Nuclear structure and excited states of superheavy nuclei

J. Luis Egido and Andrea Jungclaus

Universidad Autónoma de Madrid
Departamento de Física Teórica

UAM
UNIVERSIDAD AUTONOMA
DE MADRID

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Aim of the talk

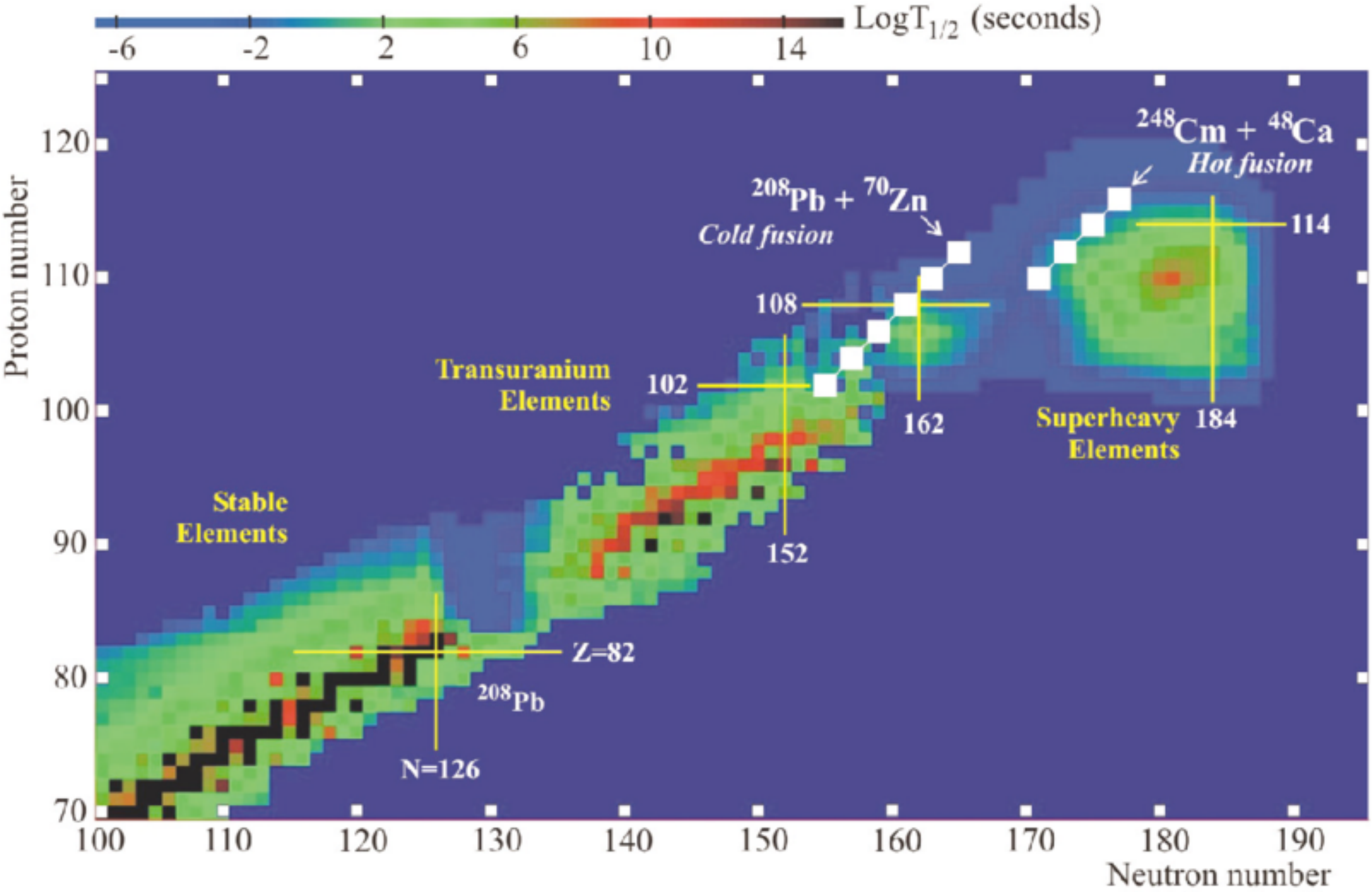
In this talk I will present the theoretical description of superheavy elements using the Finite Range density dependent Gogny interaction and sophisticated microscopical approaches namely the symmetry conserving configuration mixing theories.

Outline of the talk:

0.- **Introduction and Motivation and a short description of the theory**

1.- **Ground state deformations & shape coexistence in the Flerovium isotopes**.

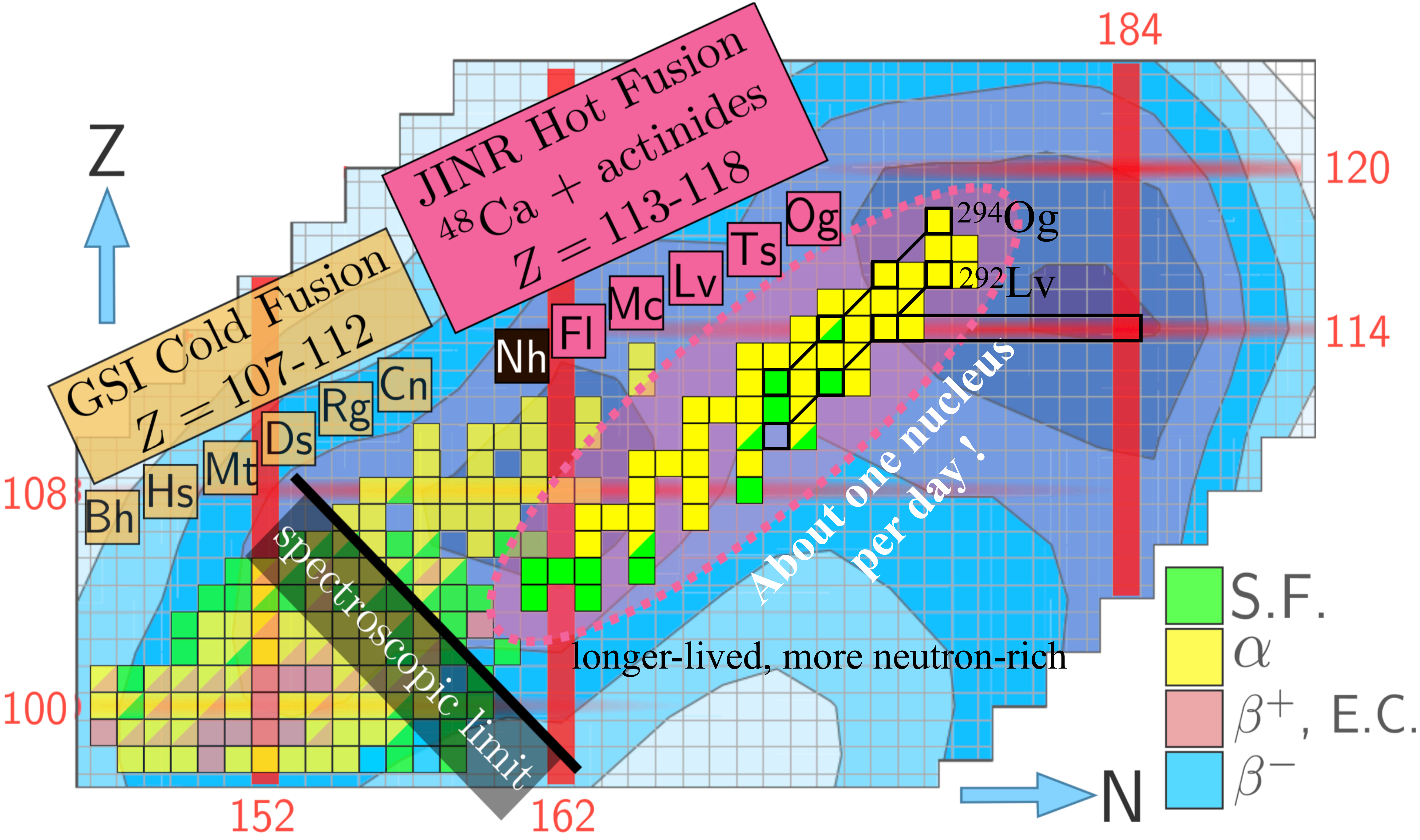
2.- **Low-Energy excited states in ^{282}Cn and along the alpha decay chain of ^{289}Fl.**



Yuri Oganessian

Pure Appl. Chem., Vol. 78, No. 5, pp. 889–904, 2006

The upper end of the chart of nuclides



^{208}Pb/^{209}Bi targets

A. Samark-Roth, PhD thesis, Lund University, 2021

Characteristics of this region of the nuclear chart

1.- Major shell closures are not well defined

2.- Coexisting prolate, oblate and triaxial minima

4.- Soft potential energy surfaces

We therefore need a theory which

1.- Explicitly considers triaxial shapes

2.- Allows for shape mixing

3.- Preserves particle number and angular momentum symmetries.

These features, together with the recently measured excited states in several SHN, is a motivation to study these interesting region of the nuclear chart.

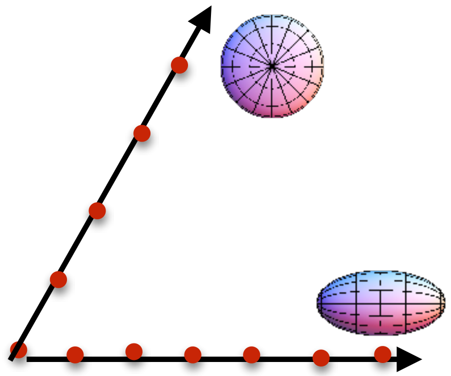
Overview of calculations in SHN

Macro-Micro (see, for example, the review of A. Sobiczewski & K. Pomorski Prog. Part. Nucl. Phys. **58** (2007)292-349)

Self-Consistent Theories: With Skyrme, Gogny or relativistic interactions

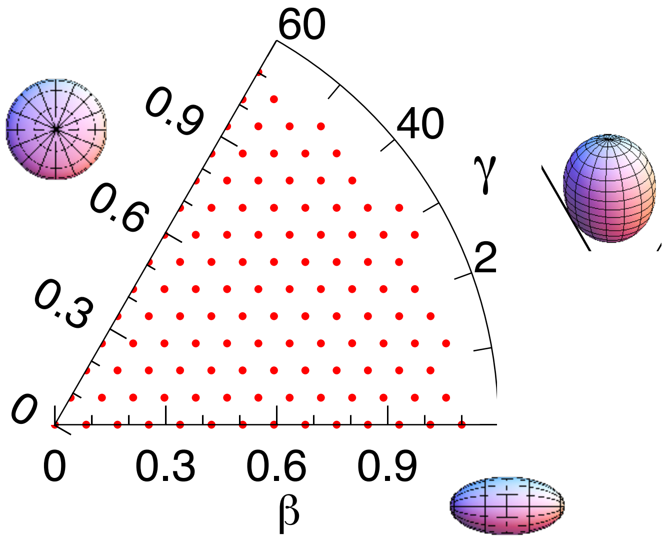
MFA
(HFB)

Axial
(Many calc.)



Triaxial
(Some calc.)

S. Cwiok, P.-H. Heenen and W. Nazarewicz
Nature 433, 705 (2005)



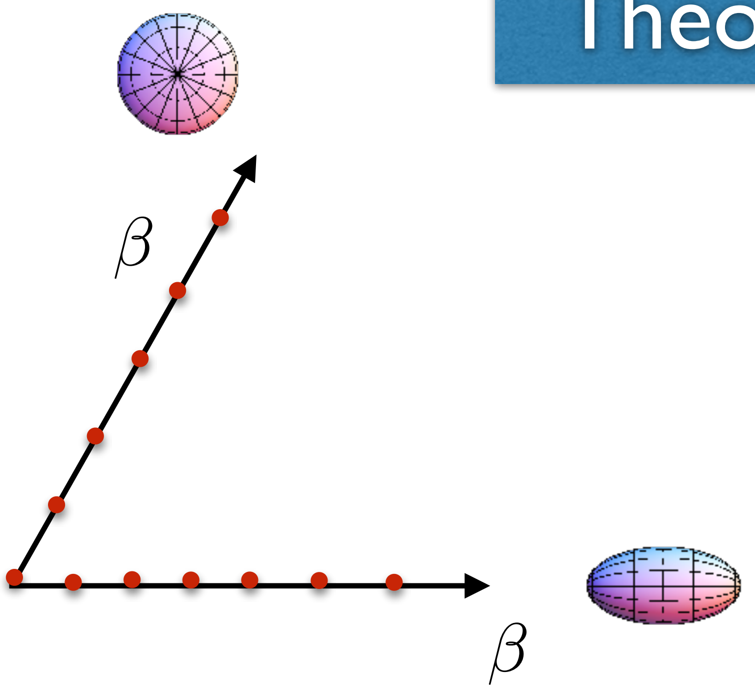
BMFA
(Shape mixing & PN+AMP)

Axial (One calc.)

