ULTRA-DENSE MATTER OF NEUTRON STARS AND SUPERNOVAE

PART II: THE EQUATION OF STATE OF HOT AND DENSE MATTER

Adapted from Fiorella Burgio's lecture
with input from A. Fantina, F. Gulminelli, M. Oertel

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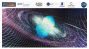
Thematic school "Gravitational waves from Neutron Stars", Aussois, June,5-9,2023



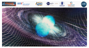
OUTLINE

1 INTRODUCTION

2 SUB-SATURATION MATTER

- Nuclear statistical equilibrium
- Beyond NSE

3 SUPRA-SATURATION MATTER



EQUATION OF STATE AND EQUILIBRIUM CONDITIONS

CONDITIONS FOR BNS MERGER REMNANTS AND CCSN

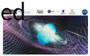
The equation of state (EoS) thermodynamically relates different quantities to close the system of hydrodynamic equations.

The number of parameters depends on equilibrium conditions :

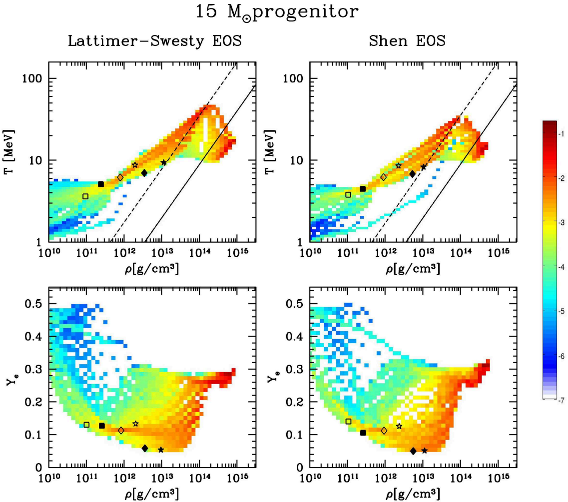
- For a cold and charge-neutral neutron star in β-equilibrium :
  EoS is $P(n_B)$ (or equivalent)
- For core collapse and neutron star merger remnants :
  - Thermal and mechanical equilibrium in general very quickly acheived except for neutrinos
    - these particles have to be treated by transport equations coupled to hydrodynamics
  - charge neutrality always fulfilled, i.e.

$$Y_e = \sum_{hadrons} n_{q,h}/n_B \equiv Y_q$$

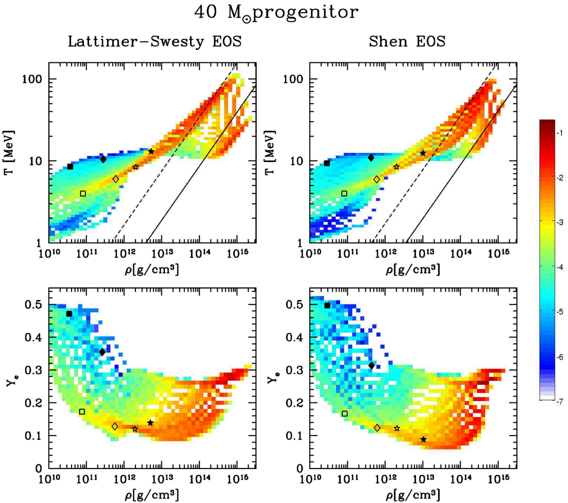
- hydrodynamical timescale $\sim 10^{-6}$ s $\longrightarrow$ β-equilibrium not always achieved



What about the temperature ?


15 $M_\odot$progenitor
Lattimer−Swesty EOS
Shen EOS
T [MeV]
100
10
1
Y_e
0.5
0.4
0.3
0.2
0.1
0
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
ρ[g/cm³]
-1
-2
-3
-4
-5
-6
-7

[A. Perego]


