Back-to-back inclusive dijets in DIS at small x: NLO results

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SUBATECH

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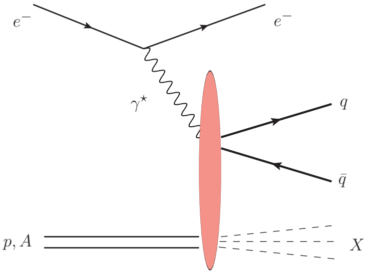
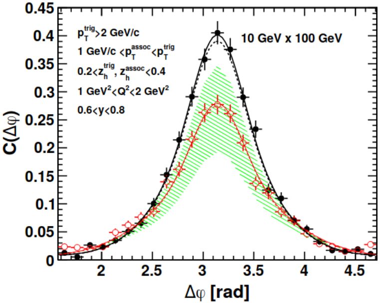
JHEP 2021 (11), 1-108, arXiv:2208.13872 and work in progress

Inclusive dijet production in DIS at small-x

$\Rightarrow$ probe of the saturated regime of QCD

$\Rightarrow$ access to the Weizsäcker-Williams gluon TMD in the back-to-back limit.





Zheng, Aschenauer, Lee, Xiao, 1403.2413

Dijets in DIS at NLO and small-x: many recent progresses!

- Dihadrons production. Bergabo, Jalilian-Marian, 2207.03606, Iancu, Mulian, 2211.04837
- Photo-production limit. Taels, Altinoluk, Beuf, Marquet, 2204.11650
- Related processes: exclusive dijet, Boussarie, Grabovsky, Szymanowski, Wallon. 1606.00419 - 1905.07371,
  Single inclusive hadron production Bergabo, Jalilian-Marian, 2210.03208 ,
  Diffractive dihadron Fucilla, Grabovsky, Li, Szymanowski, Wallon, 2211.05774

- Results from different approaches: cross-check of a challenging computation!

In this talk: NLO impact factor for inclusive dijet production in DIS

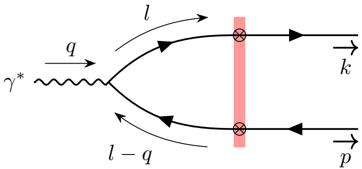
- Reliable QCD prediction requires to account for NLO corrections.
- Systematic determination of the theoretical uncertainties.
- Analytic expressions in back-to-back kinematics that simplify the numerical calculation.

Outline

- Brief overview of the computation for general kinematics
- Divergences
- Back-to-back limit at NLO: Sudakov logarithms and connection with TMD factorization.
- Preliminary numerical results

Dipole picture, CGC EFT, covariant perturbation theory

- We work in the dipole picture of DIS, large q^-.



- Covariant perturbation theory.

- CGC effective vertex:
  $= (2\pi)\delta(q^- - p^-)\gamma^- \int \mathrm{d}^2\boldsymbol{x}_\perp e^{-i(\boldsymbol{q}_\perp - \boldsymbol{p}_\perp)\boldsymbol{x}_\perp} V_{ij}(\boldsymbol{x}_\perp)$

  $\Rightarrow$ multiple gluon interactions with the target resummed via Wilson lines $V(\boldsymbol{x}_\perp)$

LO cross-section in the CGC

- Differential cross-section at leading order:

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_\perp\mathrm{d}^2\boldsymbol{p}_\perp\mathrm{d}\eta_q\mathrm{d}\eta_{\bar{q}}}\right|_{\mathrm{LO}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^6}\int\mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_\perp\boldsymbol{r}_{xx'}}e^{-i\boldsymbol{p}_\perp\boldsymbol{r}_{yy'}}\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)\mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}'_{xy})$$

- Convolution between perturbative factor describing the $\gamma^\star\to q\bar{q}$ splitting...

$$\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}'_{xy}) = 8z_q^3z_{\bar{q}}^3Q^2K_0(\bar{Q}r_{xy})K_0(\bar{Q}r_{xy'})$$

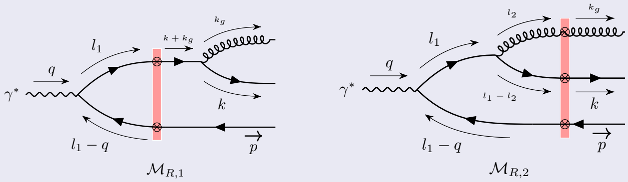
- ... and a color structure describing the interaction of $q\bar{q}$ with the dense target

