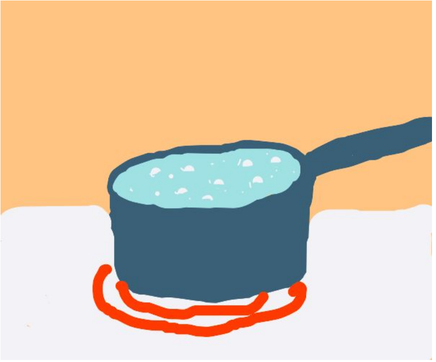
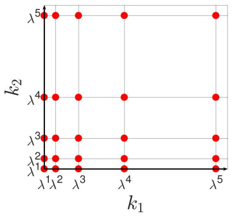
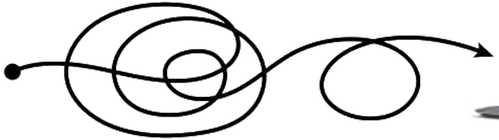
Fluid Dynamics on Logarithmic Lattices







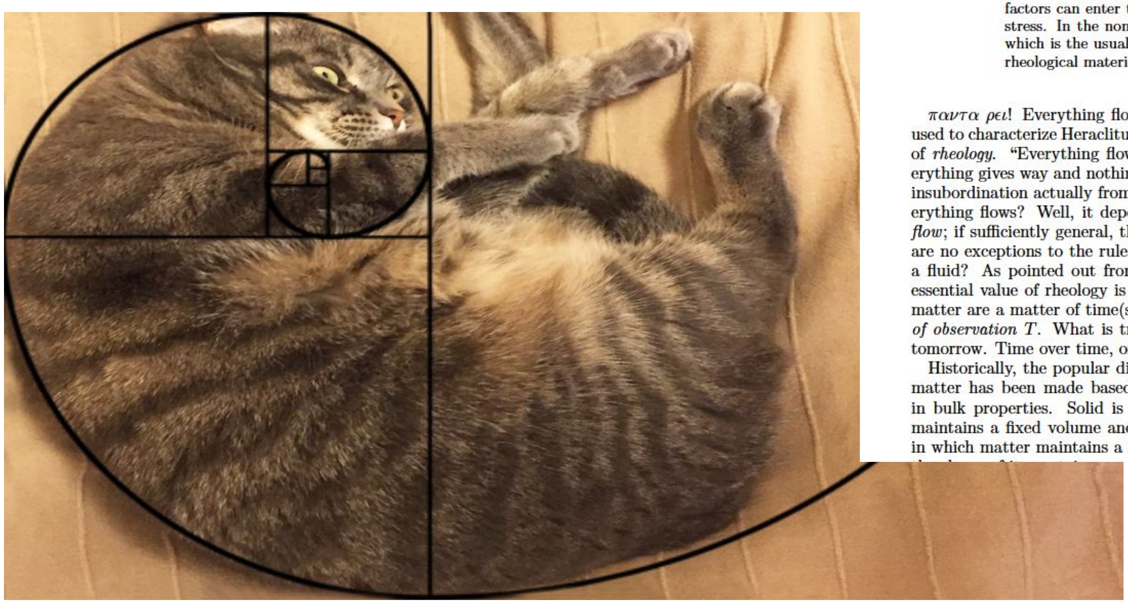
Amaury Barral
CEA/SPEC/SphynX - 02/11/22

