

The imprint of the matter-antimatter asymmetry in ν physics

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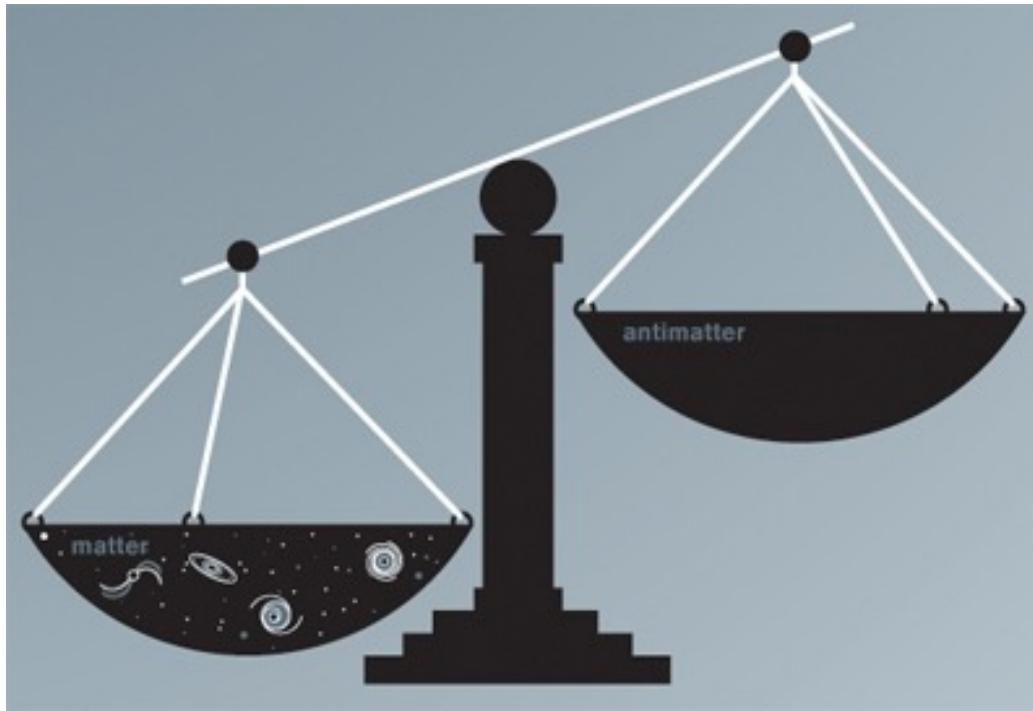
P. Hernández
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In collaboration with: J. López-Pavón, N. Rius and S. Sandner
arXiv: 2207.01651



The matter-antimatter asymmetry

The Universe seems to be made of matter



$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.21(16) \times 10^{-10}$$



Violation of CP invariance, Casymmetry, and baryon asymmetry of the universe

A. D. Sakharov

(Submitted 23 September 1966)

Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32–35 (1967) [JETP Lett. **5**, 24–27 (1967)].
Also S7, pp. 85–88]

Usp. Fiz. Nauk **161**, 61–64 (May 1991)

A. Sakharov

Three basic conditions for cosmological formation of baryonic asymmetry

- I. Absence of baryonic charge conservation.
- II. Difference between particles and antiparticles, manifesting itself in the violation of CP -invariance.
- III. Nonstationarity. Formation of BA is only possible under nonstationary conditions in the absence of local thermodynamic equilibrium.

$$n_b \sim n_{\bar{b}} \propto e^{-m_b/T}$$

The Standard Model (subtly) complies

I Baryon Number

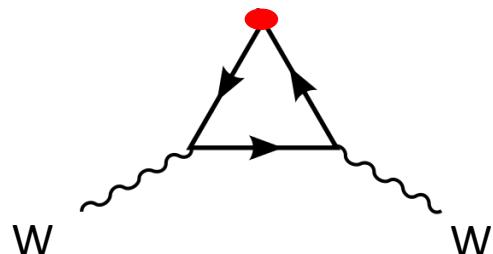
The SM Lagrangian preserves baryon number symmetry classically

$$p \not\rightarrow e^+ \pi_0$$

But the symmetry is broken by quantum vacuum effects: anomaly

t'Hooft '76, Klinkhammer, Manton '84;

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu$$



$$\text{Rate}(\mathcal{B}) \propto \begin{cases} e^{-\frac{4\pi}{\alpha_W}}, & T < T_{EW} \\ \mathcal{O}(1), & T > T_{EW} \end{cases}$$

Only B-L is conserved in the SM !

The Standard Model (subtly) complies

II CP violation

$$\mathcal{L}_{SM} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{CP\checkmark} + \underbrace{i\bar{\psi}\not{D}\psi}_{CP\checkmark} + \underbrace{\bar{\psi}_i \textcolor{red}{Y_{ij}} \phi \psi_j}_{CP \times \text{ iff } Y \neq Y^*} + \underbrace{D_\mu \phi D^\mu \phi - V(\phi)}_{CP\checkmark}$$

CP violation in the quark sector:



LHCb@ LHC

K-mesons 1964 Cronin-Fitsch

B-mesons 2001 BaBar & Belle

D-mesons 2019 LHCb

But the Standard Model fails

II CP violation

It is a subtle phenomenon that depends on many flavour parameters

$$Y_B \propto \Delta_{\text{CP}}$$

$$\Delta_{CP}^{\text{quarks}} = \left\{ \begin{array}{l} \bullet \text{ Polynomial in } Y_u, Y_d \\ \bullet \text{ Has an imaginary part} \\ \bullet \text{ It is flavour-basis invariant} \end{array} \right.$$

But the Standard Model fails

II CP violation

It is a subtle phenomenon that depends on many flavour parameters

$$Y_B \propto \Delta_{\text{CP}}$$

$$\Delta_{CP}^{\text{quarks}} = \text{Im} \left[\det \left([Y_u Y_u^\dagger, Y_d Y_d^\dagger] \right) \right] \propto J \prod_{i < j} (m_{d_i}^2 - m_{d_j}^2) \prod_{i < j} (m_{u_i}^2 - m_{u_j}^2)$$

$$J = \text{Im}[V_{ij}^* V_{ii} V_{ji}^* V_{jj}] = c_{23} s_{23} c_{12} s_{12} c_{13}^2 s_{13} \sin \delta$$

Jarlskog '85

Three non-degenerate families of up and down quarks that mix are needed for there to be CP violation !

But the Standard Model fails

Too small CP violation in quark sector

Gavela, PH, Orloff, Pene '93

$$\frac{n_b}{n_\gamma} \sim \Delta_{CP}^{quarks} \sim 10^{-20}$$

New sources of CP violation are needed !

But the Standard Model fails

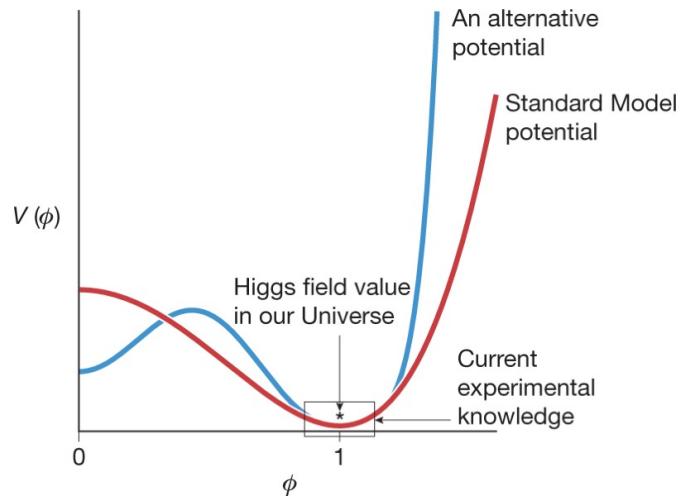
III Non-equilibrium

- First order phase transitions (EW symmetry is restored at high enough T)

It is a smooth crossover in the SM (too heavy higgs)

Kajantie, Laine, Rummukainen, Shaposhnikov '96

$$V(H) = \frac{1}{2}m_H^2 H^2 + \lambda_3 H^3 + \lambda_4 H^4$$



$$\lambda_3^{SM} = \frac{m_H^2}{\sqrt{2}v}, \lambda_4^{SM} = \frac{m_H^2}{4v^2}$$

But the Standard Model fails

III Non-equilibrium

- Expansion of the Universe when $\Gamma(T) \leq H(T)$
scattering rate < Hubble expansion

