



Small-x improved TMD factorization

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Contents of the talk

- Gluon TMDs in the small-x limit

TMDs = transverse-momentum-dependent parton distributions

non-linear QCD evolution obtained from the JIMWLK equation

- Probing the gluon TMDs at colliders

improved TMD factorization framework

unification of BFKL and TMD physics at small x

- Unpolarized and linearly-polarized gluon TMDs

using (transverse) gluon polarization to look for saturation effects

Gluon TMDs at small x

Gluon TMDs and gauge links

- the naive operator definition is not gauge-invariant

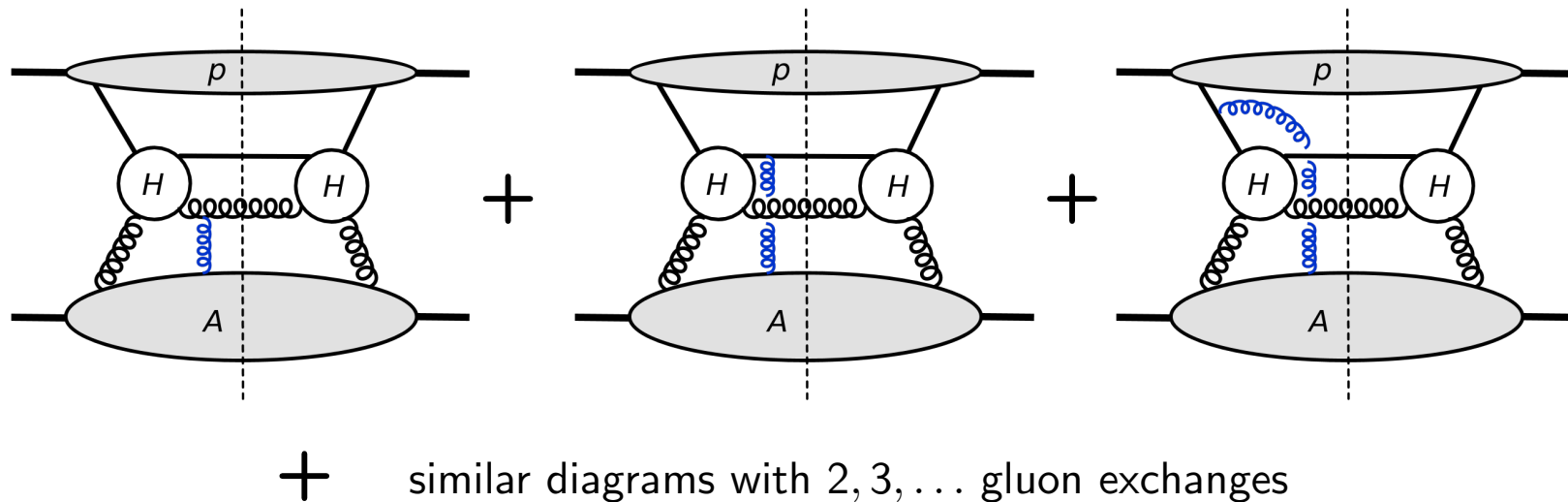
$$\mathcal{F}_{g/A}(x_2, k_t) \stackrel{\text{naive}}{=} 2 \int \frac{d\xi^+ d^2\xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) F^{i-}(0)] | A \rangle$$

Gluon TMDs and gauge links

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- a theoretically consistent definition requires to include more diagrams



They all contribute at leading power and need to be resummed.

this is done by including gauge links in the operator definition

Process-dependent TMDs

- the proper operator definition(s) some gauge link $\mathcal{P} \exp \left[-ig \int_{\alpha}^{\beta} d\eta^{\mu} A^a(\eta) T^a \right]$

$$\mathcal{F}_{g/A}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) U_{[\xi, 0]} F^{i-}(0)] | A \rangle$$

- ▶ $U_{[\alpha, \beta]}$ renders gluon distribution gauge invariant

different processes require a different gauge-link structure,
implying in turn different gluon TMDs

Process-dependent TMDs

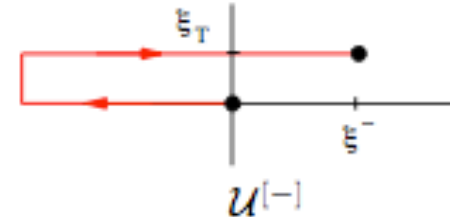
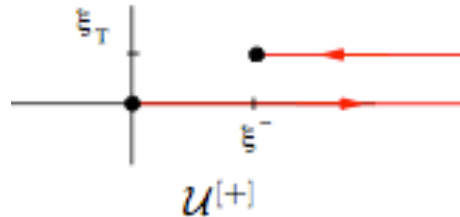
- the proper operator definition(s)

Dominguez, Xiao and Yuan (2011)

$$\mathcal{F}_{g/A}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) U_{[\xi, 0]} F^{i-}(0)] | A \rangle$$

several paths are possible for the gauge links

examples :



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several paths are possible for the gauge links

examples :



- in the large k_t limit, the process dependence of the gauge links disappears (like for the integrated gluon distribution), and a single gluon distribution is sufficient

in particular, at small- x , all the TMDs share a universal perturbative tail called the unintegrated gluon distribution:

$$\mathcal{F}_{g/A}(x_2, k_t) = UGD(x_2, k_t) + \mathcal{O}(Q_s^2/k_t^2)$$

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015)

