Ultra-dense matter of neutron stars and supernovae

Exercise session

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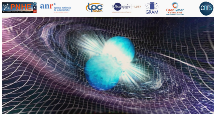
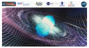


Thematic school GWsNS23: Gravitational waves from neutron stars

Outline



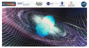
Reminder of thermodynamic identities

There are different thermodynamic potentials depending on the temperature (T)/the entropy (S), the volume (V)/the pressure (p), the particle number (N)/the chemical potential (μ), or the corresponding densities (s, n)

- the energy density $\varepsilon(s, n_i)$
- the free energy density $f(T, n_i) = \varepsilon - Ts$
- the grand canonical potential density $\omega(T, \mu_i) = \varepsilon - Ts - \sum_i \mu_i n_i$
- the conjugate variables are related via derivatives, e.g. $n_i = -\frac{\partial \omega}{\partial \mu_i}$

There is a chemical potential associated with each conserved quantity (charge, baryon number, lepton number) and the individual chemical potentials are linear combinations of μ_q, μ_B, μ_l, e.g. $\mu_{proton} = \mu_B + \mu_q$.

At zero temperature the entropy vanishes.



Equations of state in CompOSE

Stored EoS tables : grid

Grid in temperature T, baryon number density n_B and electron/charge fraction Y_e/Y_q

EOS.NB

- File with grid in baryon number density
- Two lines giving first and last grid index, then explicit listing of all grid densities in fm^{-3}

EOS.T

- File with grid in temperature
- Two lines giving first and last grid index, then explicit listing of all grid temperatures in MeV

EOS.YQ

- File with grid in hadronic charge fraction/electron fraction
- Two lines giving first and last grid index, then explicit listing of all grid fractions, dimensionless

Equations of state in CompOSE

Stored EoS tables : thermodynamic quantities

EOS.THERMO

- File with thermodynamic quantities
- First line with information about neutron/proton mass (m_n/m_p) and presence of electrons in the tables or not
- Thermodynamic information ordered with grid indices (i_T, j_{n_B}, k_{y_q})
- Each entry contains
  1. Pressure divided by baryon number density p/n_B
  2. Entropy per baryon s/N_B
  3. Scaled and shifted baryon number chemical potential $\mu_B/m_n - 1$
  4. Scaled charge chemical potential μ_q/m_n
  5. Scaled effective lepton chemical potential μ_l/m_n
  6. Scaled free energy per baryon $f/(n_B m_n) - 1$
  7. Scaled energy per baryon $e/(n_B m_n) - 1$

  and optionally other thermodynamic quantities
- Structure of a line $i_T, j_{n_B}, k_{Y_q}, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, n_{add}, \ldots$

Equations of state in CompOSE

Stored EoS tables : optional information on composition etc

EOS.COMPO

- File with information on chemical composition
  1. Particle fractions $Y_i = n_i/n_B$
  2. Information about representative nucleus(nuclei) A, Z, Y_i
  3. Phase index (there might be different phases)
- Structure of a line
  $i_T, j_{n_B}, k_{Y_q}, I_{phase}, N_{pairs}, \underbrace{I_1, Y_{I_1}, \ldots,}_{N_{pairs}\,\text{pairs}} N_{quad}, \underbrace{I_1, A_{I_1}, Z_{I_1}, Y_{I_1}, \ldots}_{N_{quad}\,\text{quadruples}}$
- Table with particle indices in the Manual, end of chapter 3

EOS.MICRO

- File with information on interaction potentials ($\to$ effective chemical potentials), effective masses, pairing gaps etc.
- Structure of a line $i_T, j_{n_B}, k_{Y_q}, N_{qty}, \underbrace{K_1, q_{K_1}, \ldots}_{N_{qty}\,\text{pairs}}$
- Combined index $K_i = 1000 I_i + J_i$ with particle index I_i and the index J_i for the quantity (cf chapter 7 of the manual)

Equations of state in CompOSE

Different families of EoS tables

Cold neutron star EoS

EoS for cold ($T = 0$) matter in β-equilibrium ; directly applicable to construct NS models, e.g. with LORENE (`https://lorene.obspm.fr`)

