Gluon Dipole Factorization and Saturation in Diffractive Dijet Production

Dionysios Triantafyllopoulos

ECT*/FBK, Trento, Italy

"Saturation at the EIC"

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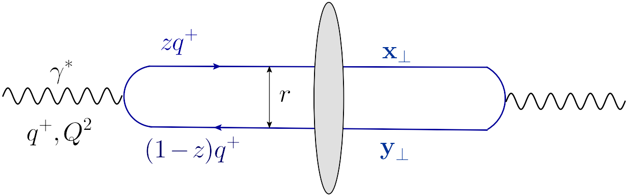
E. Iancu, A.H. Mueller, DT, 2112.06353, PRL
E. Iancu, A.H. Mueller, DT, S.Y. Wei, 2207:06268, JHEP

Dipole Picture of DIS at high energy

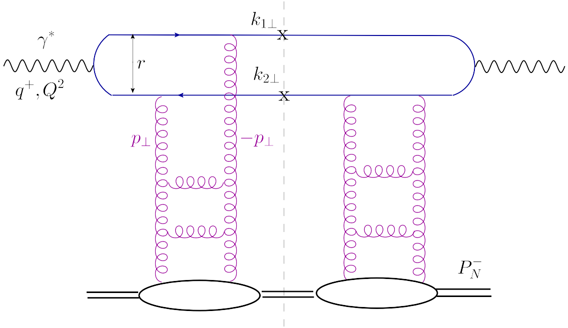
When photon coherent time larger than hadron longitudinal extent



$$\sigma_{L,T}^{\gamma^* A} = 2 \int \mathrm{d}^2 \boldsymbol{r} \int_0^1 \mathrm{d}z \left|\Psi_{L,T}(r, z; Q^2)\right|^2 \int \mathrm{d}^2 \boldsymbol{b}\, T(\boldsymbol{b}, \boldsymbol{r}, x)$$

- Probing saturation requires large dipoles $r \gtrsim 1/Q_s$
- Photon WF has support for $r\bar{Q} \lesssim 1$, with $\bar{Q}^2 = z(1-z)Q^2$
- Typical transverse momentum before scattering $k_\perp^2 \sim \bar{Q}^2$

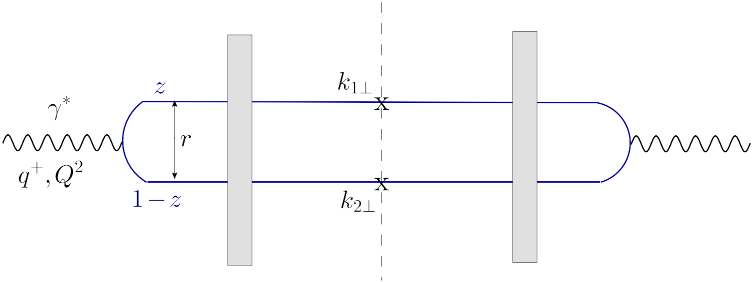
Saturation in Diffraction



$$\sigma_D \propto T^2$$

- For $Q^2 \gg Q_s^2$ aligned-jet configuration dominates:
  $r \sim 1/Q_s$ and $z \sim Q_s^2/Q^2$
- Diffractive cross section at high Q^2 determined by saturation
- Not true for inclusive cross section, dominated by $r \ll 1/Q_s$.

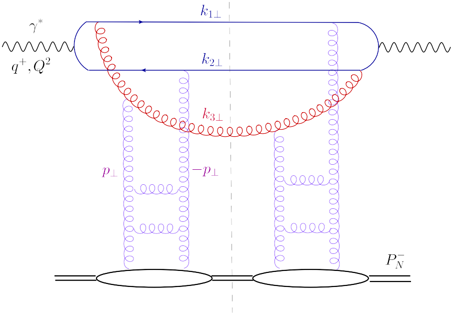
Two nearly back-to-back jets or hadrons



$$k_{1\perp} \simeq k_{1\perp} \sim \bar{Q} \gg K_\perp = |\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}| \sim Q_s$$

