Approximate symmetries in hydrodynamics

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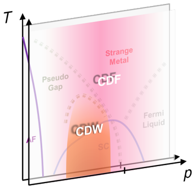
Motivation

- Spontaneous symmetry breaking leads to massless Goldstone fields, included in low-energy effective field theories

- Typically symmetries are **approximate**, and also involve some small explicit breaking $\Rightarrow$ **pinned** Goldstone fields with **small mass**

- Zero temperature EFTs well studied [Weinberg], but crucial to include finite temperature dissipative effects

- Understand the structure of **hydrodynamic EFTs with pseudo-spontaneous symmetry breaking**



Temperature vs doping [Arpaia,Ghiringhelli]

- Dynamical charge fluctuations with translational order in phase diagram of cuprate High-T_c Superconductors [Seibold et al.]

Motivation

- Pinning will lead to **damping** of Goldstones

Main result

Damping rate $\sim$ (Pinning mass)$^2 \times$ Diffusivity

- Initially observed in various holographic models, and QCD with approximate chiral symmetry [Amoretti,Areán,Goutéraux,Musso; Ammon,Baggioli,Jiménez-Alba; Donos,Martin,Pantelidou,VZ; Grossi,Soloviev,Teaney,Yan]

- Not a coincidence or artifact: **consistency of effective field theory**

- Practical application of holography!

Hydrodynamics

- Hydrodynamics describes **late-time, long wavelength** behavior of thermalizing systems compared to local equilibration scale $\ell_{th} \sim T^{-1}$ [Kovtun]
- Conservation laws/Josephson-type relations for slow modes

$$\dot{n}_a + \nabla j_a = 0$$

- Constitutive relations for currents

$$j_a = \alpha_{ab} n_b + \sigma_{ab} \nabla n_b + \lambda_{ab} \nabla^2 n_b + \cdots$$

with transport coefficients determined by UV theory

Hydrodynamics

- Equations of motion

$$\dot{n}_a + M_{ab} n_b = 0$$

- χ: Matrix of static susceptibilities, f: Thermodynamic free energy

$$\chi_{ab} = -\frac{\delta^2 f}{\delta \mu_a \delta \mu_b}$$

- Retarded Green's functions [Kadanoff,Martin]

$$G_{ab} = M_{ac}(-i\omega + M)^{-1}_{cd}\chi_{db}$$

- Physical modes correspond to poles of Green's functions

$$\det(-i\omega + M) = 0$$

Locality of hydrodynamics

- Introducing external sources

$$\delta H = -\delta\mu_a n_a$$

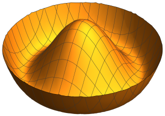
leads to modified equations of motion

$$\dot{n}_a + M_{ab}\left[n_b - \chi_{bc}\delta\mu_c\right] = 0$$

- Restrictions on M, χ from positivity of dissipation & Onsager relations...
- ... but also **locality**!

Superfluid hydrodynamics

- **Isolate condensate** $\Rightarrow$ Hydrodynamic dofs: $U(1)$ charge density n, conjugate phase (Goldstone) ϕ
- ϕ shifts under the symmetry $\Rightarrow$ only **gradients** appear in f



$$f = \frac{f_s}{2}(\nabla\phi)^2 - \frac{\chi_{nn}}{2}\delta\mu^2 + \cdots$$

  - Constitutive relation

$$j \simeq f_s \nabla\phi - D_n \nabla n$$

  - Current conservation & Josephson relation

$$\dot{n} + \nabla\cdot j = 0\,, \qquad \dot{\phi} \simeq -\frac{1}{\chi_{nn}} n + D_\phi \nabla^2\phi$$

  - Read off $M\cdot\chi$

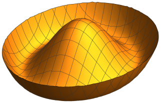
$$M\cdot\chi \simeq \begin{pmatrix} \chi_{nn} D_n q^2 & -1 \\ 1 & D_\phi/f_s \end{pmatrix}$$

  - Second sound mode

$$\omega = \pm c_s q - \frac{i}{2}(D_n + D_\phi)q^2\,, \qquad c_s^2 = \frac{f_s}{\chi_{nn}}$$

Pinning the Goldstone field

- Let us now **break the symmetry weakly** $\Rightarrow$ Introduce a (lower-gradient **mass**) term in f which breaks shift symmetry



$$f = \frac{f_s}{2}[(\nabla\phi)^2 + q_o^2\phi^2] - \frac{\chi_{nn}}{2}\delta\mu^2 + \cdots$$

  - Susceptibility matrix becomes

$$\chi(q) \simeq \begin{pmatrix} \chi_{nn} & 0 \\ 0 & \frac{1}{f_s(q^2+q_o^2)} \end{pmatrix}$$

  - Conservation law is also **weakly broken**

$$\dot{n} + \nabla \cdot j = -\Gamma n + f_s q_o^2 \phi + \cdots$$

  - Josephson relation gets **phase relaxation** term

$$\dot{\phi} \simeq -\Omega\phi - \frac{1}{\chi_{nn}} n + D_\phi \nabla^2\phi + \cdots$$

Pinning the Goldstone field

- Now

$$M \cdot \chi \simeq \begin{pmatrix} \chi_{nn}(\Gamma + D_n q^2) & -1 \\ 1 & \frac{\Omega + D_\phi q^2}{f_s(q_o^2 + q^2)} \end{pmatrix}$$

is generically **not local**

- Locality only restored if the transport parameters satisfy

$$\boxed{\Omega \simeq q_o^2 D_\phi}$$

- Sound mode acquires gap and resonance

$$\omega = \pm c_s q_o - \frac{i}{2}(\Gamma + \Omega) + \cdots = \pm c_s q_o - \frac{i}{2}\left(\Gamma + q_o^2 D_\phi\right) + \cdots$$

Holography for broken $U(1)$ symmetry

- Holography extremely useful for constructing hydrodynamics and EFTs with broken symmetries

Focus on simpler case of $U(1)$ charge relaxation: how to obtain Γ in holography?

