Full Dissipative Dynamics Description of Heavy-Ion Collisions

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NARODOWE CENTRUM BADAŃ JĄDROWYCH ŚWIERK

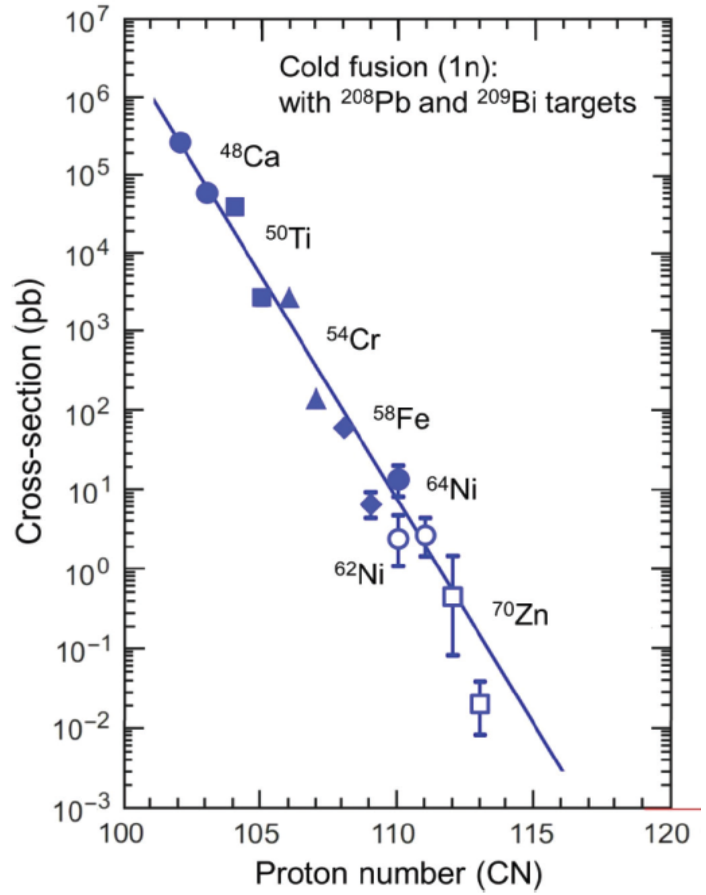
Motivations – A 30-year-old project

W. Przystupa, K. Pomorski, Nucl. Phys. A 572(1) (1994) 153

Super-heavy elements with Z > 103 do not occur in nature.
They can only be produced in the laboratory by fusing two lighter nuclei.

Our primary goal is to gain insights into the **fusion reaction mechanisms in the domain of cold synthesis reactions** (Z < 113, E* ≈ 10 – 20 MeV), in particular on the **understanding of the hindrance mechanism which prevents the formation of super-heavy nuclei**.

We propose a comprehensive dissipative dynamics **Langevin**-based formalism to describe the unrestricted motion of the systems in terms of **elongation**, **neck** and **asymmetry** variables.



Specific Goals

- Investigate the **impact of the entrance channel asymmetry parameters** on the fusion process to identify **ideal target/projectile combinations**.
- Gain a **deeper understanding of friction**, notably in scenarios involving intense friction (overdamped limit).
- Explore **the impact of stochastic/random forces** during the fusion phase.
- Analyze the patterns of **energy and angular momentum dissipation.**
- Note that this is an **ongoing work in progress**, and we will be presenting **preliminary results**.

The Langevin system of equations

- Defining **collective variables** $q_i(t)$ and their **associated moments** $p_i(t)$, the **Langevin equations** read:

$$\dot{q}_i(t) = \sum_k (\mathcal{M}^{-1})_{ik}\, p_k \qquad \leftrightarrow (P = MV)$$

$$\dot{p}_i(t) = -\frac{\partial H}{\partial q_i} - \sum_k \gamma_{ik}\, \dot{q}_k + \sum_k g_{ik}\, \xi_k(t) \qquad \leftrightarrow \left(\frac{dP}{dt} = \sum F\right)$$

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Conservative forces $(\boldsymbol{H = T + V})$

Friction forces

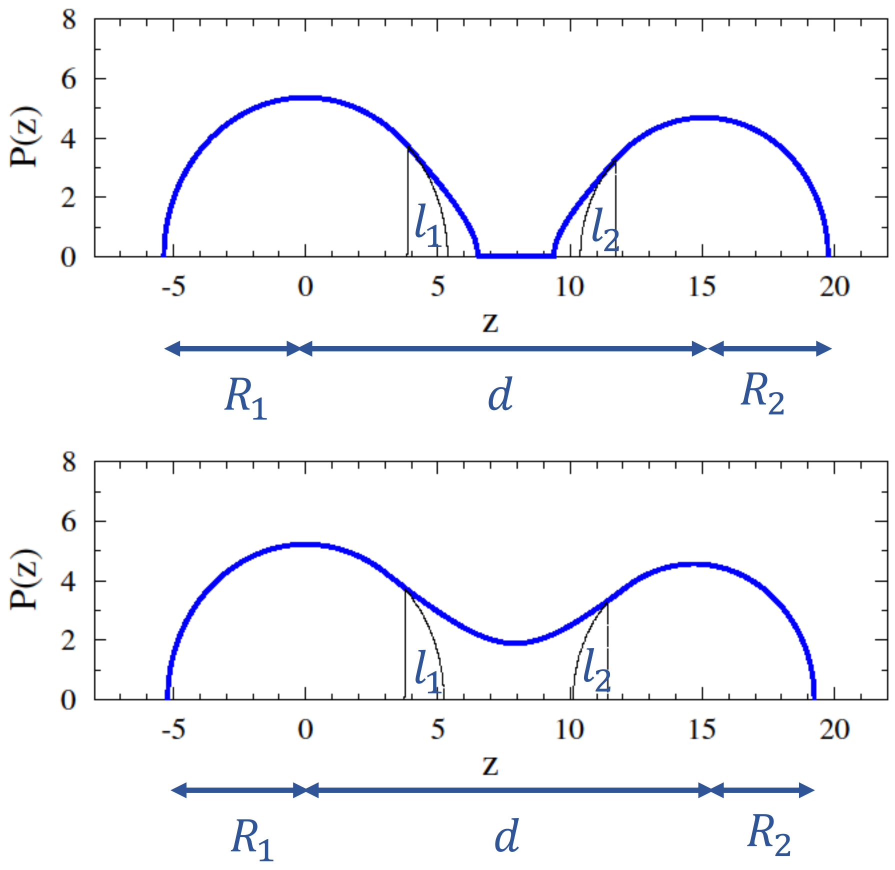
Langevin/random forces

$\longrightarrow$ A **comprehensive understanding of the dynamics process** (in comparison to the random walk f. eg.).