40 M$_\odot$progenitor
Lattimer–Swesty EOS
Shen EOS
T [MeV]
1
10
100
Y_e
0
0.1
0.2
0.3
0.4
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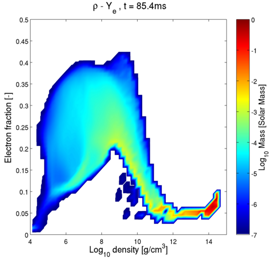
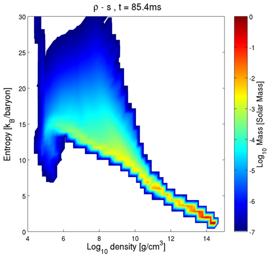
ρ[g/cm³]
-1
-2
-3
-4
-5
-6
-7

[A. Perego]


ρ - Y_e , t = 85.4ms
Electron fraction [-]
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
4
6
8
10
12
14
Log10 density [g/cm3]
Log10 Mass [Solar Mass]
0
-1
-2
-3
-4
-5
-6
-7


ρ - s , t = 85.4ms
Entropy [k_B/baryon]
0
5
10
15
20
25
30
4
6
8
10
12
14
Log10 density [g/cm3]
Log10 Mass [Solar Mass]
0
-1
-2
-3
-4
-5
-6
-7

[A. Perego]

TEMPERATURE EFFECTS IN A FERMI GAS

- Recall : Fermi-Dirac distribution function $f_{FD}(p) = \dfrac{1}{\exp((E(p)-\mu)/T)+1}$ becomes a step function in the degenerate limit, $T \ll \mu$

Temperature corrections :

- In the non-relativistic case (nucleons) :

$$\varepsilon = mn + a_1 \frac{n^{5/3}}{m} + a_2 T^2 m n^{1/3} + \cdots$$

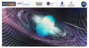
Numerical estimate for $m = 1$ GeV, $n = 0.1\ \mathrm{fm}^{-3}$ (T in MeV) : $\varepsilon[\mathrm{MeV/fm}^3] = 100 + a_1 0.86 + a_2 T^2 0.011604 + \cdots$

- In the ultra-relativistic case (electrons) :

$$\varepsilon = a_1 n^{4/3} + a_2 n^{2/3} T^2 + a_3 T^4 + \cdots$$

Numerical estimate for $n = 0.1\ \mathrm{fm}^{-3}$ (T in MeV) : $\varepsilon[\mathrm{MeV/fm}^3] = a_1 9.3 + a_2\ T^2\ 0.001 + a_3\ T^4\ 8 \times 10^{-8} + \cdots$

→ for core collapse and NS merger matter temperature effects not negligible



CONDITIONS IN CCSN AND BNS MERGERS

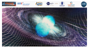
The equation of state (EoS) thermodynamically relates different quantities to close the system of hydrodynamic equations.
The number of parameters depends on equilibrium conditions :

- For a cold and charge-neutral neutron star in β-equilibrium :
  EoS is $P(n_B)$ (or equivalent)
- For core collapse and neutron star mergers :
  - charge neutrality always fulfilled
  - β-equilibrium not always achieved
  - temperature effects not negligible !

  →EoS is $P(n_B, T, Y_e)$ (or equivalent)

Very large ranges to be covered :

$$n_B = 10^{-8}\mathrm{fm}^{-3} \cdots 1\mathrm{fm}^{-3}$$
$$T = 0.2\mathrm{MeV} \cdots 150 \text{ MeV}$$
$$Y_e = 0.05 \cdots 0.5$$

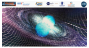


WE NEED AN EOS FOR HOT DENSE ASYMMETRIC MATTER...

- Three parts of the EoS :
  - Hadrons
  - Charged leptons, free Fermi gas coupled to hadrons only via charge neutrality
    Neutrinos are not in thermal equilibrium, can thus not be treated via EoS
  - Photons, free (massless) Bose gas with

$$p = \frac{\pi^2}{15}\frac{T^4}{3} \qquad \varepsilon = \frac{\pi^2}{15}T^4$$

In the following we will concentrate on the hadronic part.

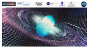


THE HADRONIC EOS

Composition of hadronic matter changes dramatically depending on baryon number density, charge fraction (asymmetry), and temperature.