All particles in the SM (even neutrinos) satisfy

$$\Gamma_{SM}(T) \geq H(T), \quad T \geq T_{EW}$$

The Standard Model+massive ν

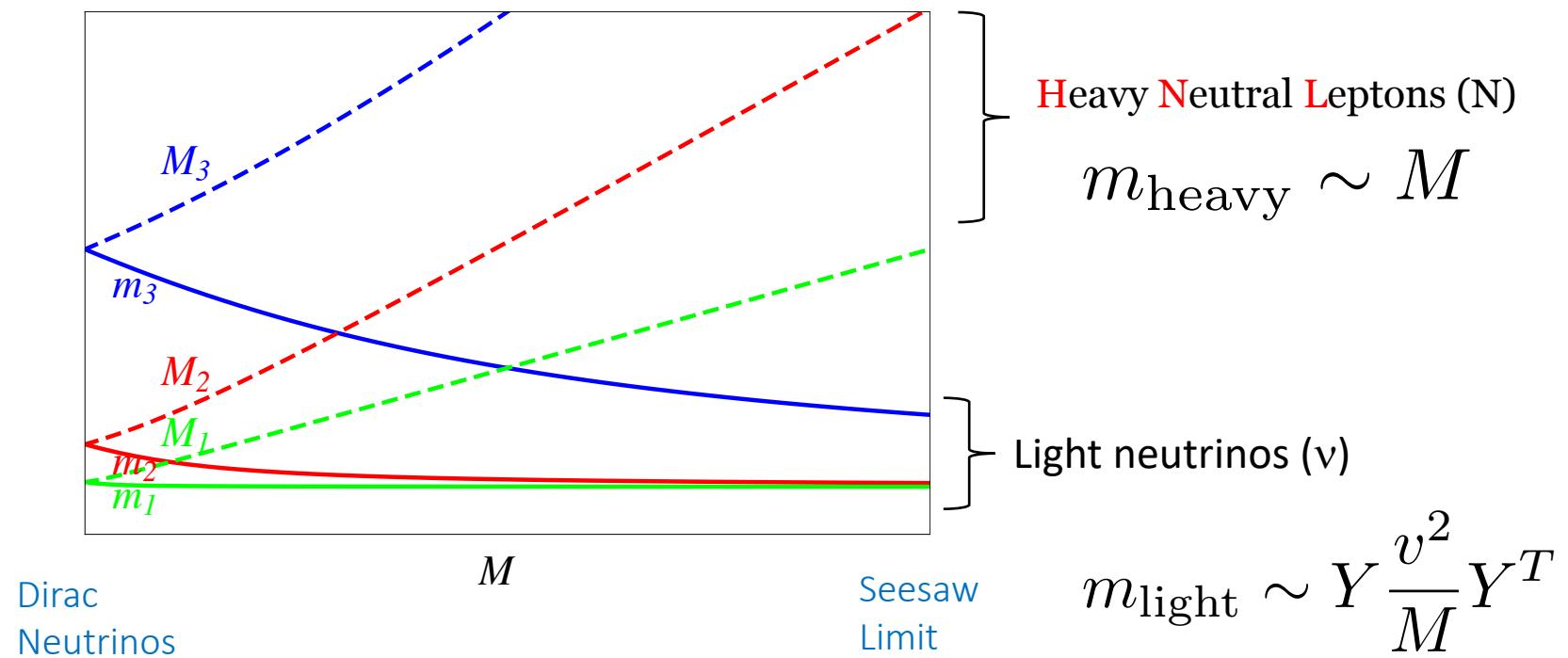
An extension of the SM table is mandatory

$(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$(\mathbf{3}, \mathbf{2})_{-\frac{1}{6}}$	$(\mathbf{1}, \mathbf{1})_{-1}$	$(\mathbf{3}, \mathbf{1})_{-\frac{2}{3}}$	$(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	$(\mathbf{1}, \mathbf{1})_0$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u_R^i	d_R^i	ν_R^1
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c_R^i	s_R^i	ν_R^2
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t_R^i	b_R^i	ν_R^3

$$\mathcal{L}_{SM} \supset \bar{\nu}_{Li} Y_{ij} H \nu_{Rj} + \bar{\nu}_{Ri} M_{ij} \nu_{Rj}^c$$

The Standard Model+massive ν

$M \neq 0 \leftrightarrow 6$ Majorana neutrinos (3 light, 3 heavy)



We can think of the heavy states as neutrino mass mediators

Fixing the light neutrino masses leaves us with a degeneracy

$$M \sim Y^2$$

The Standard Model+massive ν

New opportunities for baryogenesis!

- CP violation in the lepton sector potentially larger if $M \neq 0$: new invariants

$$\Delta_{CP}^{\text{leptons}} = \text{Im} \left\{ \text{Tr}[Y^\dagger Y M^\dagger M M^* (Y^\dagger Y)^* M] \right\}$$

Branco et al; Jenkins, Manohar; Wang, Yu Zhou...

- Heavy neutrino states might exit early/never reach thermal equilibrium at $T > T_{EW}$

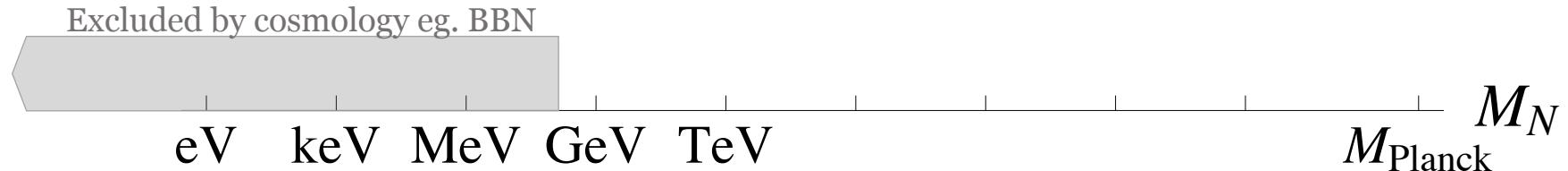
$$\Gamma_{N_i}(T) \leq H(T), T \geq T_{EW}$$

(Scattering rate < Hubble expansion rate)

The Standard Model+massive ν

Robust prediction: generation of a baryon asymmetry via **leptogenesis** for a wide range of M

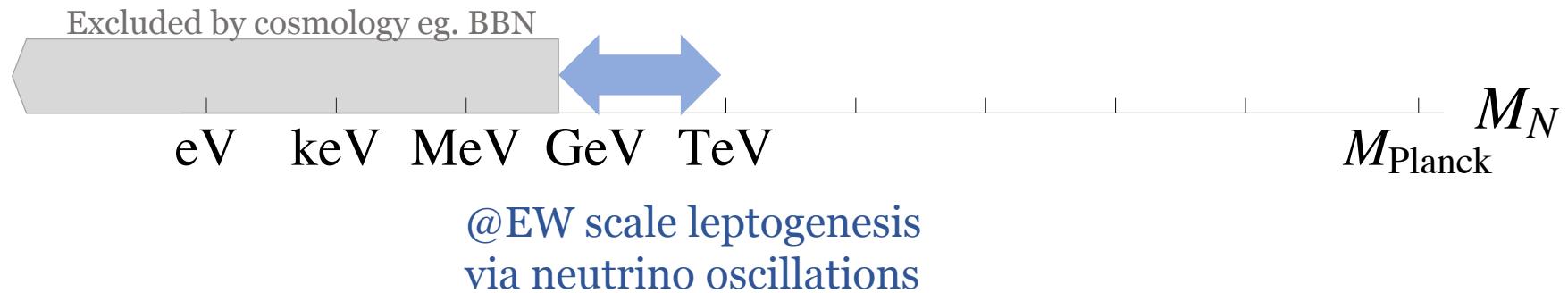
Fukugita, Yanagida; ...Abada et al;..Pilaftsis...; Akhmedov,Rubakov, Smirnov; Asaka, Shaposhnikov...



The Standard Model+massive ν

Robust prediction: generation of a baryon asymmetry via **leptogenesis** for a wide range of M

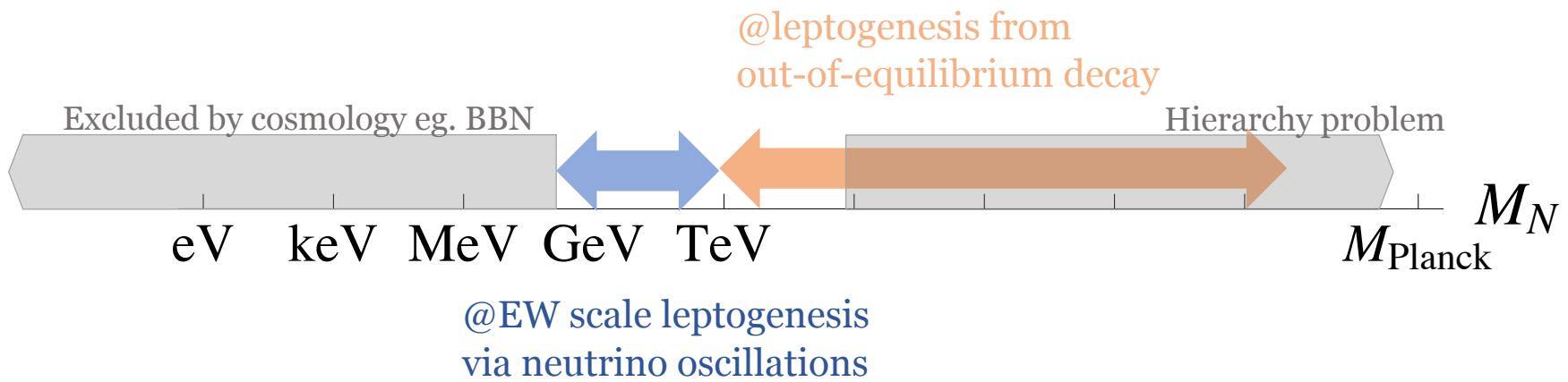
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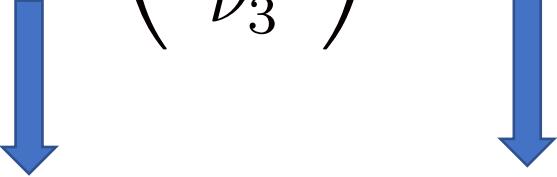
The Standard Model+massive ν

Robust prediction: generation of a baryon asymmetry via **leptogenesis** for a wide range of M

Fukugita, Yanagida; Abada et al;..Pilaftsis...; Akhmedov, Rubakov, Smirnov; Asaka, Shaposhnikov...



