Frequency dependence of the thermal dust *E/B* ratio and *EB* correlation: insights from the spin-moment expansion

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Vacher et al 2022d, arXiv:2210.14768

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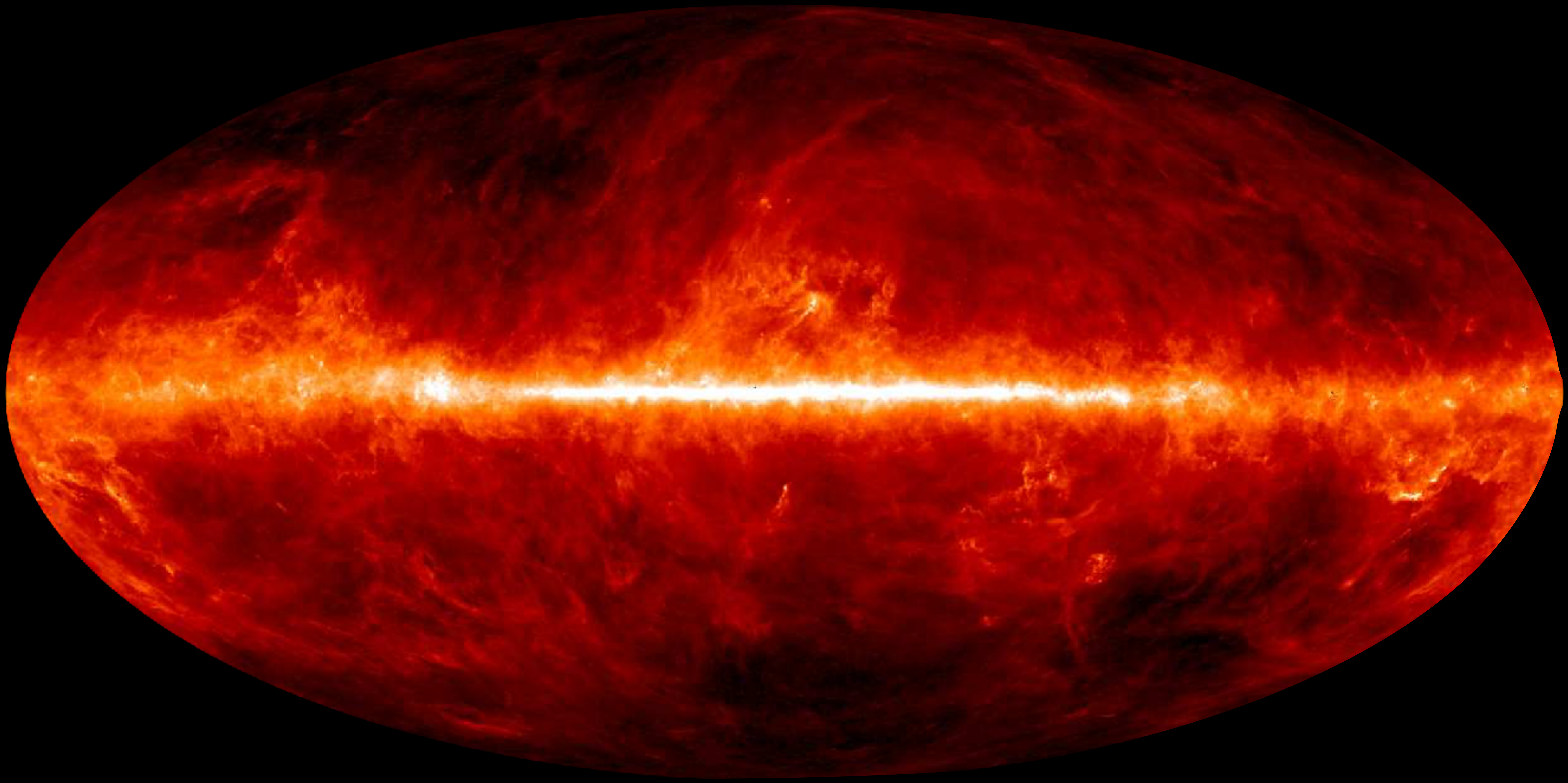
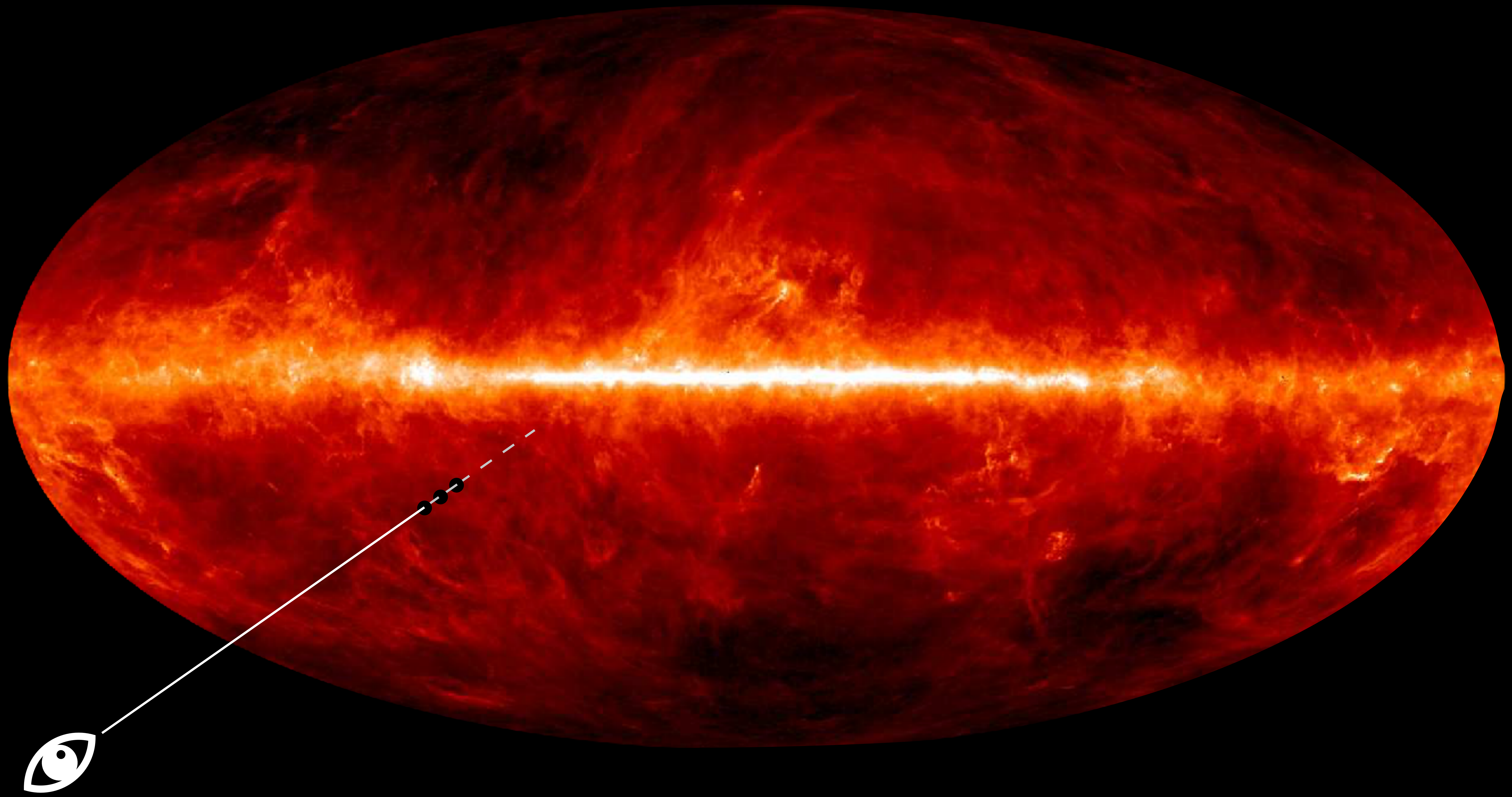
The problem of averaging



[Planck 2018 IV]

Spectral parameters (e.g. β, T for the MBB) of SEDs change with physical conditions across the sky/galaxy
(Predicted theoretically and verified observationally e.g. [Pelgrims 2021])

The problem of averaging

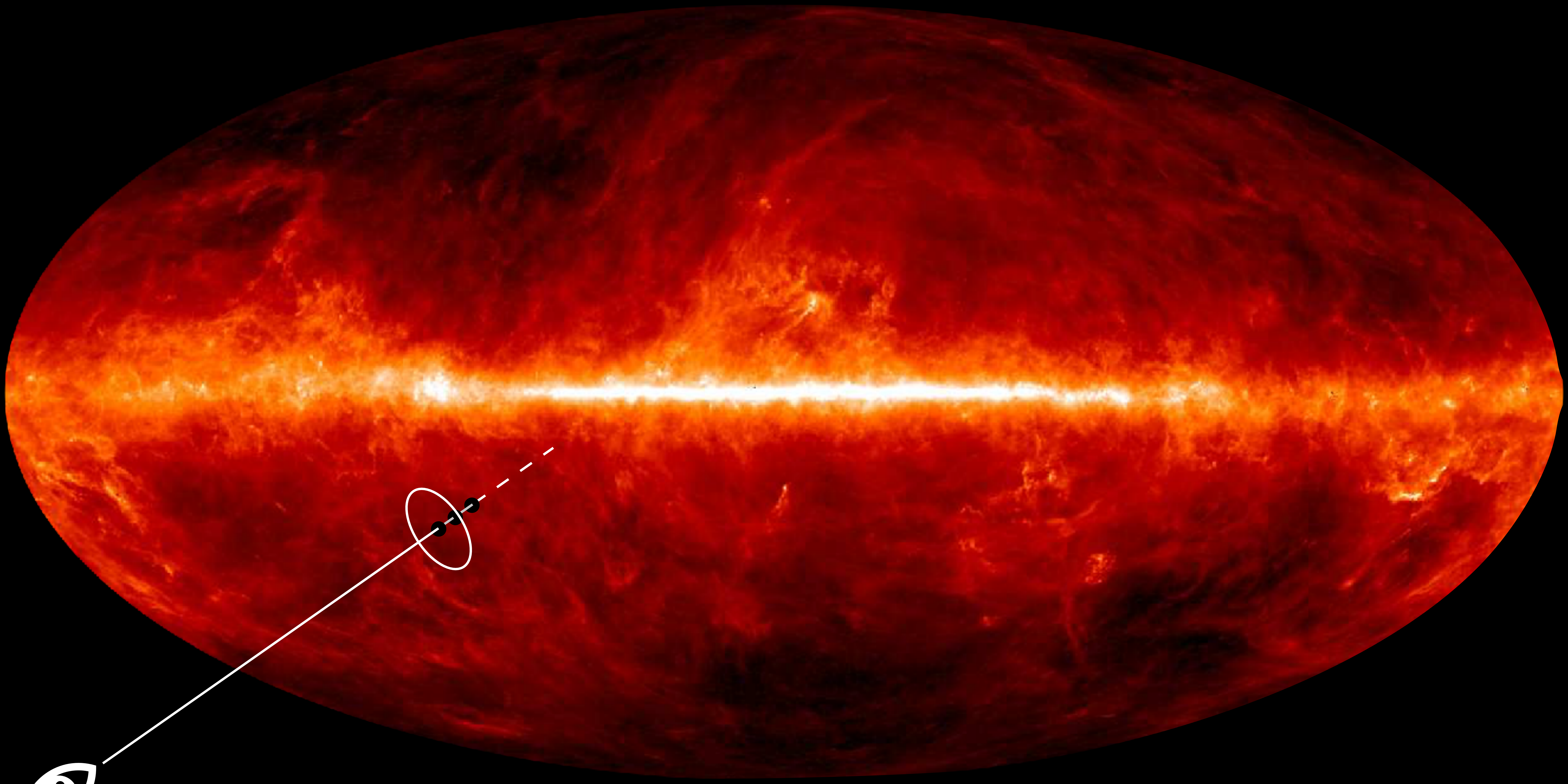


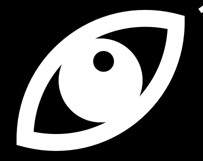
[Planck 2018 IV]

Fixed SED in *every* volume element

★ Line-of-sight average (*always there!*)