Different regimes :

- Very low densities and temperatures :
  - dilute gas of non-interacting nuclei
    → nuclear statistical equilibrium (NSE)
- Intermediate densities and low temperatures :
  - gas of interacting nuclei surrounded by free nucleons
    → approaches beyond NSE
- High densities and temperatures :
  - nuclei dissolve
    → strongly interacting (homogeneous) hadronic matter
  - potentially transition to the quark gluon plasma



Plan

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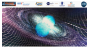
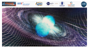
2. Sub-saturation matter
    - Nuclear statistical equilibrium
    - Beyond NSE

3. Supra-saturation matter



PLAN



Nuclear statistical equilibrium (NSE)

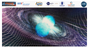
Basic assumption : mixture of nucleons (n, p) and nuclei (X) in chemical equilibrium

- chemical equilibrium expressed via equality of chemical potentials for a nucleus with Z protons and N neutrons :

$$ {}^{A}_{Z}X_N : Zp + (A-Z)n \qquad \mu_X = Z\mu_p + (A-Z)\mu_n $$

- simplest model (called NSE in the literature) assumes in addition non-interacting independent particles
  - partition function factorises : $Z = \Pi_i Z_i$
  - particles form an ideal gas (Maxwell-Boltzmann or Fermi/Bose)

Attention : Approximation not really valid below $T \sim 0.5$ MeV and low densities : nuclear reaction network necessary, determining abundances from individual reaction rates



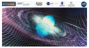
SIMPLEST MODEL

- Maxwell-Boltzmann statistics with $f_{\rm MB} = e^{-E_i/T}\ e^{\mu_i/T}$
- non-relativistic kinematics : $E_i = m_i + \frac{p_i^2}{2m_i}$
- energy density and pressure $\varepsilon = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} E_i\ f_{\rm MB}$

$$\varepsilon = \sum_i m_i n_i + \frac{3}{2}\frac{T}{(2\pi)^{3/2}}\sum_i g_i (m_i T)^{3/2} z_i = \sum_i m_i n_i + \frac{3}{2} T \sum_i n_i$$

$$p = \frac{T}{(2\pi)^{3/2}}\sum_i g_i (m_i T)^{3/2} z_i = T\sum_i n_i$$

- $g_i = (2J_i + 1)$ are the degeneracy factors
- $z_i = \exp((\mu_i - m_i)/T)$ are often called fugacities
- consider only nuclear ground states
- masses and spins of individual nuclei from data tables (e.g. NuBase 2012 evaluation with 3350 nuclides) or mass formulae



A SIMPLE EXAMPLE

- take a mixture of neutrons (n), protons (p) and deuterons ($d = {}^2\mathrm{H}$)

$$n + p \Leftrightarrow d \quad \Rightarrow \quad \mu_p + \mu_n = \mu_d$$

- $m_d = m_n + m_p - B_d$, deuteron binding energy $B_d = 2.225$ MeV

- individual number densities

$$n_i = g_i \frac{(m_i T)^{3/2}}{(2\pi)^{3/2}} \exp(\frac{\mu_i - m_i}{T})$$

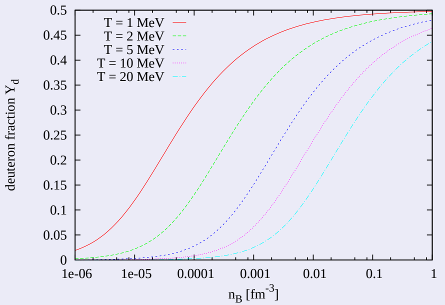
- deuteron fraction $Y_d = \dfrac{n_d}{n_B}$

- pressure and energy density

$$p = T \sum_{n,p,d} n_i$$

$$\varepsilon = \sum_{n,p,d} (m_i + \frac{3}{2} T n_i)$$

DEUTERON FRACTION FOR SYMMETRIC MATTER ($n_n = n_p$)



POSSIBLE IMPROVEMENTS ?