Heavy Neutral Leptons

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{ll} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} + U_{lh} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}$$


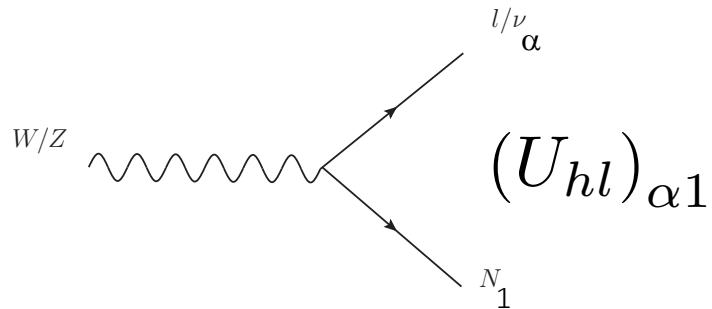
Light neutrino mixing
(~PMNS)

HNL mixing

Naïve seesaw scaling:

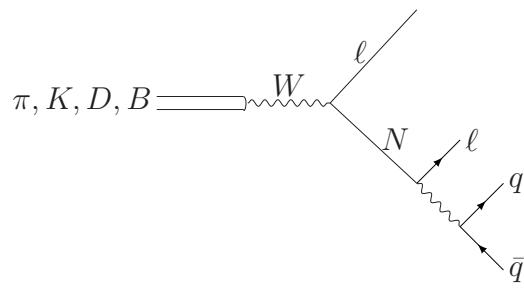
$$|U_{lh}|^2 \sim \frac{m_l}{M_N}$$

Heavy Neutral Leptons

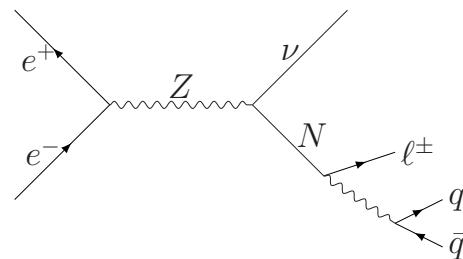


$$U^2 = \sum_{\alpha} |(U_{hl})_{\alpha 1}|^2$$

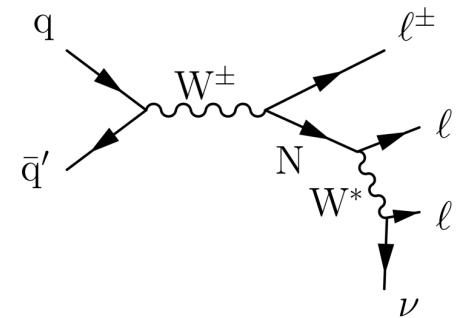
Meson decays



e+e-@Z peak

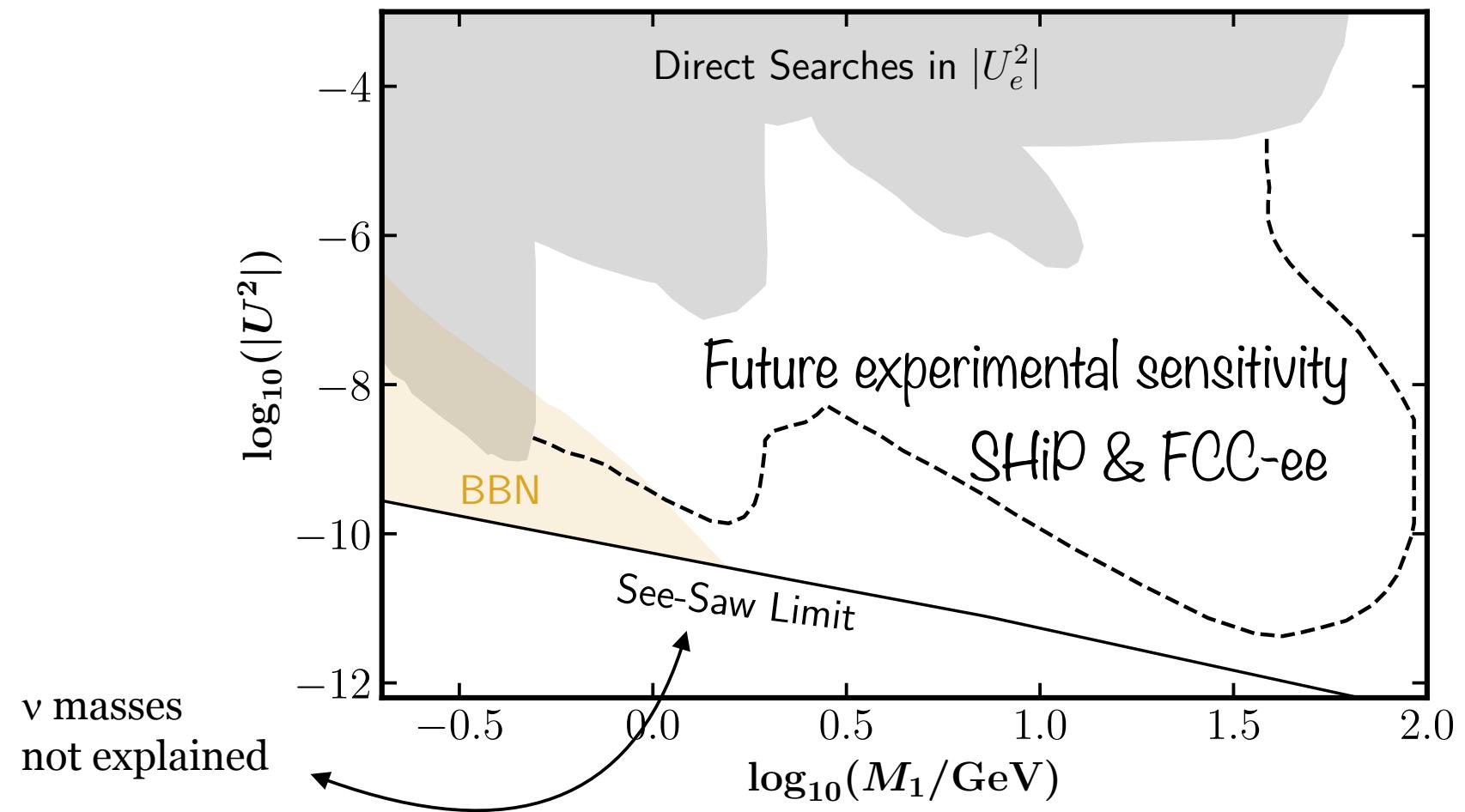


Hadron colliders



@Laboratory (fixed target, colliders) and cosmic rays

Heavy Neutral Leptons



Most of the accessible region is quite far from the seesaw limit ... |U|² ≫ $\frac{m_l}{M}$

Heavy Neutral Leptons

Strongly mixed HNL that explain neutrino masses iff **approximate** global symmetry $U(1)_L$

Wyler, Wolfenstein; Mohapatra, Valle; Branco, Grimus, Lavoura, Malinsky, Romao; Kersten, Smirnov;
Abada et al; Gavela et al....many others

Minimal Model two neutrinos:

$$L(N_1) = +1, \quad L(N_2) = -1$$

$$M = \begin{pmatrix} 0 & \Lambda \\ \Lambda & 0 \end{pmatrix} \quad Y = \begin{pmatrix} y_e & 0 \\ y_\mu & 0 \\ y_\tau & 0 \end{pmatrix}$$

Degenerate heavy neutrinos, massless light neutrinos, no CP violation...

Heavy Neutral Leptons

Strongly mixed HNL that explain neutrino masses iff **approximate** global symmetry $U(1)_L$

Wyler, Wolfenstein; Mohapatra, Valle; Branco, Grimus, Lavoura, Malinsky, Romao; Kersten, Smirnov;
Abada et al; Gavela et al....many others

$$M = \begin{pmatrix} \mu_1 & \Lambda \\ \Lambda & \mu_2 \end{pmatrix} \quad Y = \begin{pmatrix} y_e & y'_e e^{i\beta_e} \\ y_\mu & y'_\mu e^{i\beta_\mu} \\ y_\tau & y'_\tau e^{i\beta_\tau} \end{pmatrix}$$

Expansion parameters $y' \ll y, \mu \ll \Lambda, \Delta M \propto \mu$

Neutrino masses:

$$(m_\nu)_{\alpha\beta} \propto \frac{v^2}{\Lambda} (y_\alpha y'_\beta + y_\beta y'_\alpha - y_\alpha y_\beta \frac{\mu_2}{\Lambda})$$

