Status and challenges of neutrino-nucleus cross sections

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IPSA ÉCOLE D'INGÉNIEURS DE L'AIR, DE L'ESPACE ET DE LA MOBILITÉ DURABLE

SORBONNE UNIVERSITÉ

LPNHE PARIS

- Summary of my lectures at Ecole de GIF 2022

  [Ecole de Gif 2022: La Physique des Neutrinos (5-9 septembre 2022): Sections efficaces d'interaction de neutrinos]

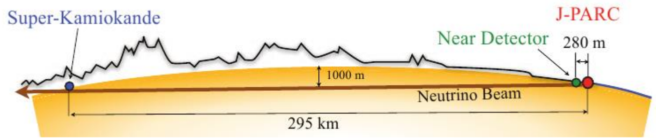
- Latest from NuINT 2022

  [NuInt 2022 (24-29 octobre 2022): Accueil · Indico (cern.ch)]

Neutrino oscillation experiments

$\nu_\alpha \to \nu_\beta$ ν Source E_ν L ν Detector

$$\nu_\alpha = U_{\alpha i}\nu_i \quad \alpha = e, \mu, \tau \quad i = 1, 2, 3$$



Reconstructed ν energy

True ν energy

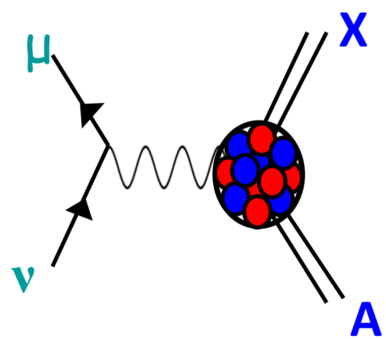
$$N_{\nu_\beta}(\overline{E_\nu}) \sim \int \Phi_{\nu_\alpha}(E_\nu) P_{\nu_\alpha \to \nu_\beta}(E_\nu, L, \{\Theta\}) \sigma_{\nu_\beta}(E_\nu) \epsilon_{\text{det.}} d(E_\nu, \overline{E_\nu}) dE_\nu$$

Number of detected events

ν flux | ν oscillation probability | ν cross section | Detector efficiency | Migration matrix

Modern accelerator-based neutrino oscillation experiments:

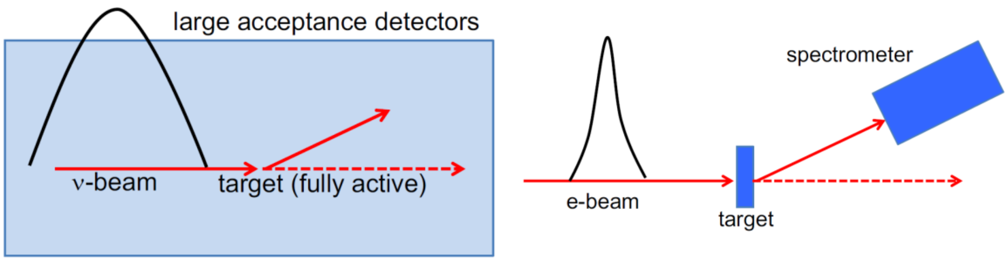
- The neutrino energy is reconstructed from the final states
- Nuclear targets (C, O, Ar, Fe…)

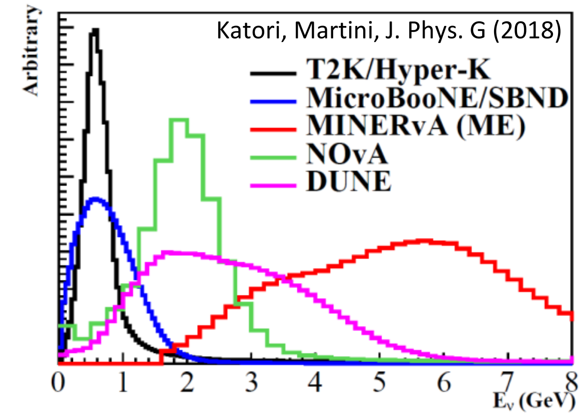


→ the knowledge of the neutrino-nucleus cross section is crucial

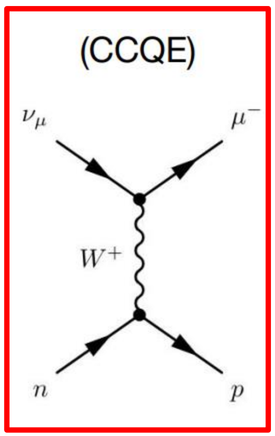
Some important points of the accelerator-based ν experiment

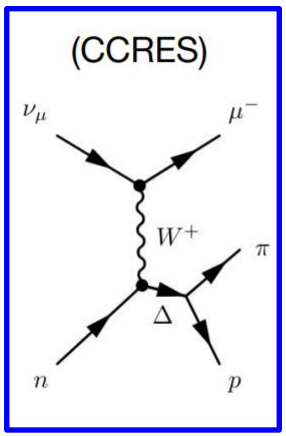
- Neutrino beams are not monochromatic (at difference with respect to electron beams)

