Perturbative quantum field theory techniques applied to fluctuating elastic membranes.

Rencontres des Jeunes Physicien(ne)s 2022 - Collège de France

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LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

An overview on perturbative field theory

Field theory – What is it?

Field theory

Anything that can be described (directly or roughly) with waves moving in a field!

Electromagnetic waves, heat waves, mechanical waves, high energy particles ...

An outstanding example; Quantum Electrodynamics (QED)

Photons are obviously waves, but electrons are too! Let's make them interact:

$$\mathcal{L}_{\text{QED}} = \underbrace{\bar{\psi}(i\not{D} - m_e)\psi}_{\text{electrons+interactions}} - \frac{1}{4}\underbrace{F^{\mu\nu}F_{\mu\nu}}_{\text{Maxwell}}$$

QED is a QFT, which is classical field theory + special relativity + quantum mechanics.

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Field theory – Quantum perturbative approach

Infinitely complicated problem → infinite sum of "simple" problems.

The famous Feynman diagrams

A visual representation of the computations ... An example of QED diagram:

Perturbative approach states that physical quantities (*probability amplitude*), e.g. how an electron truly propagates, can be computed precisely like:

Summing all infinitely possible *self-interaction* loops gives the true X value ... Hopefully, the more complicated a diagram is, the smaller it contributes!

This is how we access very high precision prediction on quantities like: "What is the probability of colliding particles A and B and get as an output particles X and Y?" or "How much, very precisely, is the magnetic moment of an electron?".

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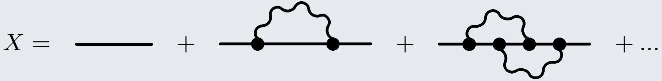

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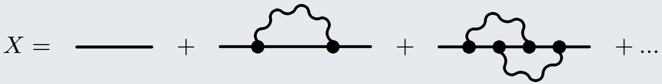

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Field theory – Statistical perturbative approach

Exactly the same formalism if fluctuations are thermal or statistical!

An example – From Ising model to ϕ^4 theory

Ising model is the simplest way to describe a magnet, i.e., ± 1 valued spins on a lattice. It is exactly solvable in 1&2 dimensions, but what to do for a physical 3D magnet?

Seen from far enough, Ising model may look like a field theory:

$$\underbrace{\mathcal{H}(S) = -J \sum_{<i,j>} S_i S_j}_{\text{Ising discreet model}} \xrightarrow[\text{smoothing}]{} \underbrace{\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4}_{\phi^4 \text{field theory}}$$

Then (IR) quantities can be computed perturbatively via diagramatic expansions too! :).

In fact, ϕ^4 model is an effective way to describe Ising model near its *critical point* ...

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Field theory – Divergences ...

Sorry, I lied ... All graphs described above with at least one closed loop are $= \infty$...

Regularization & renormalization – taming infinities

Split divergent and convergent information, e.g.:

$$= \lim_{\varepsilon \to 0} \left(\frac{A}{\varepsilon} + B \right)$$

(some crazy looking divergent diagram)

- Convergent part (B) is the *renormalized probability amplitudes* info!
- Divergent part (A) is the *critical properties* info!

Critical properties

These are violent features of a model like **phases transitions**!
Described by *fixed points* and quantified by *critical exponents/anomalous dimensions*.

Allow to answer questions like
"What happen to the magnetization of a magnet near its melting point?" or
"How does specific heat of a liquid diverges near one of its phase transition",
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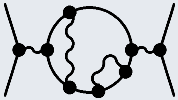

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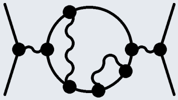

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An intuitive field theory – Fluctuating flat membranes

Fluctuating flat membranes – Physical motivation

d-dim extended objects embedded in a larger D-dim space subject to small quantum and/or thermal fluctuations.

Applications:

- cond-mat: graphene, silicene, phosphorene ...
- bio: living cells surfaces (phospholipid bilayers)
- hep: worldsheet, branes ...

