Back-to-back dijet photoproduction at NLO in the CGC

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fwo

(Gluon) TMDs

Mulders & Rodrigues (2001)
Angelez-Martinez et al. (2015)

PDFs parameterise longitudinal structure of hadron $f(x, Q^2)$

TMDs parameterise 3D momentum structure + spin correlations $f_i(x, k_\perp, Q^2)$

polarisation of gluon

polarisation of proton

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

} → this talk

$$\Gamma^{\mu\nu}(x,\mathbf{k}) = \frac{2}{p_A^-}\int \frac{\mathrm{d}\xi^+\mathrm{d}^2\boldsymbol{\xi}}{(2\pi)^3} e^{i\xi^+k^-} e^{-i\boldsymbol{\xi}\mathbf{k}} \langle p_A|\mathrm{Tr}\, F^{-\mu}(0)\mathcal{U}(0,\xi^+,\boldsymbol{\xi})F^{-\nu}(\xi^+,\boldsymbol{\xi})|p_A\rangle$$

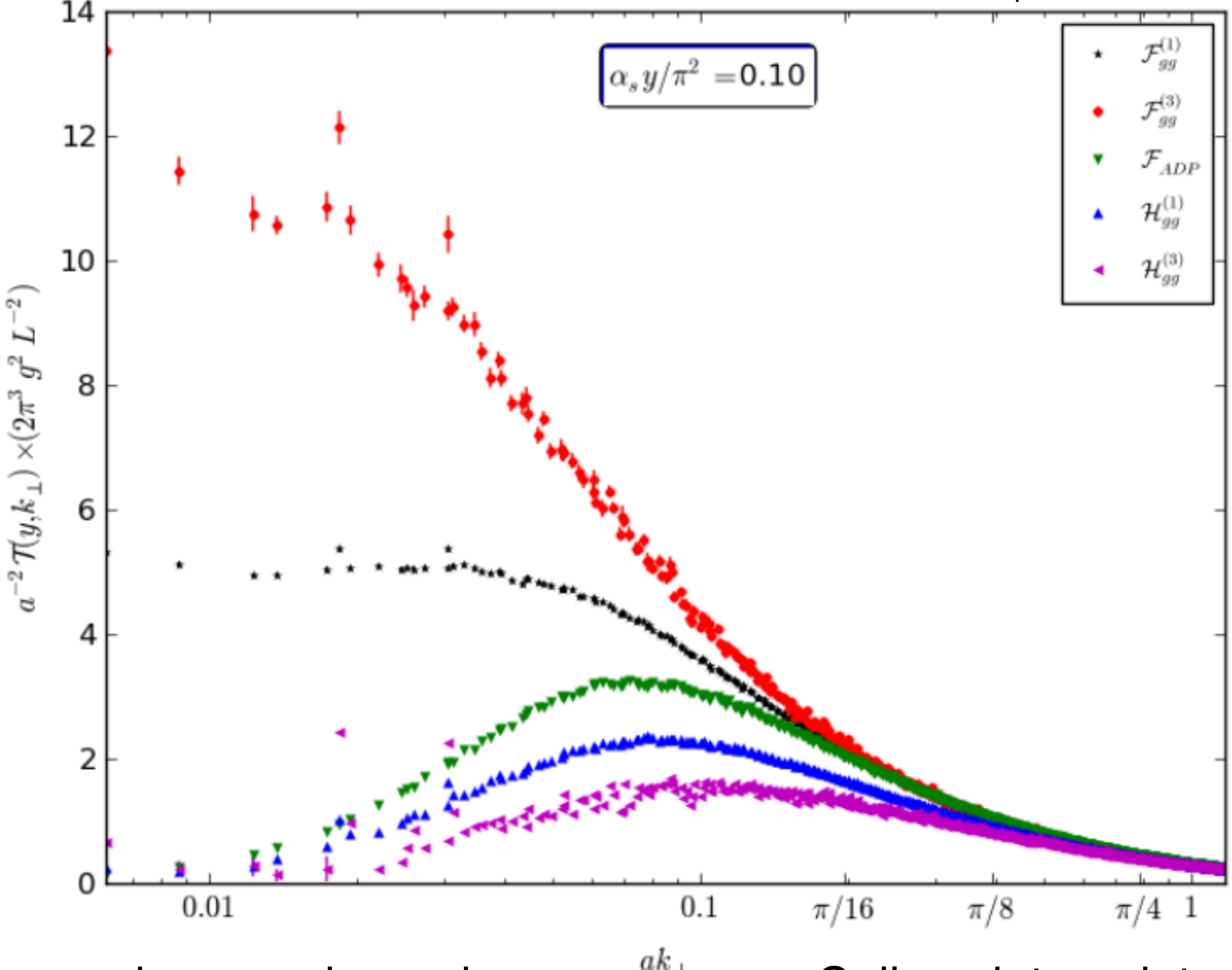
(Gluon) TMDs are process-dependent through gauge links / Wilson lines

Only low-x large-k_T tail known = unintegrated gluon distribution (UGD)

Kutak, Sapeta (2012)

Model + nonlinear high-energy evolution of low x gluon TMDs

Marquet, Roiesnel, PT (2018)



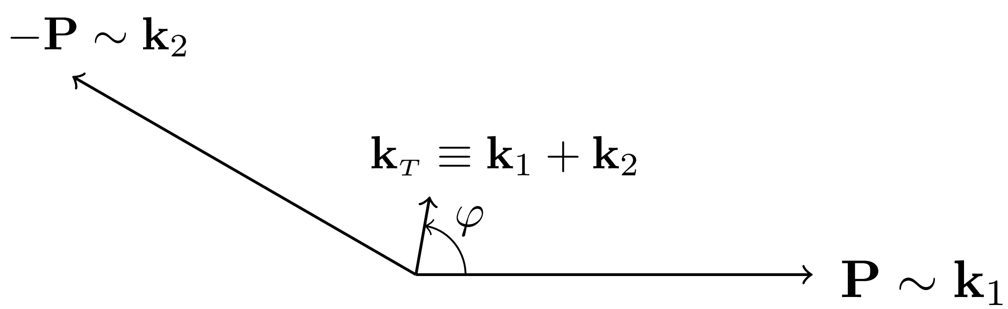
Polarisation and gauge dependence critical when $k_t \lesssim Q_s$