The problem of averaging





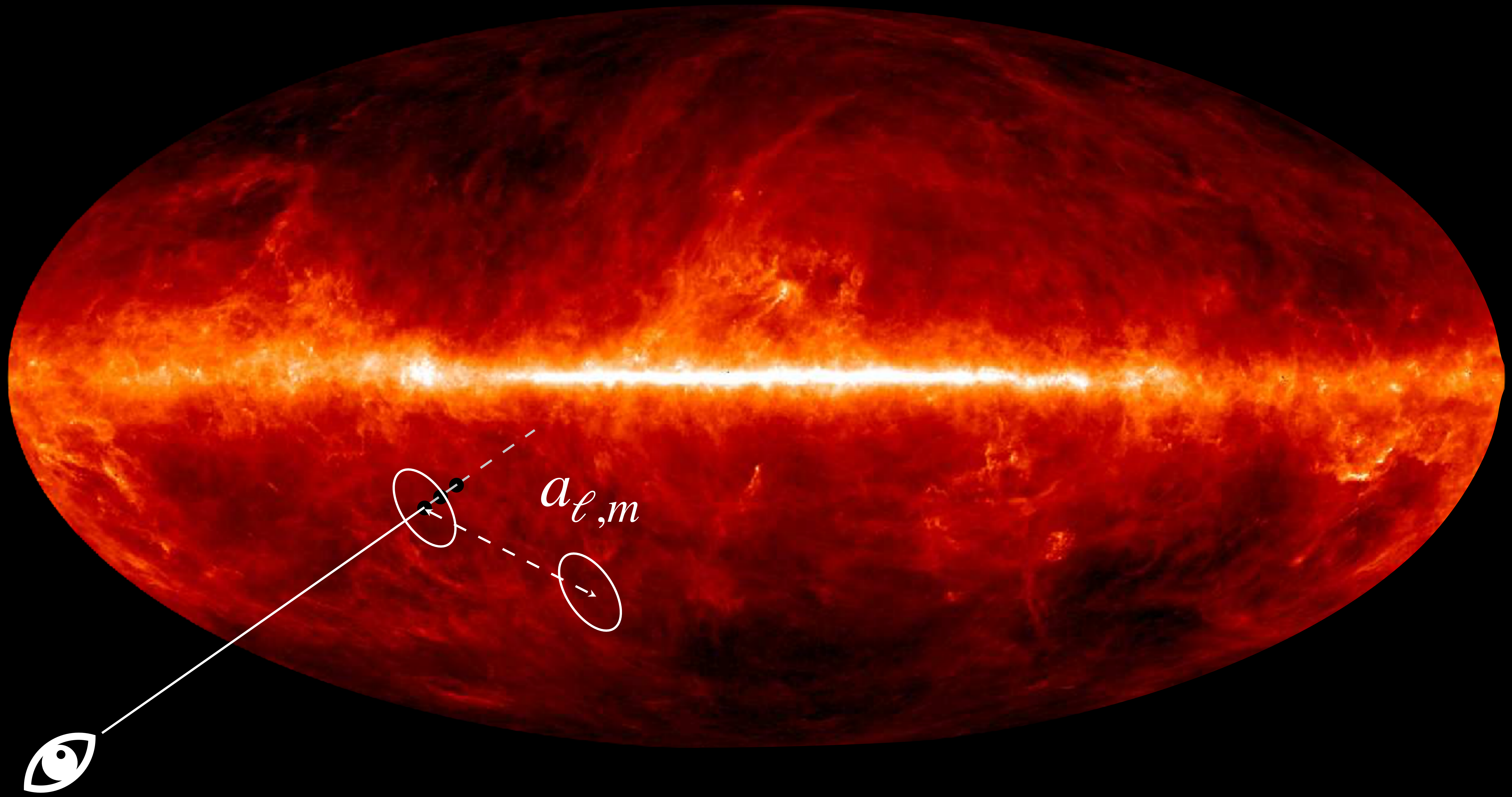
[Planck 2018 IV]

Fixed SED in *every* volume element

★ Line-of-sight average (*always there*!)

★ Experimental beam and frequency average

The problem of averaging



[Planck 2018 IV]

Fixed SED in *every* volume element

- ★ Line-of-sight average (*always there*!)
- ★ Experimental beam and frequency average
- ★ Map operations average (e.g., spherical harmonic expansion)

What are we actually averaging over?

Q and U can be united to form the complex number (**spinor**)

$$\mathscr{P}_\nu := Q_\nu + iU_\nu = P_\nu e^{2i\psi}$$

- It's module, P_ν is called the **polarized intensity**
  Under reasonable assumption, $P_\nu \propto I_\nu$ is the SED.
- It's phase, ψ is called the **polarization angle**

What are we actually averaging over?

You expect that, locally, a dust grain emits with a **modified black body (MBB)** (at CMB wavelength) [Planck 2018 XI]

$$P_\nu = A\varepsilon_\nu(\beta, T)$$

What are we actually averaging over?

You expect that, locally, a dust grain emits with a **modified black body (MBB)** (at CMB wavelength) [Planck 2018 XI]

$$P_\nu = p_0 \tau \cos(\gamma)^2 \underbrace{B_\nu^{\mathrm{Pl}}(T)\left(\frac{\nu}{\nu_0}\right)^{\beta}}_{\varepsilon_\nu(\beta, T)}$$

« **Emissivity** »

- B_ν^{Pl} Planck law/Black-body
- ν_0 reference frequency
- T temperature
- β spectral index

T, β: Spectral parameters

What are we actually averaging over?

You expect that, locally, a dust grain emits with a **modified black body (MBB)** (at CMB wavelength) [Planck 2018 XI]

$$P_\nu = p_0 \tau \cos(\gamma)^2 B_\nu^{\mathrm{Pl}}(T) \left(\frac{\nu}{\nu_0}\right)^\beta$$

A

« Weight »

- p_0 polarization fraction
- τ opacity
- γ orientation of the Galactic magnetic field in the Plane of the sky

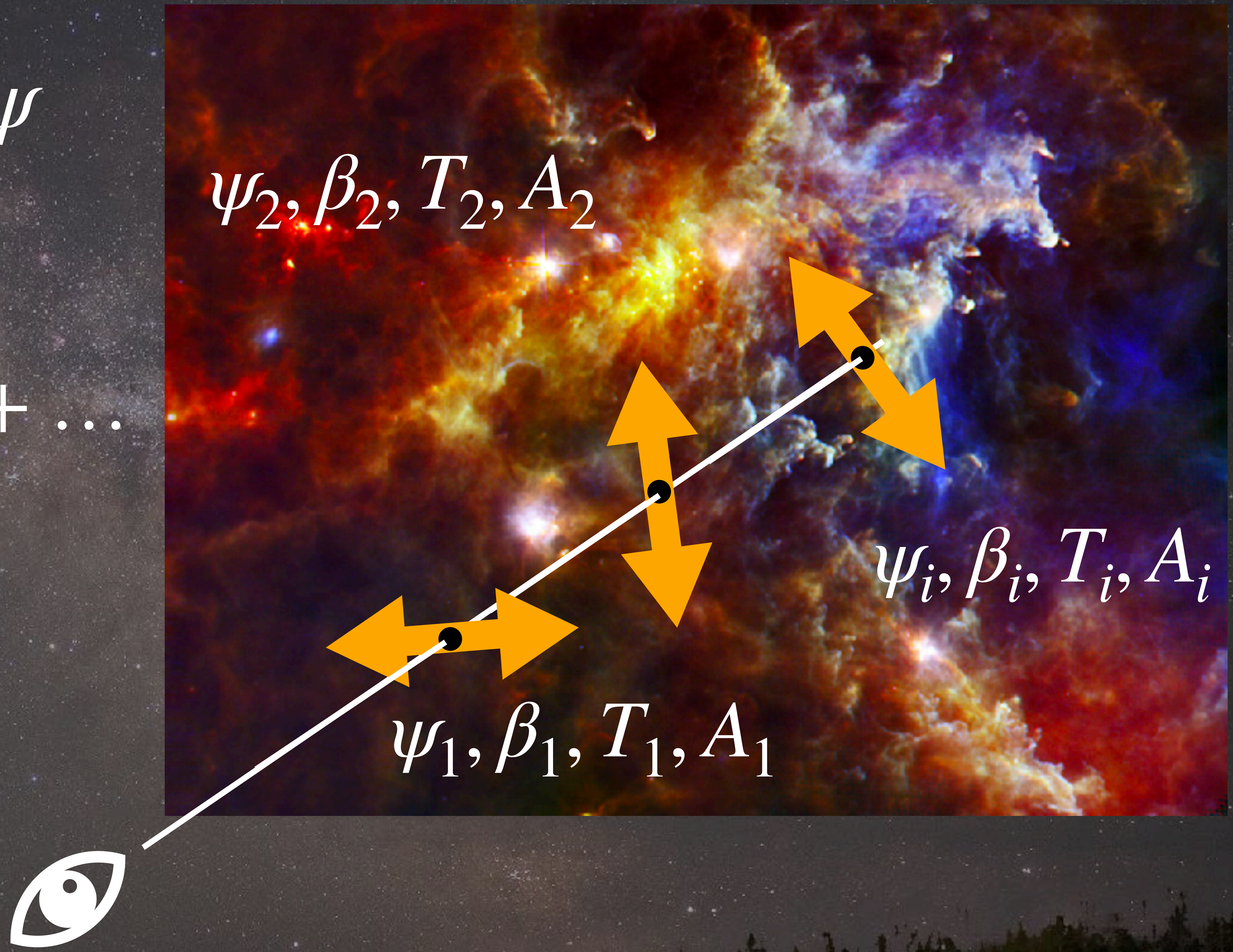
The Polarized mixing

Averaging over spinors with different ψ and β, T, A:

$$\langle \mathscr{P}_\nu \rangle = A_1 \varepsilon_\nu(\beta_1, T_1) e^{2i\psi_1} + A_2 \varepsilon_\nu(\beta_2, T_2) e^{2i\psi_2} + \ldots$$

$$\neq \bar{A} \varepsilon_\nu(\bar{\beta}, \bar{T}) e^{2i\bar{\psi}}$$

SEDs are **not linear** and they are weighted by **complex phases**!



Consequences of polarized mixing

$$\langle \mathscr{P}_\nu \rangle = A_1 \varepsilon_\nu(\beta_1, T_1) e^{2i\psi_1} + A_2 \varepsilon_\nu(\beta_2, T_2) e^{2i\psi_2} + \ldots \neq \bar{A} \varepsilon_\nu(\bar{\beta}, \bar{T}) e^{2i\bar{\psi}}$$

- **SED distortions:** $|\langle \mathscr{P}_\nu \rangle|$ is not the canonical SED (a MBB) anymore [Chluba 2017]
- **Polarisation angle frequency dependence:** $\psi_{\langle \mathscr{P}_\nu \rangle} = \psi_\nu$ [Tassis 2015]

Consequences of polarized mixing

$$\langle \mathscr{P}_\nu \rangle = A_1 \varepsilon_\nu(\beta_1, T_1) e^{2i\psi_1} + A_2 \varepsilon_\nu(\beta_2, T_2) e^{2i\psi_2} + \ldots$$

$$= \varepsilon_\nu(\bar{\beta}, \bar{T}) \left(\mathscr{W}_0 + \mathscr{W}_1^\beta \ln\left(\frac{\nu}{\nu_0}\right) + \cdots \right)$$

spin-moments: Both effects can be modeled/predicted using a moment expansion of the polarization spinor [Vacher 2022b]

Consequences of polarized mixing

$$\langle \mathscr{P}_\nu \rangle = A_1 \varepsilon_\nu(\beta_1, T_1) e^{2i\psi_1} + A_2 \varepsilon_\nu(\beta_2, T_2) e^{2i\psi_2} + \ldots$$

$$= \varepsilon_\nu(\bar{\beta}, \bar{T}) \left(\mathscr{W}_0 + \mathscr{W}_1^\beta \ln\left(\frac{\nu}{\nu_0}\right) + \cdots \right)$$

With

$$\mathscr{W}_0 = \sum_i A_i e^{2i\psi_i}$$

$$\mathscr{W}_1 = \sum_i A_i (\beta_i - \bar{\beta}) e^{2i\psi_i}$$

…

Consequences of polarized mixing

$$\langle \mathscr{P}_\nu \rangle = A_1 \varepsilon_\nu(\beta_1, T_1) e^{2i\psi_1} + A_2 \varepsilon_\nu(\beta_2, T_2) e^{2i\psi_2} + \dots$$

$$= \varepsilon_\nu(\bar{\beta}, \bar{T}) \mathscr{W}_0 \left(1 + \frac{\mathscr{W}_1^\beta}{\mathscr{W}_0} \ln\left(\frac{\nu}{\nu_0}\right) + \cdots \right)$$