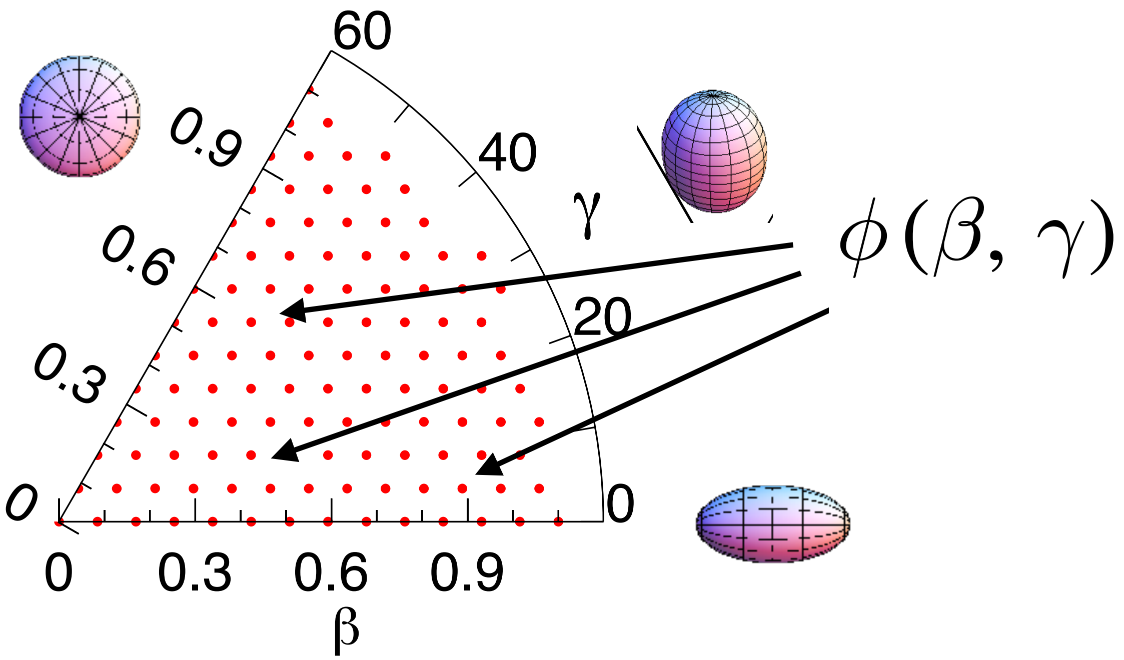
P.-H. Heenen, J. Skalski, A. Staszczak and D. Vretenar,
Nucl. Phys. A 944, 415 (2015)

Triaxial (No calc.)

Theories: Mean field and Beyond



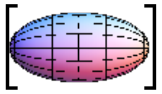
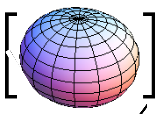
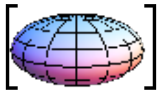
AXIAL APPROX: $\gamma = 0^{o}$ and 60^{o}



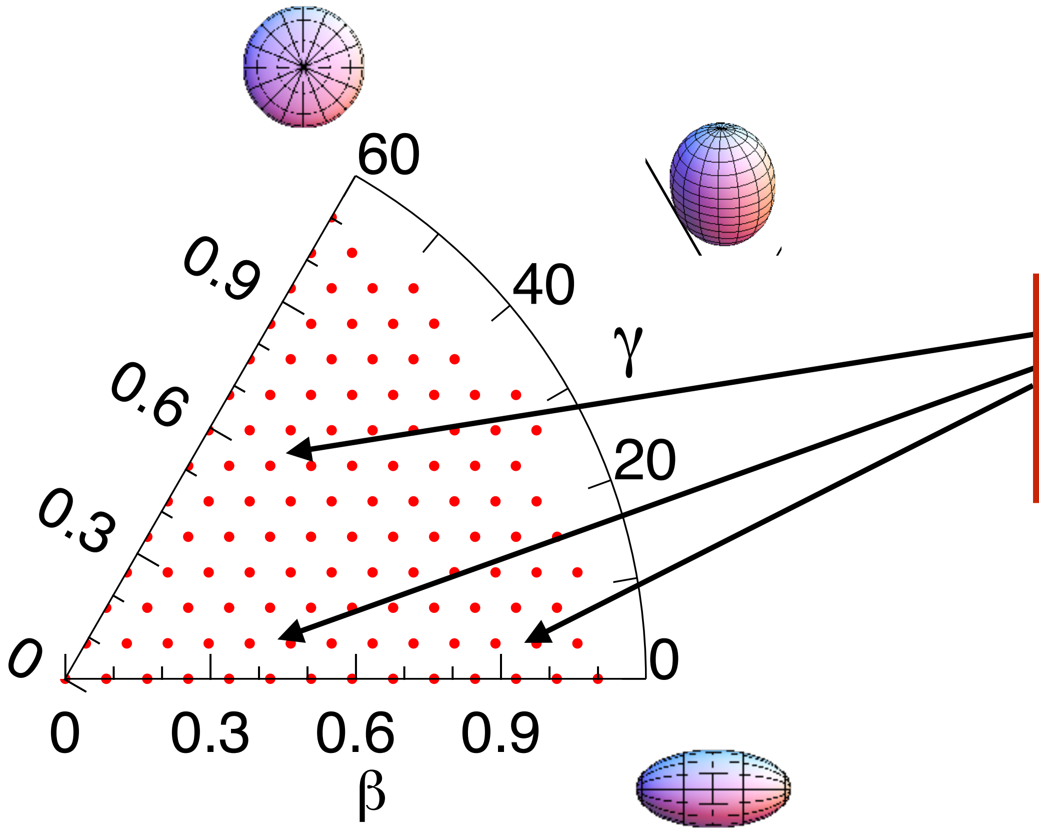
TRIAXIAL APPROX: $0^{o} \leq \gamma \leq 60^{o}$

In the symmetry-conserving configuration mixing (SCCM)

$$|\Psi^{N,I}_{M,\sigma}\rangle = \sum_{\beta,\gamma,K} f^{I}_{\sigma}(\beta, \gamma, K) P^{N} P^{I}_{MK} \, |\phi(\beta, \gamma)\rangle$$

$= e[$  $]^{I}_{M} + \ldots + f[$  $]^{I}_{M} + \ldots + g[$  $]^{I}_{M}$

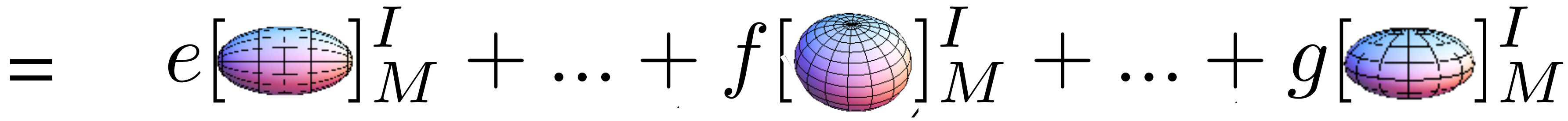
The symmetry-conserving configuration mixing (SCCM)



$\phi(\beta,\,\gamma)$ is determined in the PNVAP approach

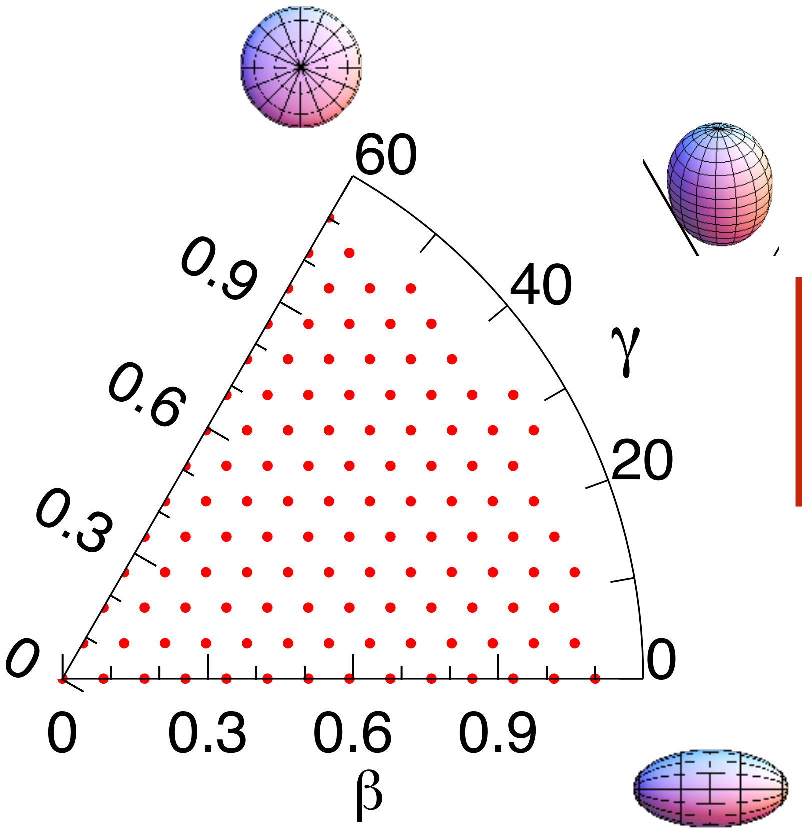
PNVAP-PES & PNAMP-PES

$$|\Psi^{N,I}_{M,\sigma}\rangle = \sum_{\beta,\gamma,K} f^{I}_{\sigma}(\beta,\,\gamma,\,K)P^{N}P^{I}_{MK}\,|\phi(\beta,\,\gamma)\rangle$$



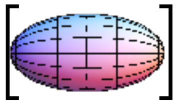
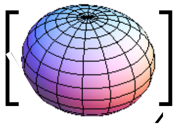
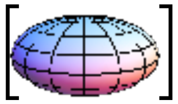
$$= e[\;]^{I}_{M} + \dots + f[\;]^{I}_{M} + \dots + g[\;]^{I}_{M}$$

The symmetry-conserving configuration mixing (SCCM)



The weights $f_{\sigma}^{I}(\beta,\gamma,K)$ are determined solving the Hill-Wheeler equation

$$|\Psi_{M,\sigma}^{N,I}\rangle = \sum_{\beta,\gamma,K} f_{\sigma}^{I}(\beta,\gamma,K) P^{N} P_{MK}^{I} |\phi(\beta,\gamma)\rangle$$

$=$ e[]$^{I}_{M}$ $+ \ldots + f$[]$^{I}_{M}$ $+ \ldots + g$[]$^{I}_{M}$

T. R. Rodriguez & J.L. E. Phys. Rev. C 81 064323 (2010)

The Gogny Interaction

J. Dechargé, D. Gogny, Phys. Rev. C 21, 1568 (1980)

In the calculations we use large configuration spaces (13 Mayor Oscillator shells, tests have been done with 17).Therefore no effective charges are needed. We use the D1S parametrisation of the Gogny force:

$$V(1,2) = \sum_{i=1}^{2} e^{-(\vec{r}_1-\vec{r}_2)^2/\mu_i^2}\,(W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau) \quad \text{central term}$$

$$+iW_0(\sigma_1+\sigma_2)\vec{k}\times\delta(\vec{r}_1-\vec{r}_2)\vec{k} \quad \text{Spin-orbit term}$$

$$+t_3(1+x_0P^\sigma)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha((\vec{r}_1+\vec{r}_2)/2) \quad \text{density-dependent term}$$