Process-dependent TMDs

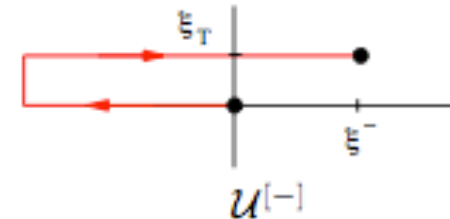
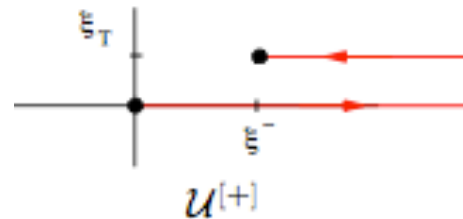
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several paths are possible for the gauge links

examples :



- one can compute the gluon TMDs at small x , using the Color Glass Condensate effective description of the dense parton content of the wave function, in terms of the large gluon field A^- :

$$\frac{\langle A | \cdot | A \rangle}{\langle A | A \rangle} \rightarrow \langle \cdot \rangle_x = \int DA^- |\phi_x[A^-]|^2 .$$

Gluon TMDs at small-x

Dominguez, CM, Xiao and Yuan (2011)

- most-known examples of gluon TMDs :

the (fundamental) dipole gluon TMD

$$\mathcal{F}_{DP}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \langle A | \text{Tr} \left[F^{i-}(\xi) \mathcal{U}^{[-]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] | A \rangle$$

the Weizsäcker-Williams gluon TMD

$$\mathcal{F}_{WW}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \langle A | \text{Tr} \left[F^{i-}(\xi) \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] | A \rangle$$

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- at small \mathbf{x} they can be written as:

$$U_{\mathbf{x}} = \mathcal{P} \exp \left[ig \int_{-\infty}^{\infty} dx^+ A_a^-(x^+, \mathbf{x}) t^a \right]$$

$$\mathcal{F}_{DP}(x_2, k_t) = \frac{4}{g^2} \int \frac{d^2\mathbf{x} d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} \left[(\partial_i U_{\mathbf{y}}) (\partial_i U_{\mathbf{x}}^\dagger) \right] \rangle_{x_2}$$

$$\mathcal{F}_{WW}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x} d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} \left[(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger \right] \rangle_{x_2}$$

these Wilson line correlators also emerge directly in CGC calculations

x evolution of the gluon TMDs

the evolution of Wilson line correlators with decreasing x can be computed from the so-called JIMWLK equation

$$\frac{d}{d \ln(1/x_2)} \langle O \rangle_{x_2} = \langle H_{JIMWLK} O \rangle_{x_2}$$

Jalilian-Marian, Iancu,
McLerran, Weigert,
Leonidov, Kovner

a functional RG equation that resums the leading logarithms in $y = \ln(1/x_2)$

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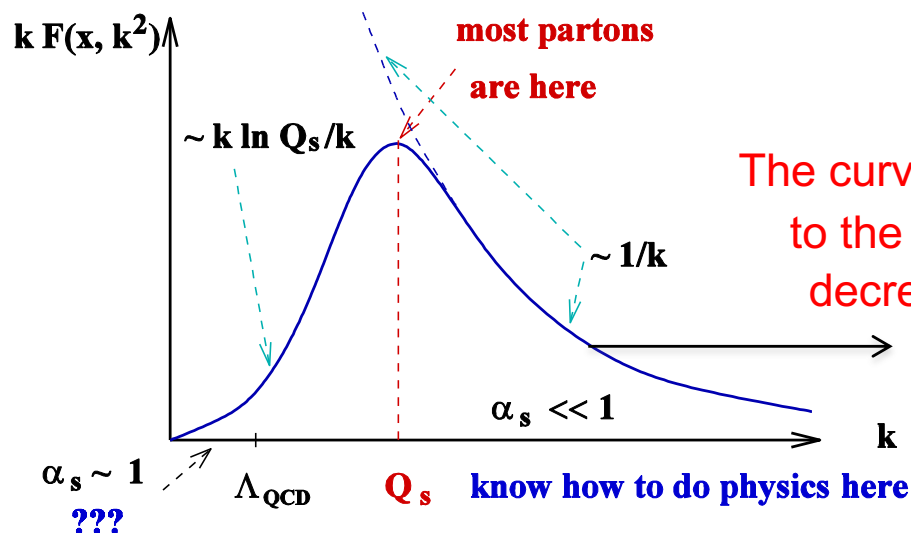
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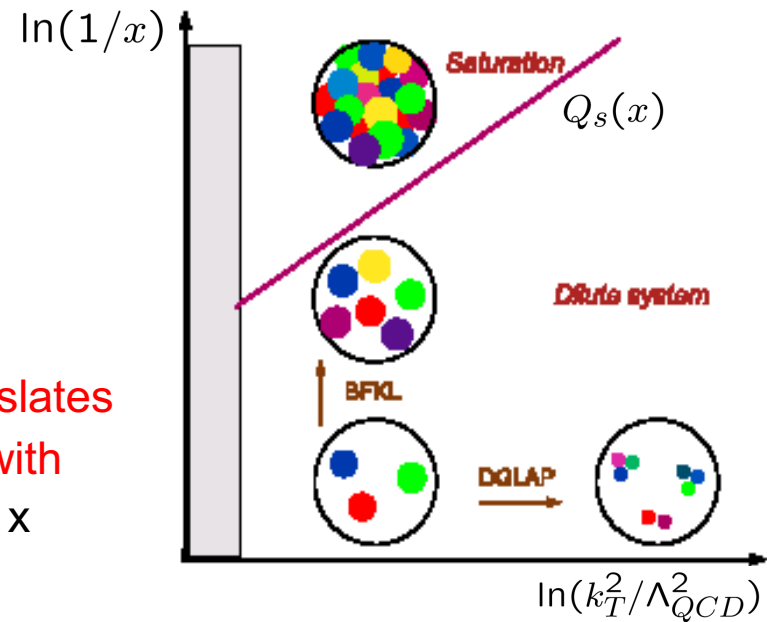
Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

a functional RG equation that resums the leading logarithms in $y = \ln(1/x_2)$

- qualitative solutions for the gluon TMDs:



The curve translates to the right with decreasing x



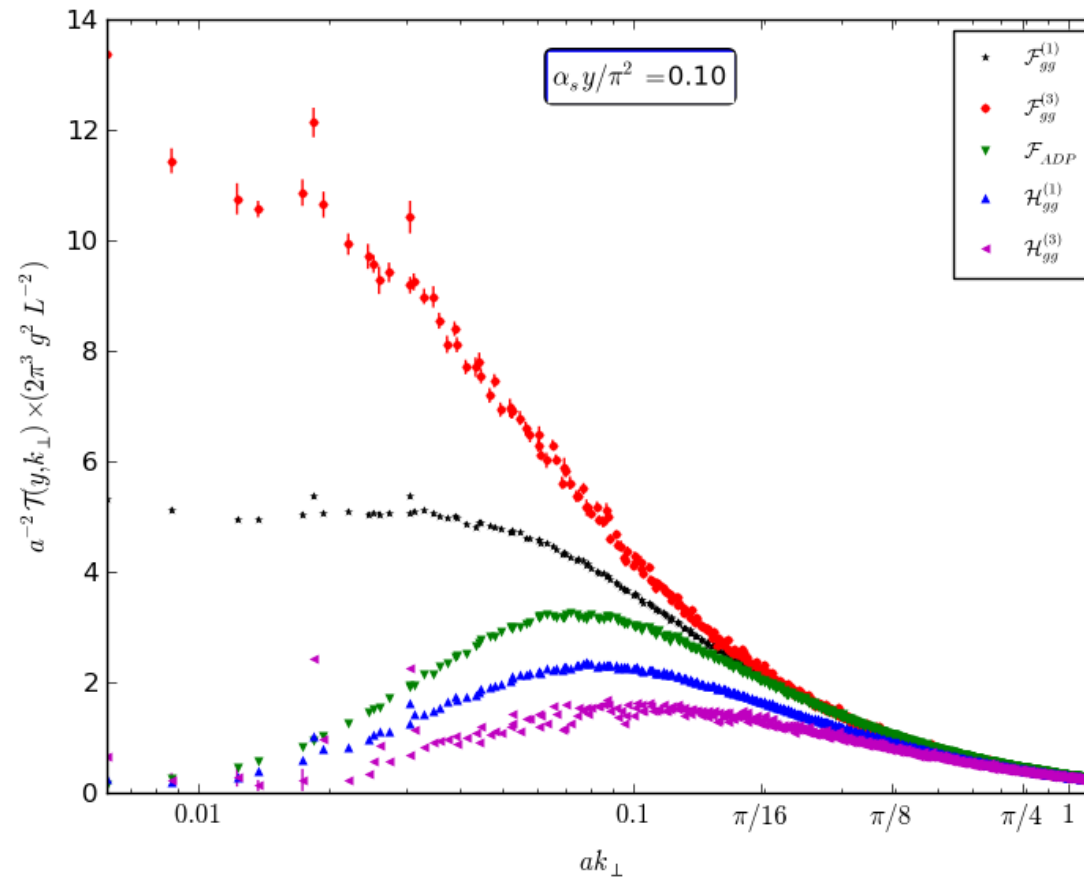
the distribution of partons as a function of x and k_T

JIMWLK numerical results

initial condition at $y=0$: MV model
evolution: JIMWLK at leading log

CM, Petreska, Roiesnel (2016)

CM, Roiesnel, Taelis (2017)



saturation effects impact the various gluon TMDs in very different ways

running-coupling JIMWLK

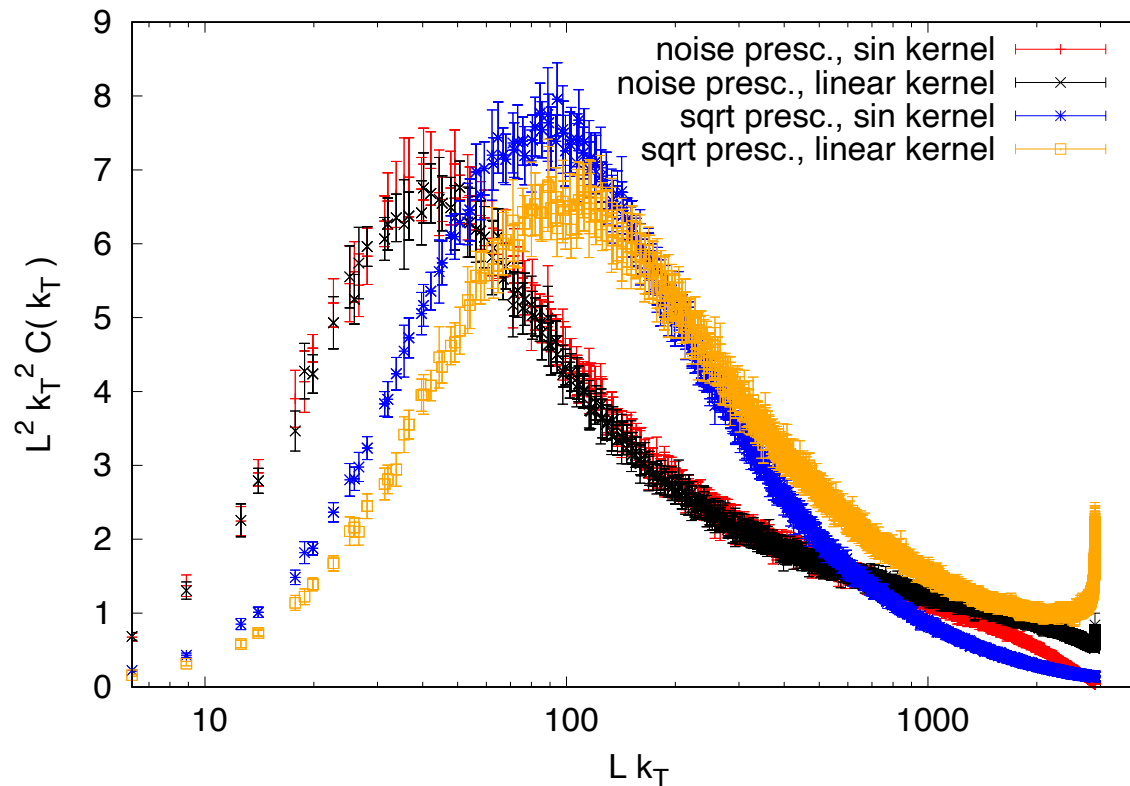
different prescriptions exist to introduce the running of the coupling in the JIMWLK equation