Collective variables adapted to fusion/fission – Shape variables

^{92}Zr + ^{64}Ni

- Axially symmetric shapes
- **Spherical** cups connected by quadratic surfaces[1]
- **Shape variables**:
  - **Distance/elongation** : $\rho = \frac{d}{R_1+R_2}$
  - **Neck/deformation** : $\lambda = \frac{l_1+l_2}{R_1+R_2}$
  - **Asymmetry**: $\Delta = \frac{R_1-R_2}{R_1+R_2}$



Bipartite
$\rho = 1.5$
$\lambda = 0.3$
$\Delta = \Delta_0$

Monopartite
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[1] J. Błocki, H. Feldmeier and W. J. Świątecki, Nucl. Phys. A 459 (1986) 145

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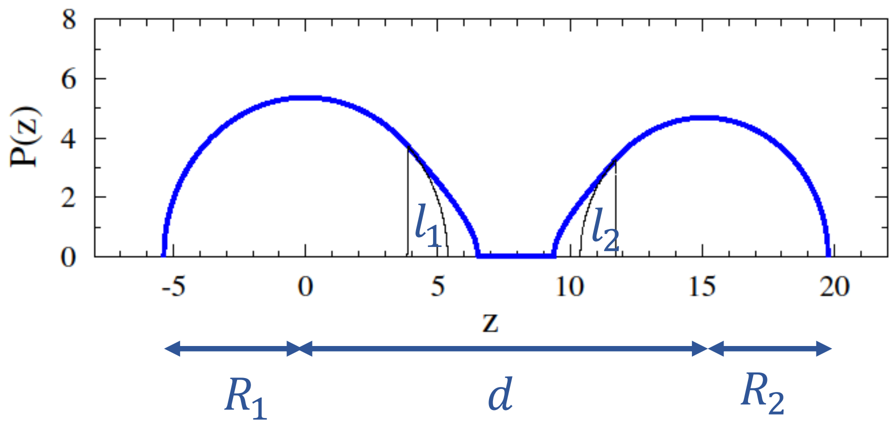
  - **Asymmetry**: $\Delta = \frac{R_1-R_2}{R_1+R_2}$
- **Scission is well-defined**: $\lambda_{\text{scission}} = 1 - \frac{1}{\rho_{\text{scission}}}$

  → **Suited to describe fusion/fission**

$^{92}Zr + ^{64}Ni$



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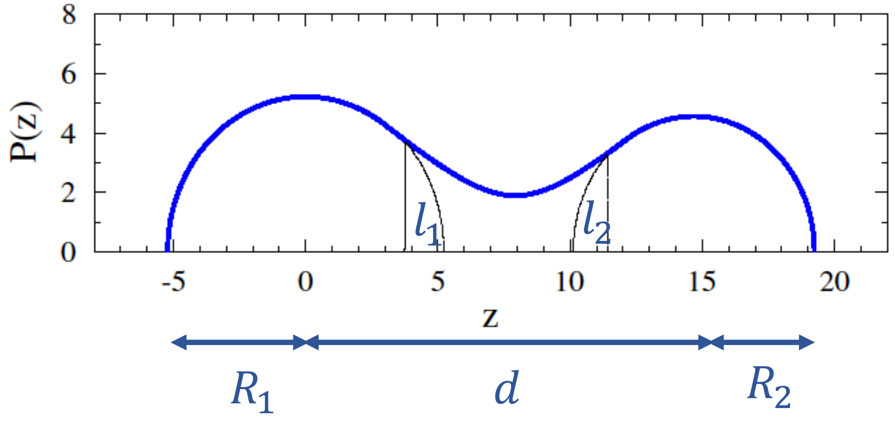

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Collective variables adapted to fusion/fission – Angle variables

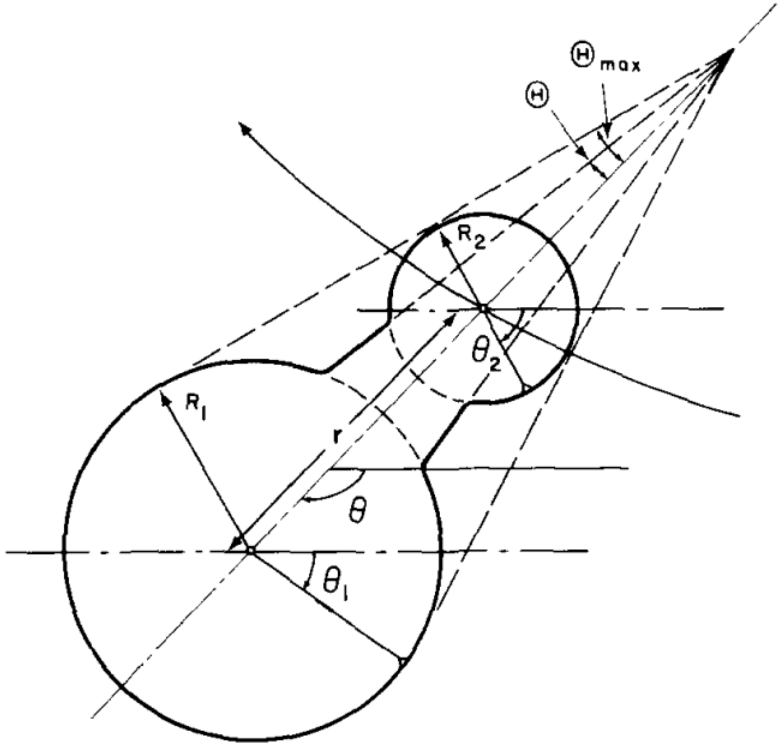
- **Collective angle variables**
  - Angle of the whole system θ
  - Angle of the first sphere θ_1
  - Angle of the first sphere θ_2

- Variations linked to angular momentum, in particular:

$$p_\Theta + p_{\theta_1} + p_{\theta_2} = -L_{init}$$

- **Exact treatment of angular momentum**

$\longrightarrow$ **Full Langevin 6-dimensional dissipative dynamics**

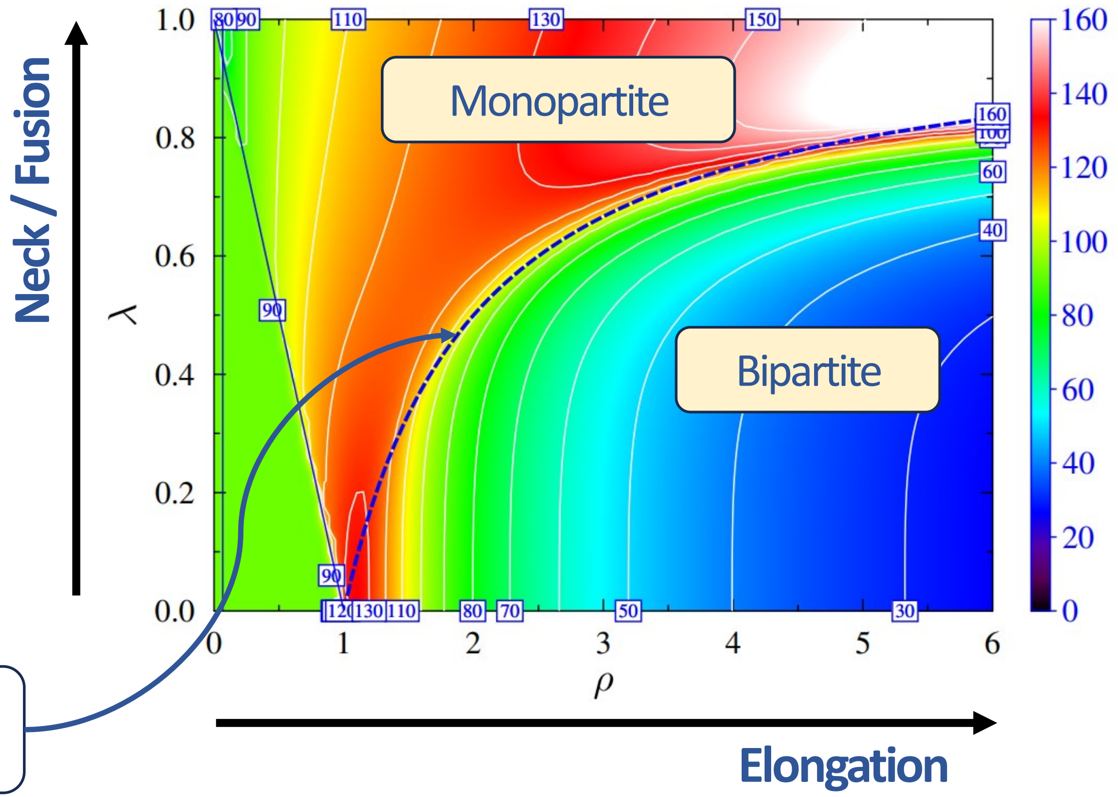


Potential Energy

- **Yukawa-plus-exponential folding potential + Coulomb**
- Parameters taken from a previous fit to experimental masses and fusion barrier heights [1]

- No shell effects at the moment.

Scission line $\lambda = 1 - \frac{1}{\rho}$

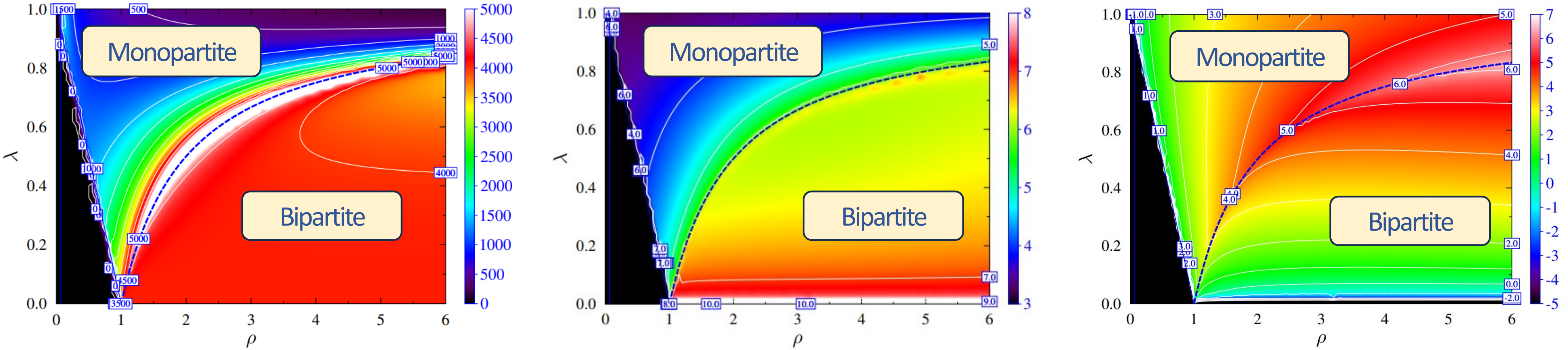


Deformation potential of $^{92}Zr + ^{64}Ni$ in MeV

[1] H. J. Krappe *et al.*, Phys. Rev. C 20 (1979) 992–1013

Mass tensor / Kinetic Energy

- **Werner-Wheeler flow approximation**:
  - **Incompressibility** (matter density is uniformly distributed)
  - The flow is **irrotational** (the moving planes remain plane)

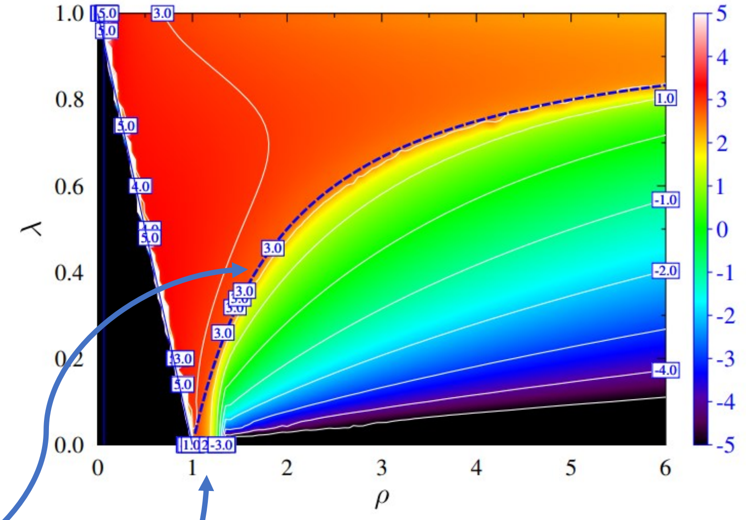


$\mathcal{M}_{\rho\rho}$ $\log(\mathcal{M}_{\Delta\Delta})$ $\log(\mathcal{M}_{\lambda\lambda})$

Friction forces

- **Proximity formalism** (for a temperature dependent friction):
  Possible matter flow/friction before contact (d = 3.2 fm)

- **Shape friction**:
  - **Wall friction** (collisions nucleons ↔ nuclear surface)
  - + **Wall-plus-window friction** (between the two fragments)



$\log(\gamma_{\rho\rho})$

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Friction **even when the system is separated** (proximity)

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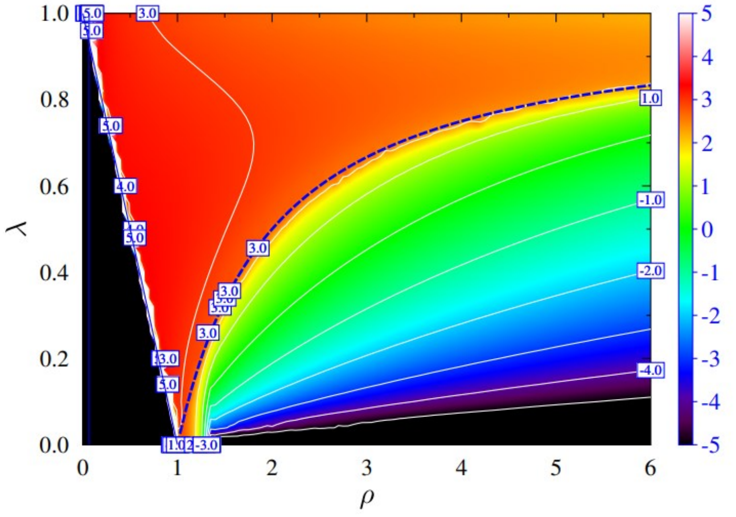