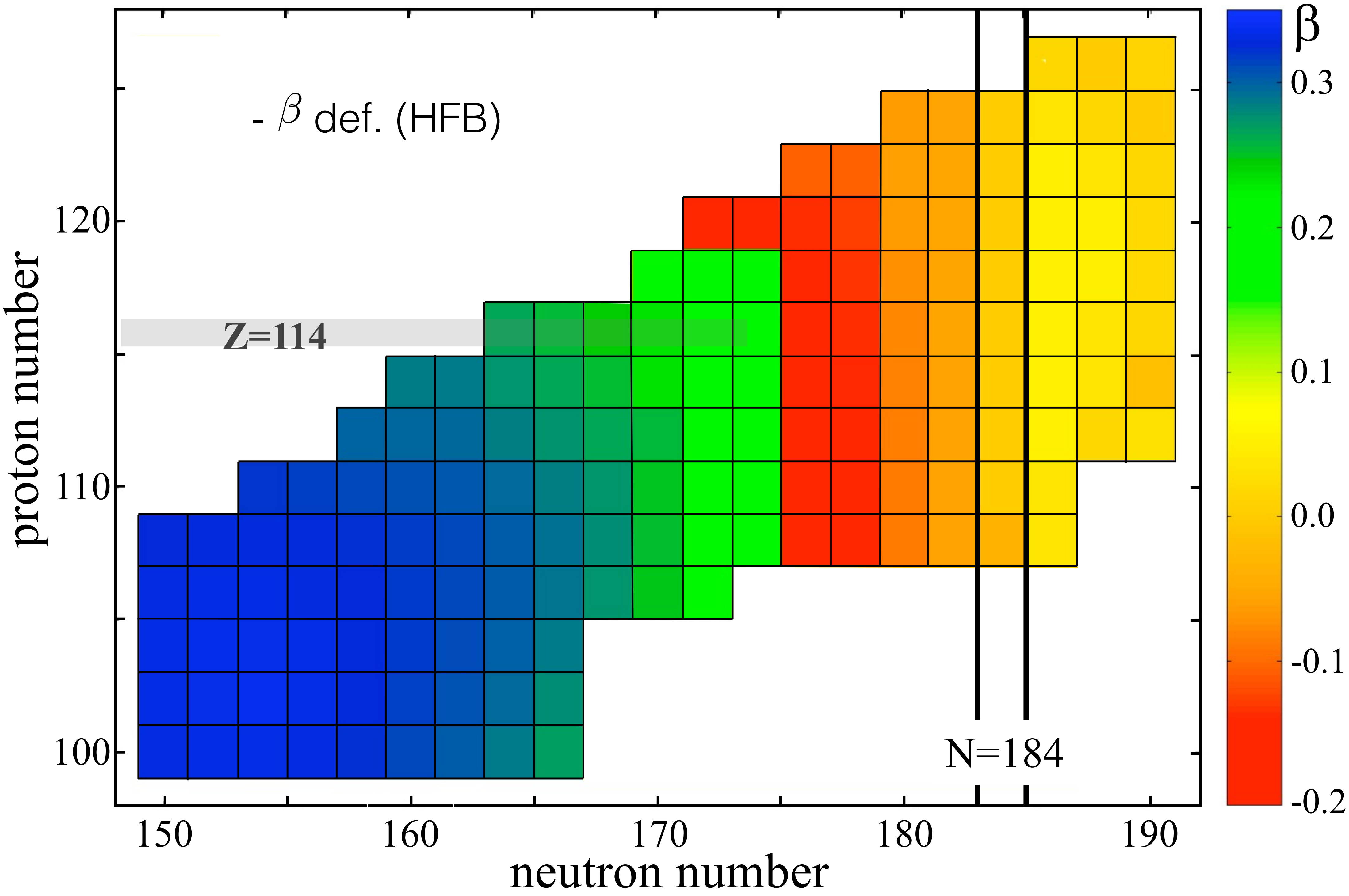
$$+V_{\text{Coulomb}}(\vec{r}_1,\vec{r}_2) \quad \text{Coulomb term}$$

D1S Parametrization (Berger et al. 1984)

i	$\mu(\text{fm})^2$	W	B	H	M
1	0,7	-1720,3	1300	-1813,53	1397,6
2	1,2	103,638	-163,48	162,81	-223,93

W_0=130 MeV fm^5
x_0=1.0, α =1/3
t_3=1390.6 MeV fm^4

Beta deformations in SHN in axial-symmetric mean-field studies



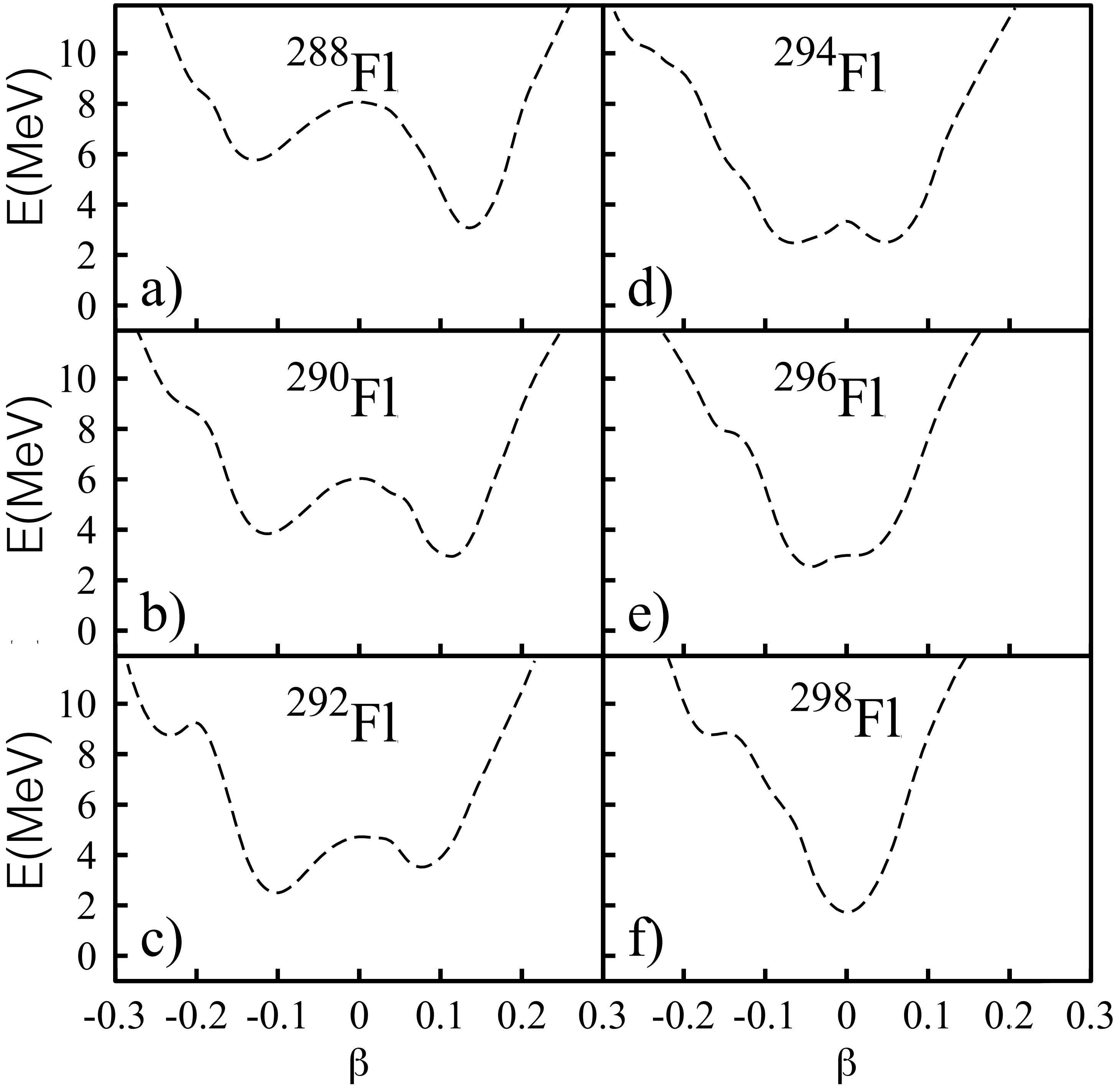
M. Warda and J.L. Egido, Phys. Rev. C 86, 014322 (2012)