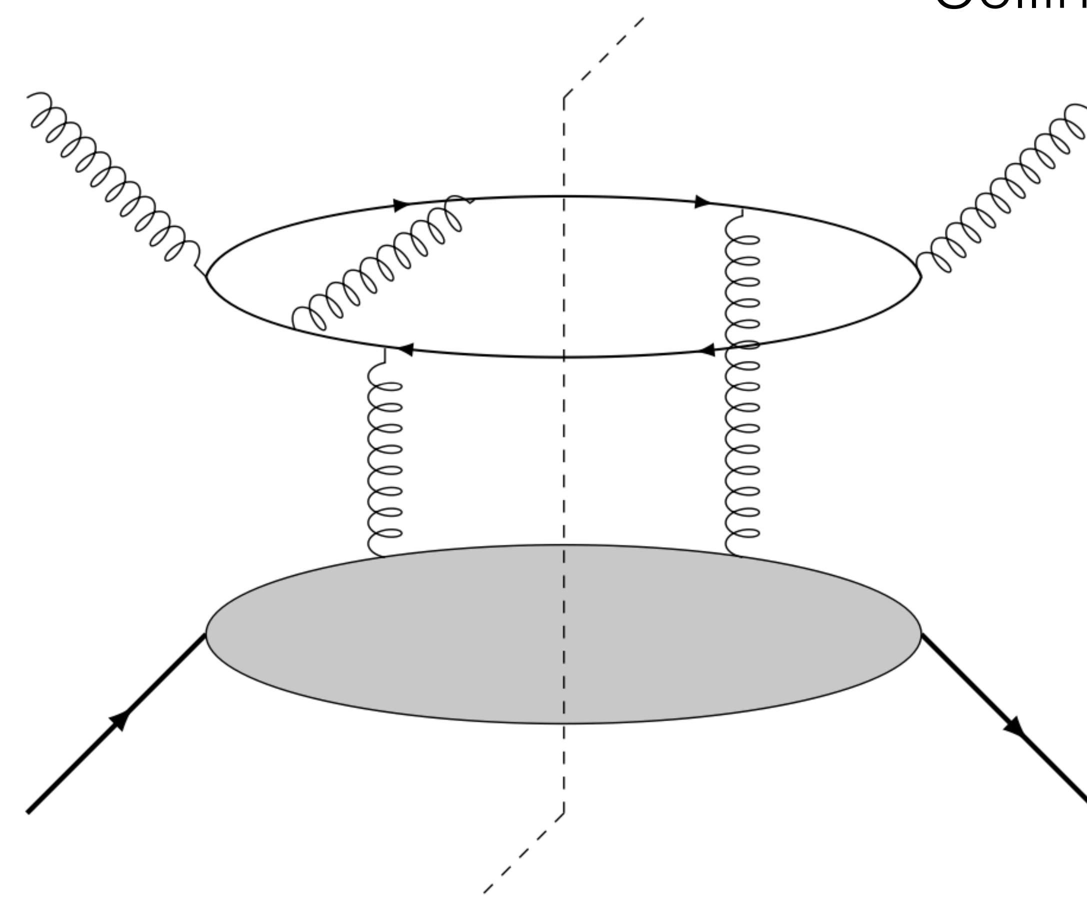
Collaps into unintegrated gluon distribution when $k_t \gg Q_s$

Transverse-momentum dependent (TMD) factorisation

Collins, Soper, Sterman ('85-'89)
Collins (2011)

$$s \sim \mathbf{P}^2 \gg \mathbf{k}_\perp^2 \gtrsim \Lambda_{\rm QCD}^2$$





$$\sigma_{\rm TMD} = \hat{\sigma}_f(\mathbf{P}^2) \otimes \mathcal{F}(x, \mathbf{k}_\perp, \mathbf{P}^2) + \hat{\sigma}_h(\mathbf{P}^2) \otimes \mathcal{H}(x, \mathbf{k}_\perp, \mathbf{P}^2) + \mathcal{O}\left(\frac{\Lambda^2}{\mathbf{P}^2}\right) + \mathcal{O}(\alpha_s^3)$$

del Castillo et al. (2021)

CSS: resum DGLAP logs $\ln(\mathbf{P}^2/\Lambda_{\rm QCD}^2)$ and Sudakov logs $\ln(\mathbf{P}^2/\mathbf{k}_\perp^2)$

Transverse-momentum dependent factorisation

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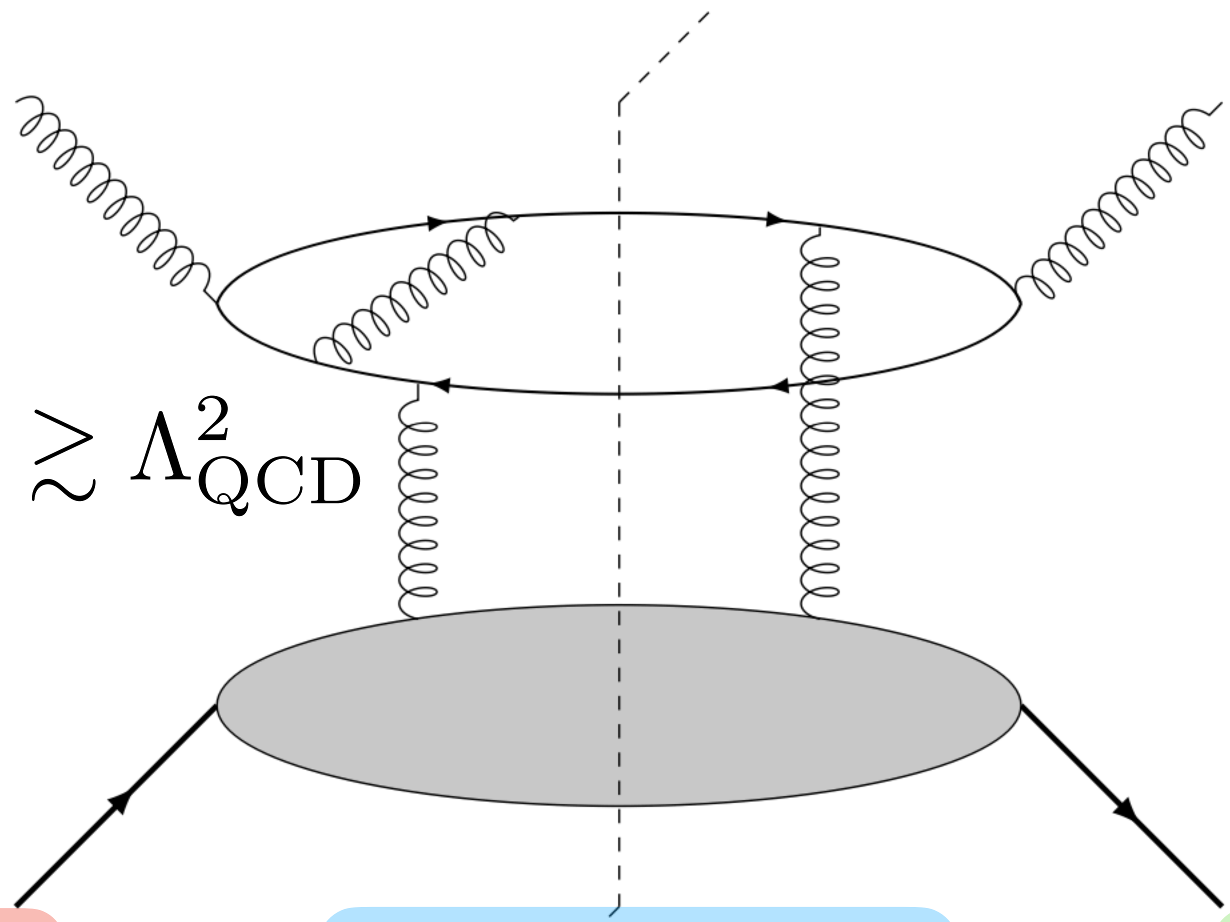


partonic cross section | unpolarised gluon TMD | linearly pol. gluon TMD

$$\sigma_{\rm TMD} = \hat{\sigma}_f(\mathbf{P}^2) \otimes \mathcal{F}(x, \mathbf{k}_\perp, \mathbf{P}^2) + \hat{\sigma}_h(\mathbf{P}^2) \otimes \mathcal{H}(x, \mathbf{k}_\perp, \mathbf{P}^2)$$

$$+ \mathcal{O}\left(\frac{\Lambda^2}{\mathbf{P}^2}\right) + \mathcal{O}(\alpha_s^3)$$