Rummukainen, Weigert (2004)

Lappi, Mäntysaari (2013)

Korczyk (2020)

Cali, Cichy, Korczyk, Kotko, Kutak, CM (2021)

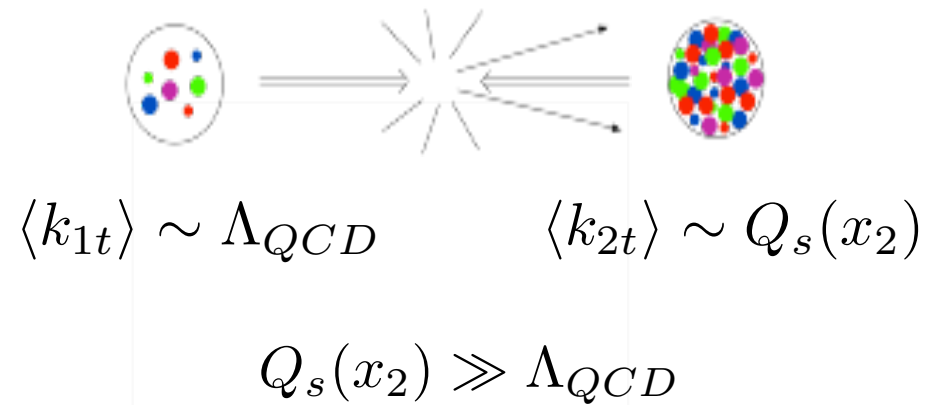
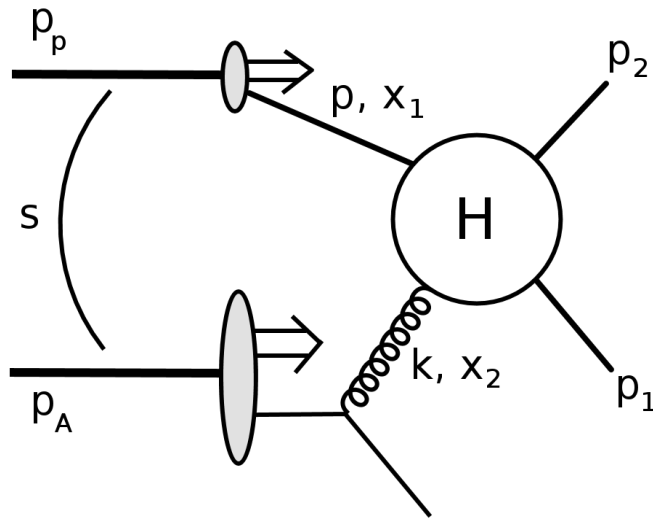


systematic effects impact the various prescriptions different ways

Probing the gluon TMDs at colliders

Dilute-dense 2-to-2 processes

- large- x projectile (proton) on small- x target (proton or nucleus)



Incoming partons' energy fractions:

$$x_1 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2})$$

$$x_2 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2})$$

$y_{1,2} \gg 0$ \rightarrow

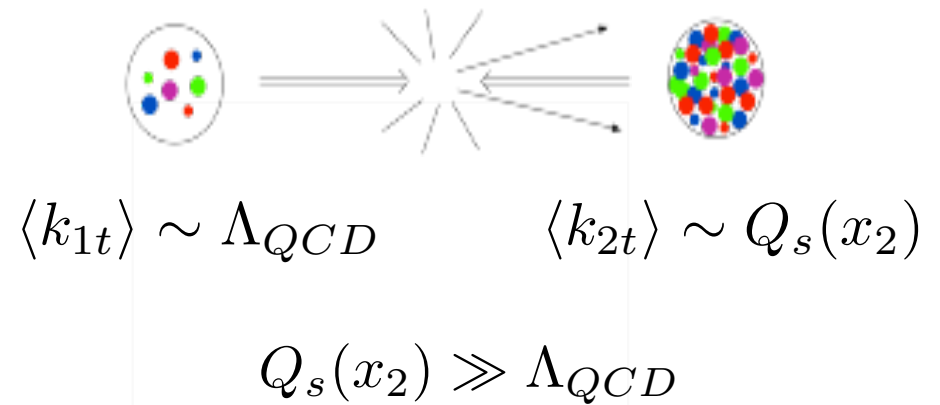
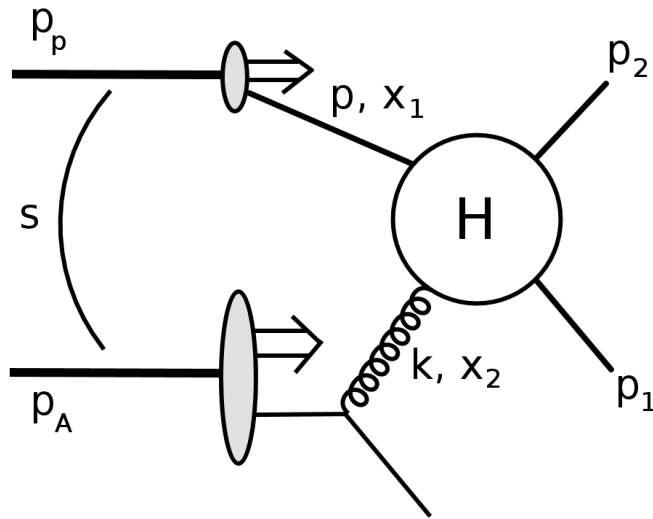
$$x_1 \sim 1$$