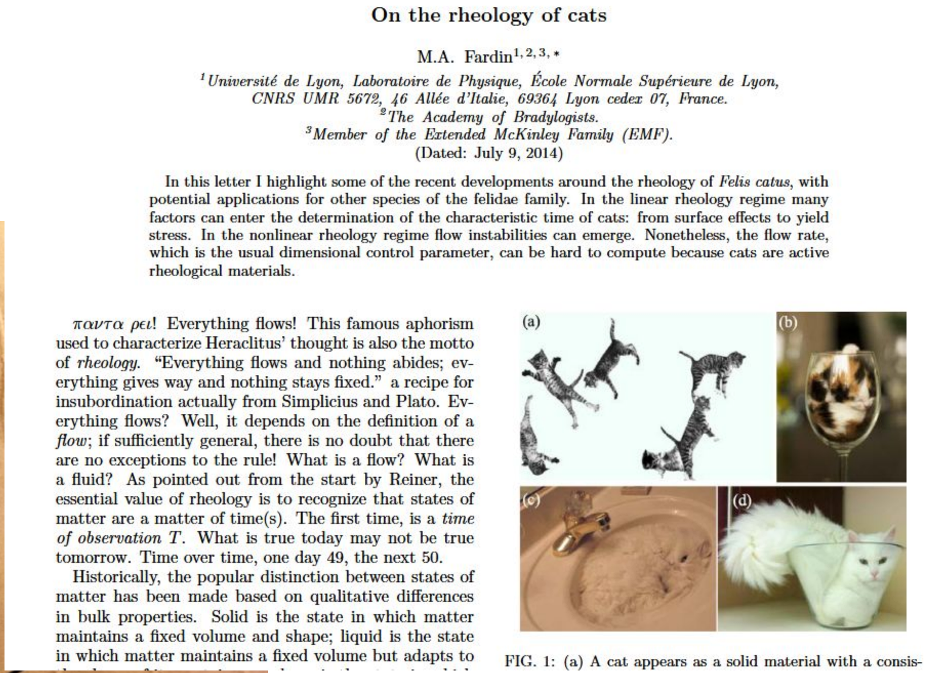
About me

- 3rd year PhD @ CEA
- With Bérengère Dubrulle & Sebastien Fromang

- PhD: "*Can we simulate the climate on a laptop ?*"

Let’s do a fluid simulation





On the rheology of cats

M.A. Fardin[1,2,3,*]

[1]*Université de Lyon, Laboratoire de Physique, École Normale Supérieure de Lyon, CNRS UMR 5672, 46 Allée d’Italie, 69364 Lyon cedex 07, France.*
[2]*The Academy of Bradylogists.*
[3]*Member of the Extended McKinley Family (EMF).*

(Dated: July 9, 2014)

In this letter I highlight some of the recent developments around the rheology of *Felis catus*, with potential applications for other species of the felidae family. In the linear rheology regime many factors can enter the determination of the characteristic time of cats: from surface effects to yield stress. In the nonlinear rheology regime flow instabilities can emerge. Nonetheless, the flow rate, which is the usual dimensional control parameter, can be hard to compute because cats are active rheological materials.

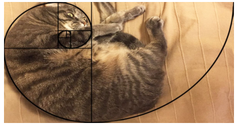
παντα ρει! Everything flows! This famous aphorism used to characterize Heraclitus’ thought is also the motto of *rheology*. “Everything flows and nothing abides; everything gives way and nothing stays fixed.” a recipe for insubordination actually from Simplicius and Plato. Everything flows? Well, it depends on the definition of a *flow*; if sufficiently general, there is no doubt that there are no exceptions to the rule! What is a flow? What is a fluid? As pointed out from the start by Reiner, the essential value of rheology is to recognize that states of matter are a matter of time(s). The first time, is a *time of observation T*. What is true today may not be true tomorrow. Time over time, one day 49, the next 50.

Historically, the popular distinction between states of matter has been made based on qualitative differences in bulk properties. Solid is the state in which matter maintains a fixed volume and shape; liquid is the state in which matter maintains a fixed volume but adapts to

(a) (b) (c) (d)