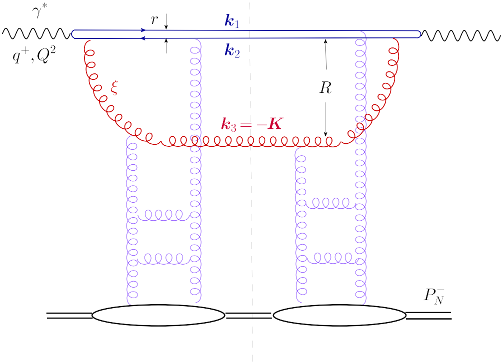
- Q^2 high, but saturation fixes dijet momentum imbalance
- Can be measured from azimuthal correlations
- Competing mechanism by Sudakov radiation

Coherent diffraction: elastic scattering, nucleus target not broken

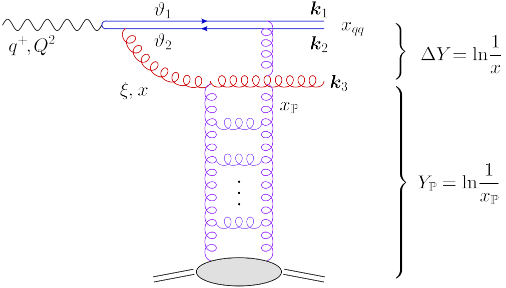


- No momentum transfer $|\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp} + \boldsymbol{k}_{3\perp}| \sim \Lambda_{\rm QCD} \sim 0$
- $k_{1\perp}, k_{2\perp} \sim Q \gg k_{3\perp} \sim Q_s$
- Soft long. momentum $k_3^+ = \xi q^+$ with $\xi \sim k_{3\perp}^2/k_{1\perp}^2 \sim Q_s^2/Q^2 \ll 1$
- Gluon jet controls imbalance

Coordinate space: $R \sim 1/k_{3\perp} \sim 1/Q_s \gg r \sim 1/k_{1,2\perp} \sim 1/Q$, effective gg dipole



- α_s penalty, but no r^2 from scattering of $q\bar{q}$ dipole
- The gg dipole scatters stronger by N_c/C_F, i.e. larger Q_s^2
- Saturation scale evaluated at the rapidity defining the gap



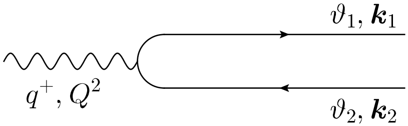
$$M_{q\bar{q}}^2 = \frac{P_\perp^2}{\vartheta_1 \vartheta_2} \sim Q^2$$

$$M_{q\bar{q}g}^2 = M_{q\bar{q}}^2 + \frac{k_{3\perp}^2}{\xi}$$

$$x = \frac{x_{q\bar{q}}}{x_{\mathbb{P}}} = \frac{Q^2 + M_{q\bar{q}}^2}{Q^2 + M_{q\bar{q}g}^2}$$

- $x_{\mathbb{P}}$: fraction of target P_N^- transferred to trijet by Pomeron
- $x_{q\bar{q}} \gtrsim x_{\rm Bj}$: fraction to hard dijet
- x: gluon splitting fraction w.r.t. Pomeron
- Typical situation $M_{q\bar{q}g}^2 \sim Q^2 \rightsquigarrow x \lesssim 1$ and $\xi \sim Q_s^2/Q^2 \ll 1$
  Maximal gap and saturation momentum

SPLITTING TO $q\bar{q}$



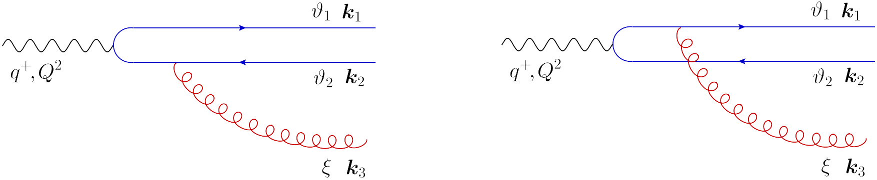
- Order g^0, Fock state expansion just $\left|q^{\alpha}_{\lambda_1}(\vartheta_1, \boldsymbol{k}_1)\, \bar{q}^{\beta}_{\lambda_2}(\vartheta_2, \boldsymbol{k}_2)\right\rangle$
- Coefficient (up to conservation δ's)

$$\psi^{i}_{\lambda_1\lambda_2}(\vartheta_1, \boldsymbol{k}_1) = \sqrt{\frac{q^+}{2}}\, \frac{ee_f}{(2\pi)^3}\, \frac{\varphi^{il}_{\lambda_1\lambda_2}(\vartheta_1)\, k_1^l}{k_{1\perp}^2 + \vartheta_1\vartheta_2 Q^2}$$