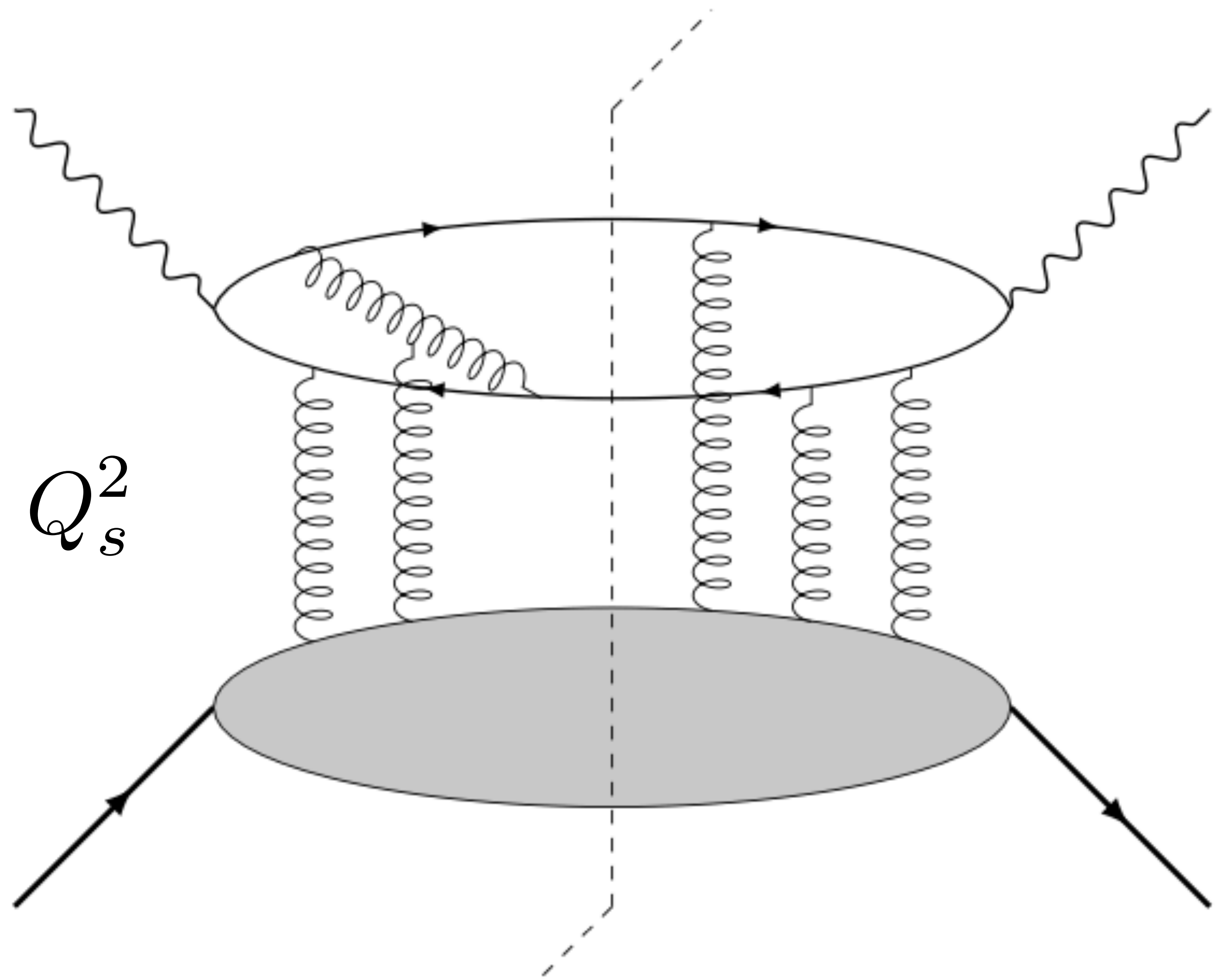
del Castillo et al. (2021)

↳ all-order proof for DY, SIDIS, and $e^+e^- \to \text{hadrons}$

CSS: resum DGLAP logs $\ln(\mathbf{P}^2/\Lambda_{\rm QCD}^2)$ and Sudakov logs $\ln(\mathbf{P}^2/\mathbf{k}_\perp^2)$

Color Glass Condensate

Mueller, McLerran, Venugopalan, Jalilian-Marian,
Kovner, Leonidov, Iancu, Weigert (1990-2001)



$s \gg \mathbf{P}^2, \mathbf{k}_\perp^2 \gtrsim Q_s^2$

$$\sigma_{\rm CGC} \neq \hat{\sigma}(\mathbf{P}^2) \otimes \mathcal{F}(x, \mathbf{k}_\perp, \mathbf{P}^2)$$

Caucal, Salazar, Venugopalan (2022)
PT, Altinoluk, Beuf, Marquet (2022)
Bergabo, Jalilian-Marian (2022)

JIMWLK: resum high-energy logs $\ln(s/\mathbf{P}^2) \sim \ln(1/x)$

CGC in the TMD limit

At LO, TMD-factorised form recovered from CGC in *correlation limit*

$$
\begin{aligned}
|\mathcal{M}_{\rm LO}|^2 = &\, 16(4\pi)\alpha_{\rm em} e_f^2 p_1^+ p_2^+ (z^2+\bar{z}^2) N_c \\
&\times \int_{\mathbf{r},\mathbf{r}',\mathbf{b},\mathbf{b}'} e^{-i\mathbf{P}_\perp\cdot(\mathbf{r}-\mathbf{r}')} e^{-i\mathbf{k}_\perp\cdot(\mathbf{b}-\mathbf{b}')} A^{\lambda'}(\mathbf{r}) A^{\lambda'}(\mathbf{r}') \\
&\times \mathrm{Tr}\Big\langle Q_{122'1'} - s_{12} - s_{2'1'} + 1 \Big\rangle \, .
\end{aligned}
$$

$$\mathbf{P}^2 \gg \mathbf{k}_\perp^2 \;\leftrightarrow\; \mathbf{b}^2, \mathbf{b}'^2 \gg \mathbf{r}^2, \mathbf{r}'^2$$

$$
\begin{aligned}
|\mathcal{M}_{\rm LO}|^2 \overset{\rm TMD}{=} &\, 16\frac{\alpha_{\rm em} e_f^2}{\pi} p_1^+ p_2^+ (z^2+\bar{z}^2) \int_{\mathbf{r},\mathbf{r}'} e^{-i\mathbf{P}_\perp\cdot(\mathbf{r}-\mathbf{r}')} \frac{\mathbf{r}\cdot\mathbf{r}'}{\mathbf{r}^2\mathbf{r}'^2} \mathbf{r}^i \mathbf{r}'^j \\
&\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}_\perp\cdot(\mathbf{b}-\mathbf{b}')} \mathrm{Tr}\Big\langle U_{\mathbf{b}} (\partial^i U_{\mathbf{b}}^\dagger)(\partial^j U_{\mathbf{b}'}) U_{\mathbf{b}'}^\dagger \Big\rangle
\end{aligned}
$$