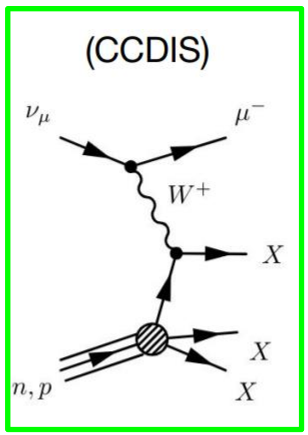


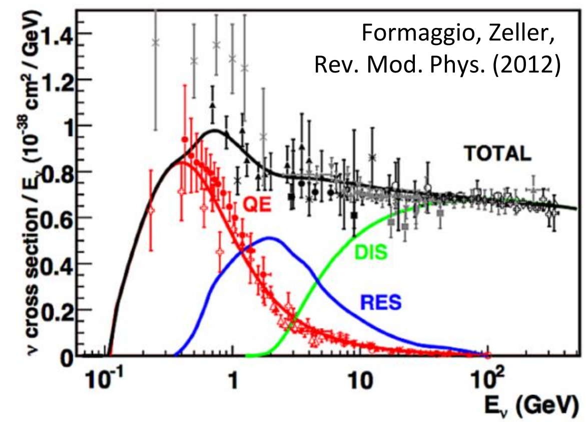


- Different reaction mechanisms contribute

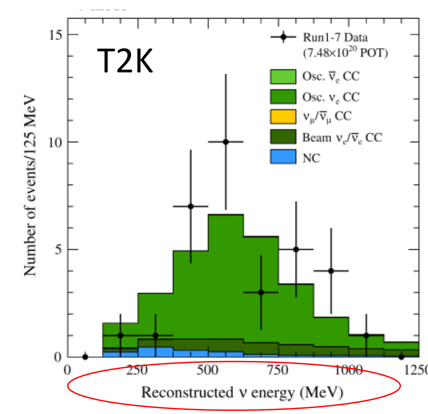




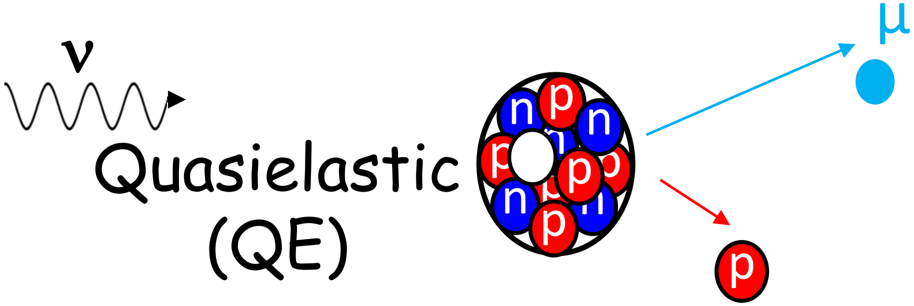


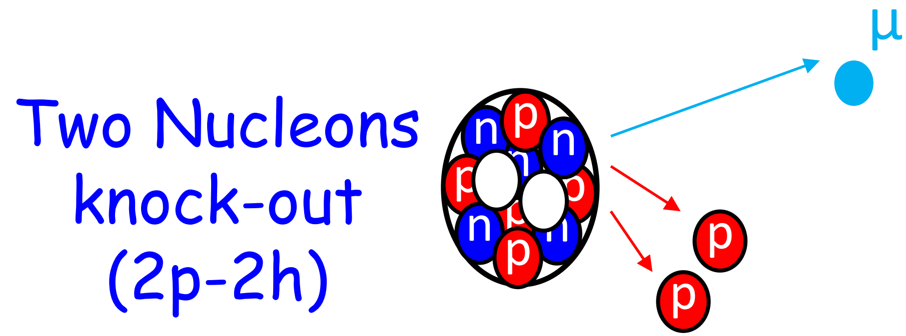


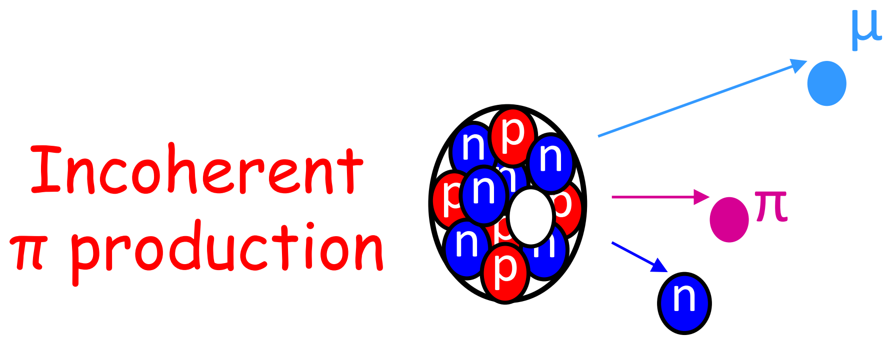
- The neutrino energy is reconstructed from the final states of the reaction (often from CCQE events)