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) = \left\langle \underbrace{Q(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)}_{quadrupole} - D(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) - \underbrace{D(\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)}_{dipole} + 1\right\rangle_Y$$

$$\text{Dipole:}\qquad D(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = \frac{1}{N_c}\langle\mathrm{Tr}(V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp))$$

NLO computation: real amplitudes

Real diagrams

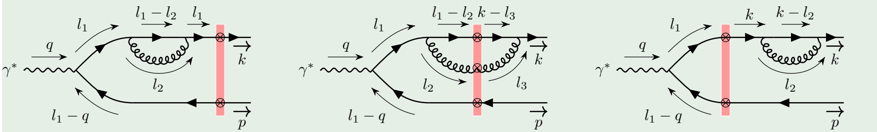


$+(q \longleftrightarrow \bar{q})$ $\quad$ $+(q \longleftrightarrow \bar{q})$

- Already computed by Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans, 1701.07143 using spinor helicities techniques.
- We recover their results.

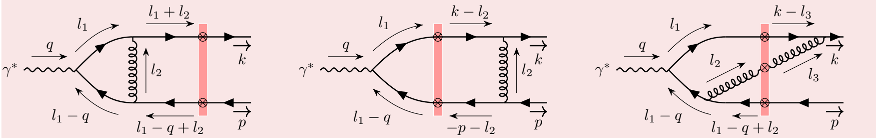
NLO computation: virtual amplitudes

Self-energies



$+(q \longleftrightarrow \bar{q})$ $\quad$ $+(q \longleftrightarrow \bar{q})$ $\quad$ $+(q \longleftrightarrow \bar{q})$

Vertex corrections



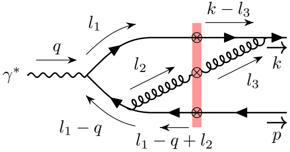
$+(q \leftrightarrow \bar{q})$

See also:

- Beuf, 1606.00777 (LCPT)
- Hänninen, Lappi, and Paatelainen 1711.08207 (LCPT)
- Boussarie, Grabovsky, Szymanowski, Wallon. 1606.00419 - 1905.07371 (exclusive dijet)
- Taels, Altinoluk, Beuf, Marquet, 2204.11650 (LCPT, $Q^2 = 0$)

Reducing the number of integrals

- Example: the dressed vertex correction for longitudinally polarized $\gamma^\star$.



$$= \frac{e e_f q^-}{\pi} \int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp \mathrm{d}^2\boldsymbol{z}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp - i\boldsymbol{p}_\perp \boldsymbol{y}_\perp} [t^a V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{z}_\perp) t_a V(\boldsymbol{z}_\perp) V^\dagger(\boldsymbol{y}_\perp) - t^a t_a]$$

$$\times \frac{\alpha_s}{\pi^2} 2(z_q z_{\bar{q}})^{3/2} Q \delta_{\sigma,-\bar{\sigma}} \int_0^{z_q} \frac{\mathrm{d}z_g}{z_g} e^{-i z_g \boldsymbol{k}_\perp / z_q \cdot \boldsymbol{r}_{zx}} \left(1 + \frac{z_g}{z_{\bar{q}}}\right) \left(1 - \frac{z_g}{z_q}\right) K_0\left(Q X_V\right)$$

$$\times \left\{ \left[1 - \frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right] \frac{\boldsymbol{r}_{zx} \cdot \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2} + i\sigma \left[\frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right] \frac{\boldsymbol{r}_{zx} \times \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2} \right\}$$

$$X_V^2 = z_{\bar{q}}(z_q - z_g)\boldsymbol{r}_{xy}^2 + z_g(z_q - z_g)\boldsymbol{r}_{zx}^2 + z_g z_{\bar{q}} \boldsymbol{r}_{zy}^2$$

Take home message

- Compact expression!
- Hopefully suitable for numerical evaluation.

Divergences

Divergences

What kind of divergence do we get?

- UV (short distance) divergences
  - internal momentum goes to ∞ or $|\boldsymbol{z}_\perp - \boldsymbol{x}_\perp| \to 0$.
  - we use dim. reg. in the transverse plane to extract the UV pole of each diagram if any.
- Rapidity divergence, "slow gluon" when $k_g^- \to 0$.
- Soft divergence $k_g^\mu \to 0$.
- Collinear divergence, $z_q \boldsymbol{k}_{\perp g} - z_g \boldsymbol{k}_\perp \to 0$ or $z_{\bar{q}} \boldsymbol{k}_{\perp g} - z_g \boldsymbol{p}_\perp \to 0$.