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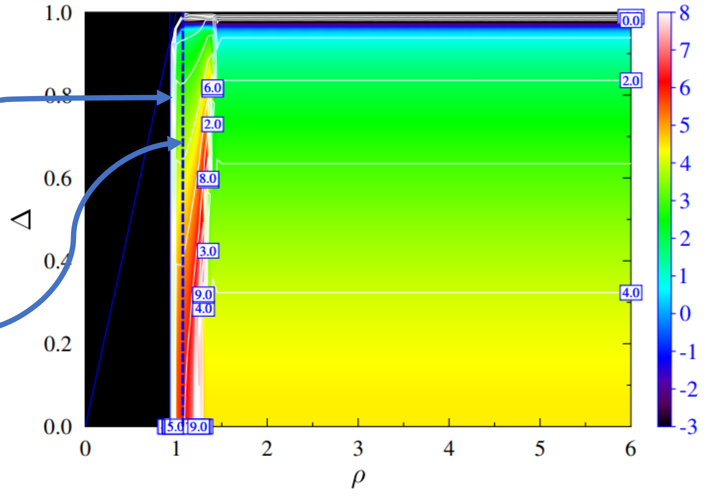

$\log(\gamma_{\Delta\Delta})$, $\lambda = 0.1$

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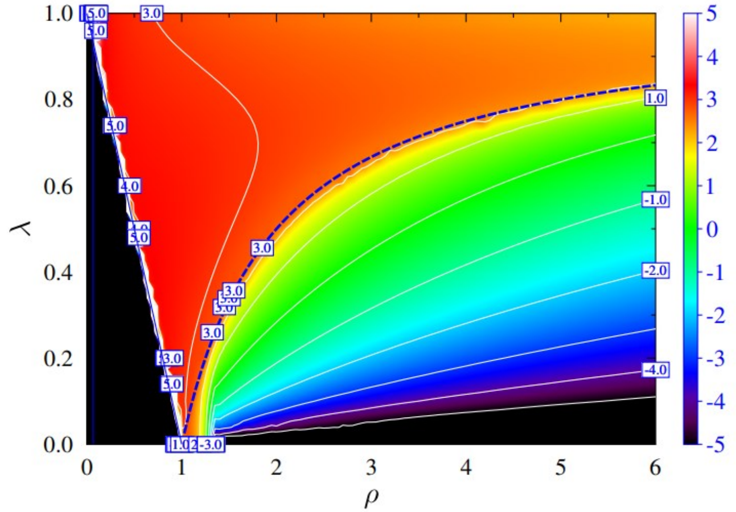
- **Angular friction**:
  - **Sliding friction**
  - **No rolling friction**



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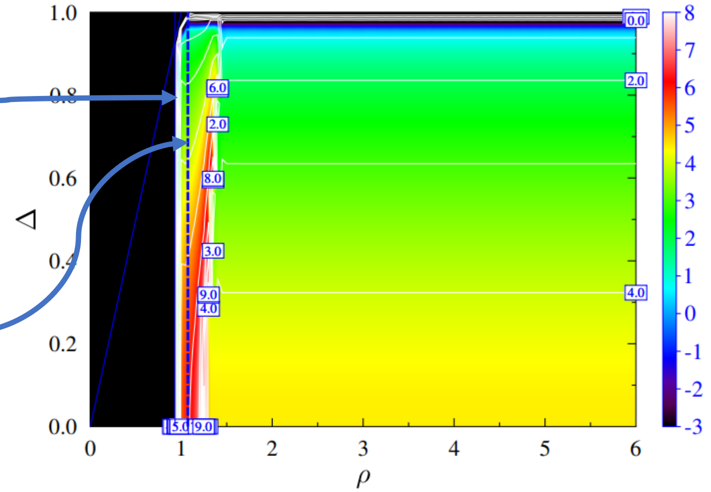


$\log(\gamma_{\Delta\Delta})$, $\lambda = 0.1$

Langevin/random forces

- **IF** we assume that the interaction between the internal vs. relative motion is **separable** + **linear** in the HO coordinates, the Langevin forces take the form:

$$F_i = \sum_k g_{ik}\xi_k(t)$$

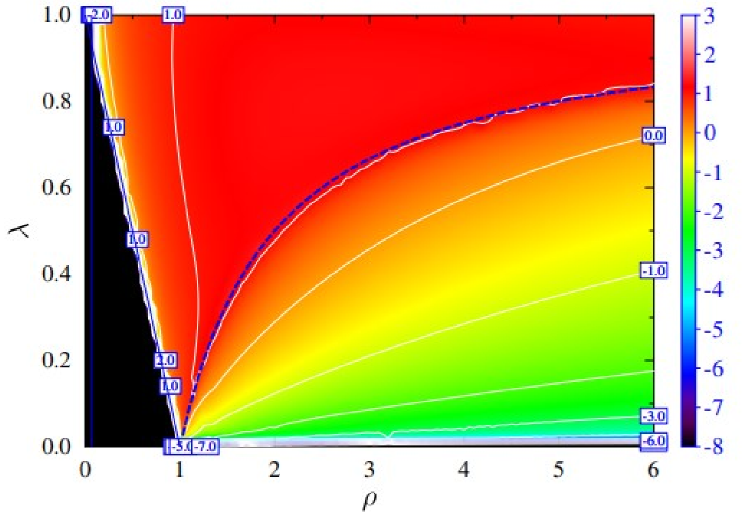
$\xi_k(t)$ are **time-dependent Gaussian random variables:**

$$\langle \xi_k(t) \rangle = 0$$
$$\langle \xi_k(t), \xi_{k'}(t') \rangle = 2\delta_{kk'}\delta(t-t')$$

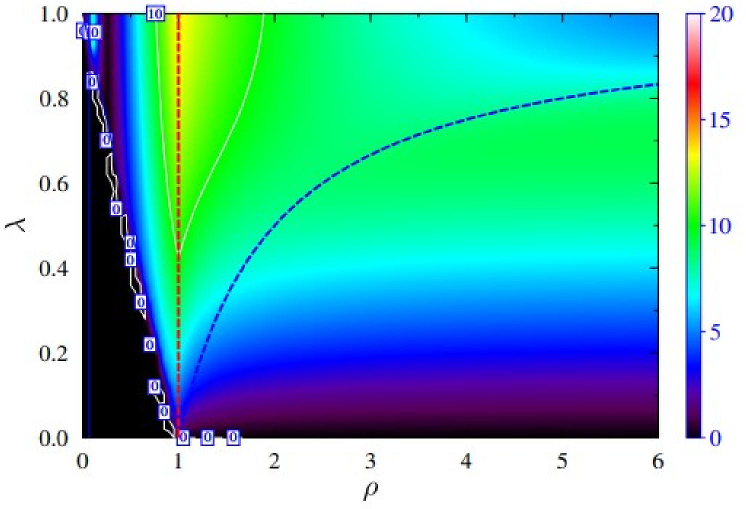
- The diffusion tensor is given by the **Einstein relation**:

$$\sum_k g_{ik}g_{kj} = D_{ij} = k_B T \gamma_{ij}\,, \qquad T = \sqrt{E^*/a}$$

E^* is the **dissipated energy**, $a = A/8$ MeV is the **level density parameter**.