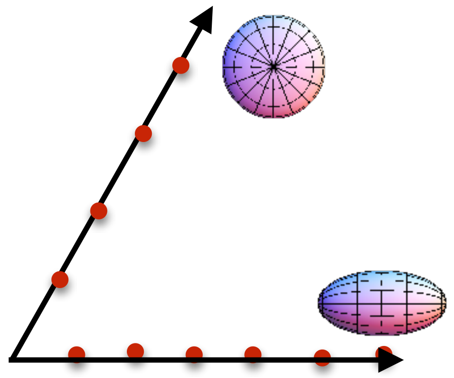
The Flerovium Isotopes

$^{288\text{-}298}\mathrm{Fl}$

The simplest approach: the axial symmetric solution in mean field

The energy versus the beta deformación

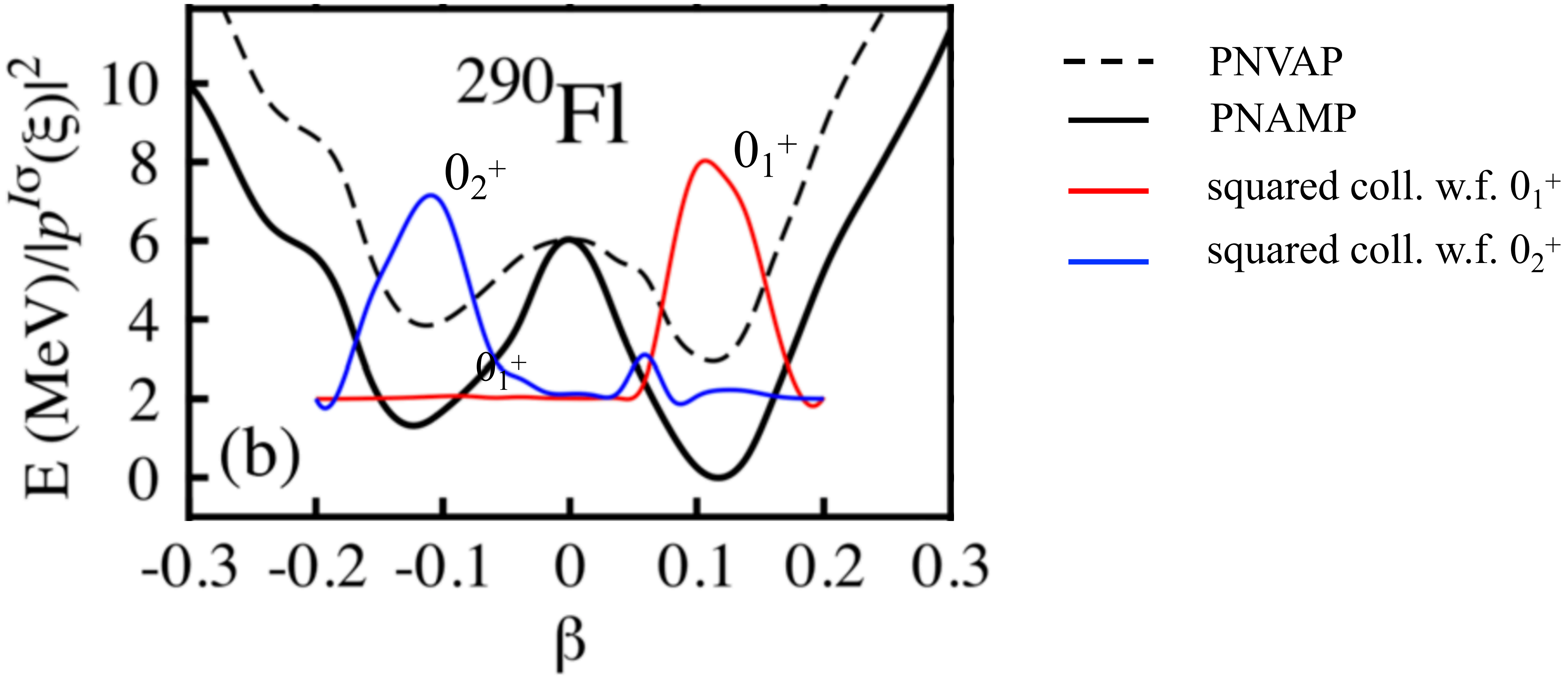




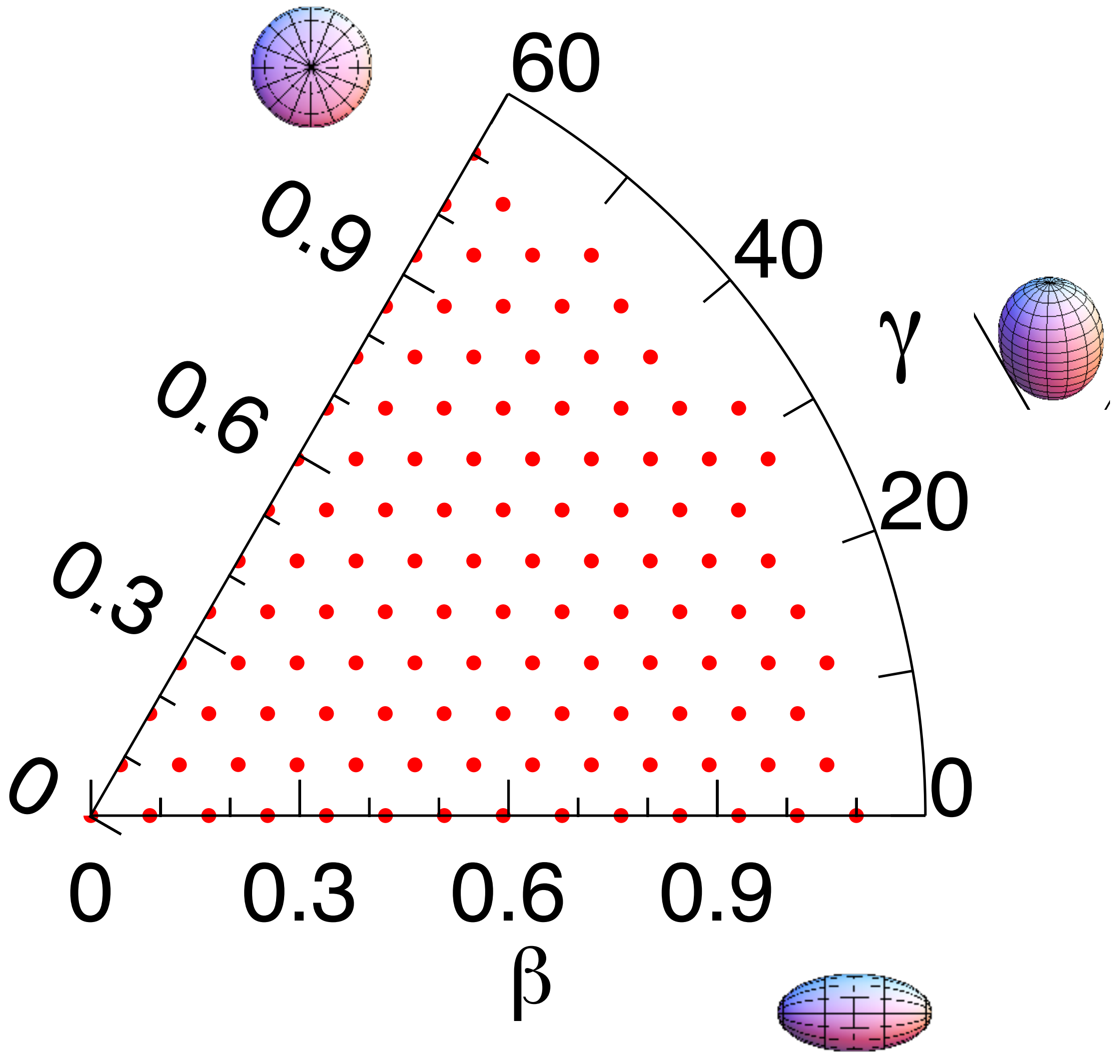
Prolate-oblate shape coexistence

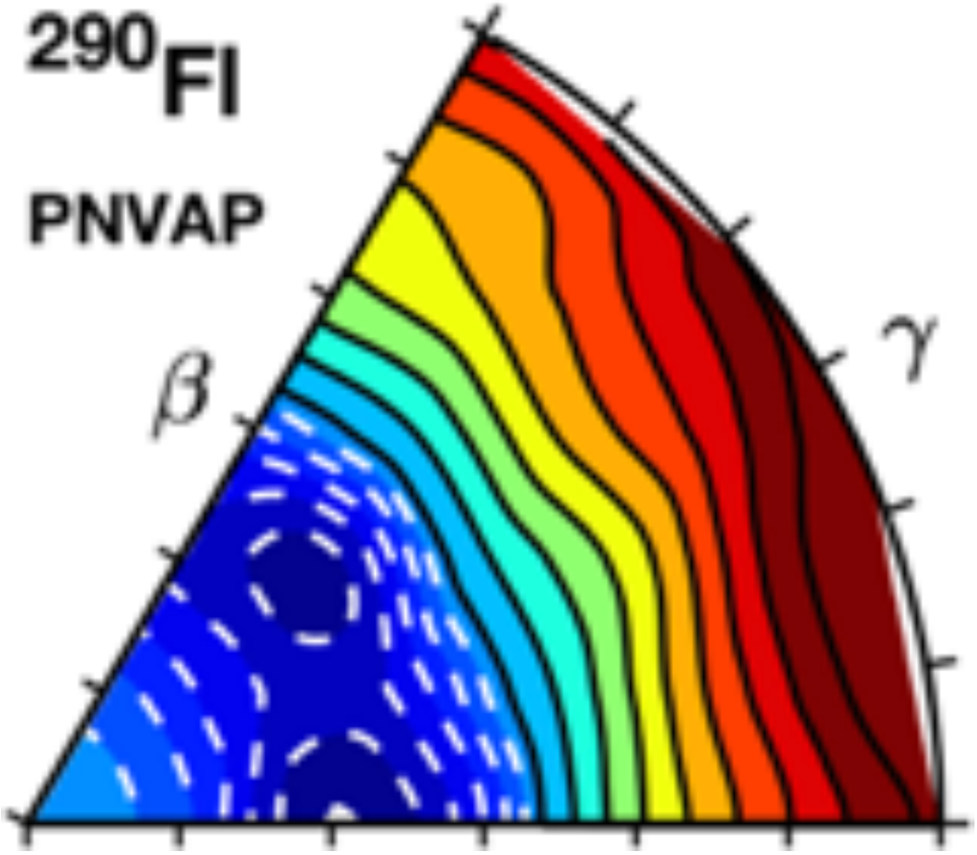
The Flerovium 290 as an example:

The full axial symmetric solution



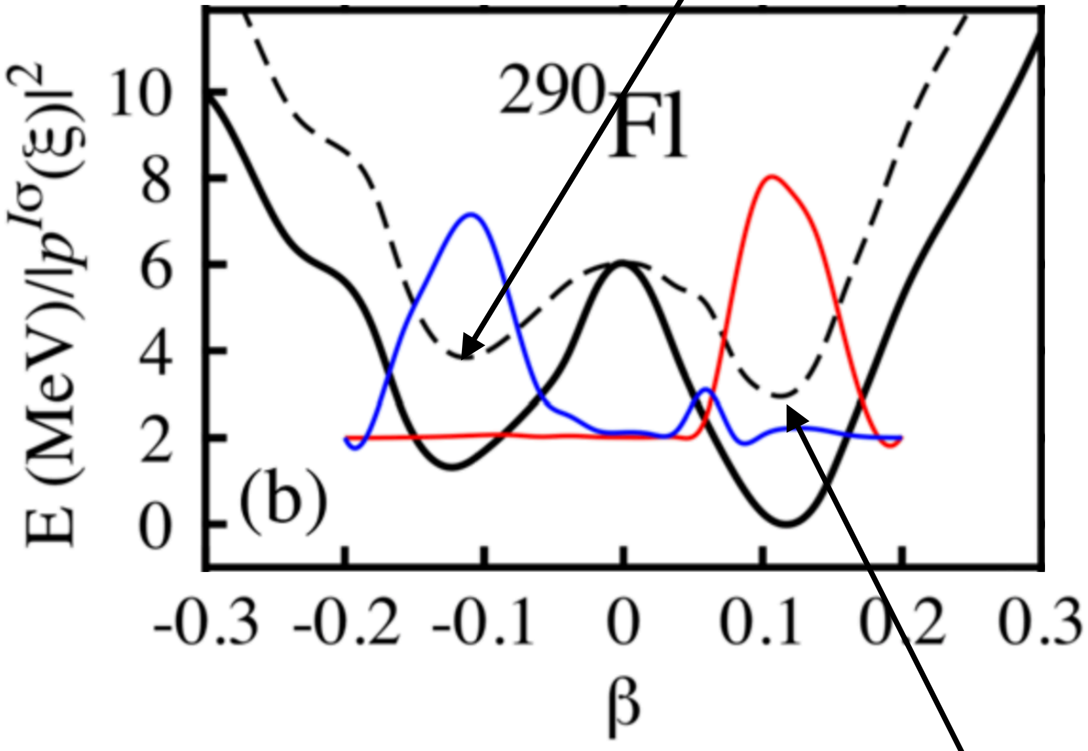
The triaxial solution: Potential energy surfaces