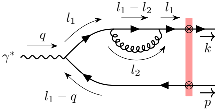
Our regularization scheme

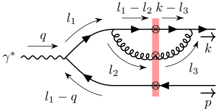
Dim. reg. in the transverse plane + lower cut-off Λ^- in the longitudinal direction:

$$\int_{\Lambda^-}^{\infty} \frac{\mathrm{d}k_g^-}{k_g^-} \mu^\varepsilon \int \frac{\mathrm{d}^{2-\varepsilon} \boldsymbol{k}_{g\perp}}{(2\pi)^{2-\varepsilon}} f(k_g^-, \boldsymbol{k}_{g\perp})$$

Cancellation of UV and IR poles

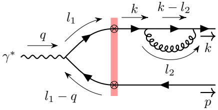
- Massless quark + universality of quark electric charge ⇒ **no need for UV renormalization**
- UV divergence cancels between free self-energy before shock-wave and dressed self energy



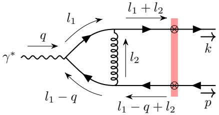


- The free self-energies after SW turn UV divergence of the free vertex correction before shock-wave into IR

Remaining $\frac{2}{\varepsilon_{\mathrm{IR}}}$ pole canceled by the real corrections for IRC safe cross-section
⇒ jets



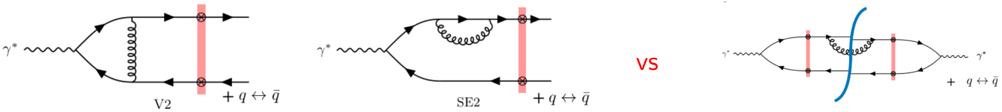
$\propto \left(\frac{2}{\varepsilon_{\mathrm{IR}}} - \frac{2}{\varepsilon_{\mathrm{UV}}}\right)$



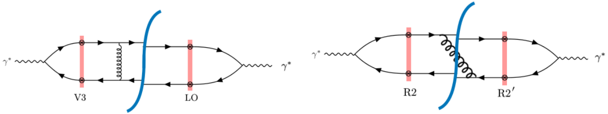
$\propto \frac{2}{\varepsilon_{\mathrm{UV}}}$

Cancellation of soft divergences

- Soft divergences: double log of the Λ^- cut-off, $\ln^2(\Lambda^-/q^-)$.
- Amplitude-level factorization of soft gluons: $\propto$ to the LO color structure or the cross-diagram color structure.
- For the LO color structure, cancel separately among the virtual diagrams and among the real (between in-cone and out-cone terms)



- For the cross color structure, cancel between real and virtual:



Cancellation of rapidity divergences

- Rapidity divergence is regularized with a **longitudinal momentum cut-off** Λ^-.
- The slow gluon phase space is divided using a factorization scale k_f^-.

- We have found:

$$
\begin{aligned}
\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_q \mathrm{d}^2\boldsymbol{p}_\perp \mathrm{d}\eta_{\bar{q}}}\right|_{\text{slow}} &= \frac{\alpha_{\text{em}} e_f^2 N_c}{(2\pi)^6}\delta(1-z_q-z_{\bar{q}}) \ln\left(\frac{z_f}{z_0}\right) \frac{\alpha_s N_c}{4\pi^2} \int \mathrm{d}\Pi_{\text{LO}} \mathcal{R}_{\text{LO}}^{\lambda}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'}) \\
\times \Bigg\langle \int \mathrm{d}^2\boldsymbol{z}_\perp \Bigg\{ & \frac{\boldsymbol{r}_{xy}^2}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2}\left(2D_{xy} - 2D_{xz}D_{zy} + D_{zy}Q_{y'x',xz} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xy}D_{y'x'}\right) \\
&+ \frac{\boldsymbol{r}_{x'y'}^2}{\boldsymbol{r}_{zx'}^2 \boldsymbol{r}_{zy'}^2}\left(2D_{y'x'} - 2D_{y'z}D_{zx'} + D_{zx'}Q_{xy,y'z} + D_{y'z}Q_{xy,zx'} - Q_{xy,y'x'} - D_{xy}D_{y'x'}\right) \\
&+ \frac{\boldsymbol{r}_{xx'}^2}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zx'}^2}\left(D_{zx'}Q_{xy,y'z} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xx'}D_{y'y}\right) \\
&+ \frac{\boldsymbol{r}_{yy'}^2}{\boldsymbol{r}_{zy}^2 \boldsymbol{r}_{zy'}^2}\left(D_{y'z}Q_{xy,zx'} + D_{zy}Q_{y'x',xz} - Q_{xy,y'x'} - D_{xx'}D_{y'y}\right) \\
&+ \frac{\boldsymbol{r}_{xy'}^2}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy'}^2}\left(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{zx'}Q_{xy,y'z} - D_{zy}Q_{y'x',xz}\right) \\
&+ \frac{\boldsymbol{r}_{x'y}^2}{\boldsymbol{r}_{zx'}^2 \boldsymbol{r}_{zy}^2}\left(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{y'z}Q_{xy,zx'} - D_{xz}Q_{y'x',zy}\right)\Bigg\}\Bigg\rangle_Y . \qquad (6.40)
\end{aligned}
$$

