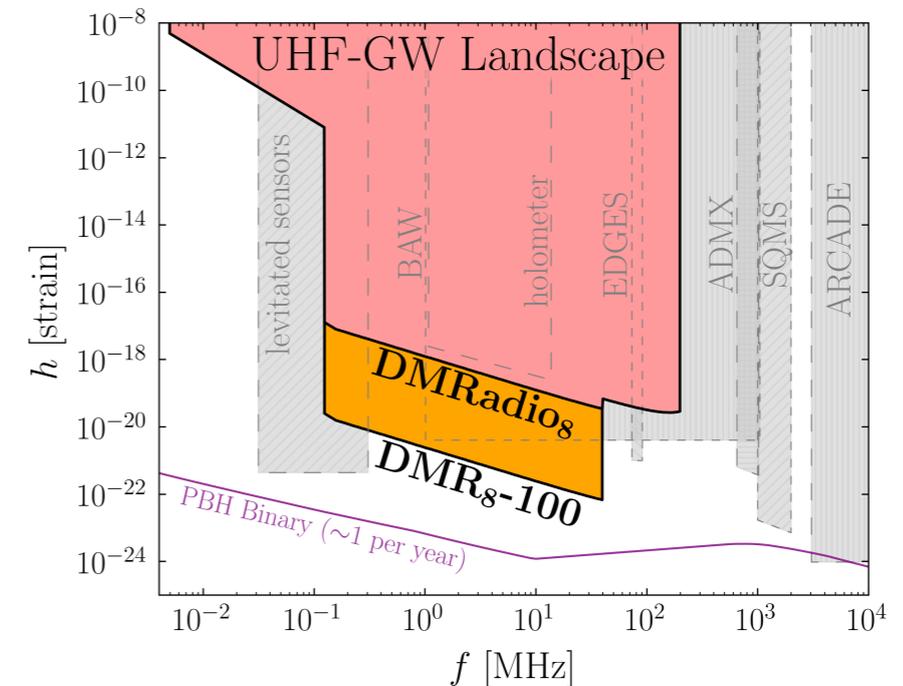
w/ Jeff Dror, Hitoshi Murayama



# High-Frequency Gravitational Waves

PRL 2022

w/ Valerie Domcke, Camilo Garcia-Cely



# Looking beyond Dark Matter in Axion Haloscopes



# Motivation

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$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a (F \tilde{F})$$

The axion

# Motivation

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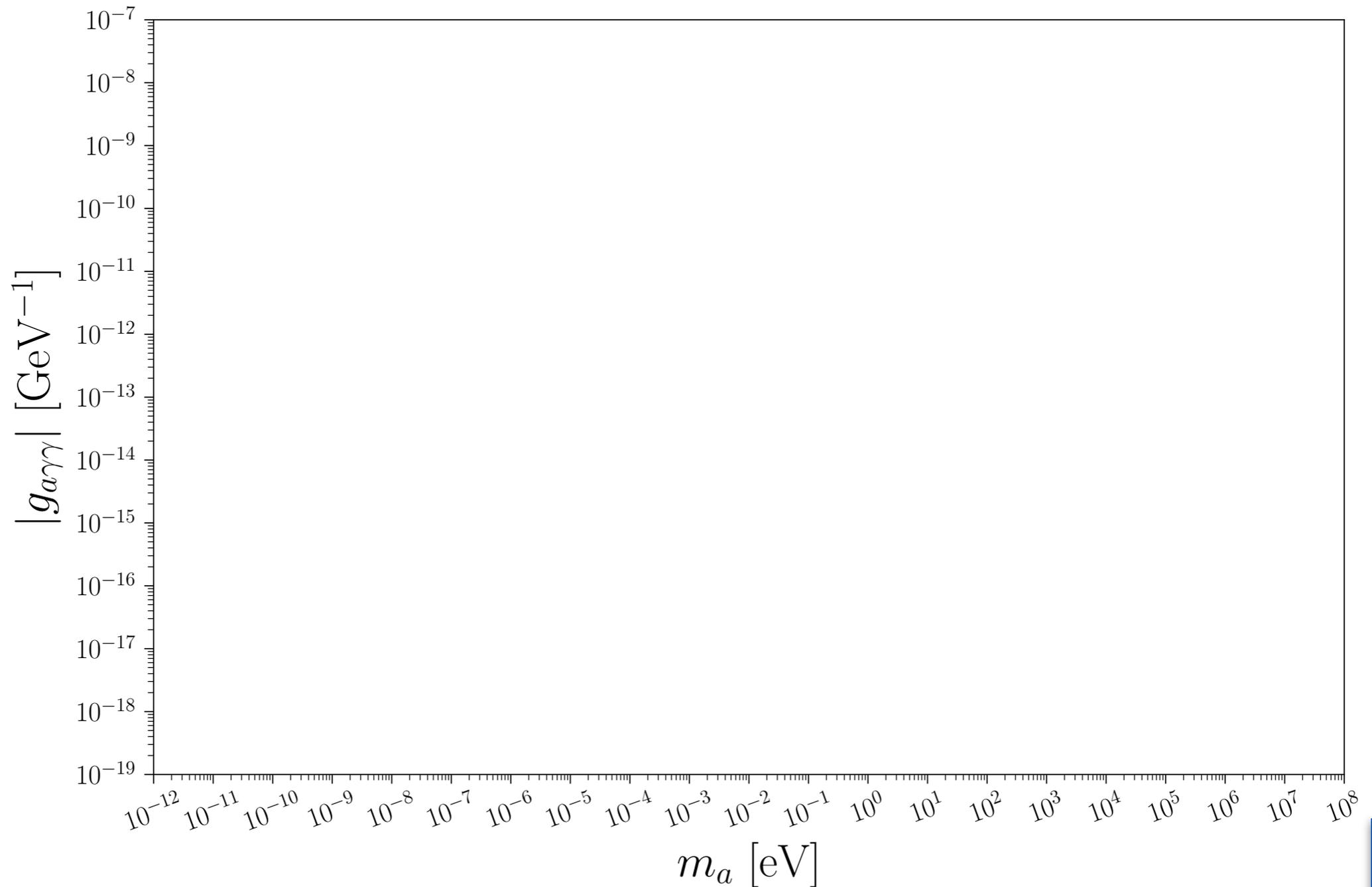
$$\mathcal{L} \supset -\frac{1}{4}g_{a\gamma\gamma}a(F\tilde{F})$$

Introduces a new source for E&M fields

$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \mathbf{J} + g_{a\gamma\gamma} \mathbf{B} \partial_t a$$

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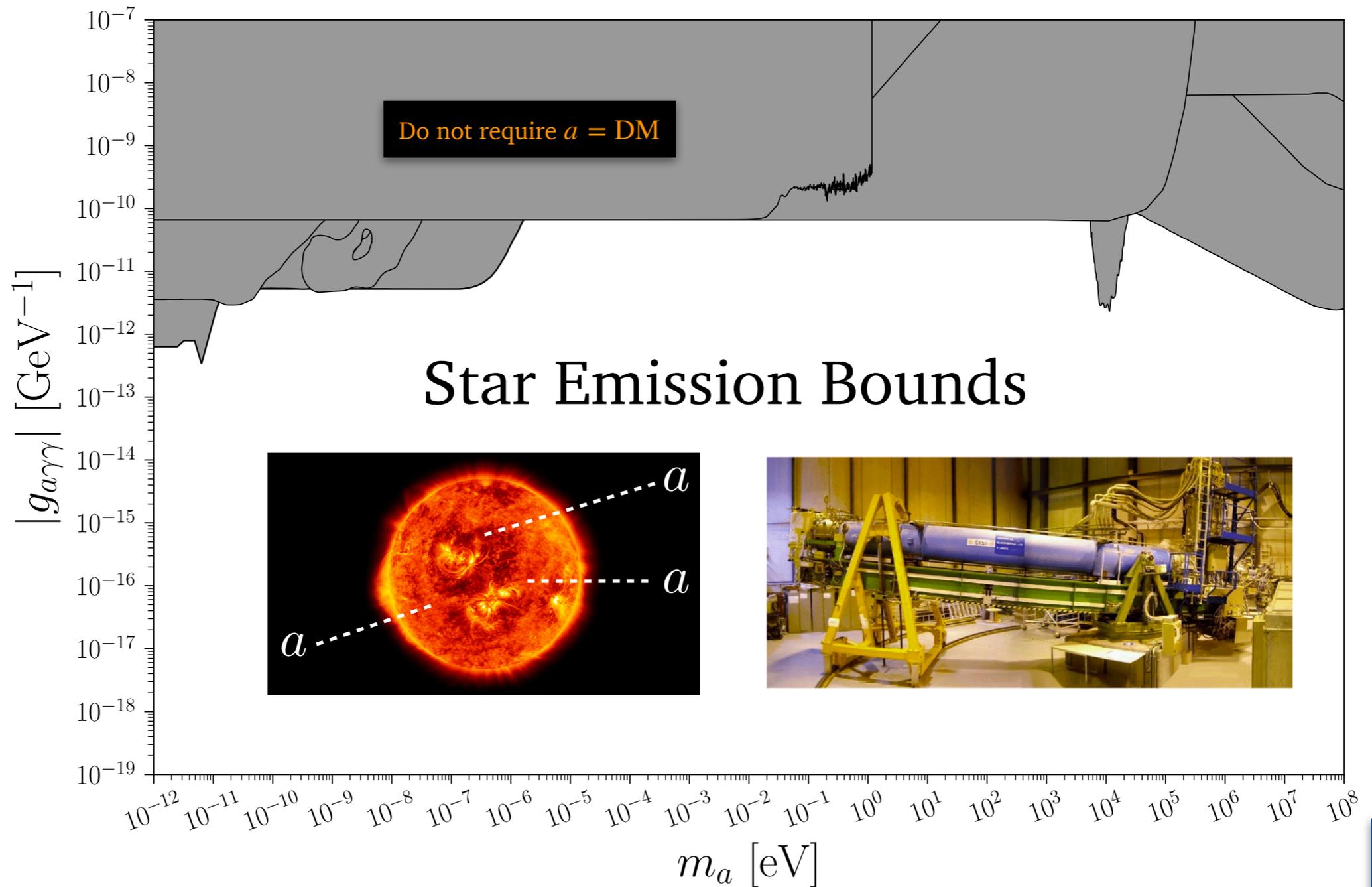


Partial summary  
[O'Hare github]



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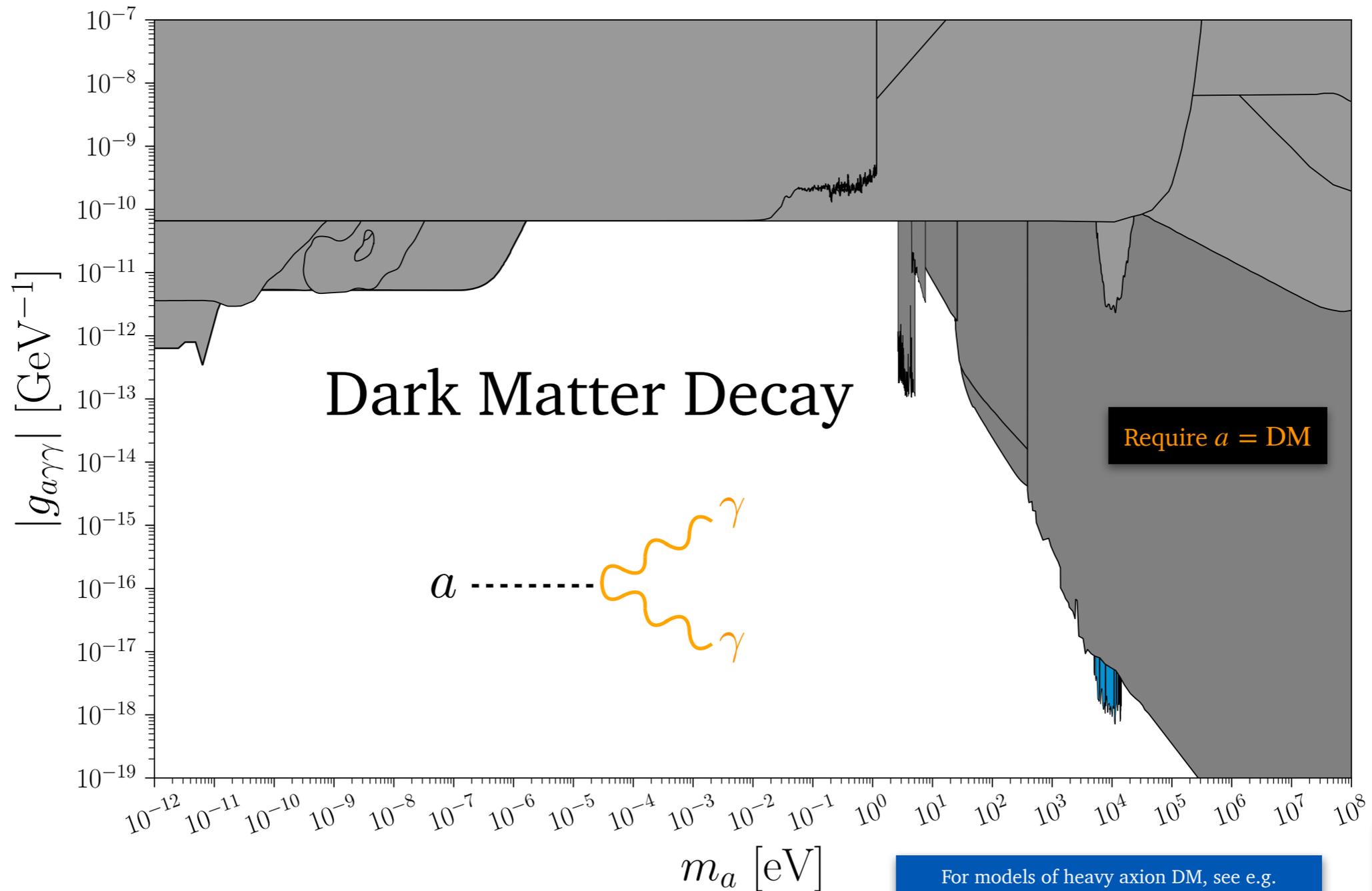
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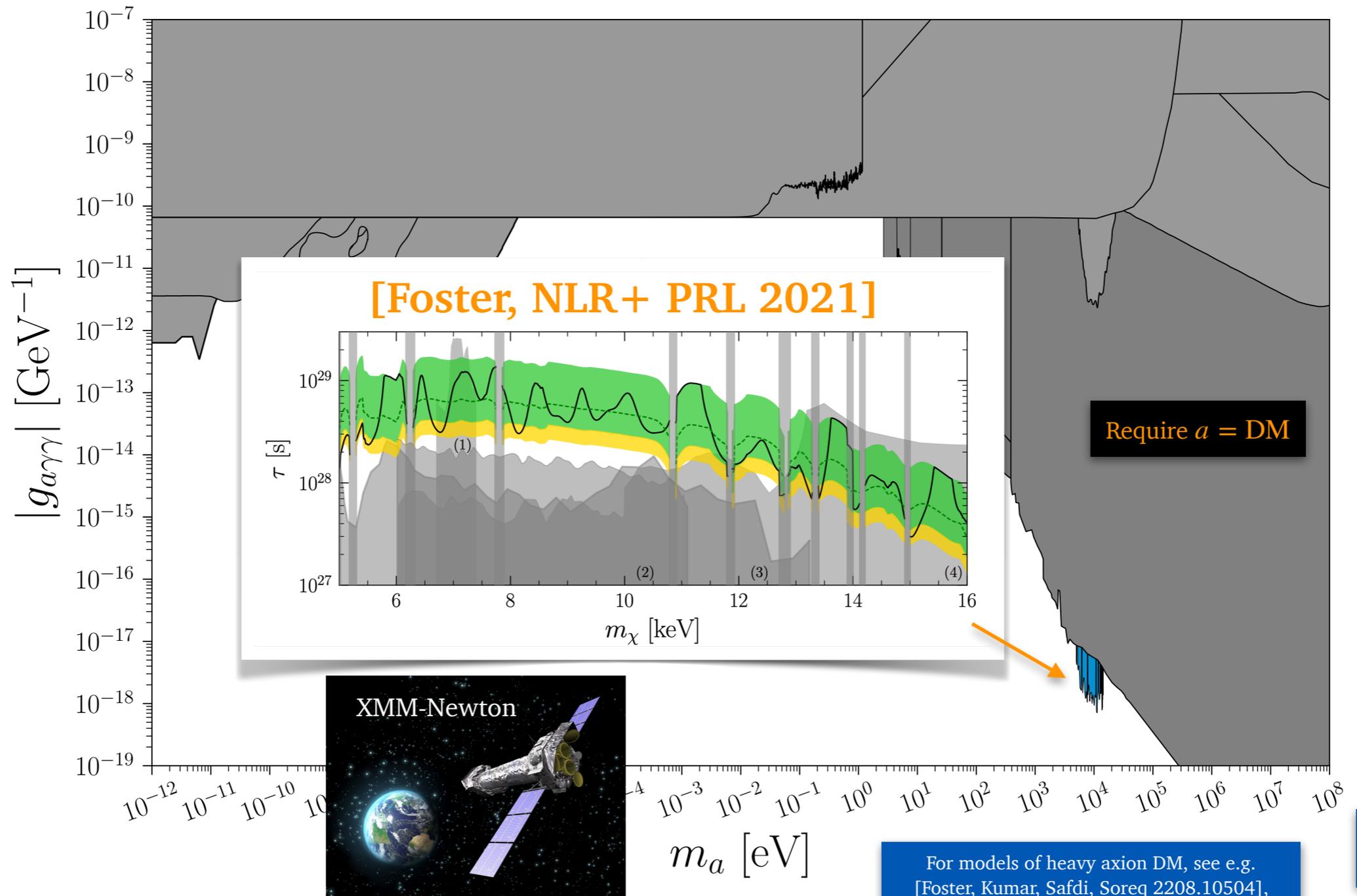
For models of heavy axion DM, see e.g.  
[Foster, Kumar, Safdi, Soreq 2208.10504],  
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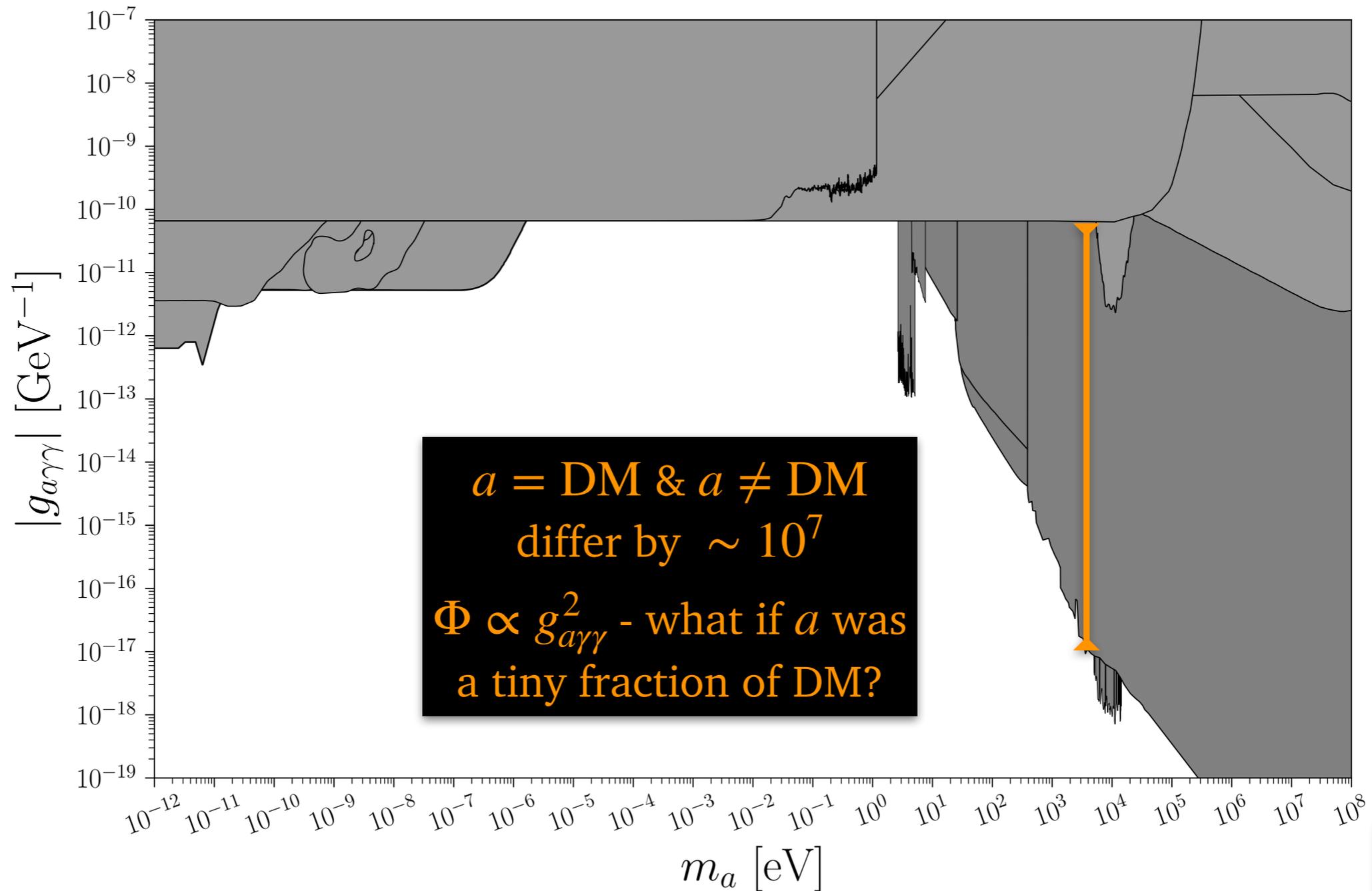
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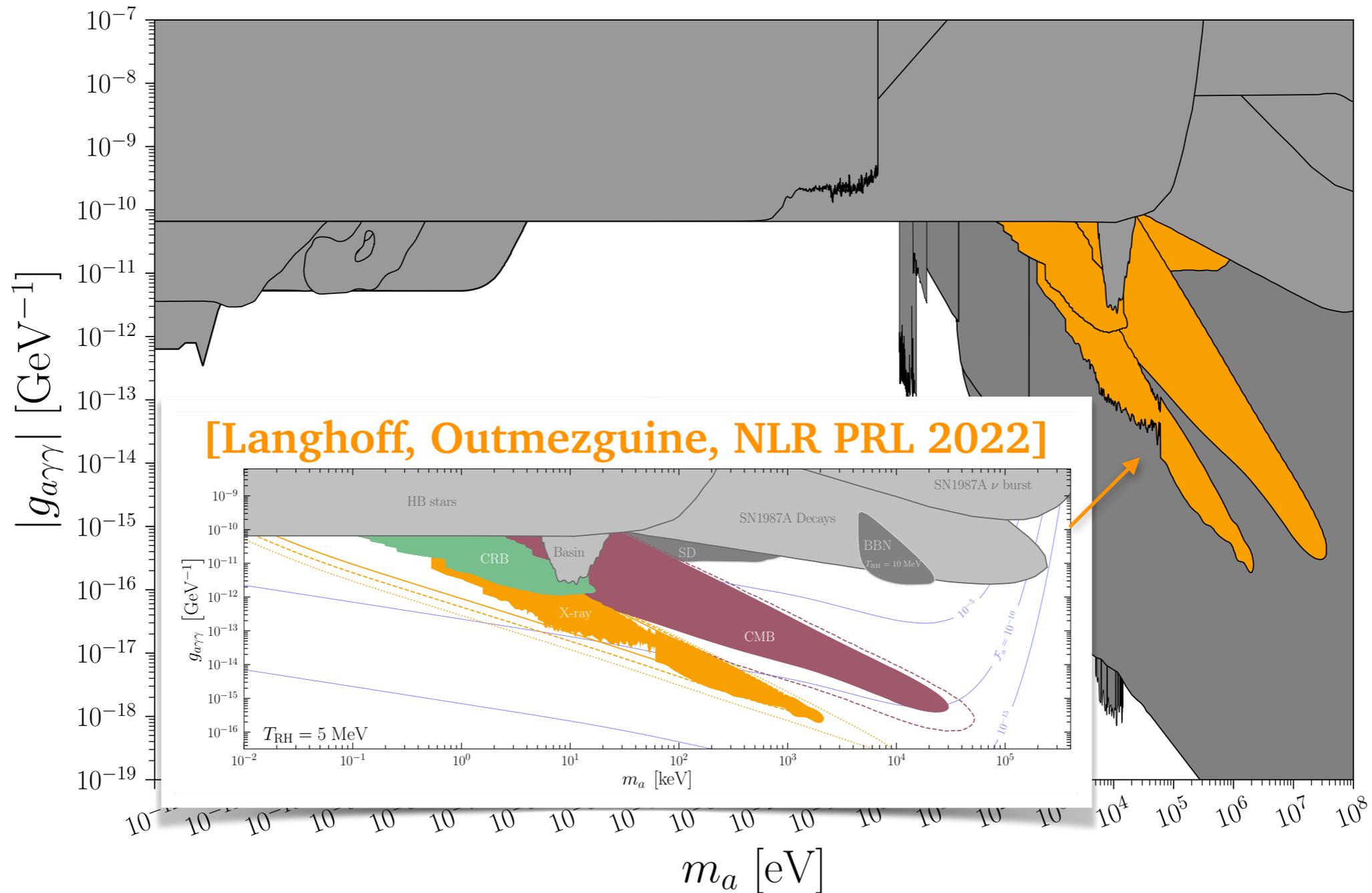
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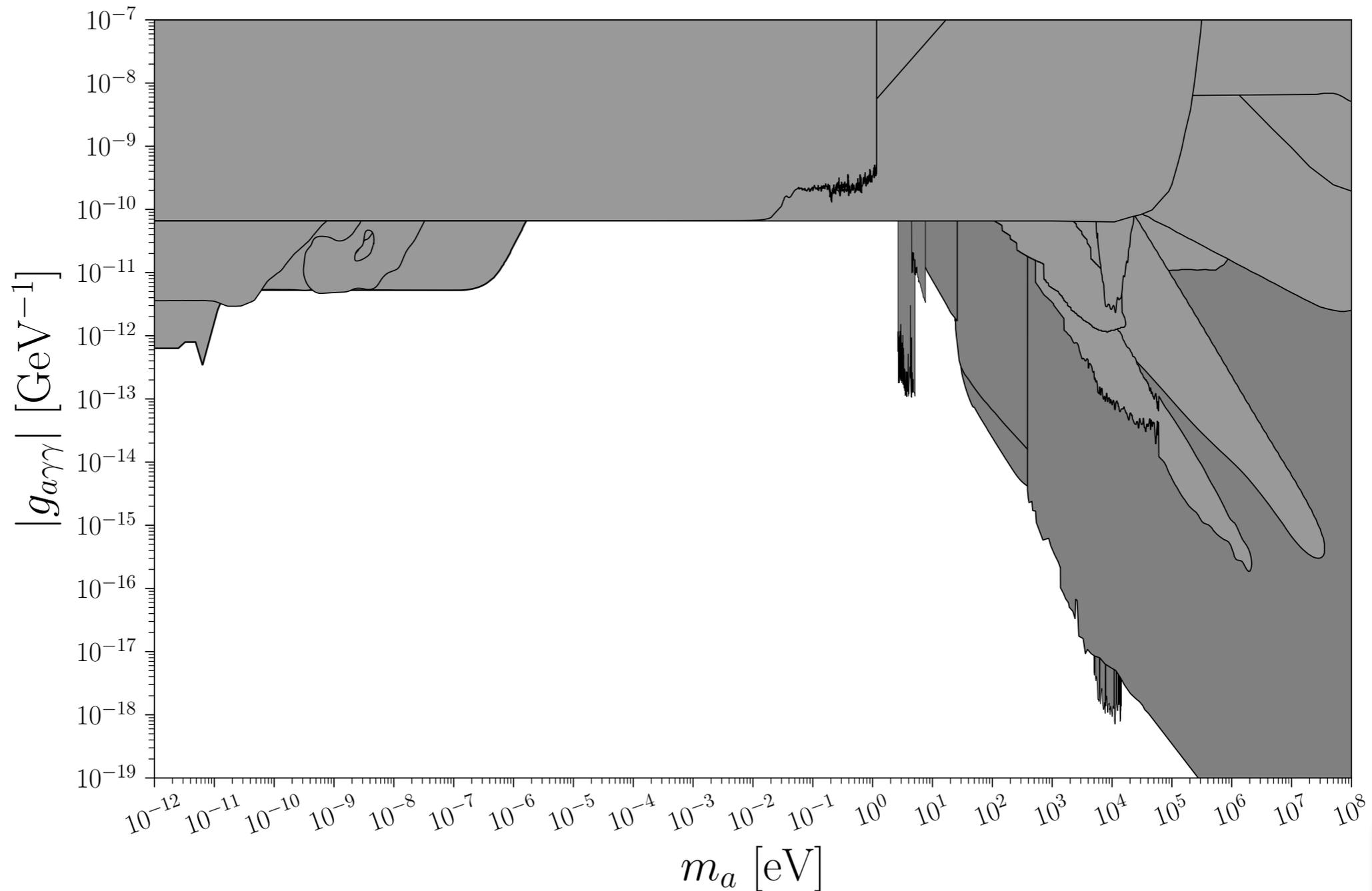
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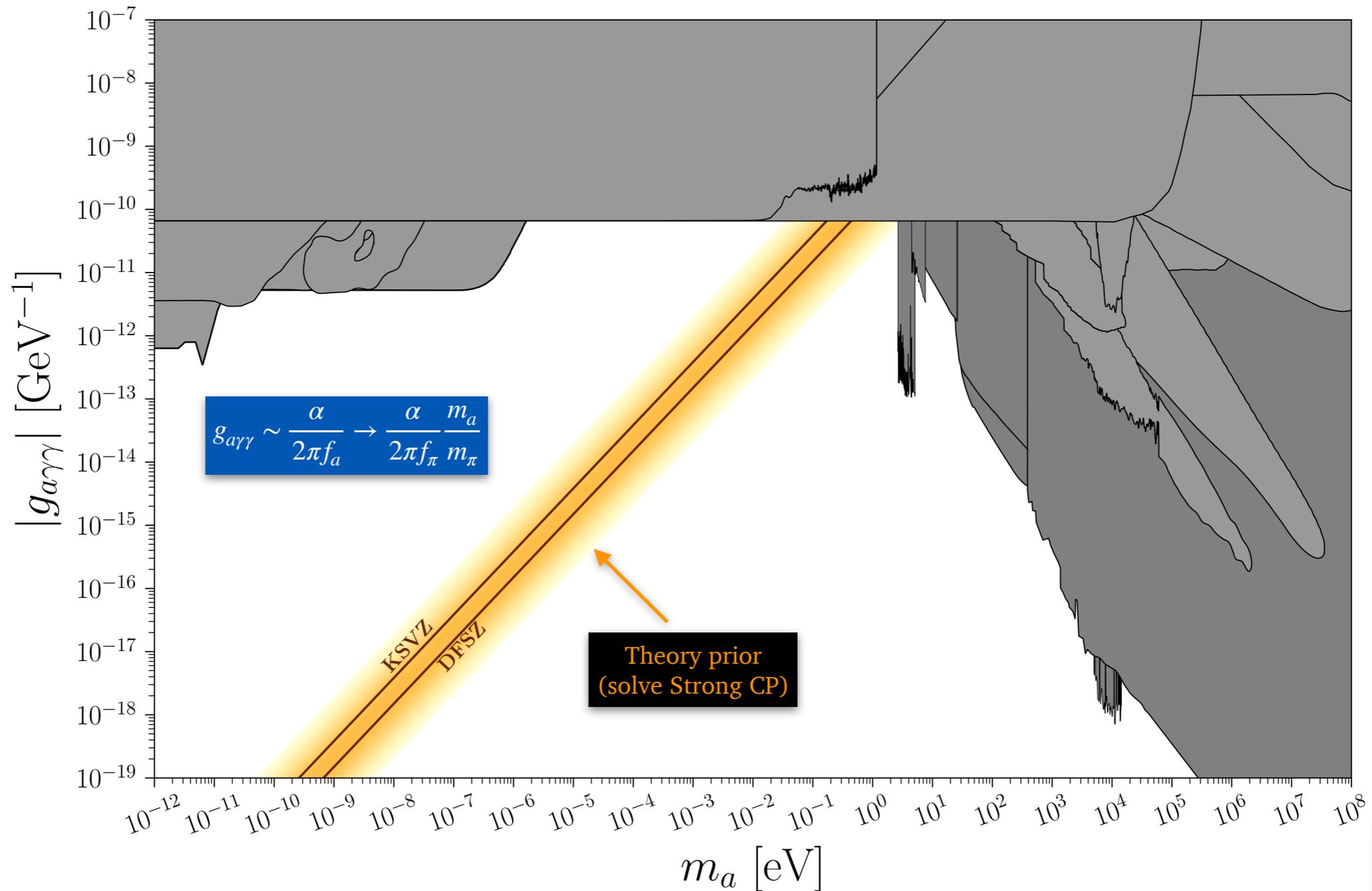
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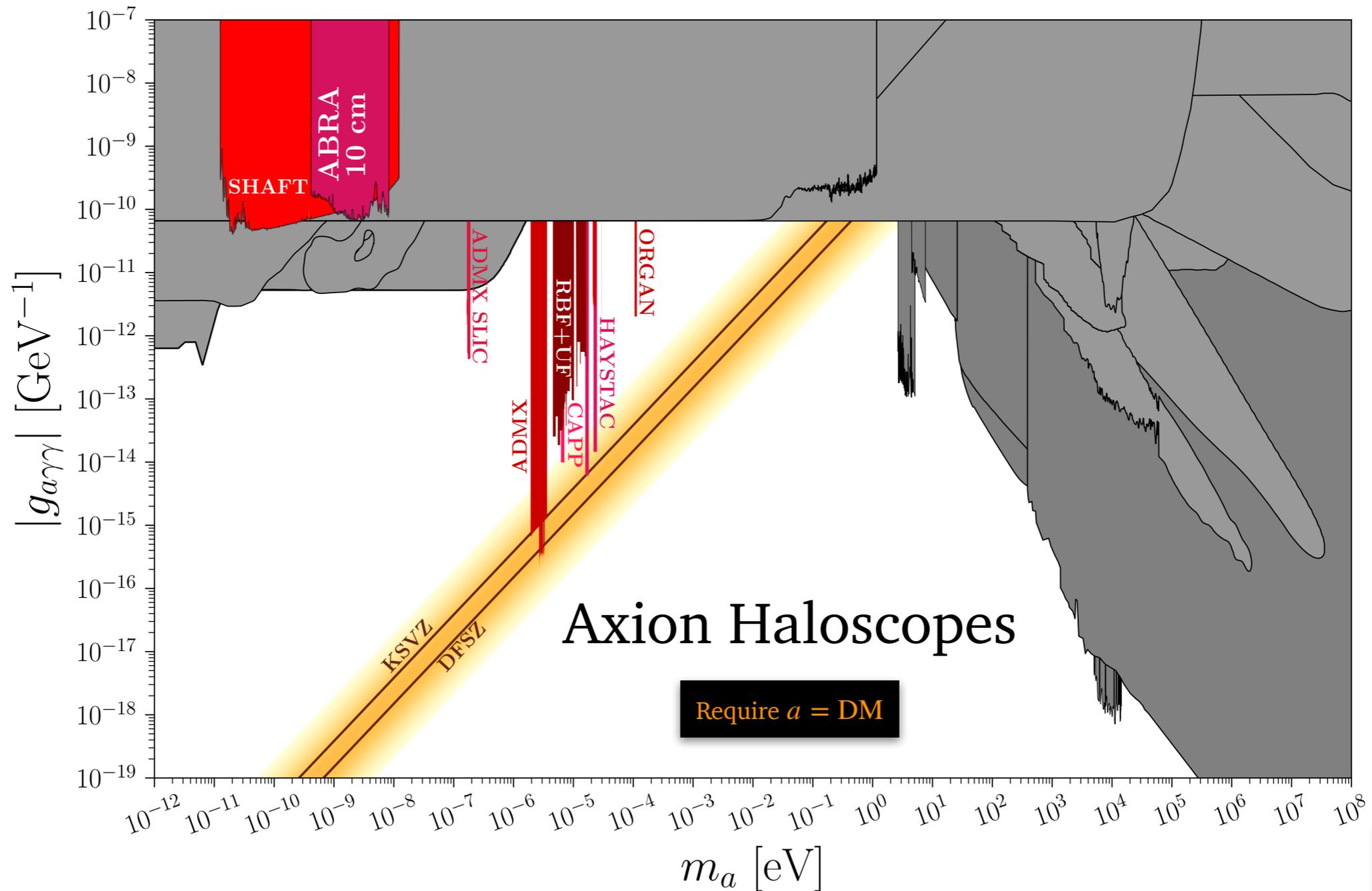
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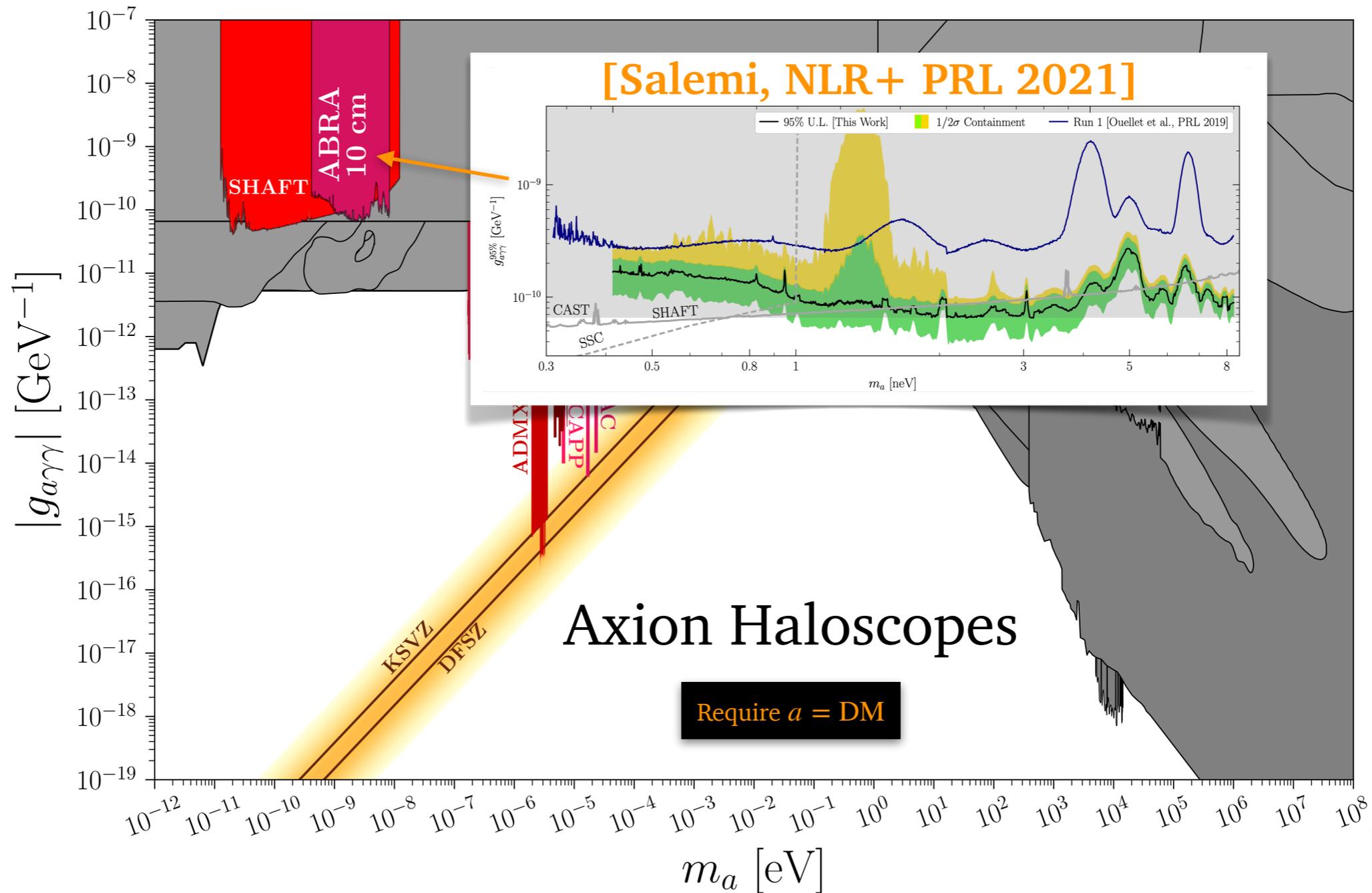
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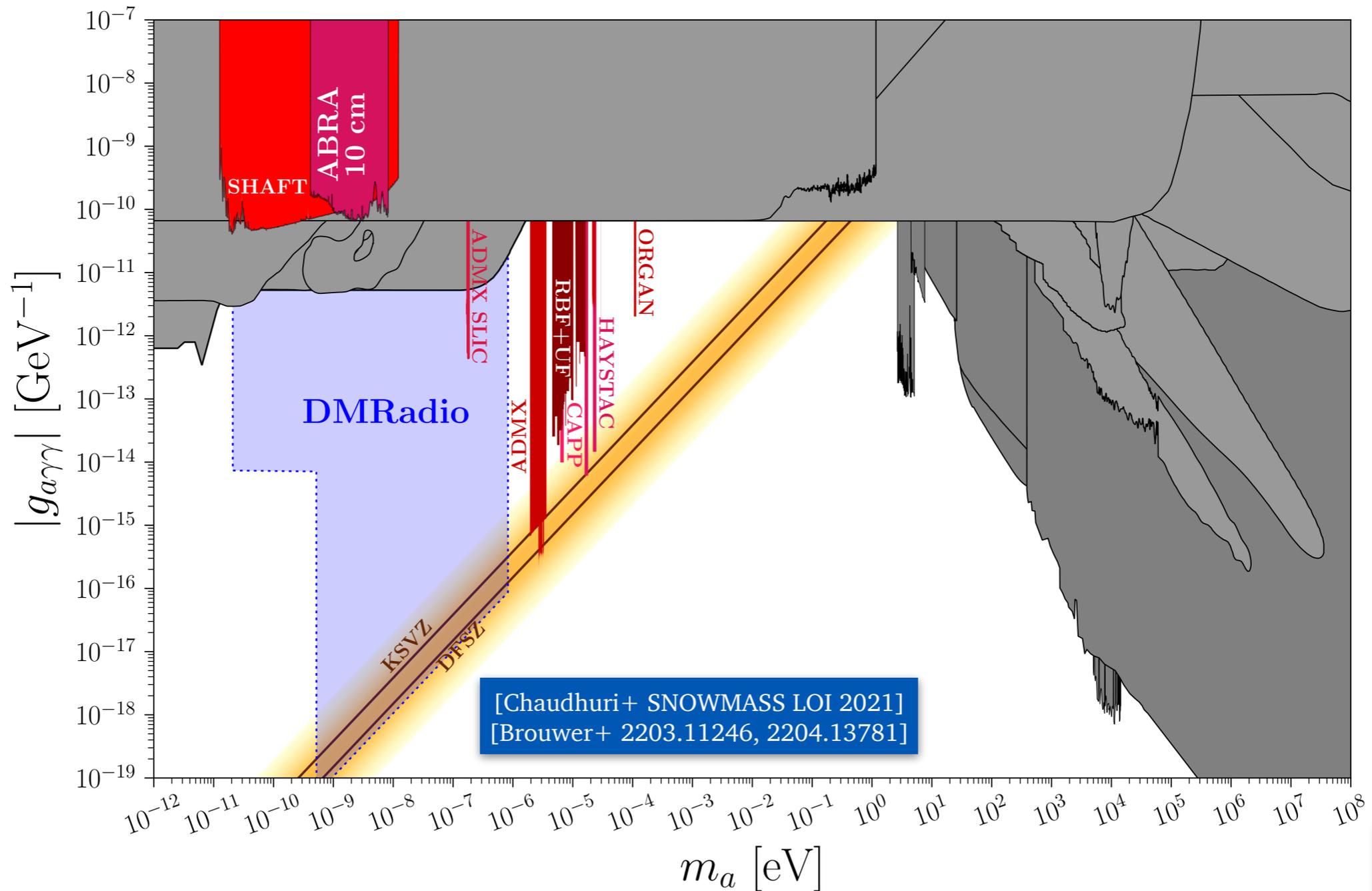
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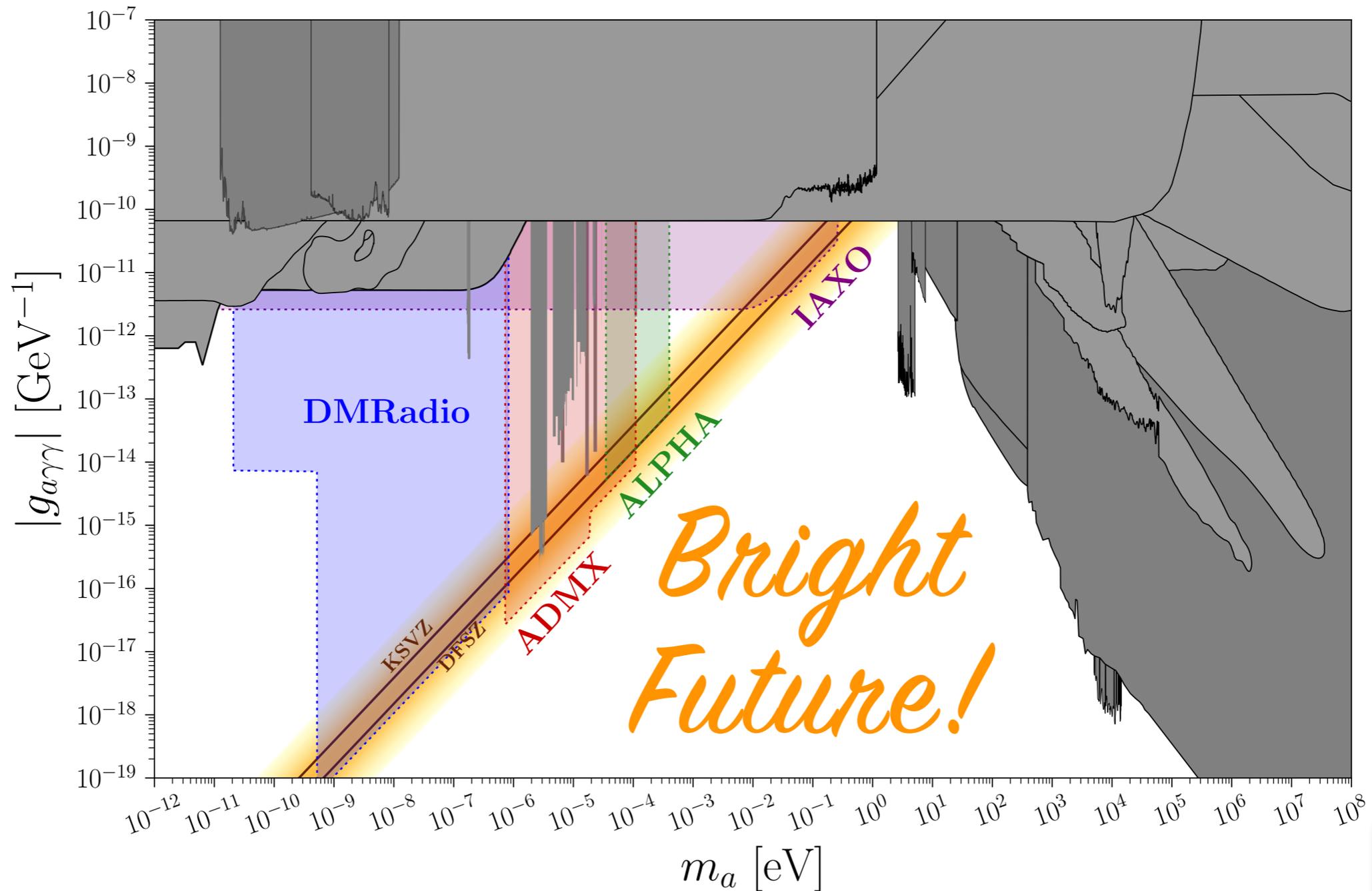
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# Outline