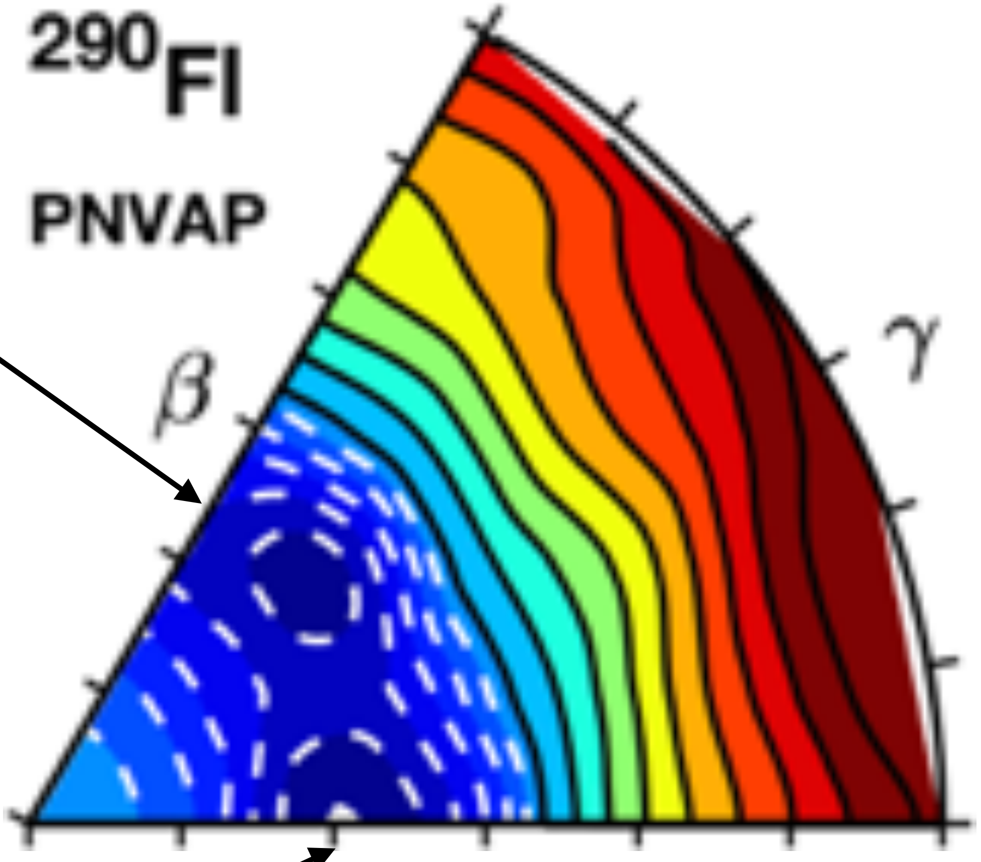




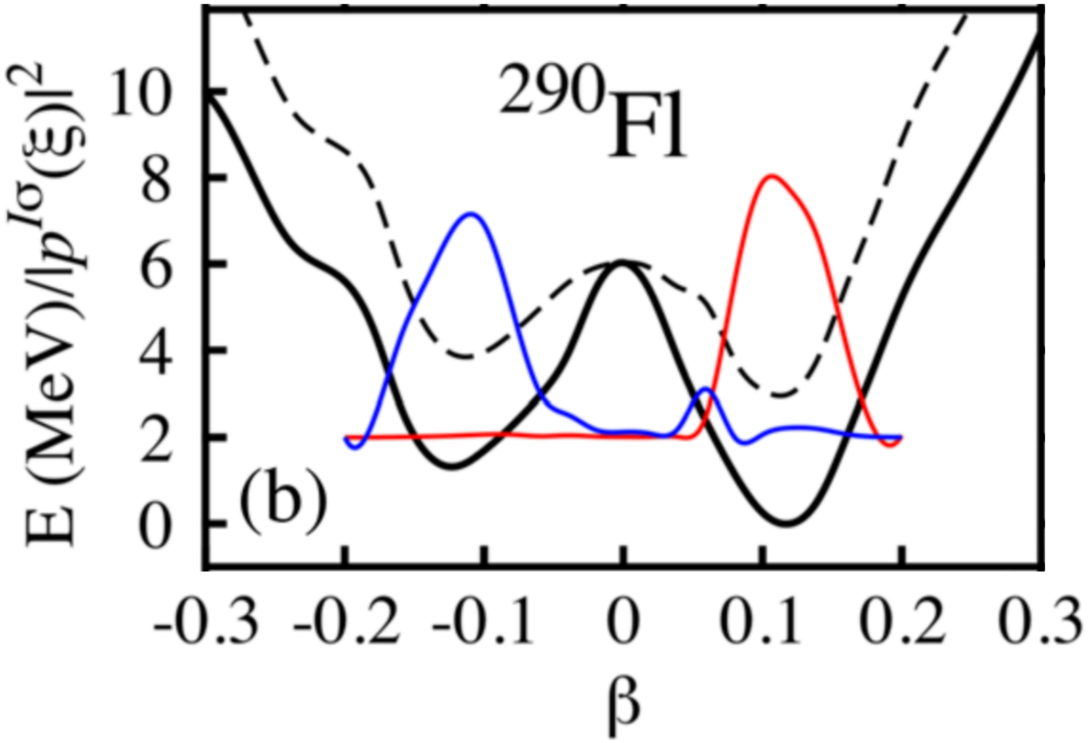
From prolate-oblate to triaxial-triaxial shape coexistence

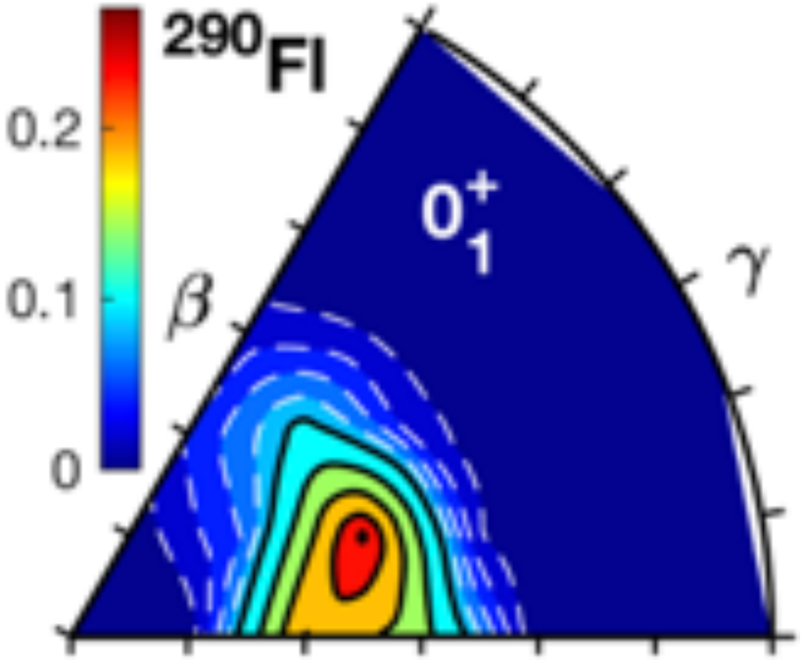
Some axial-symmetric minima are
saddle points in the β-γ plane !

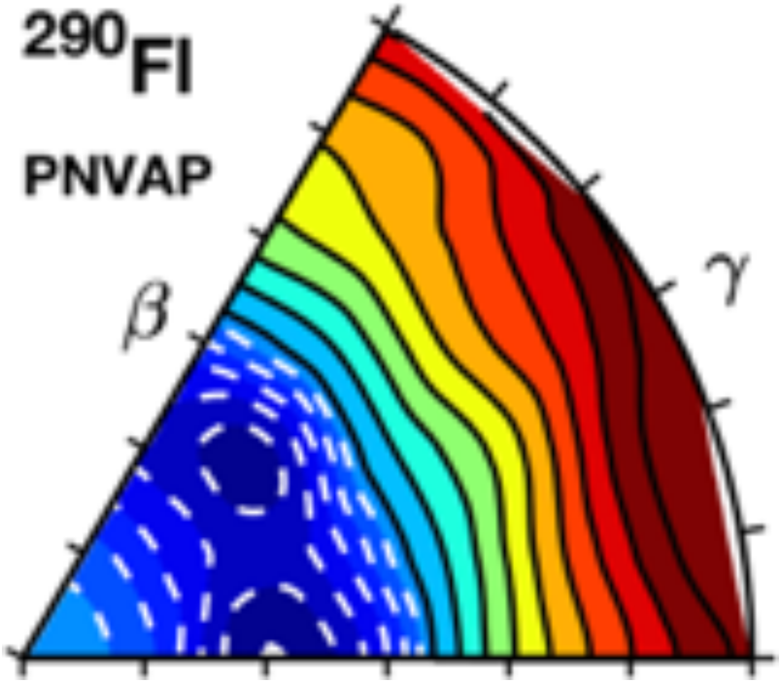


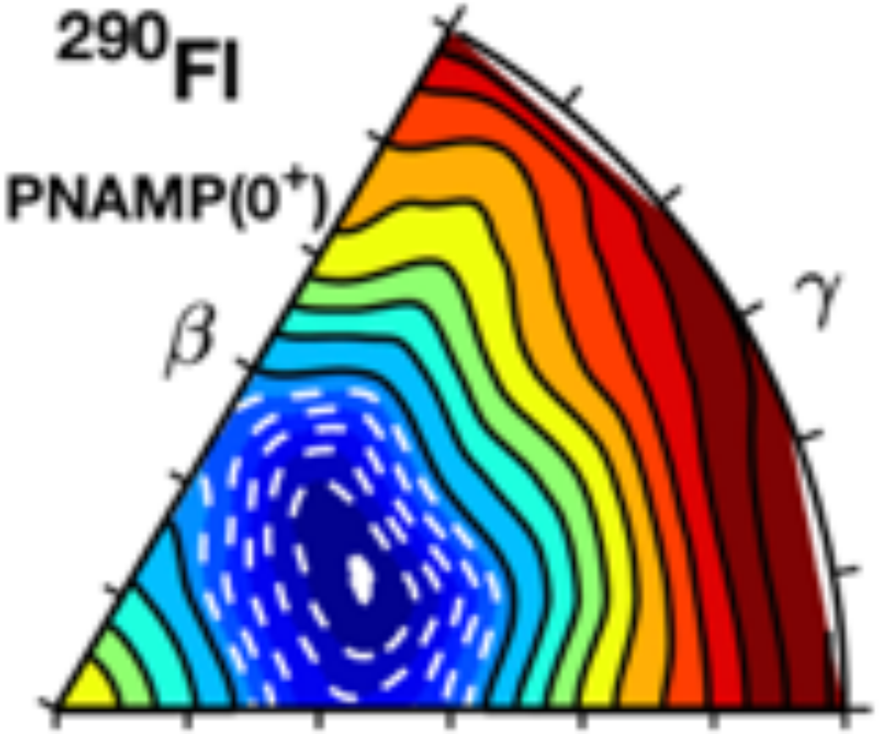


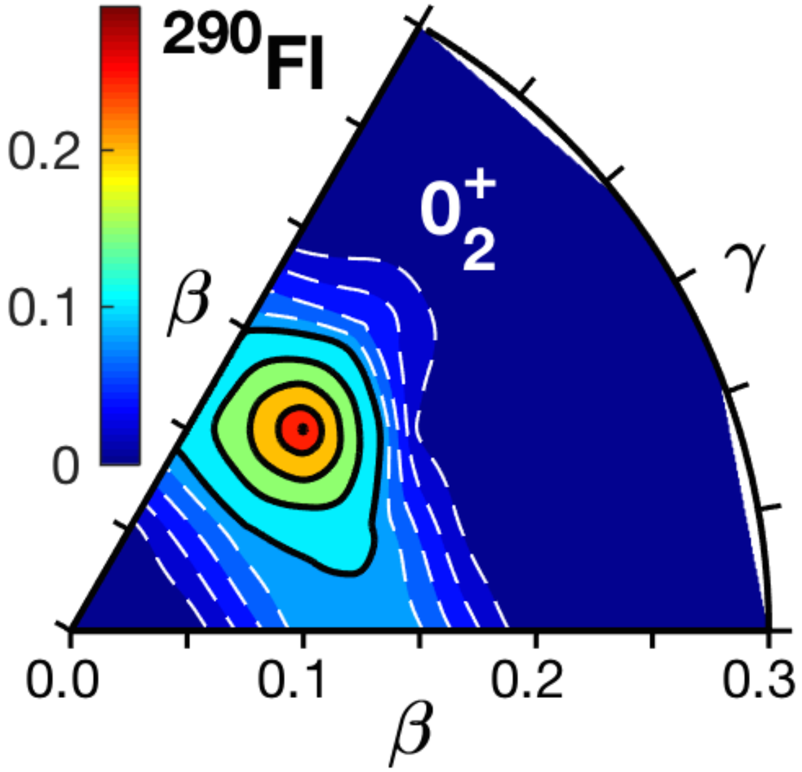
From prolate-oblate to triaxial-triaxial shape coexistence