Cancellation of rapidity divergences

- Rapidity divergence is regularized with a **longitudinal momentum cut-off** Λ^-.
- The slow gluon phase space is divided using a factorization scale k_f^-.

- We have proven:

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} = \alpha_s \ln\left(\frac{k_f^-}{\Lambda^-}\right) \underbrace{\mathcal{K}_{\mathrm{LL}} \otimes \mathrm{d}\sigma_{\mathrm{LO}}^{\gamma^\star \to q\bar{q}+X}}_{\text{action of LL JIMWLK on the LO x-section}} + \overbrace{\text{finite}}^{\Lambda^- \to 0}$$

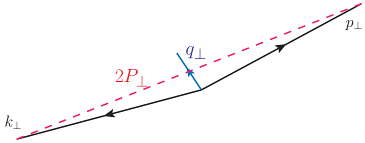
- Thus, the Λ^- dependence of the NLO impact factor is canceled by the JIMWLK evolution of the LO cross-section from Λ^- to k_f^-.

The back-to-back limit

Back-to-back limit

The back-to-back limit

- Def: $|\boldsymbol{P}_\perp| = |z_{\bar{q}}\boldsymbol{k}_\perp - z_q\boldsymbol{p}_\perp| \gg |\boldsymbol{q}_\perp| = |\boldsymbol{k}_\perp + \boldsymbol{p}_\perp|$



- LO: TMD factorization Dominguez, Marquet, Xiao, Yuan, 1101.0715

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp\mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{LO}} \propto \mathcal{H}(\boldsymbol{P}_\perp)\int \mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)}\underbrace{G_{\mathrm{WW}}(\boldsymbol{b}_\perp,\boldsymbol{b}'_\perp)}_{\left\langle\frac{1}{N_c}\mathrm{Tr}\left[\partial_i V^\dagger(\boldsymbol{b}_\perp)V(\boldsymbol{b}'_\perp)\partial_j V^\dagger(\boldsymbol{b}'_\perp)V(\boldsymbol{b}_\perp)\right]\right\rangle_Y} + \mathcal{O}\left(\frac{q_\perp}{P_\perp}\right) + \mathcal{O}\left(\frac{Q_s}{P_\perp}\right)$$

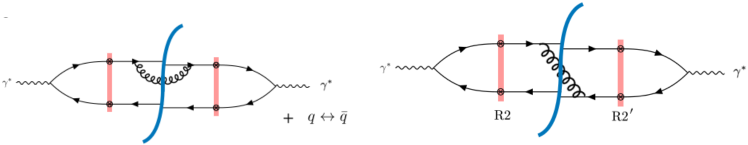
- NLO: large Sudakov logarithms vs small-x logarithm.

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp\mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{NLO}} \propto \mathcal{H}(\boldsymbol{P}_\perp)\int \mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)}$$
$$\times\left[1-\frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)^2}{c_0^2}\right)+\ldots+\alpha_s\ln\left(\frac{1}{x_{\mathrm{Bj}}}\right)\mathcal{K}_{\mathrm{LL}}\otimes\right]G_{\mathrm{WW}}(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)$$

DL computed in Mueller, Xiao, Yuan, 1308.2993 based on Higgs production in pA.

Sudakov logarithms in our computation

- Real diagrams with soft divergences.