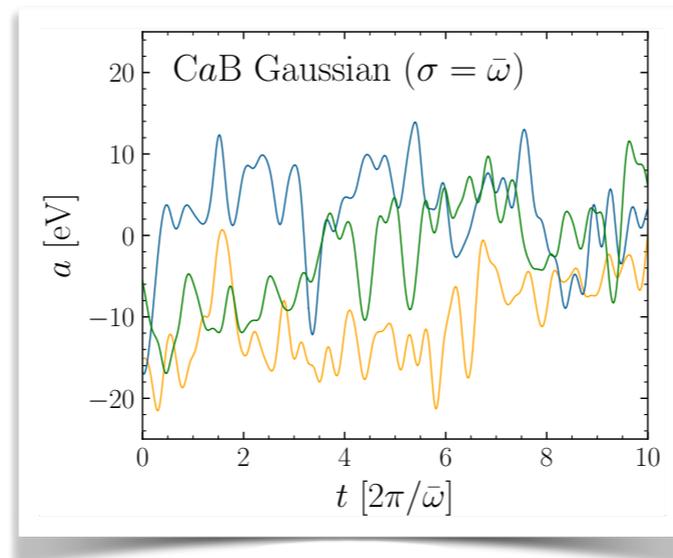
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What else might we see with these instruments?

# Outline

1. Could non-dark matter axions leave a signal?

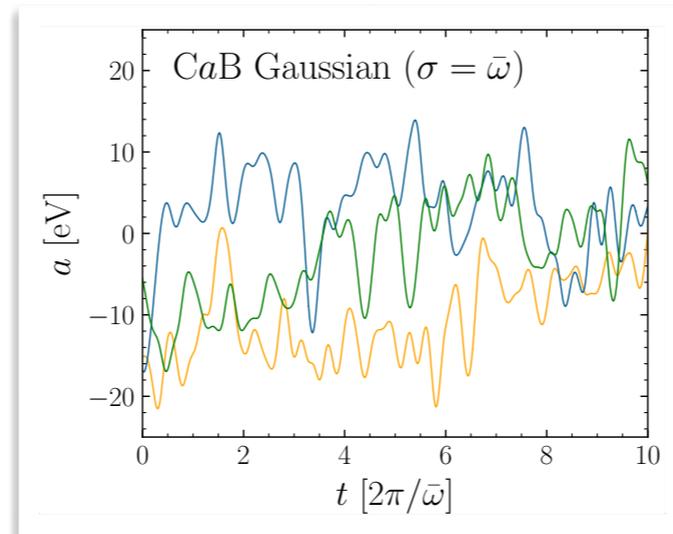
## The Cosmic Axion Background



# Outline

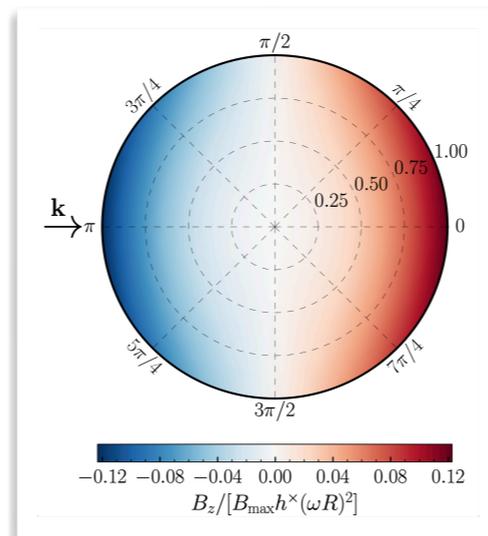
1. Could non-dark matter axions leave a signal?

## The Cosmic Axion Background



2. What other passing waves could a haloscope detect?

## High-Frequency Gravitational Waves



WARMUP

# How to Discover Dark Matter

# Axion Dark Matter

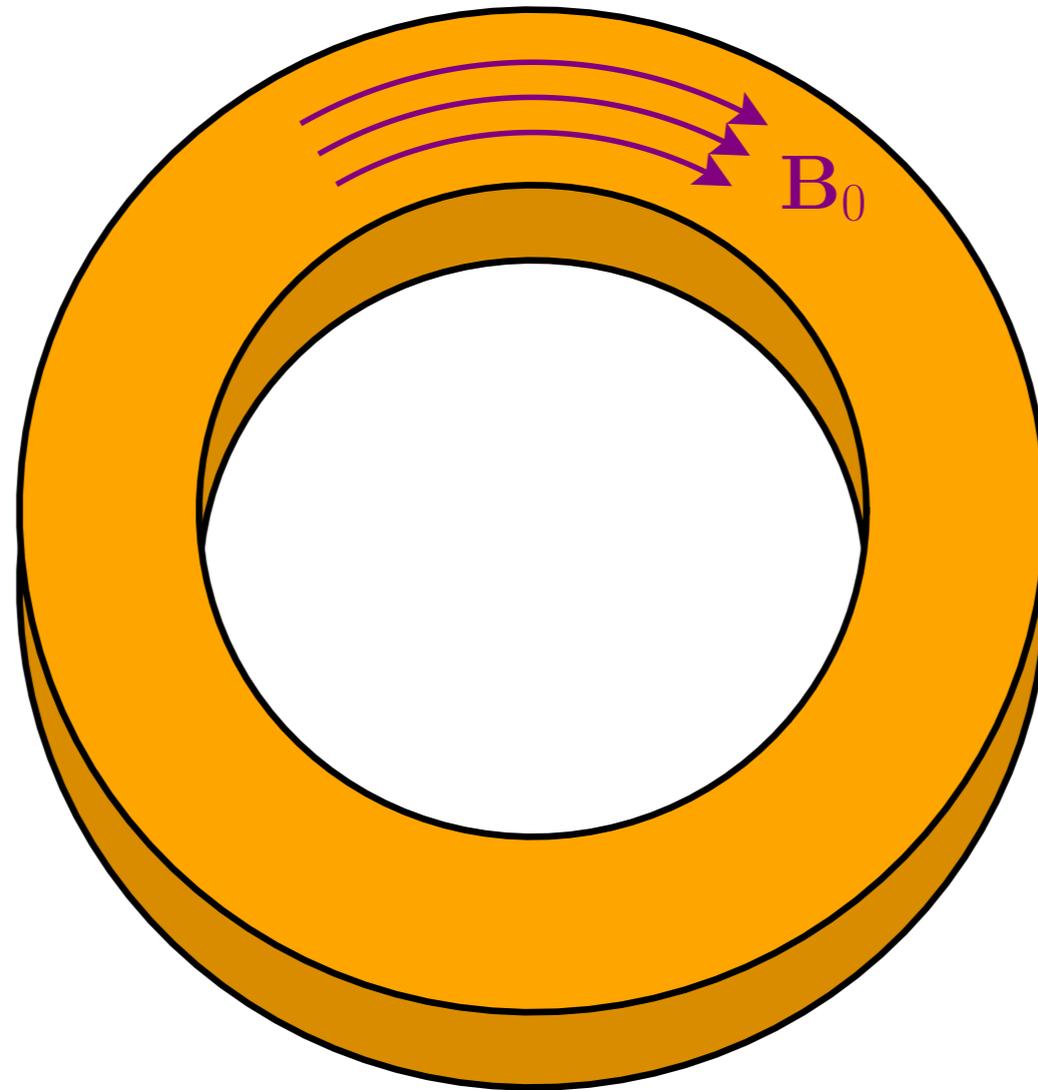
$$\mathcal{L} \supset -\frac{1}{4}g_{a\gamma\gamma}a(F\tilde{F})$$

Introduces a new source for E&M fields

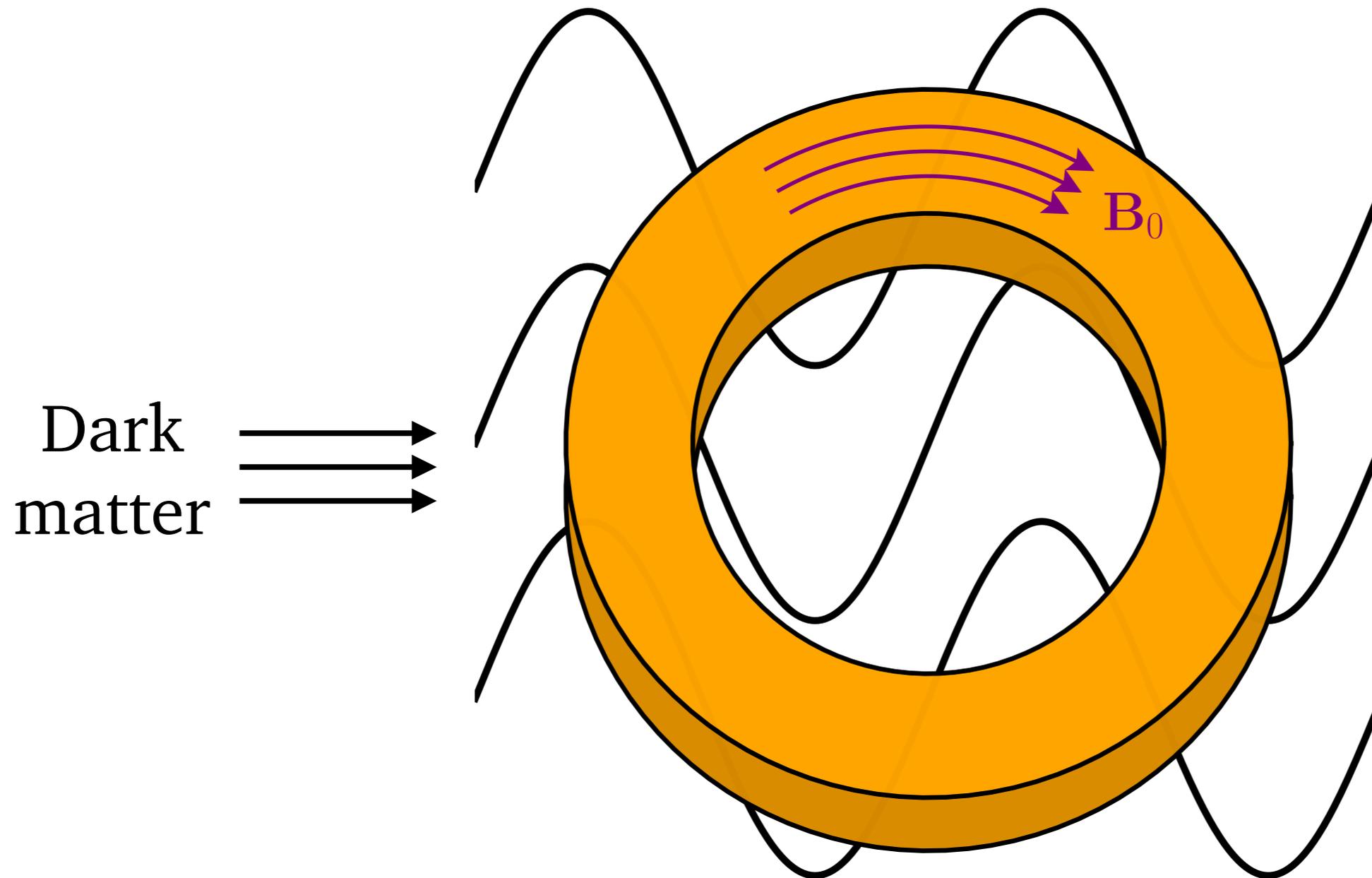
$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \mathbf{J} + g_{a\gamma\gamma} \mathbf{B} \partial_t a$$

Not shown:  $\rho_{\text{eff}} = -g_{a\gamma\gamma} \mathbf{B} \cdot \nabla a$   
and  $\mathbf{J}_{\text{eff}} = -g_{a\gamma\gamma} \mathbf{E} \times \nabla a$

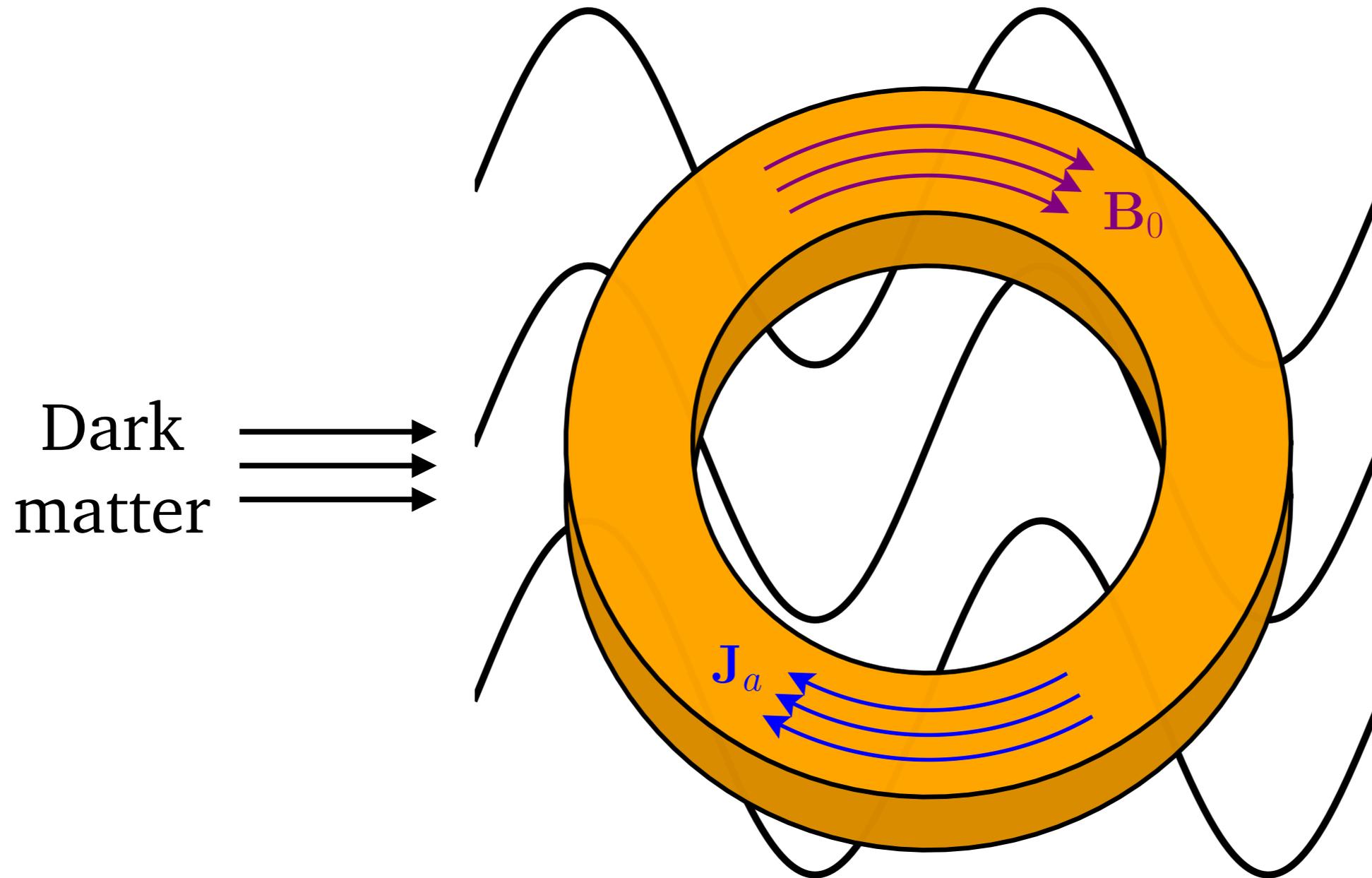
# Detection with a Toroidal Magnet



# Detection with a Toroidal Magnet

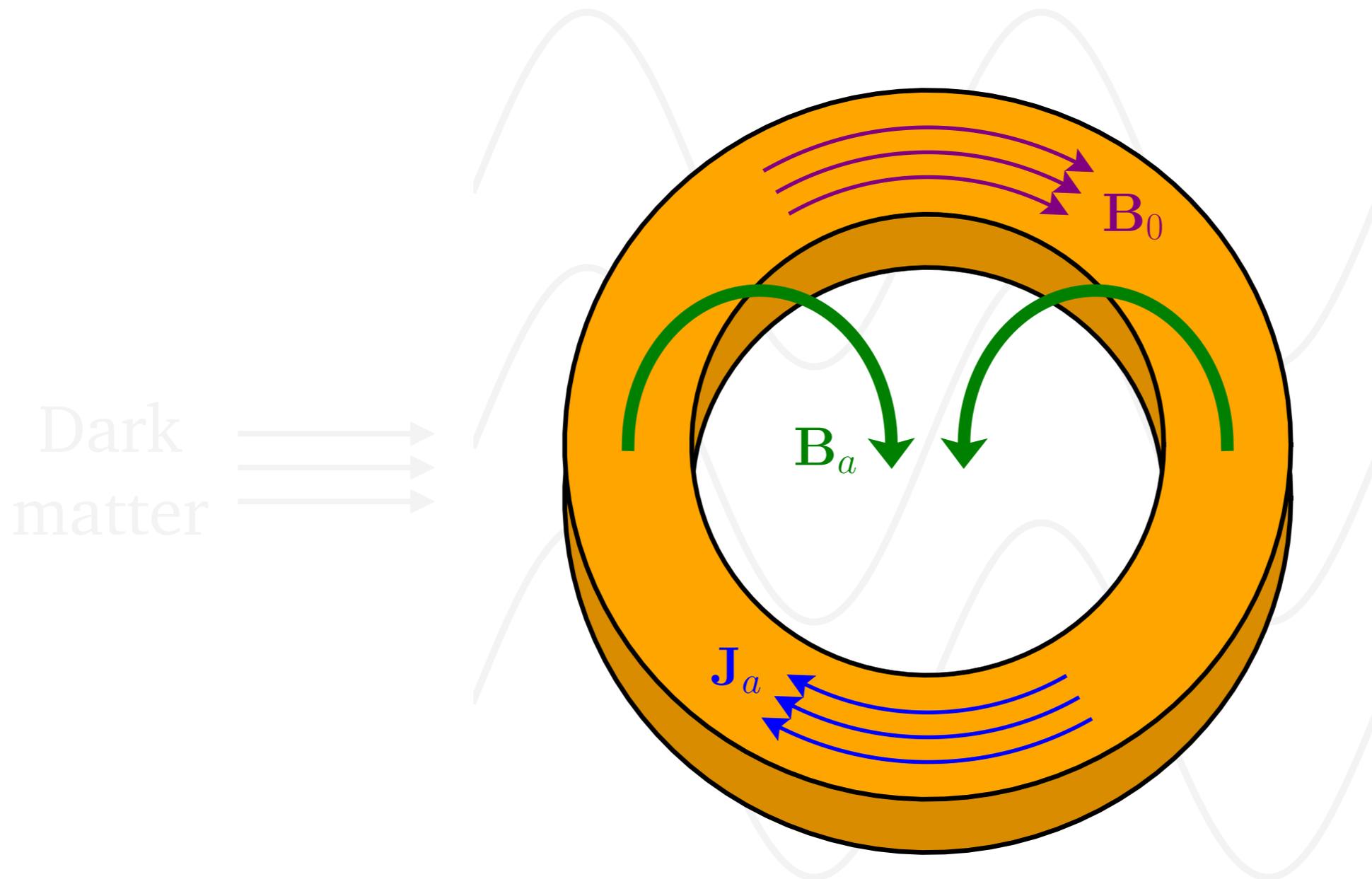


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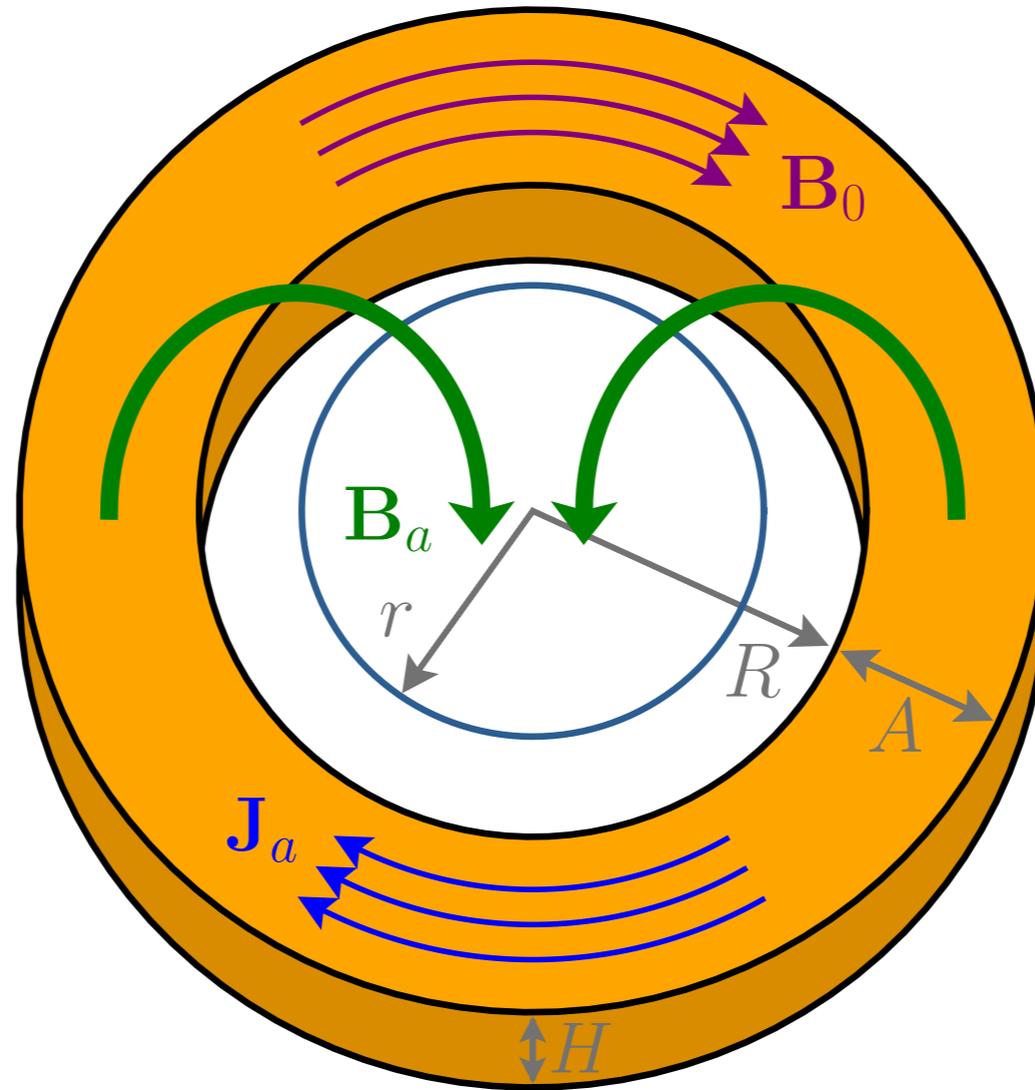
$$\mathbf{J}_a = g_{a\gamma\gamma} (\partial_t a) \mathbf{B}_0$$

# Detection with a Toroidal Magnet



$$\mathbf{J}_a = g_{a\gamma\gamma} (\partial_t a) \mathbf{B}_0$$

# Detection with a Toroidal Magnet



$$\Phi_a(t) \simeq g_{a\gamma\gamma}(\partial_t a) B_0 \pi r^2 R \ln(1 + A/R) \sim g_{a\gamma\gamma}(\partial_t a) B_0 V$$

$$H \gg R + A$$

# Cosmic Axion Background

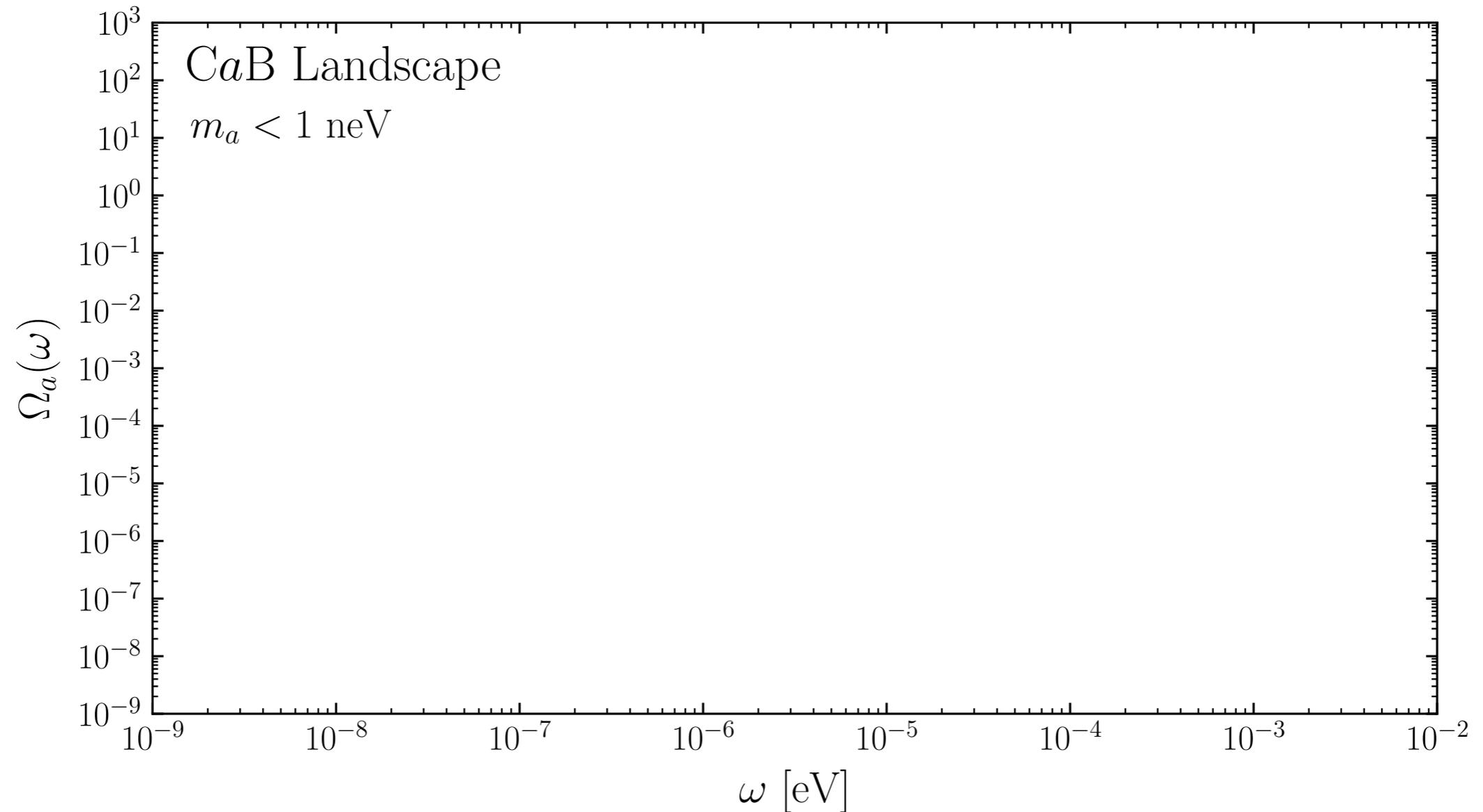
[Dror, Murayama, NLR PRD (Editors' Suggestion) 2021]

# The Cosmic Axion Background

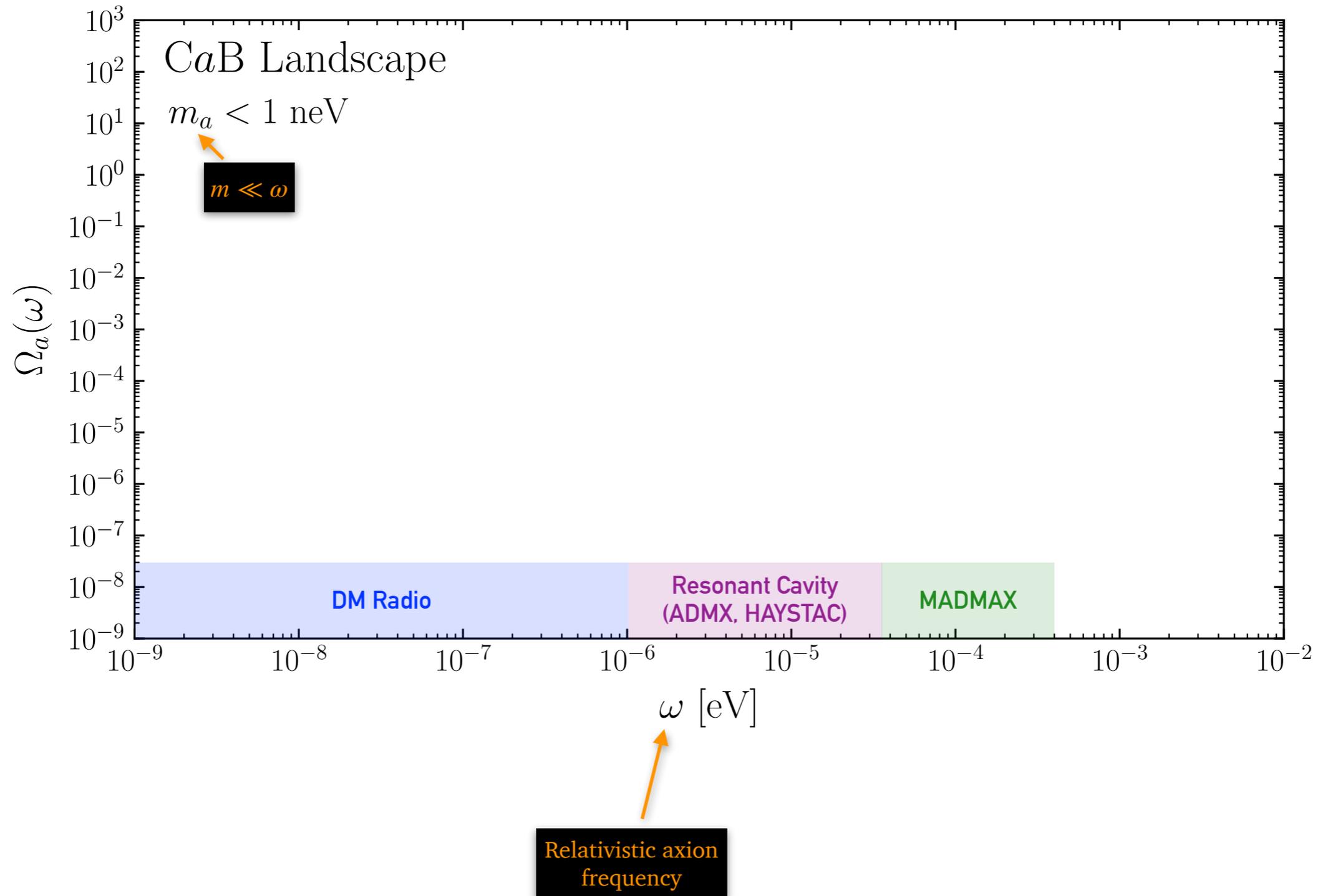
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Can we detect relativistic axions that  
are a relic of the early Universe?