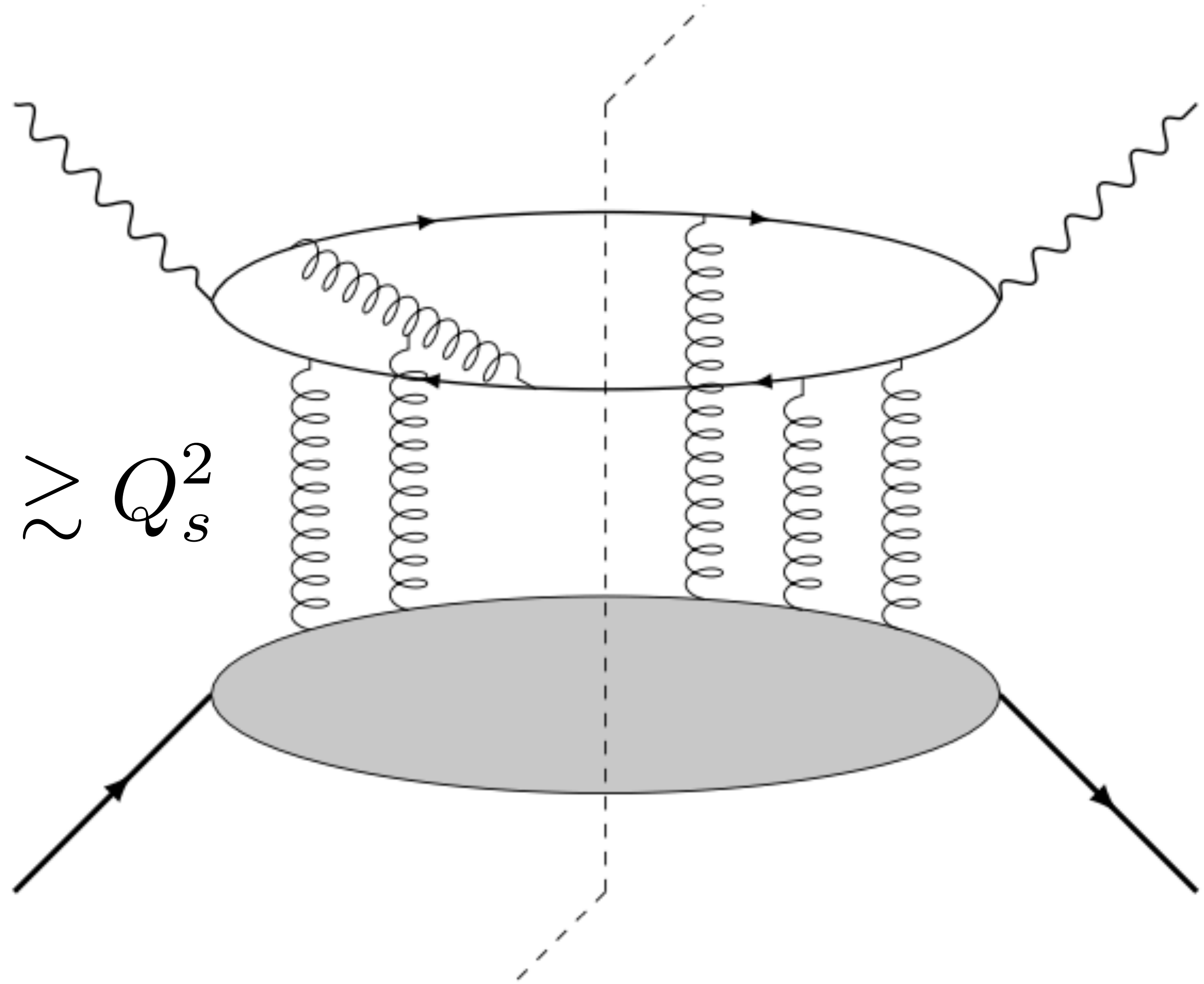
Wilson-line structure collapses into hadron tensor:

$$
\begin{aligned}
&\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}_\perp\cdot(\mathbf{b}-\mathbf{b}')} \mathrm{Tr}\Big\langle U_{\mathbf{b}} (\partial^i U_{\mathbf{b}}^\dagger)(\partial^j U_{\mathbf{b}'}) U_{\mathbf{b}'}^\dagger \Big\rangle \\
&= g_s^2 (2\pi)^3 \frac{1}{4} \left[\frac{\delta^{ij}}{2} \mathcal{F}_{\rm WW}(x_A, \mathbf{k}_\perp) + \left(\frac{\mathbf{k}_\perp^i \mathbf{k}_\perp^j}{\mathbf{k}_\perp^2} - \frac{\delta^{ij}}{2} \right) \mathcal{H}_{\rm WW}(x_A, \mathbf{k}_\perp) \right]
\end{aligned}
$$

Dominguez, Marquet, Xiao, Yuan (2011)
Altinoluk, Boussarie, Kotko (2019)

Combining low-x and Sudakov resummation

$s \gg \mathbf{P}^2 \gg \mathbf{k}_\perp^2 \gtrsim Q_s^2$



Simultaneous resummation of $\ln(s/\mathbf{P}^2)$ and $\ln(\mathbf{P}^2/\mathbf{k}_\perp^2)$?

Many approaches and implementations:

SW: Balitsky, Tarasov (2015)

HEF: Deak, Hautmann, Jung, Kutak, van Hameren, Sapeta, Hentschinski (2016-2021)