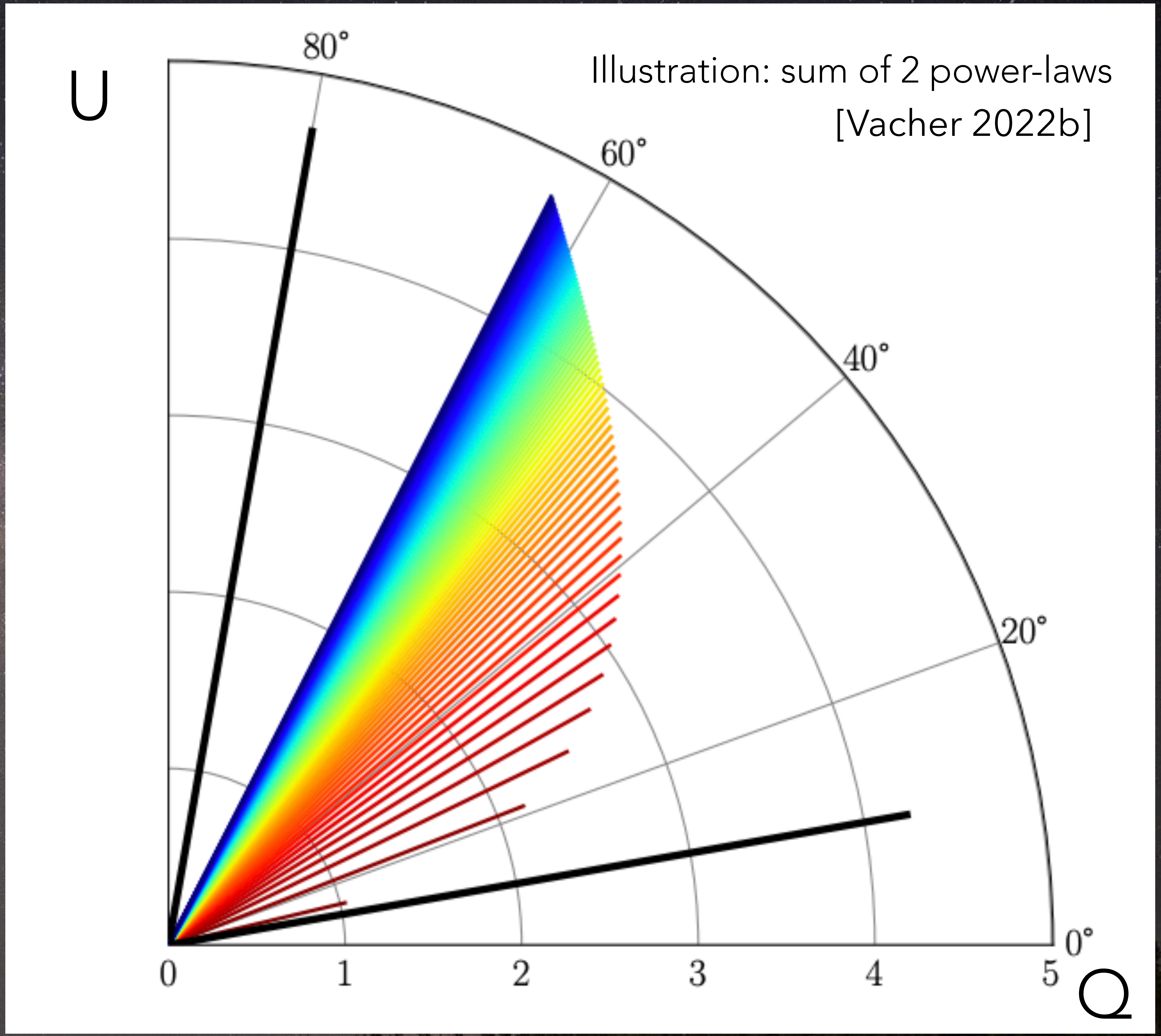
The leading order can be interpreted as a (complex) correction to the spectral index. $\bar{\beta} \rightarrow \bar{\beta} + \Delta\beta$

Spectral dependence of the polarization angle

$$\psi_\nu \simeq \psi_{\nu_0} + \frac{\mathrm{Im}(\Delta\beta)}{2} \ln\left(\frac{\nu}{\nu_0}\right)$$

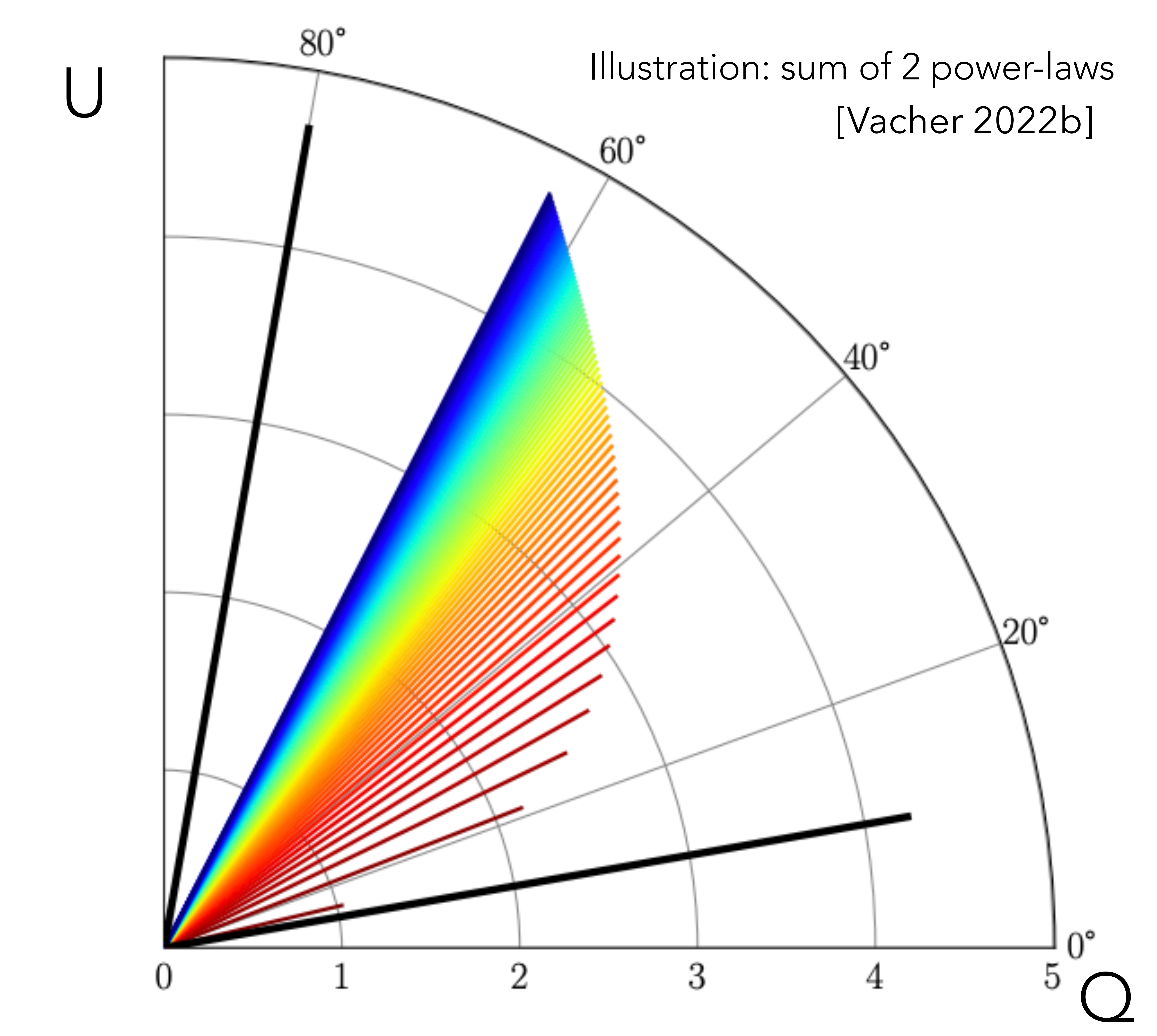
$$\Delta\beta = \frac{\mathcal{W}_1^\beta}{\mathcal{W}_0} = \frac{\sum_i A_i e^{2i\psi_i}(\beta - \bar{\beta})}{\sum_k A_k e^{2i\psi_k}}$$

$$\Delta\beta \in \mathbb{C}$$



Spectral dependence of the polarization angle

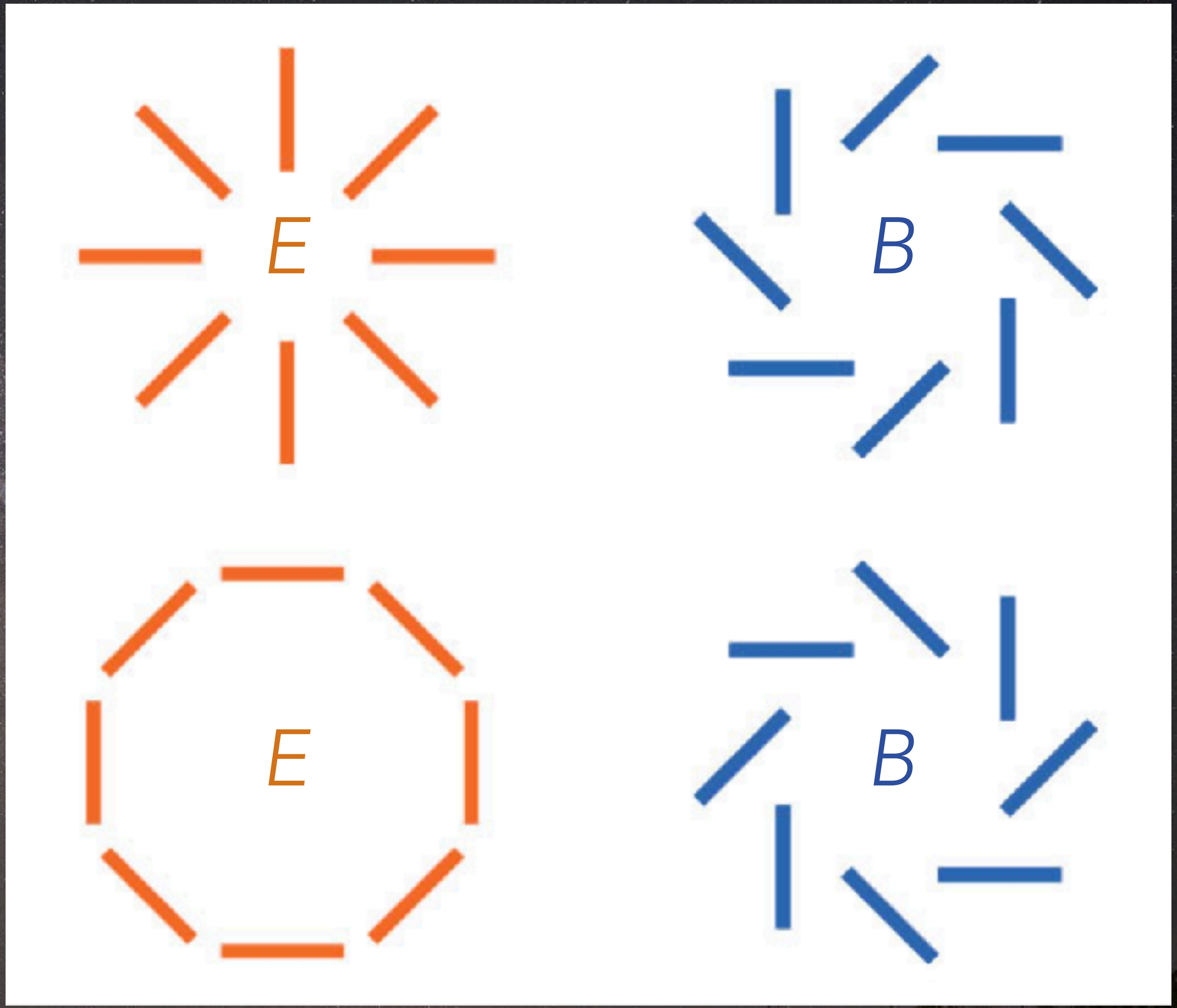
The same game can be played considering the **temperature** ($\Delta T \in \mathbb{C}$) or any other **SED** [Vacher 2022b]



The *E*- and *B*-modes

- *Quantifies **patterns** of the ψ-field*
- *Any polarized signal $\mathscr{P}_\nu(\vec{n})$ can be decomposed in **E- and B- modes** (Helmotz theorem) $E_\nu(\vec{n})$ and $B_\nu(\vec{n})$:*