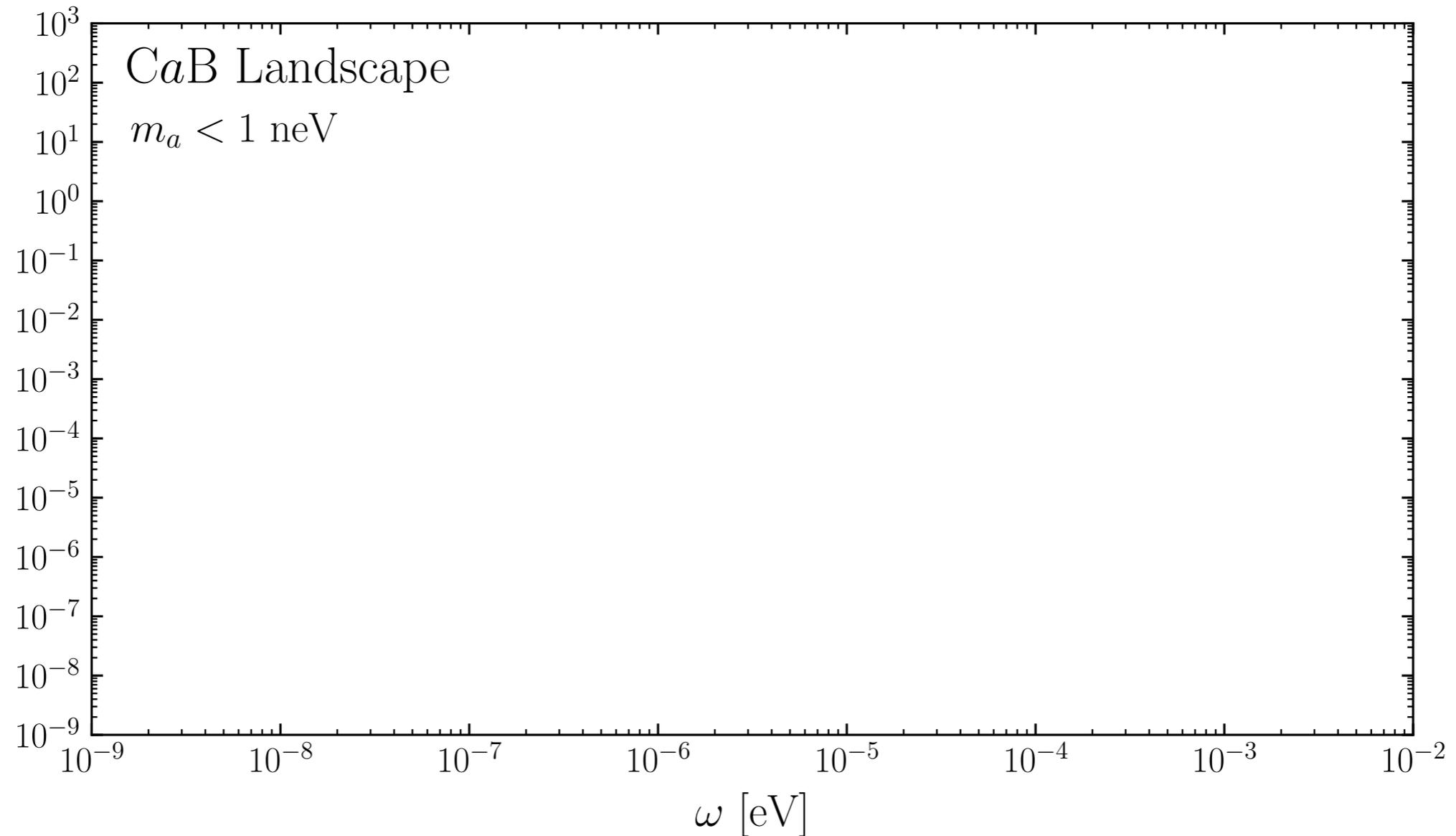
# Landscape



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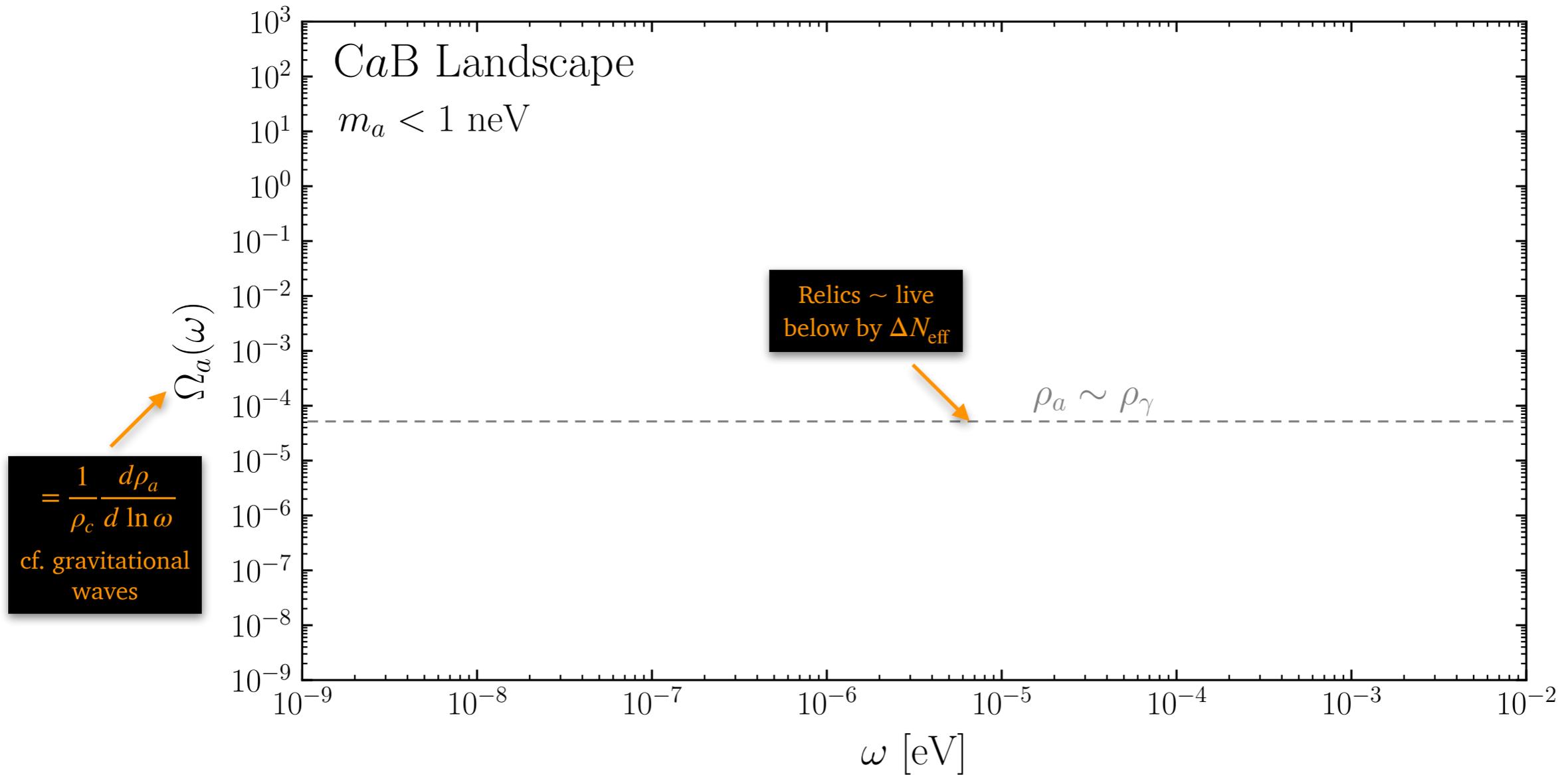


$$= \frac{1}{\rho_c} \frac{d\rho_a}{d \ln \omega}$$

cf. gravitational waves

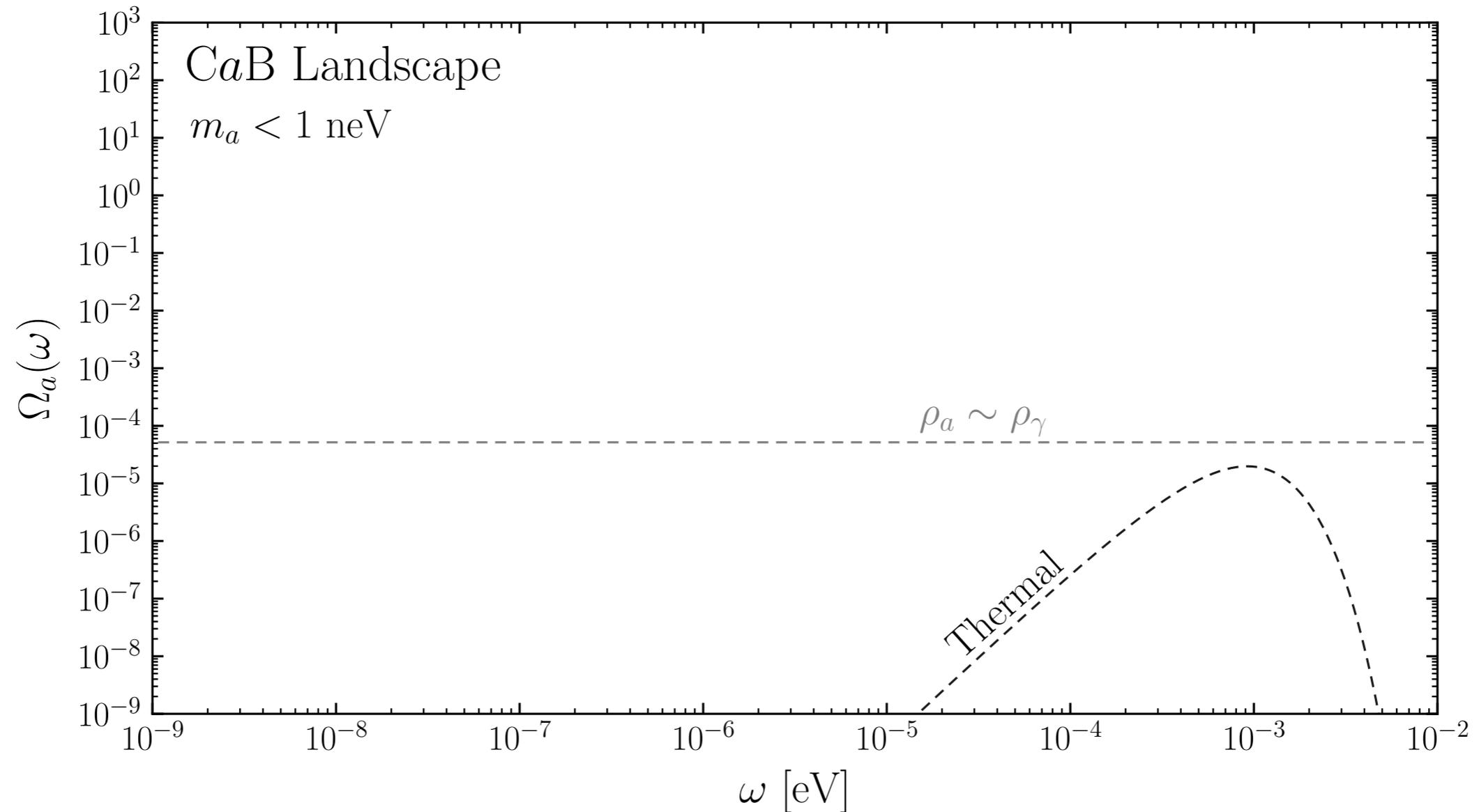
Where is  $g_{a\gamma\gamma}$ ?  
 Coming in a few slides

# Landscape

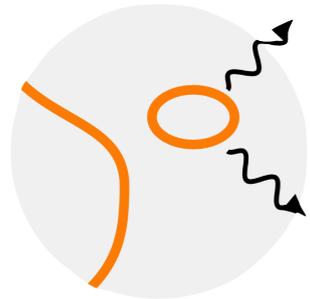
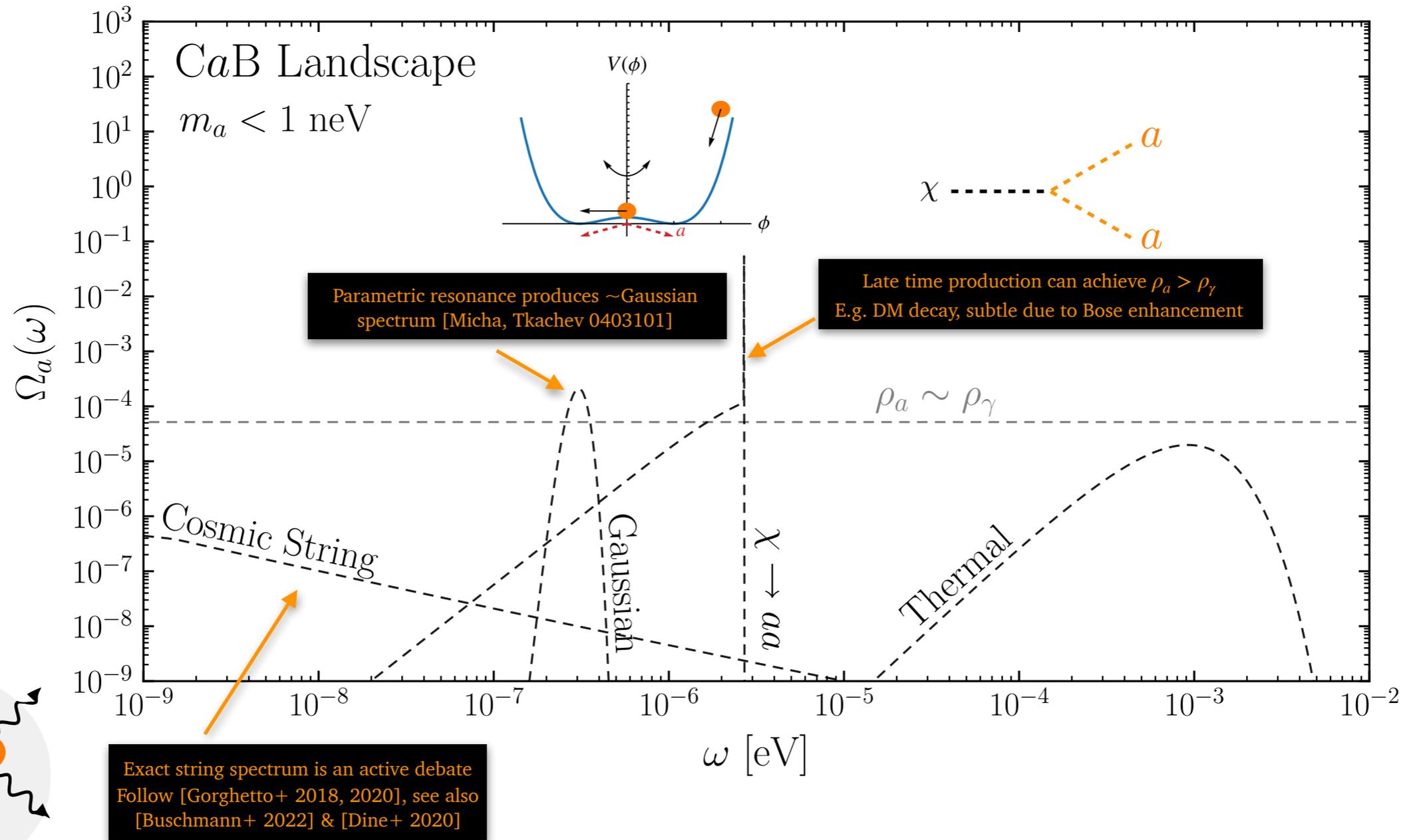


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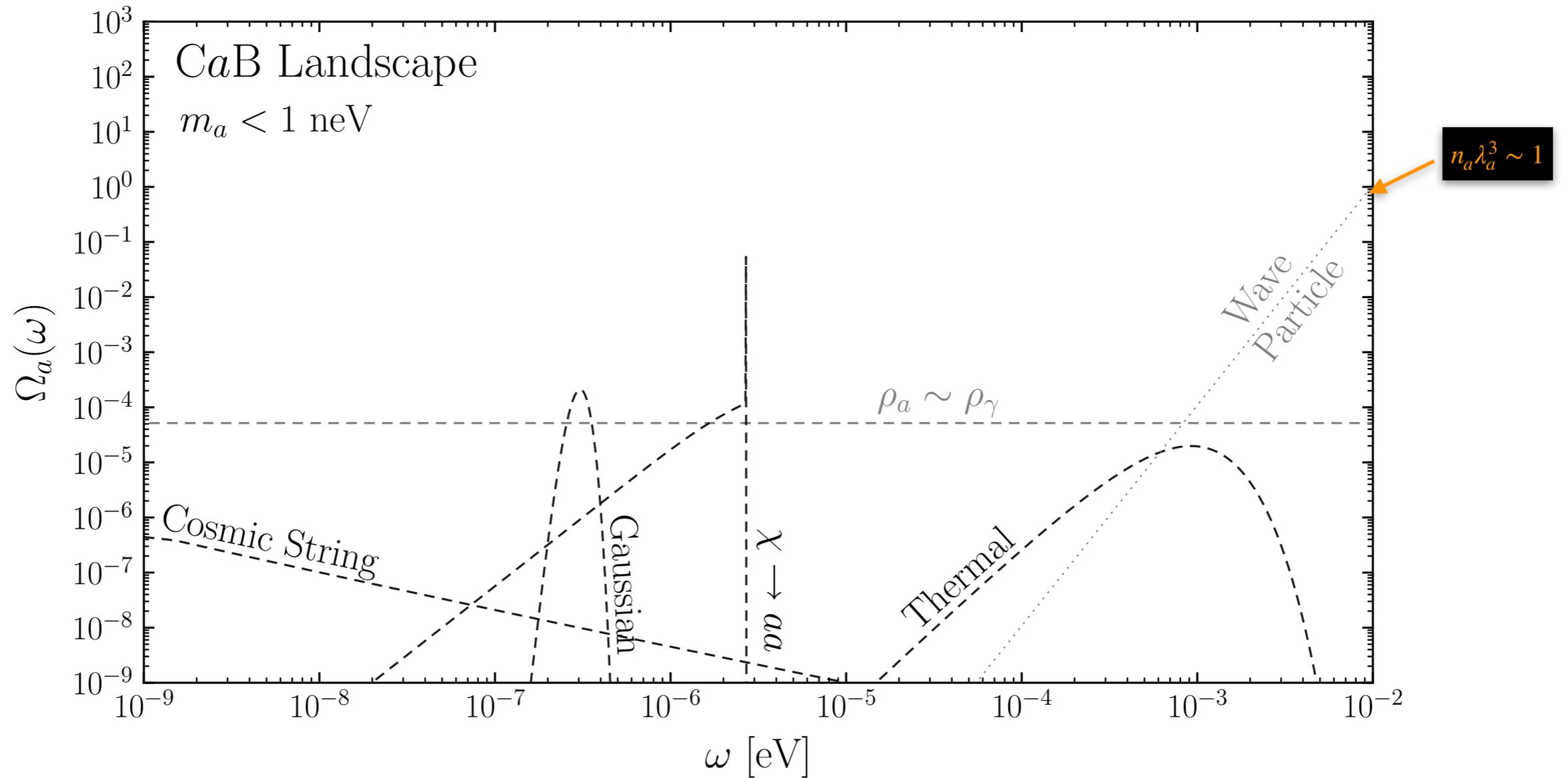
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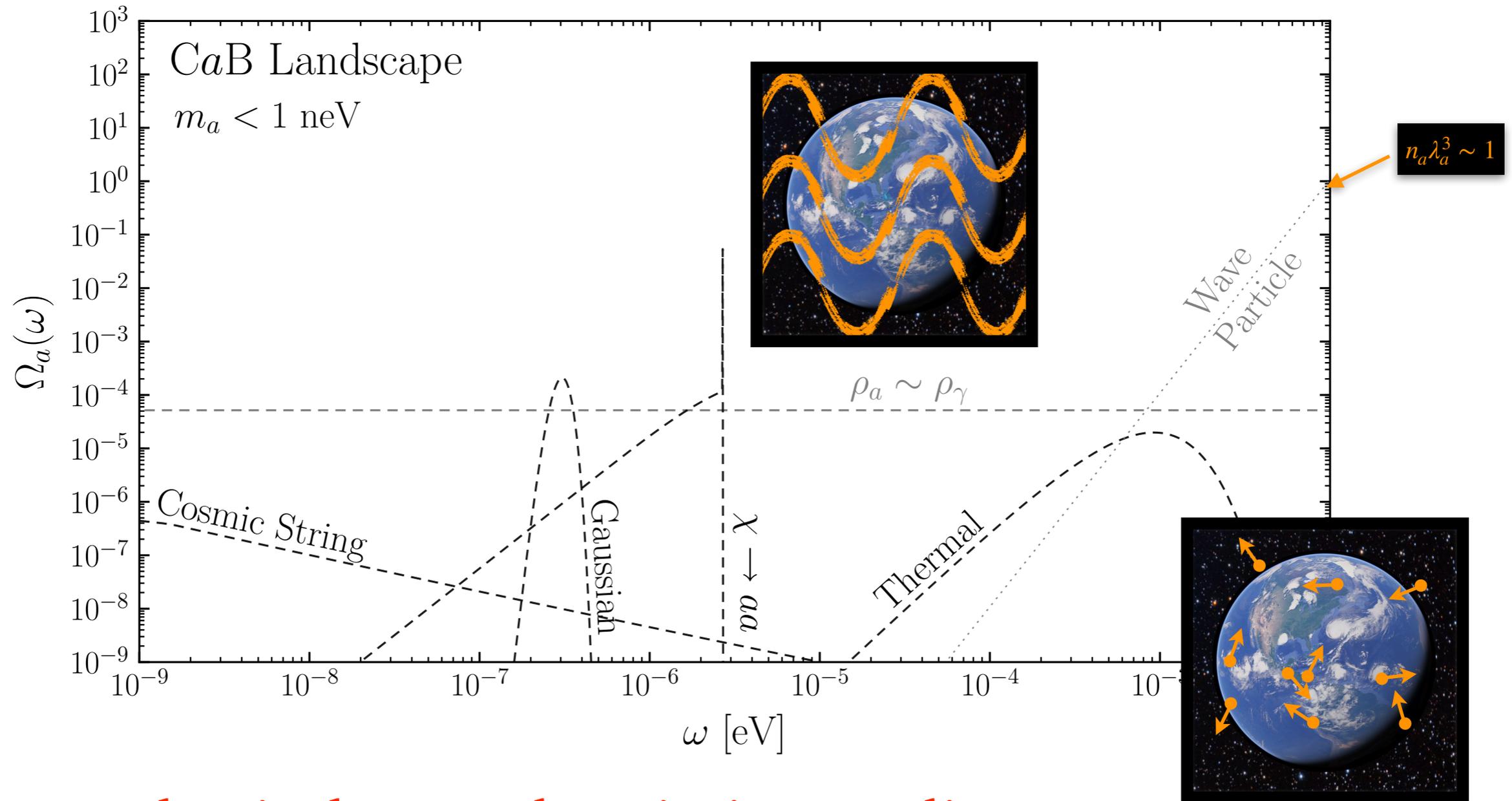
# Landscape



# Landscape



# Landscape



Classical wave description applies

# Rough Sensitivity

$$a(t) \sim \sum_i \cos(\omega_i t + \phi_i)$$

Sampled from  $p(\omega)$ , the pdf of frequency/energy

Random phase

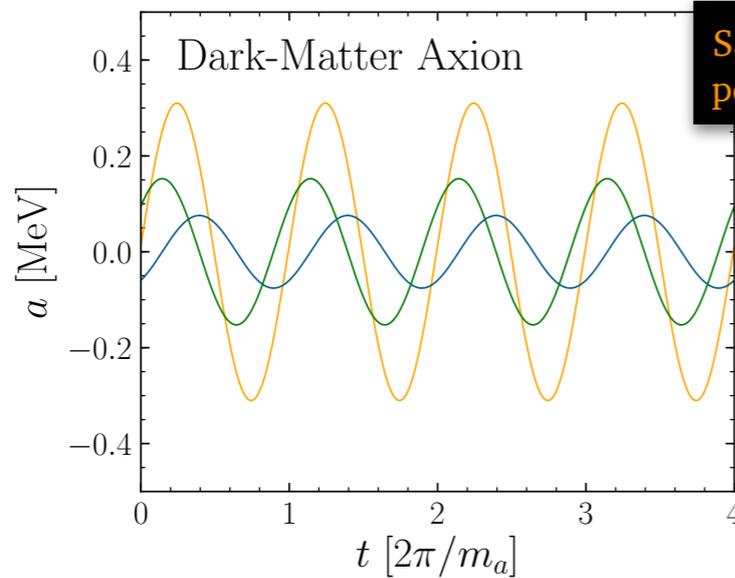
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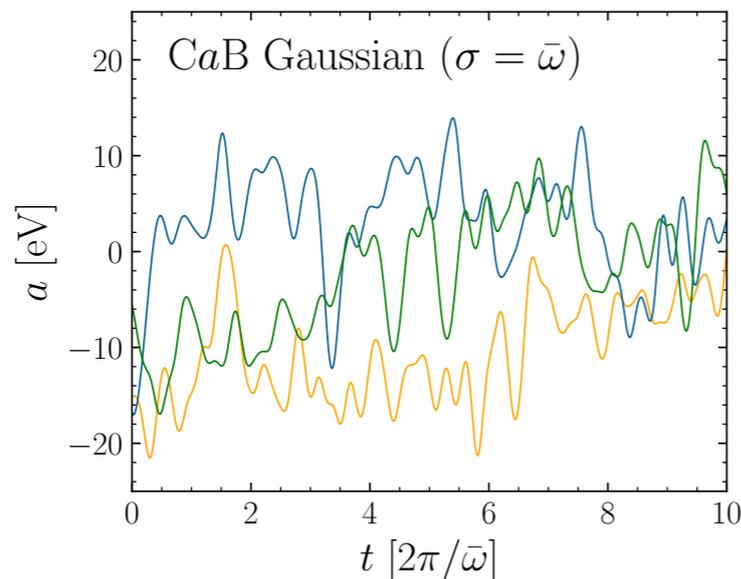
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Dark Matter



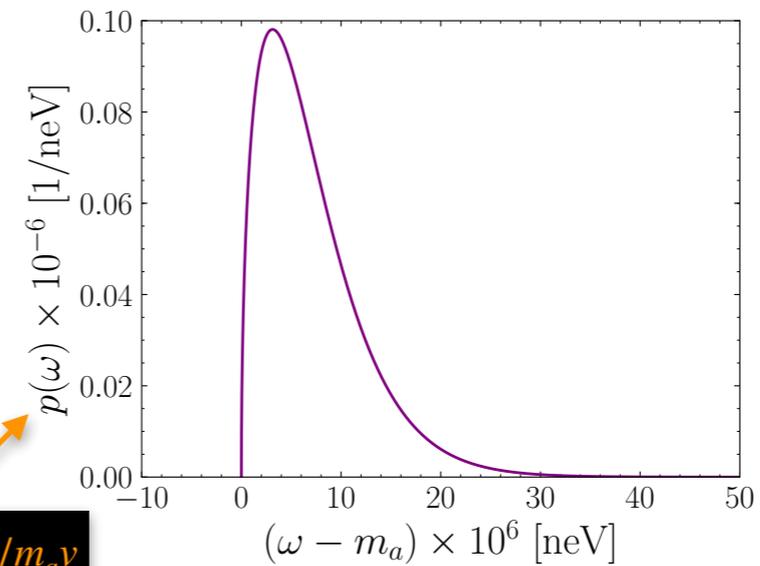
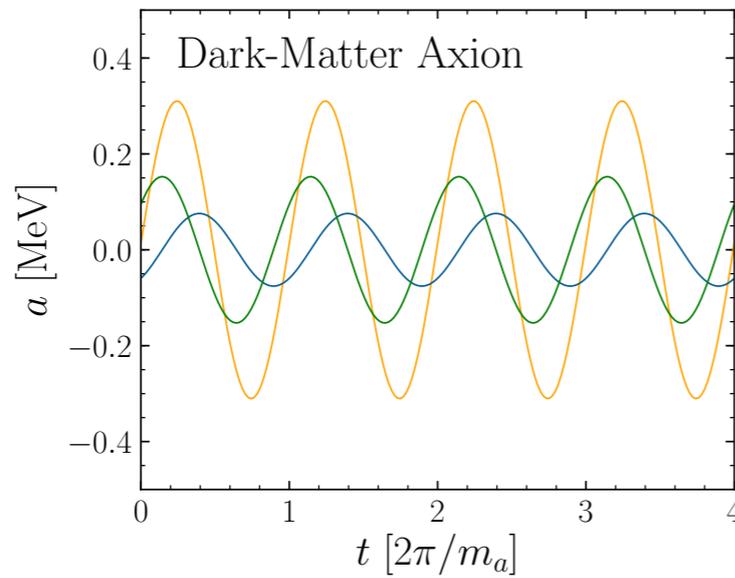
CaB



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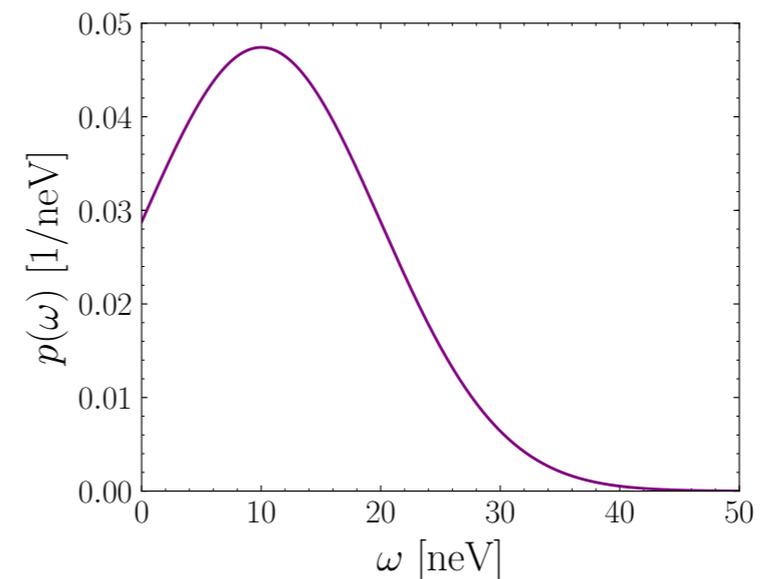
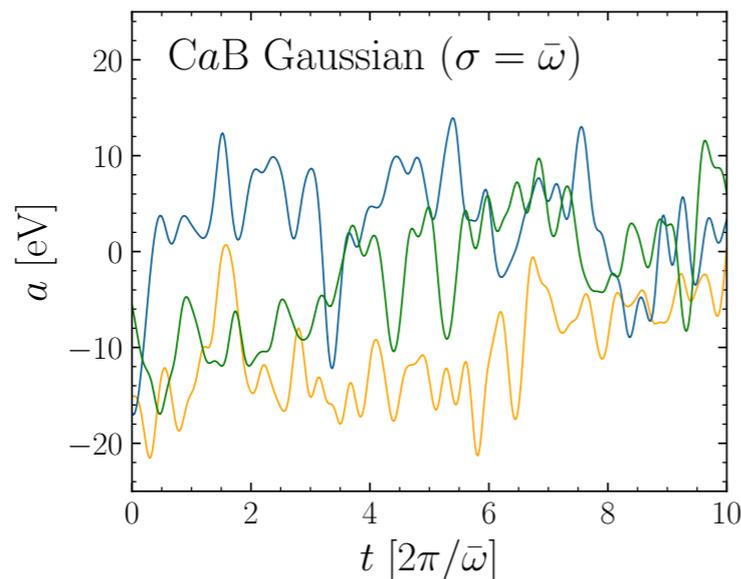
$$a(t) \sim \sum_i \cos(\omega_i t + \phi_i)$$