$\log(g_{\rho\lambda})$



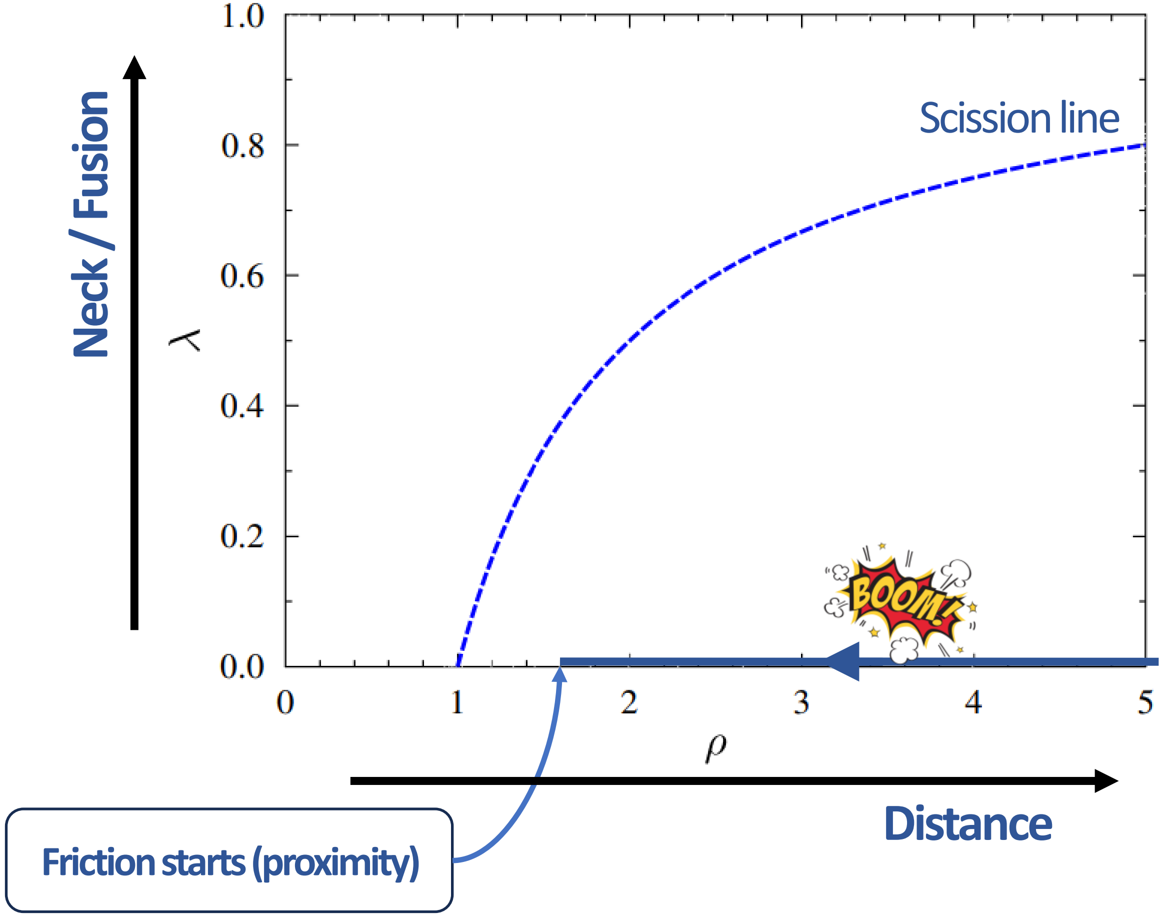
$g_{\theta_2\theta_2}$

Defining fusing and non-fusing events - I

The three main stages of the collision:

1. **A first violent deceleration** during which:
   - The **system loses most of its kinetic energy**
   - There is **almost no deformation of the nuclei**

   $(\dot{\lambda} = (\mathcal{M}_{\rho\lambda})^{-1} p_\rho < 0)$



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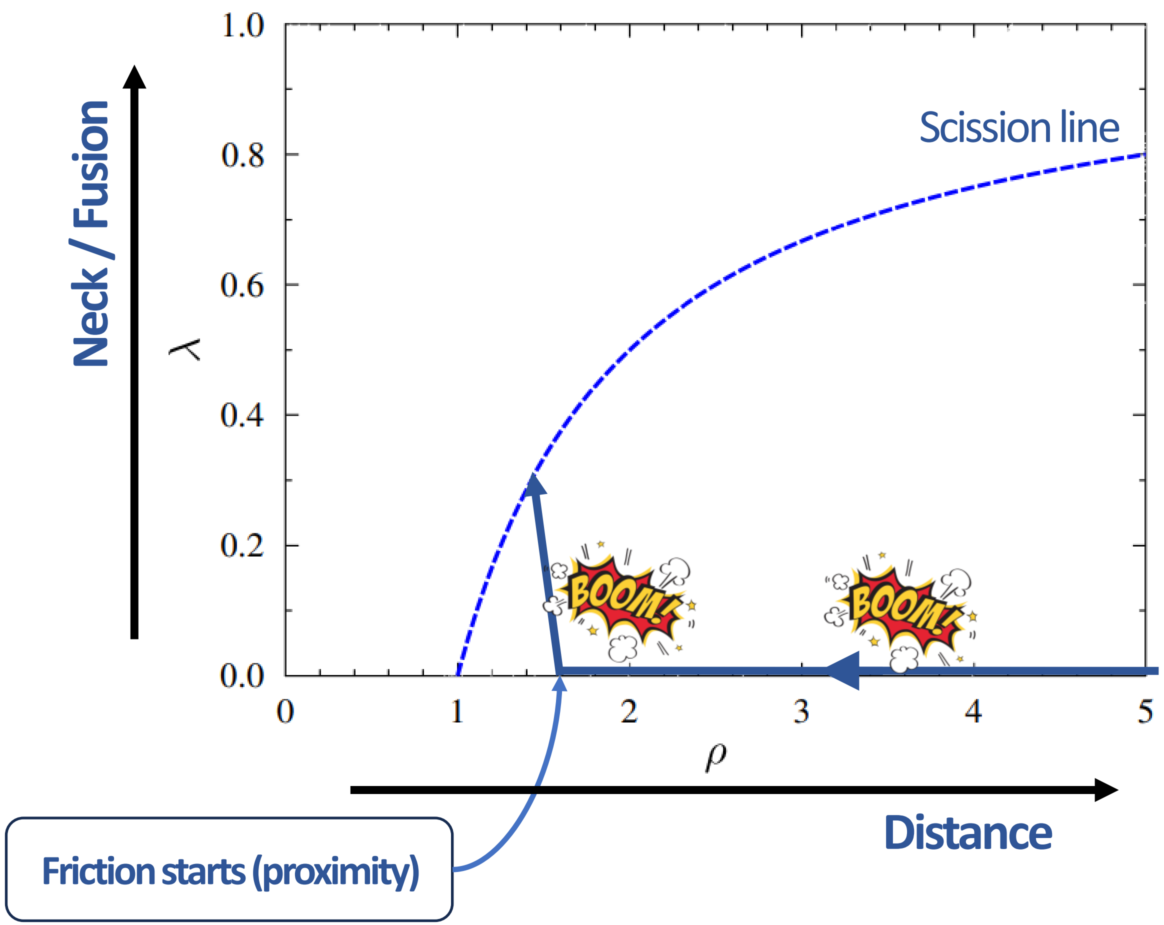
2. The **"Kiss of death"** when friction starts
   - Rapid deceleration **to the touching point**
   - **Deformation starts and remains.**