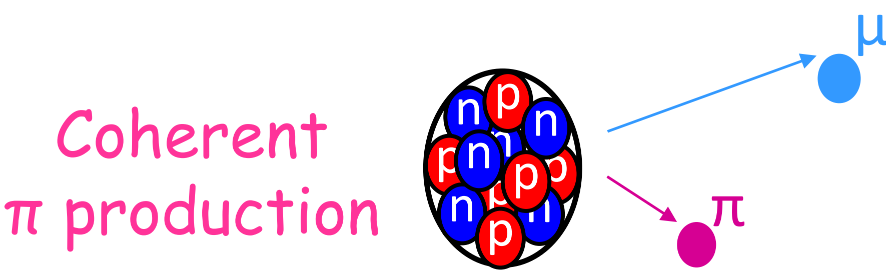


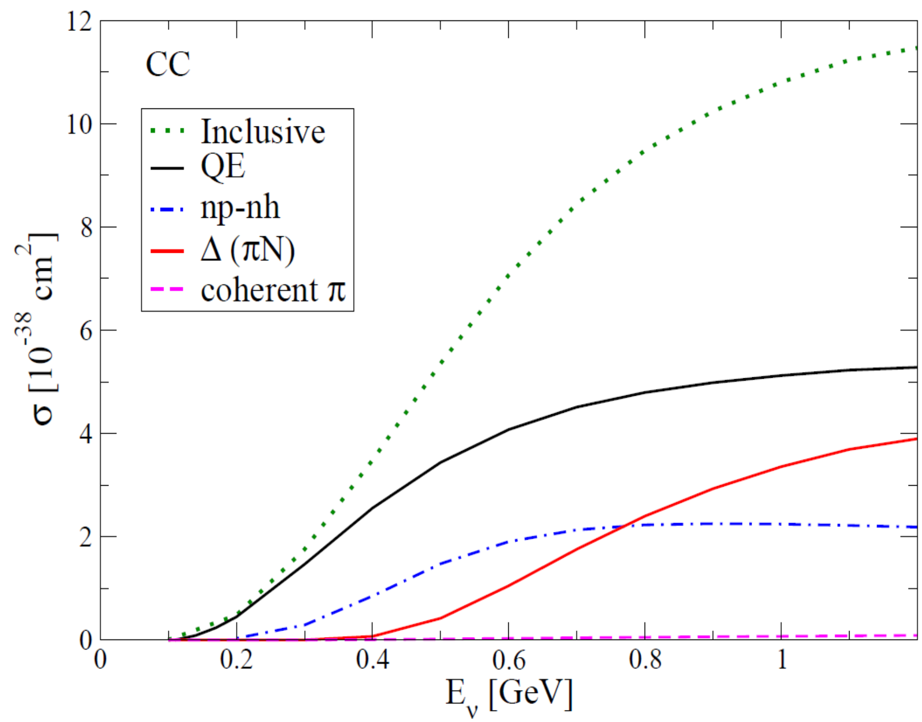
In this talk: Neutrino - nucleus interaction @ $E_\nu \sim O(1\ \text{GeV})$







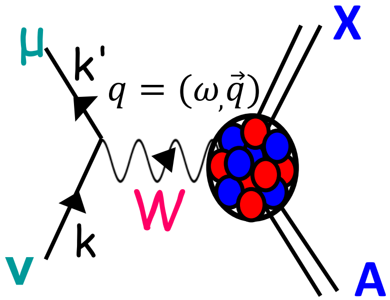




Different processes are entangled

Charged current neutrino-nucleus cross section

$$\nu_l\,(\bar{\nu}_l) + A \longrightarrow l^-\,(l^+) + X$$



$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} \cos\theta_C l_\mu J^\mu$$

Lab frame

$$\frac{d^2\sigma}{d\Omega_{k'} d\omega} = \frac{G_F^2 \cos^2\theta_C}{4\pi^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L_{\mu\nu} W^{\mu\nu}(\mathbf{q},\omega)$$

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k' \pm i\varepsilon_{\mu\nu\kappa\lambda} k^\kappa k'^\lambda$$

Leptonic tensor

$$W^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^\nu(q)|0\rangle \delta^{(4)}(p_0 + q - p_f)$$

Hadronic tensor

The "inclusive" charged current cross section is a linear combination of 5 contributions

$$\frac{d^2\sigma}{d\Omega_{k'} d\omega} = \sigma_0 \left[L_{00} W^{00} + L_{33} W^{33} + (L_{03} + L_{30}) W^{03} + (L_{11} + L_{22}) W^{11} \pm (L_{12} - L_{21}) W^{12}\right]$$

A simplified expression particularly useful for illustration:

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2 \cos^2\theta_c}{\pi} |\mathbf{k}'| E'_l \cos^2\frac{\theta}{2} \left[\frac{(\mathbf{q}^2-\omega^2)^2}{\mathbf{q}^4} G_E^2 R_\tau(\mathbf{q},\omega) + \frac{\omega^2}{\mathbf{q}^2} G_A^2 R_{\sigma\tau(L)}(\mathbf{q},\omega)\right.$$

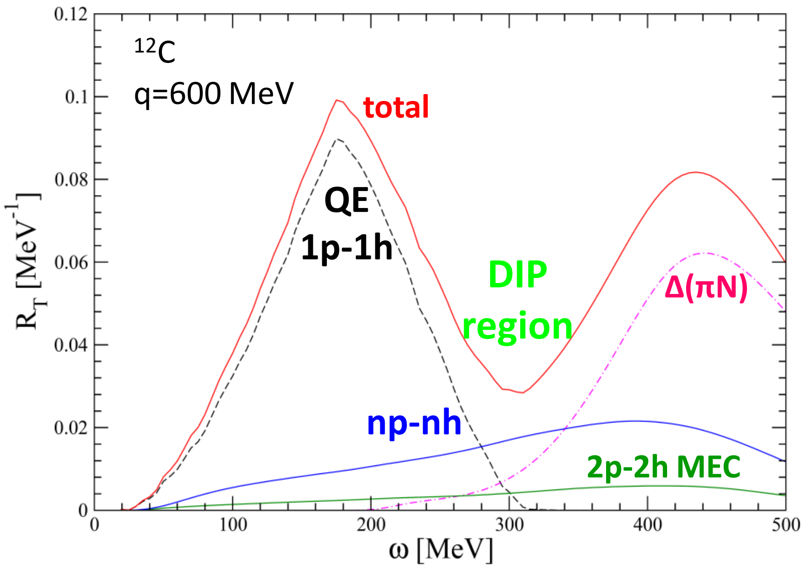
$$\left. + 2\left(\tan^2\frac{\theta}{2} + \frac{\mathbf{q}^2-\omega^2}{2\mathbf{q}^2}\right)\left(G_M^2 \frac{\mathbf{q}^2}{4M_N^2} + G_A^2\right) R_{\sigma\tau(T)}(\mathbf{q},\omega) \pm 2\frac{E_\nu + E'_l}{M_N} \tan^2\frac{\theta}{2} G_A G_M R_{\sigma\tau(T)}(\mathbf{q},\omega)\right]$$

Explicitly appear:

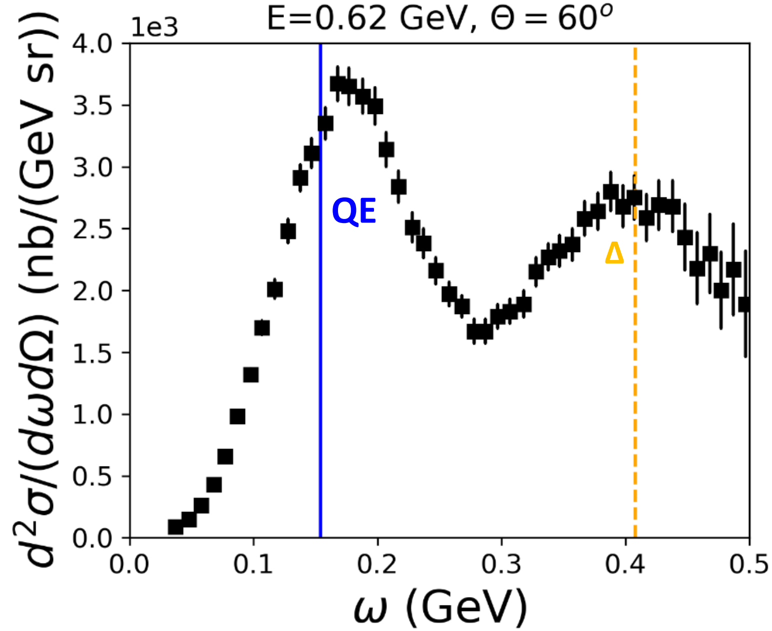
1. The different **kinematic variables** (related to the leptonic tensor)
2. The nucleon Electric, Magnetic, and Axial **form factors** (↔ nucleon properties)
3. The **nuclear response functions** (↔ nuclear dynamics)

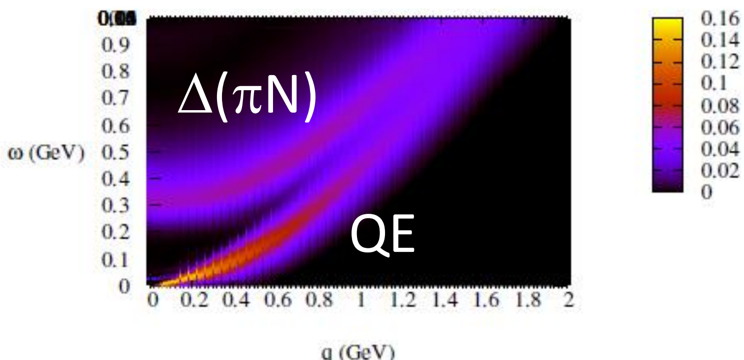
Nuclear responses and inclusive electron and ν cross sections

Nuclear Responses

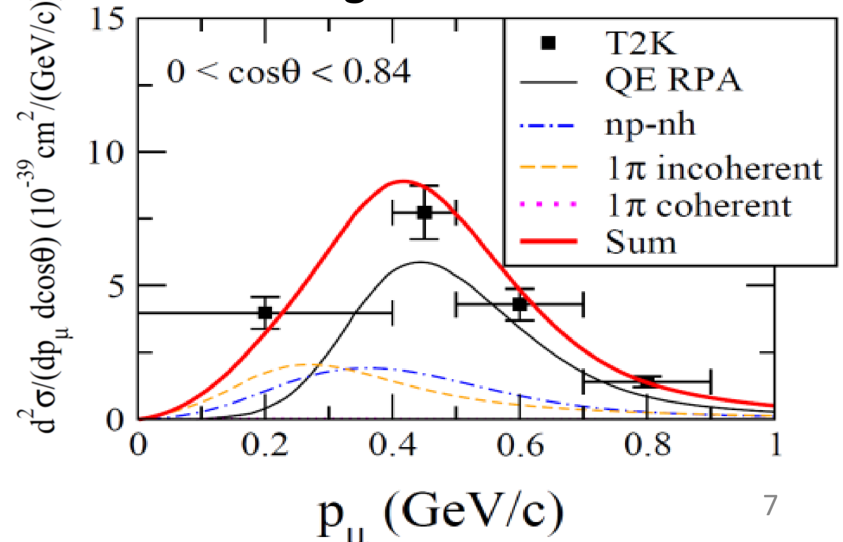


Monochromatic electron d2σ





Flux-integrated neutrino d2σ



QE peak:

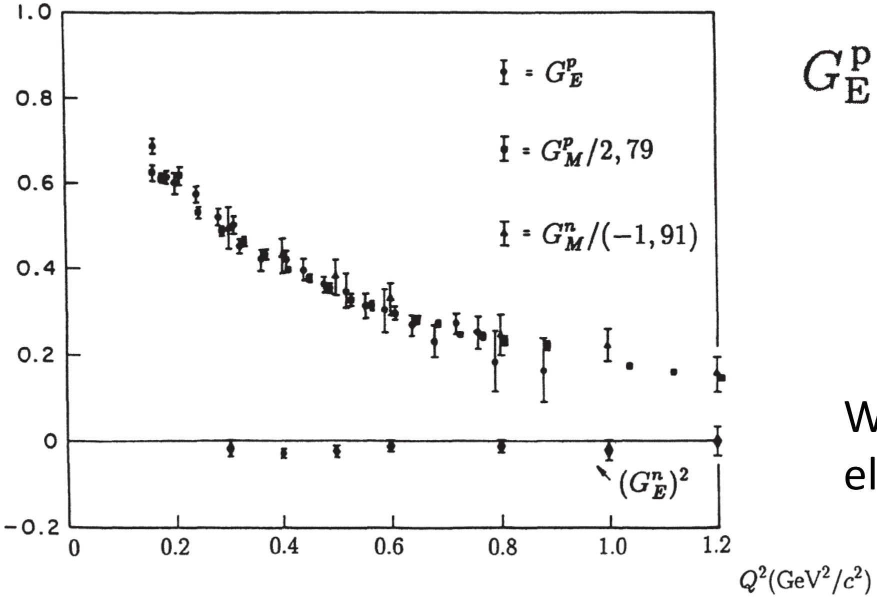
$$\omega = \sqrt{\mathbf{q}^2 + M_N^2} - M_N = \frac{Q^2}{2M_N} = \frac{\mathbf{q}^2 - \omega^2}{2M_N}$$