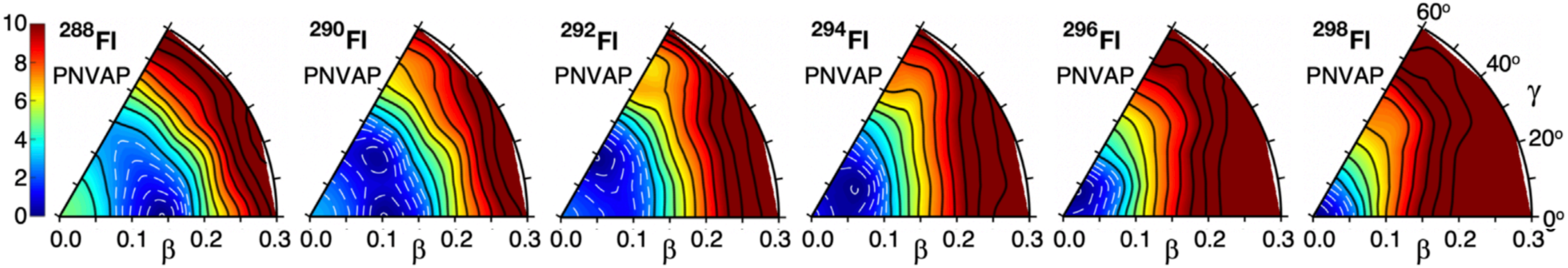






PNVAP

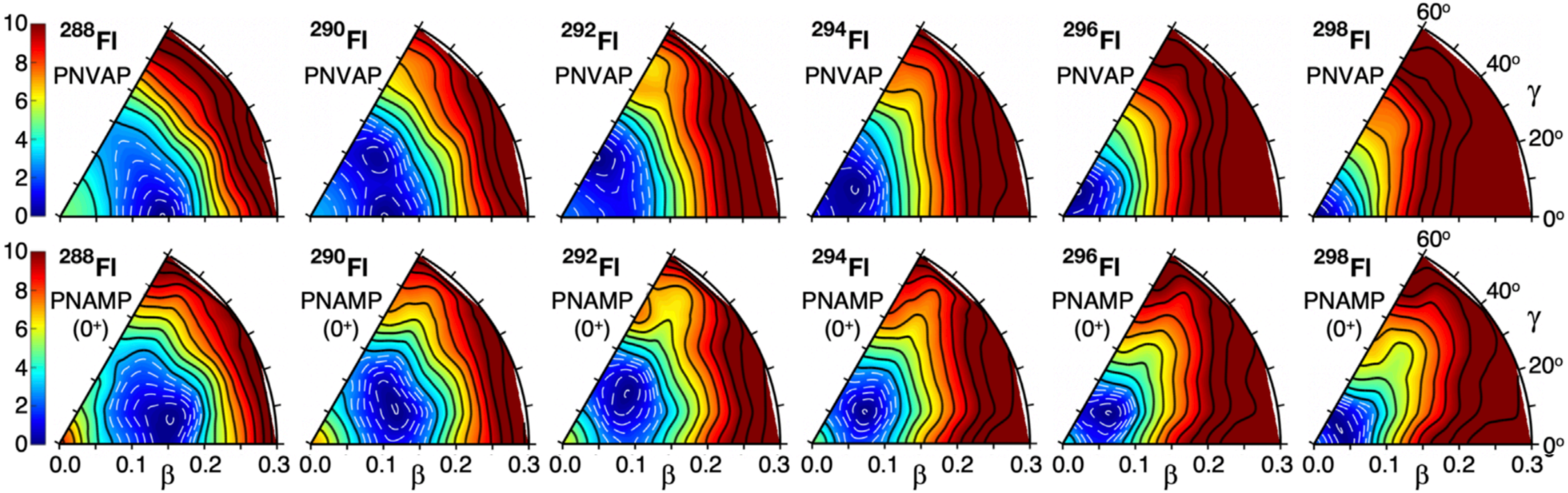
N =174 N =176 N =178 N =180 N =182 N =184



J.L. Egido and A. Jungclaus,
Phys. Rev. Lett. 125, 192504 (2020)

PNAMP

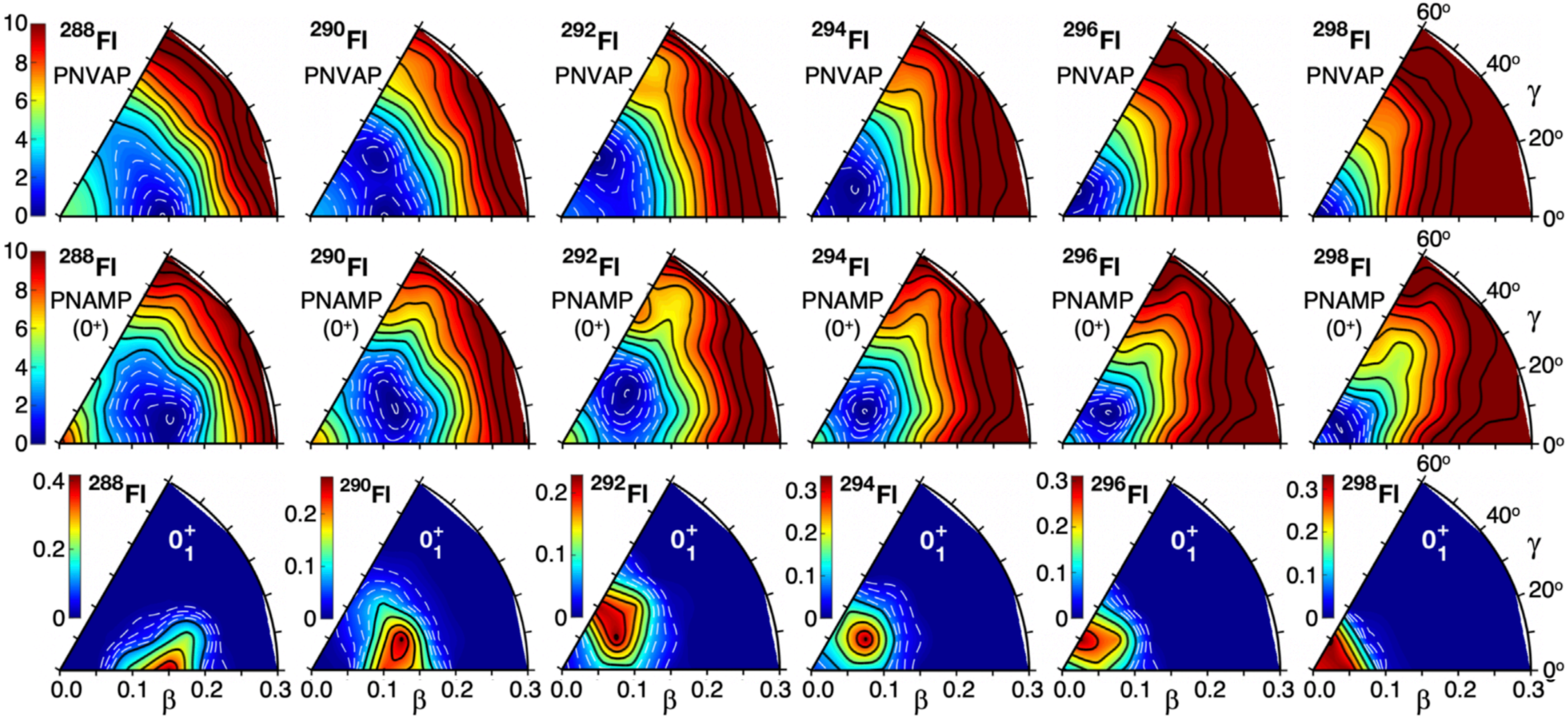
N =174 N =176 N =178 N =180 N =182 N =184



J.L. Egido and A. Jungclaus,
Phys. Rev. Lett. 125, 192504 (2020)

Collective wave functions

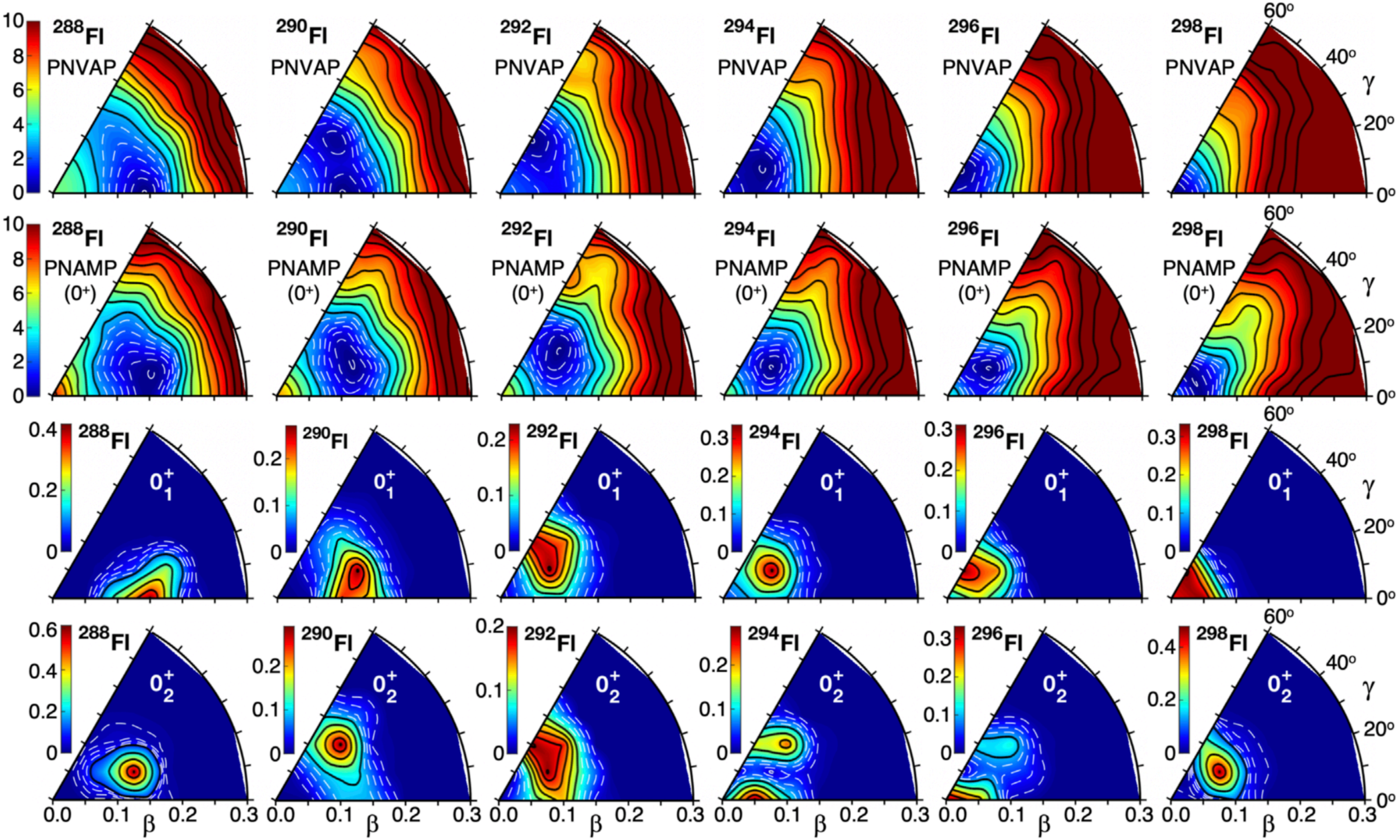
N =174 N =176 N =178 N =180 N =182 N =184



J.L. Egido and A. Jungclaus,
Phys. Rev. Lett. 125, 192504 (2020)