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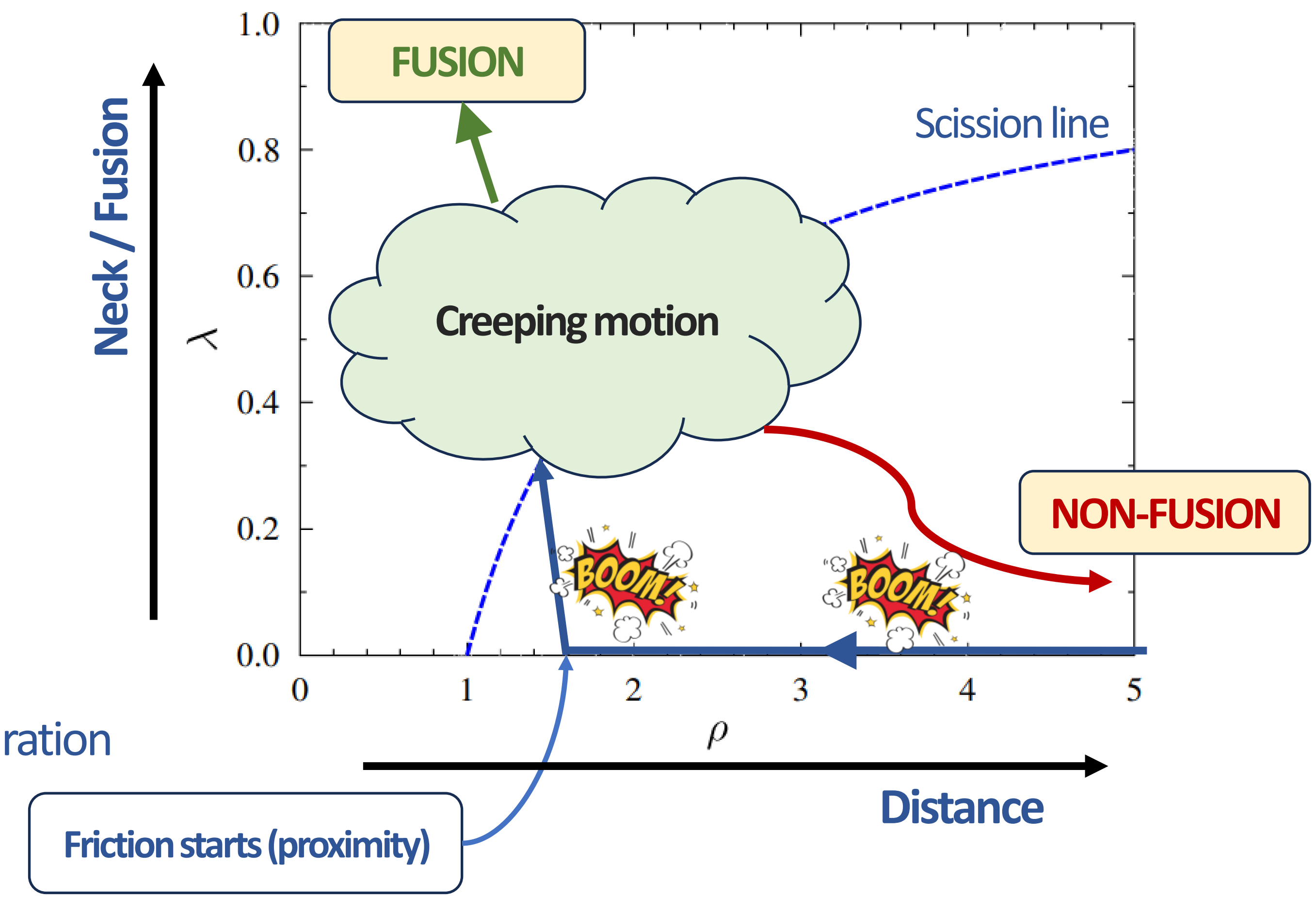
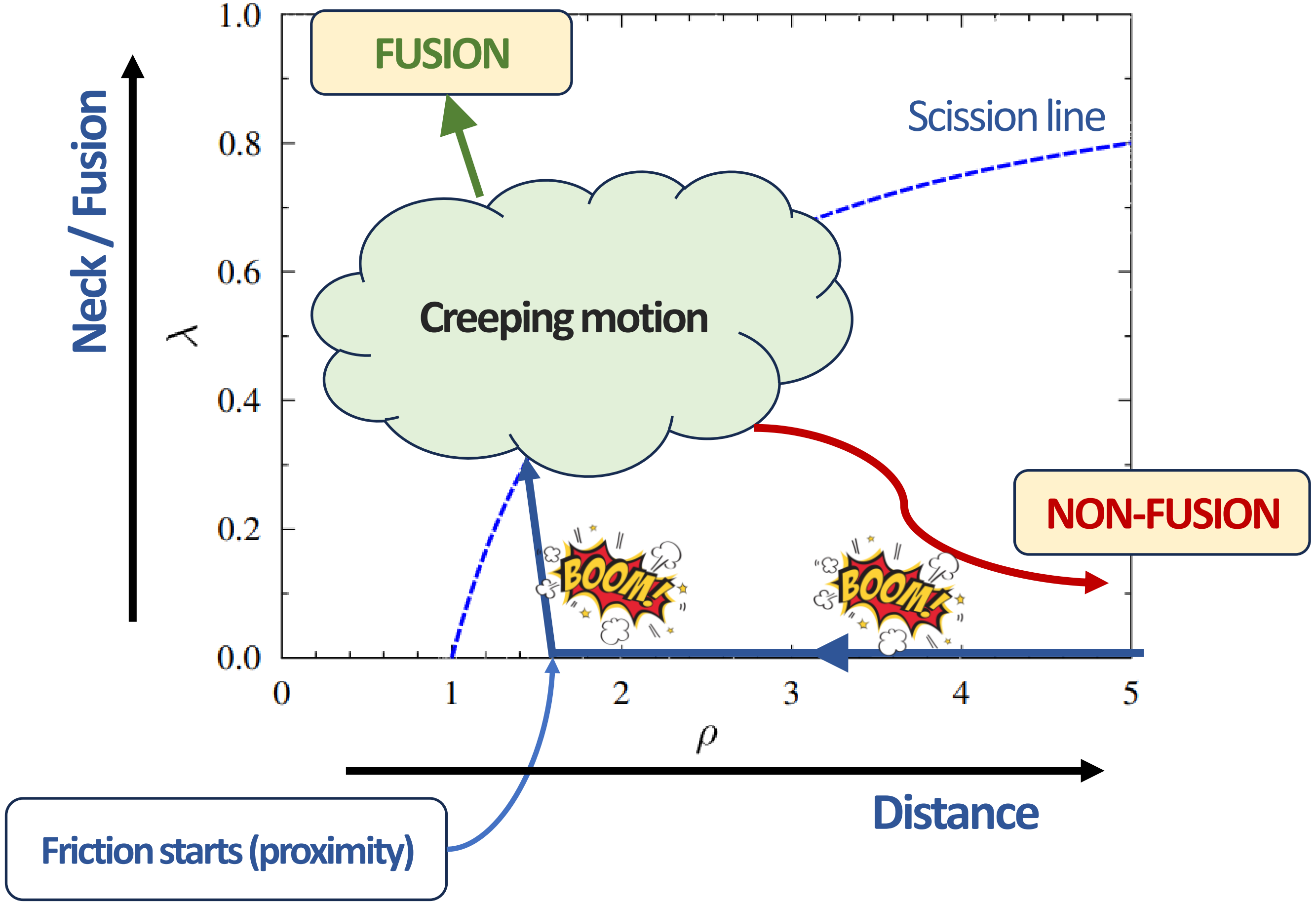
3. A **long creeping motion** that leads to fusion or separation



Defining fusing and non-fusing events - II

- **Fusing conditions**:
  - $\lambda = 1$ (half of the spheres are mixed)
  - $\rho(1-\lambda) = \Delta^2$ (window angle fully open)
  - $\rho = 0.5$
- **Non-fusing conditions**:
  - $\lambda \to \lambda_{min} = 0.1$
  - $\rho \to \rho_{max} = 3$
  - No fusion after $N_{max} = 500{,}000$ steps.