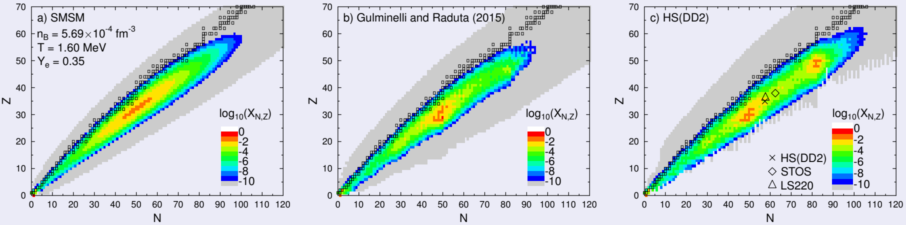
- At (very) low densities the dilute non-interacting ideal gas is a good approximation with some refinements :
  - at finite temperature excited states j of nuclei will be populated

    $$\rightarrow \quad g_i(T) = g_i(T=0) + \sum_j (2J_j + 1) \exp(-\frac{E_j}{T})$$

    in general semi-empirical formulae used
  - Coulomb corrections and surface effects
  - Mass model for "exotic" nuclei

Modeling does not only depend on the interaction between nucleons, but on the modeling of these effects

NUCLEAR ABUNDANCES WITHIN DIFFERENT MODELS (SAME THERMODYNAMIC CONDITIONS, GAS DENSITY NEGLIGIBLE) [MO+ 2017]



Possible improvements ?

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  - at finite temperature excited states j of nuclei will be populated

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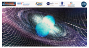
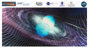
    in general semi-empirical formulae used
  - Coulomb corrections and surface effects
  - Mass model for “exotic” nuclei
- At higher densities ($n_B \sim 10^{-4}\mathrm{fm}^{-3}$) medium effects become important, the (strong) interaction of clusters and with the surrounding nucleons cannot be neglected

  In particular : Pauli exclusion principle leads to the dissolution of clusters !

  → different approaches beyond NSE



Plan



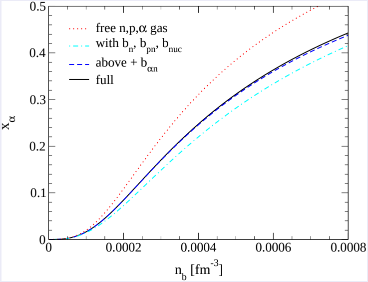
Different approaches beyond NSE

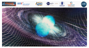
1. Virial expansion

- Idea : as far as fugacities $z_i = \exp((\mu_i - m_i)/T) \ll 1$, the (grand canonical) partition function can be expanded in terms of z_i
- bound states (clusters) and scattering states (phase shifts) can be included
- limited to $n_i \ll (m_i T/(2\pi))^{3/2}$

  $(n_i \ll 2 \times 10^{-4} T_{\mathrm{MeV}}^{3/2}$ for nucleons)

α mass fraction for symmetric matter ($n_n = n_p$)(Horowitz & Schwenk, NPA 2006)





DIFFERENT APPROACHES BEYOND NSE

1. Virial expansion

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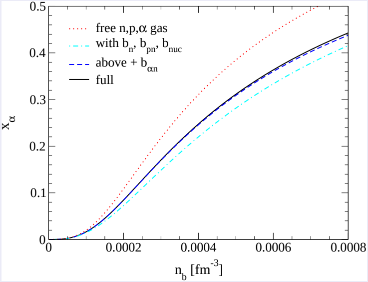
- bound states (clusters) and scattering states (phase shifts) can be included
- limited to $n_i \ll (m_i T/(2\pi))^{3/2}$

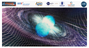
  ($n_i \ll 2 \times 10^{-4} T_{\mathrm{MeV}}^{3/2}$ for nucleons)

α MASS FRACTION FOR SYMMETRIC MATTER ($n_n = n_p$)(HOROWITZ & SCHWENK, NPA 2006)



2. Quantum statistical

- Idea : solve self-consistently for in-medium propagators and T-matrix
- medium dependent shift of binding energies (Pauli principle) and phase shifts
- dissolution of clusters at high densities (Mott effect)



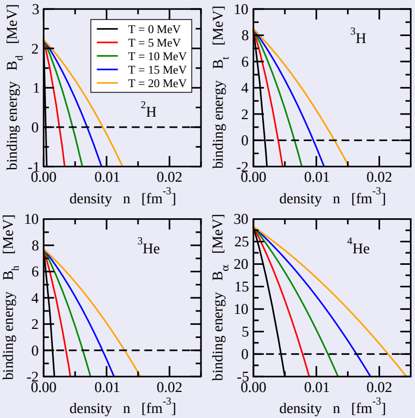
Different approaches beyond NSE

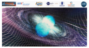
3. Generalised energy density functional approach