$$x_2 \ll 1$$

so-called "dilute-dense" kinematics

Dilute-dense 2-to-2 processes

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so-called “dilute-dense” kinematics

Incoming partons' energy fractions:

$$\begin{aligned}
 x_1 &= \frac{1}{\sqrt{s}} (|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}) \\
 x_2 &= \frac{1}{\sqrt{s}} (|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2})
 \end{aligned}
 \xrightarrow{y_1, y_2 \gg 0}
 \begin{aligned}
 x_1 &\sim 1 \\
 x_2 &\ll 1
 \end{aligned}$$

Gluon's transverse momentum (p_{1t} , p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|\cos\Delta\phi$$

relevant regime here : $|p_{1t}|, |p_{2t}| \sim \mathbf{P} \gg Q_s$

Small-x improved TMD factorization

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)

Altinoluk, Boussarie, Kotko (2019)

$$d\sigma \propto f(x_1) \sum_c H_{(c)}(\mathbf{P}, k_t) \times \text{TMD}_{(c)}(x_2, k_t)$$

←
standard collinear pdf
for the large-x projectile

off-shell
hard factors

several gluon TMDs
for the small-x target

ITMD factorization (schematically)

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$$\text{ITMD} \sim f(x_1) \sum_c \left[H_{(c)}(\mathbf{P}, 0) + \mathcal{O}\left(\frac{k_t^2}{\mathbf{P}^2}\right)_{(c)} \right] \times \left[\text{UGD}(x_2, k_t) + \mathcal{O}\left(\frac{Q_s^2(x_2)}{k_t^2}\right)_{(c)} \right]$$

↓
leading-twist
hard factors

↓
kinematic
higher twists

↓
universal
perturbative
tail

↓
leading-twist
saturation corrections

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standard collinear pdf for the large-x projectile off-shell hard factors several gluon TMDs for the small-x target

$$\text{TMD} \sim f(x_1) \sum_c \left[H_{(c)}(\mathbf{P}, 0) + \mathcal{O}\left(\frac{k_t^2}{\mathbf{P}^2}\right) \right]_{(c)} \times \left[\text{UGD}(x_2, k_t) + \mathcal{O}\left(\frac{Q_s^2(x_2)}{k_t^2}\right) \right]_{(c)}$$

↓ leading-twist hard factors
↓ kinematic higher twists
↓ universal perturbative tail
↓ leading-twist saturation corrections

improvement wrt TMD factorization is all-order resummation of kinematic twists, which allows proper matching to BFKL physics at large k_t

ITMD factorization (schematically)

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$$\text{HEF} \sim f(x_1) \sum_c \left[\underbrace{H_{(c)}(\mathbf{P}, 0) + \mathcal{O}\left(\frac{k_t^2}{\mathbf{P}^2}\right)}_{(c)} \right] \times \left[\underbrace{\text{UGD}(x_2, k_t) + \mathcal{O}\left(\frac{Q_s^2(x_2)}{k_t^2}\right)}_{(c)} \right]$$

↓ leading-twist hard factors
↓ kimenatic higher twists
↓ universal perturbative tail
↓ leading-twist saturation corrections

improvement wrt HEF factorization is all-order resummation of leading twist saturation corrections, which unveils the process-dependent TMDs and allows matching to TMD physics at low k_t

Genuine higher-twist corrections

ITMD factorization emerges from CGC calculations in the $\mathbf{P} \gg Q_s$ limit

$$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

TMD regime

$$+ (k_t/P_t)^n$$

$$|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$$

BFKL regime

$$+ (Q_s/k_t)^n$$

ITMD

$$|p_{1t}|, |p_{2t}| \gg Q_s$$

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Altinoluk, Boussarie (2019)