Neutron matter EoS

EoS tables for $Y_q = 0$

Cold matter EoS

EoS tables for $T = 0$ with different charge fractions

General purpose EoS

Tables which cover a large range of T, n_B, Y_q, as required for CCSN and BNS mergers. Most models are provided in two versions, one with the contribution of electrons, positrons and photons included and one containing only the baryonic part

Equations of state in CompOSE

Additional files

EOS.INIT

Initialisation file needed by the Compose software

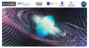
EOS.MR

Mass (solar mass)-radius (km) relation of a cold β-equilibrated and spherically symmetric NS calculated from the EoS

EOS.THERMO.NS and EOS.NB.NS

n_B and thermodynamic quantities for cold β-equilibrated matter for direct use to obtain NS models. The latter files exist only for general purpose EoS tables and have been extracted from the lowest temperature entry of the corresponding table, i.e. in general for a nonzero (but very small) temperature. eos.thermo.ns contains as additional quantity the electron fraction Y_e obtained in β-equilibrium and the enthalpy density.

Plan



Online tools vs CompOSE software

https://compose.obspm.fr/home

Two (equivalent) ways to handle CompOSE data :

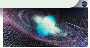
- Interpolation of tabulated data
- Extraction of selected quantities
- Calculation of additional related quantities

Online tools

Run the CompOSE software via a web interface
Results can be downloaded and visualised
Access restricted (account needed)

CompOSE software

The CompOSE software can be freely downloaded
`Fortran90` routines + a sample Makefile ; With/without HDF5 support
The compiled code (`compose`) is already embedded in the Docker image

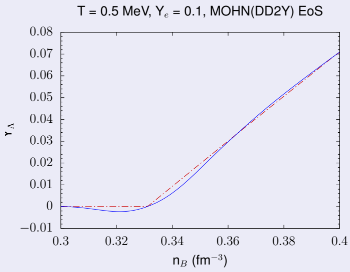


Interpolation

- Polynomial interpolation (2+1) ; direct interpolation of individual quantities (not using thermodynamic consistency)

  - Order of the interpolation can be chosen separately for T, n_B, Y_q
    - Order 3 : fifth order polyonomial, continuity of function, first and second derivative at grid points
    - Order 2 : third order polyonomial, continuity of function and first derivative at grid points
    - Order 1 : first order polyonomial, continuity of function at grid points

Interpolation order



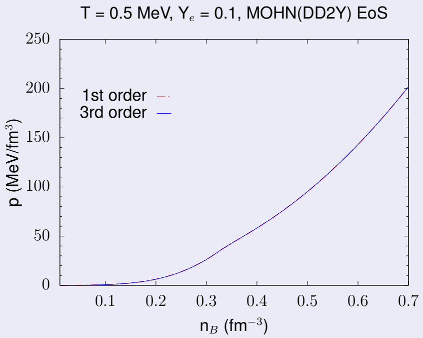
- Higher order interpolation can produce unphysical results, e.g. negative densities
- More flexible interpolation is planned

Extraction of selected quantities

- General purpose tables are large ($\sim$ hundreds MB), and perhaps you do not need all of the information stored there ...

1. Extract thermodynamic quantities fixing one/two grid variables (e.g. at constant temperature)
2. Extract data for ranges in the grid variables smaller than the originally stored data
3. Use finer/coarser grid
4. Extract selected thermodynamic quantities, e.g. pressure

Pressure for fixed T and Y_q

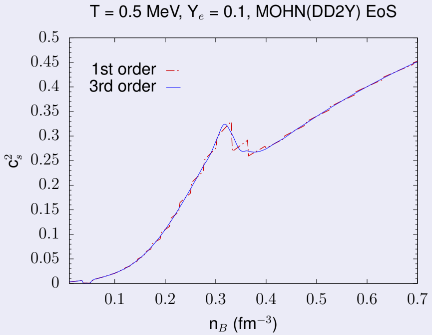


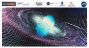
Determination of additional quantities

- Several quantities can be calculated from the EoS by using thermodynamical identities/differentiation