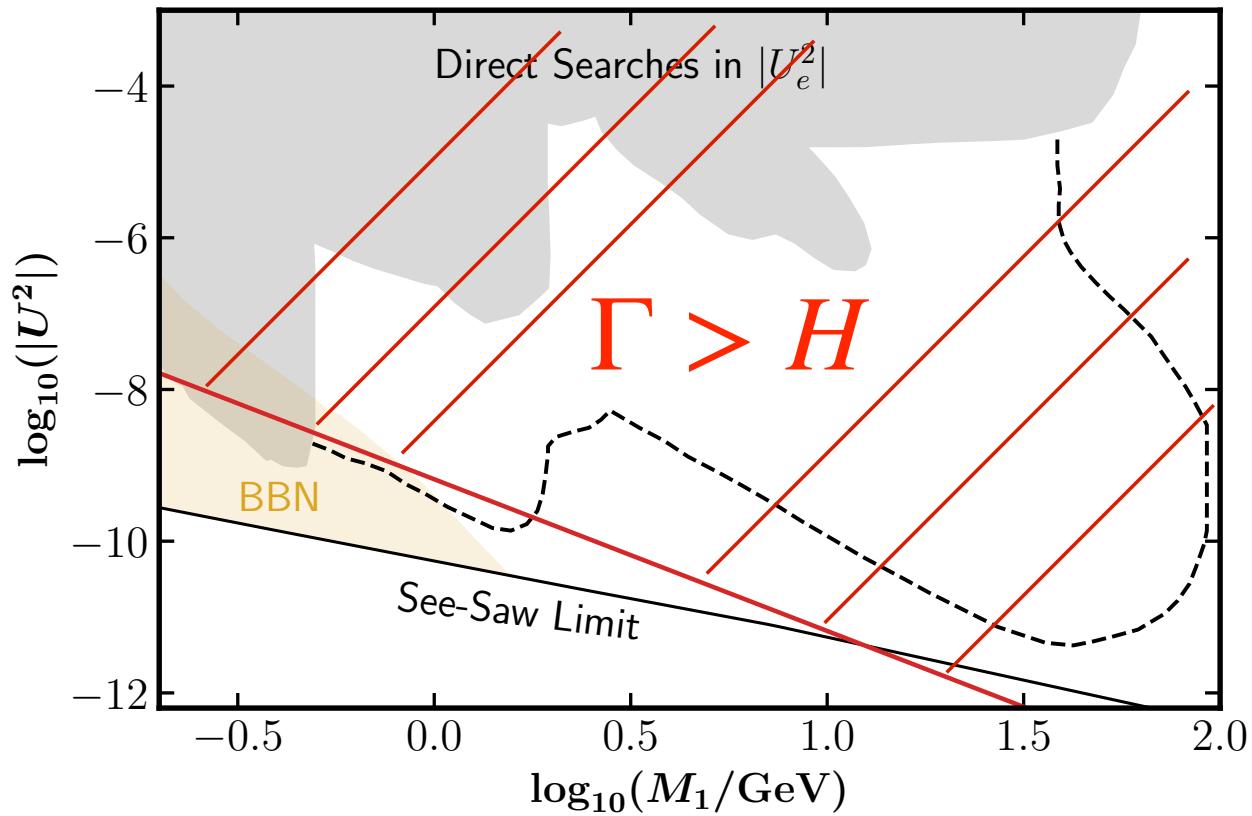
HNL Mixing:

$$U^2 \sim \frac{y^2 v^2}{2M^2}$$

Non thermal equilibration?

$\Gamma(T_{\text{EW}}) \leq H(T_{\text{EW}})$ is required

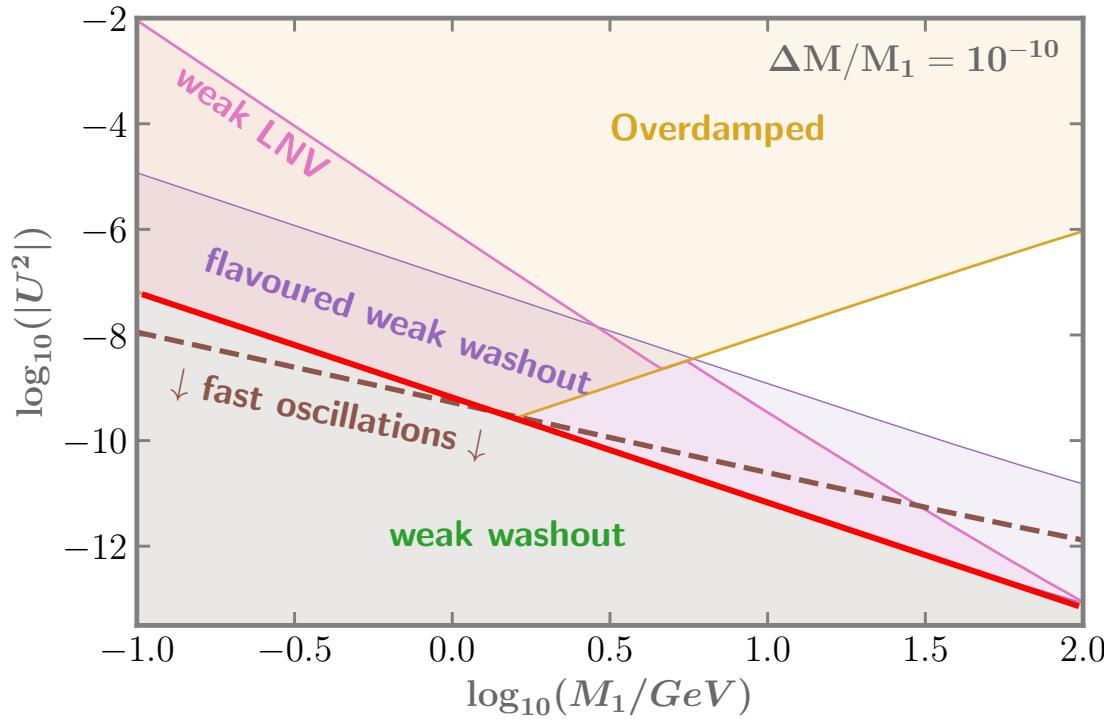
$$\Gamma \propto \text{Tr}[Y^\dagger Y] T$$



BUT approximate LN symmetry and flavour effects can lead to other slow modes

Highly-degenerate regime

Shaded regions have slow thermalizing modes



Overdamped: $\Delta M \ll M$

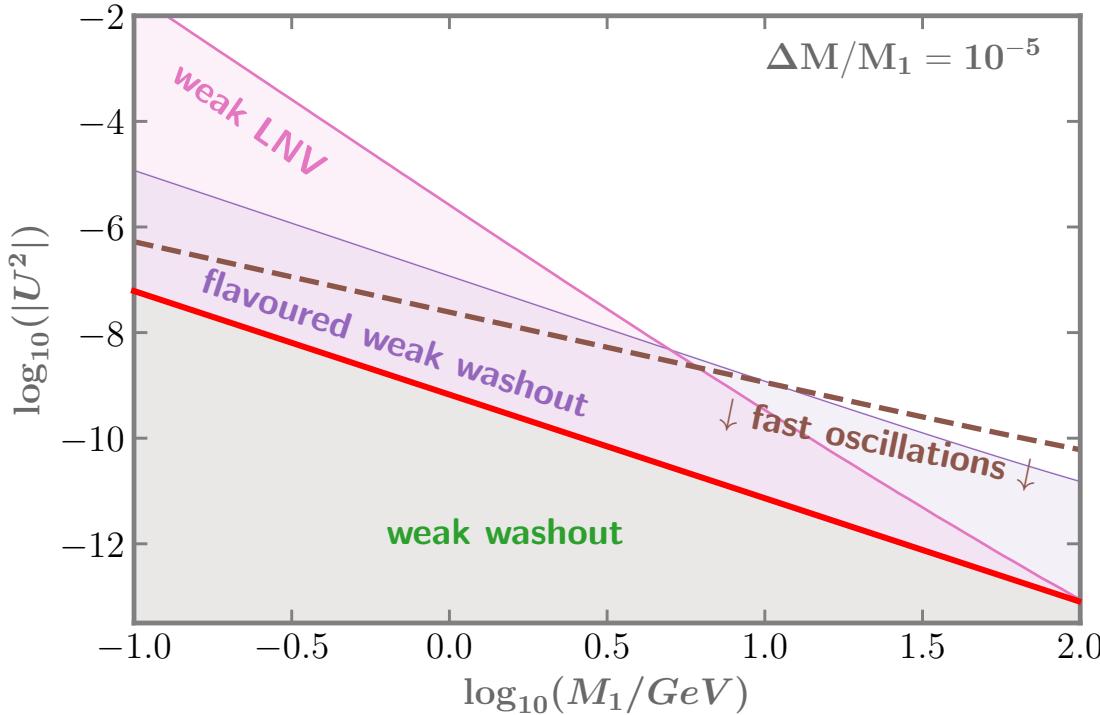
$$\Gamma_{\text{osc}}^{\text{slow}} = \epsilon^2 \Gamma$$

$$\epsilon \equiv \frac{\Gamma_{\text{osc}}}{\Gamma}$$

$$\Gamma_{\text{osc}}(T) \propto \frac{M_2^2 - M_1^2}{T}$$

Not-so-degenerate regime

Shaded regions have slow thermalizing modes



Flavoured: $y_\alpha \ll y_\beta$

wLNV: $M \ll T$

$$\Gamma_\alpha \propto (YY^\dagger)_{\alpha\alpha} T$$

$$\Gamma_M^{\text{slow}} \propto \left(\frac{M_i}{T}\right)^2 \Gamma$$

(CP violation primer)

Any CP violating observable requires the interference of at least two amplitudes that differ in **CP-even** or **CP-odd** phases