The observables

- The resolution of the Langevin equations generates **a distribution of trajectories due to the fluctuation force.**
- We use **500,000 - 1,000,000 trajectories**
- **Asymmetry is free to change**.

- The **spin distribution** is calculated as a Monte-Carlo integral on a given bin $i \equiv \ell_i$:

$$\sigma_\ell = \left(\frac{d\sigma_{fus}}{d\ell}\right)_{\ell_i} = \frac{2\pi}{k^2}\ell_i^2\frac{N_i^{fus}}{N_i^{tot}}$$

where $\ell_{init} = \ell_{max}\sqrt{x}, x$ a random number in [0,1] (for easy derived formulas).

- From the spin distribution, one can calculate:
  - The **total cross section / probability for the formation of the compound nucleus**
  - $\langle \ell \rangle, \langle \ell^2 \rangle$
  - **Excitation functions.**

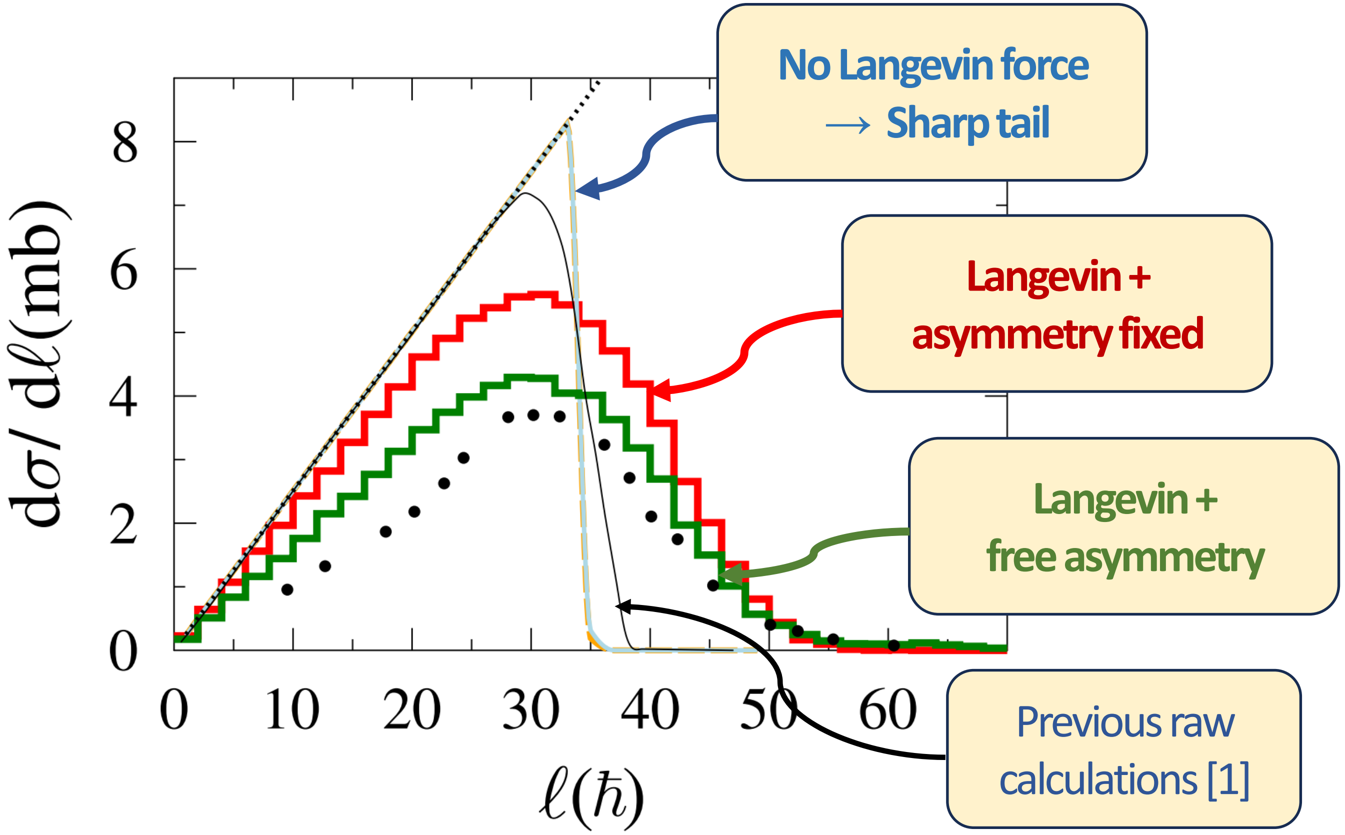
$^{64}Ni + {}^{92}Zr \longrightarrow {}^{156}Er$

- The **Langevin force brings the correct aymptotic behavior of the spin distributions**
  → Relevant for a correct description of fission.

- **By freeing the asymmetry variable, the cross sections decreases (heavy nuclei case).**

- **The experimental data[2,3] are well reproduced.**

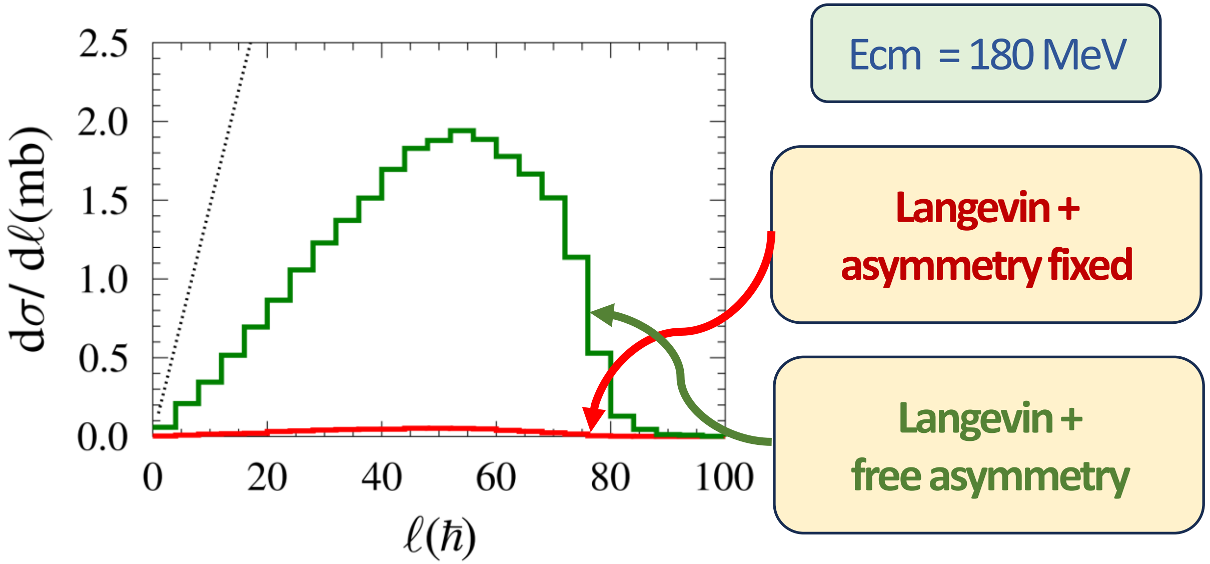
[1] W. Przystupa, K. Pomorski, Nucl. Phys. A 572(1) (1994) 153
[2] W. Kuhn *et al.*, Phy. Rev. Lett. 62 (1989) 1103
[3] A. M. Stefanini *et al.*, Phys. Lett. B 252 (1990) 43