- However: the integration over the soft gluon gives the Sudakov with a positive sign!

$$
\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star\to q\bar{q}+X} \sim \mathcal{H}(\boldsymbol{P}_\perp)\int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)}
$$

$$
\times\left[1+\frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)^2}{c_0^2}\right)+\ldots+\alpha_s\ln\left(\frac{k_f^-}{\Lambda^-}\right)\mathcal{K}_{\mathrm{LL}}\otimes\right]G_{\mathrm{WW}}(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)
$$

- Problem: overlapping phase space between soft gluons and slow gluons included in $\mathcal{K}_{\mathrm{LL}}$.

Solution: collinearly improved small-x evolution of the WW

- Kinematic improvement: impose both k_g^- and k_g^+ ordering (lifetime ordering).
  $\Longrightarrow$ Resum large transverse double logarithms to all orders.
  $\Longrightarrow$ Solve the instability of NLO B-JIMWLK evolution.
  Beuf, 1401.0313, Taels, Altinoluk, Beuf, Marquet, 2204.11650
- In practice, add an additional constraint in the LL evolution kernel

$$k_g^+ \geq k_f^+ \Longrightarrow k_g^- \leq \frac{\boldsymbol{k}_{g\perp}^2}{Q_f^2} k_f^-$$

  with $Q_f^2 \sim Q^2 \sim \boldsymbol{P}_\perp^2$.
- With this modification $\mathcal{K}_{\mathrm{LL}} \to \mathcal{K}_{\mathrm{LL,coll}}$, one recovers the expected double logarithm.

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} \sim \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)}$$
$$\times \left[1 - \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \ldots + \alpha_s \mathcal{K}_{\mathrm{LL,coll}} \otimes\right] G_{\mathrm{WW}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)$$

Sudakov resummation at single log accuracy

- Exponentiation of the Sudakov logarithms $G_{\mathrm{WW}}(\boldsymbol{r}_{bb'}) \to G_{\mathrm{WW}}(\boldsymbol{r}_{bb'})\mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2)$

$$\mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) = \exp\left(-\int_{c_0^2/\boldsymbol{r}_{bb'}^2}^{\boldsymbol{P}_\perp^2} \frac{\mathrm{d}\mu^2}{\mu^2}\frac{\alpha_s(\mu^2)N_c}{\pi}\left[\frac{1}{2}\ln\left(\frac{\boldsymbol{P}_\perp^2}{\mu^2}\right) + \frac{C_F}{N_c}s_0 - s_f\right]\right)$$

- Double and **single** Sudakov logarithms with **exact** N_c dependence:
- Dijet geometry single log s_0

$$s_0 = \ln\left(\frac{2(1+\cosh(\Delta Y_{12}))}{R^2}\right) + \mathcal{O}(R^2)$$

See also Hatta, Xiao, Yuan, Zhou, 2106.05307

- Single log from the interplay between small-x and Sudakov resummation:

$$s_f = \ln\left(\frac{\boldsymbol{P}_\perp^2 x_{\mathrm{Bj}}}{z_1 z_2 Q^2 c_0^2 x_f}\right)$$

$\Rightarrow$ Dependence on the rapidity factorization scale x_f at small x !?

Finite terms in α_s in the back-to-back limit

Azimuthal anisotropies from soft gluon radiations

- We can also access pure α_s (and non power suppressed) corrections.
- Some of them are coming from soft gluon radiations. Hatta, Xiao, Yuan, Zhou, 2010.10774
- Azimuthally averaged x-section sensitive to the linearly polarized gluon TMD at NLO!

$$\langle \mathrm{d}\sigma \rangle = \ldots + \mathcal{H}(\boldsymbol{P}_\perp) \times \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{h}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2)$$
$$\times \frac{\alpha_s}{\pi} \left\{ \frac{N_c}{2} + C_F \ln(R^2) - \frac{1}{2N_c} \ln(z_1 z_2) \right\}$$

- The $\cos(2\phi)$ anisotropy is also sensitive to the unpolarized gluon TMD.