- Idea : include light clusters explicitely in the EDF with medium dependent binding energies fitted to QS data
- How to treat heavy clusters ? → same principle with phenomelogical form for binding energies (competition between electron screening and Pauli blocking)

Typel 2018, Typel & Pais 2017, Fischer+ 2020

in-medium cluster binding energies(Typel et al. PRC 2010)



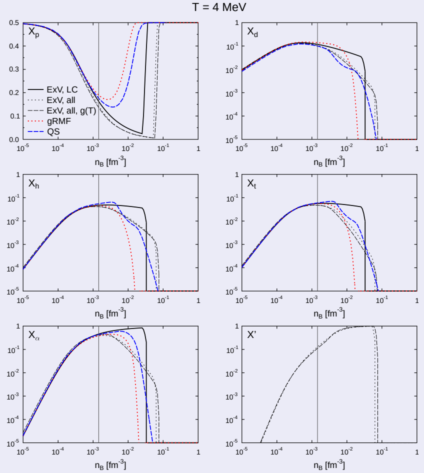


DIFFERENT APPROACHES BEYOND NSE

3. Generalised energy density functional approach
   - Idea : include light clusters explicitely in the EDF with medium dependent binding energies

4. Phenomenolgical excluded volume
   - Idea : mimic medium effects (Pauli principle) by excluding the volume occupied by a cluster for all other clusters
   - cluster dissolution not well described since medium modifications of cluster properties not included

COMPARISON OF LIGHT CLUSTER ABUNDANCES FOR SYMMETRIC MATTER (HEMPEL+ PRC 2011)

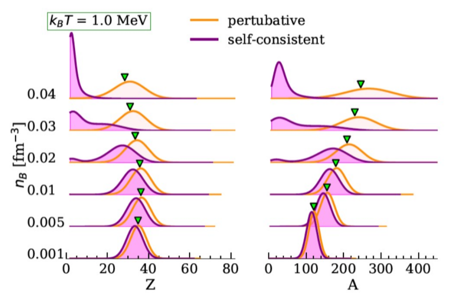


DIFFERENT APPROACHES BEYOND NSE

5. Microscopic modeling of in-medium effects

- Idea : in-medium effects are naturally included in a microscopic density functional calculation in a WS cell (type HFB, ETF, . . .)
- "Cluster" defined after subraction of uniform nucleon background
- Optimal cluster distribution by minimizing the free energy of a mixture of WS configurations

CLUSTER DISTRIBUTIONS (DINH THI+ A&A 2023)



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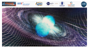


Choice of the interaction for bulk matter

Ab-initio calculations at finite temperature very demanding

→ only few results for restricted $n_B, T_{,} Y_q$ ranges

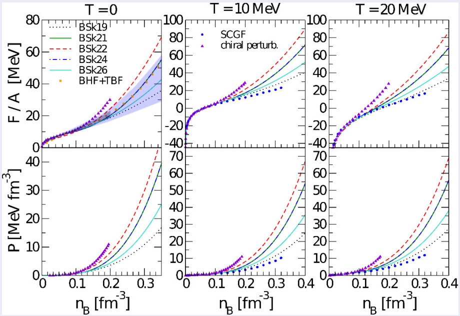
→ mainly phenomenological models used

Question : Can we use the same (phenomenological) effective interactions in the whole (T, n_B, Y_q) range ?