Δ peak:

$$\omega = \sqrt{\mathbf{q}^2 + M_\Delta^2} - M_N = \frac{Q^2}{2M_N} + \frac{M_\Delta^2 - M_N^2}{2M_N}$$

The Form Factors

Vector form factors



$$Q^2 = q^2 - \omega^2$$

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{2.79} = \frac{G_M^n(Q^2)}{-1.91} = G^{\mathrm{dipole}}(Q^2)$$

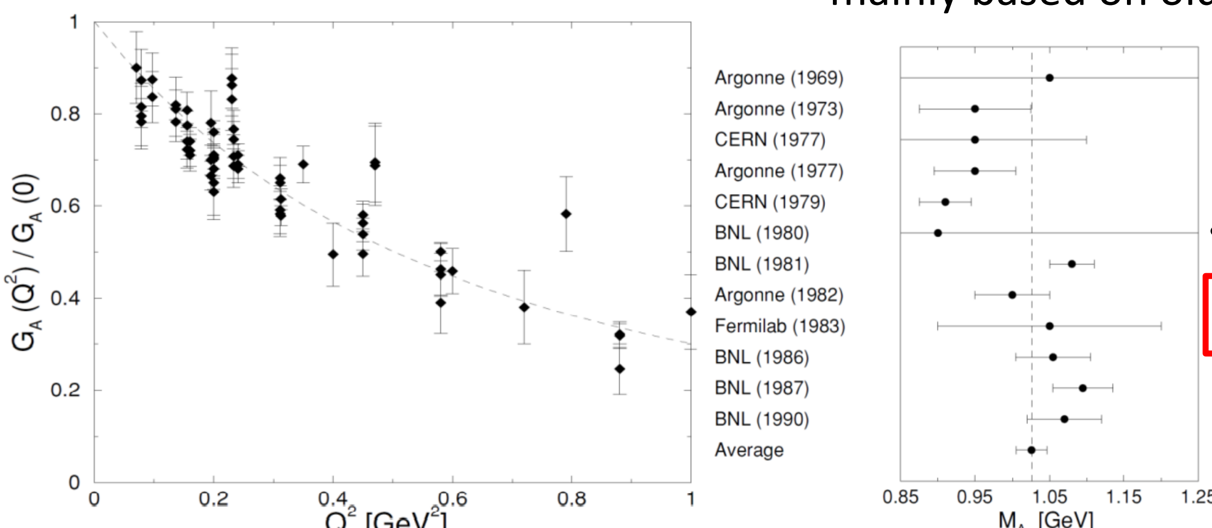
$$G^{\mathrm{dipole}}(Q^2) = \left(1 + \frac{Q^2}{0.71\ (\mathrm{GeV}/c)^2}\right)^{-2}$$

Global **dipole**-like behavior

Weak vector form factors are well constrained by electron scattering experiments (CVC)

Axial form factor

Q^2 evolution of the axial form factor is less well-known, mainly based on old bubble chamber data



V. . Bernard, J.Phys. G28 (2002) R1-R35

$$G_A(Q^2) = g_A(1 + Q^2/M_A^2)^{-2}$$

$g_A = 1.26$ from neutron β decay

$$M_A = (1.026 \pm 0.021)\ \ GeV/c^2$$

from ν-^{2}H (bubble-chamber) CCQE
and
from π electroproduction

CCQE, CCQE-like and CC0π

MiniBooNE CC Quasielastic cross section on Carbon and the M_A puzzle

First Measurement of Muon Neutrino Charged Current Quasielastic (CCQE) Double Differential Cross Section

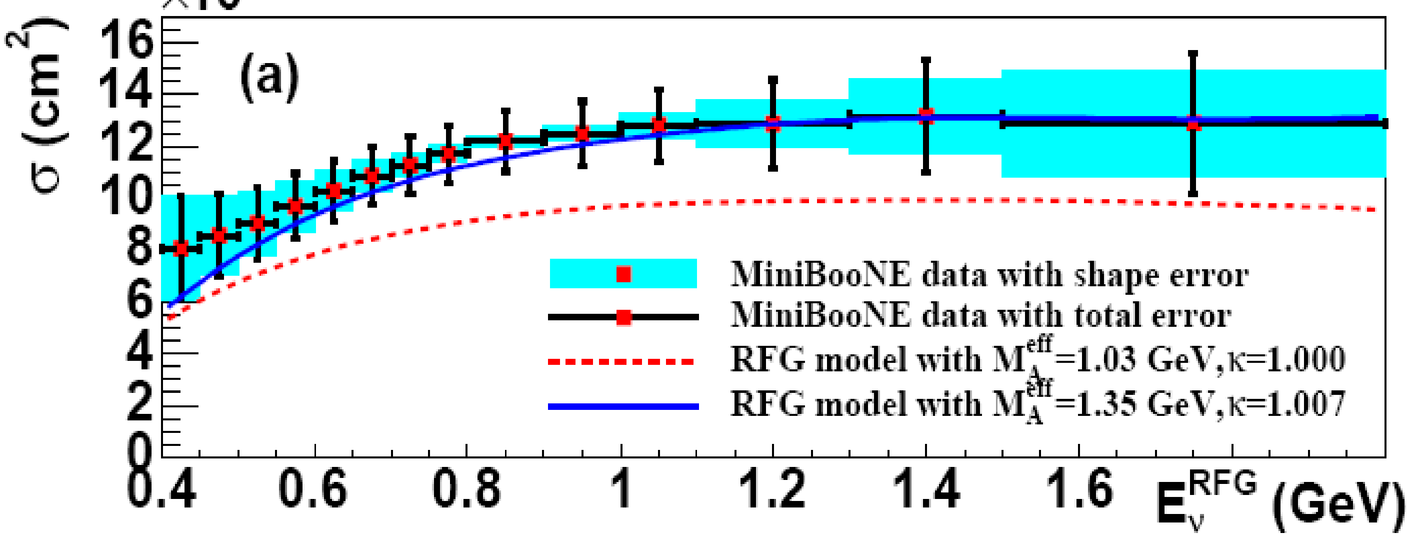
Cite as: AIP Conference Proceedings **1189**, 139 (2009); https://doi.org/10.1063/1.3274144
Published Online: 02 December 2009