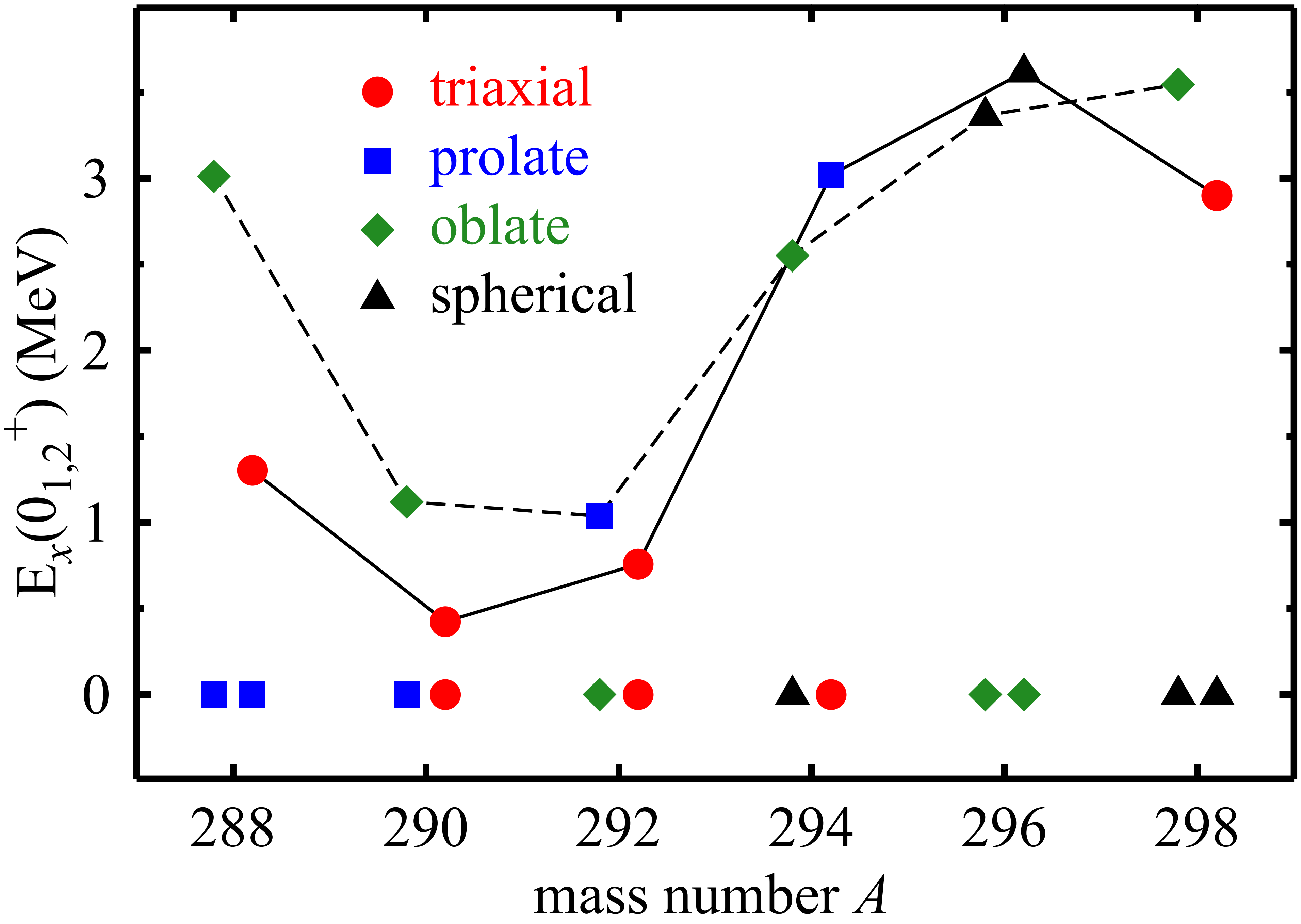
Collective wave functions

N =174 N =176 N =178 N =180 N =182 N =184

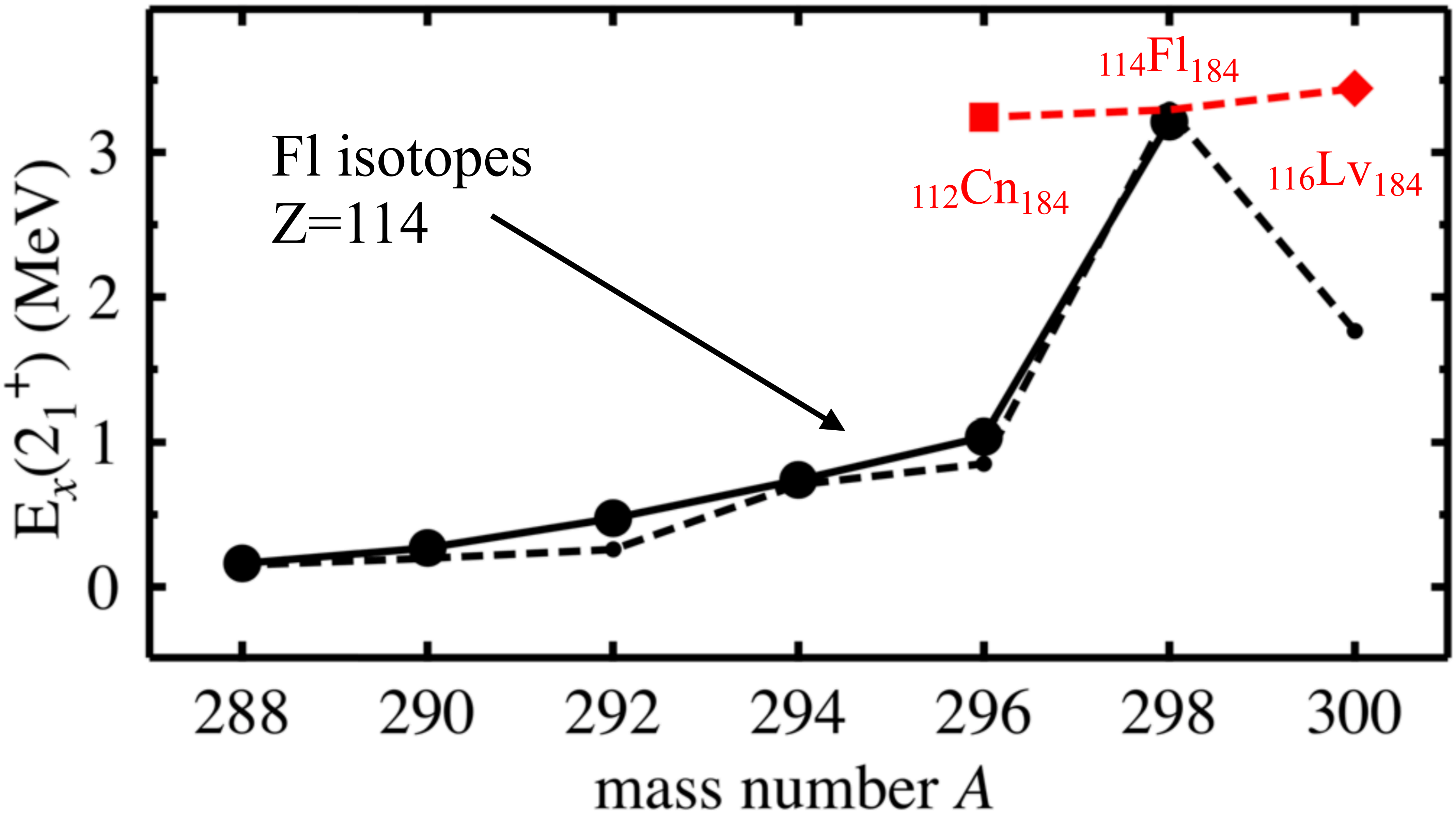


J.L. Egido and A. Jungclaus,
Phys. Rev. Lett. 125, 192504 (2020)

Excitation energies of the 0^+_2 states and their shapes (AXIAL & TRIAXIAL)

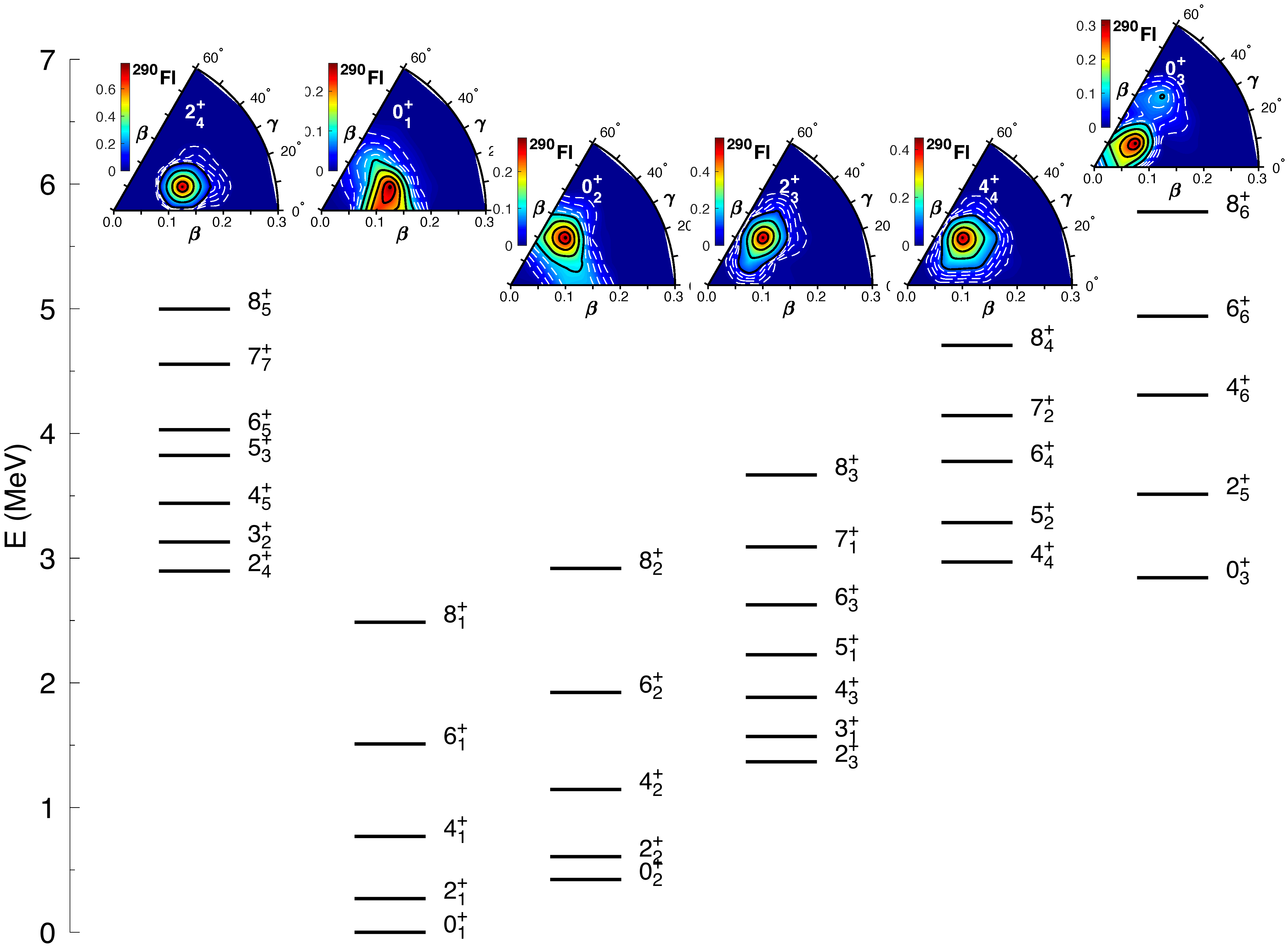


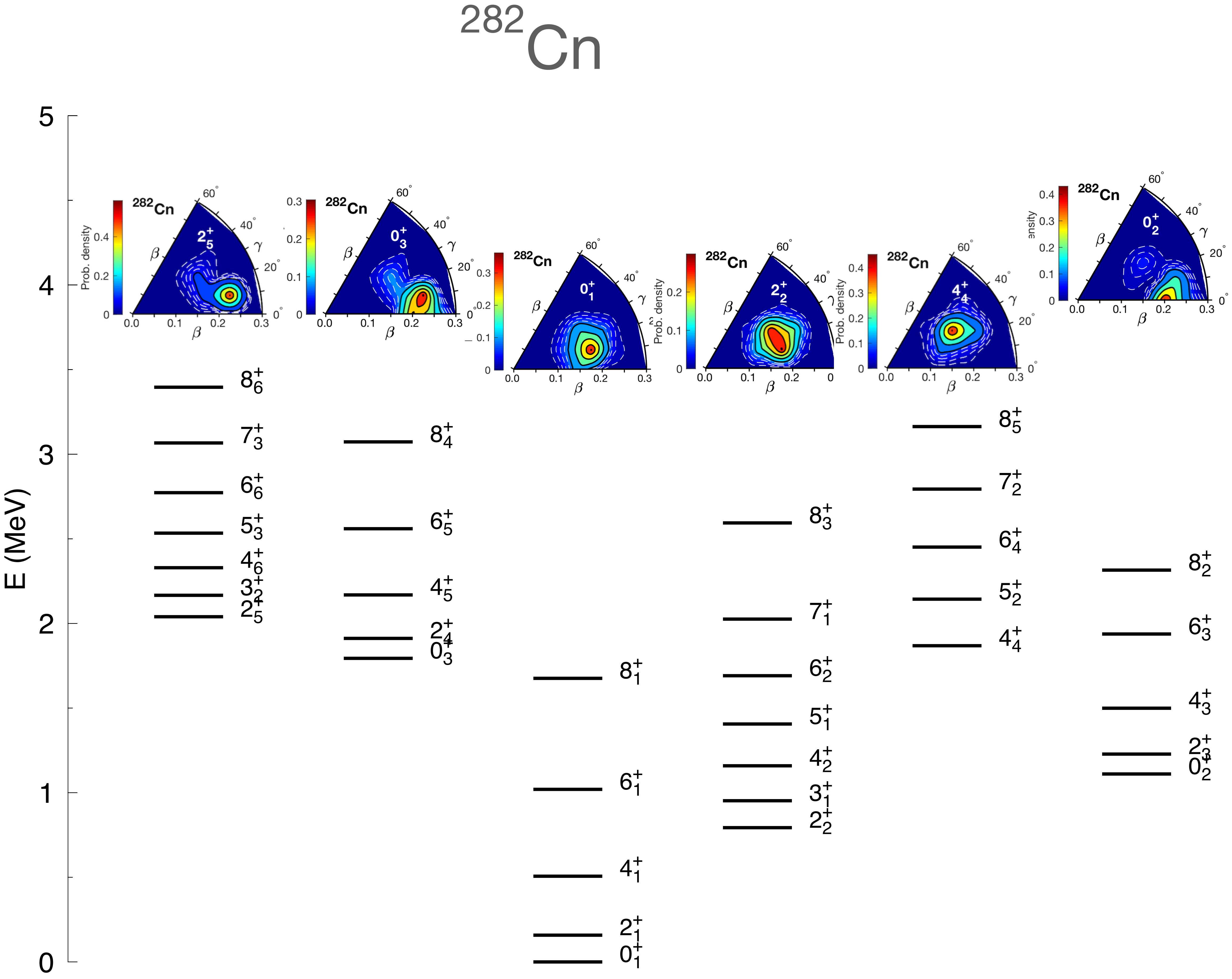
2_1^+ excitation energy as indicator for shell gaps