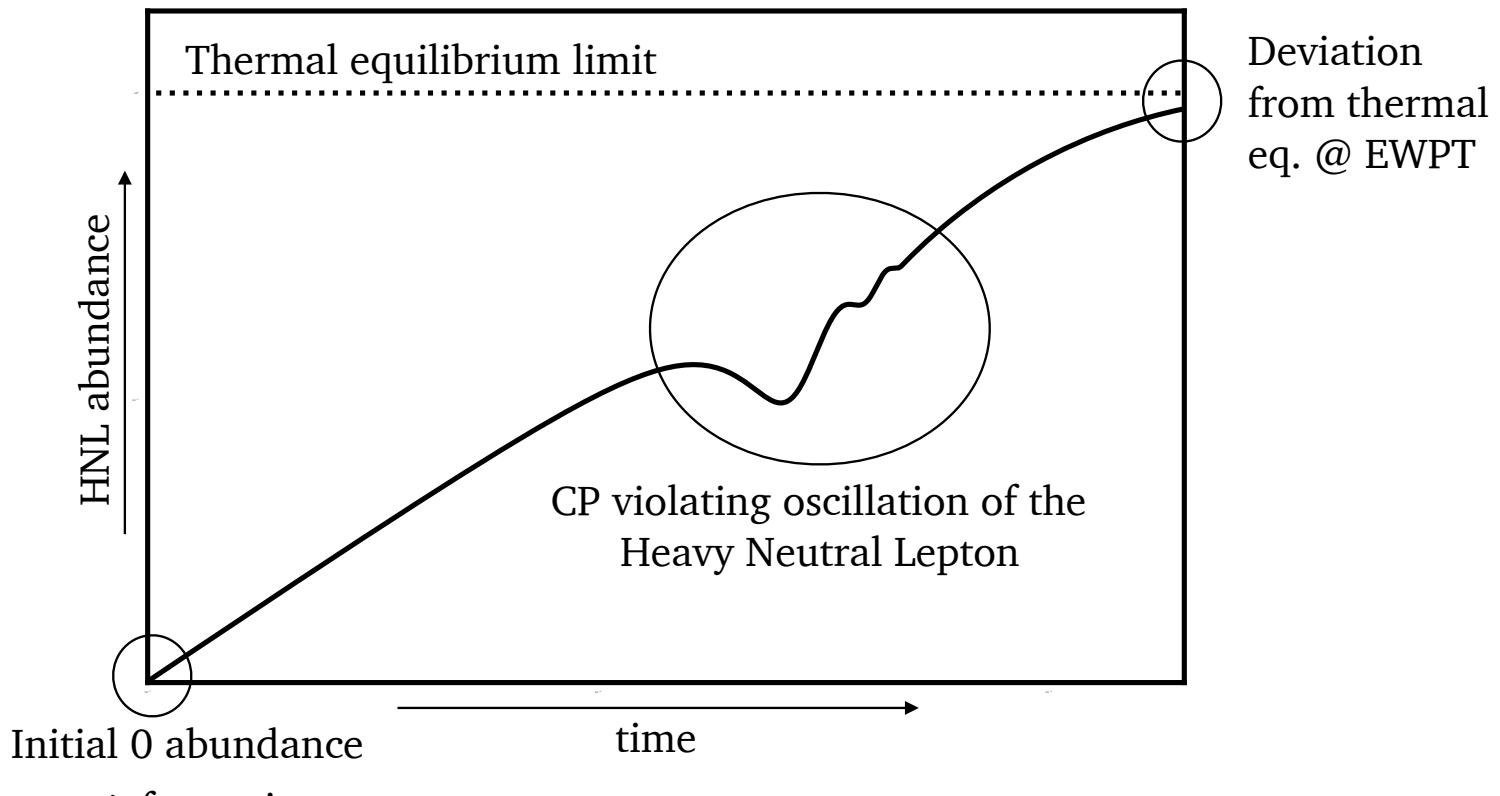
$$\Delta_{CP} \sim |A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2}|^2 - |A_1 e^{i\phi_1} e^{-i\delta_1} + A_2 e^{i\phi_2} e^{-i\delta_2}|^2$$

Vanishes if $|\phi_2 - \phi_1| = 0$ or $|\delta_2 - \delta_1| = 0$

In the context of ARS leptogenesis:

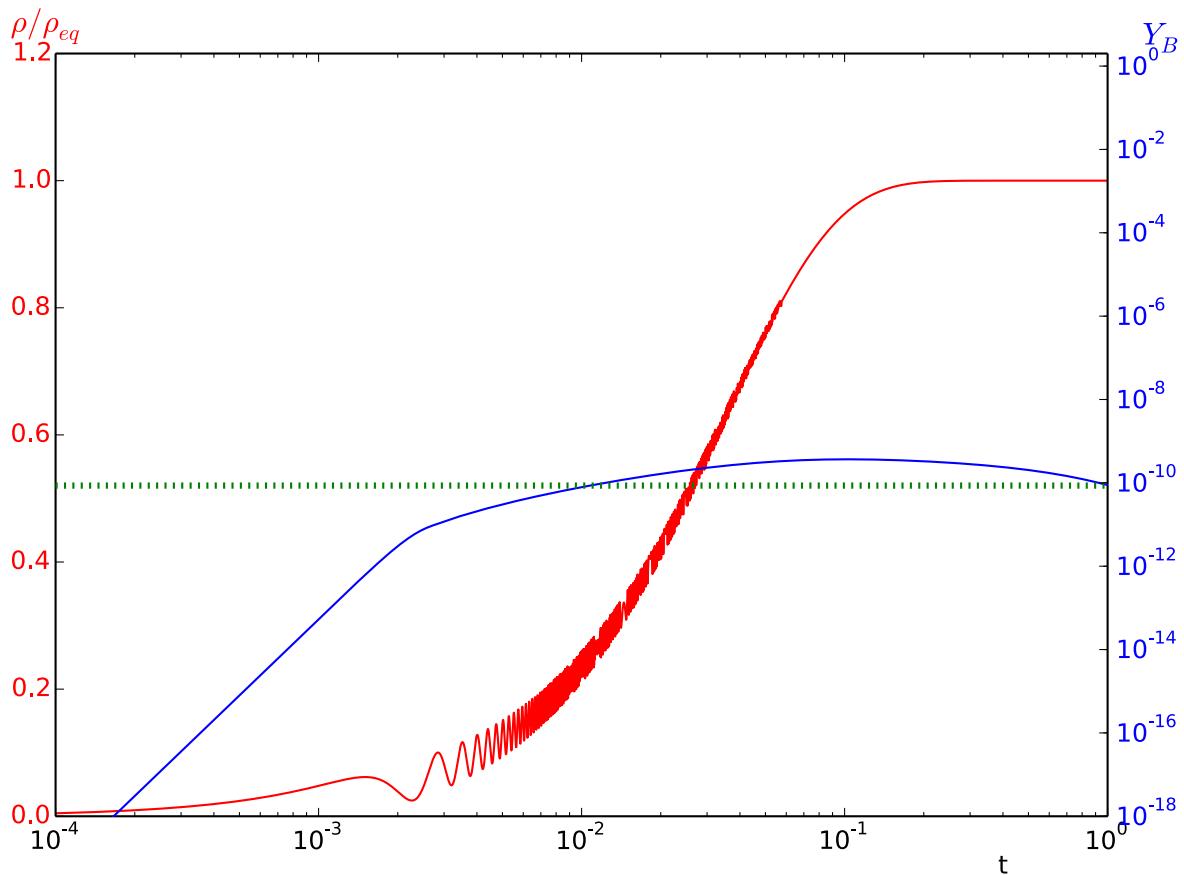
$\Delta\phi$ Oscillations/space-time phases

Freeze-in Leptogenesis



Courtesy of S. Sandner

Freeze-in Leptogenesis



The Standard Model+massive ν

Quantum kinetic equations describe the evolution of the N density matrix and the $B/3-L_\alpha$ chemical potentials

Raffelt-Sigl

$$\frac{d\rho_N(k)}{dt} = -i[H, \rho_N(k)] - \underbrace{\frac{1}{2} \{\Gamma_N^a, \rho_N\} + \frac{1}{2} \{\Gamma_N^p, 1 - \rho_N\}}_{\text{Scattering}}$$

↓
Oscillations

$$\bar{\rho}_N(H \rightarrow H^*)$$

$$\frac{d\mu_{B/3-L_\alpha}}{dt} = f(\rho_N, \rho_{\bar{N}}, \mu_{B/3-L_\alpha})$$

Stiff non-linear system with several relevant time scales: hard to explore parameter space !

Many numerical studies before

Antusch et al.; Abada et al. ; Drewes et al.; Ghiglieri, Laine; PH et al ;Klaric et al.