- temperature effects on the interaction small, enter only via the kinetic energy terms

→ Basic assumption : the same (effective) interaction can be used throughout the entire EoS range

EoS of neutron matter in a Skyrme mean field compared with microscopic calculations(A. Fantina)



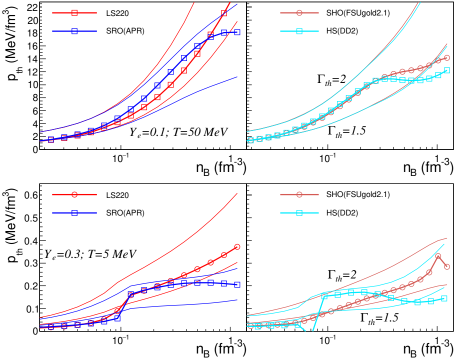
THERMAL EFFECTS MAINLY VIA EFFECTIVE MASSES

- Main thermal effect via distribution function in kinetic energy

SINGLE PARTICLE ENERGIES

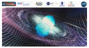
$$\varepsilon = \frac{\vec{p}^2}{2m^*} + m^* - \mu^*$$

- Attention : different definition of effective masses and chemical potentials in non-relativistic/relativistic models (Landau/Dirac) masses



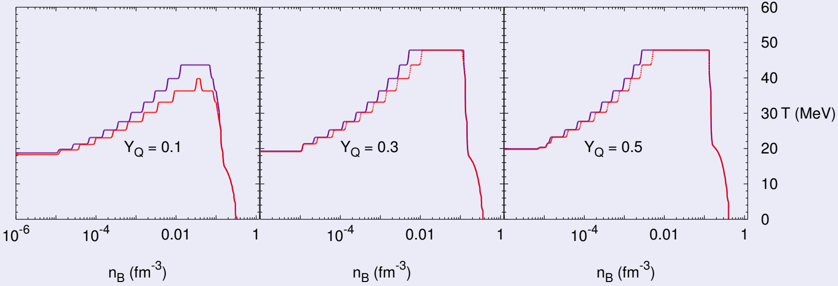
[Raduta+2022]

- Overall thermal effects small in energy and larger in pressure
- Often used Γ-law does not reproduce density dependence of thermal pressure



WHERE DO HYPERONS APPEAR ?

REGIONS WITH AN OVERALL HYPERON FRACTION $> 10^{-4}$ [MARQUES ET AL 2017]

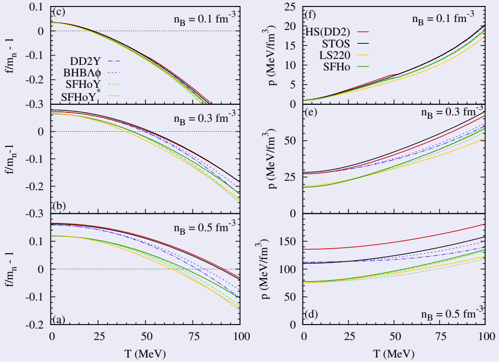


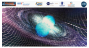
- Bump slightly below saturation due to competition between hyperons and light clusters
- Low charge fraction favors hyperons
- No hyperons included in inhomogeneous matter (→ poster by Tiago Custodio)

Homogeneous hot and dense matter

- Temperature effects in favor of appearance of additional particles (hyperons, mesons, . . .)
- Choose values of the parameters compatible with constraints on the EoS and hypernuclear data
- Effect on thermodynamic quantities not negligible

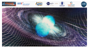
Thermodynamic quantities as function of T for $Y_e = 0.3$ [Fortin+ 2018]





EXERCISE SESSION THIS AFTERNOON

If you did not yet do, please download the Docker image :
`https://hub.docker.com/r/pdavis422/gw-summer-school-compose`



SOME REFERENCES FOR FURTHER READING

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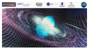

SOME USEFUL SOFTWARE

- If you want to know more and test for example rotating neutron stars with your favorite EoS, there are two publicly available codes :
    - The RNS code written by Nick Stergioulas.

      See `http://www.gravity.phys.uwm.edu/rns/`
    - The LORENE library developped at Meudon mainly by E. Gourgoulhon, P. Grandclément, J.-A. Marck, J. Novak. K. Taniguchi.

      See `http://www.lorene.obspm.fr`
- `http://www.Stellarcollapse.org` is a website aimed at providing resources supporting research in stellar collapse, core-collapse supernovae, neutron stars, and gamma-ray bursts
- Tables of realistic EoS for neutron stars and core collapse are available on different web sites, e.g.
    - Compose (the Compstar project), `https://compose.obspm.fr`
    - EOSDB, web site by Chikako Ishizuka, `http://asphwww.ph.noda.tus.ac.jp/eos-gate/`