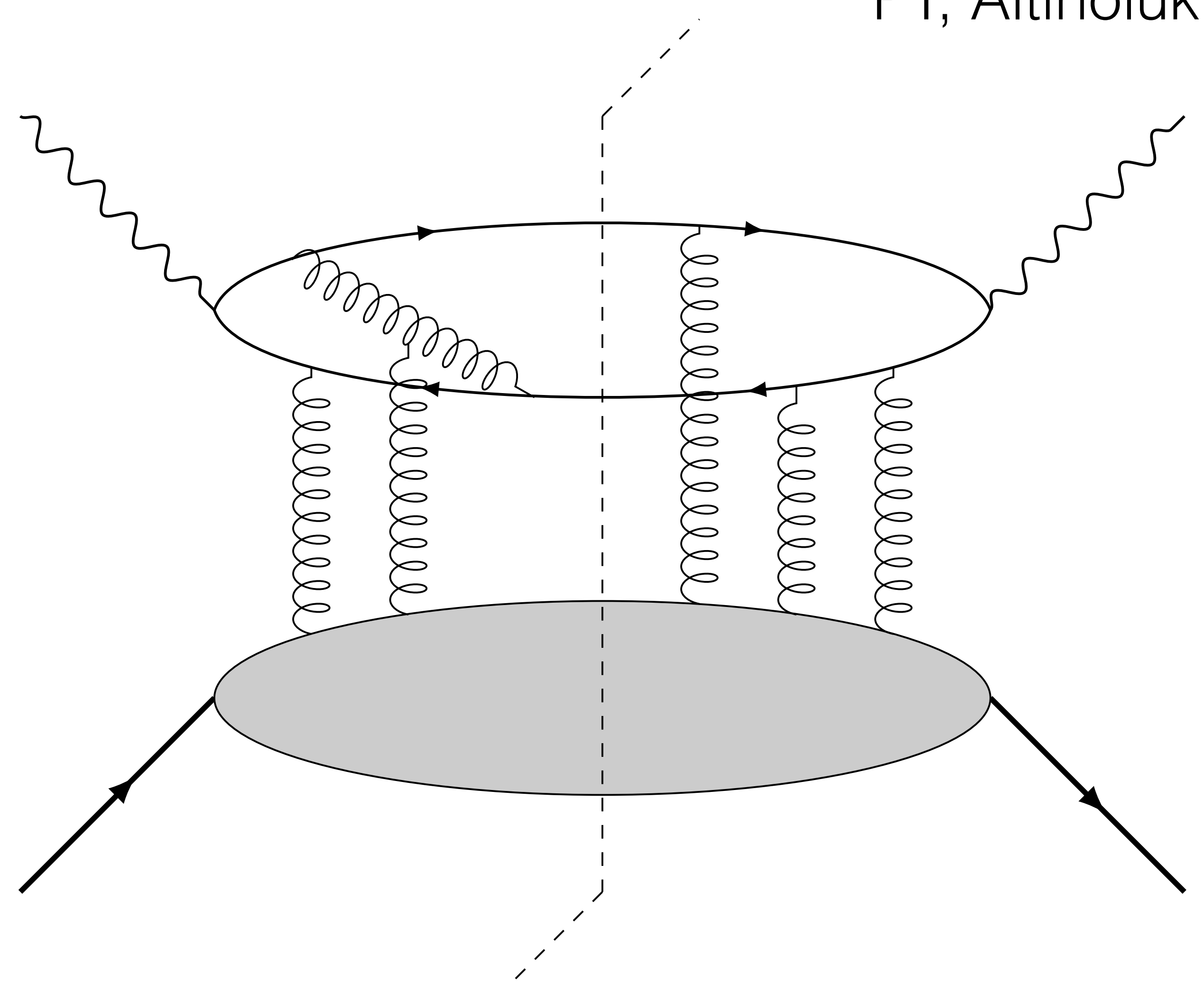
BFKL: Nefedov (2021)

PB: Hautmann, Hentschinski, Keersmaekers, Kusina, Kutak, Lelek (2022)

CGC: Mueller, Xiao, Yuan (2011); Xiao, Yuan, Zhou (2017); Stasto, Wei, Xiao, Yuan (2018); PT, Altinoluk, Beuf, Marquet (2022); Caucal, Salazar, Schenke, Venugopalan (2022)

Dijet photoproduction at NLO in the CGC

PT, Altinoluk, Beuf, Marquet (2022)



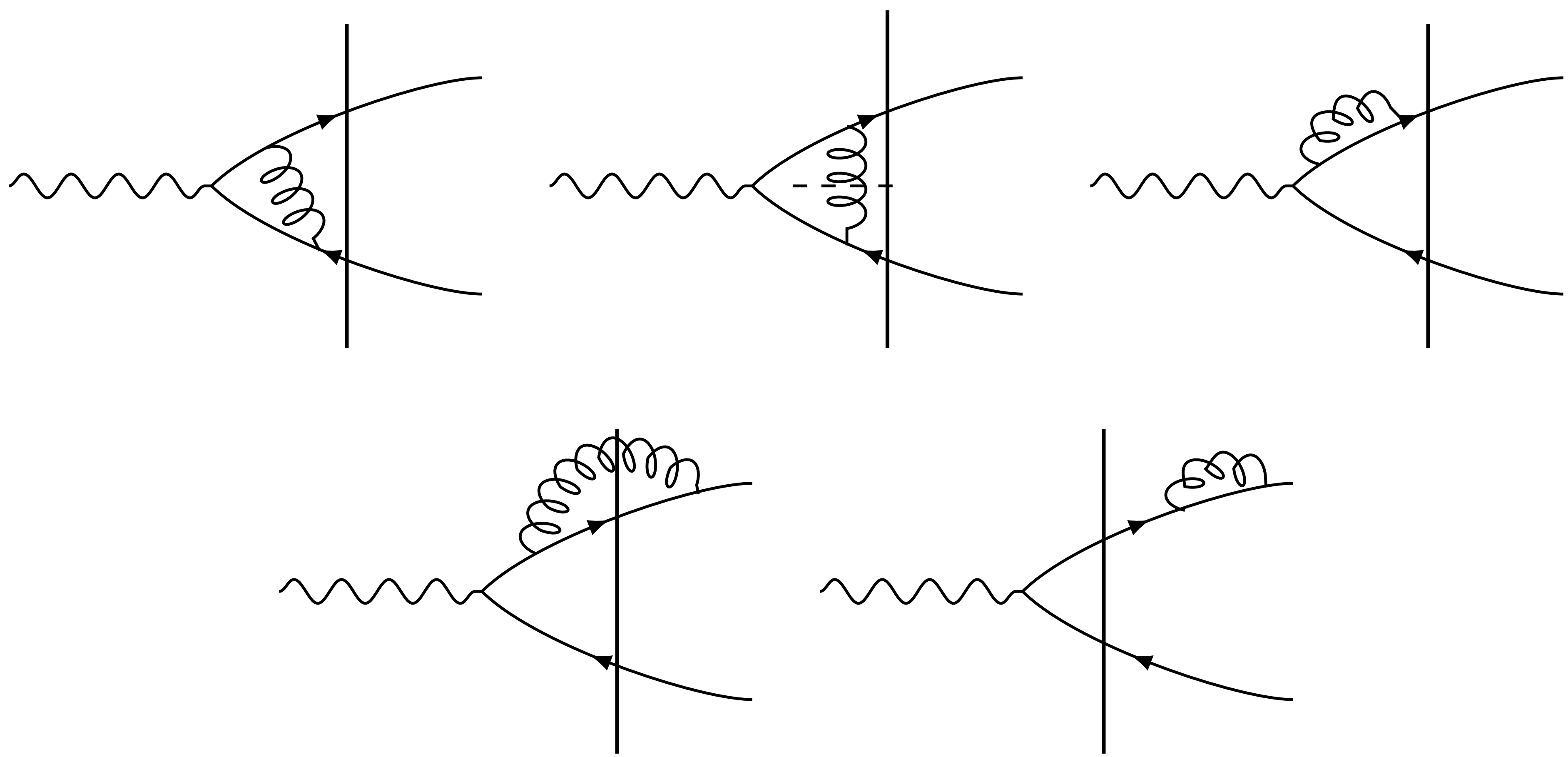
Framework: dipole formulation of CGC, light-cone perturbation theory

$$
{}_f\langle(\mathbf{q})[\vec{p}_1]_{s_1};(\bar{\mathbf{q}})[\vec{p}_2]_{s_2}|\hat{F}-1|(\boldsymbol{\gamma})[\vec{q}]_\lambda\rangle_i
= \langle(\mathbf{q})[\vec{p}_1]_{s_1};(\bar{\mathbf{q}})[\vec{p}_2]_{s_2}|\mathcal{U}(+\infty,0)(\hat{F}-1)\mathcal{U}(0,-\infty)|(\boldsymbol{\gamma})[\vec{q}]_\lambda\rangle
$$

Dipole picture: Mueller (1990)
LCPT: Bjorken, Kogut, Soper (1971)
Inclusive DIS: Beuf (2016-2017)