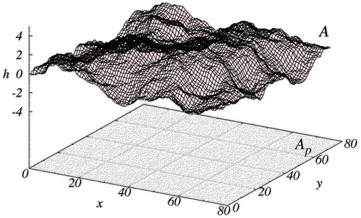

Figure: Generic fluctuating membrane

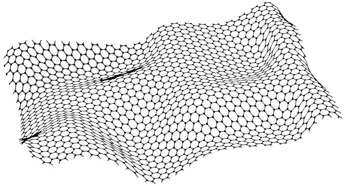

Figure: Fluctuating graphene

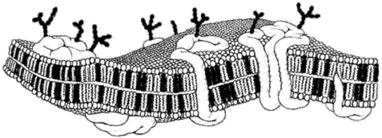

Figure: Cell bi-layered membrane

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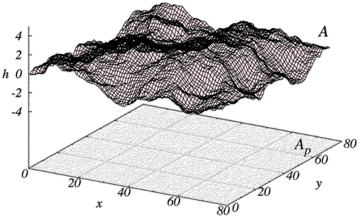

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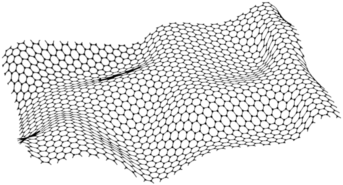

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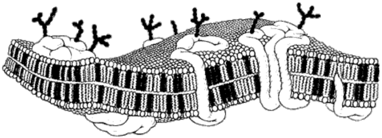

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Fluctuating flat membranes – An intuitive field theory

Model parametrization

Fields parametrization:

- $\vec{u}(\vec{x}) \equiv$ longitudinal displ. (P-wave) (phonon)
- $h(\vec{x}) \equiv$ height displ. (S-wave) (flexuron)
- $\vec{R}(\vec{x}) = (\vec{x} + \vec{u}(\vec{x}), h(\vec{x})) \equiv$ coordinates with $\vec{x} = (x, y)$ and $\vec{u}(\vec{x}) = (u_x(\vec{x}), u_y(\vec{x}))$

Action

$$S[\vec{u}, h] = \int \mathrm{d}^2x \left[\frac{1}{2} (\Delta h)^2 + \frac{\mu}{(4\pi)^2} (u_{ij})^2 + \frac{\lambda}{2(4\pi)^2} (u_{ii})^2 \right]$$

$$\text{with} \quad u_{ij} \approx \frac{1}{2} [\partial_i u_j + \partial_j u_i + \partial_i h \, \partial_j h]$$

$u_{ij} \equiv$ stress tensor ; fluctuations with respect to the flat configuration $\vec{R}_0(\vec{x}) = (\vec{x}, 0)$
λ, $\mu \equiv$ coupling constants $\equiv$ Lamé coefficients

Fluctuating flat membranes – An intuitive field theory

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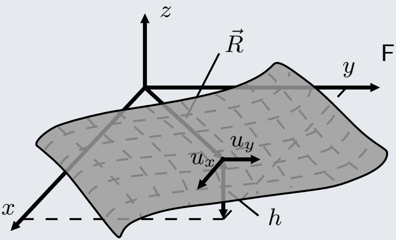

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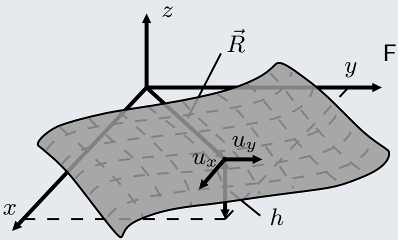

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Fluctuating flat membranes – What to compute?

What to compute?

- $(\lambda, \mu) \equiv$ Lamé coefficients at a stable (scale invariant) fixed point.
- $\eta \equiv$ elastic critical exponent $\equiv$ field anomalous dimension.

A papersheet subject to small deformations acquire anomalous rigidity and elasticity!

All other mechanical quantities are accessible with λ, μ and η only:

Quantities derived from λ and μ:

- Young modulus
- Poisson ratio (negative!)
- bulk modulus
- s-wave sound velocity ...