FIG. 1: (a) A cat appears as a solid material with a consis-

The climate is like a cat



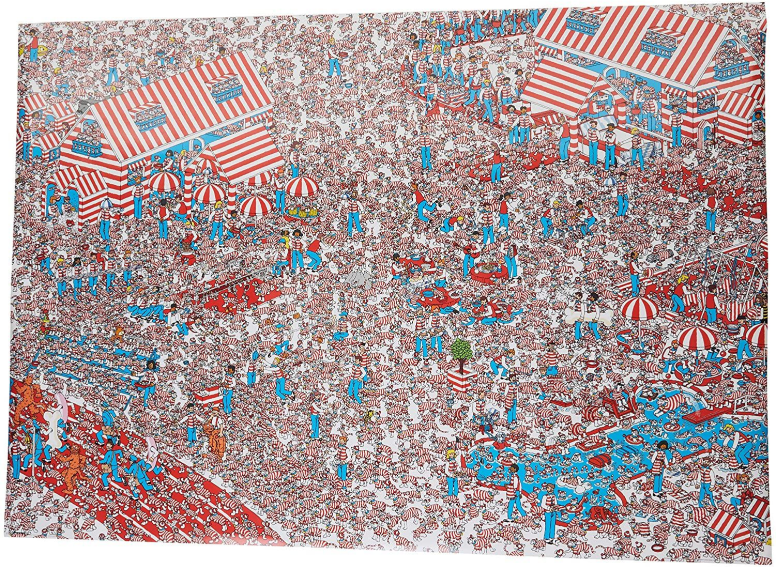
Large scales of interest

- GCM: L~ 6000km

Relevant scale for viscosity

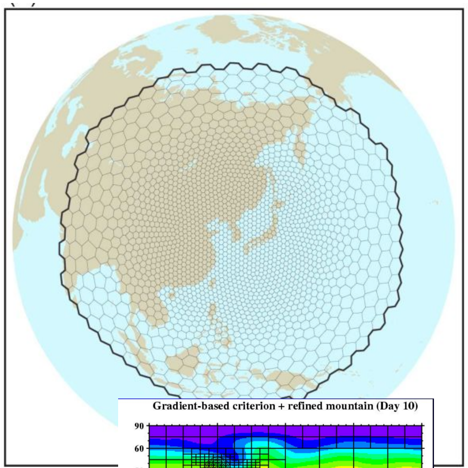
- Kolmogorov scale Eta ~ 1mm (water)

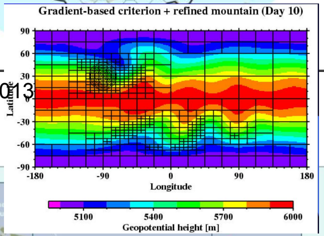
N=(L/eta)^3 >> 1 !!



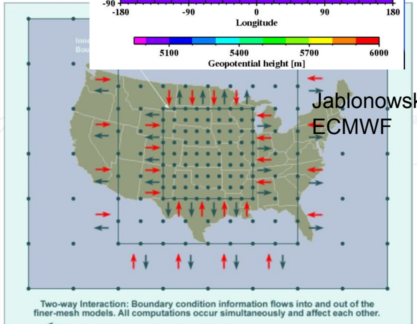
How to deal with small scales ?