$$+ (Q_s/P_t)^n$$

CGC

- CGC and ITMD can be compared numerically

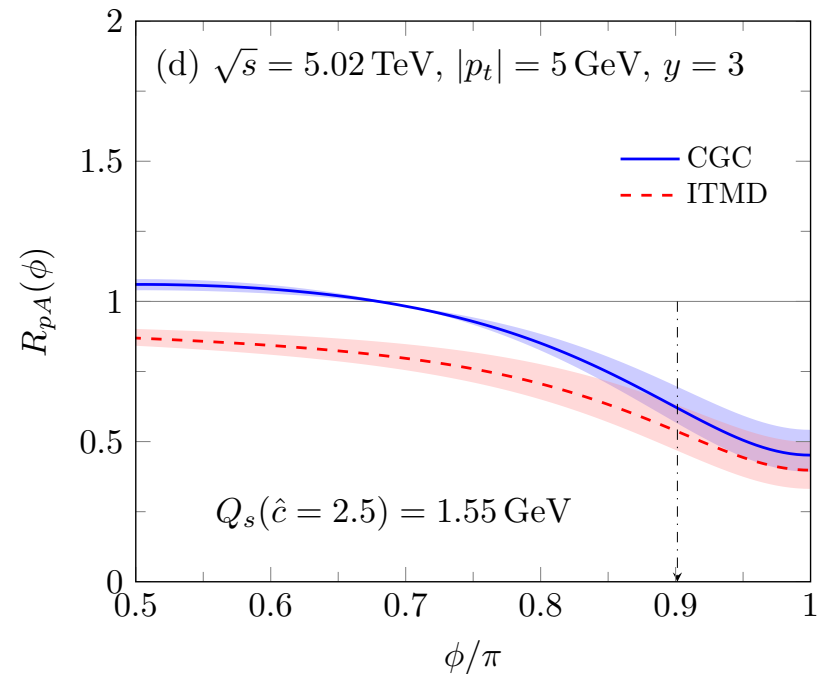
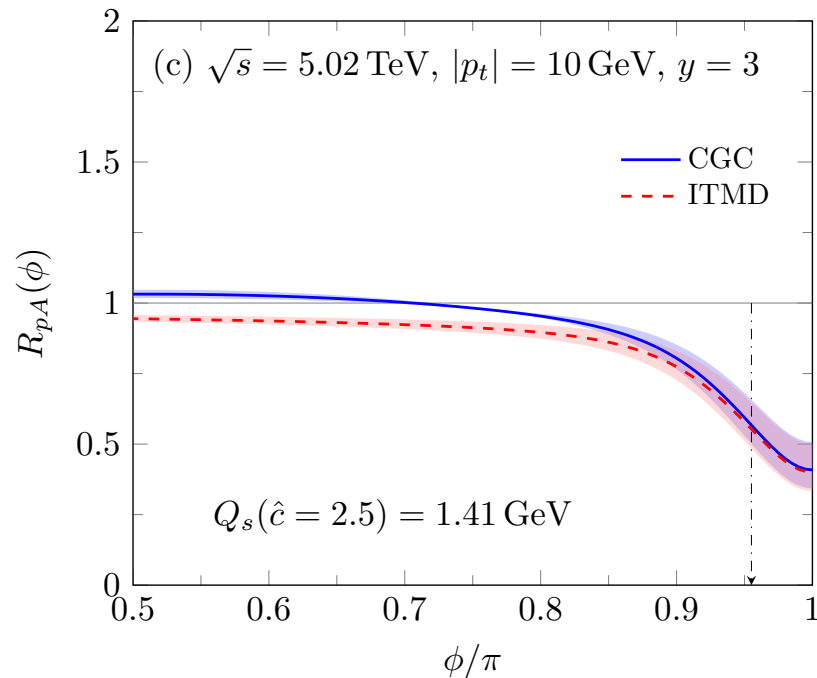
Fujii, CM, Watanabe (2020) Boussarie, Mäntysaari, Salazar, Schenke (2021)

Genuine higher-twist corrections

the genuine-twist corrections start to matter when the jet transverse momenta get closer to Q_s

for the $gA \rightarrow q\bar{q} + X$ final state

Fujii, CM, Watanabe (2020)



Unpolarized & linearly-polarized gluon TMDs

Spin physics and TMDs

TMDs are crucial to describe hard processes in polarized collisions
(e.g. Drell-Yan and semi-inclusive DIS)

8 leading-twist TMDs

Sivers function






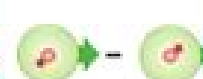


correlation between transverse spin of the nucleon and transverse momentum of the quark

Boer-Mulders function

correlation between transverse spin and transverse momentum of the quark in unpolarized nucleon

nucleon polarization

quark polarization

	U	L	T
U	f_1 number density q 		f_{1T}^\perp Sivers 
L		g_1 helicity Δq 	g_{1T} 
T	h_1^\perp Boer Mulders 	h_{1L}^\perp 	h_1 transversity  h_{1T}^\perp 

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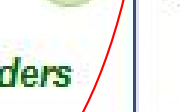
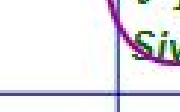
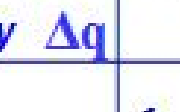

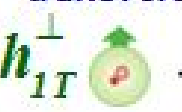
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correlation between transverse spin of the nucleon and transverse momentum of the quark

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quark polarization

I discuss those for gluons

Generic definitions of gluon TMDs

I consider only hadronic/nuclear states that are *unpolarized*

$$2 \int \frac{d\xi^+ d^2 \boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ixp_A^- \xi^+ - ik_t \cdot \boldsymbol{\xi}_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \boldsymbol{\xi}_t) F^{j-}(0)] | A \rangle$$

$$= \frac{\delta_{ij}}{2} \mathcal{F}(x, k_t) + \left(\frac{k_i k_j}{k_t^2} - \frac{\delta_{ij}}{2} \right) \mathcal{H}(x, k_t)$$

↓
unpolarized gluon TMD

↓
linearly-polarized gluon TMD

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unpolarized gluon TMD

linearly-polarized gluon TMD

- TMDs appear in cross-sections by pairs:

several pairs may be involved depending on the process

$$d\sigma \propto f(x_1) \sum_c H_{(c)}^{ij}(\mathbf{P}, k_t) \left[\frac{1}{2} \delta^{ij} \mathcal{F}_{(c)}(x_2, k_t) + \left(\frac{k^i k^j}{k_t^2} - \frac{1}{2} \delta^{ij} \right) \mathcal{H}_{(c)}(x_2, k_t) \right]$$