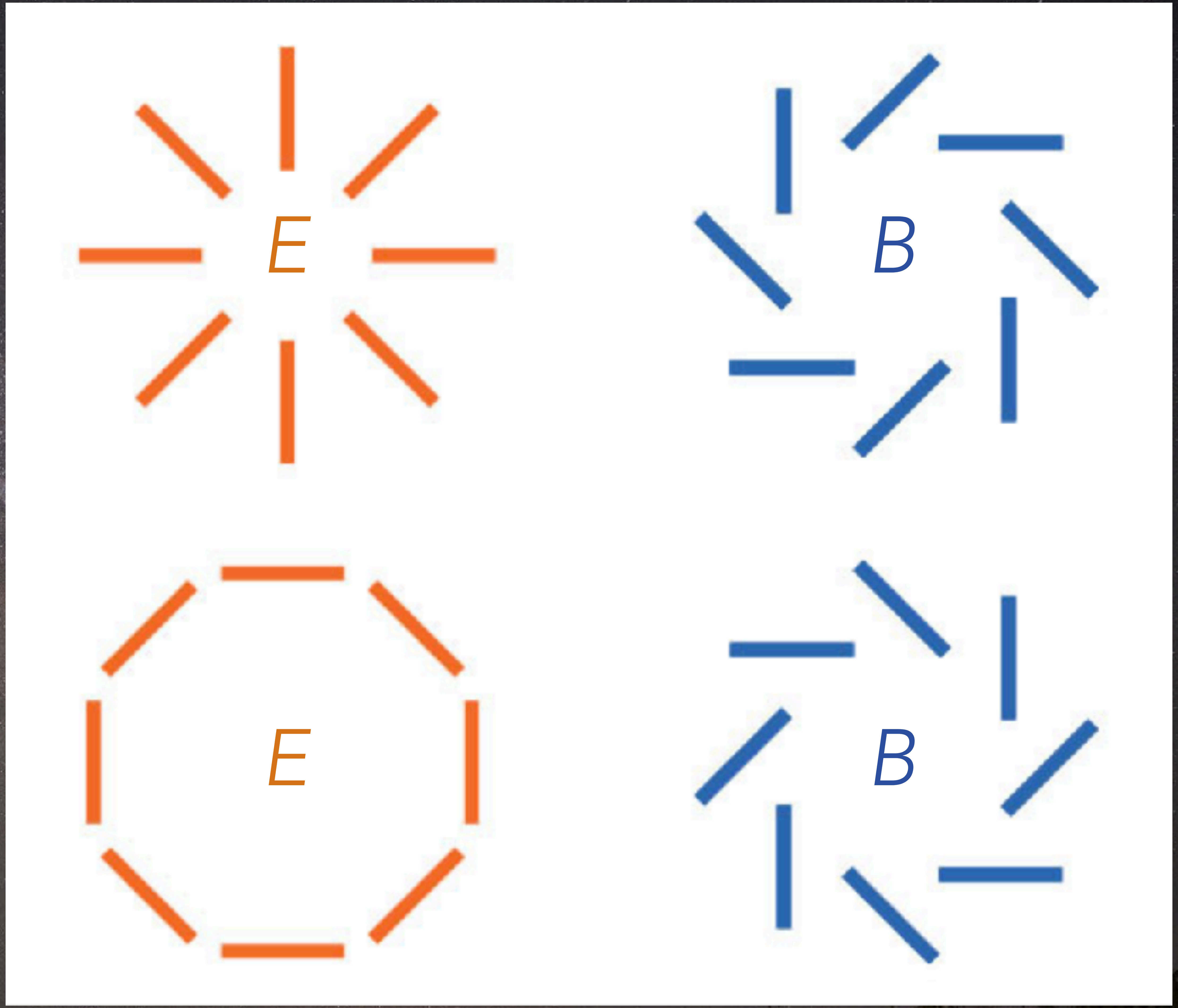
$$E + iB = -\bar{\eth}^2 \mathscr{P}_\nu$$



The *E*- and *B*-modes

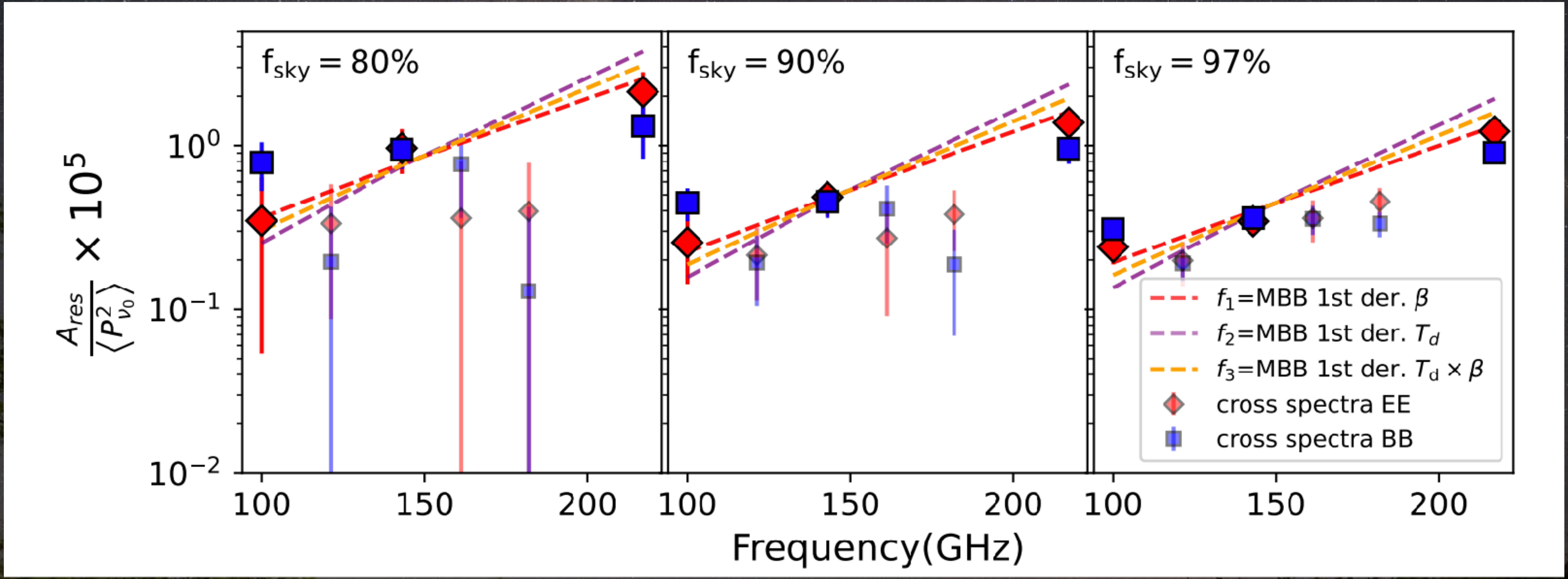
- Three **angular power-spectra** in polarization: $\mathscr{D}_\ell^{EE}$, $\mathscr{D}_\ell^{BB}$ and $\mathscr{D}_\ell^{EB}$, Written « *EE* », « *BB* » and « *EB* »

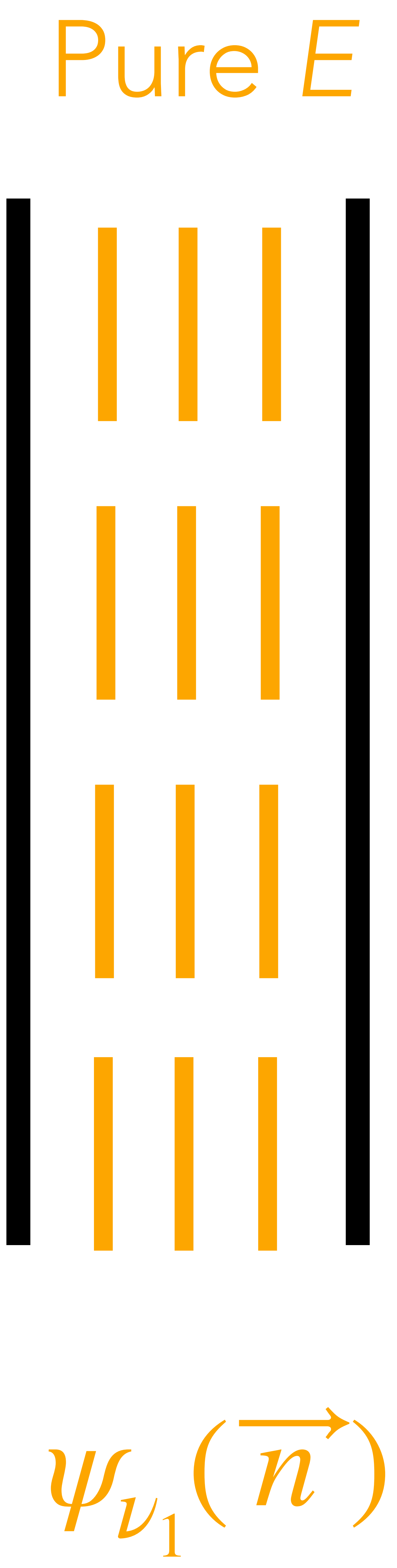
$$\mathscr{D}_\ell^{XX'} = \frac{\ell(\ell+1)}{2\pi} \sum_m (X)^*_{\ell m} (X)_{\ell m}$$



The *E*- and *B*-modes

Different spectral behavior for E- and B- modes in Planck data [Ritacco 2022]





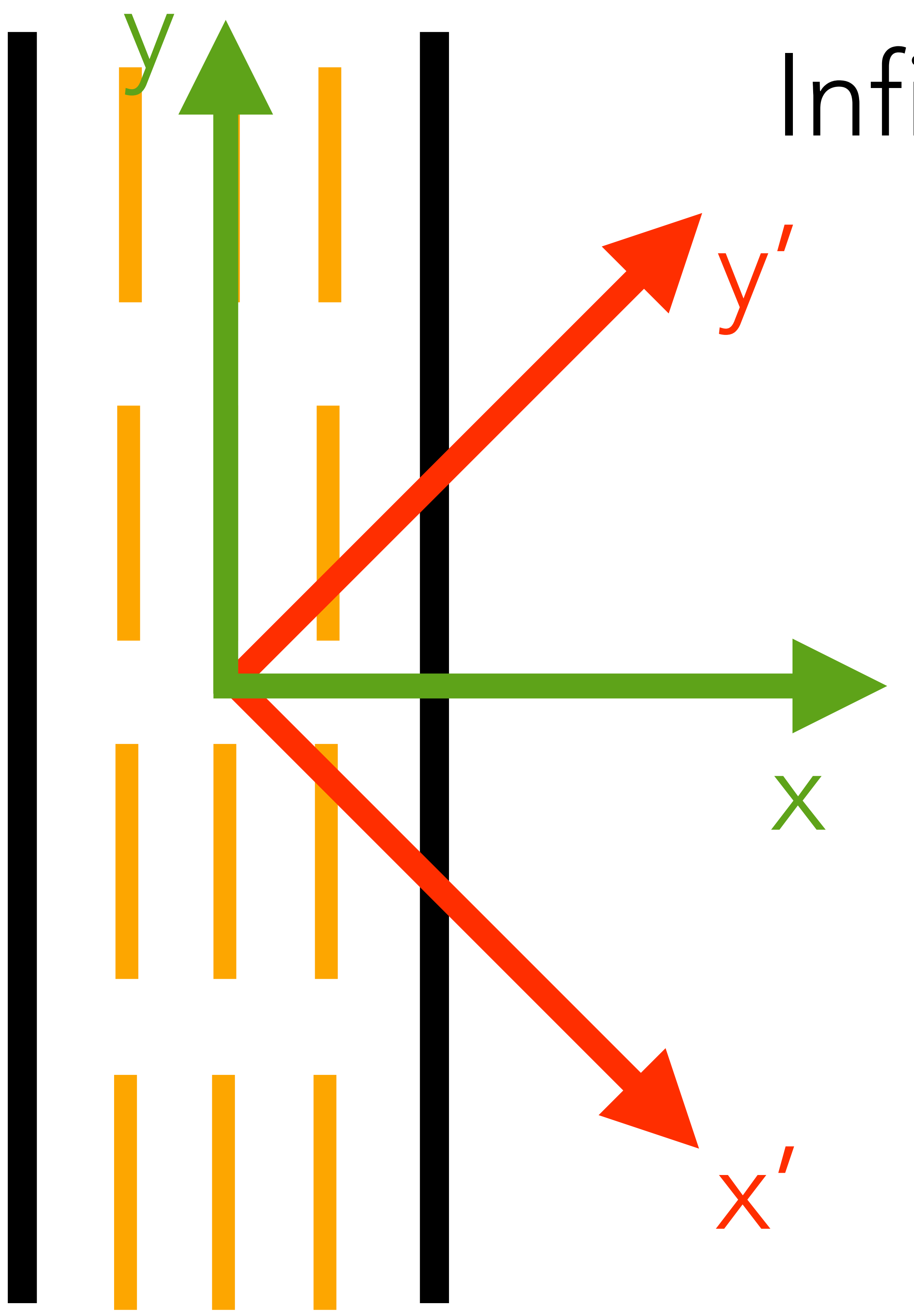
Infinite filament in front of a null background

$$E_\nu \propto Q,$$

$$B_\nu \propto U = 0$$

[Zaldariaga 2001]