Our latest refinements: inclusion of **full** M/T corrections to the scattering rates, $v(T)$ close to T_{EW}

Numerical code available on [GitHub](#)



Stephan Sandner

The screenshot shows a GitHub repository page for the project "amiqs". The top navigation bar includes "Code" (selected), "Go to file", "Add file", and "About". Below the navigation, a commit history lists:

Commit	Date	Message
stefan minor cout change	da31593 11 days ago	minor cout change
amiqs	11 days ago	
rates.cpp	3 months ago	analytical LN param., detailed testing routine
.gitignore	3 months ago	initial commit
README.md	3 months ago	DOI
kinetic_equation.nb	last month	Mathematica
logo_transparent.png	3 months ago	renaming

The "About" section notes: "Calculates the baryon asymmetry generated via right handed neutrino oscillations. Code used in <https://arxiv.org/abs/2207.01651>". It also shows statistics: 4 stars, 1 watching, and 2 forks. The "Releases" section lists "amiqs v1.0 (Latest) on 20 Jul". The "Packages" section indicates "No packages published Publish your first package". The "Languages" section shows C++ (78.4%), Mathematica (21.3%), and CMake (0.3%).

State-of-the-art kinetic equations

$$\begin{aligned}
xH_u \frac{dr_N}{dx} &= -i[\langle H \rangle, r_N] - \frac{\langle \gamma_N^{(0)} \rangle}{2} \{Y^\dagger Y, r_N - 1\} - x^2 \frac{\langle s_N^{(0)} \rangle}{2} \{MY^T Y^* M, r_N - 1\} \\
&\quad + \langle \gamma_N^{(1)} \rangle Y^\dagger \mu Y - x^2 \langle s_N^{(1)} \rangle MY^T \mu Y^* M \\
&\quad - \frac{\langle \gamma_N^{(2)} \rangle}{2} \{Y^\dagger \mu Y, r_N\} + x^2 \frac{\langle s_N^{(2)} \rangle}{2} \{MY^T \mu Y^* M, r_N\}, \\
xH_u \frac{dr_{\bar{N}}}{dx} &= -i[\langle H^* \rangle, r_{\bar{N}}] - \frac{\langle \gamma_N^{(0)} \rangle}{2} \{Y^T Y^*, r_{\bar{N}} - 1\} - x^2 \frac{\langle s_N^{(0)} \rangle}{2} \{MY^\dagger Y M, r_{\bar{N}} - 1\} \\
&\quad - \langle \gamma_N^{(1)} \rangle Y^T \mu Y^* + x^2 \langle s_N^{(1)} \rangle MY^\dagger \mu Y M \\
&\quad + \frac{\langle \gamma_N^{(2)} \rangle}{2} \{Y^T \mu Y^*, r_{\bar{N}}\} - x^2 \frac{\langle s_N^{(2)} \rangle}{2} \{MY^\dagger \mu Y M, r_{\bar{N}}\}, \\
xH_u \frac{d\mu_{B/3-L_\alpha}}{dx} &= \frac{\int_k \rho_F}{\int_k \rho'_F} \left[\frac{\langle \gamma_N^{(0)} \rangle}{2} (Y r_N Y^\dagger - Y^* r_{\bar{N}} Y^T) - x^2 \frac{\langle s_N^{(0)} \rangle}{2} (Y^* M r_N M Y^T - Y M r_{\bar{N}} M Y^\dagger) \right. \\
&\quad - \mu_\alpha \left(\langle \gamma_N^{(1)} \rangle Y Y^\dagger + x^2 \langle s_N^{(1)} \rangle Y M^2 Y^\dagger \right) + \frac{\langle \gamma_N^{(2)} \rangle}{2} \mu_\alpha (Y r_N Y^\dagger + Y^* r_{\bar{N}} Y^T) \\
&\quad \left. + x^2 \frac{\langle s_N^{(2)} \rangle}{2} \mu_\alpha (Y M r_{\bar{N}} M Y^\dagger + Y^* M r_N M Y^T) \right]_{\alpha\alpha}, \tag{4.2}
\end{aligned}$$

Towards an analytical understanding

PH, Lopez-Pavon, Rius, Sandner, arxiv: 2207.01651

- Identify the different non-thermal regimes and their characteristic time-scales
- Set up a perturbative approximation of the equations exploiting $y' \ll y$, $\mu \ll \Lambda$
- Identify the CP invariants that control the parameter dependences of Y_B

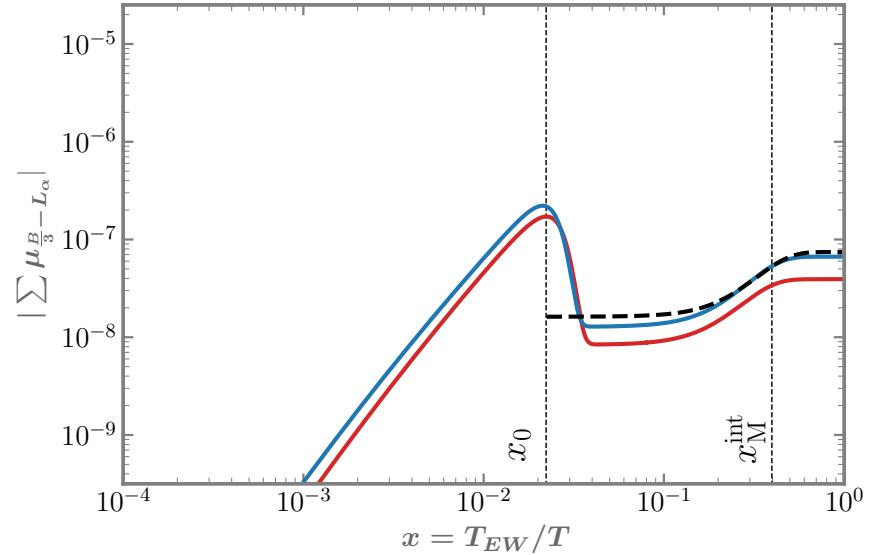
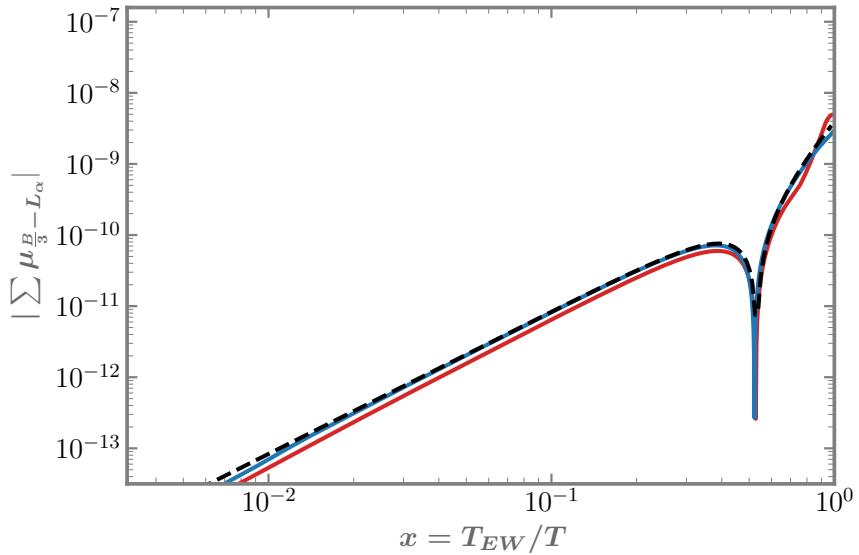
Branco et al, Manohar, Jenkins

$$I_0 = \text{Im} \left(\text{Tr}[Y^\dagger Y M^\dagger M Y^\dagger Y_l Y_l^\dagger Y] \right)$$

$$I_1 = \text{Im} \left(\text{Tr}[Y^\dagger Y M^\dagger M M^* (Y^\dagger Y)^* M] \right)$$

- Write the CP invariants in terms of observable parameters: find bounds and correlations implied by the matter-antimatter asymmetry

Analytical vs numerical solution in overdamped regime



$$\left(\sum_{\alpha} \mu_{B/3-L_\alpha} \right)^{\text{ov-wLNV}} \approx \frac{\kappa x^2}{6\gamma_0 + \kappa\gamma_1} \frac{\gamma_0^2}{\gamma_0^2 + 4\omega^2} \frac{c_H M_P^*}{T_{EW}^3} \left(\Delta_{\text{LNC}}^{\text{ov}} - \frac{24}{5} \frac{s_0 x^3}{T_{EW}^2} \Delta_{\text{LNV}}^{\text{ov}} \right)$$

$$\Delta_{\text{LNC}}^{\text{ov}} = \frac{1}{[\text{Tr}(Y^\dagger Y)]^2} \sum_{\alpha} \frac{1}{(YY^\dagger)_{\alpha\alpha}} \sum_{i < j} (M_j^2 - M_i^2) \text{Im} \left[Y_{\alpha j}^* Y_{\alpha i} (Y^\dagger Y)_{ij} \right]$$

$$\Delta_{\text{LNV}}^{\text{ov}} = \frac{1}{[\text{Tr}(Y^\dagger Y)]^2} \sum_{\alpha} \sum_{i < j} (M_j^2 - M_i^2) M_i M_j \text{Im} \left[Y_{\alpha j} Y_{\alpha i}^* (Y^\dagger Y)_{ij} \right]$$

Connecting to observables

For Inverted Ordering:

Gavela, Hambye, PH, DH

$$Y = \begin{pmatrix} y_e & y'_e e^{i\beta_e} \\ y_\mu & y'_\mu e^{i\beta_\mu} \\ y_\tau & y'_\tau e^{i\beta_\tau} \end{pmatrix}$$

m_ν

$$\left\{ \begin{array}{l} Y_{\alpha 1} = \frac{e^{-i\theta/2}y}{\sqrt{2}} \left(U_{\alpha 2}^* \sqrt{1+\rho} + U_{\alpha 1}^* \sqrt{1-\rho} \right), \\ Y_{\alpha 2} = \frac{e^{i\theta/2}y'}{\sqrt{2}} \left(U_{\alpha 2}^* \sqrt{1+\rho} - U_{\alpha 1}^* \sqrt{1-\rho} \right) + \frac{\Delta M}{4M} \frac{e^{-i\theta/2}y}{\sqrt{2}} \left(U_{\alpha 2}^* \sqrt{1+\rho} + U_{\alpha 1}^* \sqrt{1-\rho} \right) \end{array} \right.$$

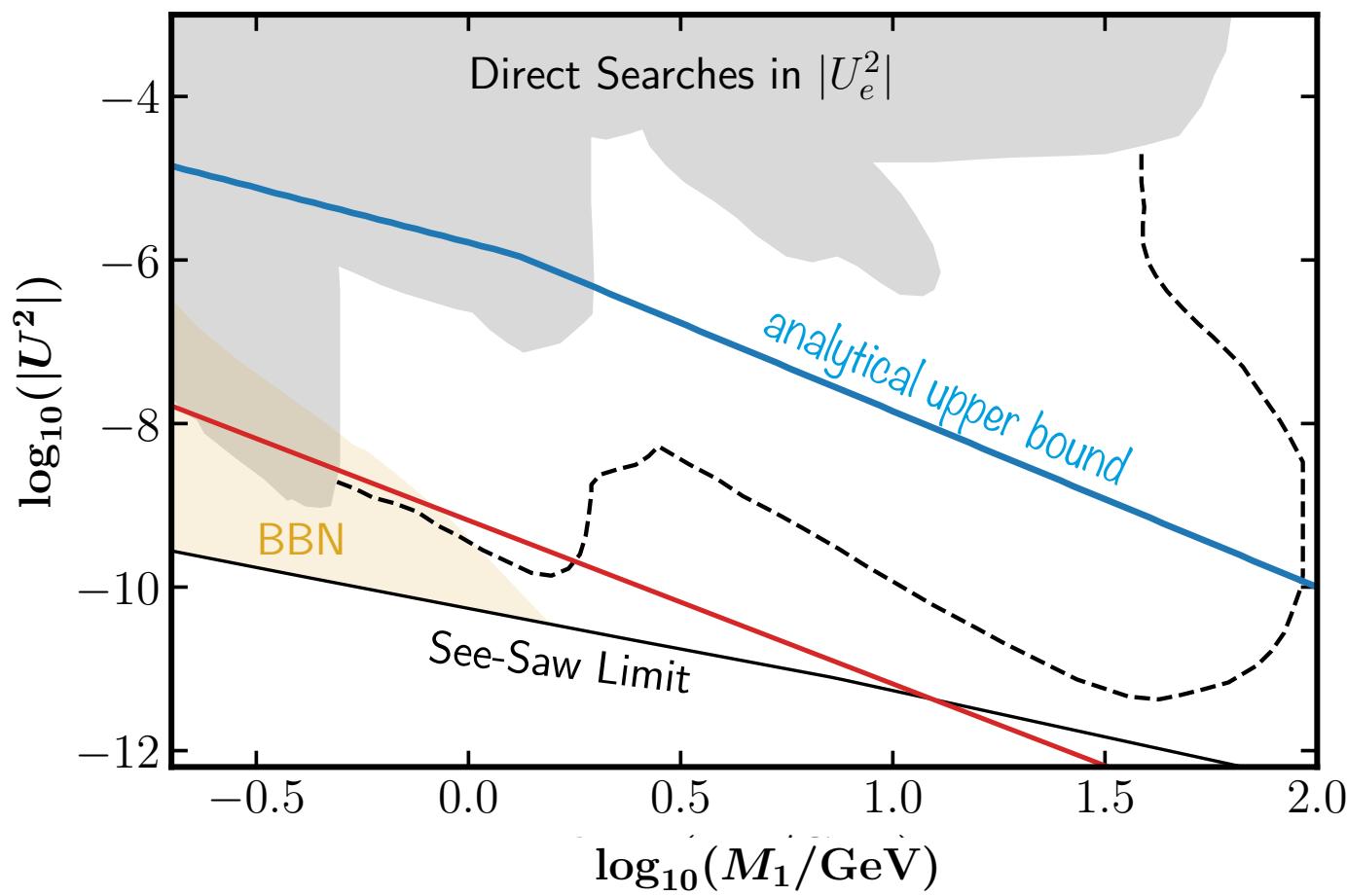
$$\rho = \frac{\sqrt{\Delta m_{\text{atm}}^2} - \sqrt{\Delta m_{\text{sol}}^2}}{\sqrt{\Delta m_{\text{atm}}^2} + \sqrt{\Delta m_{\text{sol}}^2}}, \quad y' = \frac{M}{2v^2y} \left(\sqrt{\Delta m_{\text{atm}}^2} + \sqrt{\Delta m_{\text{sol}}^2} \right)$$

$$\frac{\Delta_{\text{LNC}}^{\text{ov}}}{M_2^2 - M_1^2} \approx \frac{v^2 \sqrt{\Delta m_{\text{atm}}^2}}{8M^3U^4} \frac{(1 + 3c_\phi \sin 2\theta_{12})(c_\theta s_\phi \sin 2\theta_{12} + s_\theta \cos 2\theta_{12})}{-1 + c_\phi^2 \sin^2 2\theta_{12}}$$

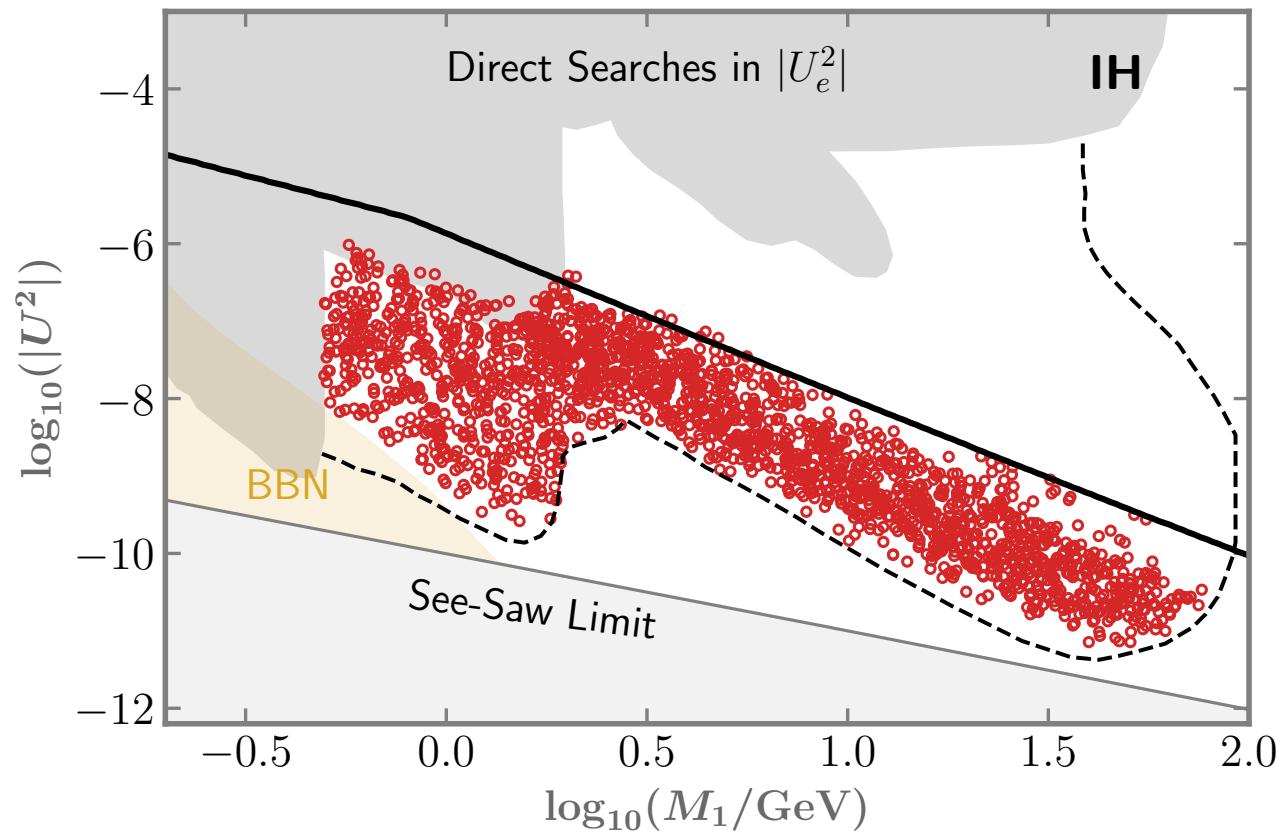
$$\frac{\Delta_{\text{LNV}}^{\text{ov}}}{M_1 M_2 (M_2^2 - M_1^2)} \approx -\frac{\sqrt{\Delta m_{\text{atm}}^2}}{8MU^2} r^2 s_\theta$$

Depend on **M, U, θ** not accessible at low energies, and the **PMNS Majorana phase φ**