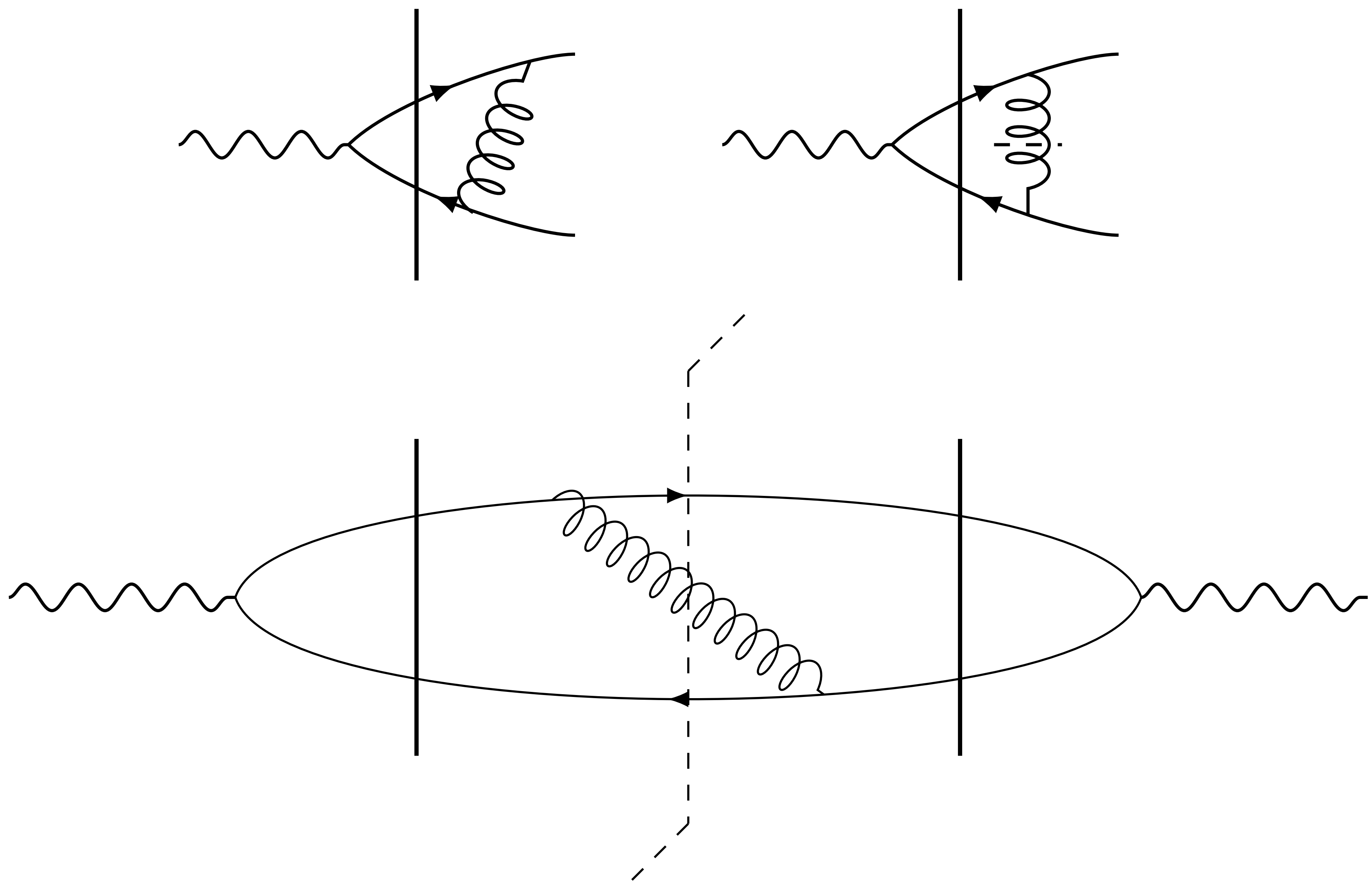
DIS: Caucal, Salazar, Venugopalan (2022)
Dihadron: Bergabo, Jalilian-Marian (2022)
Diffraction: Boussarie et al. (2016);
Fucilla, Li, et al. (2022)

UV divergences



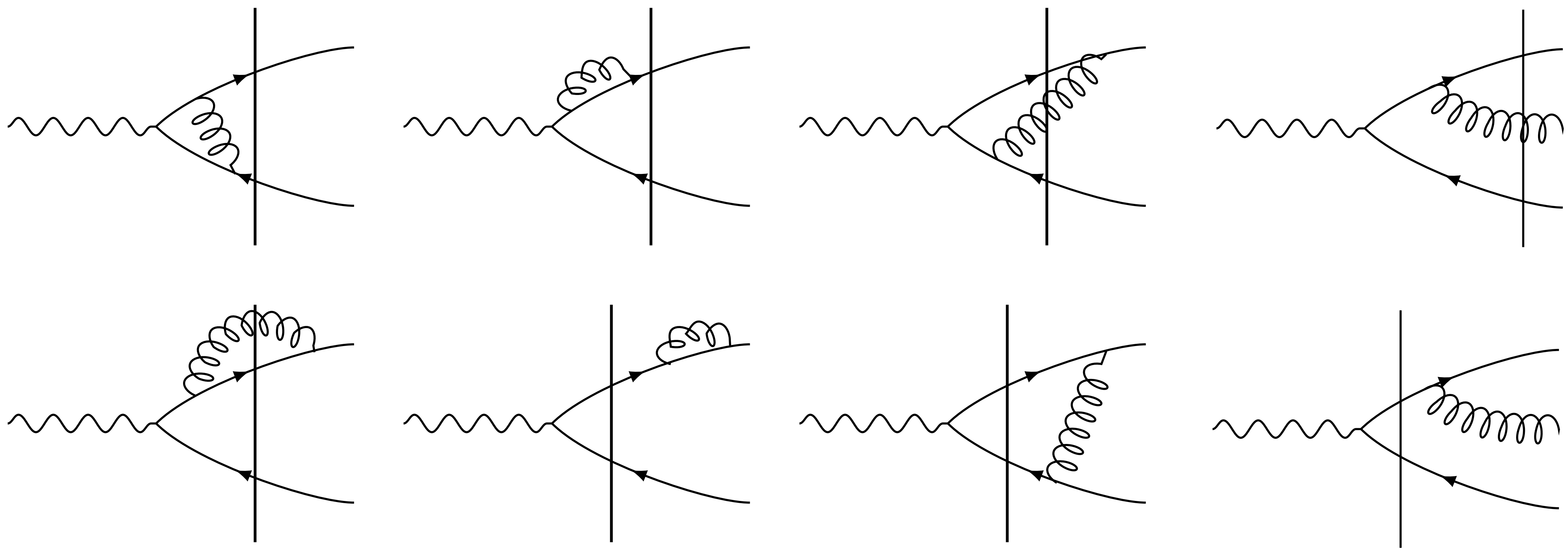
$\mathbf{k}_\perp \to \infty$ in loops, regulated with dimensional regularisation, no leftover logarithms

Soft divergences



$(k^+, \mathbf{k}_\perp) \to 0$ in final state, regulated with dimensional regularisation, no leftover logarithms