- List of all quantities which can be computed given in the manual, chapter 7
- Examples : (free) enthalpy, adiabatic index, sound speed, specific heats, ...
- Numerical finite difference derivatives, order depends on the order chosen for interpolation

Sound speed as function of n_B



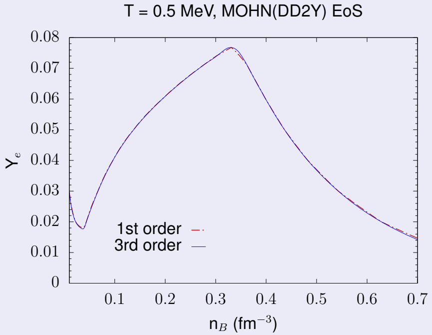


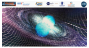
β-EQUILIBRIUM

- In some situations it is interesting to extract the EoS for matter in weak (β)-equilibrium

- Obtained at a given temperature and density by root finding :
  $\mu_l = 0$, i.e. assuming matter transparent to neutrinos
- Only possible for tables covering a range in Y_e sufficiently large
- Needs electrons to be present in the table
- `eos.beta`, `eos.thermo.ns`, `eos.nb.ns` extracted always at the lowest temperature entry

Y_e in β-equilibrium



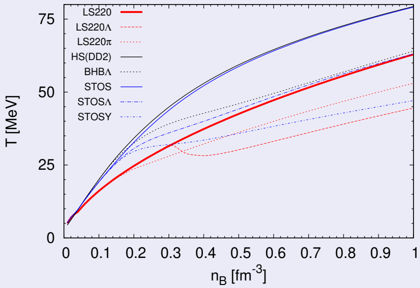


Temperature vs entropy per baryon

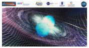
- Temperature used as grid parameter, but often EoS at constant entropy per baryon required

- Function extracts results at chosen entropy per baryon by simple root finding
- Cannot invent data : lowest density often > lowest density entry of the tables, depends on chosen s/n_B
- Attention : if you want the value of s/n_B be given in the output tables, ask explicitely for it

Temperature at constant $s/n_B = 2k_B$ [Oertel+2016]



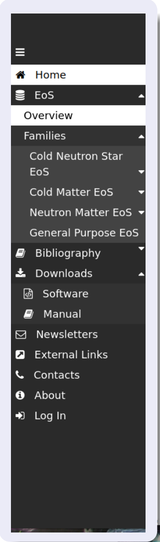
Plan



Structure of the web site I

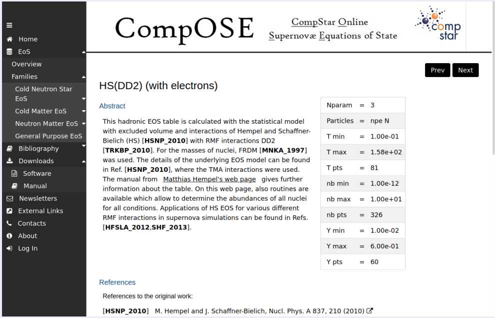
- Different families of EoS tables (see before) with search function
  - Cold neutron star EoS
  - Cold Matter EoS
  - Neutron Matter EoS
  - General purpose EoS

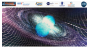
  with different sub-families (particle content $\rightarrow$ method for calculation of EoS for homogeneoues matter $\rightarrow$ inhomogeneous matter)
- Bibliography
- Downloads
  - Software
  - Manual
- Newsletter, external links, Contacts, . . .



Structure of the web site II

- One page for each EoS table with
  - Information on grid (minimum/maximum values, number of grid points)
  - Abstract
  - References to original work (links)
  - Data sheet (covers key properties of the EoS such as nuclear matter parameters)
  - Files with EoS data
  - Mass-radius plot (if available)
  - Tables with key matter and NS properties
  - Button for online computation with this EoS