Dark Matter



$p(\omega) = f(v)/m_a v$

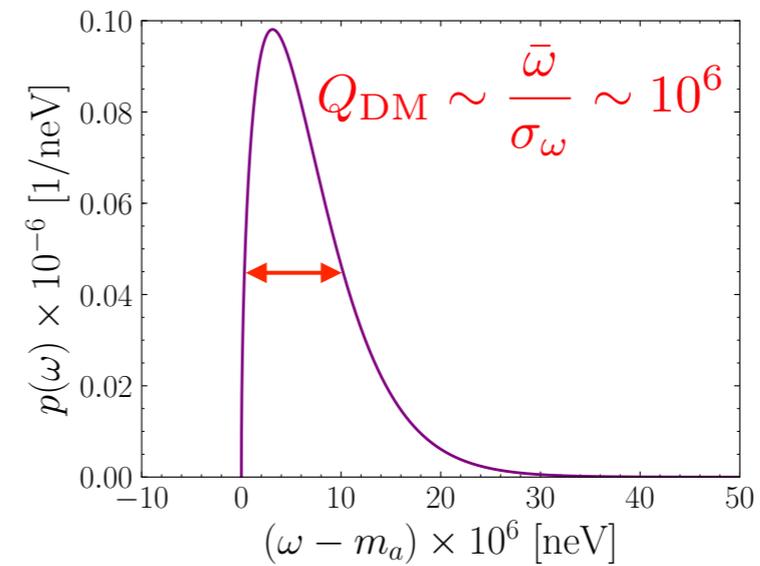
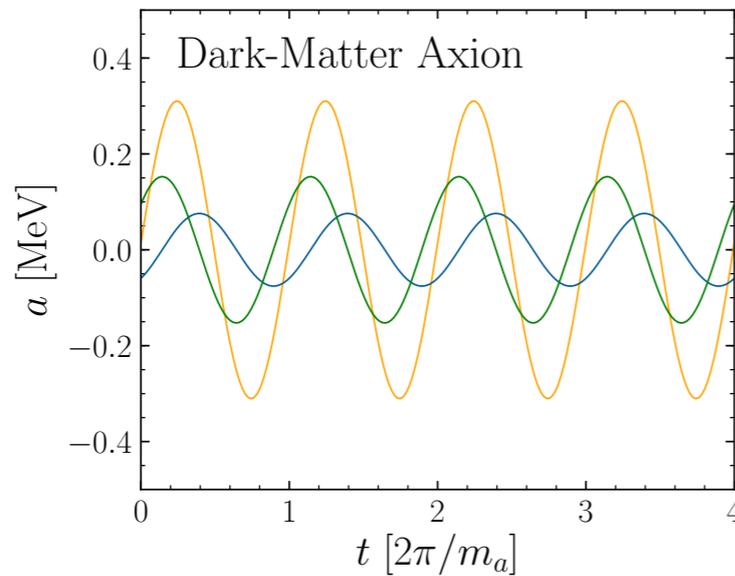
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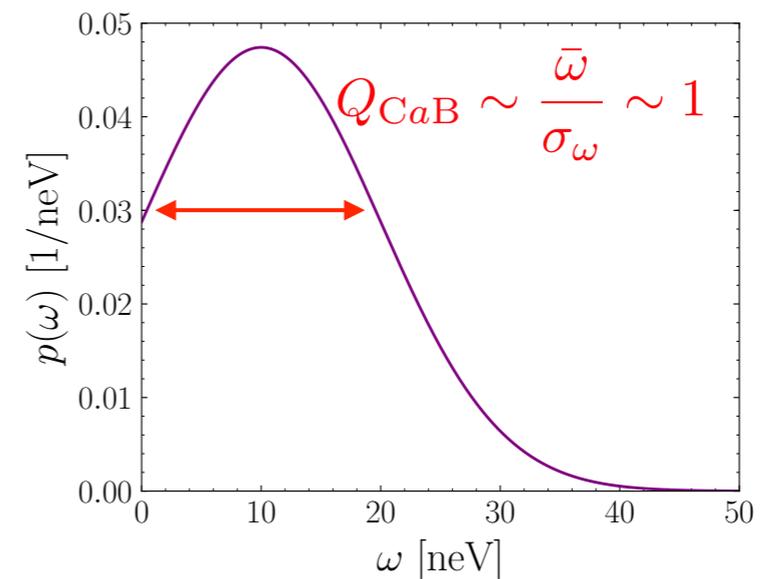
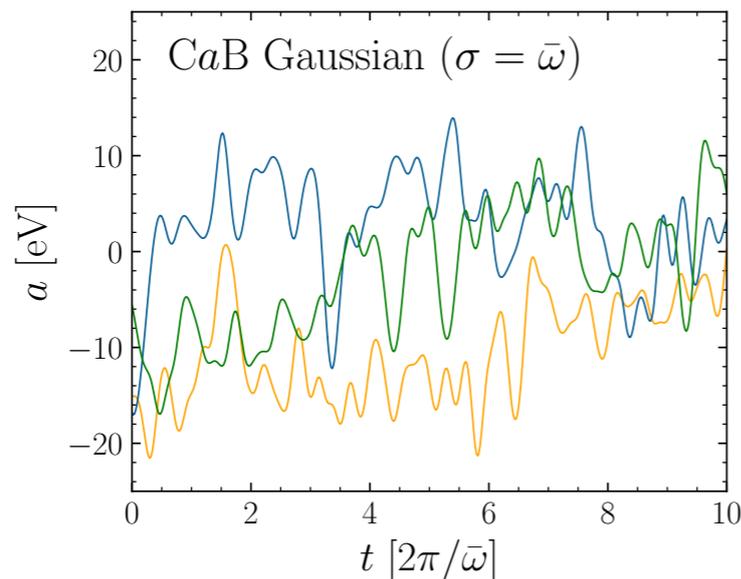
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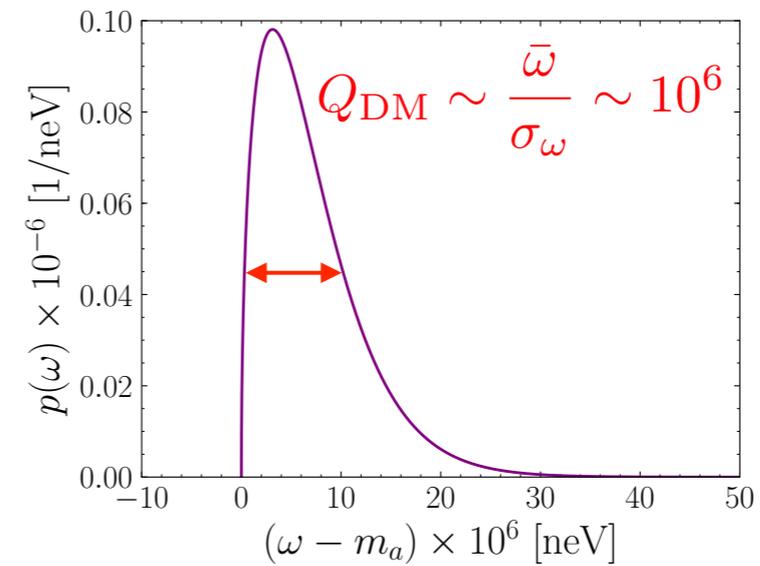
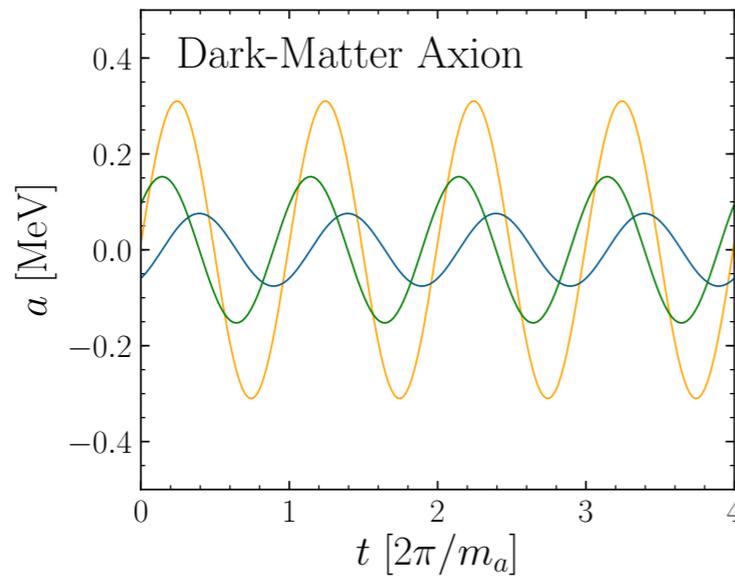
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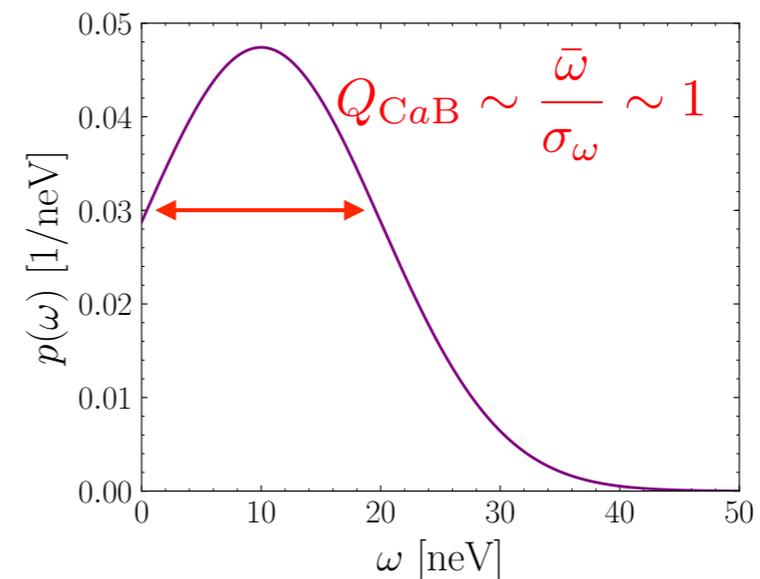
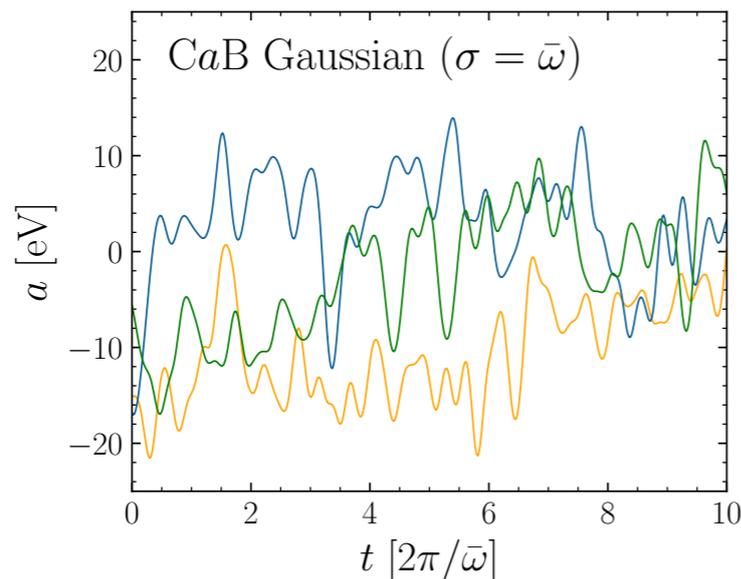
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Dark Matter



Broad signal - existing searches remove as background  
Resolved with ADMX

CaB



# Rough Sensitivity

$$a(t) \sim \sum_i \cos(\omega_i t + \phi_i)$$

Accessible power in the axion field

Power spectral density -  
measures power at a  
given frequency

$$\langle S_{g\partial a}(\omega) \rangle = \pi g_{a\gamma\gamma}^2 \rho_a \frac{\omega}{\bar{\omega}} p(\omega)$$

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Accessible power in the axion field

$$\langle S_{g\partial a}(\bar{\omega}) \rangle = \frac{\pi g_{a\gamma\gamma}^2 \rho_a Q_a}{\bar{\omega}}$$

Power spectral density -  
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Approximate  $p(\omega) \sim Q_a/\bar{\omega}$

Recall  $Q_a \sim \bar{\omega}/\sigma_\omega$

# Rough Sensitivity

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Estimate sensitivity by matching power  $P_{\text{DM}} = P_{\text{CaB}}$

# Rough Sensitivity

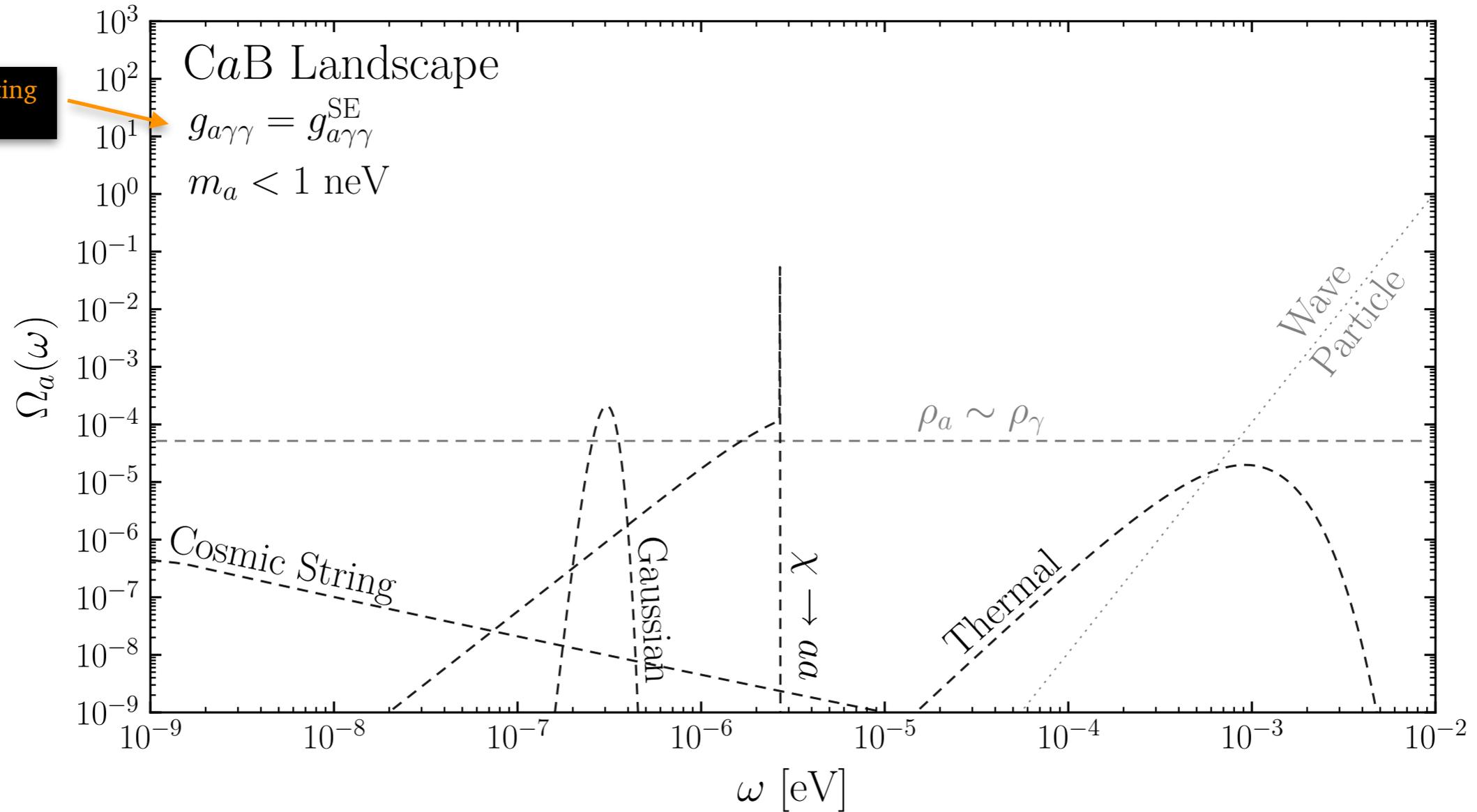
$$\rho_a = \rho_{\text{DM}} \left( \frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}^{\text{SE}}} \right)^2 \sqrt{\frac{Q_{\text{DM}}}{Q_{\text{CaB}}}}$$

Lose:  $\rho_a \ll \rho_{\text{DM}}$       Win:  $g_{a\gamma\gamma}^{\text{DM}} \ll g_{a\gamma\gamma}^{\text{SE}}$       Lose:  $Q_{\text{CaB}} \ll Q_{\text{DM}}$

Parametric scaling confirmed by detailed calculations for both resonant and broadband instruments

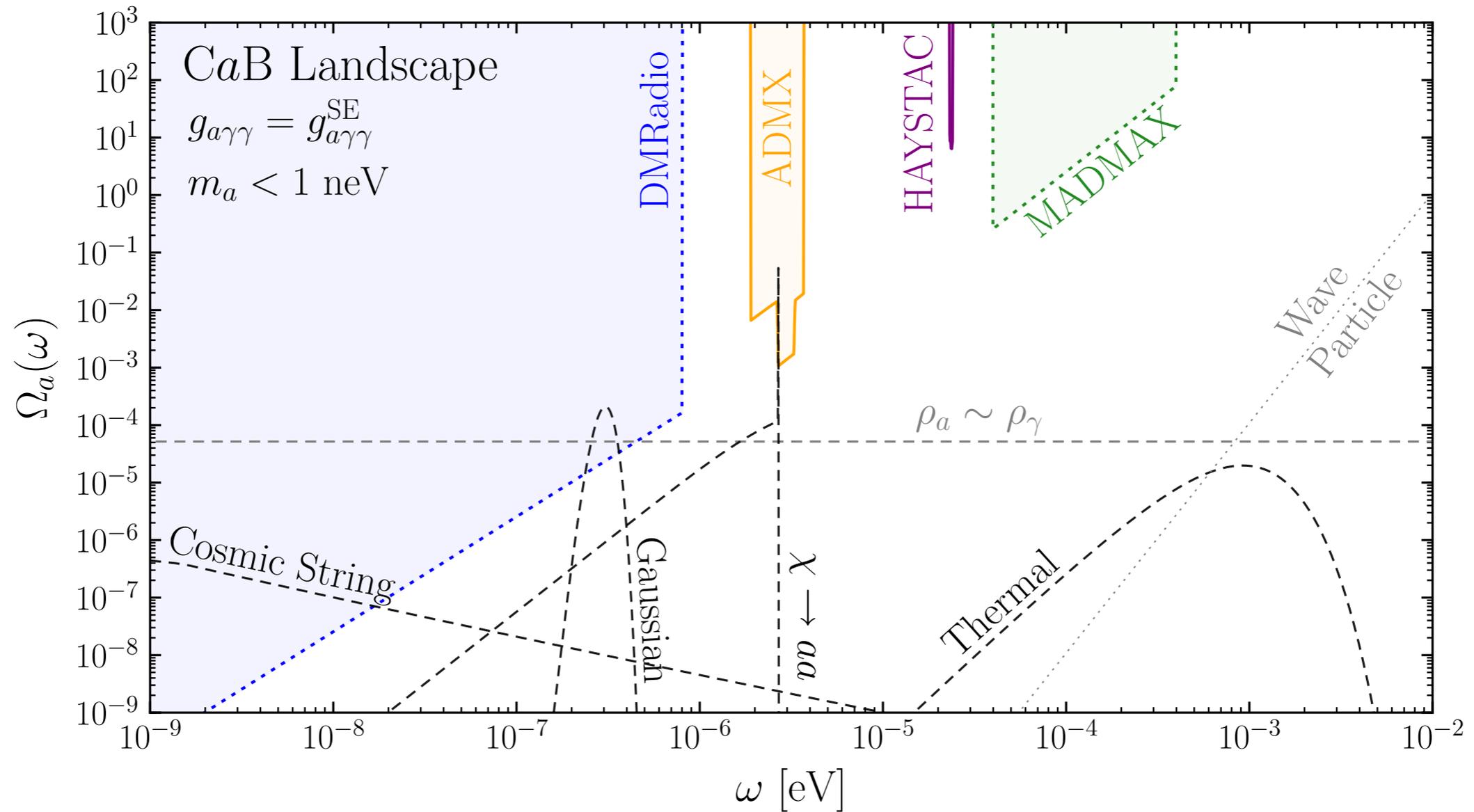
Detection requires enhanced  $g_{a\gamma\gamma}$  e.g.  
 [Choi, Im 1511.00132]  
 [Farina+ 1611.09855]  
 [Agrawal+ 1709.06085]  
 [Dror, Leedom 2008.02279]

# Experimental Landscape

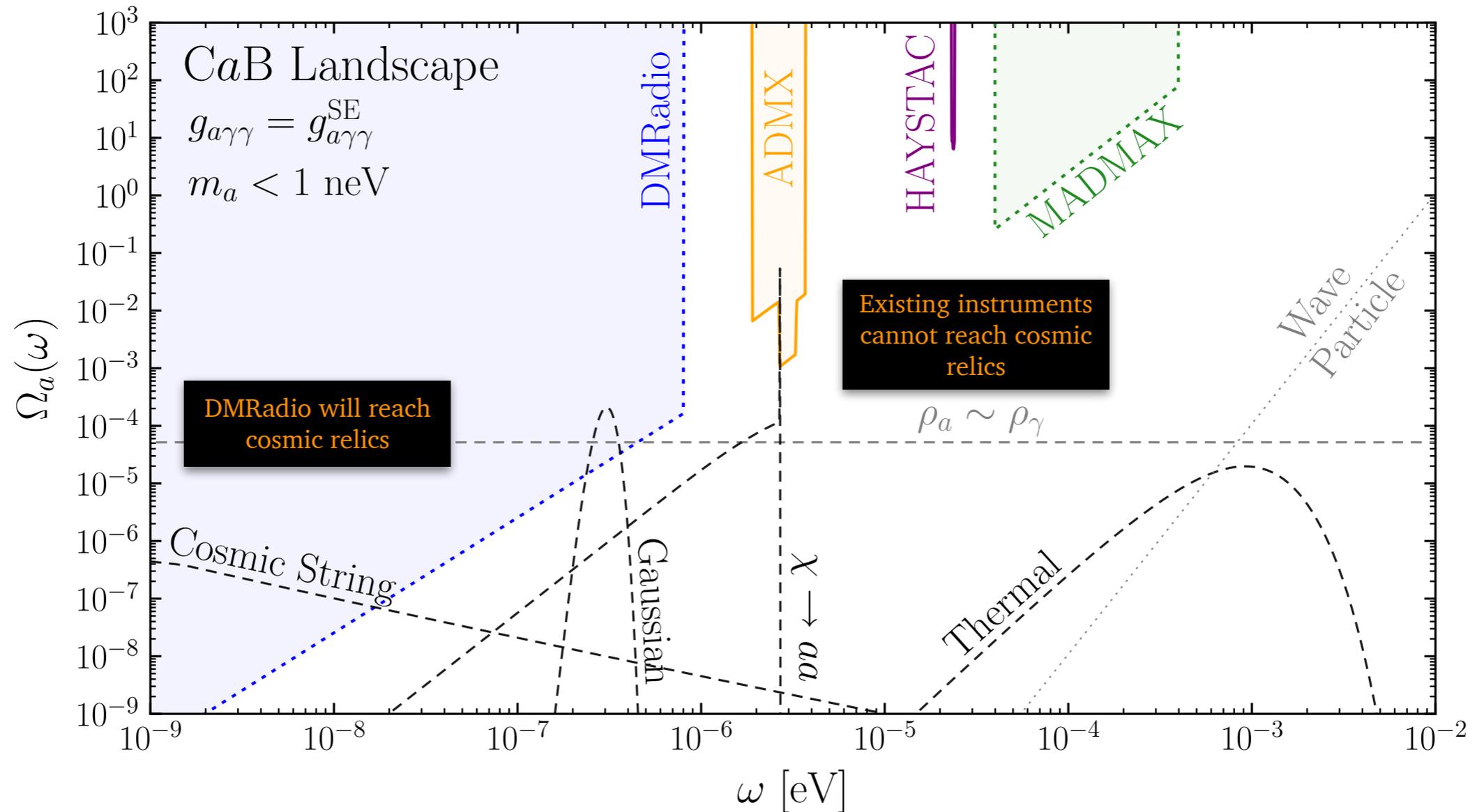


Saturate existing bounds

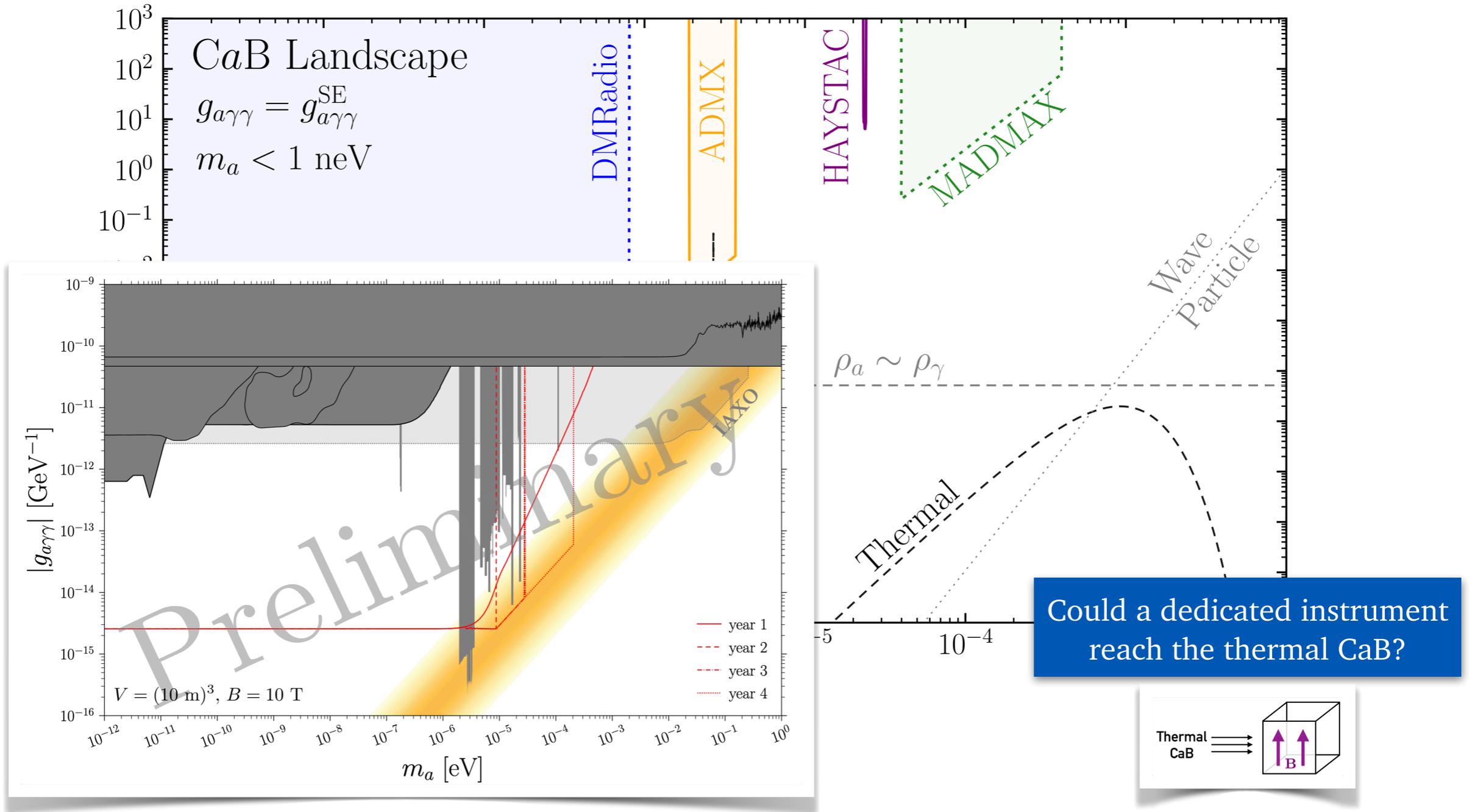
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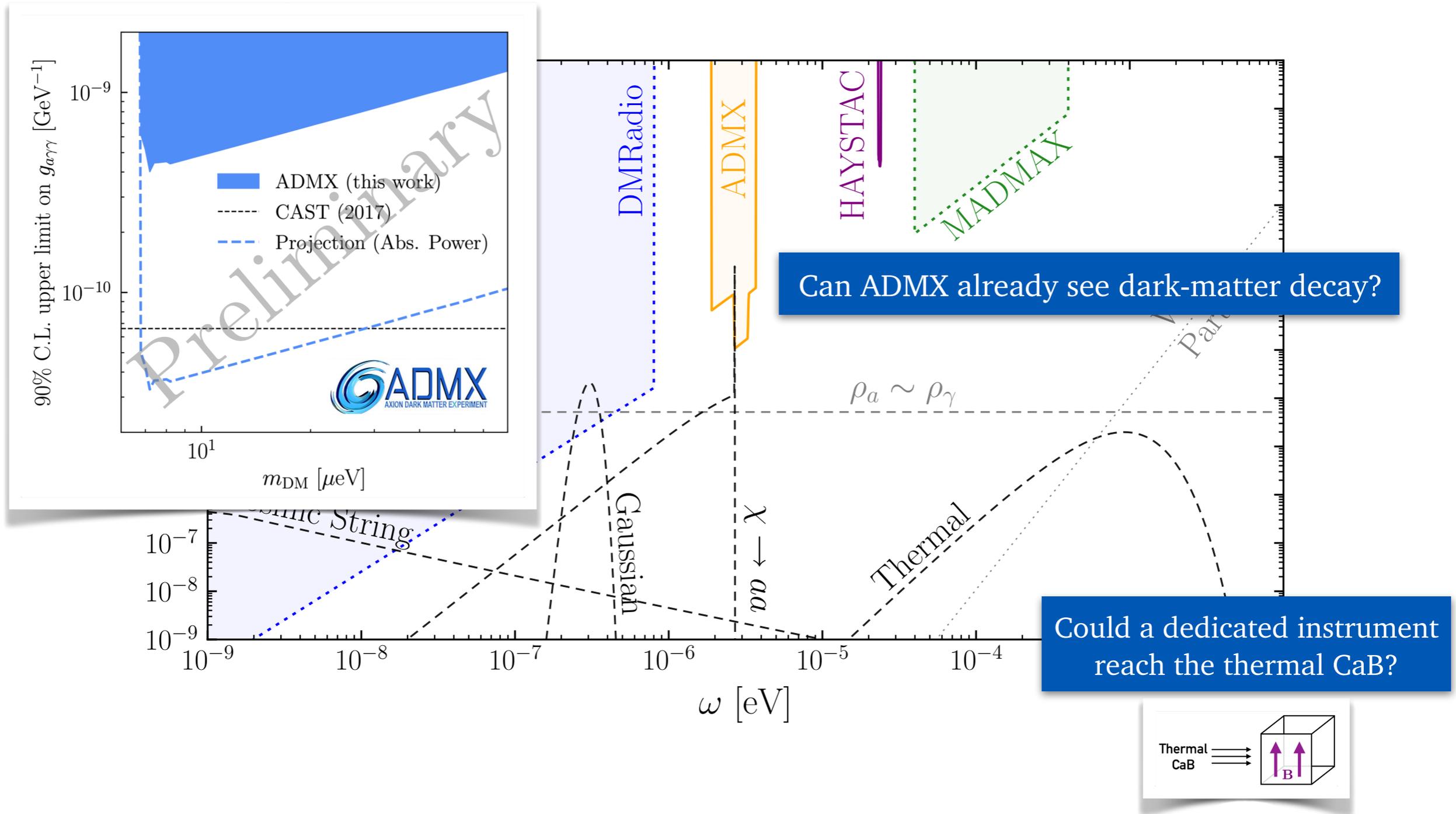
# Experimental Landscape



# Future Directions

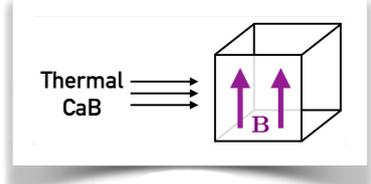
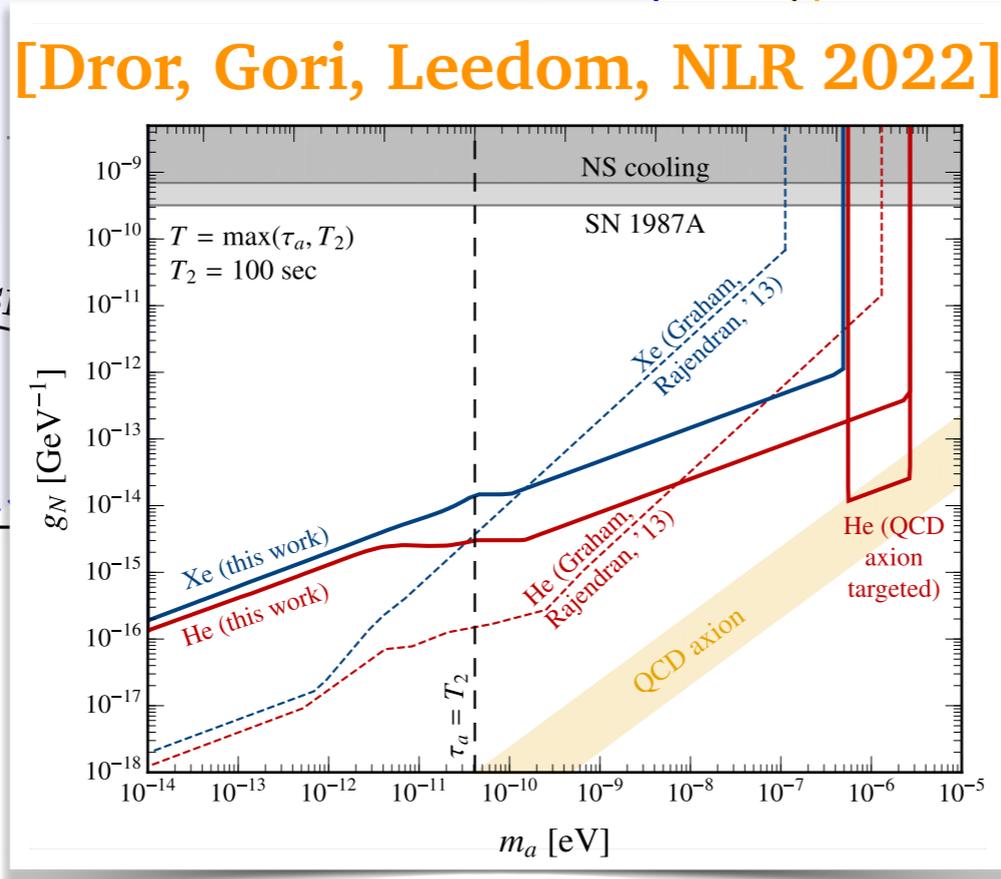
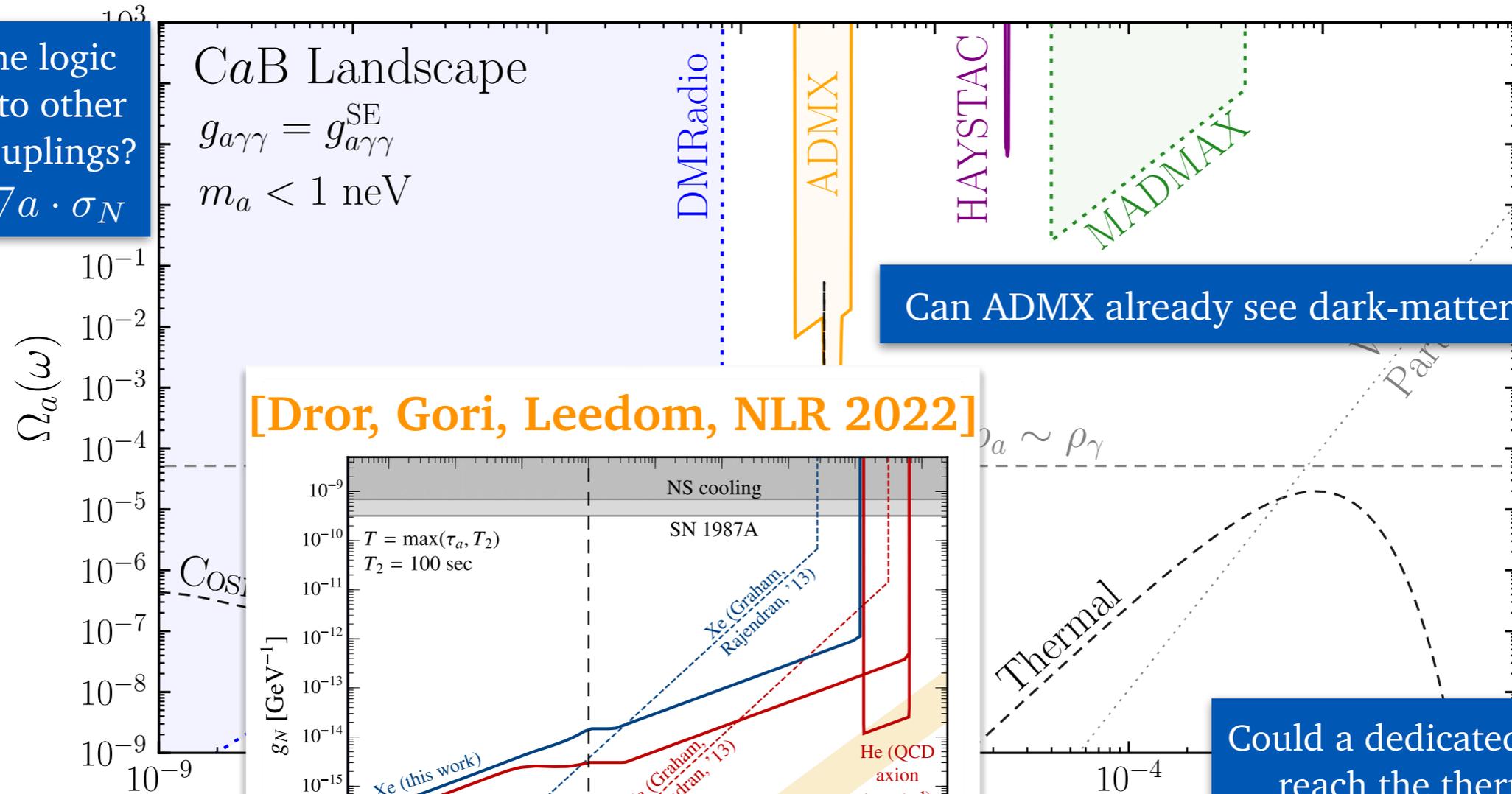


# Future Directions



# Future Directions

Does the logic extend to other axion couplings?  
 $g_{aNN} \nabla a \cdot \sigma_N$

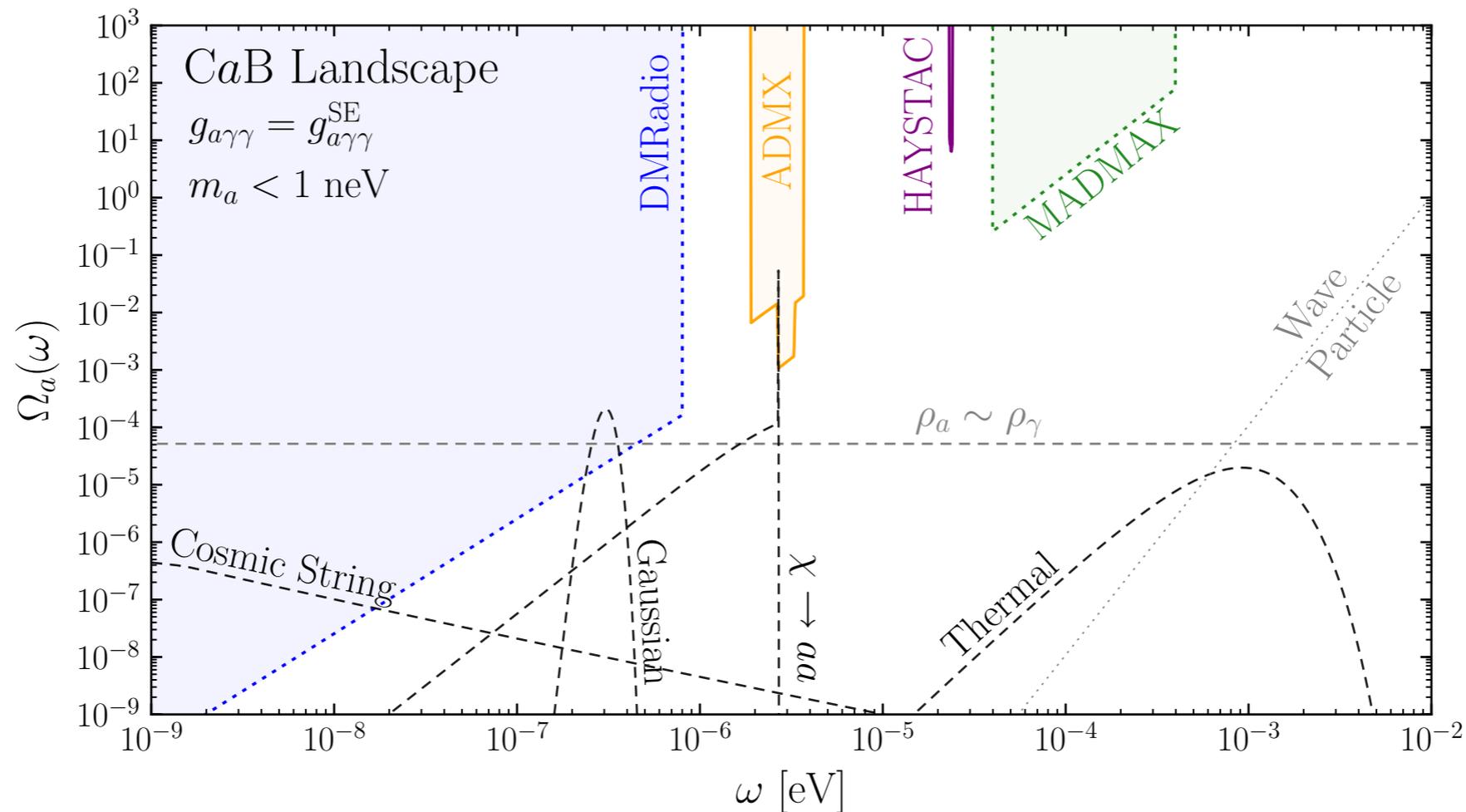


[Dror, Murayama, NLR 2021]



# Summary

The Cosmic Axion Background could emerge as we drive towards dark matter



# High-Frequency Gravitational Waves

[Domcke, Garcia-Cely, NLR PRL 2022]

# High-Frequency Gravitational Waves

Is an axion haloscope also a gravitational-wave telescope?