Teppei Katori and MiniBooNE collaboration

PHYSICAL REVIEW D **81**, 092005 (2010)

First measurement of the muon neutrino charged current quasielastic double differential cross section

AIP Conf. Proc. 1189: 139-144 (2009); Phys. Rev. D 81, 092005 (2010)



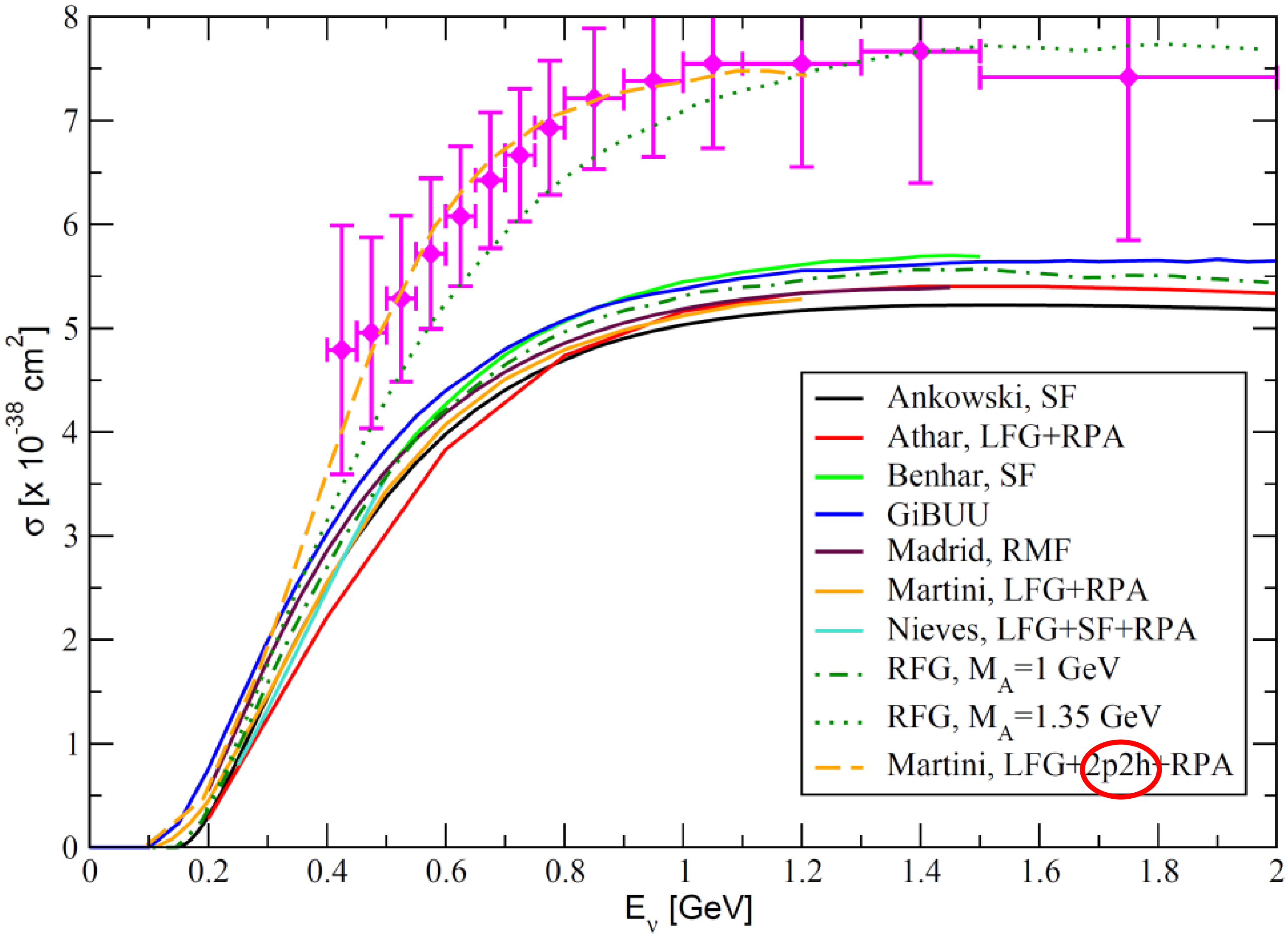
Comparison with a prediction based on Relativistic Fermi Gas (**RFG)**
using **M_A=1.03 GeV** (standard value) reveals a discrepancy

In the Relativistic Fermi Gas (RFG) model an axial mass of **1.35 GeV** is needed to account for data

puzzle??