- Variable size grid



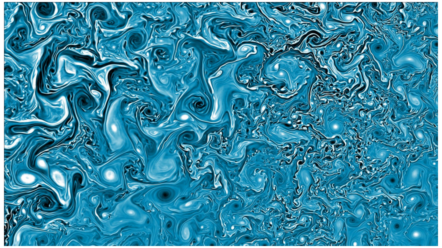


Collins 2013
InTech

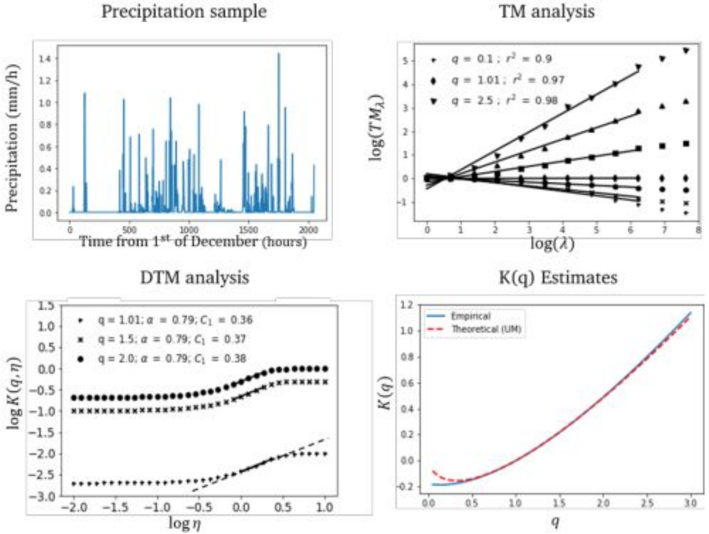


Jablonowski 2004
ECMWF

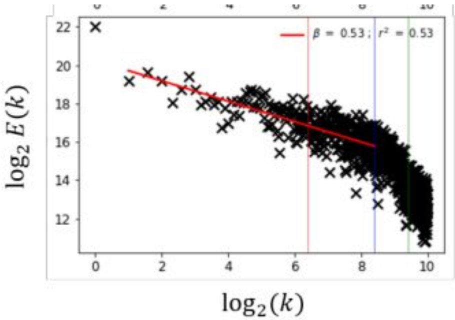
- Parametrization



- Statistical extrapolations (ex. multifractal)

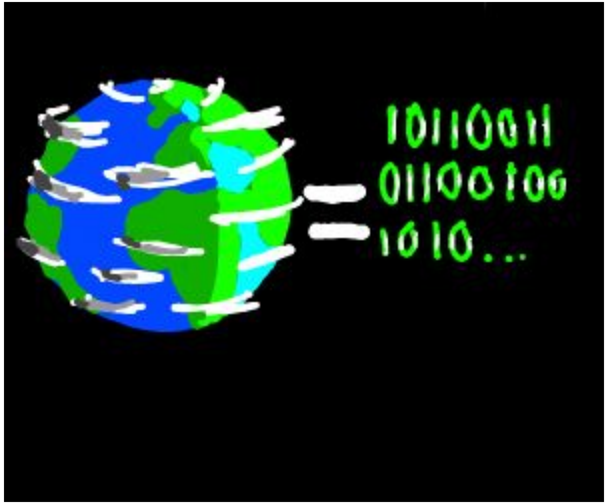


Brochet 2020



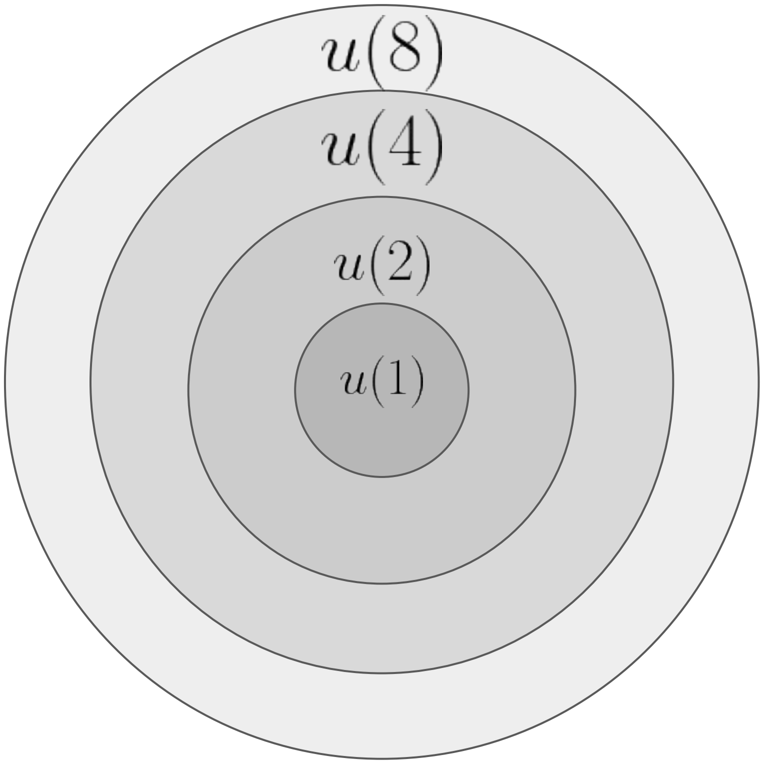
How to deal with small scales ?