# Gravitational Wave Electrodynamics

$$S \supset \int d^4x \sqrt{-g} \left( -\frac{1}{4} F^2 \right)$$

Work with a linearized metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

Induces terms of the schematic form

$$hF^2$$

[Gertsenshtein 1962]  
[Boccaletti+ 1970]  
[Raffelt, Stodolsky 1988]

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Induces terms of the schematic form

$$h F^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

[Gertsenshtein 1962]  
[Boccaletti+ 1970]  
[Raffelt, Stodolsky 1988]

Clear analogy with  
axion electrodynamics

# Gravitational Wave Electrodynamics

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

0th order estimate by pushing this analogy

See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]



# Gravitational Wave Electrodynamics

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

0th order estimate by pushing this analogy

$$h \sim g_{a\gamma\gamma} a$$

See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]



# Gravitational Wave Electrodynamics

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

0th order estimate by pushing this analogy

$$h \sim g_{a\gamma\gamma} a \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_a f_a}$$

See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]



# Gravitational Wave Electrodynamics

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

0th order estimate by pushing this analogy

$$h \sim g_{a\gamma\gamma} a \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_a f_a} \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_\pi f_\pi}$$

QCD Axion

See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]

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See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]

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Far from bound on cosmological sources

$$h \lesssim 10^{-27} \left( \frac{1 \text{ MHz}}{f} \right) \Delta N_{\text{eff}}^{1/2}$$

Only sensitive to late time sources  
e.g. PBH Binary signal within reach

See also [Berlin, Blas, Tito D'Agnolo, Ellis,  
Harnik, Kahn, Schutte-Engel 2021]

# Gravitational Wave Electrodynamics

As for the axion, induce new E&M sources

$$\nabla \cdot \mathbf{E} = -\nabla \cdot \mathbf{P}$$

$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \nabla \times \mathbf{M} + \partial_t \mathbf{P}$$

although with a more complicated form

$$P_i = -h_{ij}E_j + \frac{1}{2}hE_i + h_{00}E_i - \epsilon_{ijk}h_{0j}B_k$$

$$M_i = -h_{ij}B_j - \frac{1}{2}hB_i + h_{jj}B_i + \epsilon_{ijk}h_{0j}E_k$$

Axion equivalent

$$\mathbf{P} = g_{a\gamma\gamma}a\mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma}a\mathbf{E}$$

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$$M_i = -h_{ij} B_j - \frac{1}{2} h B_i + h_{jj} B_i + \epsilon_{ijk} h_{0j} E_k$$

Following [Berlin, Ellis+ 2021] work in proper detector frame, where

Axion equivalent  
 $\mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$

$$h \sim \omega^2 + \mathcal{O}(\omega^3)$$

See also [Fortini and Gualdi 1982],  
 [Marzlin 1994], [Rakhmanov 2014]

# Proper Detector Frame

TT gauge: GW is a plane wave  $\sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Proper Detector Frame: more involved

$$h_{00} = \omega^2 F(\mathbf{k} \cdot \mathbf{r}) \mathbf{b} \cdot \mathbf{r}, \quad b_j \equiv r_i h_{ij}^{\text{TT}} \Big|_{\mathbf{r}=0},$$

$$h_{0i} = \frac{1}{2} \omega^2 [F(\mathbf{k} \cdot \mathbf{r}) - iF'(\mathbf{k} \cdot \mathbf{r})] \left( \hat{\mathbf{k}} \cdot \mathbf{r} b_i - \mathbf{b} \cdot \mathbf{r} \hat{k}_i \right),$$

$$h_{ij} = -i\omega^2 F'(\mathbf{k} \cdot \mathbf{r}) \left( |\mathbf{r}|^2 h_{ij}^{\text{TT}} \Big|_{\mathbf{r}=0} + \mathbf{b} \cdot \mathbf{r} \delta_{ij} - b_i r_j - b_j r_i \right),$$

$$F(\xi) = (e^{i\xi} - 1 - i\xi) / \xi^2 = -1/2 + \mathcal{O}(\xi)$$

# Proper Detector Frame

TT gauge: GW is a plane wave  $\sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

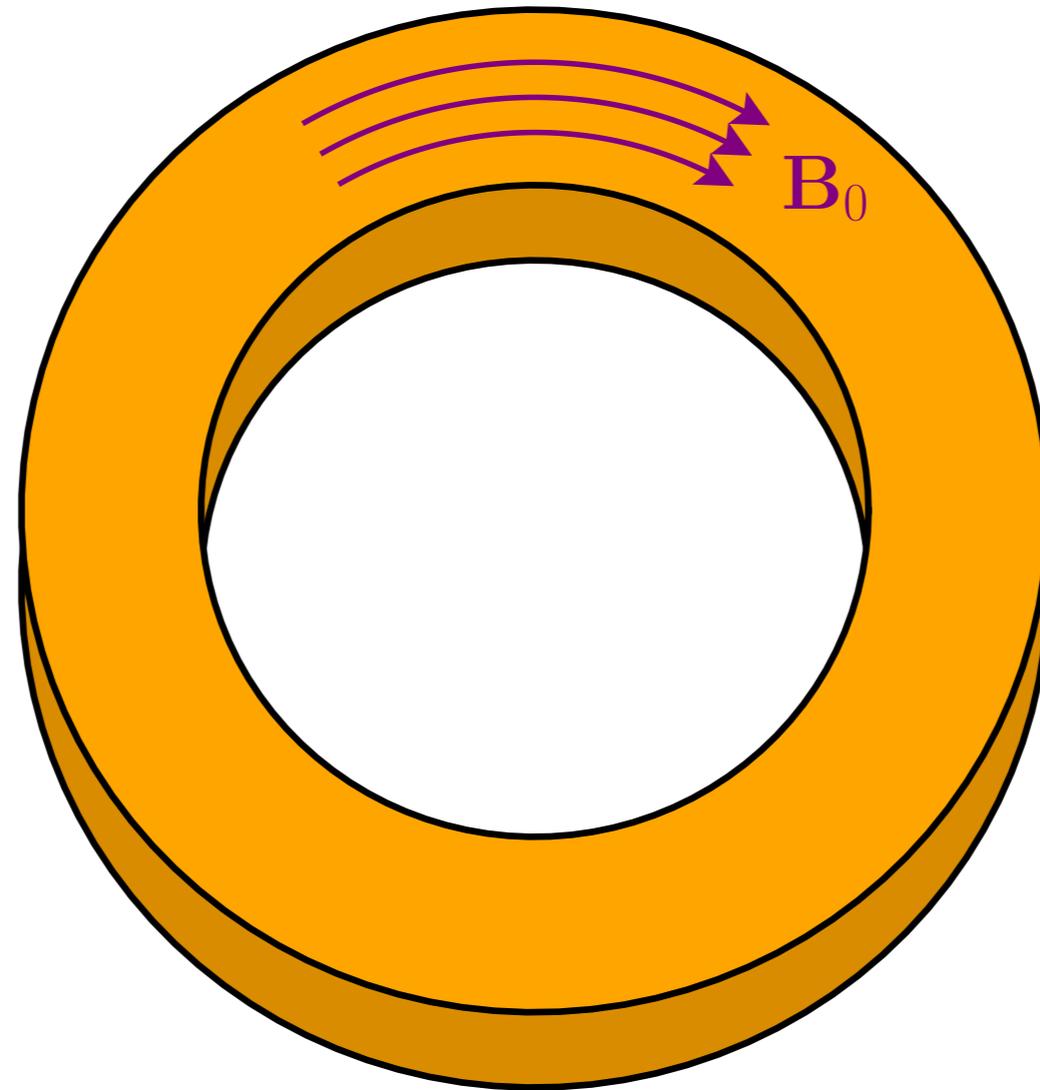
Proper Detector Frame: more involved

$$\begin{aligned}
 h_{00} &= \omega^2 F(\mathbf{k}\cdot\mathbf{r}) \\
 h_{0i} &= \frac{1}{2}\omega^2 [F(\xi) - \xi^i F'(\xi)] \\
 h_{ij} &= -i\omega^2 F'(\xi) \xi^i \xi^j + \dots
 \end{aligned}$$

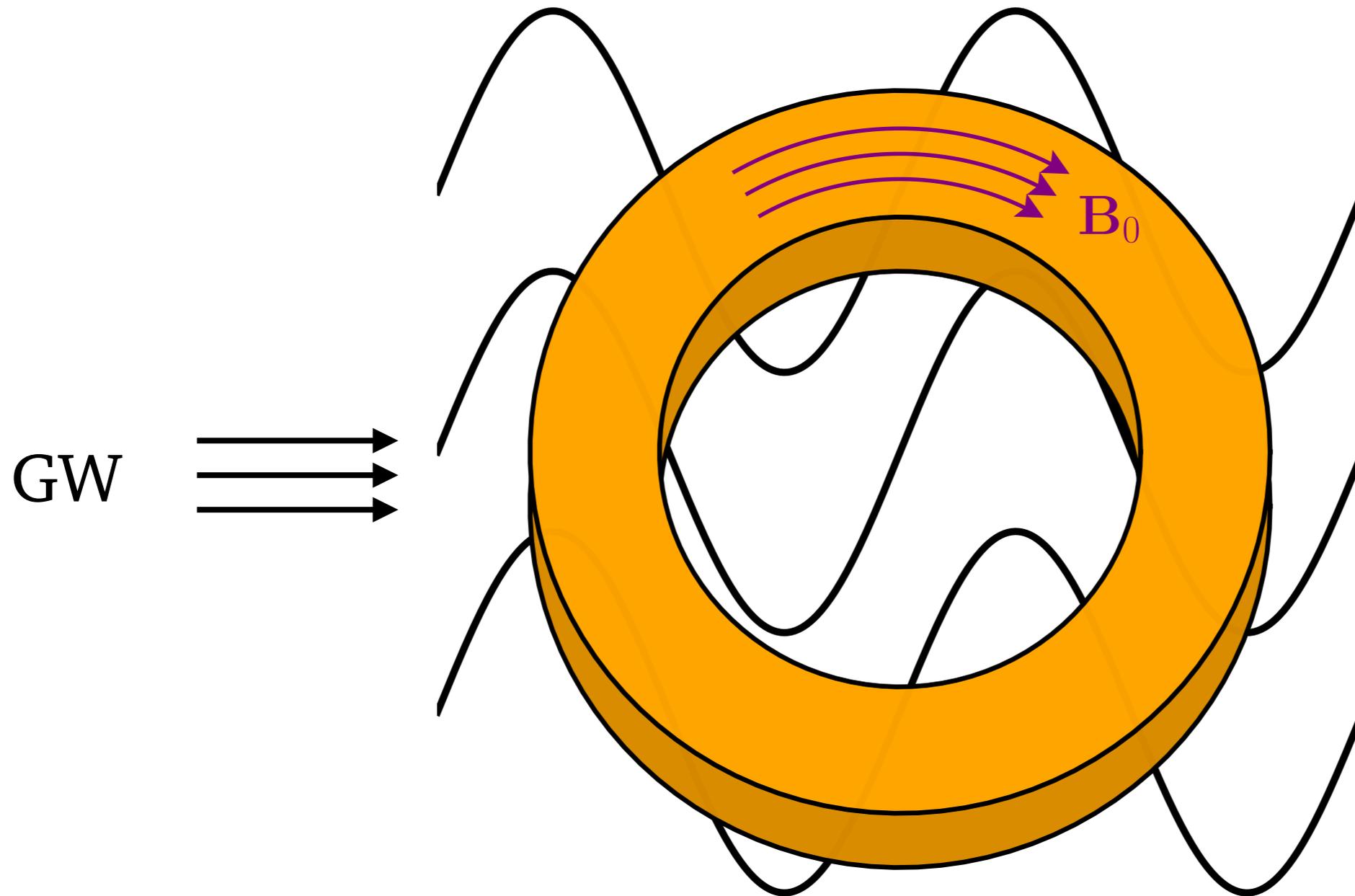
$$\begin{aligned}
 g_{\mu\nu}(x) &= \underbrace{g_{\mu\nu}(x_0)}_{=\eta_{\mu\nu}} + \underbrace{(x-x_0)^\alpha \partial_\alpha g_{\mu\nu}(x_0)}_{=0 \text{ } (\because \Gamma_{\nu\rho}^\mu(x_0)=0)} \\
 &\quad + \underbrace{(x-x_0)^\alpha (x-x_0)^\beta \partial_\alpha \partial_\beta g_{\mu\nu}(x_0)}_{\mathcal{O}(\omega^2 R^2)} + \dots
 \end{aligned}$$

$$F(\xi) = (e^{i\xi} - 1 - i\xi)/\xi^2 = -1/2 + \mathcal{O}(\xi)$$

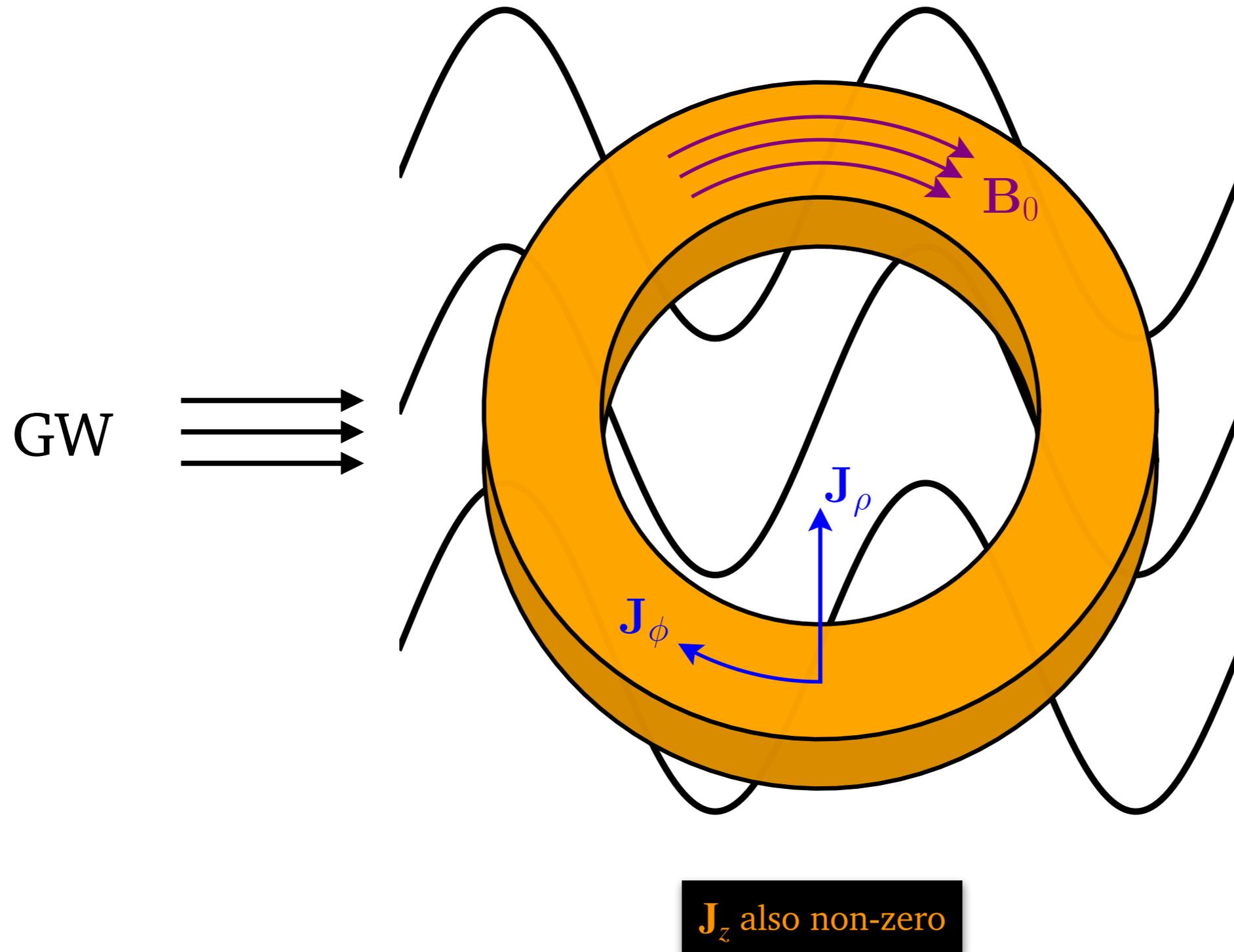
# Detection Strategy



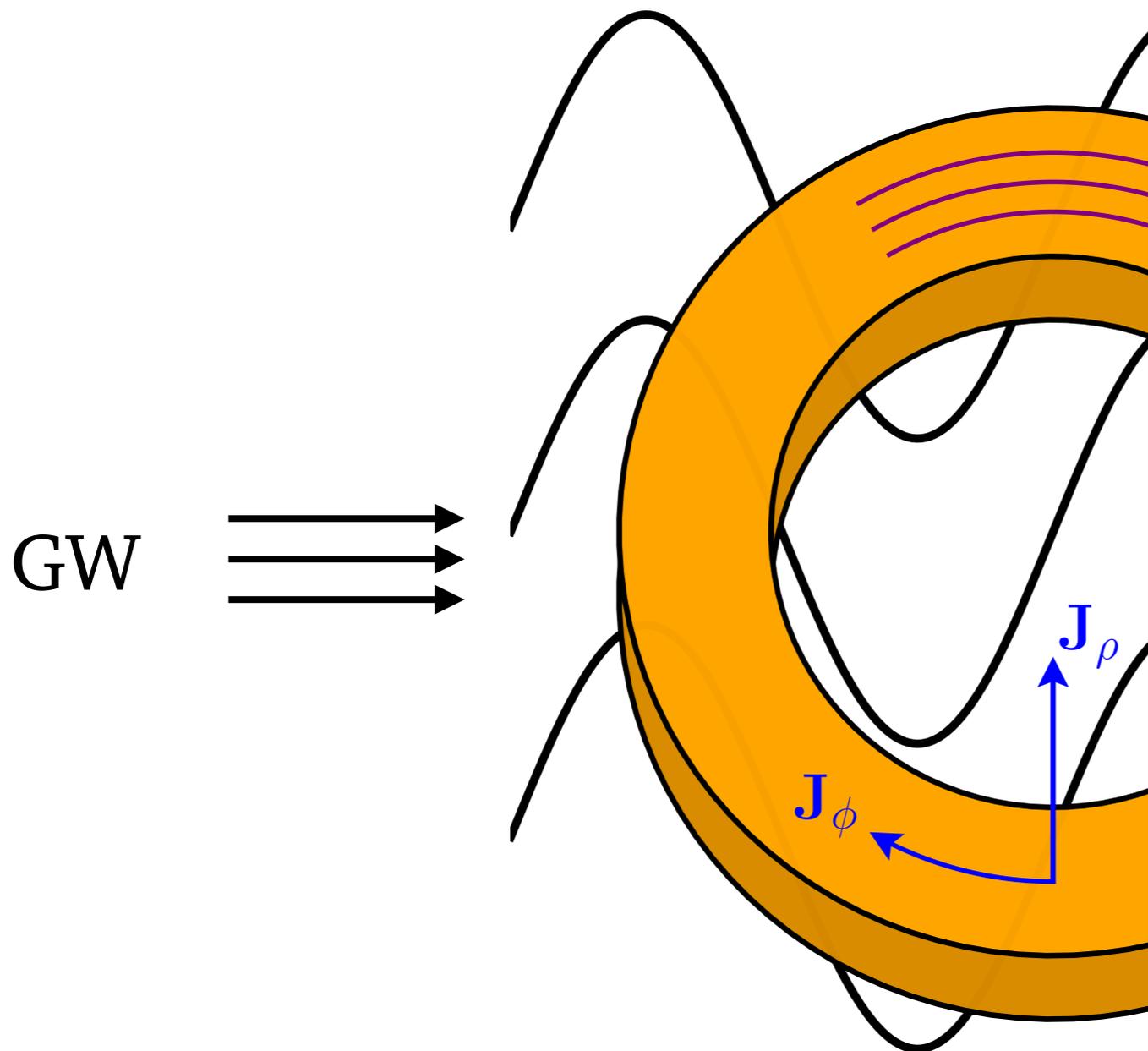
# Detection Strategy



# Detection Strategy



# Detection Strategy



$$j_\phi = \frac{\omega^2 B_{\max} R}{\rho} \left[ \frac{e^\kappa}{\kappa} - \frac{2e^\kappa}{\kappa^2} + \frac{2(e^\kappa - 1)}{\kappa^3} \right] (z h_{\rho\phi}^{\text{TT}}|_{\mathbf{r}=0} - \rho h_{\phi z}^{\text{TT}}|_{\mathbf{r}=0}),$$

$$j_\rho = \frac{\omega^2 B_{\max} R}{\rho} \left( \left[ -\frac{1}{2} - \frac{1}{\kappa} + \frac{2e^\kappa}{\kappa^2} + \frac{2(1 - e^\kappa)}{\kappa^3} \right] (\rho h_{\rho z}^{\text{TT}}|_{\mathbf{r}=0} + z h_{z z}^{\text{TT}}|_{\mathbf{r}=0}) \right. \\ \left. + \left[ \frac{e^\kappa}{\kappa} + \frac{2}{\kappa^2} + \frac{2(1 - e^\kappa)}{\kappa^3} \right] (z h_{\rho\rho}^{\text{TT}}|_{\mathbf{r}=0} + z h_{z z}^{\text{TT}}|_{\mathbf{r}=0}) \right. \\ \left. + i k_z \left[ \frac{1}{2\kappa} + \frac{1}{2\kappa^2} - \frac{1 + 2e^\kappa}{\kappa^3} + \frac{3(e^\kappa - 1)}{\kappa^4} \right] r_i r_j h_{ij}^{\text{TT}}|_{\mathbf{r}=0} \right),$$

$\kappa = i\mathbf{k} \cdot \mathbf{r}$

$$h_{\rho\rho}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left( -h^+ (\sin^2(\phi - \phi_h) - \cos^2(\phi - \phi_h) \cos^2 \theta_h) + 2h^\times \cos \theta_h \cos(\phi - \phi_h) \sin(\phi - \phi_h) \right),$$

$$h_{\rho\phi}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left( -h^+ (1 + \cos^2 \theta_h) \sin(\phi - \phi_h) \cos(\phi - \phi_h) + h^\times \cos(2(\phi - \phi_h)) \cos \theta_h \right),$$

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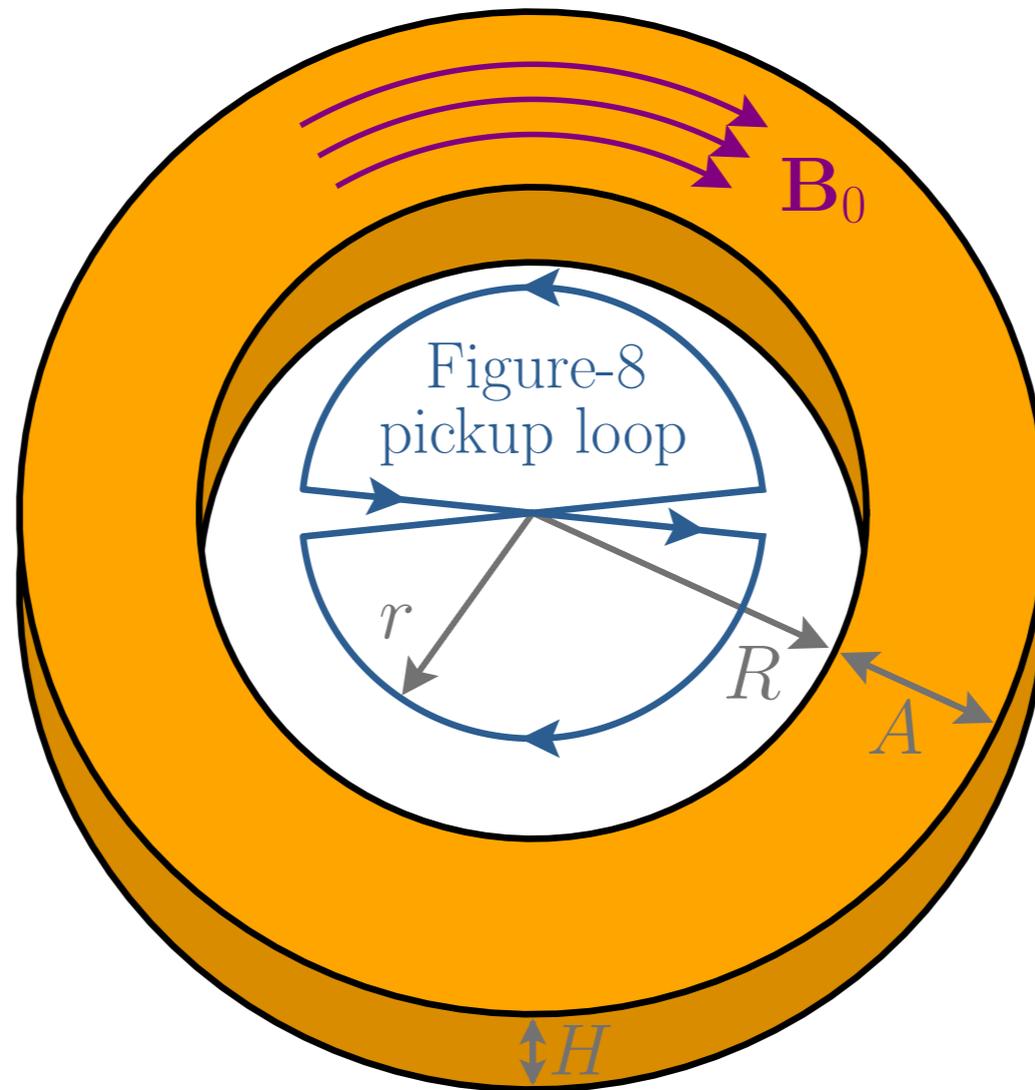
$$h_{\phi z}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left( h^+ \cos \theta_h \sin \theta_h \sin(\phi - \phi_h) - h^\times \sin \theta_h \cos(\phi - \phi_h) \right),$$

$$h_{z z}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} h^+ \sin^2 \theta_h.$$

Cf. axion:  $\mathbf{J}_a = g_{a\gamma\gamma}(\partial_t a)\mathbf{B}_0$

$\mathbf{J}_z$  also non-zero

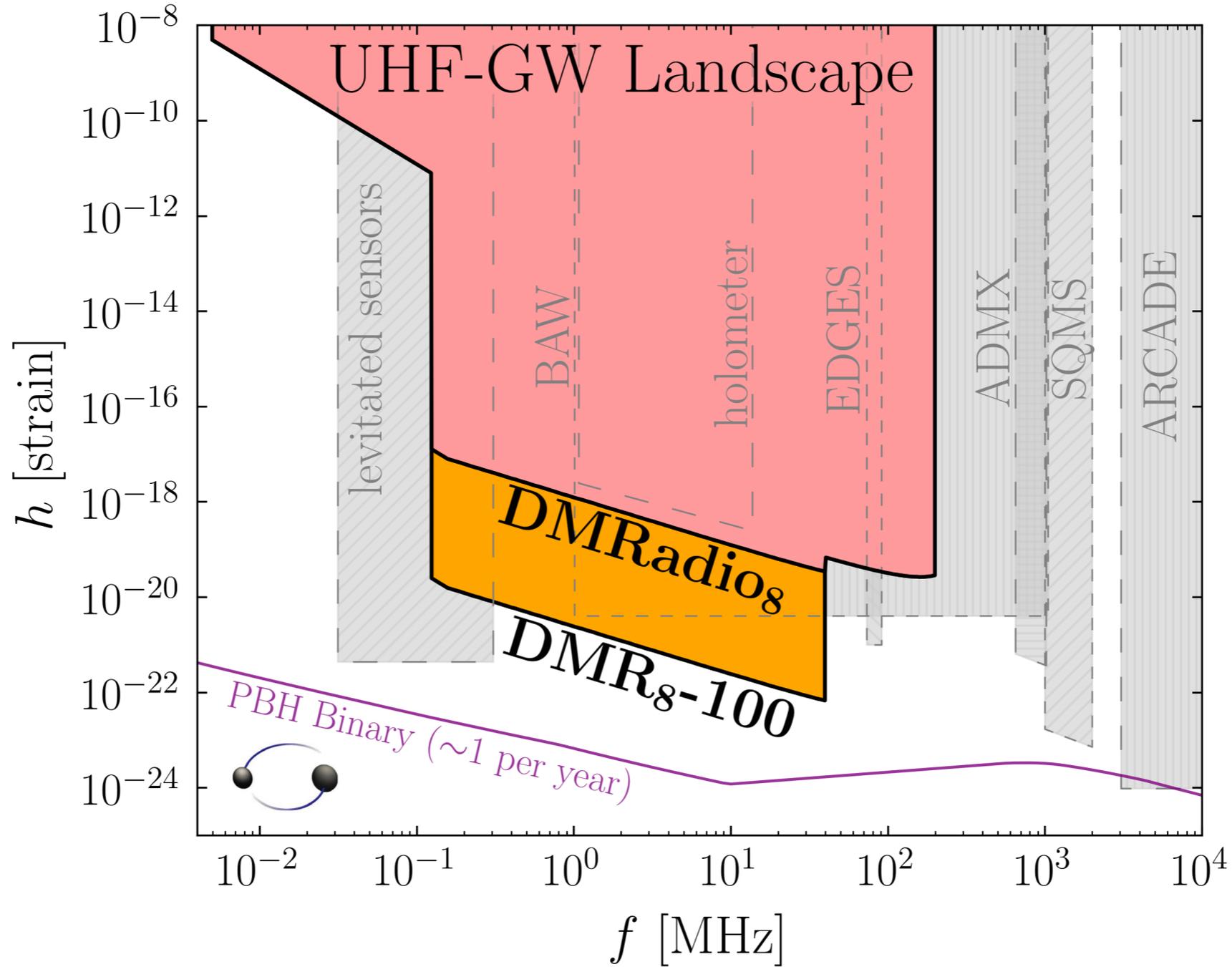
# Optimized Reach



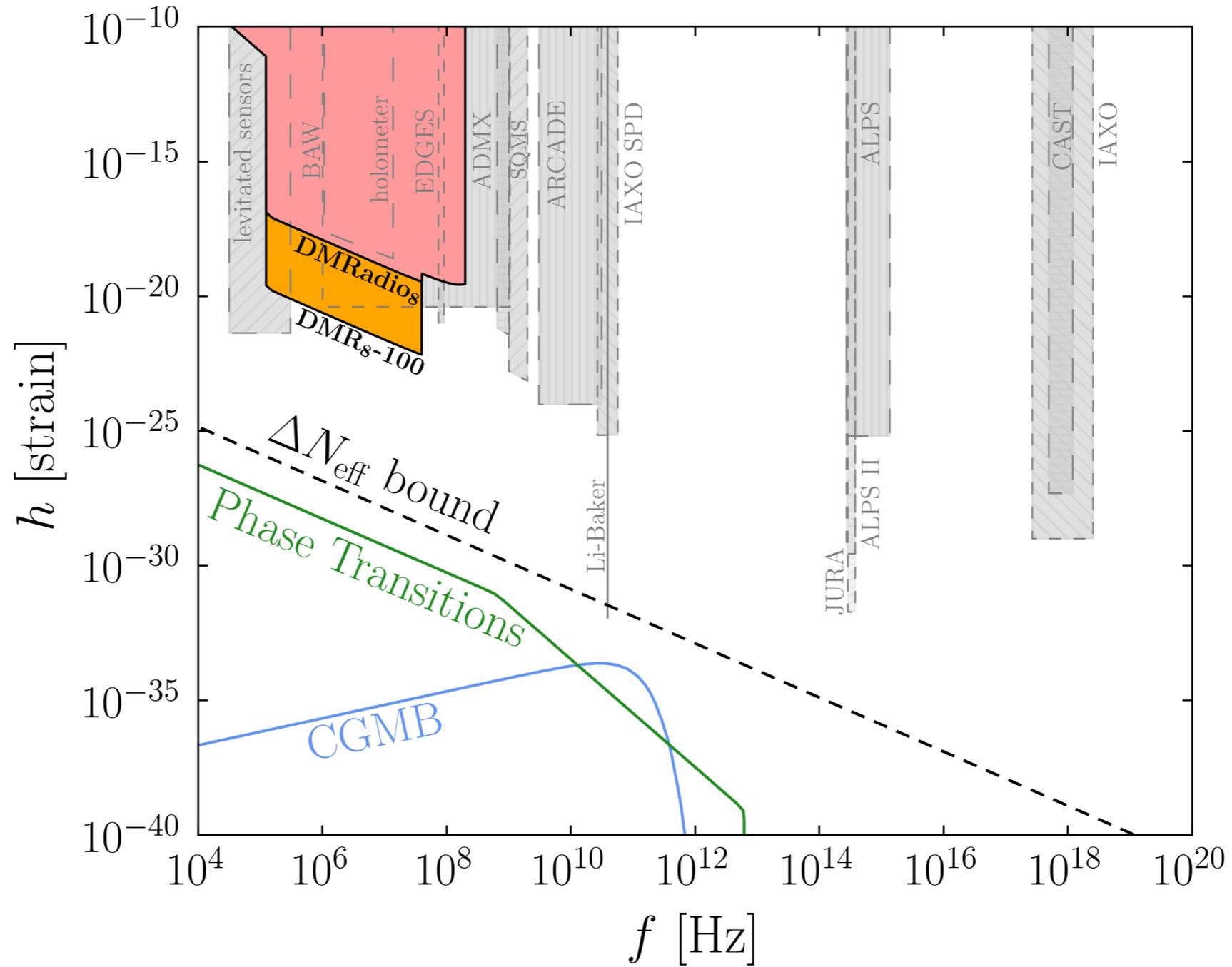
Circular loop is not optimal  
Why? [Ongoing work] due to  
symmetry (especially parity)

$$\Phi_h(t) \simeq \frac{e^{-i\omega t}}{3\sqrt{2}} \omega^2 B_0 r^3 R \ln(1 + A/R) s_{\theta_h} (h^\times s_{\phi_h} - h^+ c_{\theta_h} c_{\phi_h}) \sim \omega^2 h B_0 V^{4/3}$$