Comparison of different theoretical models for Quasielastic

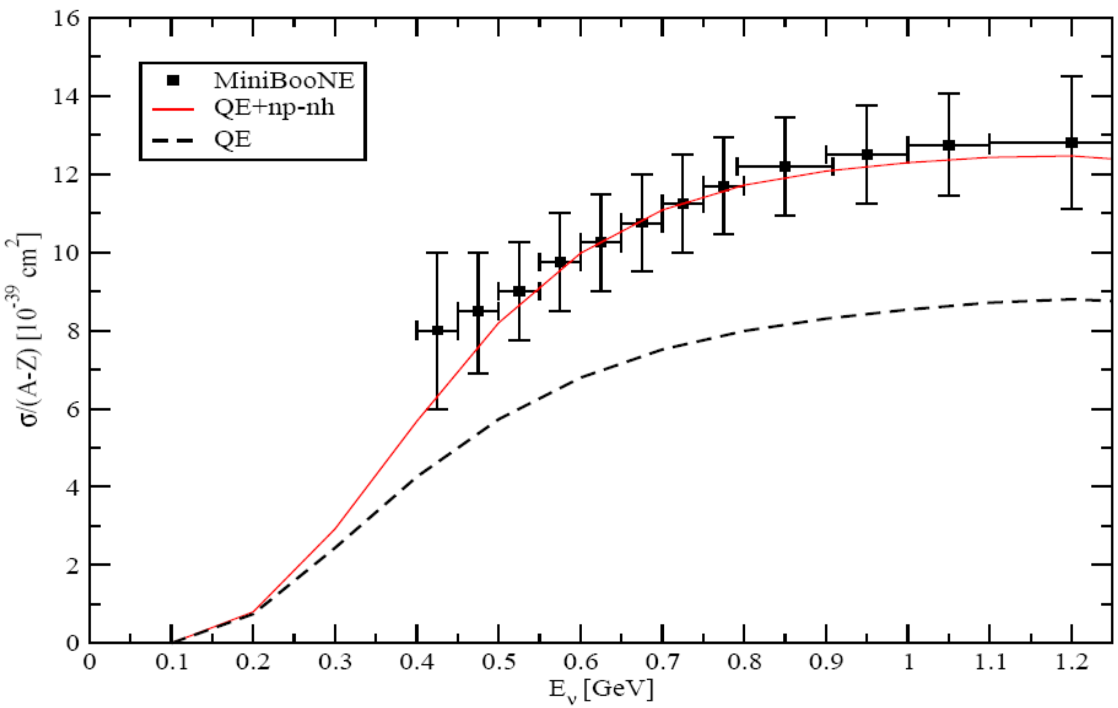
L. Alvarez-Ruso , arXiv:1012.3871 (Neutrino 2010)

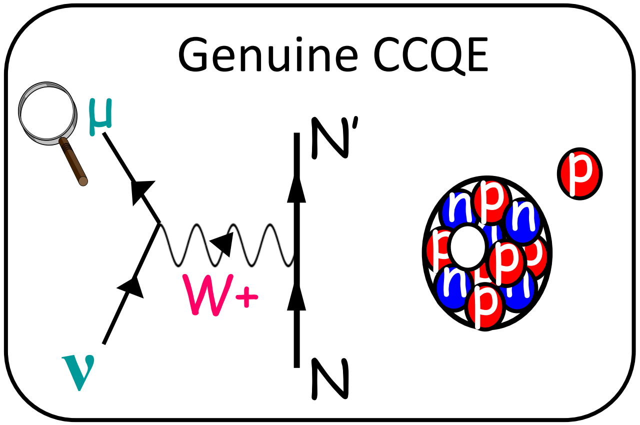


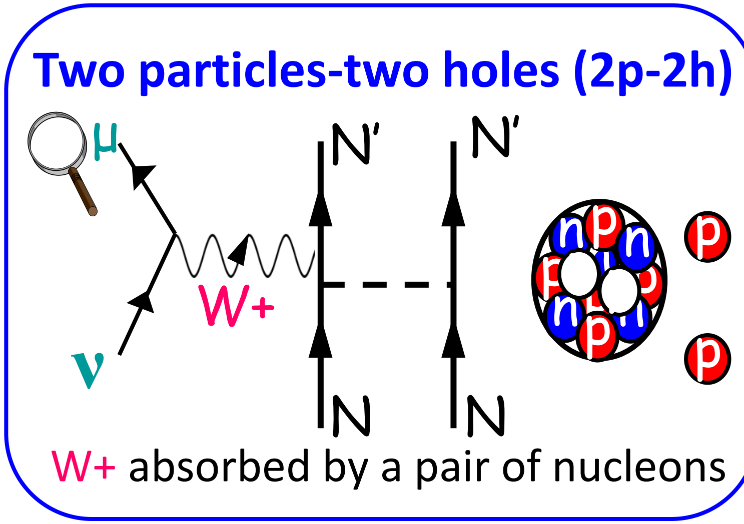
puzzle??

An explanation of this puzzle

Inclusion of the multinucleon emission channel
(np-nh = 2p-2h + 3p-3h)







CCQE-like = Genuine CCQE + np-nh

M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C 80 065501 (2009)

Agreement with MiniBooNE without increasing M_A

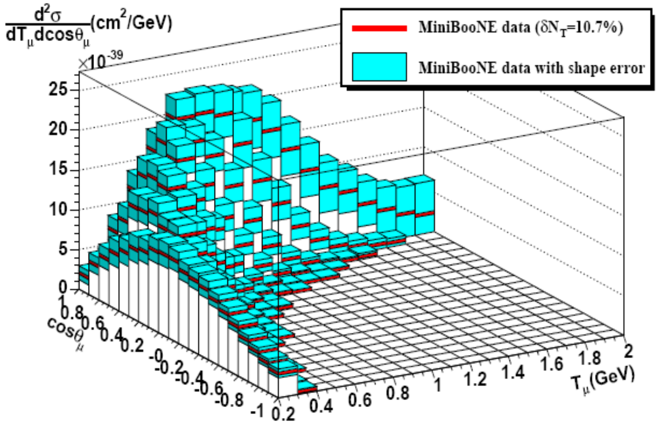
⇨ MiniBooNE measured CCQE-like, not genuine CCQE