Rapidity divergences

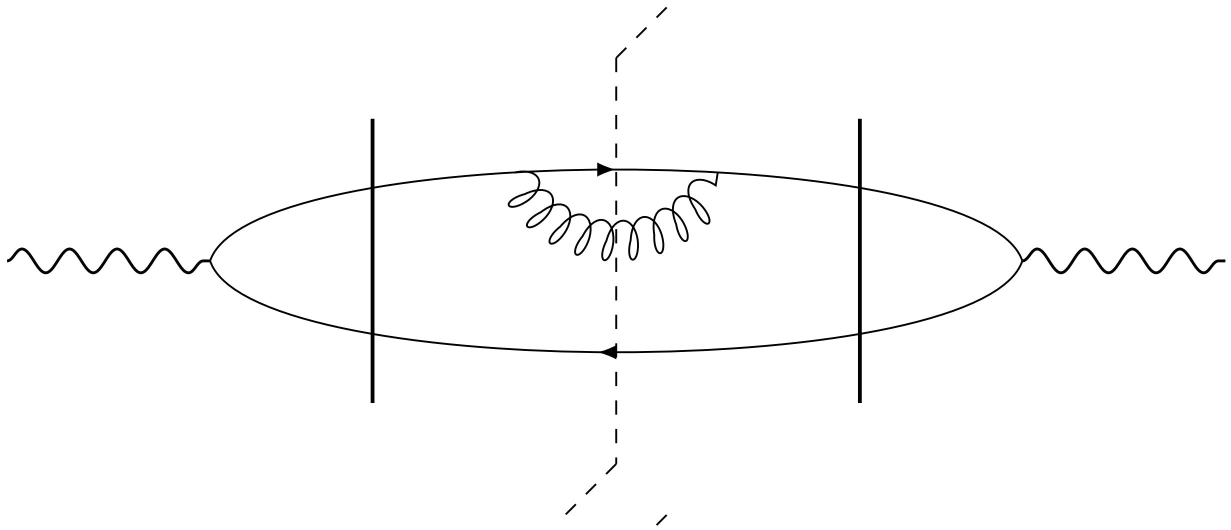


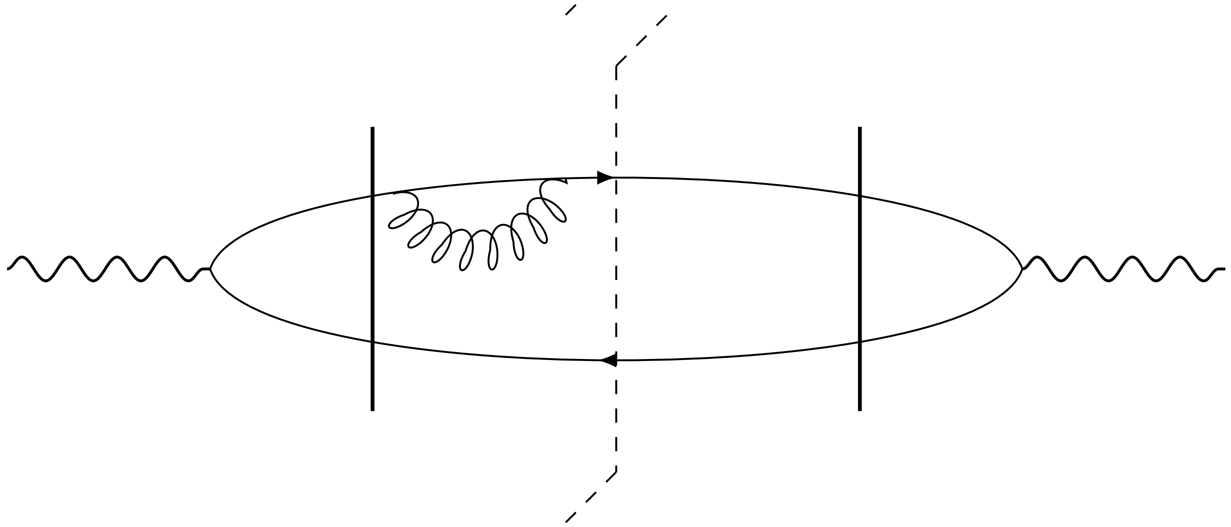
$k^+ \to 0$, regulated with cutoff $k^+_{\min}$, ‘renormalisation scale’ k^+_f, absorbed into JIMWLK evolution of LO cross section

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int_{k^+_{\min}}^{k^+_f} \frac{\mathrm{d}p_3^+}{p_3^+} \hat{H}_{\mathrm{JIMWLK}} \mathrm{d}\sigma_{\mathrm{LO}} + \int_{k^+_{\min}}^{+\infty} \frac{\mathrm{d}p_3^+}{p_3^+} \left[\mathrm{d}\tilde{\sigma}_{\mathrm{NLO}} - \theta(k^+_f - p_3^+) \hat{H}_{\mathrm{JIMWLK}} \mathrm{d}\sigma_{\mathrm{LO}} \right]$$

Dominguez, Mueller, Munier, Xiao (2011)