All crit. exponents depend only on η:

- bending/rigidity modulus
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We computed the Lamé coefficients (μ, λ) and the critical exponent η **analytically** with high precision:

Flexuron propagator $\equiv$ Phonon propagator $\equiv$ 

- First order (hand computations) [Aronovitz & Lubensky, '88] (4 integrals)

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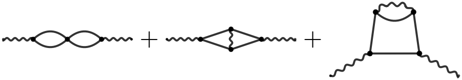
$\Pi_2 =$ 

Fluctuating flat membranes – Computations

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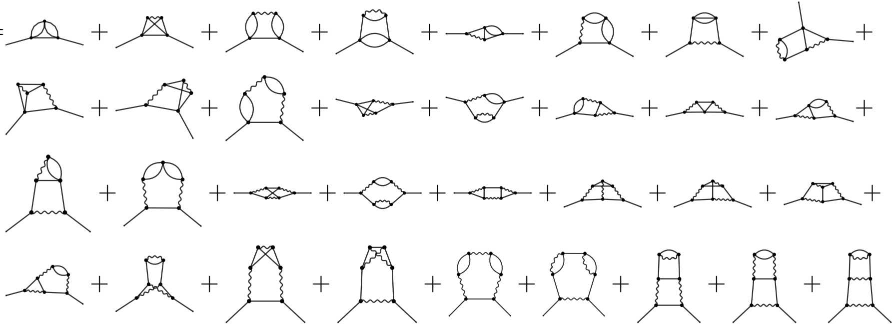

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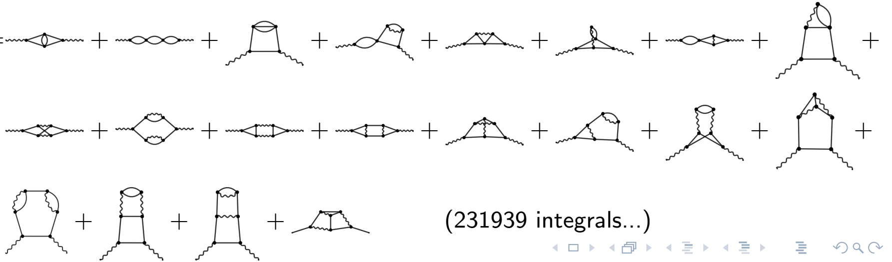

(231939 integrals...)

Fluctuating flat membranes – Results

Analytical results exact order by order in perturbation theory:

$$\lambda = -\underbrace{\frac{4}{25}}_{\text{1st order}} + \underbrace{\frac{56}{3125}}_{\text{2nd order}} - \underbrace{\frac{2(1703808\zeta_3 - 3032351)}{48828125}}_{\text{3rd order}} + \ldots$$

$$= -0.160 + 0.018 + 0.040 + \ldots$$

$$\mu = +\underbrace{\frac{12}{25}}_{\text{1st order}} - \underbrace{\frac{88}{3125}}_{\text{2nd order}} + \underbrace{\frac{6(1847808\zeta_3 - 2076601)}{48828125}}_{\text{3rd order}} + \ldots$$

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Fluctuating flat membranes – Results

This leads the fundamental numbers:

$$\lambda = -0.102 \qquad \mu = 0.470 \qquad \eta = 0.888$$

From which we compute all desired property of the membrane, e.g.:

Bulk modulus $\equiv K = \lambda + \mu = 0.368$

Young modulus $\equiv E = \dfrac{4\mu(\lambda+\mu)}{\lambda+2\mu} = 0.825$

Poisson ratio $\equiv \nu = \dfrac{\lambda}{\lambda+2\mu} = -0.121$

P-wave modulus $\equiv M = \lambda + 2\mu = 0.837$

Anomalous roughness $\equiv \zeta = \dfrac{2-\eta}{2} = 0.556$

Anomalous rigidity $\equiv \eta = 0.888$ etc

These are the experimental results that would be obtained by averaging the mechanical properties of a membrane over a large number of (fractal) configurations.

Fluctuating flat membranes – Results

This leads the fundamental numbers:

$$\lambda = -0.102 \qquad \mu = 0.470 \qquad \eta = 0.888$$

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Thank you for your attention :).