- *Energy* denominator, off-shellness of $q\bar{q}$ fluctuation

$$E_{q\bar{q}} - E_\gamma = \frac{1}{2q^+}\left(\frac{k_{1\perp}^2}{\vartheta_1} + \frac{k_{1\perp}^2}{\vartheta_2} + Q^2\right) = \frac{1}{2q^+\vartheta_1\vartheta_2}\left(k_{1\perp}^2 + \vartheta_1\vartheta_2 Q^2\right)$$

Splitting to $q\bar{q}g$, Strict $\xi \to 0$ Limit ("Large" M_X)



- Order g, Fock state expansion $\left| q^{\alpha}_{\lambda_1}(\vartheta_1, \boldsymbol{k}_1)\, \bar{q}^{\beta}_{\lambda_2}(\vartheta_2, \boldsymbol{k}_2)\, g^{a}_{j}(\xi, \boldsymbol{k}_3) \right\rangle$
- Coefficient (up to conservation δ's and $t^{\alpha\beta}$)

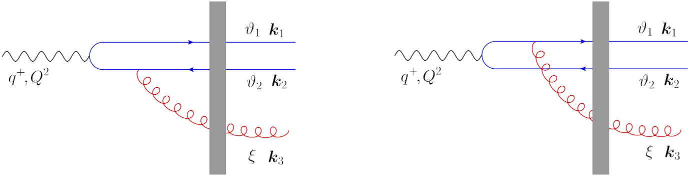
$$\Psi^{ij}_{\lambda_1\lambda_2} = -\frac{e e_f g q^+}{(2\pi)^6} \frac{1}{\sqrt{\xi}}\, \varphi^{il}_{\lambda_1\lambda_2}(\vartheta_1) \left[\frac{k_1^l}{k_{1\perp}^2 + \bar{Q}^2} + \frac{k_2^l}{k_{2\perp}^2 + \bar{Q}^2} \right] \frac{k_3^j}{k_{3\perp}^2}$$

Expect cancellations since $\boldsymbol{k}_1 + \boldsymbol{k}_2$ is the small momentum

- *Energy* denominator

$$E_{q\bar{q}g} - E_{\gamma} = \frac{1}{2q^+} \left(\frac{k_{1\perp}^2}{\vartheta_1} + \frac{k_{2\perp}^2}{\vartheta_2} + \frac{k_{3\perp}^2}{\xi} + Q^2 \right) \simeq \frac{k_{3\perp}^2}{2k_3^+}$$

- Eikonal gluon emission: trivially factorization



Partons keep transverse coordinate $\boldsymbol{x}$,$\boldsymbol{y}$,$\boldsymbol{z}$ fixed during scattering

$$\Psi^{ij,\,\alpha\beta}_{\lambda_1\lambda_2}(\vartheta_1,\boldsymbol{x},\vartheta_2,\boldsymbol{y},\xi,\boldsymbol{z}) \;=\; \frac{ee_f g q^+}{(2\pi)^4}\,\frac{1}{\sqrt{\xi}}\,\varphi^{il}_{\lambda_1\lambda_2}(\vartheta_1)\,\frac{r^l}{r}\,\bar{Q}K_1(\bar{Q}r)$$

$$\times \left\{ \left[\frac{(\boldsymbol{x}-\boldsymbol{z})^j}{(\boldsymbol{x}-\boldsymbol{z})^2} - \frac{(\boldsymbol{y}-\boldsymbol{z})^j}{(\boldsymbol{y}-\boldsymbol{z})^2}\right] U^{ab}(\boldsymbol{z})V(\boldsymbol{x})t^b V^\dagger(\boldsymbol{y})\right.$$

$$\left. - \left[\frac{(\boldsymbol{x}-\boldsymbol{z})^j}{(\boldsymbol{x}-\boldsymbol{z})^2}\, t^a V(\boldsymbol{x})V^\dagger(\boldsymbol{y}) - \frac{(\boldsymbol{y}-\boldsymbol{z})^j}{(\boldsymbol{y}-\boldsymbol{z})^2}\, V(\boldsymbol{x})V^\dagger(\boldsymbol{y})t^a\right] \right\}_{\alpha\beta}$$