Spin distribution of $^{64}Ni + {}^{92}Zr \rightarrow {}^{156}Er$ at E_{cm} = 138.8 MeV

Preliminary calculations for $^{48}Ca + ^{208}Pb \longrightarrow ^{256}No$

- **By freeing the asymmetry variable, the cross sections increases by a factor 10 – 20!**
- Upcoming analyses will determine if during the fusion process, the asymmetry of the system decreases (hindrance).



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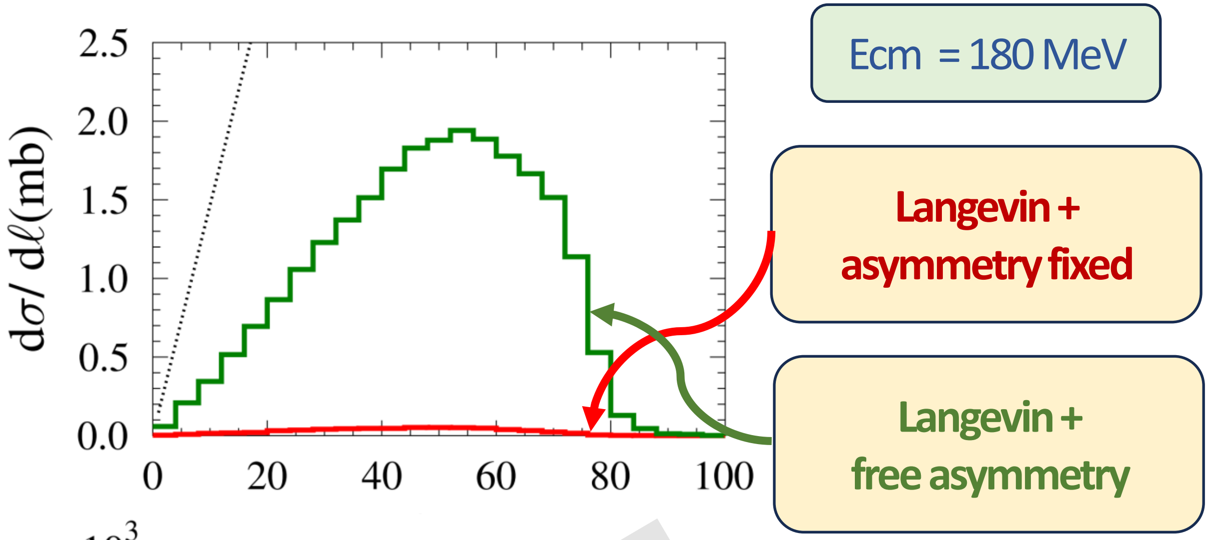
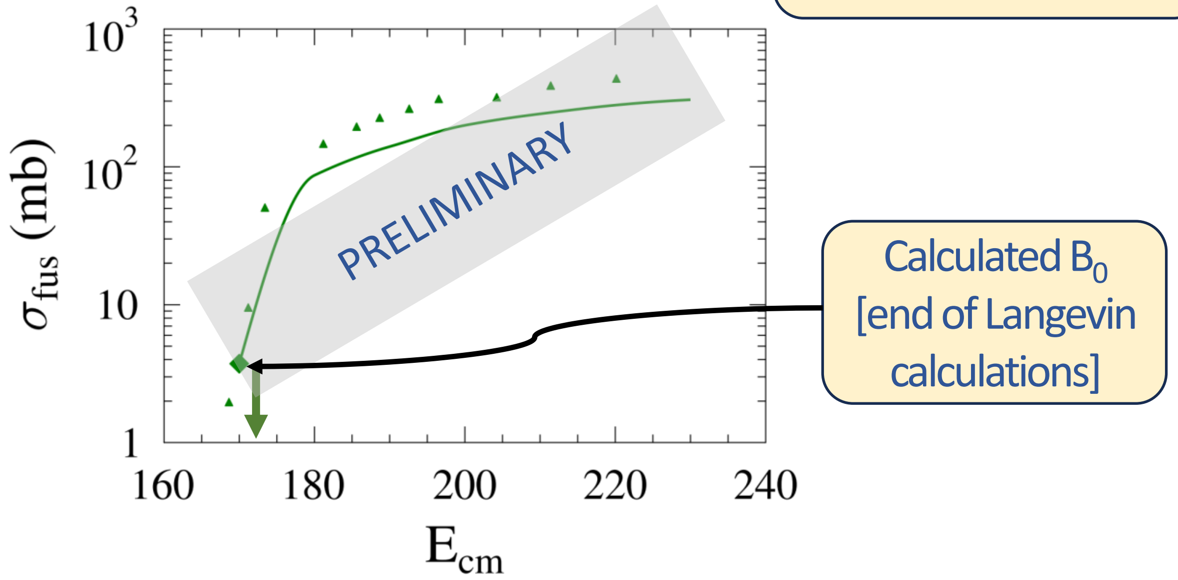
- **Preliminary excitation functions show a relative good agreement with the experimental data**[1] (considering the simple macroscopic potential).
- **Upcoming calculations involving heavier systems (^{50}Ti, ^{54}Cr) to study the hindrance mechanism.**





[1] K. Banerjee *et al.*, Phy. Rev. Lett. 122 (2019) 232503

Summary and Perspectives

- We have derived a fully **6-dimensional dissipative dynamics Langevin**-based formalism to describe the **unrestricted motion of the systems** in terms of **elongation**, **neck** and **asymmetry** variables.

- Thanks to a correct treatment of the different stages of fusion, we obtained the **correct tail behavior of the spin distributions**, which will be **crucial for future fission calculations**.

- In the very near future, we will tackle the **hindrance problem** by comparing the **^{48}Ca/^{50}Ti/^{54}Cr + ^{208}Pb** systems.

- We are planning to make the following improvements of the formalism:
  - The addition of **shell effects** for a **fully microscopic-macroscopic picture**
  - The testing of different forms of stochastic noises (**color noises**), which will allow us to explore **memory effects** as the process evolves.

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Thank you for your attention!