Generic definitions of gluon TMDs

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unpolarized gluon TMD

linearly-polarized gluon TMD

- TMDs comes by pair, expect dipole type TMDs for which $\mathcal{F} = \mathcal{H}$
- at small x , $\mathcal{F} = \mathcal{H} = \text{UGD}$ in the BFKL regime, for all TMD types :

$$\mathcal{F}(x_2, k_t) - \mathcal{H}(x_2, k_t) = \mathcal{O}(Q_s^2/k_t^2)$$

Processes sensitive to \mathcal{H}

- ITMD factorization may be rewritten

$$d\sigma \propto f(x_1) \sum_c \left[H_{(c)}^{ns}(\mathbf{P}, k_t) \mathcal{F}_{(c)}(x_2, k_t) + H_{(c)}^h(\mathbf{P}, k_t) \left(\mathcal{H}_{(c)}(x_2, k_t) - \mathcal{F}_{(c)}(x_2, k_t) \right) \right]$$

↓

projections onto
“non-sense” polarization

$$H_{(c)}^{ns} = H_{(c)}^{ij} k^i k^j / k_t^2$$

↓

= 0 in BFKL regime

projections onto linear polarization

$$H_{(c)}^h = H_{(c)}^{ij} (k^i k^j / k_t^2 - \delta^{ij} / 2)$$

Processes sensitive to \mathcal{H}

- ITMD factorization may be rewritten

$$d\sigma \propto f(x_1) \sum_c \left[H_{(c)}^{ns}(\mathbf{P}, k_t) \mathcal{F}_{(c)}(x_2, k_t) + H_{(c)}^h(\mathbf{P}, k_t) \left(\mathcal{H}_{(c)}(x_2, k_t) - \mathcal{F}_{(c)}(x_2, k_t) \right) \right]$$

↓
projections onto
“non-sense” polarization

$$H_{(c)}^{ns} = H_{(c)}^{ij} k^i k^j / k_t^2$$

↓
= 0 in BFKL regime
projections onto linear polarization

$$H_{(c)}^h = H_{(c)}^{ij} (k^i k^j / k_t^2 - \delta^{ij} / 2)$$

- processes for which the $H_{(c)}^h$ hard factors are non-zero:

- dijets in deep inelastic scattering (e+p or e+A)
- heavy-quark pair production (in photo-production or p+A collisions)
- trijets or more

Altinoluk, Boussarie, CM, Taels (2019 - 2020)

Altinoluk, CM, Taels (2021)

linearly-polarized gluons come with a $\cos(2\phi)$ modulation
(at small k_t / \mathbf{P}) where ϕ is the angle between k_t and \mathbf{P}

Dijets in deep inelastic scattering

- looking at $\Delta\phi$ distributions:

Altinoluk, CM, Taelis (2021)