Helicity part, K_1 Bessel, WW kernels, scattering after/before gluon emission via Wilson lines

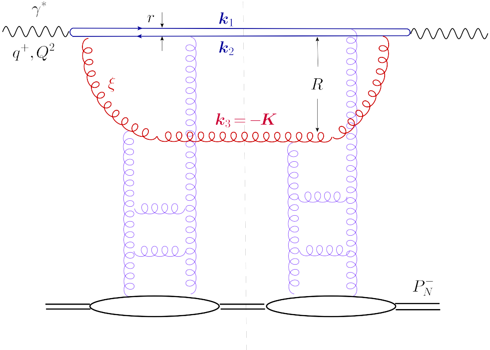
Diffractive Projection

- Color structure of amplitude $\mathcal{O}^a_{\alpha\beta}\,\big|q^\alpha \bar{q}^\beta g^a\big\rangle$
- Isolate color singlet in amplitude

$$\mathbb{P}_D\,\mathcal{O}^a_{\alpha\beta} \;\equiv\; \frac{1}{C_F N_c}\mathrm{tr}\big[t^c \mathcal{O}^c\big]\, t^a_{\alpha\beta}\,.$$

- Trijet cross section proportional to $\dfrac{1}{\xi}\big|\tilde{\mathcal{A}}^{lj}_{q\bar{q}g}\big|^2$

$$\tilde{\mathcal{A}}^{lj}_{q\bar{q}g} = \int \frac{\mathrm{d}^2\boldsymbol{x}}{2\pi}\int \frac{\mathrm{d}^2\boldsymbol{y}}{2\pi}\, e^{-i\boldsymbol{k}_1\cdot\boldsymbol{x}-i\boldsymbol{k}_2\cdot\boldsymbol{y}}\,\frac{r^l}{r}\,\bar{Q}K_1(\bar{Q}r)$$
$$\times\int \frac{\mathrm{d}^2\boldsymbol{z}}{2\pi}\, e^{-i\boldsymbol{k}_3\cdot\boldsymbol{z}}\left[\frac{(\boldsymbol{x}-\boldsymbol{z})^j}{(\boldsymbol{x}-\boldsymbol{z})^2}-\frac{(\boldsymbol{y}-\boldsymbol{z})^j}{(\boldsymbol{y}-\boldsymbol{z})^2}\right]\left[\mathcal{S}_{q\bar{q}g}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})-\mathcal{S}_{q\bar{q}}(\boldsymbol{x},\boldsymbol{y})\right]$$



- New momentum variables: $\boldsymbol{P} = \vartheta_2 \boldsymbol{k}_1 - \vartheta_1 \boldsymbol{k}_2, \;\; \boldsymbol{K} \equiv \boldsymbol{k}_1 + \boldsymbol{k}_2$
- "Correlation limit": $P_\perp \gg K_\perp \Leftrightarrow r \ll R$
- $\mathcal{S}_{q\bar{q}g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) - \mathcal{S}_{q\bar{q}}(\boldsymbol{x}, \boldsymbol{y}) \to -\mathcal{T}_{gg}(\boldsymbol{R})$
- Large distance emission from a dipole source factorizes

$$\frac{(\boldsymbol{x}-\boldsymbol{z})^j}{(\boldsymbol{x}-\boldsymbol{z})^2} - \frac{(\boldsymbol{y}-\boldsymbol{z})^j}{(\boldsymbol{y}-\boldsymbol{z})^2} \simeq \frac{r^i}{R^2}\left(\delta^{ij} - \frac{2R^iR^j}{R^2}\right)$$