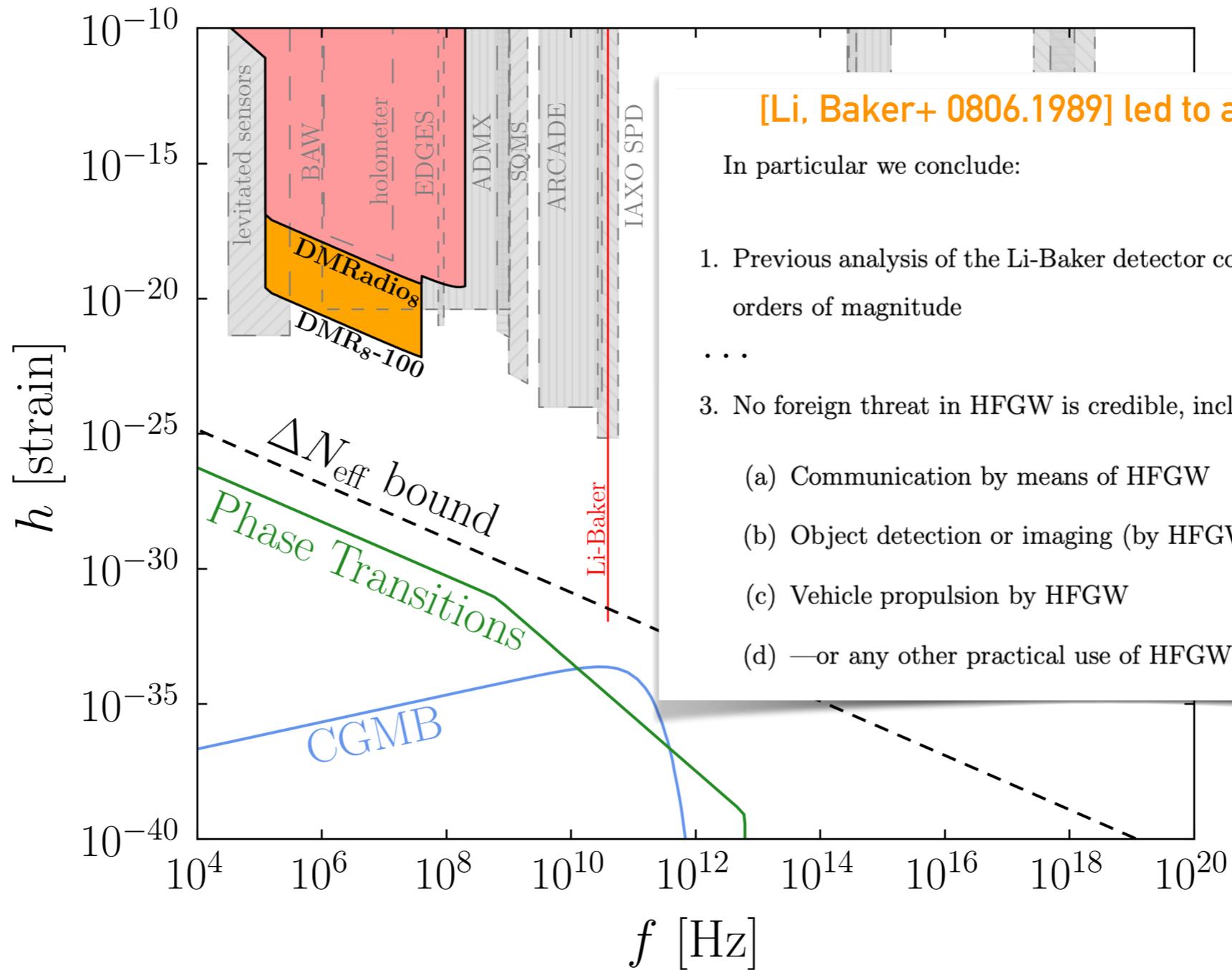
# Optimized Reach



# Future Directions



# Future Directions

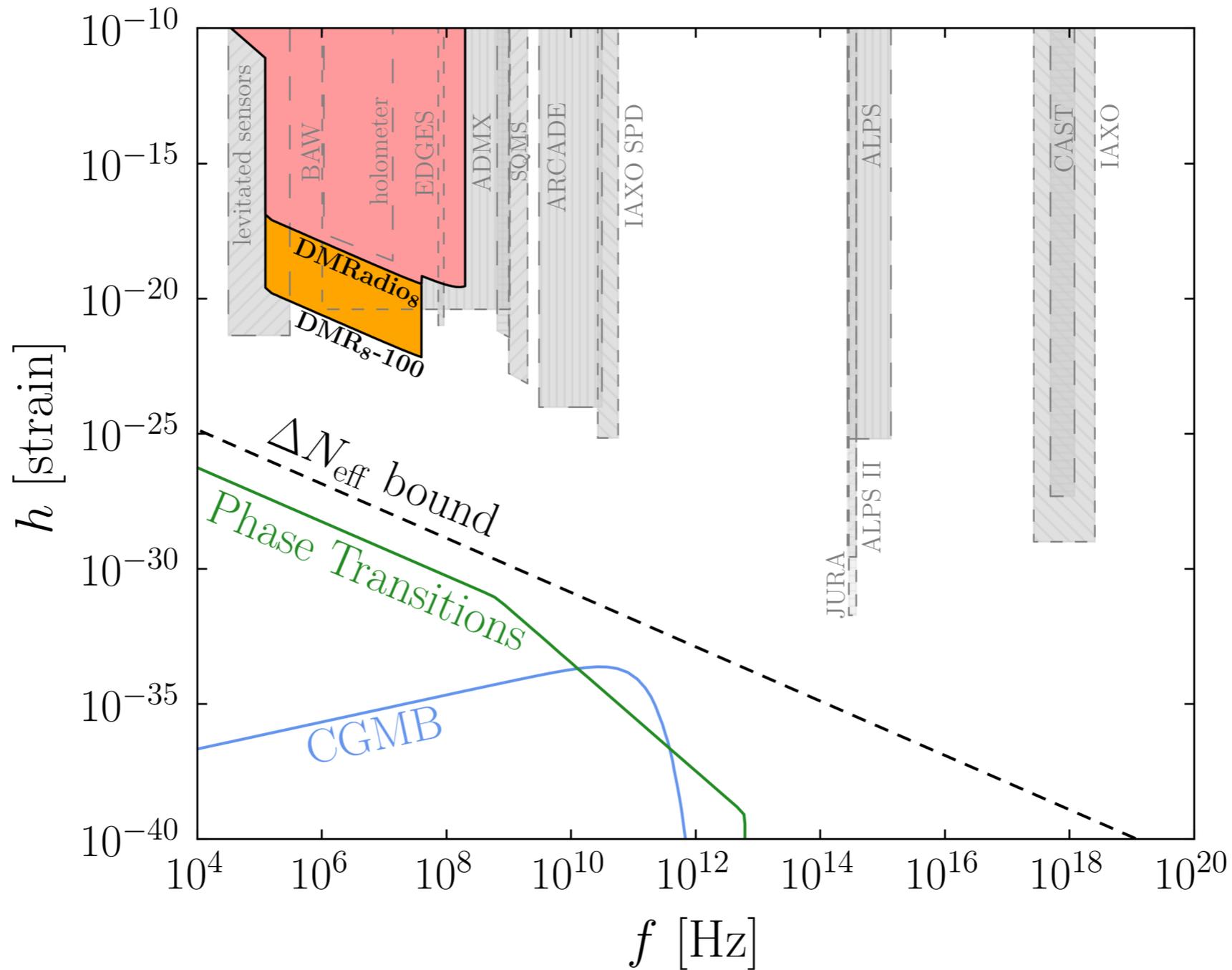


[Li, Baker+ 0806.1989] led to a JASON report

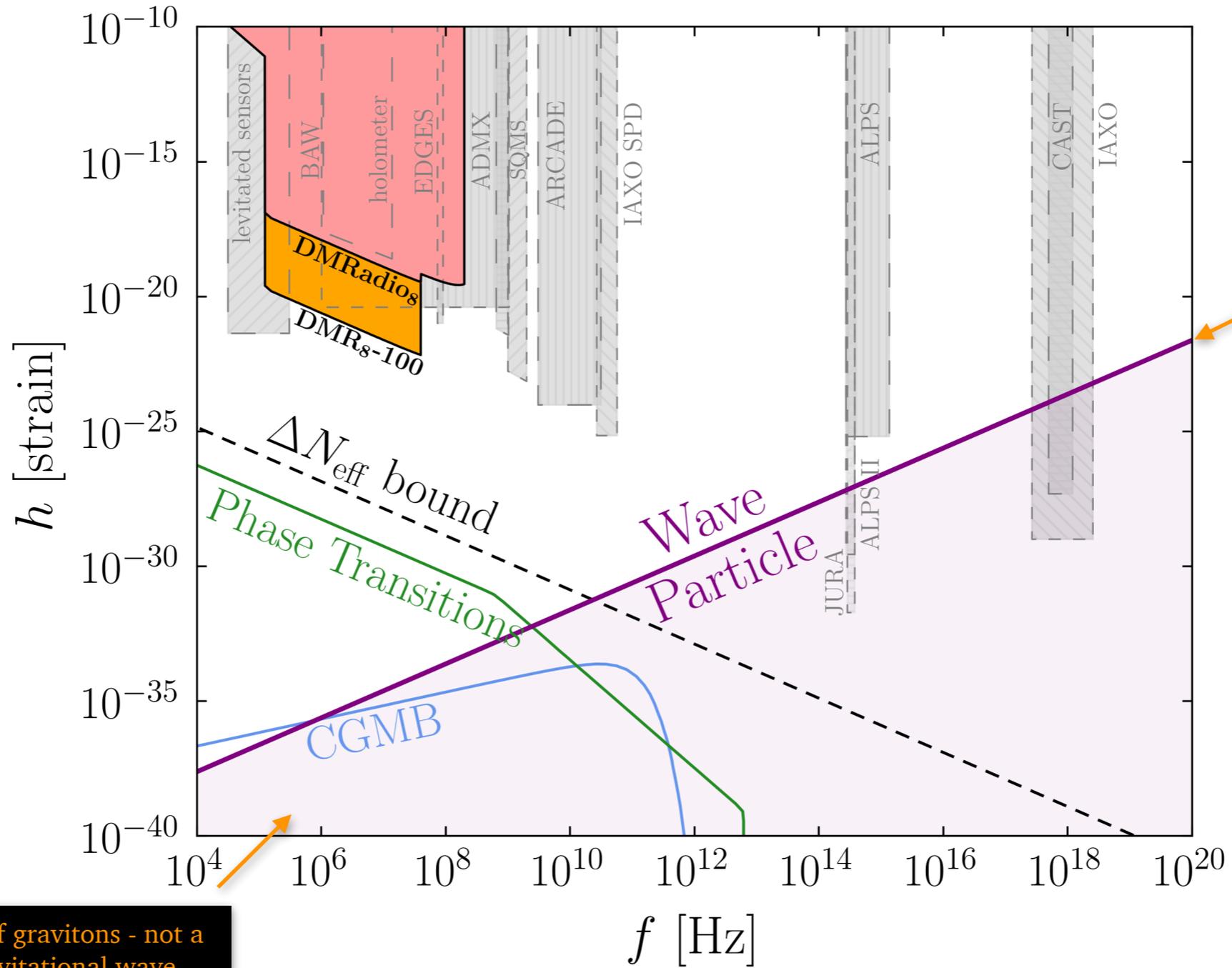
In particular we conclude:

1. Previous analysis of the Li-Baker detector concept is incorrect by many orders of magnitude
- ...
3. No foreign threat in HFGW is credible, including:
  - (a) Communication by means of HFGW
  - (b) Object detection or imaging (by HFGW radar or tomography)
  - (c) Vehicle propulsion by HFGW
  - (d) —or any other practical use of HFGW.

# Future Directions



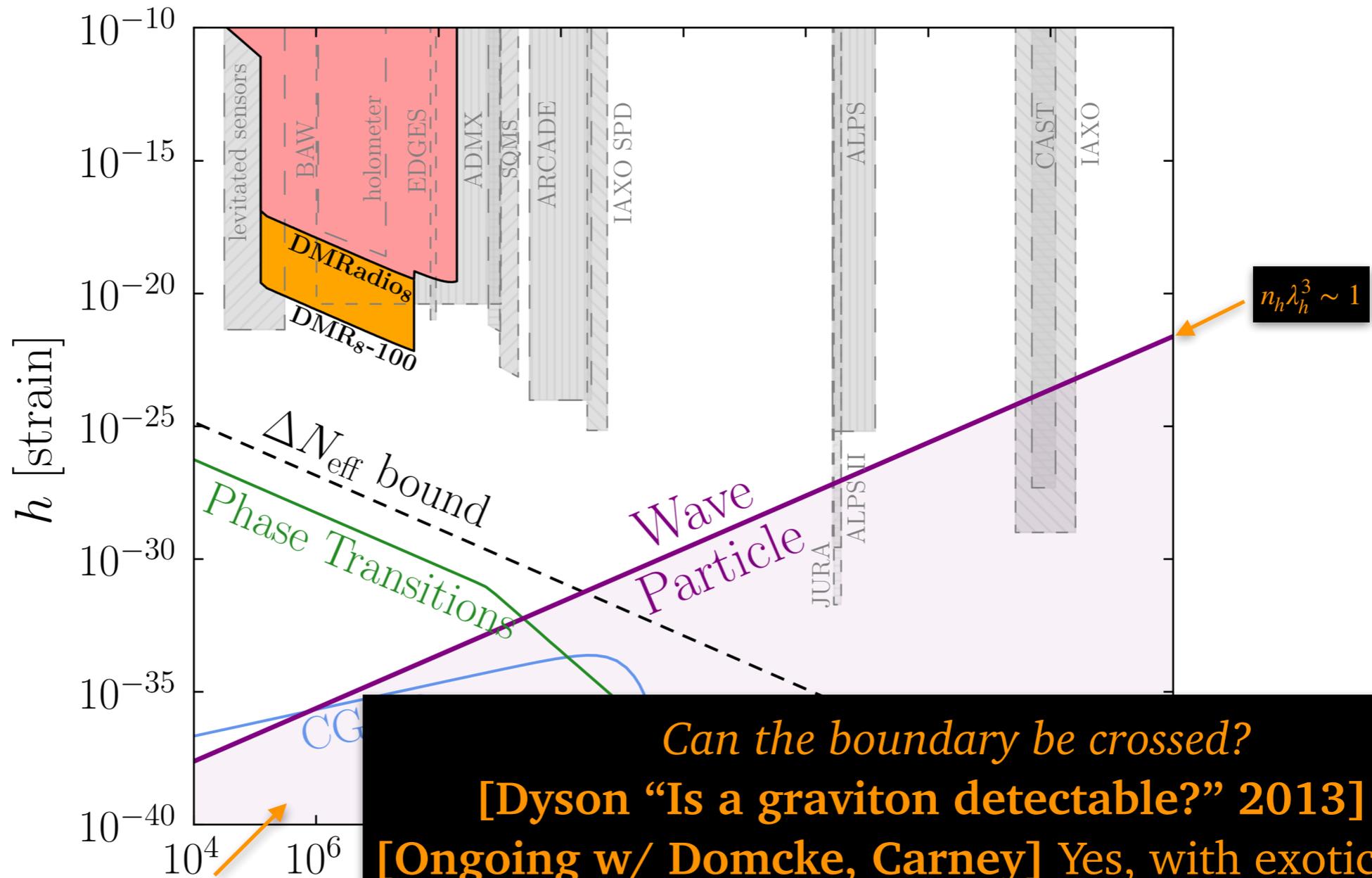
# Future Directions



Gas of gravitons - not a gravitational wave

$n_h \lambda_h^3 \sim 1$

# Future Directions

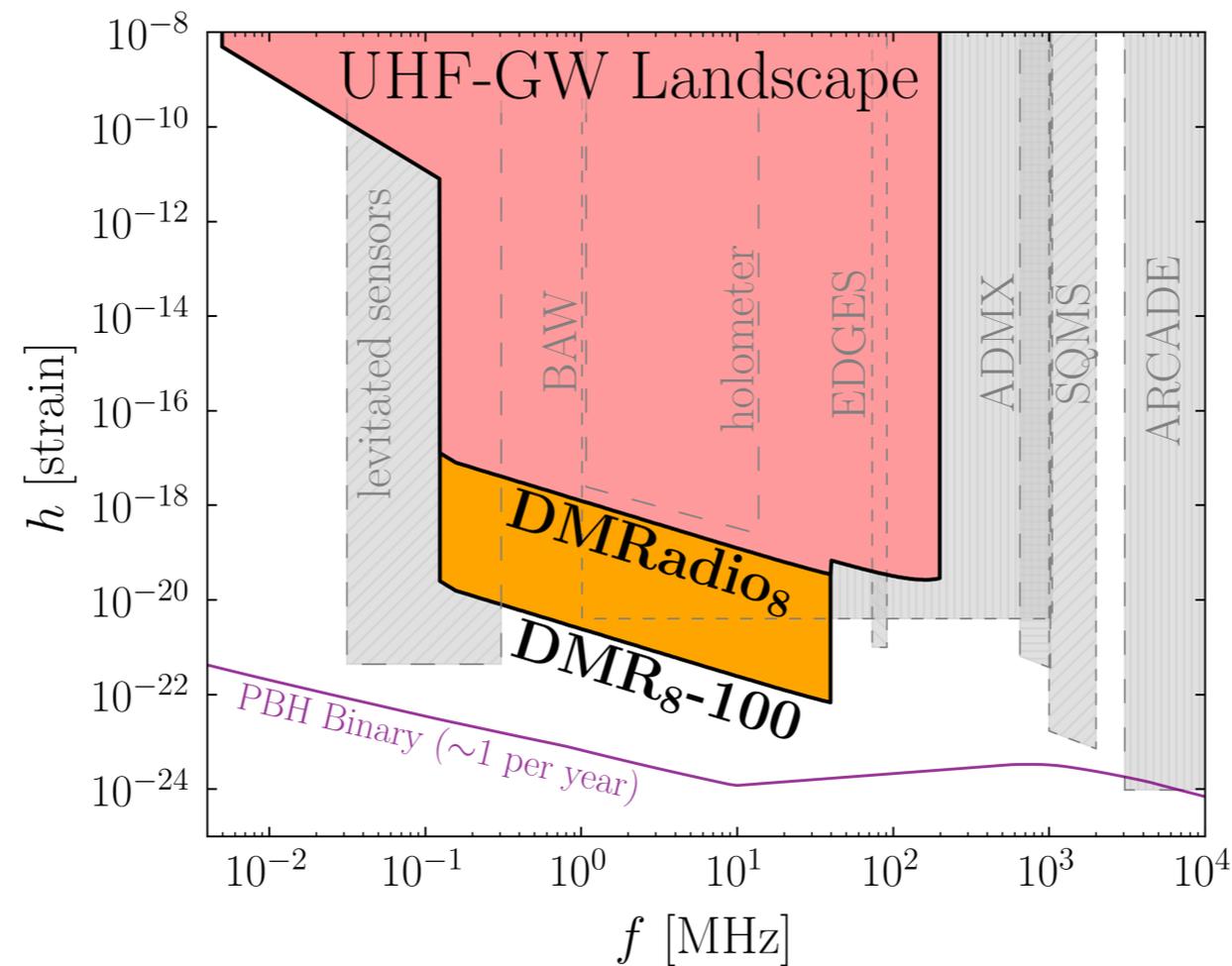


Gas of gravitons - not a gravitational wave

*Can the boundary be crossed?*  
**[Dyson “Is a graviton detectable?” 2013] No**  
**[Ongoing w/ Domcke, Carney] Yes, with exotic sources**  
**Would this prove that gravity is quantum? No**

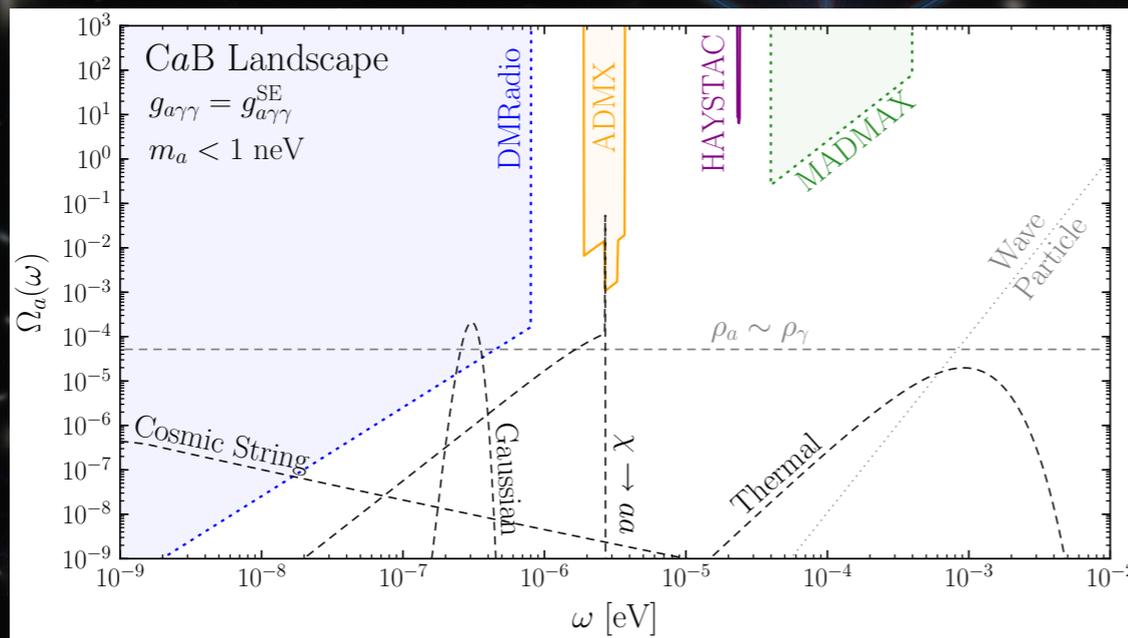
# Summary

Axion haloscopes are  
gravitational wave telescopes

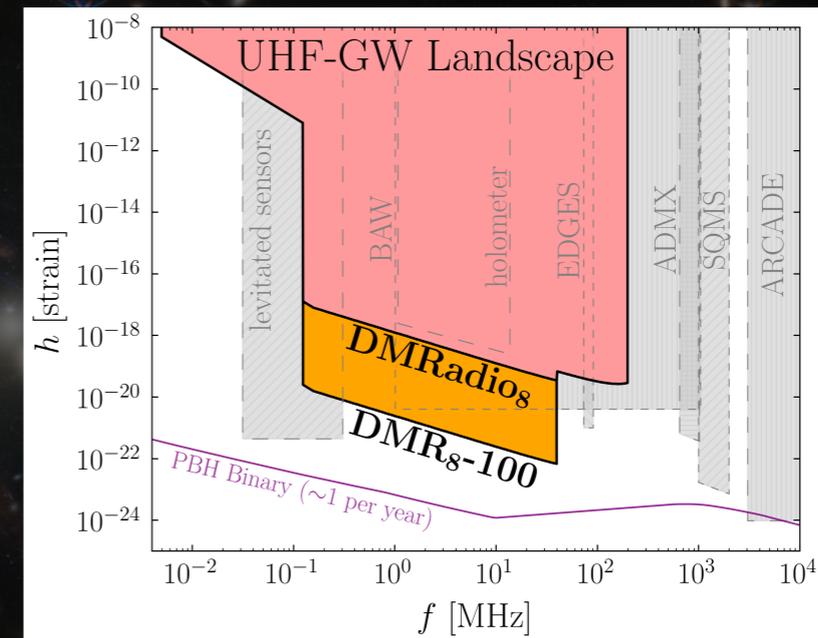


# Conclusion

Our deepening search for dark matter opens a path to many new discoveries



[Dror, Murayama, NLR PRD 2021]



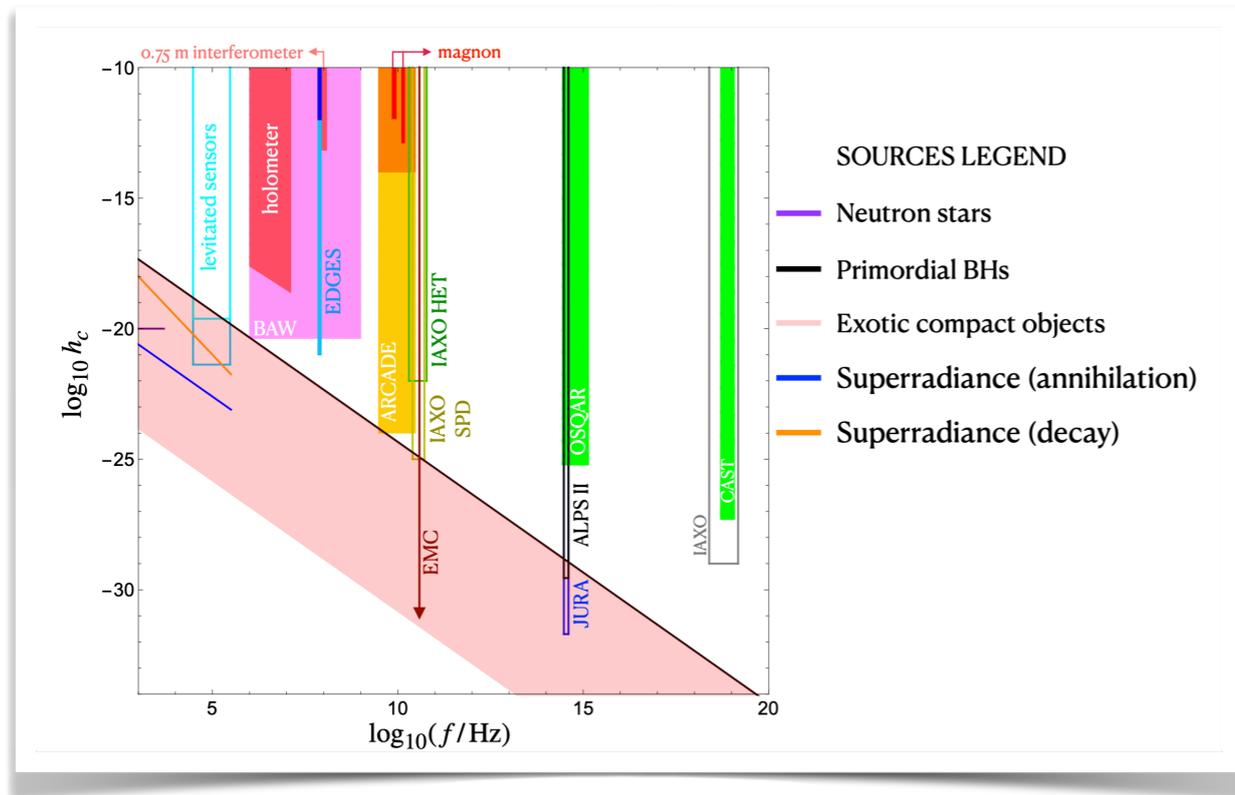
[Domcke, Garcia-Cely, NLR PRL 2022]



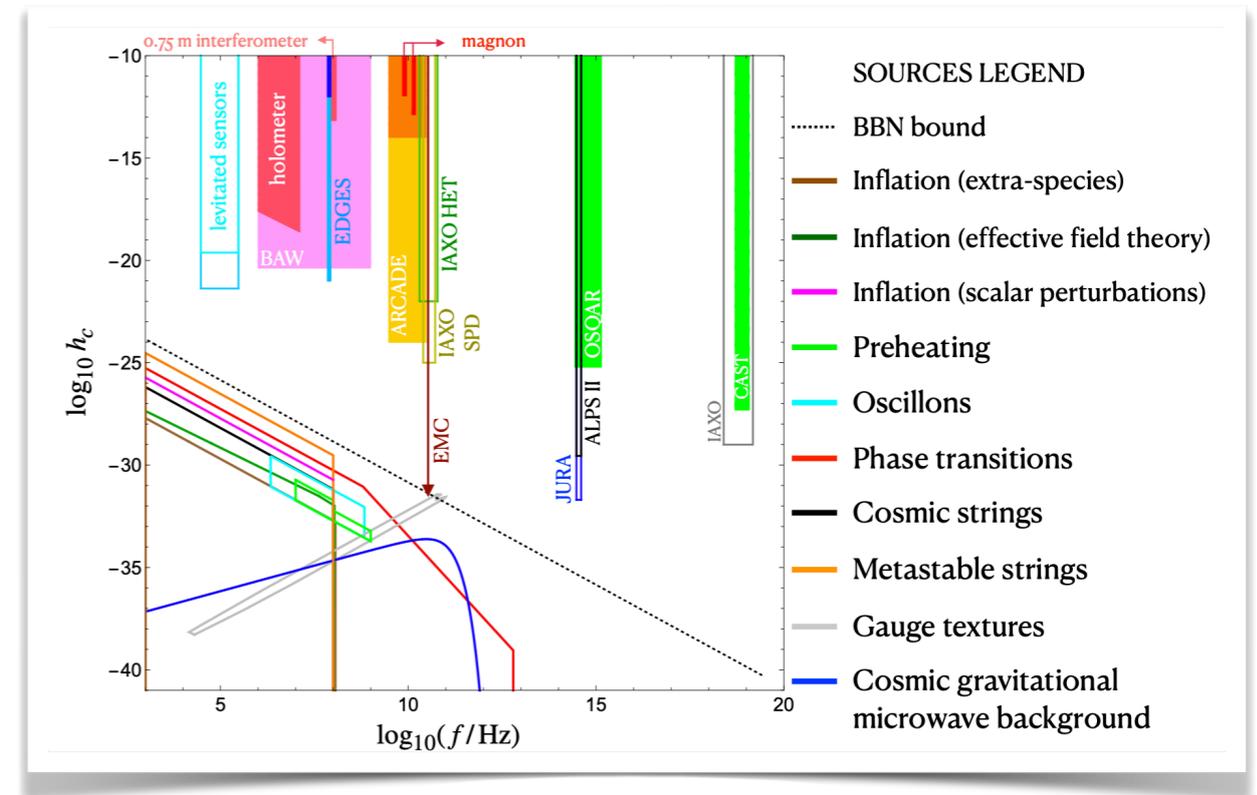
Backup Slides

# Sources

## Late Universe



## Early Universe



[Aggarwal+ 2020]

# Sources: PBH Binaries

Nearby PBH merger would produce a detectable signal

$$h_{+, \times}^{\text{PBH}}(f, m_{\text{PBH}}, D) \simeq 1.3 \times 10^{-23} \left( \frac{10 \text{ kpc}}{D} \right) \left( \frac{m_{\text{PBH}}}{10^{-5} M_{\odot}} \right)^{5/3} \left( \frac{f}{100 \text{ MHz}} \right)^{2/3}$$

Cosmological rate is roughly set by

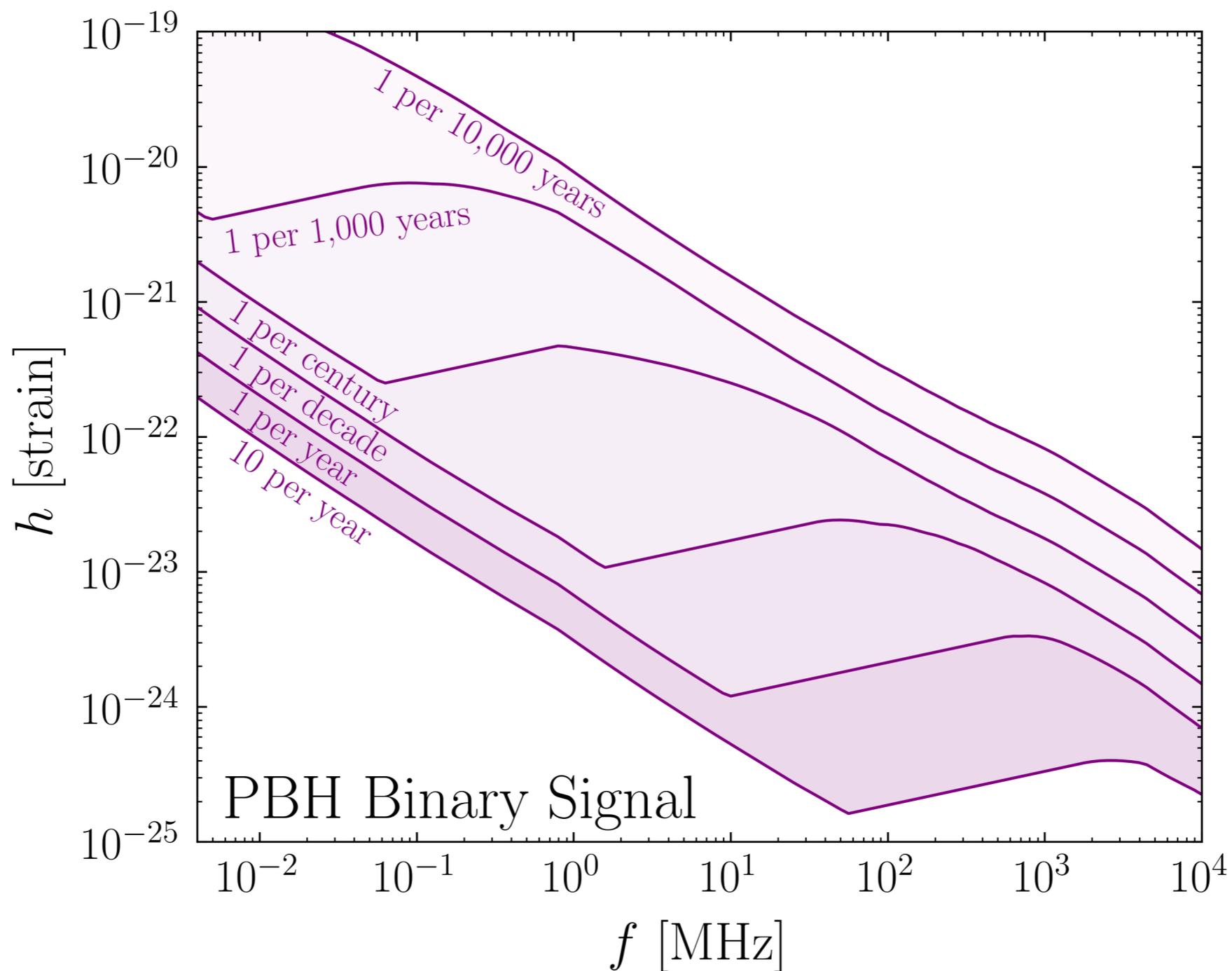
$$R_0(m_{\text{PBH}}, f_{\text{PBH}}) \simeq 6.6 \times 10^{-8} \text{ kpc}^{-3} \text{ yr}^{-1} f_{\text{PBH}}^{53/37} \left( \frac{m_{\text{PBH}}}{10^{-5} M_{\odot}} \right)^{-32/37} S_{\text{early}}(f_{\text{PBH}}) S_{\text{late}}(f_{\text{PBH}})$$

$$S_{\text{early}}(f_{\text{PBH}}) = \min \left\{ 1, \left( \frac{f_{\text{PBH}}}{0.01} \right)^{1/2} \right\}$$

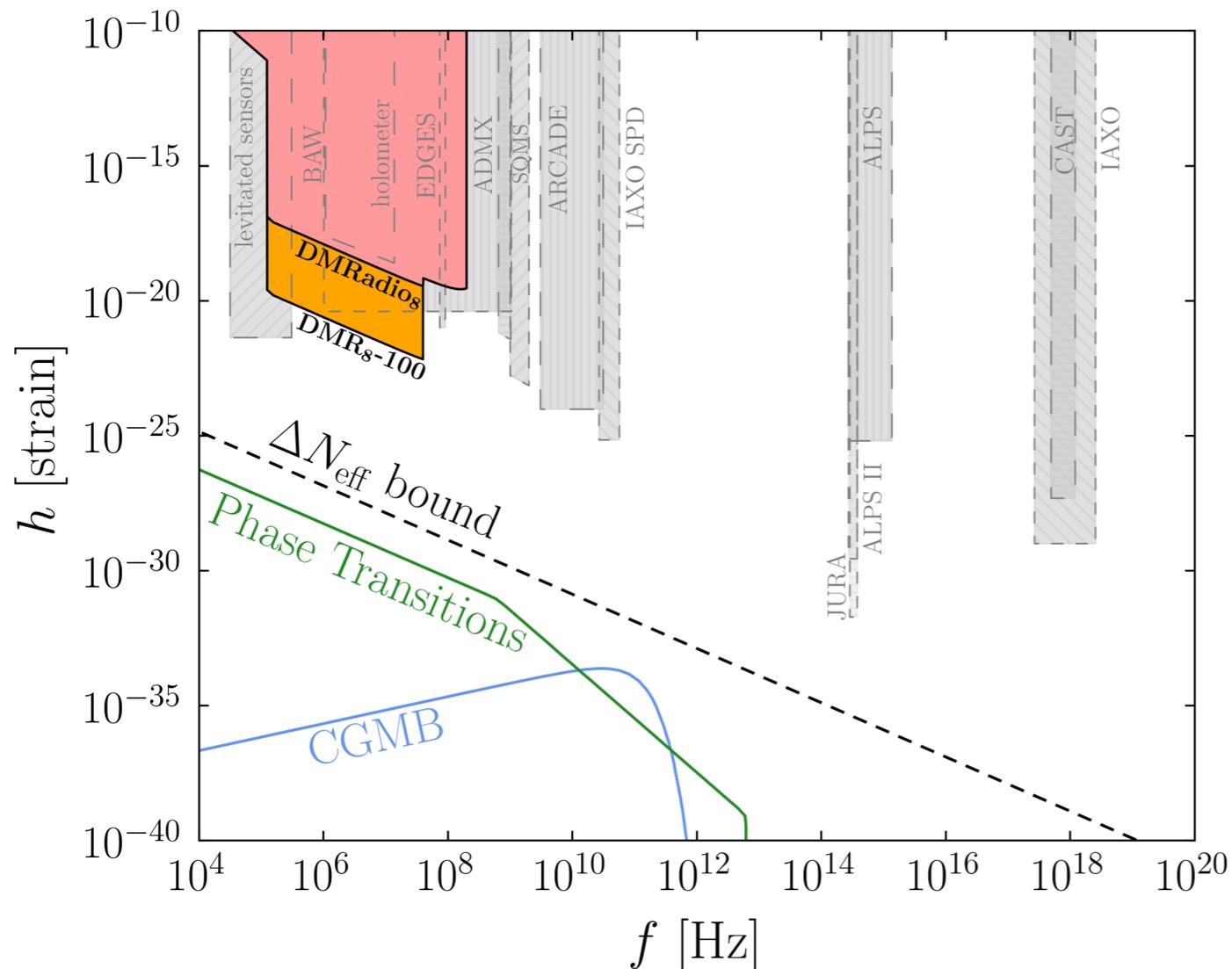
$$S_{\text{late}}(f_{\text{PBH}}) = \min \left\{ 1, 9.6 \times 10^{-3} f_{\text{PBH}}^{-0.65} e^{0.03 \ln^2 f_{\text{PBH}}} \right\}$$