BFKL

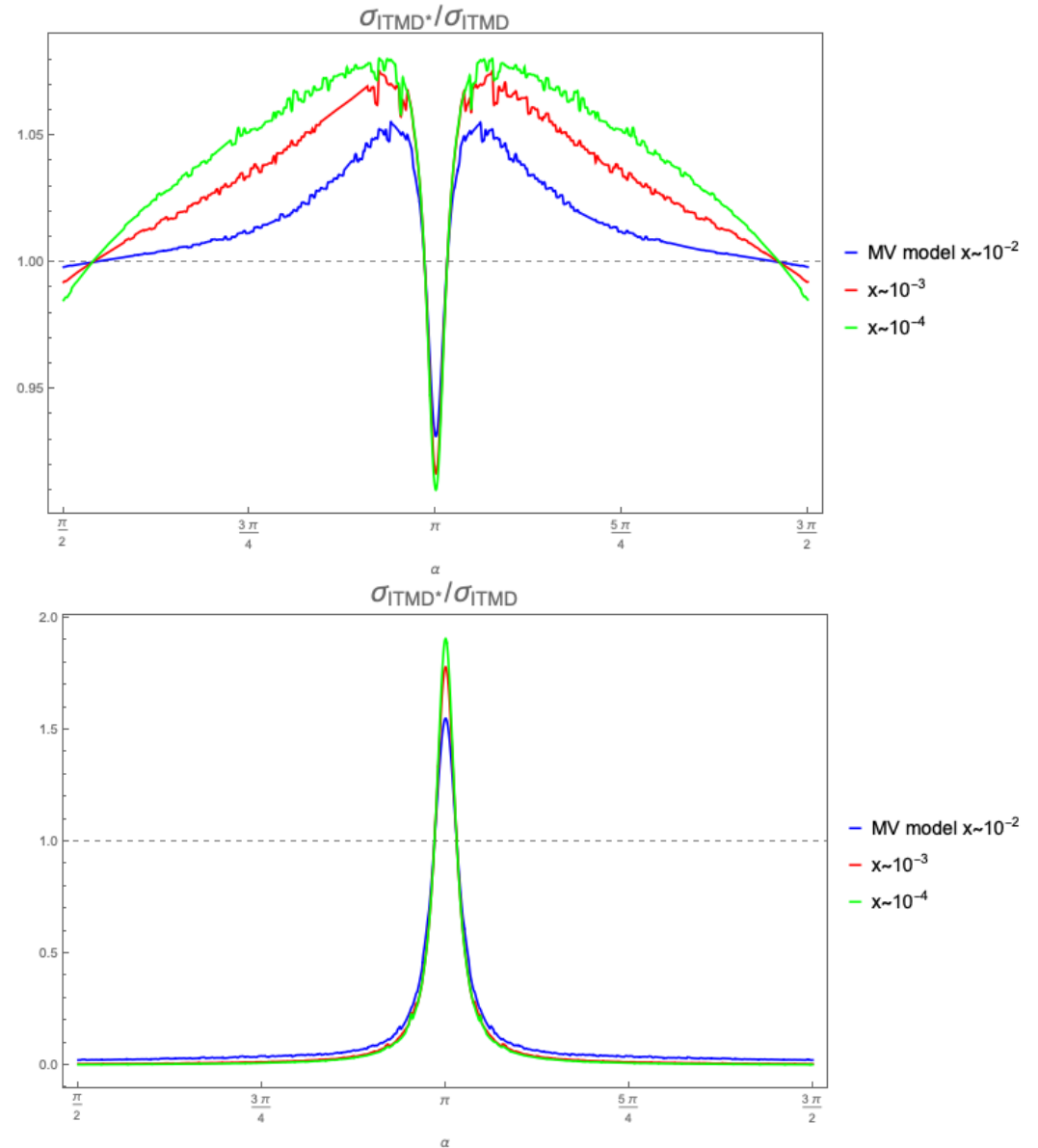
BFKL + saturation

for transverse photon

BFKL

BFKL + saturation

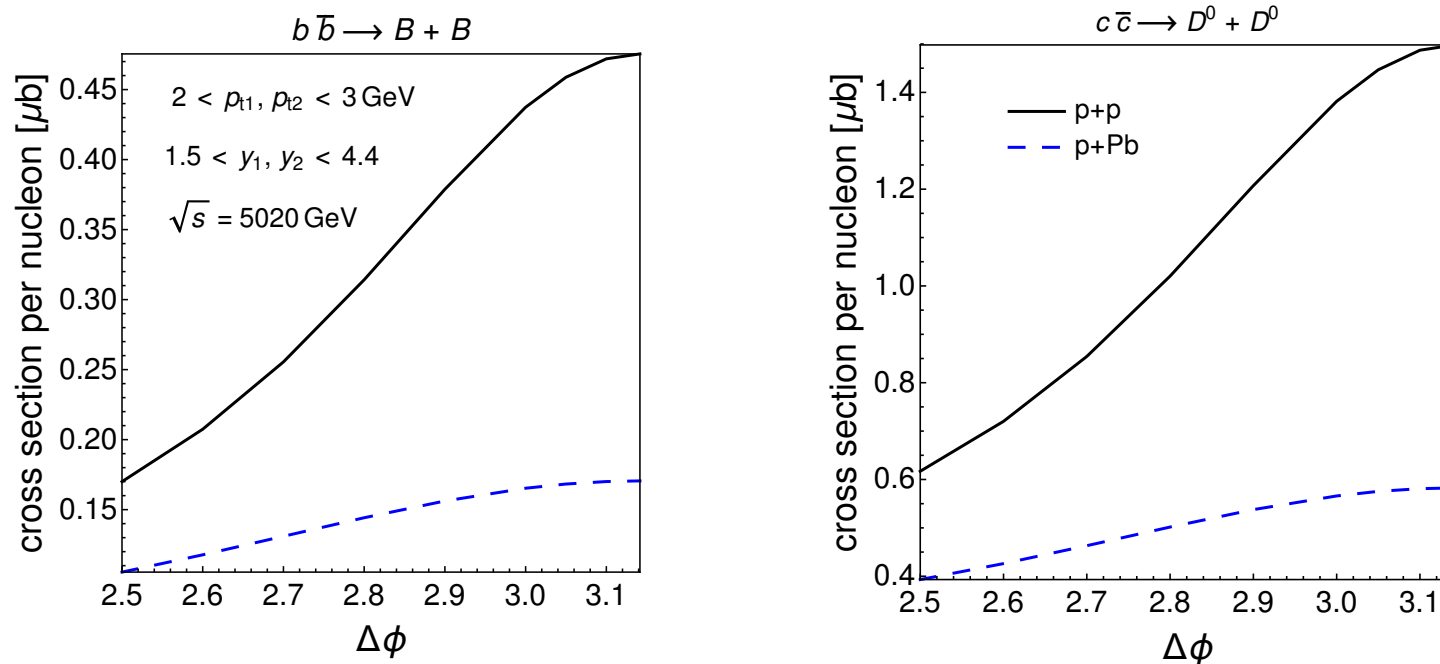
for longitudinal photon



Forward $Q\bar{Q}$ pair in p+A collisions

- preliminary study performed for HL-LHC yellow report

CM, Giacalone (2018)



ITMD hard factors to be implemented

important away from $\Delta\phi = \pi$, when $k_t \sim \mathbf{P}$

soft-gluon resummation to be added as well

important near $\Delta\phi = \pi$, when $\log(\mathbf{P}/k_t)$ becomes large

Conclusions I

- ITMD factorization emerges from CGC calculations after neglecting $O(Q_S/P_t)$ terms (so-called genuine higher-twist corrections) where P_t is the hard scale
- it resums $(Q_S/k_t)^n$ and $(k_t/P_t)^n$ terms, where k_t is the semi-hard scale, and therefore encompasses other frameworks that account for either, but not both
- from the TMD perspective, the improvement is the matching to BFKL at high k_t , due to the additional resummation of the $(k_t/P_t)^n$ terms (so-called kinematical higher-twist corrections)
- from the BFKL/HEF/ k_t -factorization perspective, the improvement is the matching to TMD factorization at low k_t due to the additional resummation of the $(Q_S/k_t)^n$ terms (leading-twist saturation corrections)

Conclusions II

- different processes involve different gluon TMDs, with different operator definitions
- each operator definition provides an unpolarized gluon TMD and a linearly-polarized one
- the various gluon TMDs coincide at large transverse momentum, in the linear regime
- however, they differ significantly from one another at low transverse momentum, in the non-linear saturation regime
- one could use small-x gluons which are not fully linearly polarized to look for saturation effects