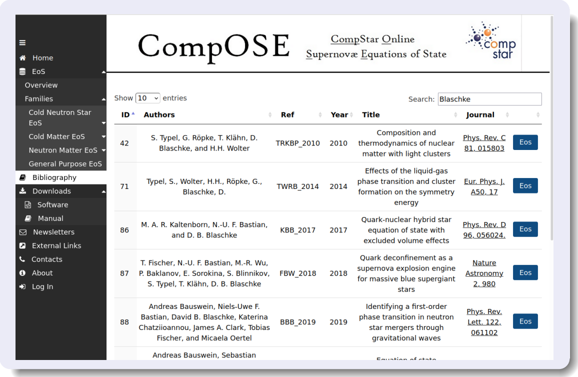


Documentation

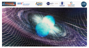
- Detailed manual with instructions for users and contributors

  [Typel+2013, Typel+2022]

- Quick guides for users and providers
- Bibliography with search function (references all orginal publications related to available data)
- Data sheet with detailed information about each EoS model



ID	Authors	Ref	Year	Title	Journal	
42	S. Typel, G. Röpke, T. Klähn, D. Blaschke, and H.H. Wolter	TRKBP_2010	2010	Composition and thermodynamics of nuclear matter with light clusters	Phys. Rev. C 81, 015803	Eos
71	Typel, S., Wolter, H.H., Röpke, G., Blaschke, D.	TWRB_2014	2014	Effects of the liquid-gas phase transition and cluster formation on the symmetry energy	Eur. Phys. J. A50, 17	Eos
86	M. A. R. Kaltenborn, N.-U. F. Bastian, and D. B. Blaschke	KBB_2017	2017	Quark-nuclear hybrid star equation of state with excluded volume effects	Phys. Rev. D 96, 056024.	Eos
87	T. Fischer, N.-U. F. Bastian, M.-R. Wu, P. Baklanov, E. Sorokina, S. Blinnikov, S. Typel, T. Klähn, D. B. Blaschke	FBW_2018	2018	Quark deconfinement as a supernova explosion engine for massive blue supergiant stars	Nature Astronomy 2, 980	Eos
88	Andreas Bauswein, Niels-Uwe F. Bastian, David B. Blaschke, Katerina Chatziioannou, James A. Clark, Tobias Fischer, and Micaela Oertel	BBB_2019	2019	Identifying a first-order phase transition in neutron star mergers through gravitational waves	Phys. Rev. Lett. 122, 061102	Eos
	Andreas Bauswein, Sebastian			Equation of state		

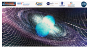


Plan

Instructions (1)

Using the Compose software : installation and generation of the parameter files

1. Once the Docker image has been downloaded and started (instructions are here : `https://hub.docker.com/r/pdavis422/gw-summer-school-compose`), download and unpack the desired EoS `eos.zip` into the "inputs" directory on your host machine. Note : The `compose` executable should be in the same directory.
2. Run `./compose` in the (container) terminal. You have then three options
   1. Task 1 : you will be guided to prepare a `eos.quantities` file containing information about the desired output quantities
   2. Task 2 : you will be guided to prepare a `eos.parameters` file containing information about the grid, T vs s/n_B, β-equilibrium or not and the interpolation order
   3. Task 3 : run the code with the previously defined parameters files for output and grid. Output is then contained in the file `eos.table`. The first three columns contain T (MeV), n_B (fm^{-3}) and Y_e/Y_q, the remaining ones the quantities asked for in `eos.quantities`



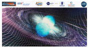
Instructions (2)

Using the TOV solver provided

Note : If you use the TOV solver provided within the Docker container, the `ns_tov` executable should be in the same "inputs" directory where you have run the `compose` code.

1. To generate the EoS table to be provided to the TOV solver, run the `compose` code to generate a table with the format : T (MeV), n_B (fm^{-3}), Y_e/Y_q, p (MeV fm^{-3}), E (internal energy per baryon, MeV)
2. Run the `ns_tov` with the command :
   `./ns_tov` ⟨ `eos_input` ⟩ ⟨ `tov_output` ⟩

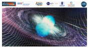
Remark : the `eos_compose` class in LORENE works with the `eos.nb` and `eos.thermo` files for cold NS matter EoS. The files `eos.thermo.ns` and `eos.nb.ns` generated for the general purpose EoS work fine, too, but they describe β-equilibrated matter at the lowest temperature entry