Account for Milky Way overdensity and obtain a rate

# Sources: PBH Binaries



# Sources: Phase Transitions



Temperature of PT

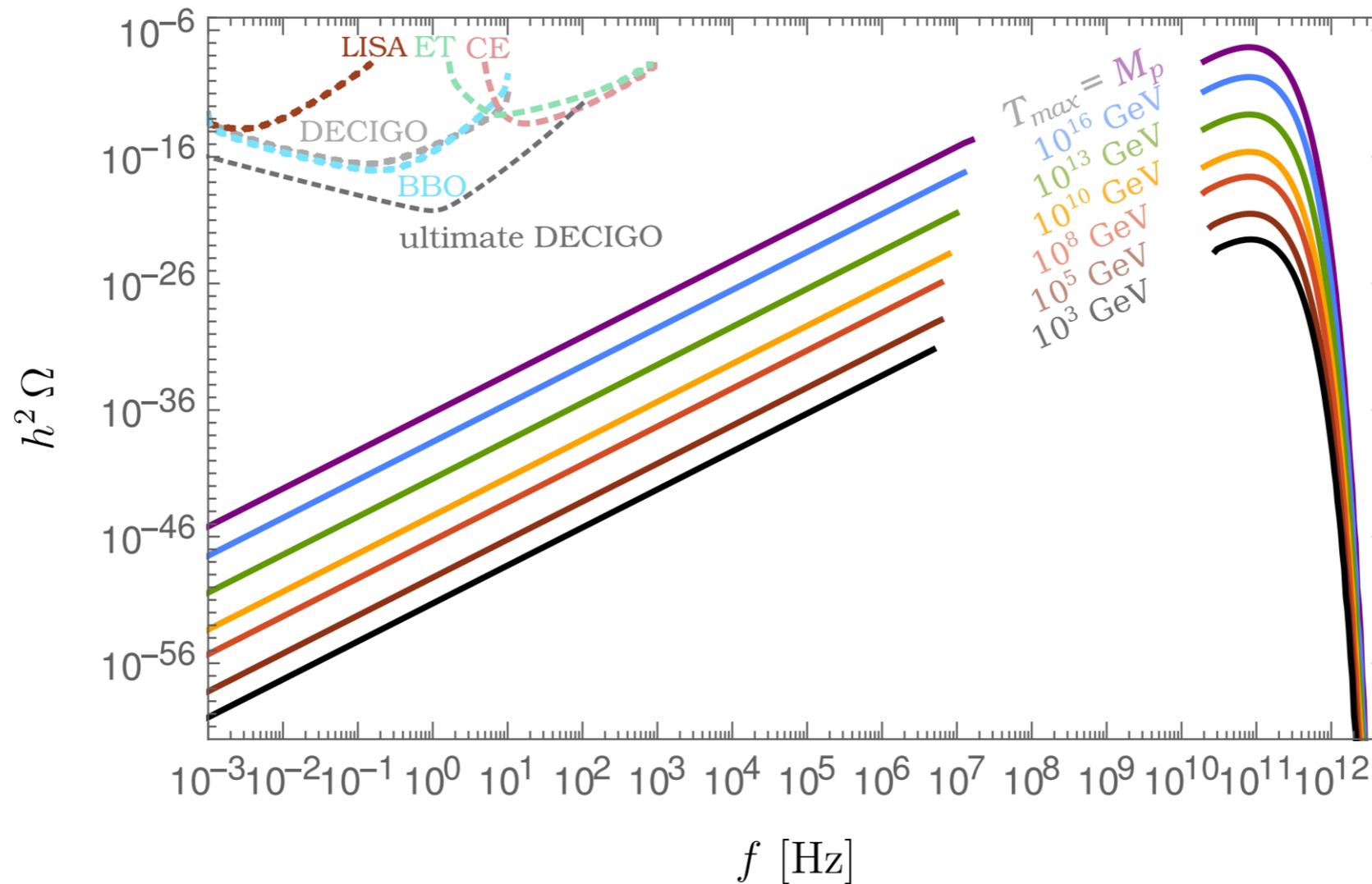
$$f \simeq 3 \text{ GHz} \left( \frac{1}{\epsilon_\star} \right) \left( \frac{T_\star}{10^{16} \text{ GeV}} \right)$$

$$\epsilon_\star = \lambda_\star H_\star$$

$$\lambda_\star = \text{GW wavelength at PT}$$

# Sources: CGMB

Blackbody only if  $T_{RH} > M_{Pl}$  - arises from non-thermal emission



[Ringwald, Schütte-Engel, Tamarit 2020]

# Proper Detector Frame

In the TT gauge GW satisfies

$$h_{0\mu} = 0, \quad h^{\mu}_{\mu} = 0, \quad \partial^{\mu} h_{\mu\nu} = 0$$

Treating the GW as a plane wave, takes the form

$$h_{ij}^{\text{TT}} = \left[ (U_i U_j - V_i V_j) h^+ + (U_i V_j + V_i U_j) h^{\times} \right] \frac{e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}}{\sqrt{2}},$$

$$\hat{\mathbf{k}} = s_{\theta_h} \hat{\mathbf{e}}_{\rho}^{\phi_h} + c_{\theta_h} \hat{\mathbf{e}}_z, \quad \mathbf{V} = \hat{\mathbf{e}}_{\phi}^{\phi_h}, \quad \mathbf{U} = \mathbf{V} \times \hat{\mathbf{k}}$$

Incident angle  $(\theta_h, \phi_h)$

Convention

TT frame is not a locally inertial coordinate system:  
description of experimental apparatus complex

# Proper Detector Frame

Use Fermi normal coordinates

Locally inertial coordinates  
along a geodesic [Fermi 1922]

$$h_{ij} = -2 \sum_{n=0}^{\infty} \frac{n+1}{(n+3)!} \hat{R}_{ikjl, m_1 \dots m_n} r_k r_l r_{m_1} \dots r_{m_n},$$

$$h_{0i} = -2 \sum_{n=0}^{\infty} \frac{n+2}{(n+3)!} \hat{R}_{0kil, m_1 \dots m_n} r_k r_l r_{m_1} \dots r_{m_n},$$

$$h_{00} = -2 \sum_{n=0}^{\infty} \frac{n+3}{(n+3)!} \hat{R}_{0k0l, m_1 \dots m_n} r_k r_l r_{m_1} \dots r_{m_n}$$

$\hat{R}$  is evaluated at the  
coordinate origin

[Fortini and Gualdi 1982], [Marzlin 1994], [Rakhmanov 2014]

# Proper Detector Frame

Proper detector frame:  
Fermi normal coordinates transformed to the non-inertial reference frame of the detector

[Ni, Zimmermann 1978]

Non-inertial corrections (Earth's gravity, Coriolis effect, etc) are irrelevant at higher frequencies - effectively can just use Fermi normal coordinates

# Proper Detector Frame

All orders currents for a toroidal magnetic field

$$j_\phi = \frac{\omega^2 B_{\max} R}{\rho} \left[ \frac{e^\kappa}{\kappa} - \frac{2e^\kappa}{\kappa^2} + \frac{2(e^\kappa - 1)}{\kappa^3} \right] (z h_{\rho\phi}^{\text{TT}}|_{\mathbf{r}=0} - \rho h_{\phi z}^{\text{TT}}|_{\mathbf{r}=0}),$$

$$j_\rho = \frac{\omega^2 B_{\max} R}{\rho} \left( \left[ -\frac{1}{2} - \frac{1}{\kappa} + \frac{2e^\kappa}{\kappa^2} + \frac{2(1 - e^\kappa)}{\kappa^3} \right] (\rho h_{\rho z}^{\text{TT}}|_{\mathbf{r}=0} + z h_{zz}^{\text{TT}}|_{\mathbf{r}=0}) \right. \\ \left. + \left[ \frac{e^\kappa}{\kappa} + \frac{2}{\kappa^2} + \frac{2(1 - e^\kappa)}{\kappa^3} \right] (z h_{\rho\rho}^{\text{TT}}|_{\mathbf{r}=0} + z h_{zz}^{\text{TT}}|_{\mathbf{r}=0}) \right. \\ \left. + i k_z \left[ \frac{1}{2\kappa} + \frac{1}{2\kappa^2} - \frac{1 + 2e^\kappa}{\kappa^3} + \frac{3(e^\kappa - 1)}{\kappa^4} \right] r_i r_j h_{ij}^{\text{TT}}|_{\mathbf{r}=0} \right),$$

$$\kappa = i\mathbf{k} \cdot \mathbf{r}$$

$$h_{\rho\rho}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left( -h^+ (\sin^2(\phi - \phi_h) - \cos^2(\phi - \phi_h) \cos^2 \theta_h) + 2h^\times \cos \theta_h \cos(\phi - \phi_h) \sin(\phi - \phi_h) \right),$$

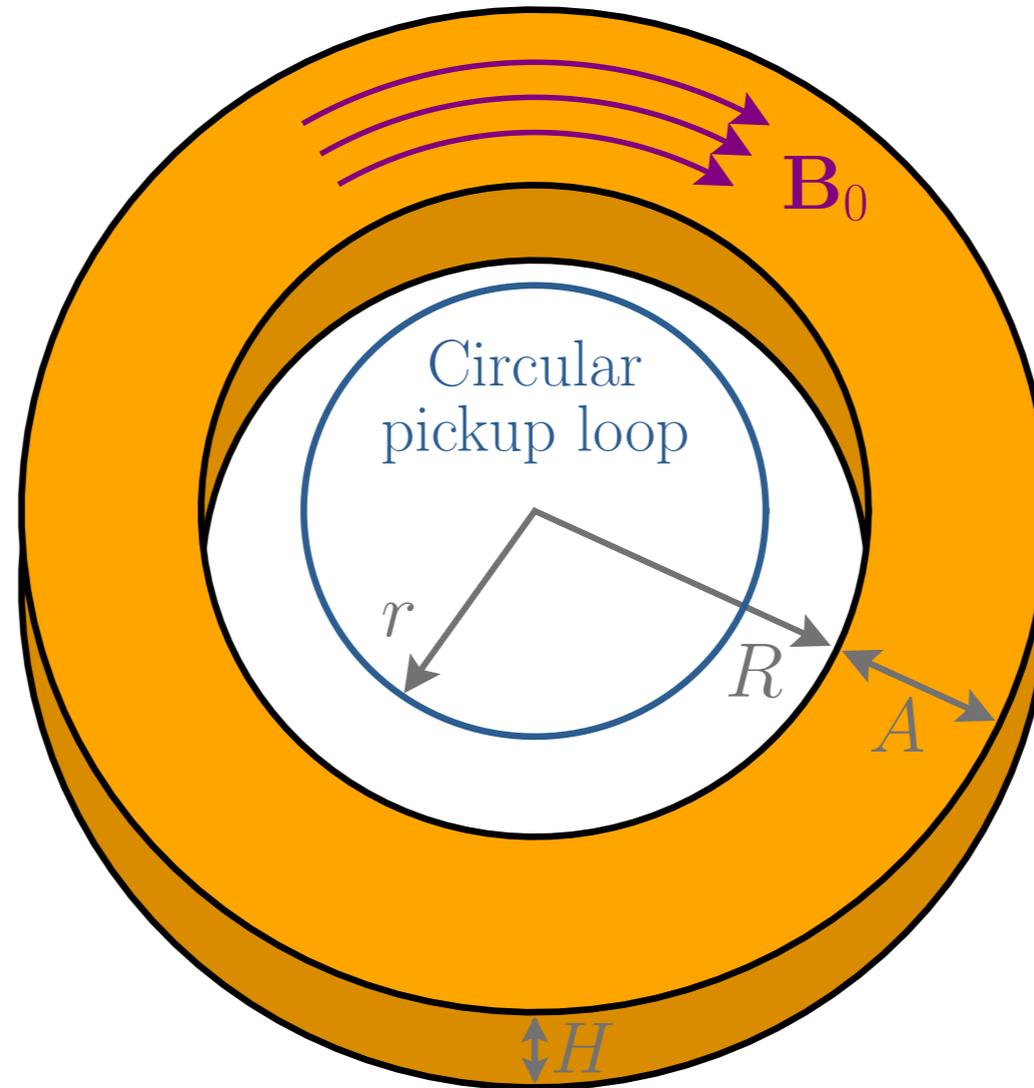
$$h_{\rho\phi}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left( -h^+ (1 + \cos^2 \theta_h) \sin(\phi - \phi_h) \cos(\phi - \phi_h) + h^\times \cos(2(\phi - \phi_h)) \cos \theta_h \right),$$

$$h_{\rho z}^{\text{TT}}|_{\mathbf{r}=0} = -\frac{e^{-i\omega t}}{\sqrt{2}} \left( h^+ \cos \theta_h \sin \theta_h \cos(\phi - \phi_h) + h^\times \sin \theta_h \sin(\phi - \phi_h) \right),$$

$$h_{\phi z}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left( h^+ \cos \theta_h \sin \theta_h \sin(\phi - \phi_h) - h^\times \sin \theta_h \cos(\phi - \phi_h) \right),$$

$$h_{zz}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} h^+ \sin^2 \theta_h.$$

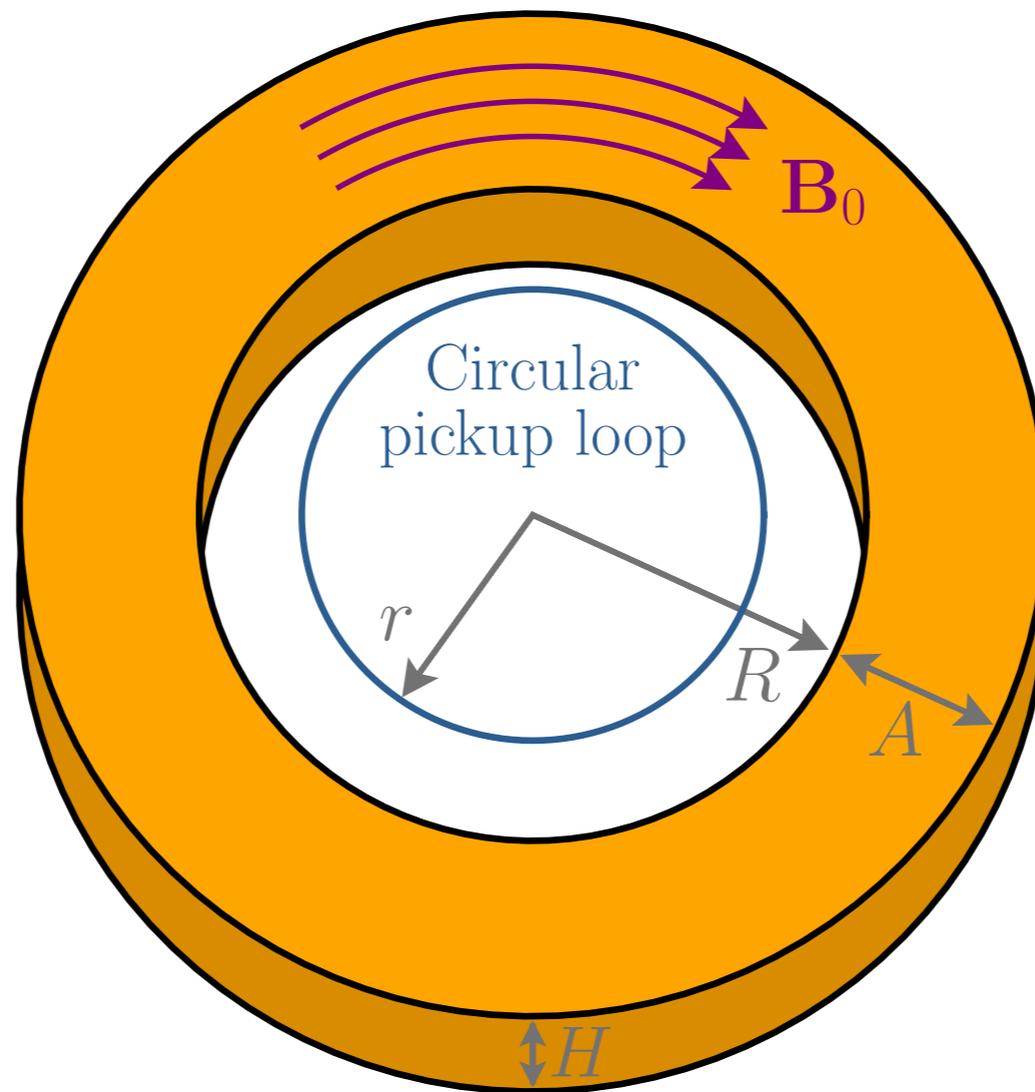
# Detection Strategy



cf. axion  
 $\Phi_a(t) \sim g_{a\gamma\gamma} (\partial_t a) B_0 V$

$$\Phi_h(t) \simeq \frac{ie^{-i\omega t}}{16\sqrt{2}} \omega^3 h^\times B_0 \pi r^2 R A (A + 2R) \sin^2 \theta_h \sim \omega^3 h B_0 V^{5/3}$$

# Detection Strategy

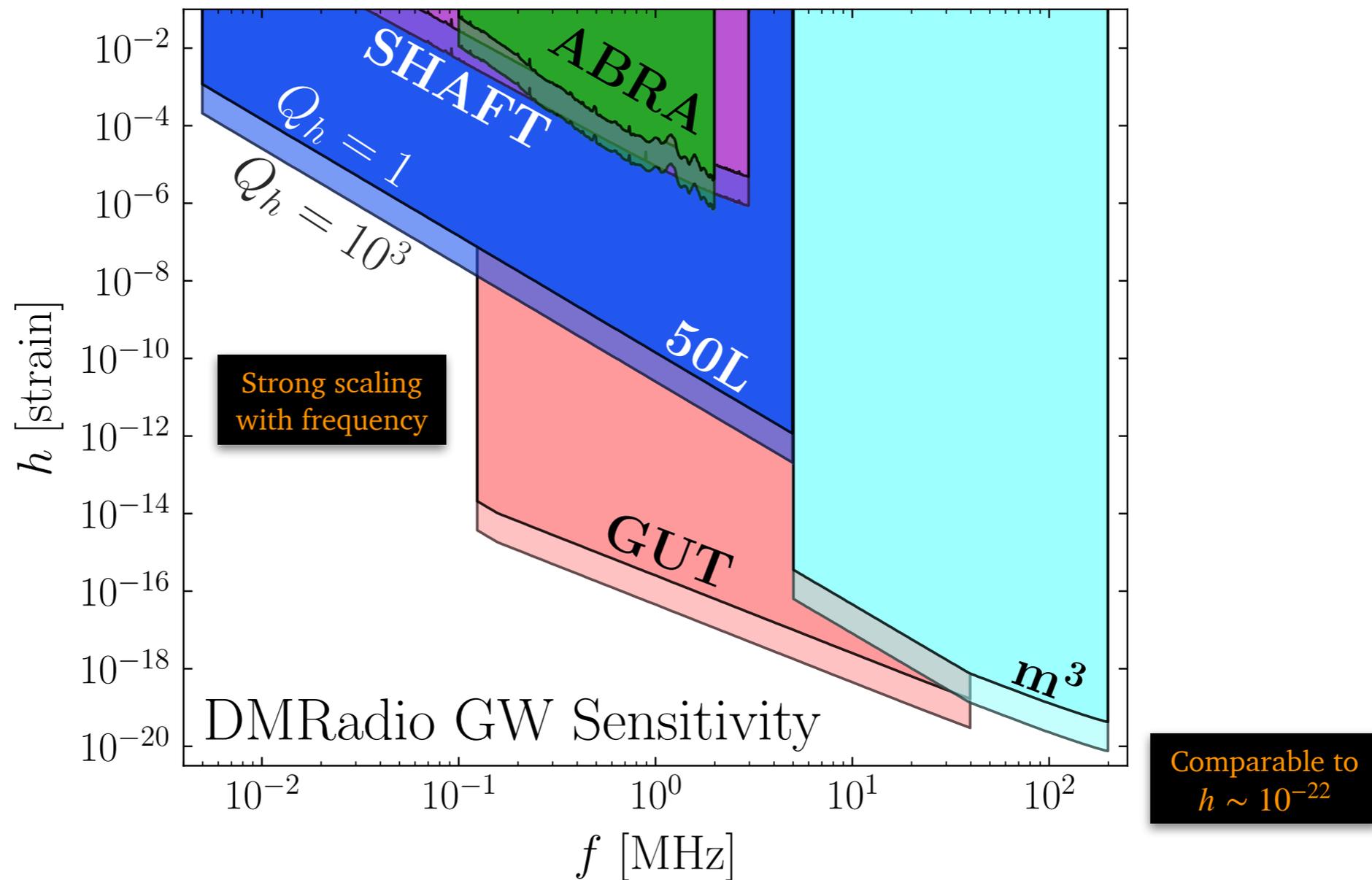


$\mathcal{O}(\omega^2)$  contribution  
has vanished!

$$\Phi_h(t) \simeq \frac{ie^{-i\omega t}}{16\sqrt{2}} \omega^3 h^\times B_0 \pi r^2 R A (A + 2R) \sin^2 \theta_h \sim \omega^3 h B_0 V^{5/3}$$

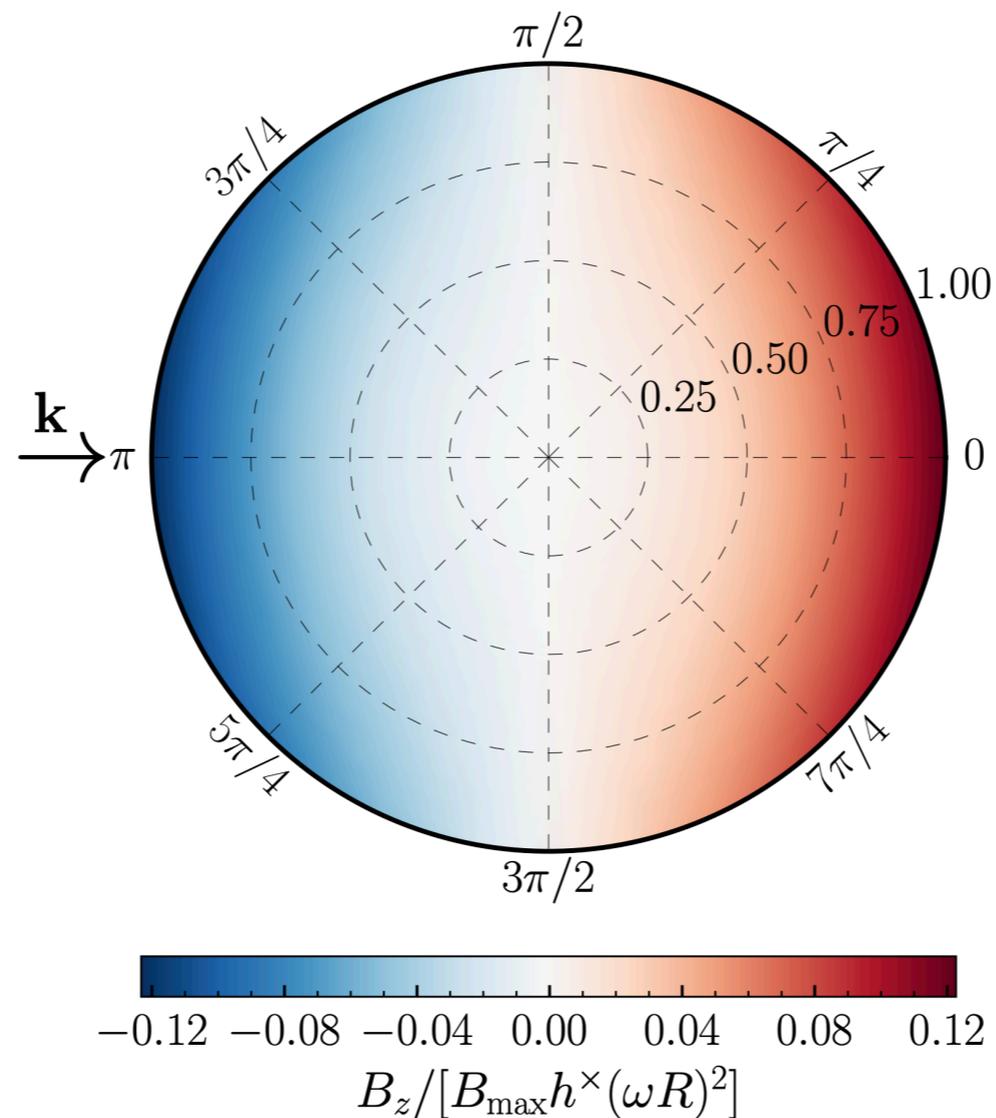
# Circular pickup loop reach

Sensitivity set by  $\Phi_h = \Phi_a (Q_a/Q_h)^{1/4}$



# Optimized Reach

What happened to the leading contribution?



Leading contribution vanishes when integrated over a circular pickup loop

*But why?* [Ongoing work] due to symmetry (especially parity)

# What could CAST see?

Detecting gravitons requires an extremely large flux

$$p(g \rightarrow \gamma) \simeq 4.6 \times 10^{-35} \left( \frac{B}{9 \text{ T}} \right)^2 \left( \frac{L}{9.26 \text{ m}} \right)^2$$

Could be produced by a nearby PBH inspiral, naively require

$$m \simeq 2 \times 10^{-15} M_{\odot}, \quad d \simeq 1.4 \times 10^6 \text{ m} \simeq 0.2 R_{\oplus}$$

At this mass, PBHs can be 100% of DM [Carr, Kühnel 2021]  
Final merger generate  $f \sim \text{keV}$  gravitons

Distance incorporates the extremely short duration of such a merger

But in principle possible, unlike examples Dyson gave

# What could CAST see?

---

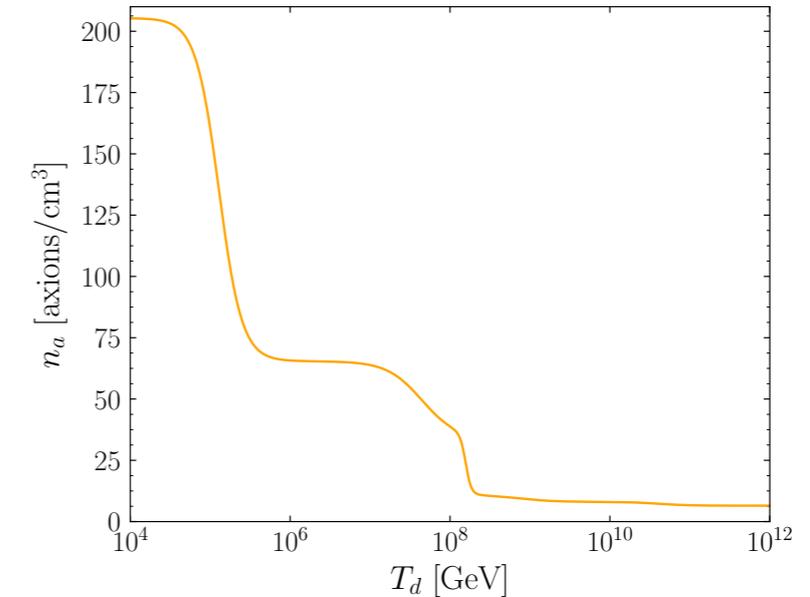
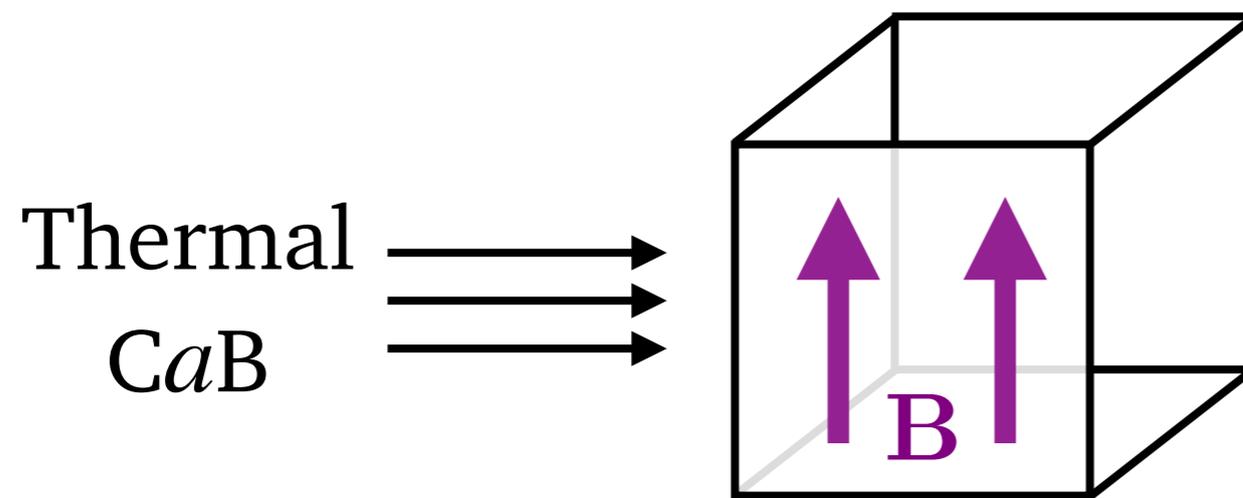
Would such a detection prove a GW is made of quanta? **No**

Even if the GW was perfectly antibunched,  $1\sigma$  evidence for a quantized signal would require the following number of events

$$N \gtrsim p(g \rightarrow \gamma)^{-2} \simeq 10^{70}$$

Cf. the proposal on the last slide:  $N = 1$

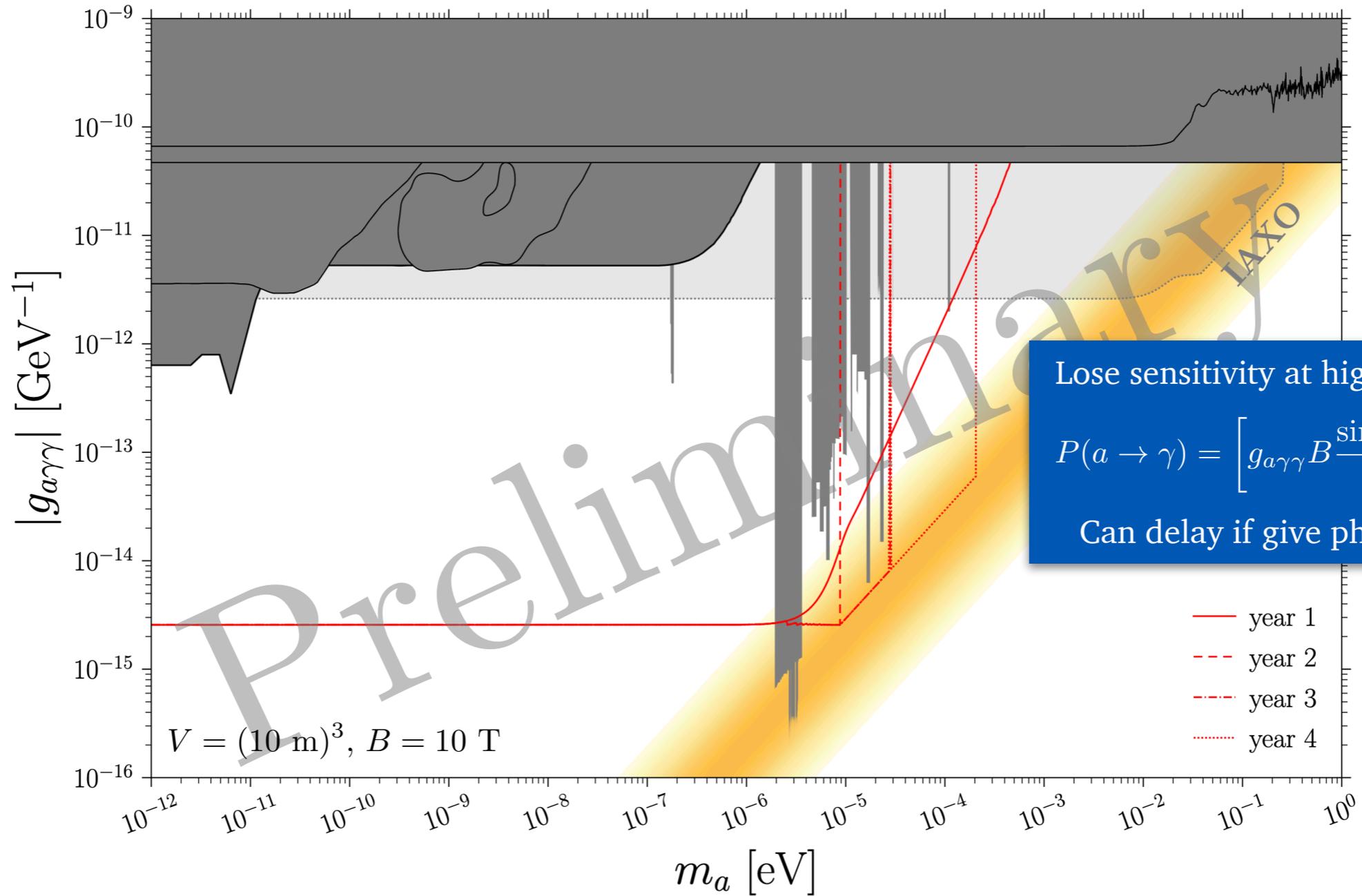
# Detecting the Thermal $CaB$



$$N(a \rightarrow \gamma) \simeq 2.5 \times 10^9 \left( \frac{g_{a\gamma\gamma}}{10^{-10} \text{ GeV}^{-1}} \right)^2 \left( \frac{B}{10 \text{ T}} \right)^2 \left( \frac{L}{10 \text{ m}} \right)^4 \left( \frac{t_{\text{exp}}}{1 \text{ year}} \right)$$

Potentially detectable!

# Detecting the Thermal $CaB$



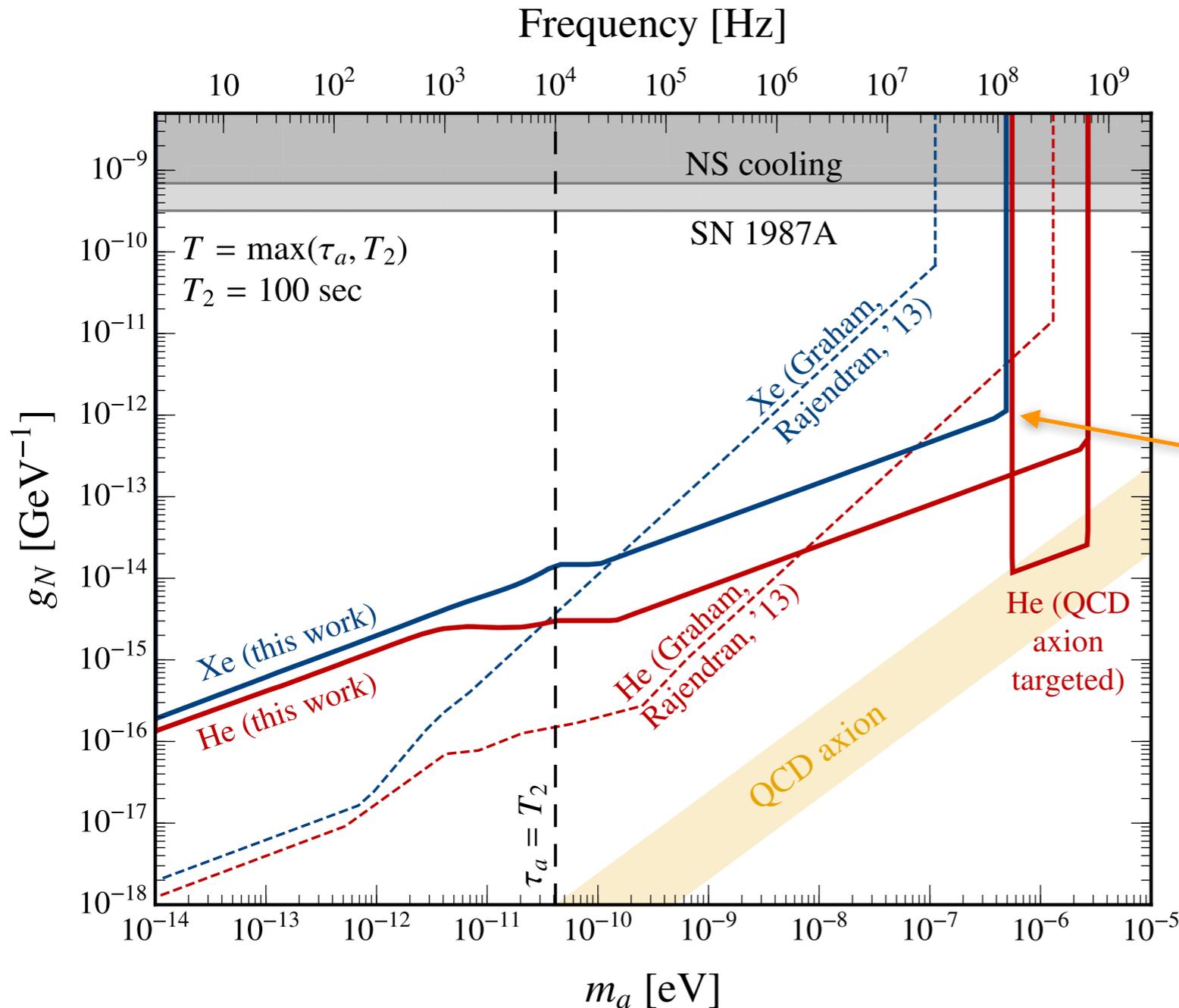
Lose sensitivity at higher masses as

$$P(a \rightarrow \gamma) = \left[ g_{a\gamma\gamma} B \frac{\sin(m_a^2 L / 2\omega_a)}{m_a^2 / \omega_a} \right]^2$$