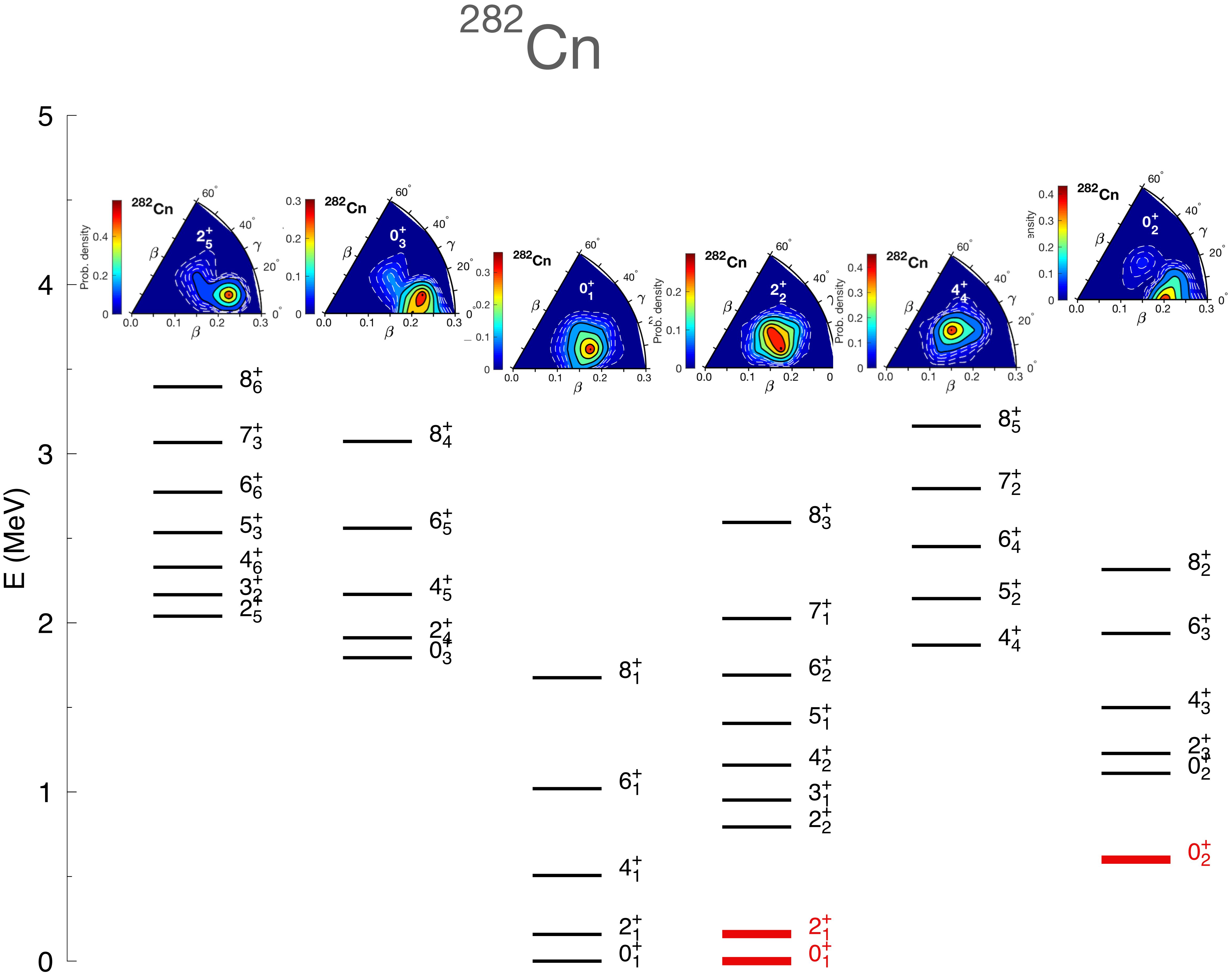
With the Gogny force also the triaxial calculations predict a shell closure at N=184, but not at Z=114 !

^{290}Fl


E (MeV)
0
1
2
3
4
5
6
7
0^+_1
2^+_1
4^+_1
6^+_1
8^+_1
0^+_2
2^+_2
4^+_2
6^+_2
8^+_2
2^+_3
3^+_1
4^+_3
5^+_1
6^+_3
7^+_1
8^+_3
4^+_4
5^+_2
6^+_4
7^+_2
8^+_4
0^+_3
2^+_5
4^+_6
6^+_6
8^+_6
2^+_4
3^+_2
4^+_5
5^+_3
6^+_5
7^+_7
8^+_5
290Fl
β
γ
0.0
0.1
0.2
0.3
0°
20°
40°
60°

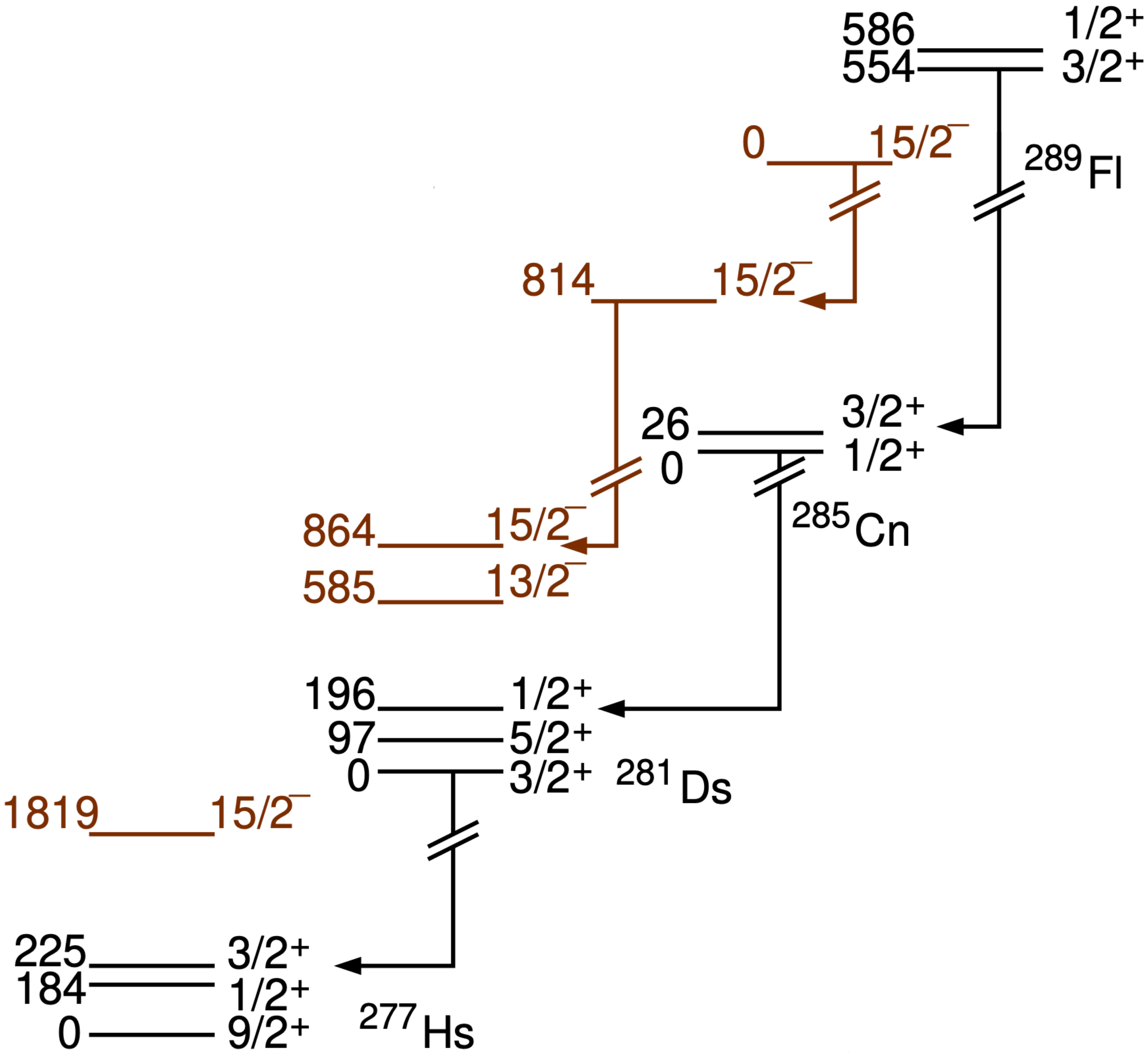


^{282}Cn
E (MeV)
0
1
2
3
4
5
8^+_6
7^+_3
6^+_6
5^+_3
4^+_6
3^+_2
2^+_5
8^+_4
6^+_5
4^+_5
2^+_4
0^+_3
8^+_1
6^+_1
4^+_1
2^+_1
0^+_1
8^+_3
7^+_1
6^+_2
5^+_1
4^+_2
3^+_1
2^+_2
8^+_5
7^+_2
6^+_4
5^+_2
4^+_4
8^+_2
6^+_3
4^+_3
2^+_3
0^+_2
^{282}Cn
Prob. density
β
γ
0.0
0.1
0.2
0.3
0°
20°
40°
60°
2^+_5
0^+_3
0^+_1
2^+_2
4^+_4
0^+_2



^{282}Cn
E (MeV)
0
1
2
3
4
5
8_6^+
7_3^+
6_6^+
5_3^+
4_6^+
3_2^+
2_5^+
8_4^+
6_5^+
4_5^+
2_4^+
0_3^+
8_1^+
6_1^+
4_1^+
2_1^+
0_1^+
8_3^+
7_1^+
6_2^+
5_1^+
4_2^+
3_1^+
2_2^+
2_1^+
0_1^+
8_5^+
7_2^+
6_4^+
5_2^+
4_4^+
8_2^+
6_3^+
4_3^+
2_3^+
0_2^+
0_2^+
^{282}Cn
2_5^+
0_3^+
0_1^+
2_2^+
4_4^+
0_2^+
Prob. density
β
γ
0°
20°
40°
60°
0.0
0.1
0.2
0.3
0.4

The alpha decay of the 289Fl chain



D.M. Cox et al., Phys. Rev. C 107, L021301(2023)

Conclusions

- State-of-the-art symmetry conserving configuration mixing calculations provide a rich variety of nuclear shapes in SHN. We predict six different ground state deformations for the six Flerovium isotopes studied at variance with axial calculations.

- We predict a new shape coexistence in ^{290}Fl. Two 0^+ triaxial states are predicted to coexist within less than 500 keV.

- We have calculated the first excited states for the decay chains of ^{282}Cn and ^{290}Fl. The predicted energies are in reasonable agreement with the experimental available values

Additional Reading:

1. A. Samark-Roth et al, Phys. Rev. Lett. 126, 032503 (2021
2. J.L.E. and A. Jungclaus, Phys. Rev. Lett 126, 032503 (2021)
3. D.M. Cox et al, Phys. Rev. C 107, L021301(2023)
4. A. Samark-Roth et al, Phys. Rev. C 107, 024301 (2023)