Exercise 1a

Application : Compare different EoSs

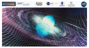
1. Generate tables for cold β-equilibrated matter for the RG(SLy4) and the PCP(BSk24) (based on non-relativistic Skyrme models), the GPPVA(DD2) EoS and the R(DD2YDelta) 1.2-1.1 EoS (based on relativistic mean-field models), the BL(chiral) (based on ab-initio model) with unified crust, OOS(DD2-FRG) with vector interaction (2 flavors) (based on relativistic mean-field model, with quark matter)
2. Compare pressure as function of energy density for the different EoSs
3. Compare the composition versus baryon number density or versus energy density (particle content, abundance of nucleons and hyperons, ...)
4. Generate tables for cold β-equilibrated matter for BL(chiral), the BL(chiral) with unified crust, and the BL(chiral) with crust (non-unified) and compare pressure as function of energy density



Exercise 1b

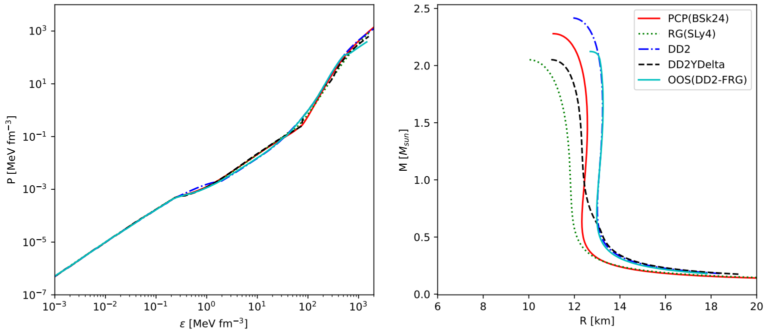
Application : Obtain the M-R relation for different cold β-equilibrated EoS

1. Use your preferred TOV solver or the one provided with the Docker image to obtain M-R relations for the chosen EoSs (see Exercise 1a, points 1 and 4) and compare
2. (Optional) Use the TOV solver to obtain Λ-M relations for the chosen EoSs and compare



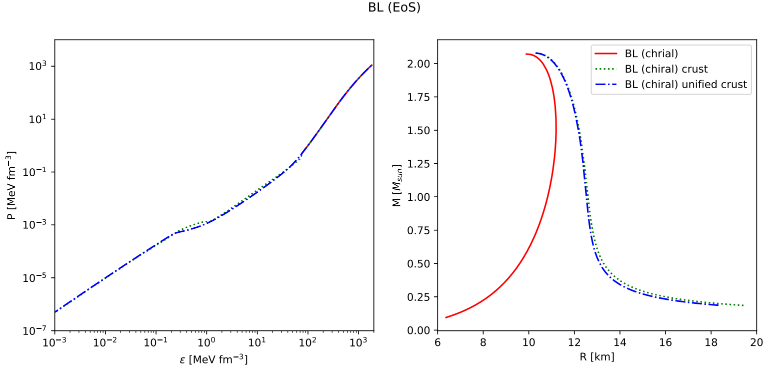
Exercise 1 : results (1)

EoSs for β-equilibrated matter and M-R relations



Exercise 1 : results (2)

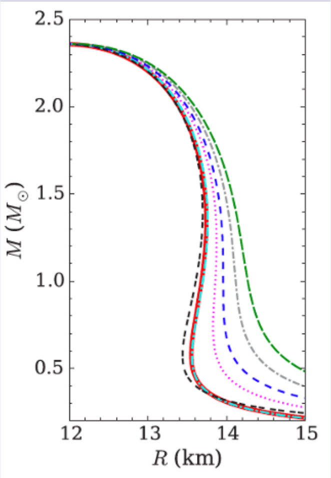
EoSs for β-equilibrated matter and M-R relations



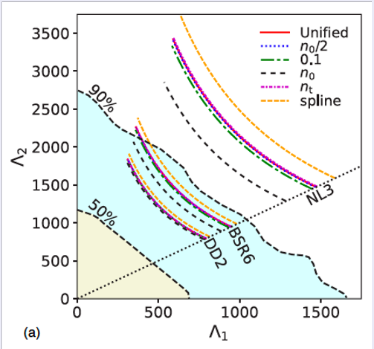
Exercise 1 : results (3)

Remark : In the case of BL EoS there is not a lot of difference between a non-unified and unified crust, but this is not always the case !