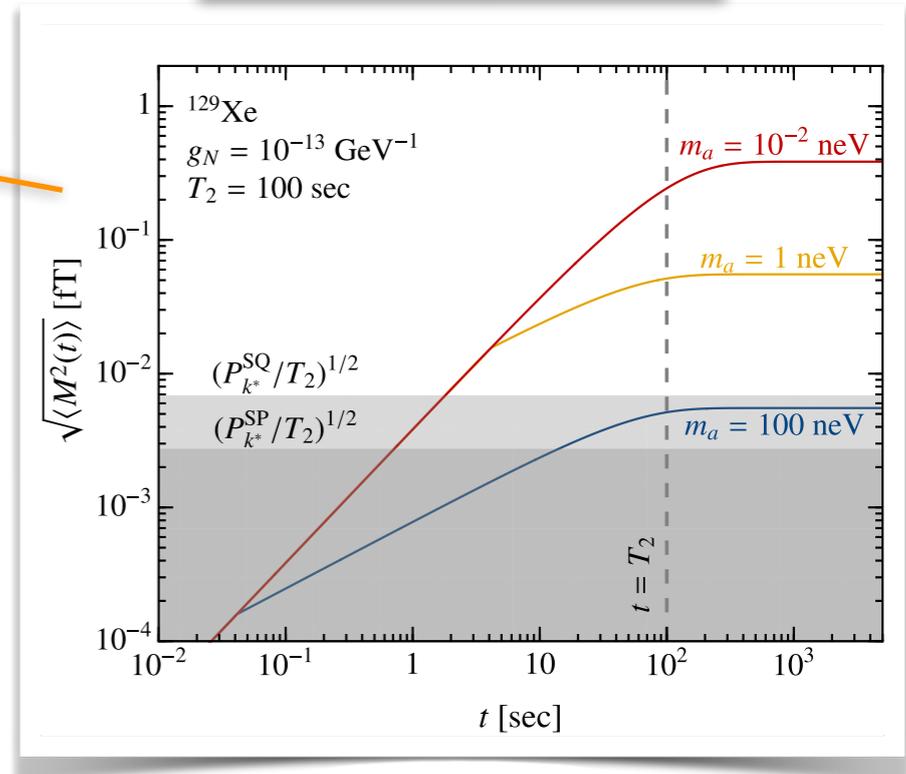
Can delay if give photon a mass

# Axion NMR

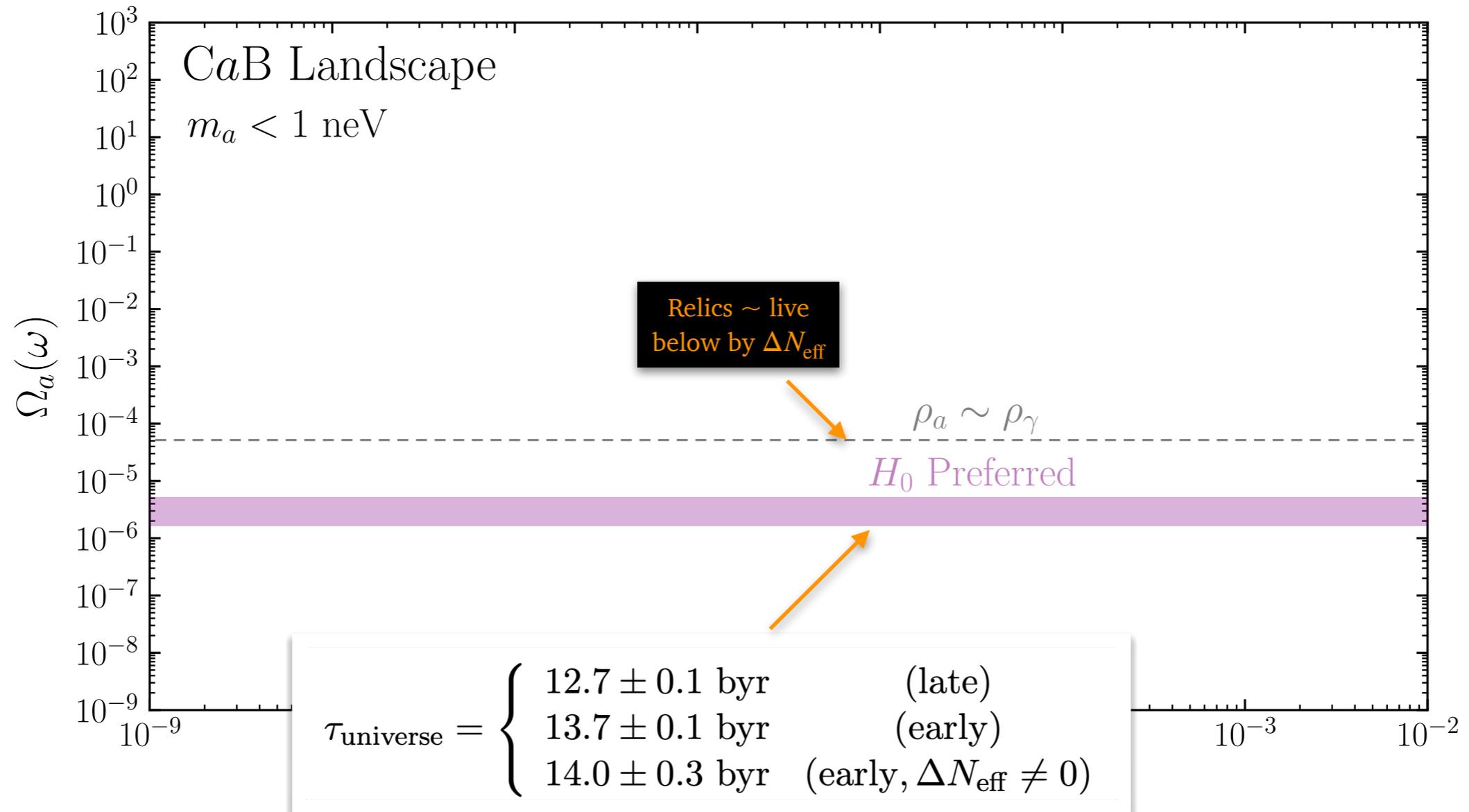
$$\mathcal{L} \supset g_N (\partial_\mu a) \bar{N} \gamma^\mu \gamma_5 N$$



Axion signal continues to grow for  $\tau_a < T < T_2$



# Hubble Tension



See e.g. [Valentino+ “In the Realm of the Hubble tension – a Review of Solutions” 2021]  
 Not a finalist in the  $H_0$  Olympics  
 [Schoneberg+ 2021]



# Bose Enhancement

Relevant when  $f_a \gg 1$

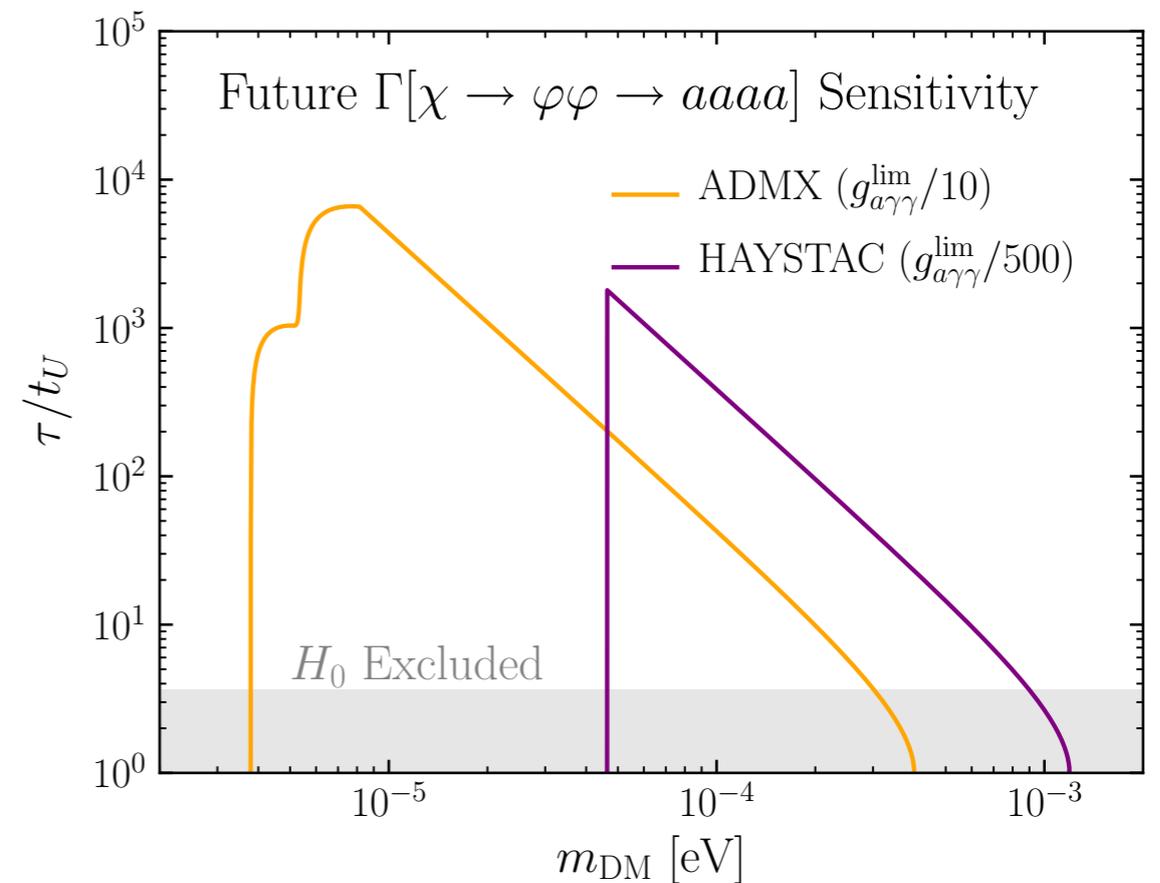
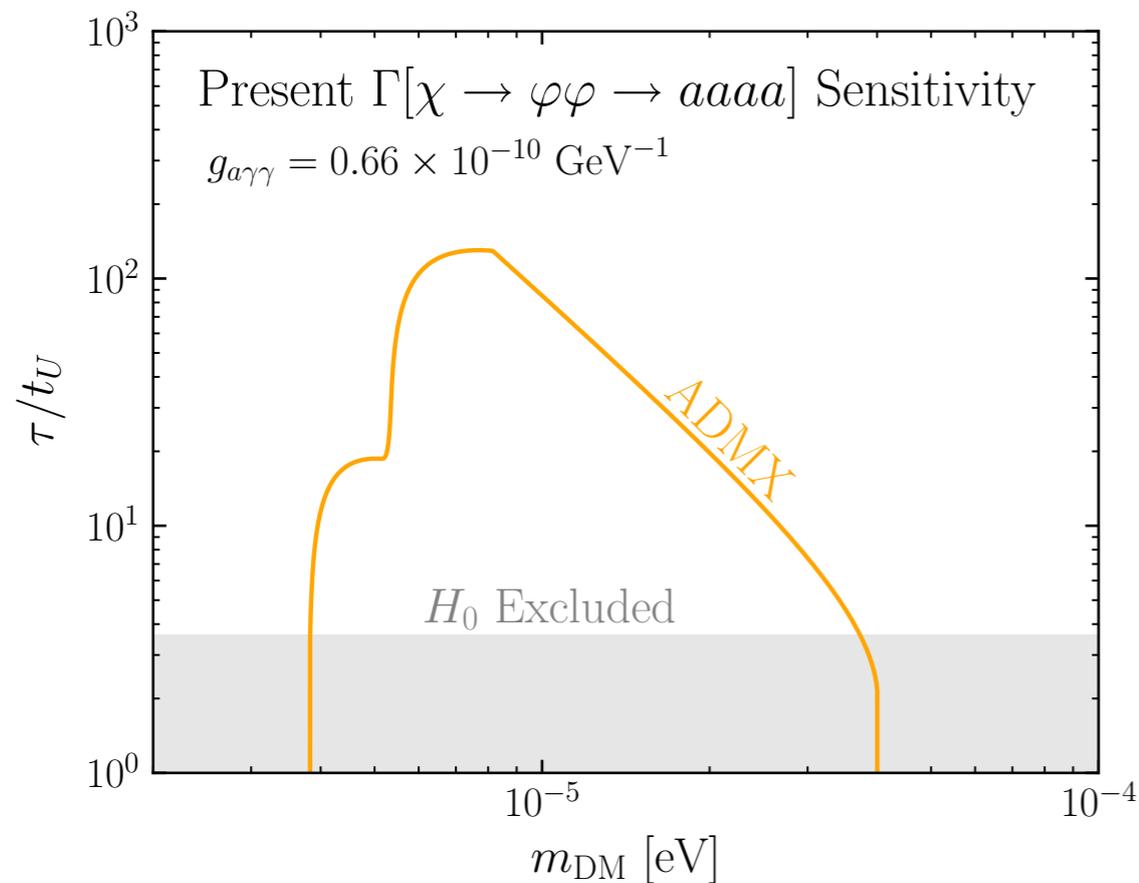
$$f_a = \frac{2\pi^2}{\omega^3} \frac{d\rho_a}{d\omega} \simeq 4 \times 10^{10} \left( \frac{Q_a}{1} \right) \left( \frac{\rho_a}{\rho_\gamma} \right) \left( \frac{\bar{\omega}}{1 \mu\text{eV}} \right)^{-4}$$

Large over the entire range we consider

# Bose Enhancement

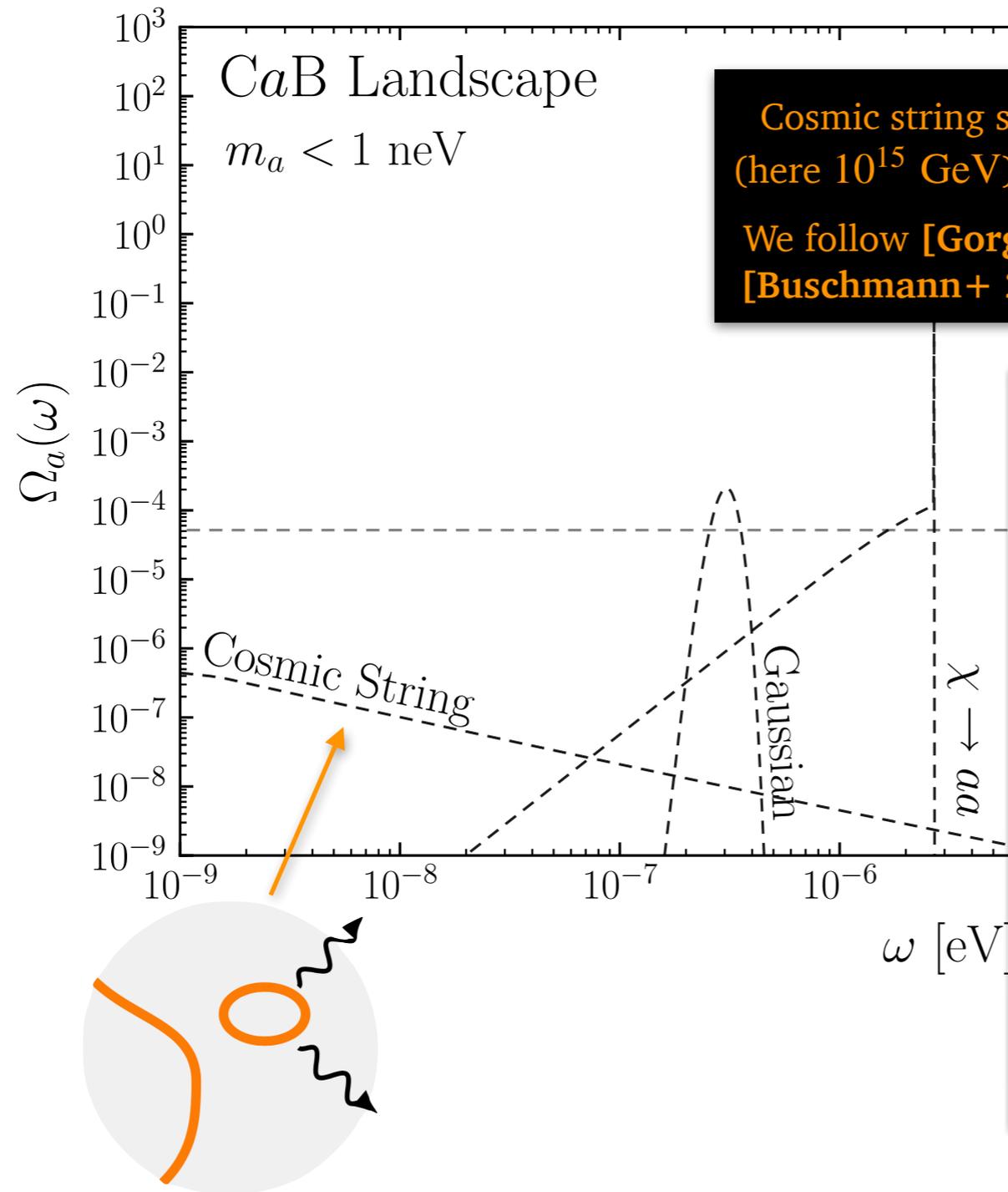
Can lift with cascade decays

Exploit daily modulation of the signal

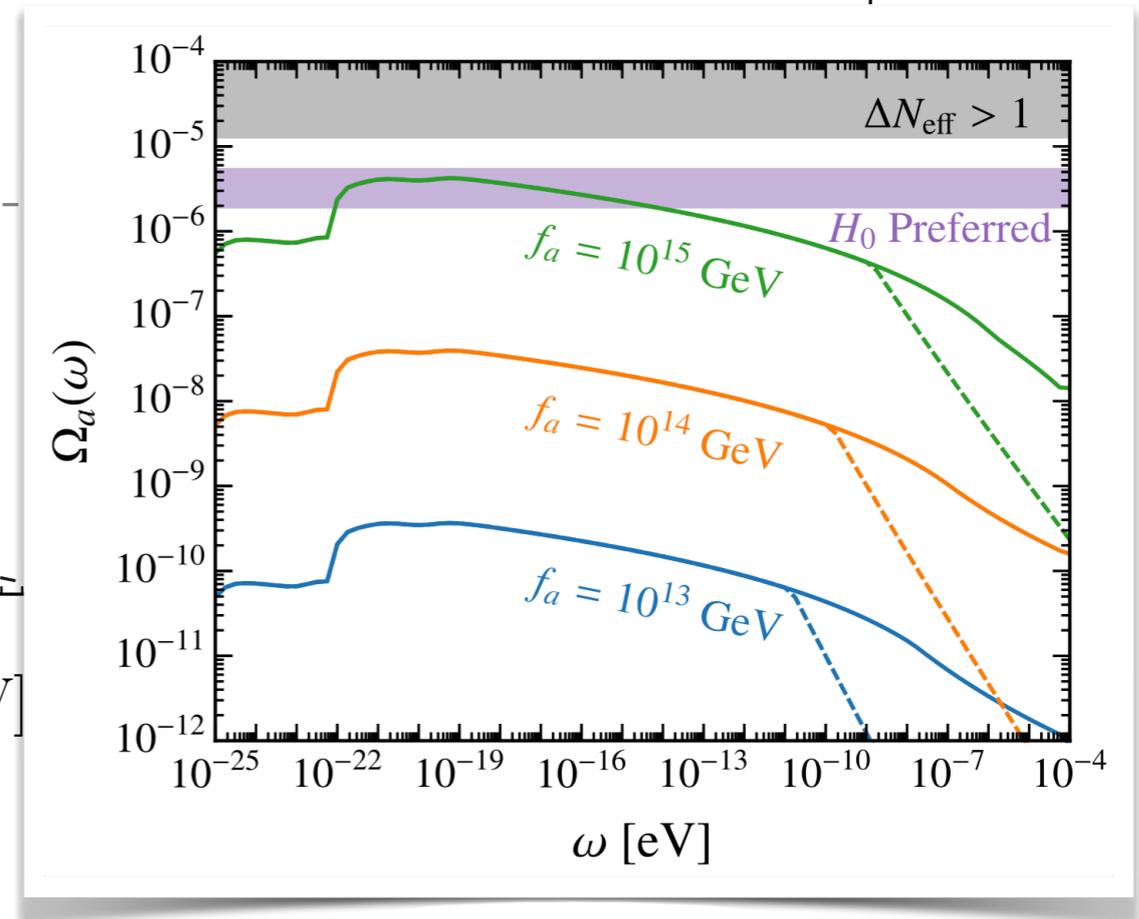


Finalizing this analysis with ADMX

# Cosmic Strings



Cosmic string spectrum depends on the symmetry breaking scale  $f_a$  (here  $10^{15} \text{ GeV}$ ) and the exact distribution is an area of active debate  
 We follow [Gorghetto, Hardy, Villadoro 2018, 2020], but see also [Buschmann+ 2021] & [Dine, Fernandez, Ghalsasi, Patel 2020]



# Parametric Resonance

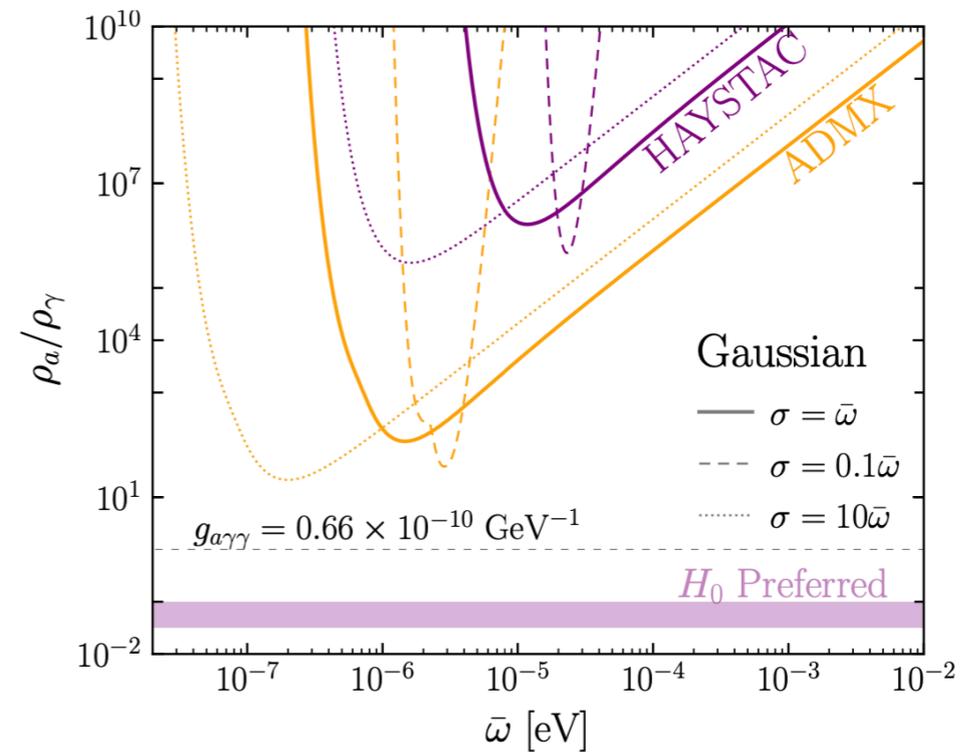
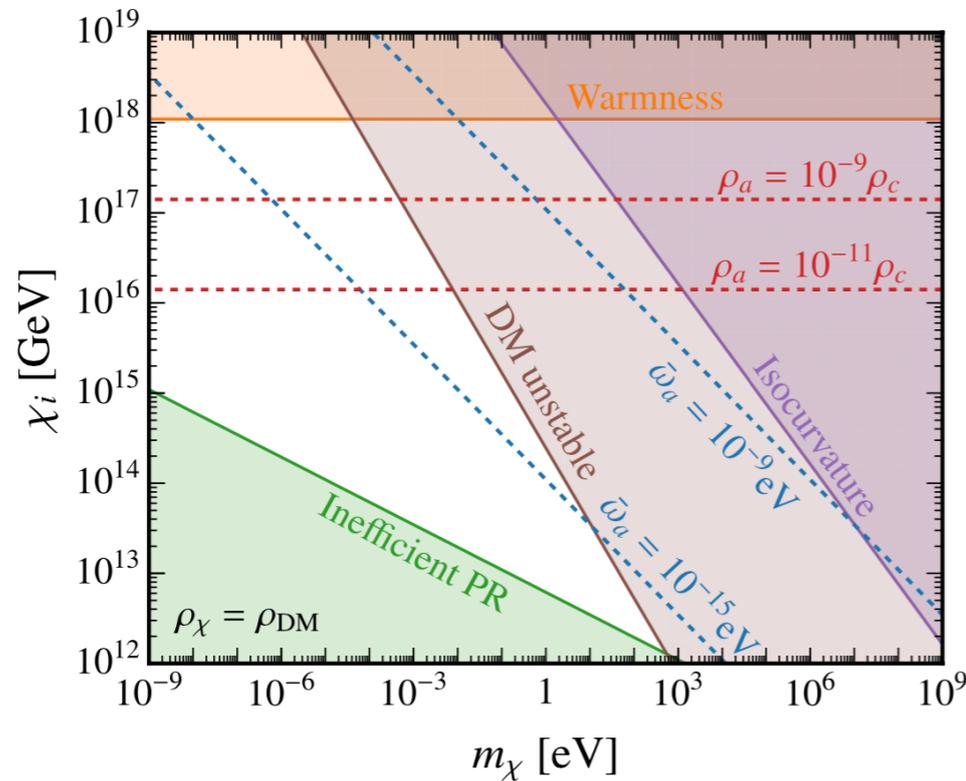
$$V(\Phi) = \lambda^2 \left( |\Phi|^2 - f_a^2/2 \right)^2$$

Oscillations when  $m_{\chi}^{\text{eff}}(\chi_i) \simeq \lambda \chi_i \sim H$

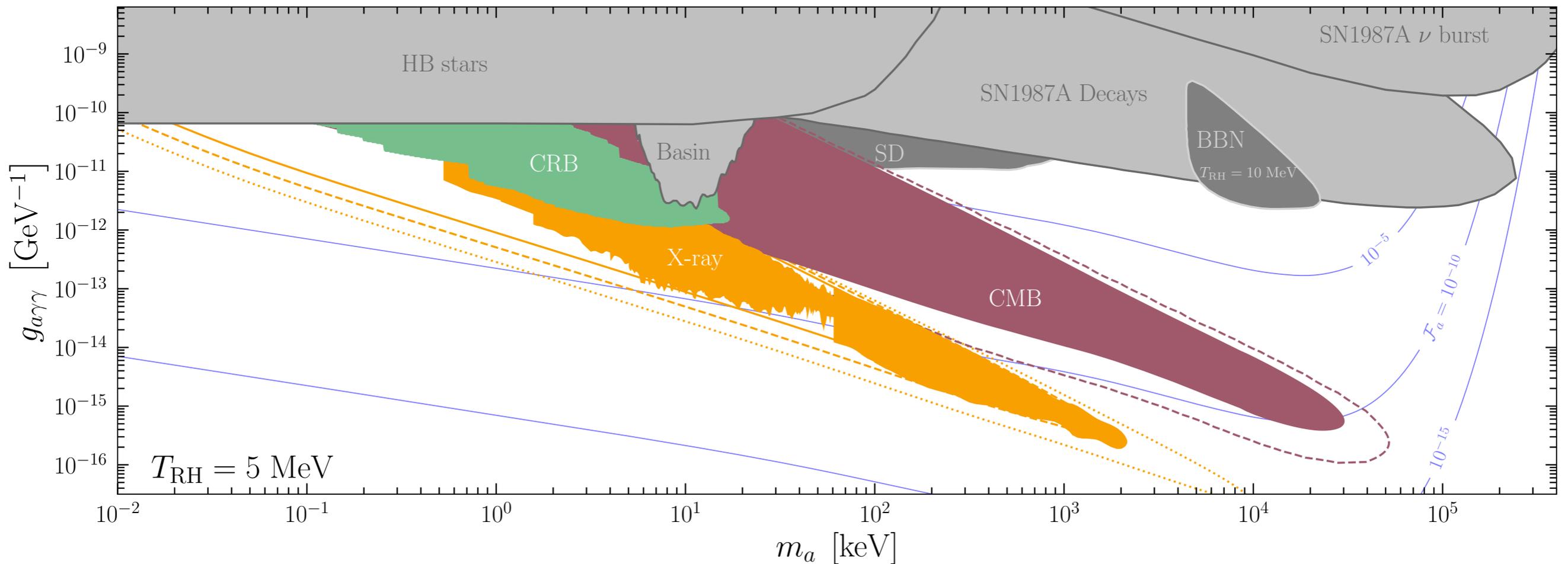
Typical energy:  $\bar{\omega}_a \sim m_{\chi}^{\text{eff}}(\chi_i) \left( \frac{s(T_0)}{s(T_{\text{osc}})} \right)^{1/3} \sim 10^{-15} \text{ eV} \left( \frac{m_{\chi}^{\text{eff}}(\chi_i)}{\text{MeV}} \right)^{1/2}$

Energy density:  $\Omega_a \sim 3 \times 10^{-7} \left( \frac{\chi_i}{M_{\text{Pl}}} \right)^2$  ← detectable?

Assume  $\chi$  dark matter



# The Irreducible Axion



# The Irreducible Axion

Take  $m_a = 10$  keV

Early Universe: photon conversion ( $\gamma e \rightarrow ae$ )  
freezes-in axions

$$\mathcal{F}_a \simeq 10^{-4} \left( \frac{g_{a\gamma\gamma}}{g_{a\gamma\gamma}^{\text{HB}}} \right)^2 \left( \frac{T_{\text{RH}}}{5 \text{ MeV}} \right) \quad (1)$$

$= \rho_a / \rho_{\text{DM}}$

UV dominated

# The Irreducible Axion

X-ray constraints at  $\sim 10$  keV require

$$\tau_{\text{DM}} \gtrsim 10^{29} \text{ s} \Rightarrow g_{a\gamma\gamma}^{\text{DM}} \lesssim 7 \times 10^{-19} \text{ GeV}^{-1} \simeq 10^{-8} g_{a\gamma\gamma}^{\text{HB}}$$

Must satisfy  $\rho_a/\tau_a \lesssim \rho_{\text{DM}}/\tau_{\text{DM}}$  and  $\tau^{-1} \propto g_{a\gamma\gamma}^2$ , so

$$\mathcal{F}_a \lesssim \frac{\tau_a}{\tau_{\text{DM}}} = \left( \frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}} \right)^2 \quad (2)$$

# The Irreducible Axion

$$\mathcal{F}_a \simeq 10^{-4} \left( \frac{g_{a\gamma\gamma}}{g_{a\gamma\gamma}^{\text{HB}}} \right)^2 \left( \frac{T_{\text{RH}}}{5 \text{ MeV}} \right) \quad (1)$$

$$\mathcal{F}_a \lesssim \frac{\tau_a}{\tau_{\text{DM}}} = \left( \frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}} \right)^2 \quad (2)$$

Combine (1) and (2)

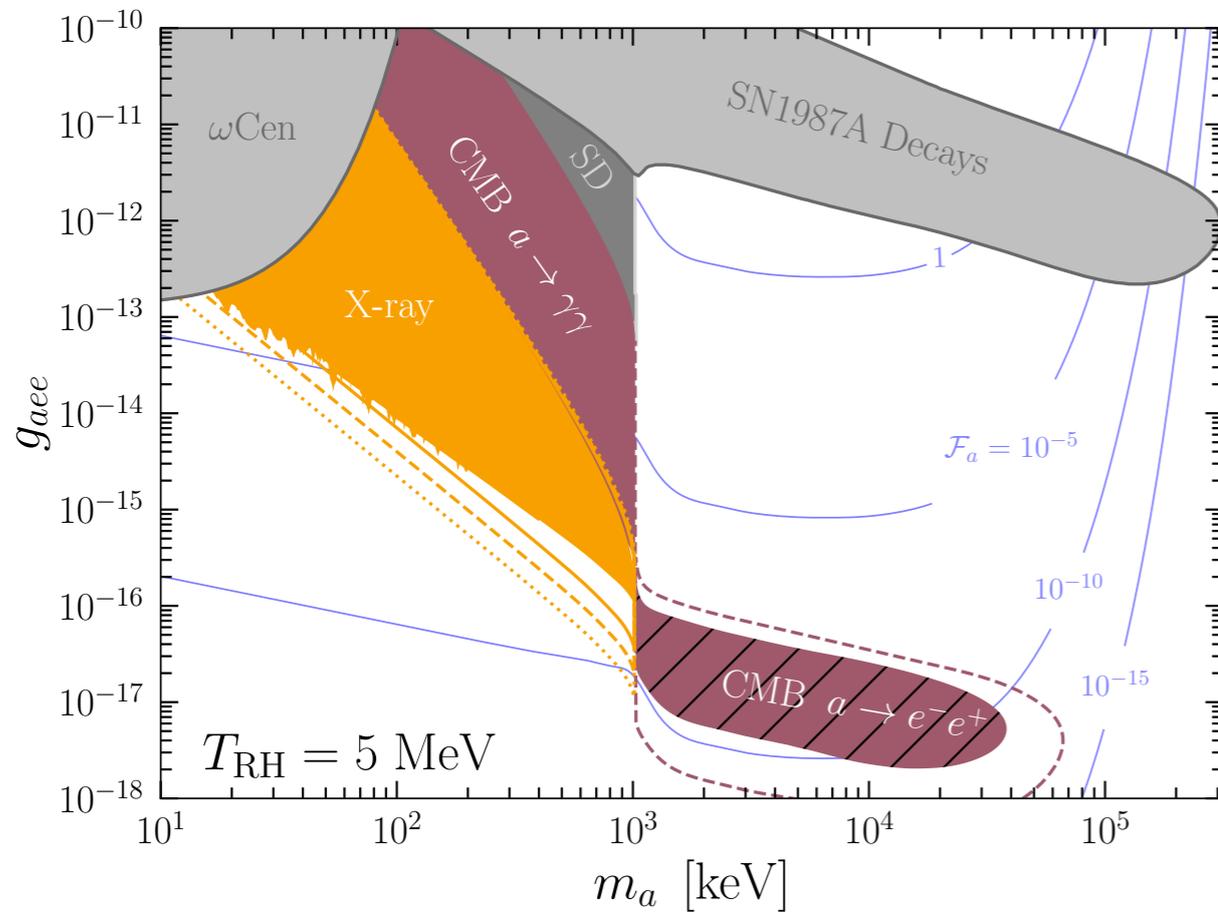
$$10^{-4} \left( \frac{g_{a\gamma\gamma}}{g_{a\gamma\gamma}^{\text{HB}}} \right)^2 \lesssim \left( \frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}} \right)^2$$

$$\Rightarrow \frac{g_{a\gamma\gamma}}{g_{a\gamma\gamma}^{\text{HB}}} \lesssim \left[ 10^4 \left( \frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}^{\text{HB}}} \right)^2 \right]^{1/4} \simeq (10^{-12})^{1/4} \simeq 10^{-3}$$

$$\Rightarrow g_{a\gamma\gamma} \lesssim 7 \times 10^{-14} \text{ GeV}^{-1} \ll g_{a\gamma\gamma}^{\text{HB}}$$

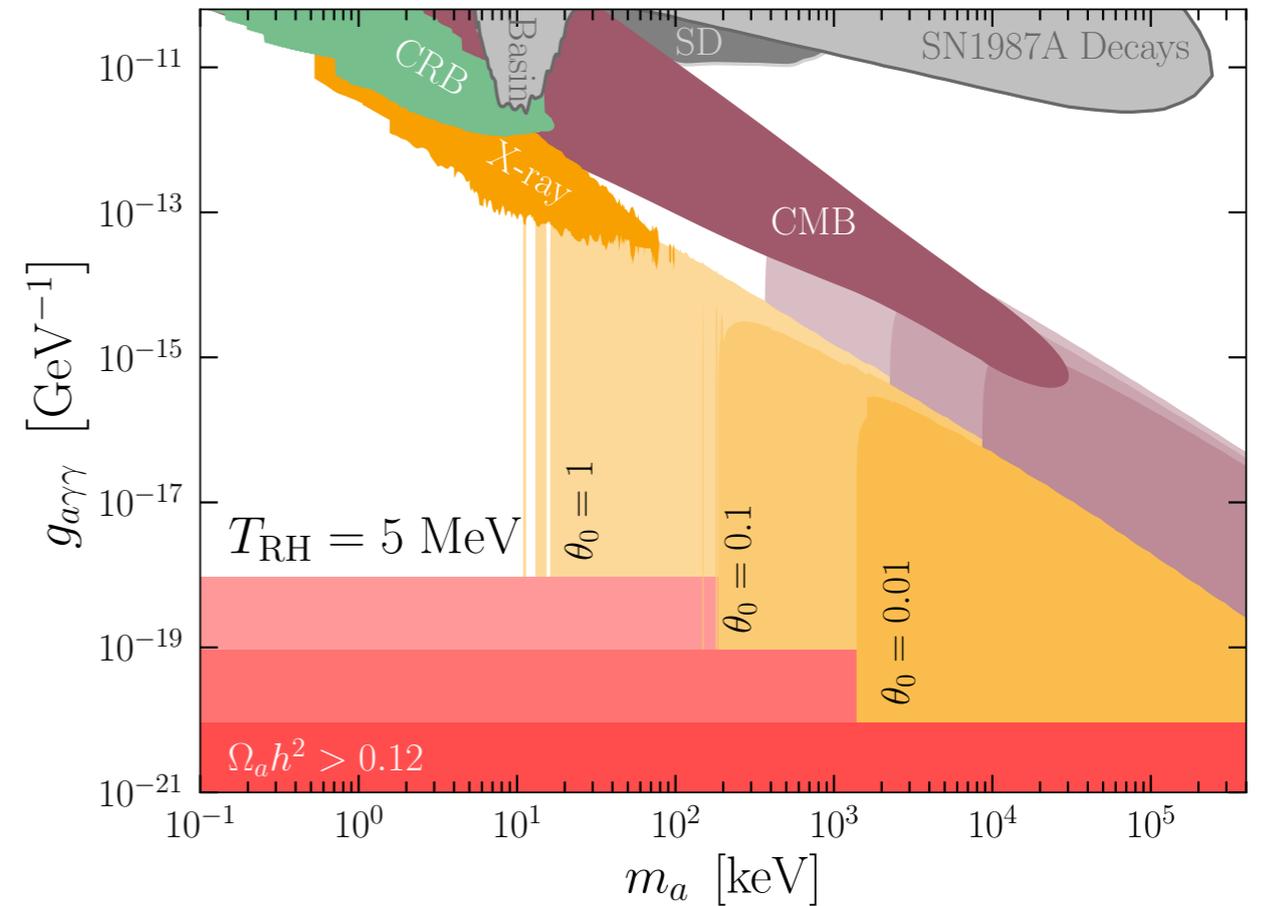
# The Irreducible Axion

## Electron Couplings



$$\frac{g_{aee}}{2m_e} (\partial_\mu a) \bar{e} \gamma^\mu \gamma_5 e$$

## With misalignment



# QCD Axion Mass

At  $T = 0$  one can compute the axion effective potential

$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \left( \frac{a}{2f_a} \right)}$$

Expanding out, we find

$$m_a = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} \simeq 0.5 \frac{m_\pi f_\pi}{f_a}$$

[Cortona, Hardy, Vega, Villadoro 2015]

# Axion Dark Matter

In an FRW universe, the axion evolves according to

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

For early times,  $H \gg m_a$ ,  $a(t) = a_0$

At late times,  $H \ll m_a$

$$a = \left[ \frac{R(H = m_a)}{R(t)} \right]^{3/2} a_0 \cos(m_a t)$$

The energy density,  $\rho \propto a^2$ , behaves like CDM

$$\rho(t) = \rho(H = m_a) \left[ \frac{R(H = m_a)}{R(t)} \right]^3$$

See, e.g. [Hook 2018]

# Axion Dark Matter

Signal Power of the QCD Axion

$$P_a \simeq 5 \text{ yW} \quad \text{at } 5 \text{ GHz}$$

$10^{-24}$

For a detailed discussion see [Brubaker 2018]

# SMACS 0723

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Cluster of galaxies about 4 billion light years away