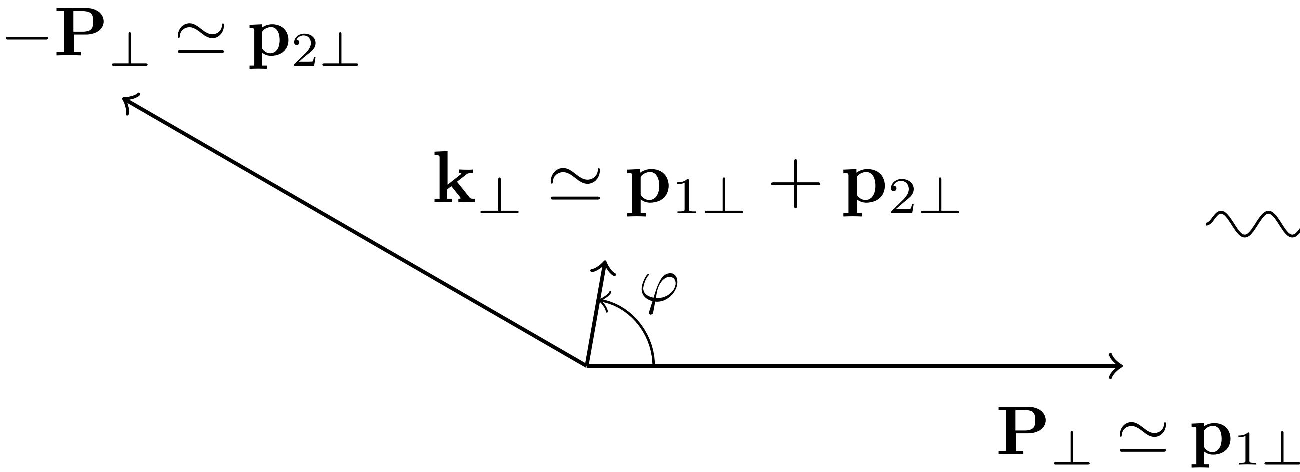
Collinear-soft divergences

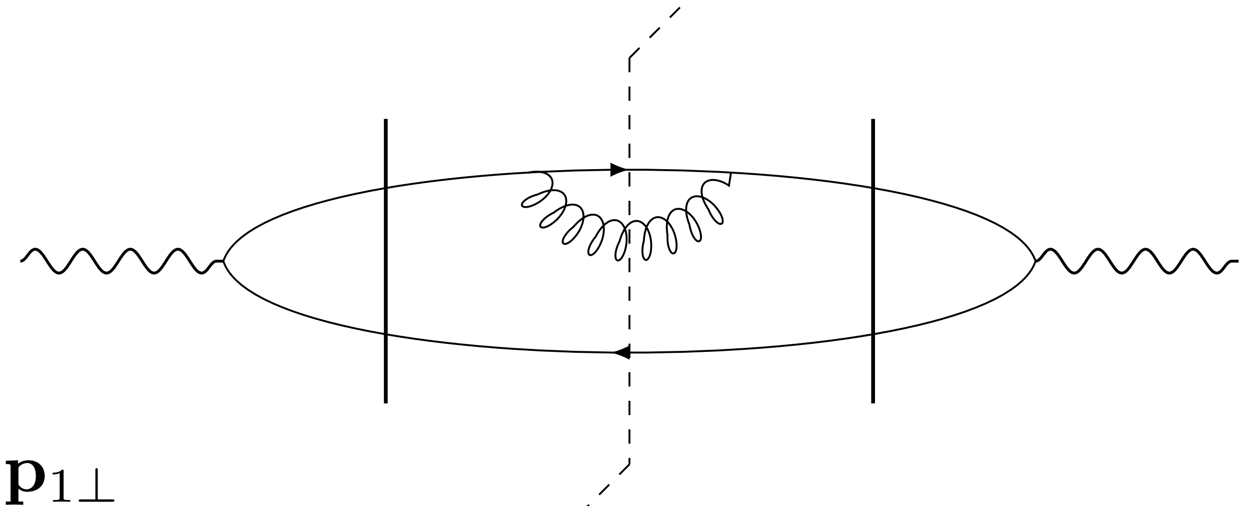




Mix of dimensional regularisation and cutoff method
Collinear divergences cancel between inside-jet radiation and self-energy
Leftover soft divergences cancel between radiation in-and outside the jet

Back-to-back limit: Sudakov logarithms





Remnants of soft-collinear generate Sudakov double log with wrong sign!

$$\mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{NLO}} = \mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{LO}} \times \frac{\alpha_s N_c}{4\pi} \ln\left(\frac{\mathbf{P}_\perp^2 (\mathbf{b}-\mathbf{b}')^2}{c_0^2}\right)^2 \qquad (\mathbf{b}-\mathbf{b}')^2 \sim 1/\mathbf{k}_\perp^2$$

… but in our framework hard to distinguish soft $(k^+, \mathbf{k}_\perp) \to 0$ and rapidity $k^+ \to 0$ divergences

oversubtraction of high-energy logs via JIMWLK?

Kinematically consistent low-x resummation

High-energy evolution along p^+ in interval $k^+_{\min} \to k^+_f$

'Naive' approach: strong ordering in p^+ only, implicitely assumes $s \to \infty$

More realistic approach calls for additional ordering in p^-, and additional renormalisation scale k_f^-

Implementing this ordering in final-state diagrams with suitable choice $k_f^+ = \dfrac{p_{j1}^+ p_{j2}^+}{q^+}$ and $k_f^- = \dfrac{\mathbf{P}_\perp^2}{2k_f^+}$ exactly compensates for wrong sign!

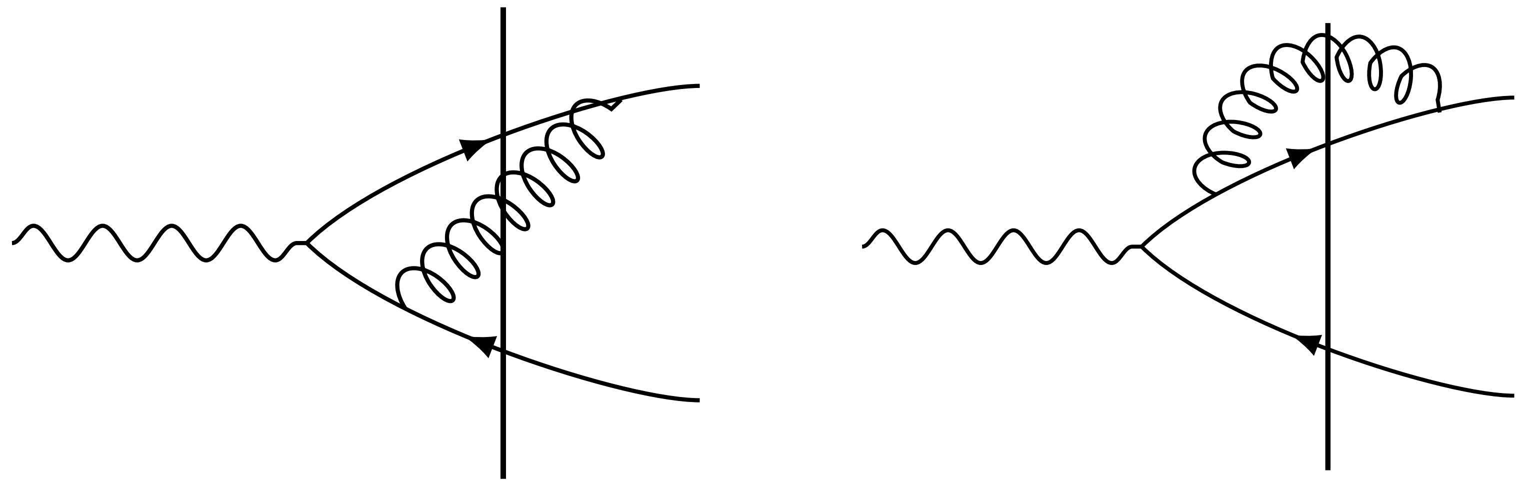
We end up with expected:

$$\mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{NLO}} = \mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{LO}} \times -\frac{\alpha_s N_c}{4\pi} \ln\left(\frac{\mathbf{P}_\perp^2 (\mathbf{b}-\mathbf{b}')^2}{c_0^2}\right)^2$$

Beyond large-N_c and double log: see Paul Caucal's talk

Ciafaloni ('88); Andersson, Gustafson, Samuelsson ('96); Kwiecinski, Martin, Sutton ('96); Salam ('98); Motyka, Stasto (2009); Kutak, Golec-Biernat, Jadach (2011); Beuf (2014); Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2019); Hatta, Iancu (2016) Nefedov (2022)

Breaking of TMD factorisation (?)



$$\mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{NLO}} = \mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{LO}} \times -\frac{\alpha_s N_c}{4\pi} \ln\left(\frac{\mathbf{P}_\perp^2(\mathbf{b}-\mathbf{b}')^2}{c_0^2}\right)^2 + \text{fact. breaking}$$

(Could rigorous power-counting à la SCET provide some insights?)

Outlook

Computed full NLO dijet photoproduction cross section in CGC

Recover correct Sudakov logs in TMD limit provided kinematical improved JIMWLK → consistent way to perform high-energy and (the perturbative part of) CSS evolution

Appearance of factorisation breaking terms beyond LO

→ At EIC, where Q_s^2 is small and only Weizsäcker-Williams gluon TMDs: ignore twist corrections $Q_s^2/\mathbf{P}^2$ and hope linearly polarised gluon contribution is small = ITMD (c.f.r. Cyrille's talk)

→ At LHC, gauge-dependence crucial and saturation scale larger, but there TMD factorisation is broken for most processes…

Thanks for your attention !