- Bulk Maxwell field $\Leftrightarrow$ dual QFT with conserved $U(1)$ current

$$\partial_\mu j^\mu = 0$$

- Softly **break bulk gauge symmetry** $\Leftrightarrow$ QFT with charge relaxation

$$\partial_\mu j^\mu \simeq -\Gamma n$$

$\Rightarrow$ We consider bulk **Proca theory**

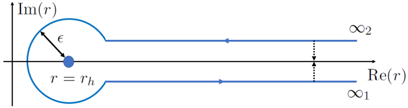
$$S_{\rm bulk} = -\int d^5x\sqrt{-g}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu\right]$$

Holography for broken $U(1)$ symmetry

- Schwarzschild-AdS bulk geometry $\Leftrightarrow$ neutral, thermal state in QFT

$$ds^2 = 2drdv - r^2\left(1 - \frac{r_h^4}{r^4}\right)dv^2 + r^2\delta_{ij}dx^i dx^j$$

- Prescription for **Schwinger-Keldysh closed time path contour**: complexify radial coordinate and analytically continue around the horizon [Glorioso,Crossley,Liu]



- We then partially solve the bulk eoms, in order to derive the **finite temperature SK action** for the hydrodynamics of broken $U(1)$ symmetry!
  - Action principle $\Rightarrow$ includes thermal fluctuations, interactions between modes, is manifestly local and consistent, leads to Onsager relations and 2nd law,... [Crossley,Glorioso,Liu]

- Transport coefficients given by **horizon quantities** $\Rightarrow$ horizon encodes dissipation [Kovtun,Son,Starinets; Iqbal,Liu; Donos,Gauntlett]

Outlook

Many applications:

- QCD, nematic/hexatic liquid crystals, (anti-)ferromagnets, Wigner crystal/Charge density waves (in magnetic fields), strange metallic transport, ...

Future directions:

- Derive SK action for holographic superfluids and charge density waves
- Non-linear response and loop corrections with SK action
- Consequences for order parameter fluctuations near phase transitions & systems with higher-form symmetries
- Implications for strange metal phenomenology
- ...

Thank You!

Extra slides: Schwinger-Keldysh EFT [Crossley,Glorioso,Liu]

- Real time dynamics in thermal state on a closed-time path contour; ϕ_1 and ϕ_2 on each leg

$$\phi_r = \frac{1}{2}(\phi_1 + \phi_2) \qquad \phi_a = \phi_1 - \phi_2$$

- Shift symmetry $\Rightarrow \phi_{a,r}$ enter the superfluid action with derivatives

$$S_{eff} = \chi_{nn} \int \left(\dot{\phi}_a \dot{\phi}_r - c_s^2 \nabla\phi_a \nabla\phi_r\right) + \left(D_n \nabla^2 \phi_a \dot{\phi}_r + \frac{D_\phi}{c_s^2} \ddot{\phi}_a \dot{\phi}_r\right) + \mathcal{O}(\partial^3, \phi_a^2) + \cdots$$

- Currents are computed by Noether procedure and obey eoms

$$J^\mu \equiv \frac{\delta S_{eff}}{\delta(\partial_\mu \phi_a)} \qquad \partial_\mu J^\mu = 0$$

- **Breaking** the symmetry allows for only **two** new terms

$$\delta S_{eff} = -\chi_{nn} \int q_o^2 \phi_a \phi_r + \Gamma \phi_a \dot{\phi}_r + \cdots$$

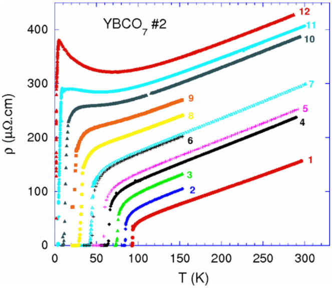
Extra slides: Strange metallic transport

- Expect diffusivities in strongly-correlated materials to saturate **Planckian bound** [Hartnoll]

$$D \simeq \frac{\hbar}{k_B T} c_s^2$$

- Resistivity for CDWs

$$\rho_{\rm dc} = \frac{m^\star}{ne^2}\left(\Gamma_\pi + \frac{q_o^2}{\Omega}\right) \simeq \frac{m^\star}{ne^2}\left(\Gamma_\pi + \frac{k_B T}{\hbar}\right)$$



Resistivity vs temperature for irradiated single crystal $YBCO_7$ [Rullier-Albenque,Alloul,Tourbot]

- Γ_π from conventional scattering (Umklapp, disorder, el-ph interactions)
- Slope of the linear term **independent** from disorder
- Natural mechanism for T-**linear** resistivity