TMD Factorization and Cross Section

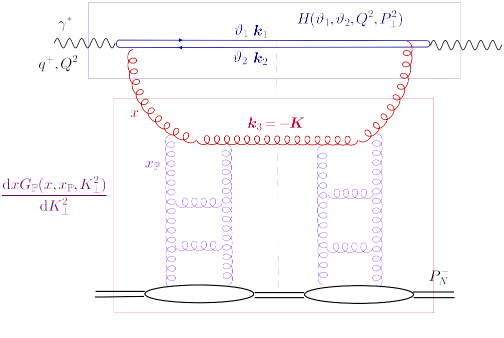
Do FT's to go back to momentum space

$$\mathcal{A}^{lj}_{q\bar{q}g} = \underbrace{\frac{1}{P_\perp^2+\bar{Q}^2}\left(\delta^{li} - \frac{2P^lP^i}{P_\perp^2+\bar{Q}^2}\right)}_{\text{hard factor}} \underbrace{\left(\frac{K^iK^j}{K_\perp^2} - \frac{\delta^{ij}}{2}\right)\mathcal{G}(K_\perp, Y_{\mathbb{P}})}_{\text{semi}-\text{hard factor}}$$

$$\mathcal{G}(K_\perp, Y_{\mathbb{P}}) = 2\int_0^\infty \frac{\mathrm{d}R}{R}\, J_2(K_\perp R)\, \mathcal{T}_{gg}(R, Y_{\mathbb{P}})$$

Real, symmetric, traceless dimensionless distribution.

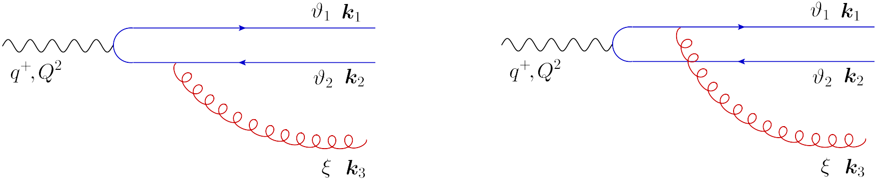
Contains all QCD dynamics



Straightforward to square (traceless $\times$ traceless $\rightsquigarrow$ diagonal)

$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma_{T,L}^* A \to q\bar{q}gA}}{\mathrm{d}\vartheta_1 \mathrm{d}\vartheta_2 \mathrm{d}^2\boldsymbol{P} \mathrm{d}^2\boldsymbol{K} \mathrm{d}Y_{\mathbb{P}}} = \underbrace{H_{T,L}(\vartheta_1, \vartheta_2, \bar{Q}^2, P_\perp^2)}_{\text{hard factor}} \underbrace{\frac{\mathrm{d}xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}}_{\text{semi}-\text{hard factor}}$$

TMD factorization, "first principles" result for Pomeron UGD

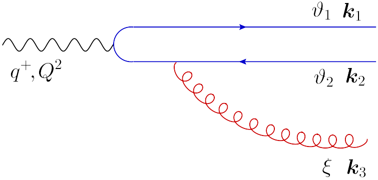


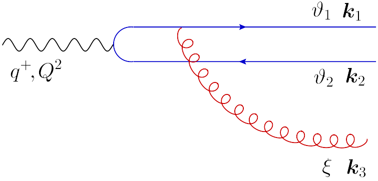
$$\Psi^{lj}_{\rm reg} = \left[\frac{k_1^l \left(k_3^j + \frac{\xi}{1-\vartheta_1}\, k_1^j\right)}{k_{1\perp}^2 + \bar{Q}^2} + \frac{k_2^l \left(k_3^j + \frac{\xi}{1-\vartheta_2}\, k_2^j\right)}{k_{2\perp}^2 + \bar{Q}^2}\right] \frac{1}{k_{3\perp}^2 + \mathcal{M}^2}$$

j: gluon pol., $\mathcal{M}^2 = \xi \left(\frac{k_{1\perp}^2}{\vartheta_1} + \frac{k_{2\perp}^2}{\vartheta_2} + Q^2\right)$ gluon "off-shellness"

- Both gluon vertex and energy denominator violate factorization
- Switch to hard and semi-hard momenta $\boldsymbol{P}$ and $\boldsymbol{K}$
- Expand for $K_\perp \ll P_\perp$ and $\xi \sim K_\perp^2/P_\perp^2$, leading terms cancel
- Add contribution from instantaneous quark propagator

Gluon Dipole Wavefunction ("Small" M_X)