Pure E

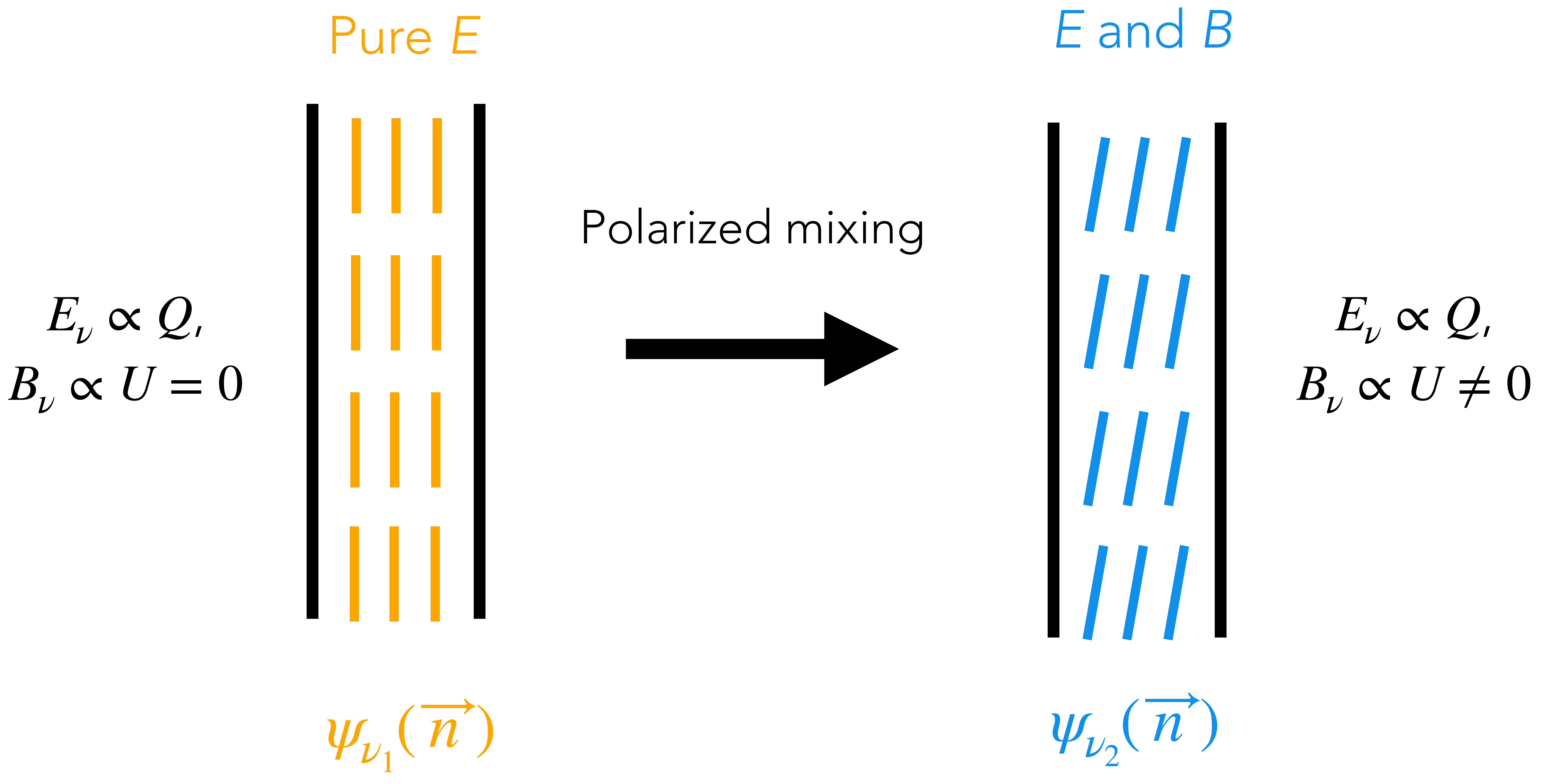


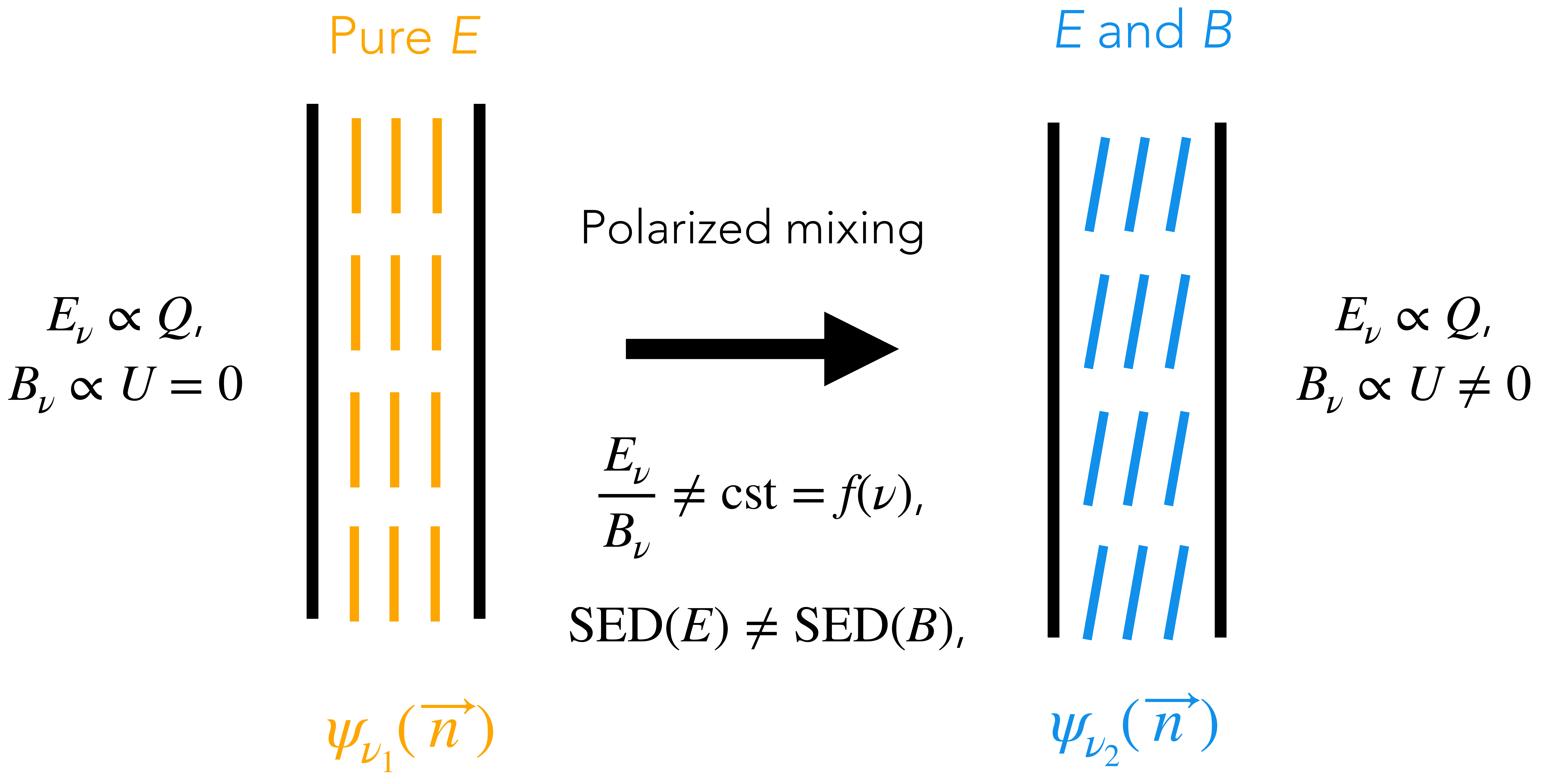
$\psi_{\nu_1}(\overrightarrow{n})$

Infinite filament in front of a null background

$$E_\nu \propto Q,$$
$$B_\nu \propto U = 0$$

[Zaldariaga 2001]


Pure E
E and B
Polarized mixing
$E_\nu \propto Q,$
$B_\nu \propto U = 0$
$E_\nu \propto Q,$
$B_\nu \propto U \neq 0$
$\psi_{\nu_1}(\overrightarrow{n})$
$\psi_{\nu_2}(\overrightarrow{n})$



Pure E
E and B
Polarized mixing
$E_\nu \propto Q$,
$B_\nu \propto U = 0$
$\frac{E_\nu}{B_\nu} \neq \text{cst} = f(\nu)$,
$\text{SED}(E) \neq \text{SED}(B)$,
$E_\nu \propto Q$,
$B_\nu \propto U \neq 0$
$\psi_{\nu_1}(\overrightarrow{n})$
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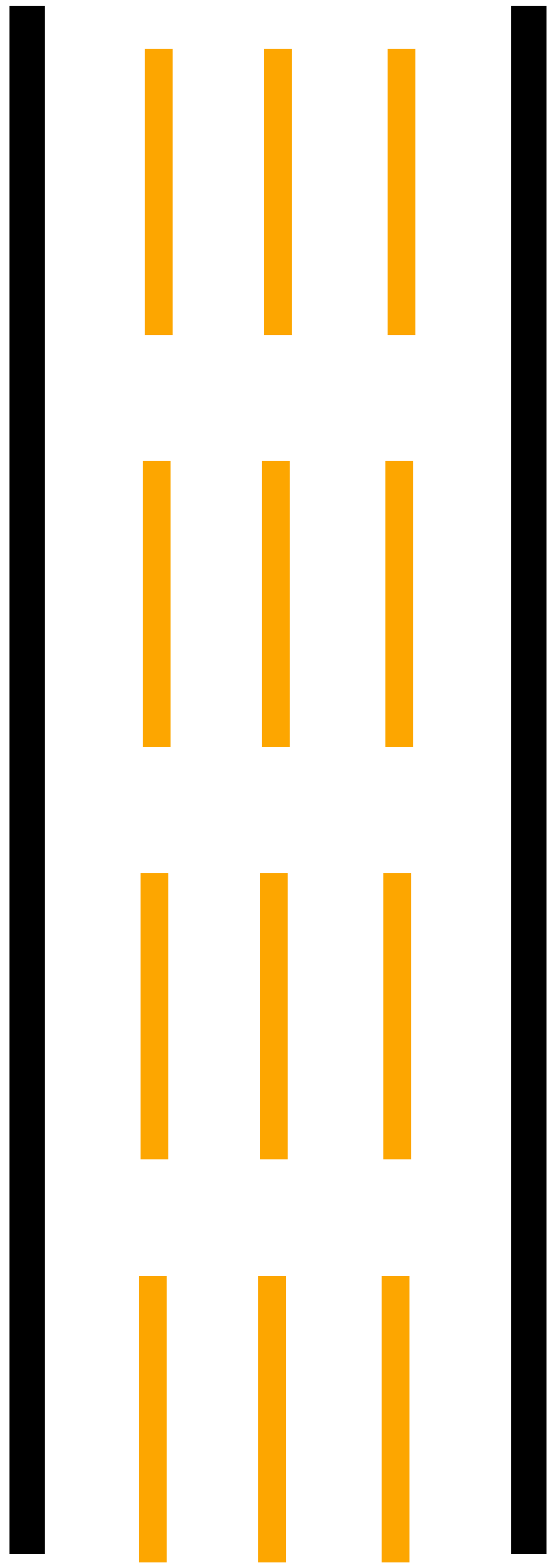
Pure E