Upper bound on the HNL mixing



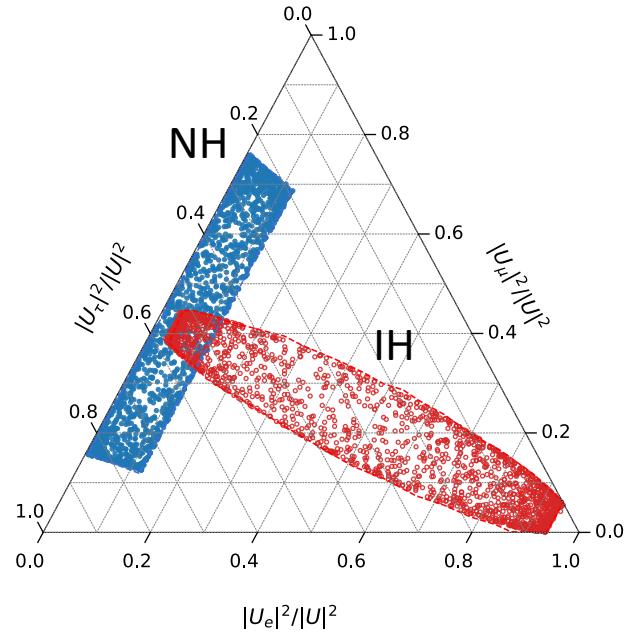
Upper bound on the HNL mixing



Numerical scan within the sensitivity region of SHIP and FCCee

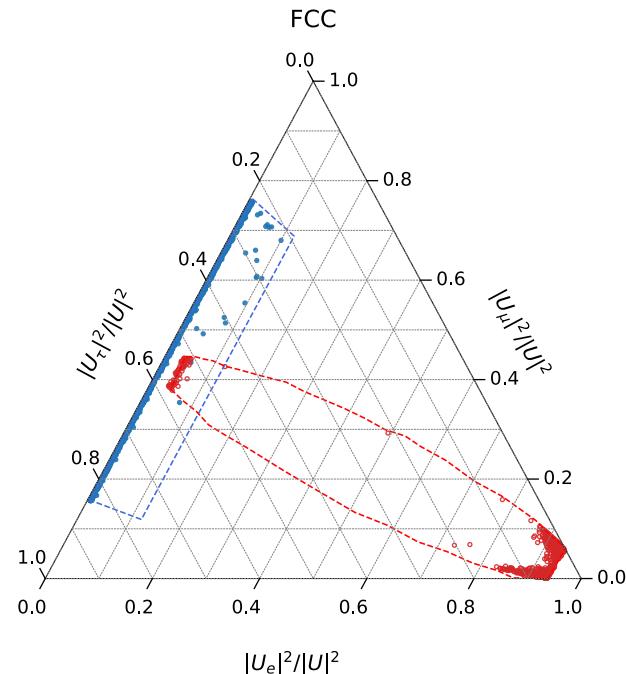
Implications for HNL mixings

In the not-so-degenerate case Y_B constrains significantly flavour ratios because flavour effects are necessary



Constrained by ν masses

$$\frac{\Delta M}{M} = 10^{-2}$$



$$+ \frac{n_b}{n_\gamma} \sqrt{ }$$

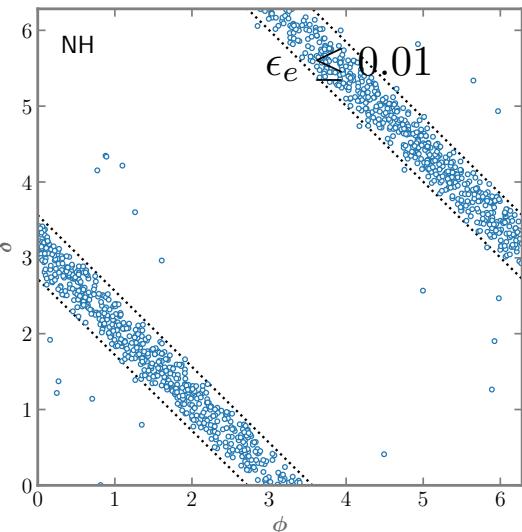
Implications for PMNS CP violation

In the not-so-degenerate case strong correlations with U_{PMNS} CP phases because flavour effects are necessary

$$\frac{\Delta M}{M} = 10^{-2} \quad \frac{n_b}{n_\gamma} \sqrt{\quad}$$



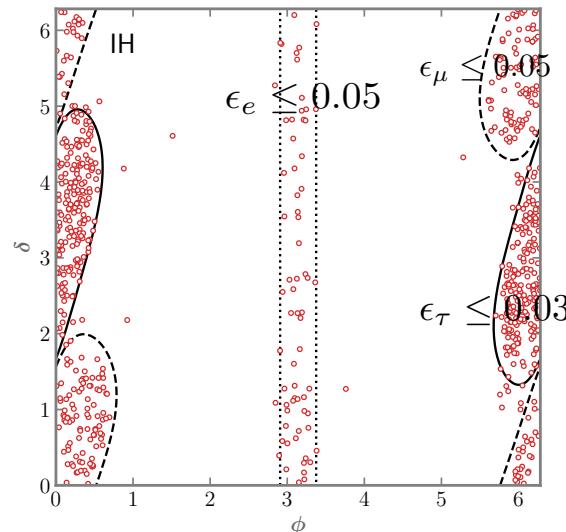
CP violation in
 ν oscillations



$\delta + \phi$ in $\beta\beta 0\nu$



Majorana phase

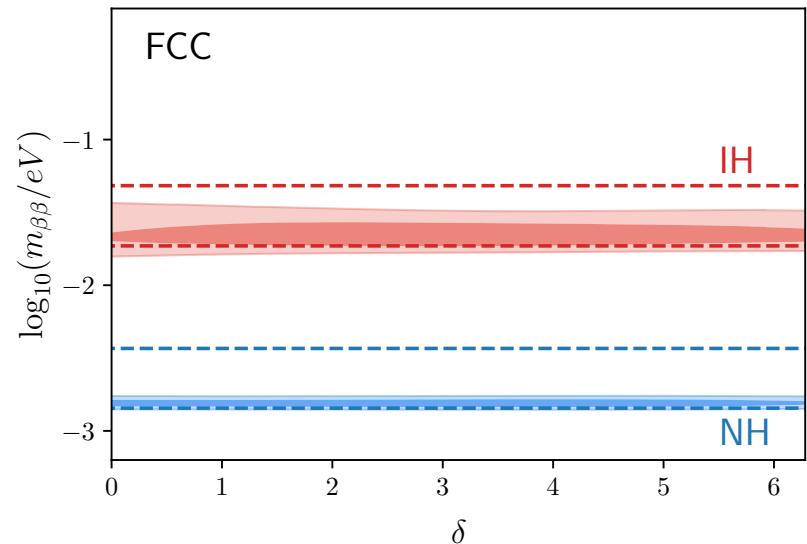
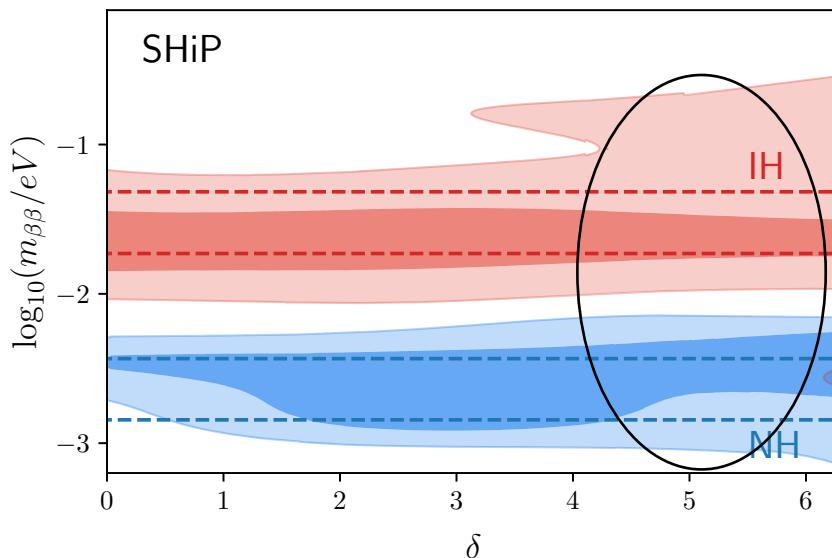


Implications for $\beta\beta0\nu$

$$m_{\beta\beta} = \left| \sum_{i=\text{light}} U_{ei}^2 m_i + \sum_{I=\text{heavy}} \Theta_{eI}^2 M_I \mathcal{M}(M_I) / \mathcal{M}(0) \right|$$

$$\frac{\Delta M}{M} = 10^{-2}$$

$$\frac{n_b}{n_\gamma} \sqrt{ }$$



- HNL effects on the amplitude within SHiP: no trivial dependence on phases
- Flavour effects needed for Y_B constrain the light contribution within FCC

Conclusions

- Neutrino mass models robustly predict a matter-antimatter asymmetry even at accessible scales
- Searches for the neutrino mass mediators (HNLs) could provide essential info about CP violation, the non-thermal condition and Y_B
- We have identified the CP invariants involved in low-scale leptogenesis
- CP invariants allow connecting Y_B to other observables: neutrino masses, CP violation in neutrino oscillations, $\beta\beta0\nu$ and HNL masses and mixings
- Methods developed can be applied to more complex scenarios: understanding how Y_B constrains the more complex parameter space