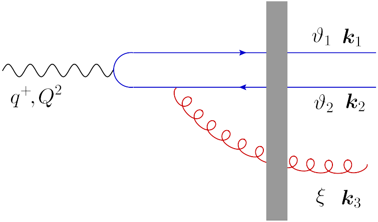


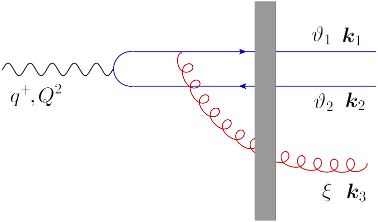


$$\Psi^{lj} = \frac{1}{P_\perp^2 + \bar{Q}^2}\left(\delta^{li} - \frac{2P^l P^i}{P_\perp^2 + \bar{Q}^2}\right)\frac{K^i K^j - (\delta^{ij}/2)K_\perp^2}{K_\perp^2 + \mathcal{M}^2}$$

- Virtuality $\mathcal{M}^2 = \xi\left(\frac{P_\perp^2}{\vartheta_1\vartheta_2} + Q^2\right)$
- Is there a recoil since ξ is not too small?

$$\frac{|\Delta \boldsymbol{y}|}{r} \sim \frac{\xi}{\theta_2}\,\frac{R}{r} \sim \xi P_\perp/K_\perp \quad \text{small when} \quad \xi \ll K_\perp/P_\perp \ll 1$$





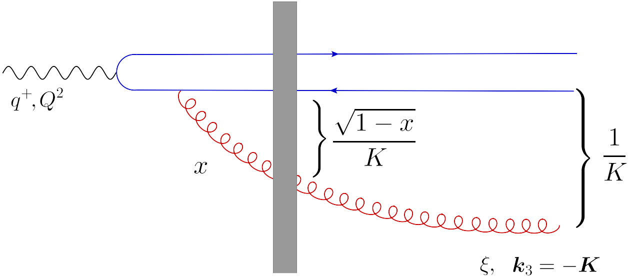
Fourier Transform $\boldsymbol{K} \to \boldsymbol{R}$, insert amplitude $\mathcal{T}_{gg}(R)$, inverse FT

$$\mathcal{A}^{lj}_{q\bar{q}g} = \underbrace{\frac{1}{P_\perp^2 + \bar{Q}^2}\left(\delta^{li} - \frac{2P^lP^i}{P_\perp^2 + \bar{Q}^2}\right)}_{\text{hard factor}} \underbrace{\left(\frac{K^iK^j}{K_\perp^2} - \frac{\delta^{ij}}{2}\right)\mathcal{G}(x, K_\perp, Y_{\mathbb{P}})}_{\text{semi-hard factor}}$$

$$\mathcal{G}(x, K_\perp, Y_{\mathbb{P}}) = \mathcal{M}^2 \int_0^\infty \mathrm{d}R\, R\, J_2(K_\perp R) K_2(\mathcal{M}R) \mathcal{T}_{gg}(R, Y_{\mathbb{P}})$$

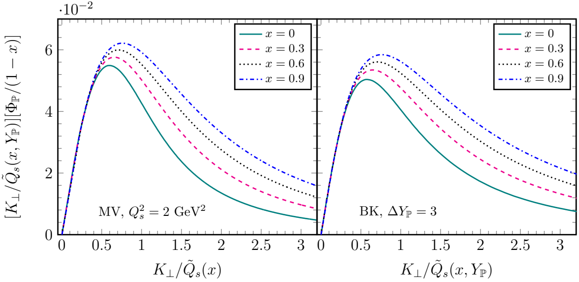
- Hyperbolic Bessel K_2 reflecting the off-shellness
- Factorization: $\mathcal{M}^2 = \dfrac{x}{1-x} K_\perp^2$ in terms of target fractions

Effective Saturation Momentum



- Virtuality limits the gg dipole size R
- Diffusion $\sqrt{\tau_q/\tau_g} \sim K_\perp/(\sqrt{\xi}Q) \sim \sqrt{1-x}$
- Effective saturation momentum $\tilde{Q}_s^2(x, Y_{\mathbb{P}}) = (1-x)Q_s^2(Y_{\mathbb{P}})$

The Pomeron UGD



$$\tilde{Q}_s^2(x) = (1-x)Q_s^2$$

$$\frac{\mathrm{d}xG_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} = (1-x)\begin{cases} 1 & \text{for} \quad K_\perp \ll \tilde{Q}_s \\ \dfrac{\tilde{Q}_s^4}{K_\perp^4} & \text{for} \quad K_\perp \gg \tilde{Q}_s \end{cases}$$