E and B

$E_\nu \propto Q,$
$B_\nu \propto U = 0$



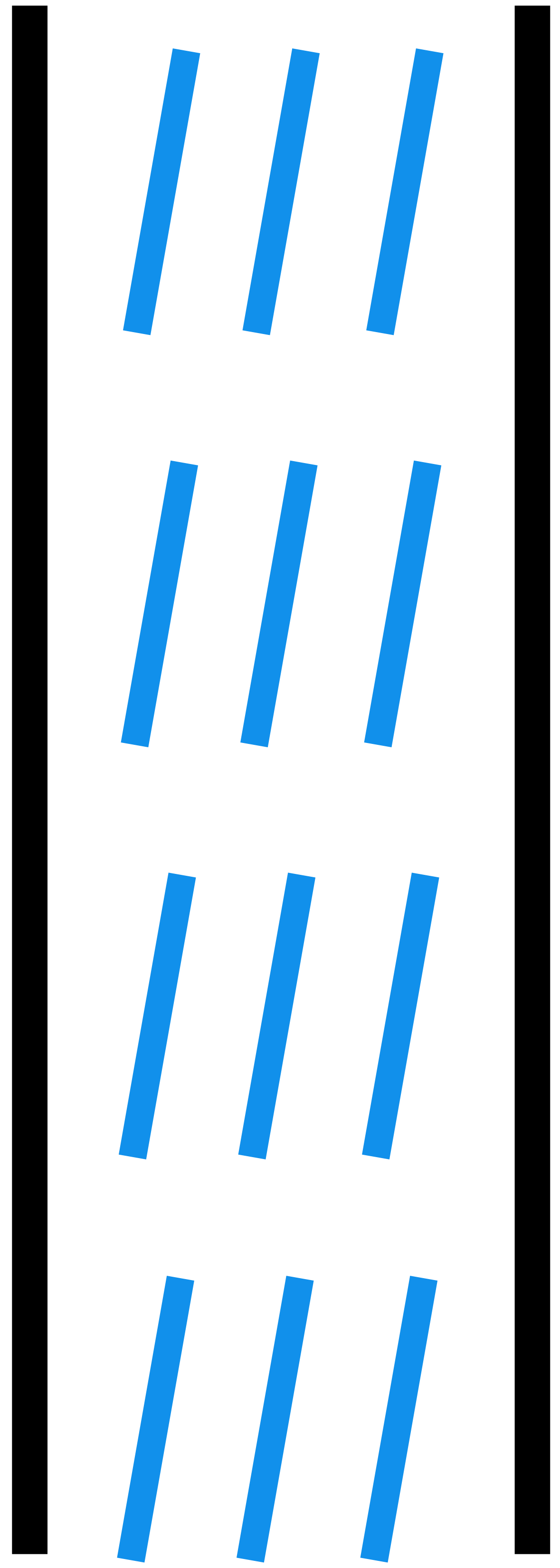
Polarized mixing

$$\frac{E_\nu}{B_\nu} \neq \mathrm{cst} = f(\nu),$$

$$\mathrm{SED}(E) \neq \mathrm{SED}(B),$$

$$\mathrm{SED}(EE) \neq \mathrm{SED}(BB) \neq \mathrm{SED}(EB),$$

$$EE/BB = f(\nu),$$



$E_\nu \propto Q,$
$B_\nu \propto U \neq 0$

From *Q* and *U* to *E* and *B*

$$\langle E_\nu + iB_\nu \rangle = -\,\bar{\eth}^2 \langle P_\nu \rangle$$

$$= \varepsilon_\nu(\bar{\beta}, \bar{T}) \left(\mathbb{W}_0 + \mathbb{W}_1^\beta \ln\left(\frac{\nu}{\nu_0}\right) + \ldots \right),$$

With: $\mathbb{W}_k^\beta = -\,\eth^2 \mathscr{W}_k^\beta$

E and B should be treated together as real and complex components of a **single complex number**! (as Q and *U*)

From Q and U to E and B

$$\langle E_\nu + iB_\nu \rangle = -\,\bar{\eth}^2 \langle P_\nu \rangle$$

$$= \varepsilon_\nu(\bar{\beta}, \bar{T})\mathbb{W}_0 \left(1 + \frac{\mathbb{W}_1^\beta}{\mathbb{W}_0} \ln\left(\frac{\nu}{\nu_0}\right) + \dots \right),$$

The complex phase of $E + iB$ will become frequency dependent:

$E/B = f(\nu)$

From *E* and *B* to *EE*, *EB* and *BB*

$$\langle \mathcal{D}_\ell^{XX'} \rangle = \varepsilon_\nu\left(\bar{\beta}, \bar{T}\right)^2 \left(\mathcal{D}_\ell^{\mathbb{W}_{0,X}\mathbb{W}_{0,X'}} + \left[\mathcal{D}_\ell^{\mathbb{W}^\beta_{1,X}\mathbb{W}_{0,X'}} + \mathcal{D}_\ell^{\mathbb{W}_{0,X}\mathbb{W}^\beta_{1,X'}} \right] \ln\left(\frac{\nu}{\nu_0}\right) + \mathcal{D}_\ell^{\mathbb{W}^\beta_{1,X}\mathbb{W}^\beta_{1,X'}} \ln\left(\frac{\nu}{\nu_0}\right)^2 + \ldots \right)$$

With X,X′ $\in \{E,B\}$

Knowing the β, T, ψ, A distributions, one can compute the **spin-moments maps** $\mathcal{W}_k^\beta$ and predict the behavior of $\langle \mathcal{D}_\ell^{XX'} \rangle$

From *E* and *B* to *EE*, *EB* and *BB*

$$\langle \mathcal{D}_\ell^{XX'} \rangle = \varepsilon_\nu\left(\bar{\beta}, \bar{T}\right)^2 \mathcal{D}_\ell^{\mathbb{W}_{0,X}\mathbb{W}_{0,X'}} \left(1 + \frac{\left[\mathcal{D}_\ell^{\mathbb{W}^\beta_{1,X}\mathbb{W}_{0,X'}} + \mathcal{D}_\ell^{\mathbb{W}_{0,X}\mathbb{W}^\beta_{1,X'}} \right]}{\mathcal{D}_\ell^{\mathbb{W}_{0,X}\mathbb{W}_{0,X'}}} \ln\left(\frac{\nu}{\nu_0}\right) \right.$$

$$\left. + \mathcal{D}_\ell^{\mathbb{W}^\beta_{1,X}\mathbb{W}^\beta_{1,X'}} \ln\left(\frac{\nu}{\nu_0}\right)^2 + \ldots \right)$$

The **orange term** (largest one) can be interpreted as a ℓ dependent correction to β: $\bar{\beta} \to \bar{\beta}_\ell^{XX'}$

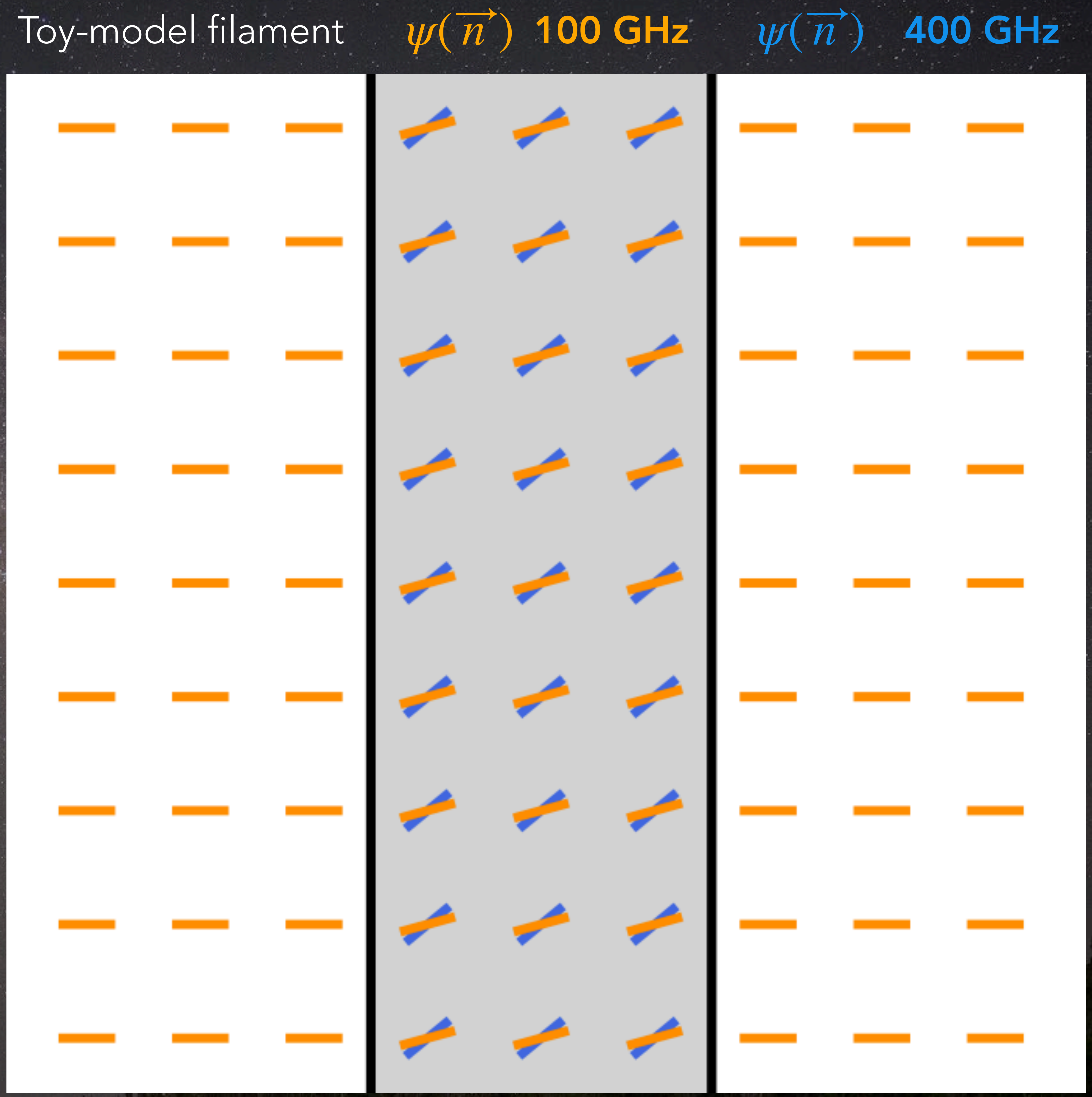
Hence, after corrections: $\bar{\beta}_\ell^{EE} \neq \bar{\beta}_\ell^{BB} \neq \bar{\beta}_\ell^{EB}$

A Toy-model filament

Simple model:

Filament in front of a background

Sums of 2 MBB in the filament



A Toy-model filament

Simple model:

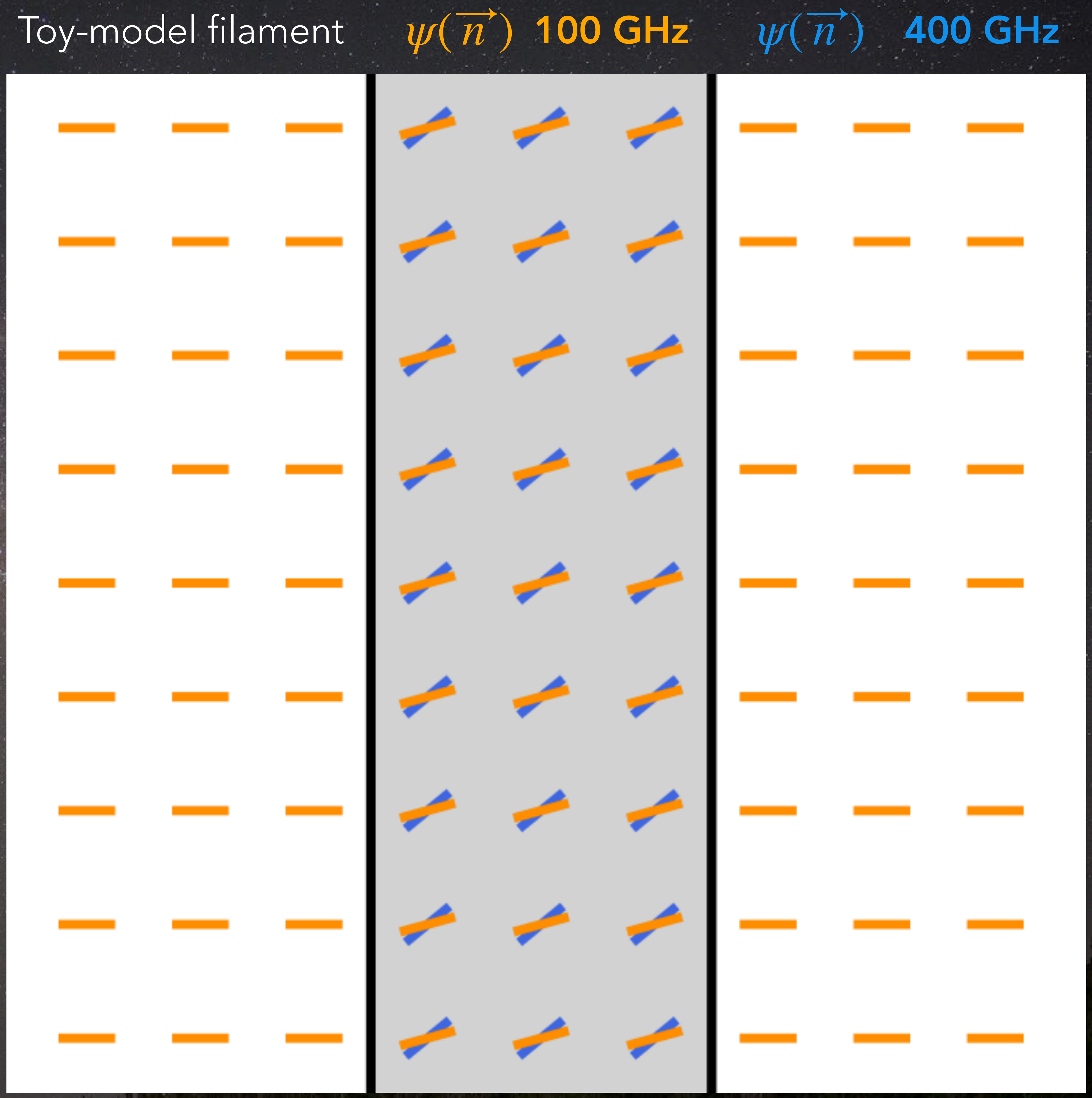
Filament in front of a background

Sums of 2 MBB in the filament

Non zero *EB*:

Phenomenon of « **magnetic misalignment**»

[Clark 2021],[Cukierman 2022]



A Toy-model filament

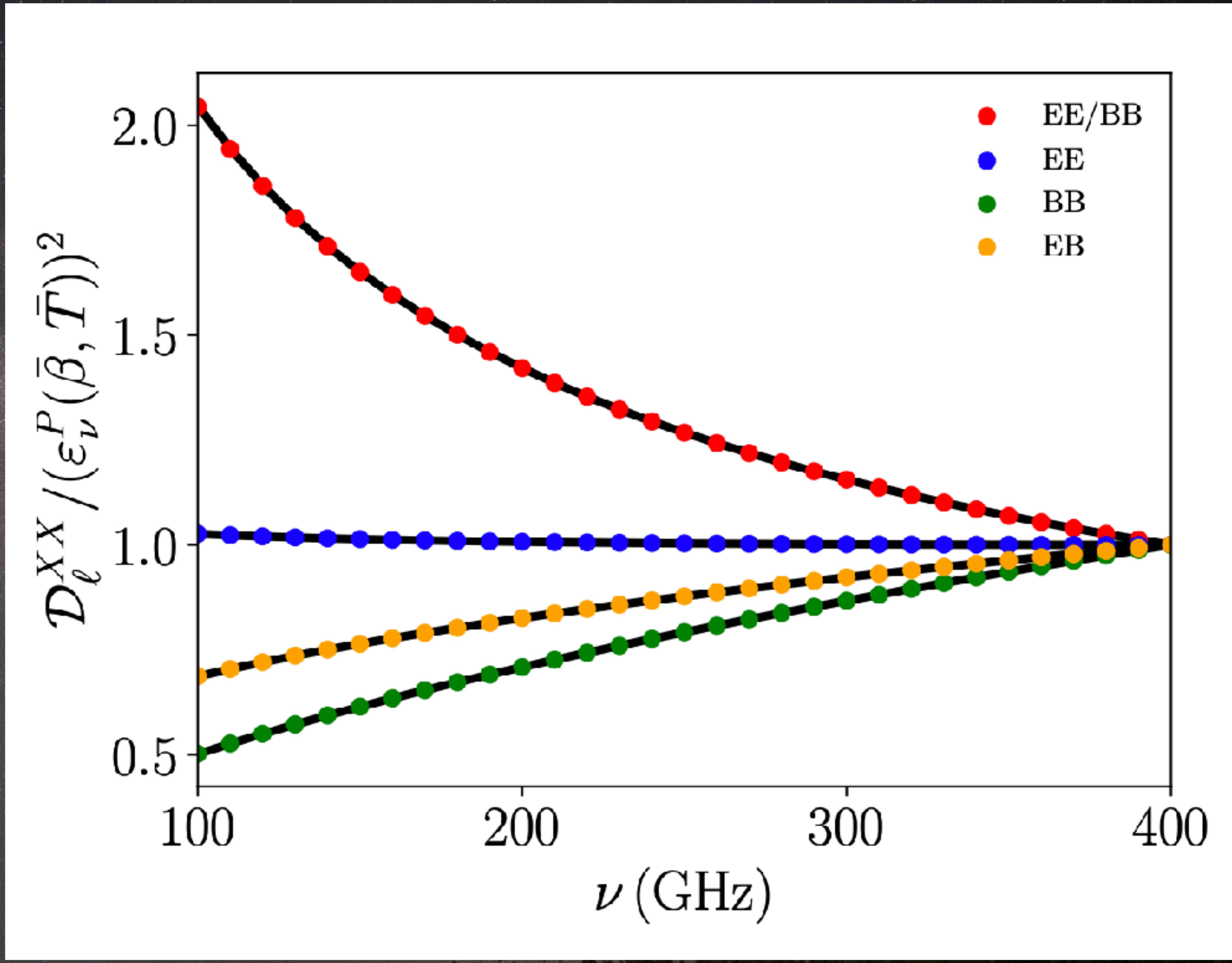
Simple model:

Filament in front of a background

Sums of 2 MBB in the filament

Spin moments:

Allow to **understand** and **model** the **spectral dependence** of the **polarized power-spectra**



PySM models

Considering the **PySM** models:

- `d0`: single MBB with constant β and T over the sky
- `d1`: single MBB with varying β and T over the sky
- `d10`: refined version of `d1`
- `d12`: 6 layer MBB with different β and T over the sky

Using the Planck **galactic mask (PLA)** with $f_{\mathrm{sky}} = 0.8$ and a 2° apodisation scale.

A single bin of $\ell \in \{2{,}200\}$, $n_{\mathrm{side}} = 128$, purification of *E*- and *B*-modes

PySM models

Considering the **PySM** models:

- `d0`: single MBB with constant β and T over the sky
- `d1`: single MBB with varying β and T over the sky
- `d10`: refined version of `d1`
- `d12`: 6 layer MBB with different β and T over the sky

⚠️ Not expected to reproduce the reality of the dust *EB* signal

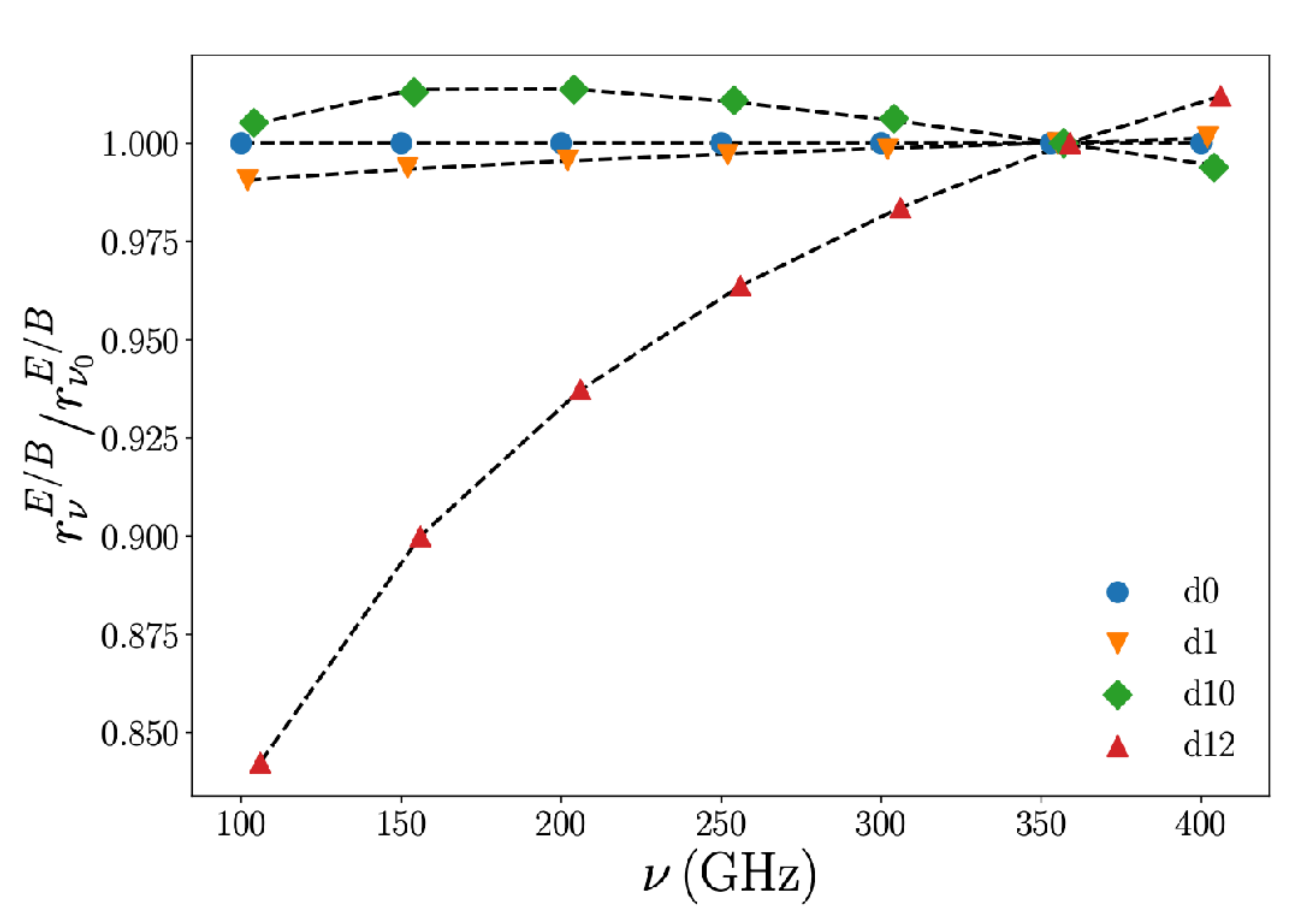
But provide still a **good illustration** of our points

⚠️ Amplitudes of the effects will change strongly depending on the ℓ **range and $f_{\rm sky}$ considered**

PySM models

$$r_\nu^{E/B} = \frac{\mathcal{D}_\ell^{EE}}{\mathcal{D}_\ell^{BB}}$$

- Is a **function of frequency** as expected!
- **Spin moments** = good model
- *EE/BB* is a probe of **polarized mixing** **independent** of the canonical SED (MBB)