Impact on non-unified EoS on M-R (Fortin et al. 2016)



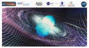
Impact on non-unified EoS on Λ_1-Λ_2 relation (Suleiman et al. 2021)



Exercise 2

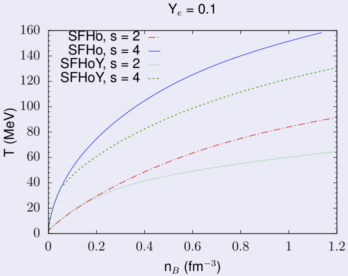
Application : Generating a table at fixed entropy per baryon from a general purpose table

1. Generate a table at fixed $s/n_B = 2k_B$ and $s/n_B = 4k_B$ and $Y_e = 0.1$ for the SFHo and the SFHoY general purpose EoS tables as function of baryon number density. Compare the temperatures in both models.



Exercise 2 : results

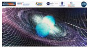
Temperature vs n_B at constant s/n_B



Exercise 3

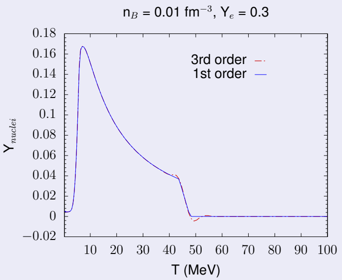
Application : Extracting information on composition and effective masses

1. Generate a table at fixed $n_B = 0.01$ fm^{-3} and $Y_e = 0.3$ for the SFHoY general purpose EoS tables and extract pressure, energy density, nucleonic, hyperonic fractions as well as information about the available nuclei. At which temperature matter becomes homogeneous ?
2. (Optional) Generate a table where instead of the particle fractions you have as output the effective (Dirac) masses of n, p and Λ (in units of their respective free masses). The latter are needed for calculating neutrino reaction rates in dense matter [Reddy+1998,Roberts+2012,Martinez-Pinedo+2012, ...]

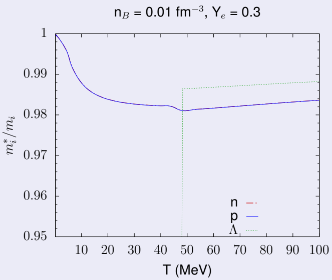


Exercise 3 : results

Sum of fractions of different nuclei



Effective masses



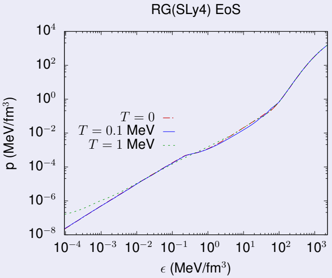
Exercise 4 (optional)

Application : Compare the EoS at $T = 0$ and at finite temperature

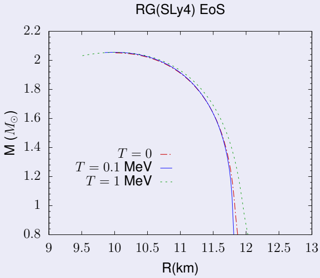
1. Generate tables for the β-equilibrated RG(SLy4) EoS for the following temperatures : $T = 0$, $T = 0.1$ MeV, $T = 1$ MeV
2. Compare pressure as function of energy density and composition as a function of energy density (or baryon number density)
3. Use a TOV solver to obtain M-R relations for the RG(SLy4) EoS at zero and finite temperature and compare

Exercise 4 : results

EoS for β-equilibrated matter



M-R relation



Thanks !

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Stefan Typel (TU Darmstadt), T. Klähn (California State University), C. Ishizuka (Hokkaido University), M. Manicini (Tours), M. Servillat (Paris Observatory), J. Novak (Paris Observatory), J.-Y. Giot (Paris Observatory), H. Pais (Coimbra), L. Tolos (Barcelona), V. Dexheimer (Kent State), C. Providencia (Coimbra), A. Raduta (Bucarest), D. Chatterjee (Pune)

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