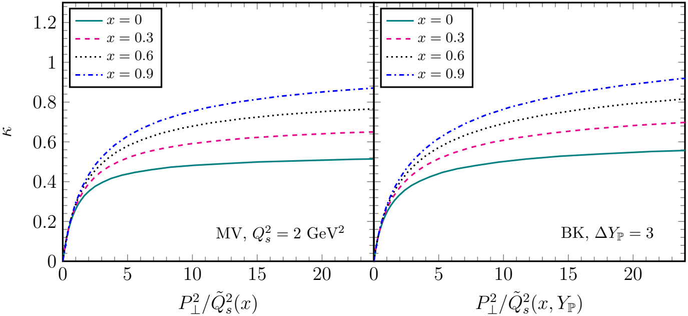
Modulo smooth functions of x and $K_\perp^2$, slightly softer power with BK.

- Strong, "integrable" decrease at large momenta
- Approximate scaling after dividing by $1-x$

$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma^*_{T,L}A\to q\bar{q}gA}}{\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}Y_{\mathbb{P}}} = H_{T,L}(\bar{Q}^2, P_\perp^2)\, xG_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)$$

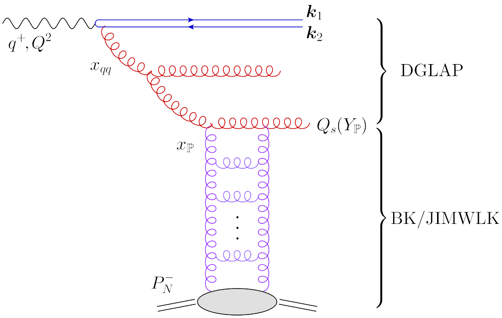
$$xG_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2) = \pi \int_0^{P_\perp^2} \mathrm{d}K_\perp^2 \frac{\mathrm{d}xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} \simeq \pi\kappa(x, P_\perp^2)(1-x)^2 Q_s^2(Y_{\mathbb{P}})$$

$K_\perp \sim Q_s$ dominates, saturation determines PDF at *hard* scale $P_\perp$



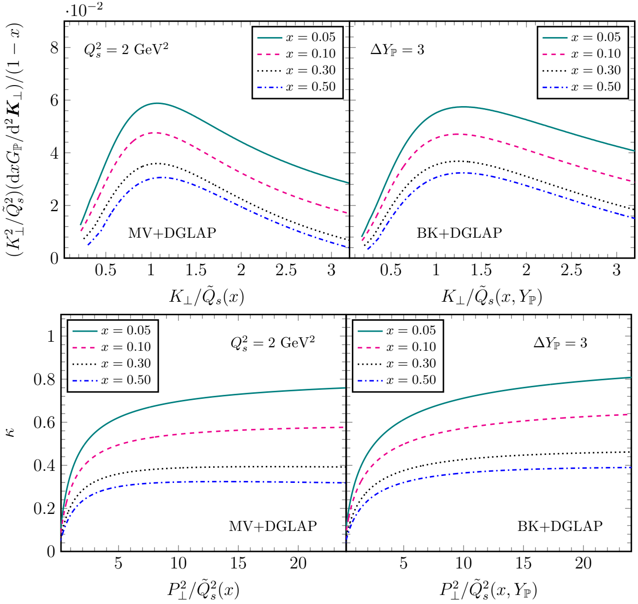
Converging as inverse power of $P_\perp$, smooth function of x

Evolution



- High energy (BK/JIMWLK) evolution of the target
- DGLAP evolution of the Integrated Pomeron
  - Initial condition by saturation
  - Eliminates possible Sudakov effects
- Remarkable case incorporating both types of evolution

DGLAP Evolution



Conclusions

- *Hard* dijet production process sensitive to saturation
- Significance of soft gluon
  - α_s penalty, but no r^2 from scattering of $q\bar{q}$
  - Leads to large gg dipole $R \sim 1/Q_s$
  - Provides for imbalance of dijet
- Saturation relevant even when not measuring imbalance
- BK/JIMWLK as initial condition for DGLAP
- Integrate over all phase space $\rightsquigarrow$ $q\bar{q}g$ component of DSF
- Similar process in Ultra Peripheral AA collisions
- Sudakov? Feasible at EIC?