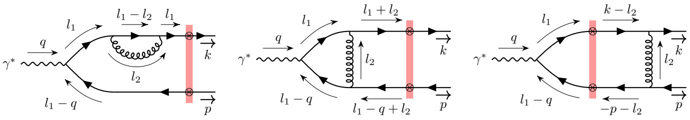
$$\langle \cos(2\phi) \mathrm{d}\sigma \rangle = \ldots + \mathcal{H}(\boldsymbol{P}_\perp) \times \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \cos(2\theta) \hat{G}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2)$$
$$\times \frac{\alpha_s}{\pi} \left\{ N_c + 2C_F \ln(R^2) - \frac{1}{N_c} \ln(z_1 z_2) \right\}$$

NLO hard factors

- Some other pure α_s corrections involve **new** "hard factors" and do not break TMD factorization:

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{NLO}} \propto \alpha_s \mathcal{H}^{ij}_{\mathrm{NLO}}(\boldsymbol{P}_\perp)\int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp\cdot(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)} G^{ij}_{\mathrm{WW}}(\boldsymbol{b}_\perp,\boldsymbol{b}'_\perp)$$

- They come from virtual graphs in which the gluon does not cross the SW:



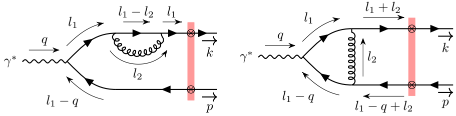
NLO hard factors

- Remarkably, they can be computed fully analytically!
- They have the form

$$\mathcal{H}_{\mathrm{NLO}}(\boldsymbol{P}_\perp) = \mathcal{H}_{\mathrm{LO}}(\boldsymbol{P}_\perp) \times f\left(\frac{Q}{P_\perp}, z_1, z_2\right)$$

with f expressed in terms of logarithms and dilogarithms.

- Example:



$$\mathcal{H}^{\lambda=\mathrm{L},ij}_{\mathrm{SE}_1+\mathrm{V}_1}(\boldsymbol{P}_\perp) = \mathcal{H}^{\lambda=\mathrm{L},ij}_{\mathrm{LO}}(\boldsymbol{P}_\perp) \times \left[4 - 4\ln\left(\frac{u}{c_0}\right) - 4\ln\left(1 + \frac{1}{u^2}\right)\right]$$

with $u = \sqrt{z_1 z_2}\,Q/P_\perp$.

TMD factorization at NLO at small-x

- Can we put the full result in a factorized TMD form?
- The small-x evolution of the WW is not closed.

  Dominguez, Mueller, Munier, Xiao, 1108.1752
- At NLO, non-trivial color correlators, e.g.

$$\frac{N_c}{2}\langle 1 - D_{y'x'} + Q_{zy,y'x'}D_{xz} - D_{xz}D_{zy}\rangle_Y$$

  which does not reduce to the WW gluon TMD, unless at least $Q_s^2 \ll \boldsymbol{k}_{g\perp}^2 \sim$ dilute limit.
- Maybe there is an other argument to neglect these complicated finite terms beyond the dilute limit...

Preliminary numerical results

- For the numerics, one may first focus on contributions which are naturally proportional to G_{WW} including
  - Sudakov double and single logs,
  - $\mathcal{O}(\alpha_s)$ finite terms that do not break TMD factorization.

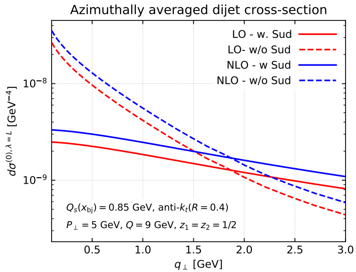
- Requires numerical solution of a "collinearly improved" evolution of the WW $\Rightarrow$ challenging!

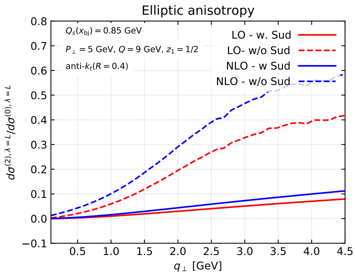
- Rely on the Gaussian approximation and use collinearly improved BK instead.

Some plots with many caveats

- Include:
  - Sudakov with double and single log **but** at fixed coupling.
  - **All** finite terms that do not manifestly break TMD factorization.

- Does not include:
  - Proper small-x evolution. The WW is parametrized by $Q_s(x_{\mathrm{Bj}})$ in the Gaussian approx.
  - Factorization breaking terms.





Summary and outlook

- **Proof of UV and IR finiteness** of the dijet cross-section in the CGC.
- **Proof of JIMWLK factorization** of the rapidity divergence for a process with non-trivial final state.
- Back-to-back limit: Sudakov double and single log at exact N_c, and impact factor.
- Necessity to use a collinearly improved small-x evolution to find the correct Sudakov double log.
- Towards a numerical evaluation of the impact factor with saturation corrections: **very challenging...**
  But recent progresses with the complete evaluation of all non factorization breaking terms!