- Direct numerical simulations: ??
  - Natural lead: use Fourier space



Shell models

$$u(\lambda^n \leq |k| < \lambda^{n+1}) \approx u_n$$



- Isotropic
- Algebra determined by conservation laws

Non-linear terms

$$\mathrm{d}z_n^+/\mathrm{d}t + \nu_p k_n^2 z_n^+ + \nu_m k_n^2 z_n^- = k_n \sum_{IJ} A_{IJ} z_{n+I}^+ z_{n+J}^-,$$

Gloaguen 1985
Physica 17D

$$E^{\pm} = \sum_n (z_n^{\pm})^2/2 = E^{\mathrm{T}} \pm C \tag{2.9}$$

are conserved), we see that only two independent coefficients remain. Denoting them by α and β, the equations for the z_n^+ finally read

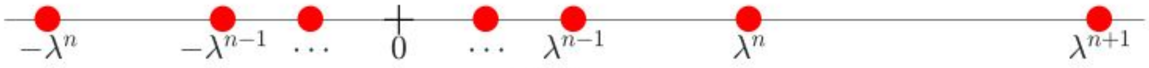
$$\begin{aligned} \mathrm{d}z_n^+/\mathrm{d}t &+ \nu_p k_n^2 z_n^+ + \nu_m k_n^2 z_n^- \\ &= \alpha(k_n z_{n-1}^+ z_{n-1}^- - k_{n+1} z_{n+1}^+ z_n^-) \\ &\quad + \beta(k_n z_{n-1}^+ z_n^- - k_{n+1} z_{n+1}^+ z_{n+1}^-). \end{aligned} \tag{2.10}$$

New framework: log lattices

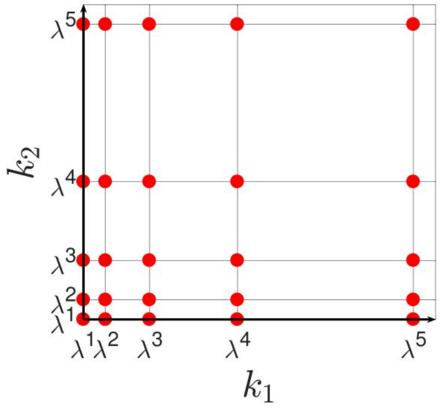
- 1D: same as shell models

$$k_n \in (\pm 1, \pm\lambda, \pm\lambda^2, \ldots, \pm\lambda^N)$$

Campolina 2020



- 2D, 3D: resolved in k-space



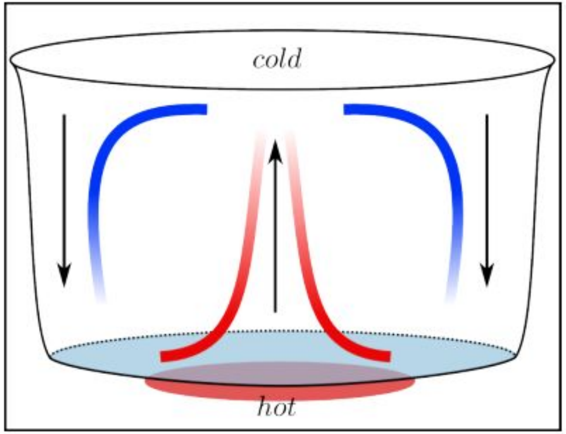
Mathematical problems:

- Conserve triadic interactions k=p+q

$$\widehat{F(x) \cdot G(x)} = (\hat{F} * \hat{G})(k)$$

- Keep the symmetries of the NS equation

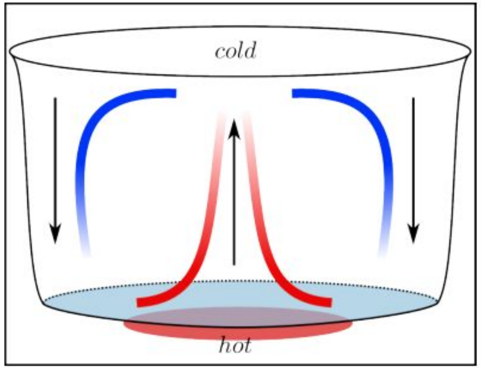
The Rayleigh-Bénard system



$$\begin{aligned} \boldsymbol{U}_t + \boldsymbol{U} \cdot \nabla \boldsymbol{U} + \nabla P &= \mathrm{Pr}\,(\Delta \boldsymbol{U} + \mathbf{k}\,\mathrm{Ra}\,\Theta)\,, \\ \Theta_t + \boldsymbol{U} \cdot \nabla \Theta &= \Delta \Theta + w\,, \end{aligned}$$

$$\mathrm{Pr} \equiv \nu/\kappa \text{ and } \mathrm{Ra} \equiv \alpha g H^3 \Delta T \,(\nu\kappa)^{-1}\,.$$

Rayleigh-Bénard: Heat transfer scaling



How does heat transfer scale with Pr, Ra ?

The Nusselt number is defined as the dimensionless heat flux

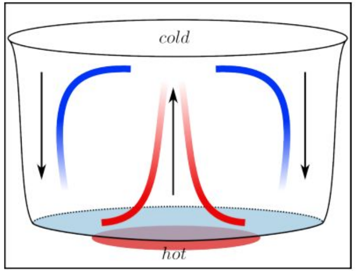
$$Nu = \frac{1}{\kappa \Delta L^{-1}} \left(\langle u_3 T \rangle_{A,t}(z) - \kappa \langle \partial_3 T \rangle_{A,t}(z) \right) = \frac{\langle u_3 \theta \rangle_{A,t}(z)}{\kappa \Delta L^{-1}} - 1 \quad (9)$$

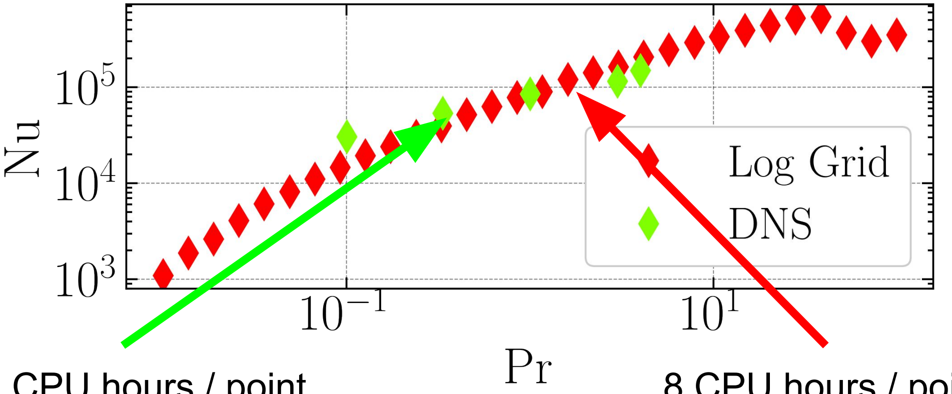
Still an open question!