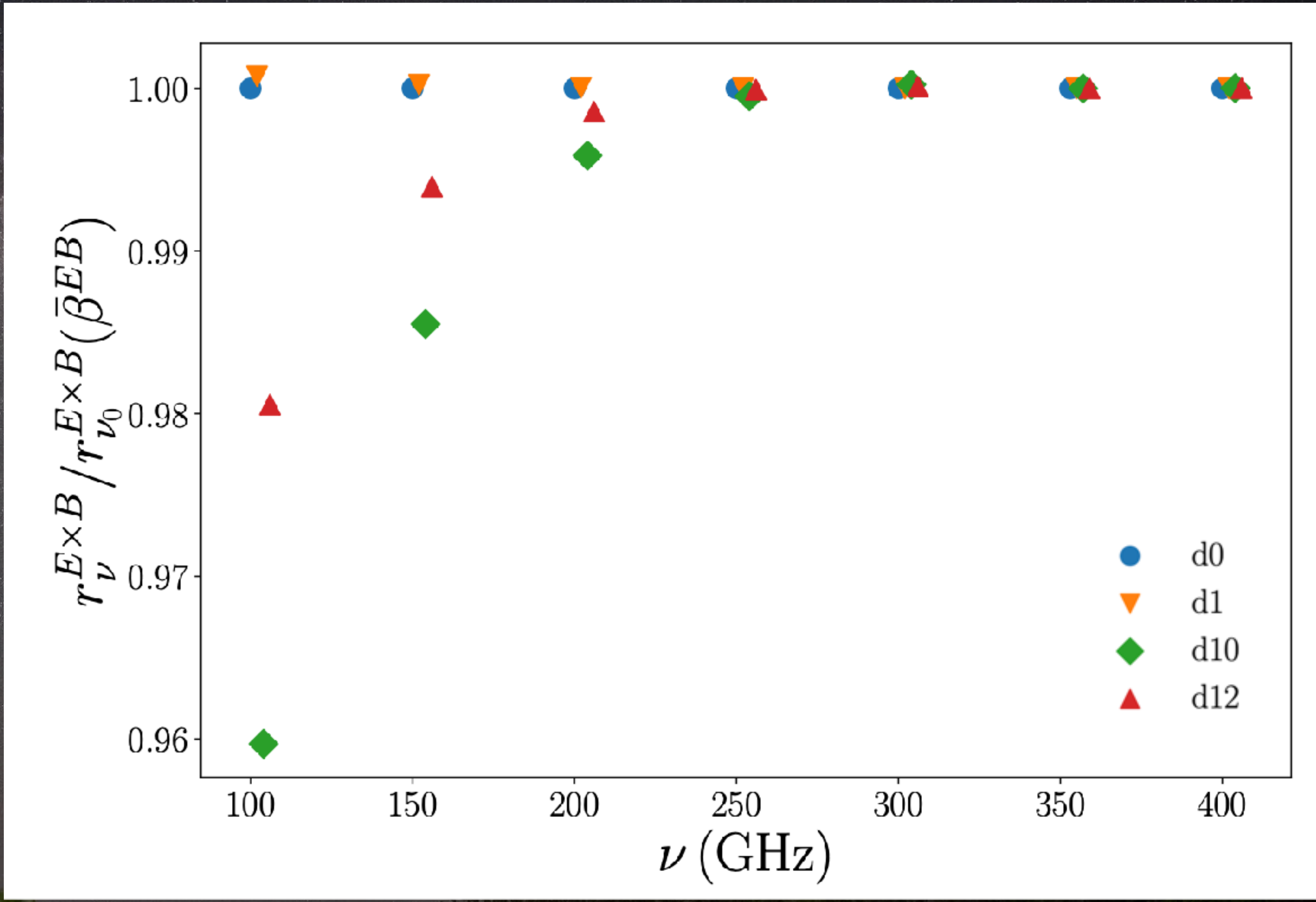


PySM models

$$r_\nu^{E\times B} = \frac{\mathcal{D}_\ell^{EB}}{\varepsilon^2(\bar{\beta},\bar{T})^2}$$

Looking at **distortions** from MBB of *EB* signal.

$\bar{\beta}$ and $\bar{T}$ are fitted over the *EB* signal

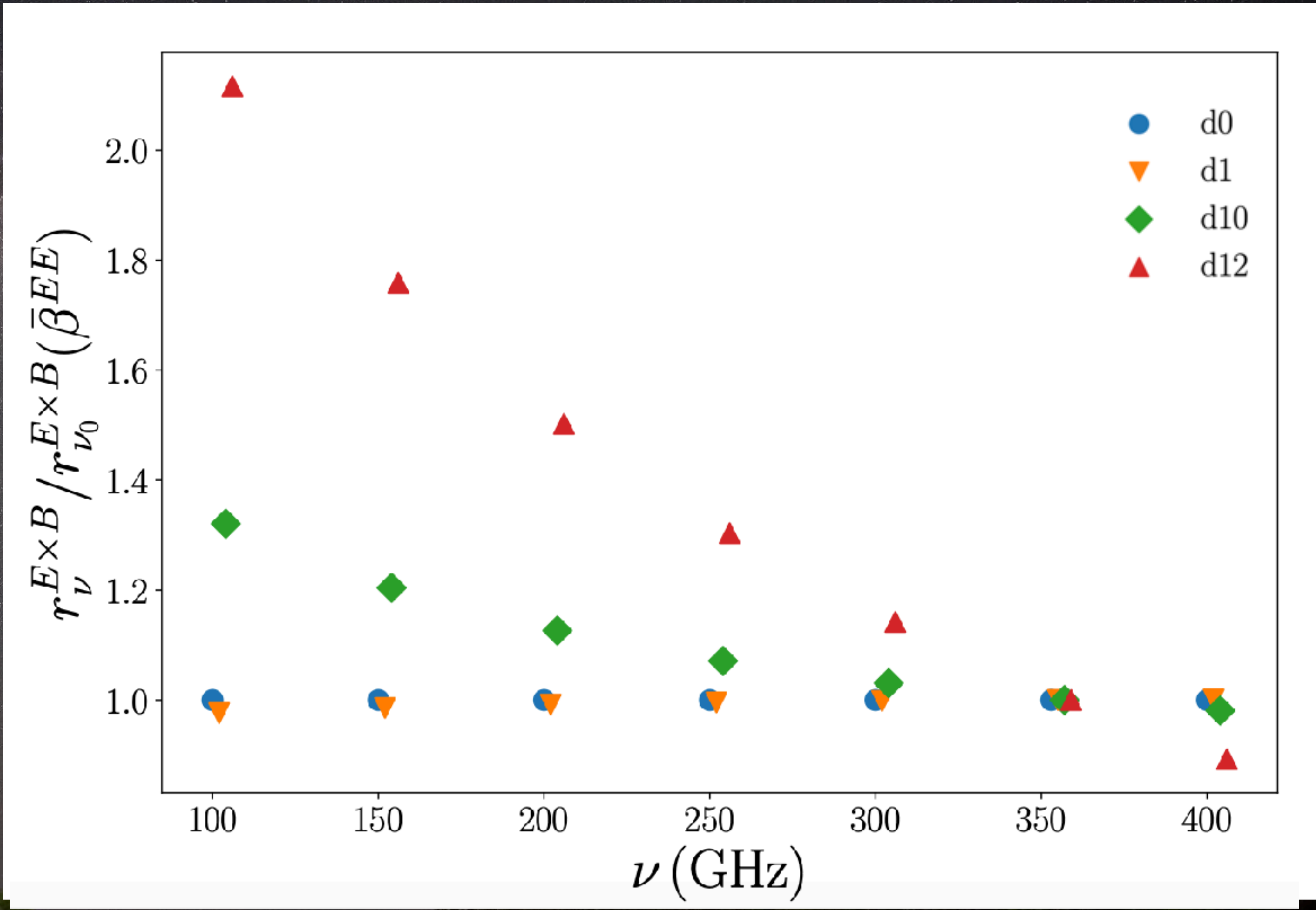


PySM models

$$r_\nu^{E\times B} = \frac{\mathcal{D}_\ell^{EB}}{\varepsilon^2(\bar{\beta},\bar{T})^2}$$

Looking at **distortions** from MBB of *EB* signal.

$\bar{\beta}$ and $\bar{T}$ are fitted over the *EE* signal



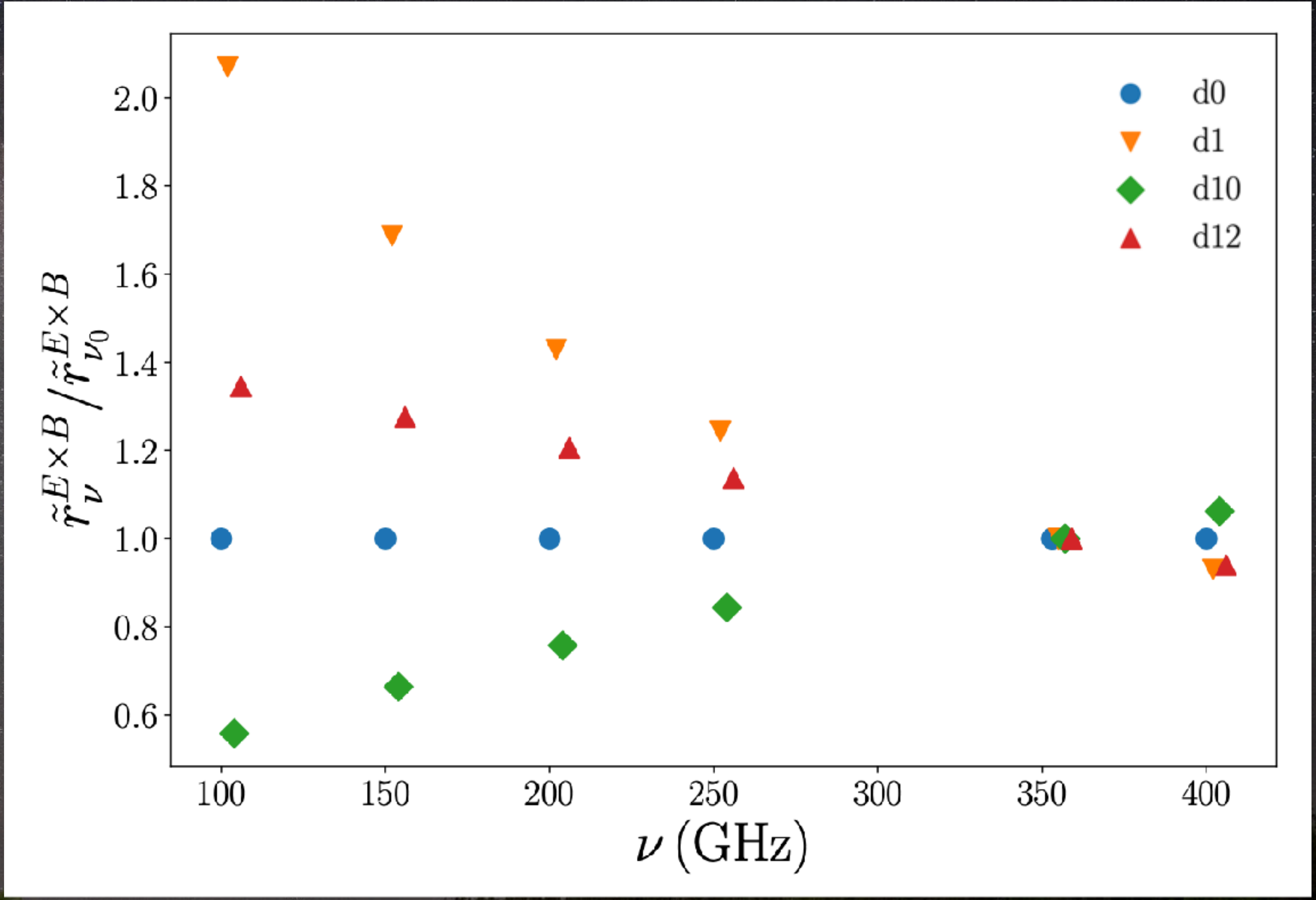
PySM models

Using the simple **model**:

$$\tilde{\mathscr{D}}_\ell^{EB} = \frac{\mathscr{D}_\ell^{EE}\mathscr{D}_\ell^{TB}}{\mathscr{D}_\ell^{TE}}$$

and looking at **deviations**:

$$\tilde{r}_\nu^{E\times B} = \frac{\mathscr{D}_\ell^{EB}}{\tilde{\mathscr{D}}_\ell^{EB}}$$



Conclusions

When averaging over different polarized signal (**polarized mixing**):

- *EE*, *BB* and *EB* will have **different SEDs** and hence **different pivots** spectral parameters $\bar{\beta}_\ell$ and $\bar{T}_\ell$. (Observed in Planck data [Ritacco et al (2022)])
- **EE/BB** will become **frequency dependent** (no matter what the canonical SED is) and provides a model independent probe of spatial variations of spectral parameters and polarization angles
- ***EB* is distorted**. *EE* or *EE*x*TB*/*TE* can not be used as **proxies for *EB***.
- Spin-moment expansion allows to **model** the SEDs, suggesting a **common treatment** for *E* and *B* (as for *Q* and *U*)

All these considerations can be applied to any SEDs (**synchrotron**)

photo: Slovinsky

First conclusions

In the presence of polarized mixing (i.e. average of different polarized signals) :

Pixel level

- $|\mathscr{P}_\nu|$, Q_ν and U_ν are **not MBBs** anymore (SED distortions)
- ψ becomes **frequency dependent** $\leftrightarrow$ Q and U have different moments
- If Q and U are treated independently: $\bar{\beta}^Q \neq \bar{\beta}^U$

Power spectra level

- EE, BB and EB are **not MBBs** squared anymore (SED distortions)
- EE/BB becomes **frequency dependent** $\leftrightarrow$ EE and BB have different moments
- If E and B are treated independently: $\bar{\beta}_\ell^{EE} \neq \bar{\beta}_\ell^{BB} \neq \bar{\beta}_\ell^{EB}$