Table 1: Scaling predictions for RB observables in the ultimate state of convection
(°) With the logarithmic correction $\mathcal{L}$ from Grossmann & Lohse (2011, 2012) (*) For Pr < 0.15 (**) For 0.15 < Pr $\lesssim$ 1 (§) Variational bound
[1]: *Roche (2020)* [2]: *Grossmann & Lohse (2000, 2011)* [3]: *Kraichnan (1962)*
[4]: *Doering & Constantin (1996)* [5]: *Shraiman & Siggia (1990)*

	Experimental[1]	GL[2]	Kraichnan[3]	Exact
Nu ~	$\mathrm{Ra}^{0.25} \lesssim \mathrm{Nu} \lesssim \mathrm{Ra}^{0.5}$	(°) $\mathrm{Ra}^{1/2}\,\mathrm{Pr}^{1/2}\,\mathcal{L}(\mathrm{Re})$	(*) $\mathrm{Ra}^{1/2}\,\mathrm{Pr}^{1/2}\log(\mathrm{Ra})^{-3/2}$ (**) $\mathrm{Ra}^{1/2}\,\mathrm{Pr}^{-1/4}\log(\mathrm{Ra})^{-3/2}$	(§) $\leqslant \mathrm{Ra}^{1/2}$[4]
Re ~	$\mathrm{Ra}^{1/2}$	$\mathrm{Ra}^{1/2}\,\mathrm{Pr}^{-1/2}$	(*) $\mathrm{Ra}^{1/2}\,\mathrm{Pr}^{-1/2}\log(\mathrm{Ra})^{-1/2}$ (**) $\mathrm{Ra}^{1/2}\,\mathrm{Pr}^{-3/4}\log(\mathrm{Ra})^{-1/2}$	
ϵ_θ ~		$(\mathrm{Re}\,\mathrm{Pr})^{1/2}$ to Re Pr		RB: $\kappa L^{-2}(\Delta T)^2(\mathrm{Nu}-1)$[5] HRB: Nu
ϵ_u ~		$\mathrm{Re}^3\,\mathrm{Pr}^2$		RB: $\nu^3 L^{-4}(\mathrm{Nu}-1)\,\mathrm{Ra}\,\mathrm{Pr}^{-2}$[5] HRB: Nu Ra

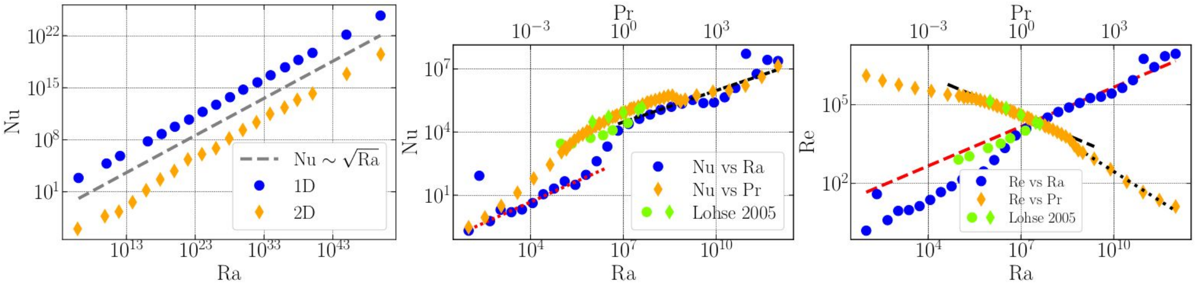
Rayleigh-Bénard: comparison





450 CPU hours / point
~100Mb / Snapshot
C@Supercalculator

8 CPU hours / point
~50Mb RAM, ~2Mb/Snapshot
Cython@My laptop



"Ultimate Regime" $\mathrm{Nu} \sim \sqrt{\mathrm{Ra\,Pr}}$

Going further ?

- Rotating / Quasi-Geostrophic flows (beta-plane)
- Reversible Navier-Stokes *(-> G. Costa)*
- Thermal noise in NS near the Kolmogorov scale
- Singularity strip in Euler/NS3D *(-> Q. Pikeroen)*
- Impact of the grid scaling
- Anisotropy
- Inverse Fourier visualization
- Shear *(-> C. Campolina)*
- Non-periodic BC
- CFL condition
- Comparison with DNS/LES