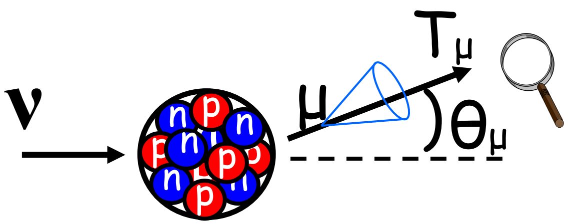
Flux-integrated double differential cross section

Unfolding matrix to remove detector effects — Number of observed events — Background contribution

$$\left(\frac{d^2\sigma}{dT_l \cos\theta}\right)_i = \frac{\sum_j U_{ij}(d_j - b_j)}{\Phi \cdot T \cdot \epsilon_i \cdot (\Delta T_l, \Delta\cos\theta)_i}$$

Total integrated flux — Number of target nucleons in the Fiducial Volume — Efficiency — Bin widths





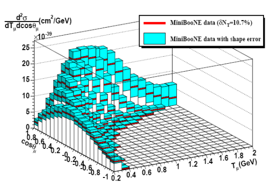
- Function of two measured variables
- Less model dependent than $\sigma(E_\nu)$: free from the neutrino energy reconstruction problem
- Flux dependent

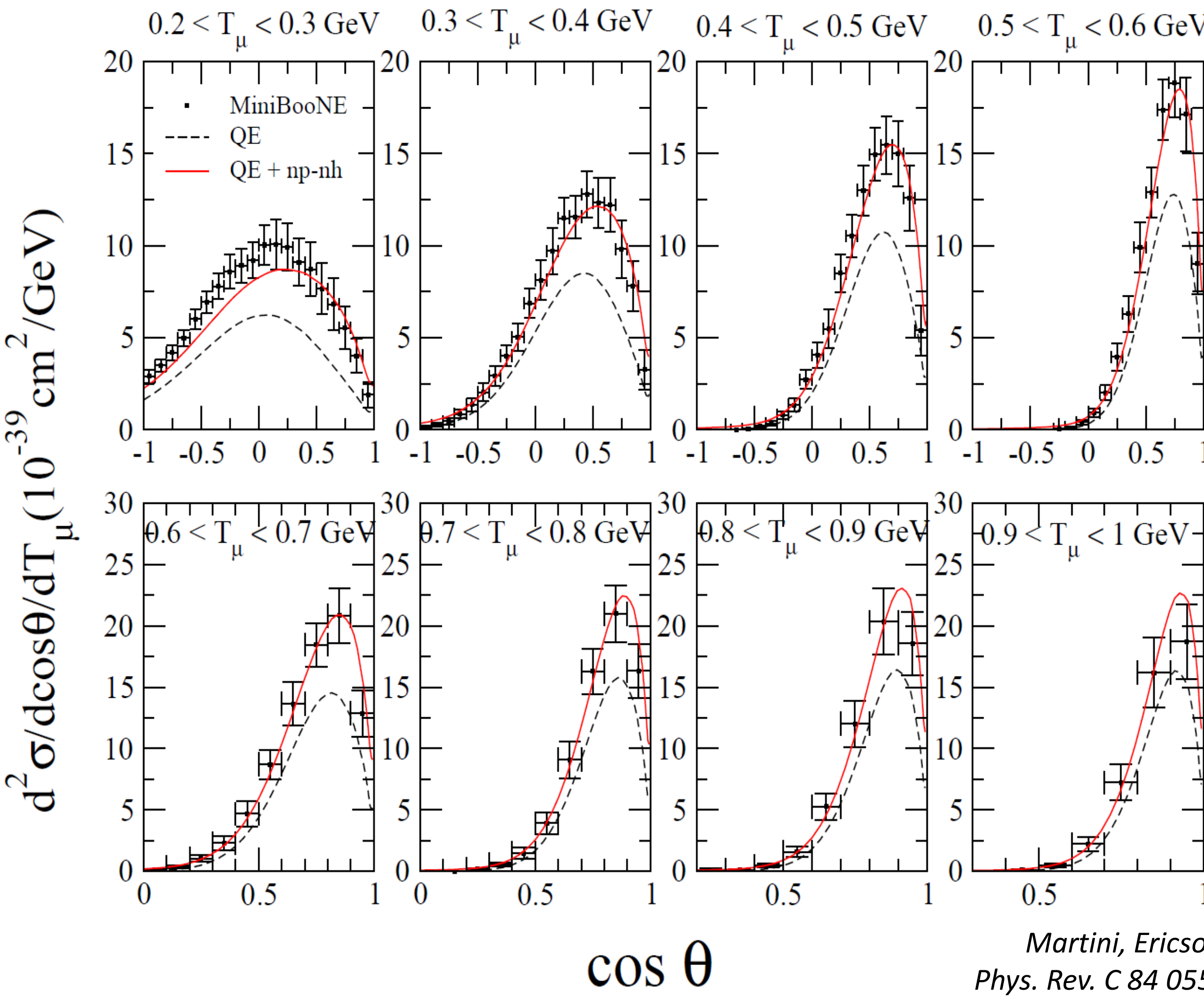
Flux-integrated differential cross section is where theorists and experimentalists meet for ν interaction

MiniBooNE CCQE-like flux-integrated double differential cross section

$$\frac{d^2\sigma}{dT_l\ d\cos\theta} = \frac{1}{\int \Phi(E_\nu)\ dE_\nu} \int dE_\nu \left[\frac{d^2\sigma}{d\omega\ d\cos\theta}\right]_{\omega=E_\nu-E_l} \Phi(E_\nu)$$

ν

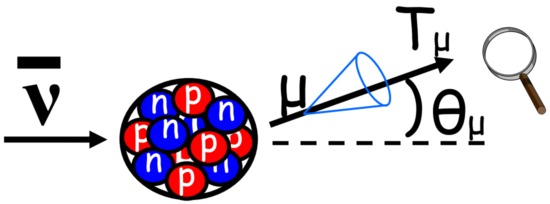




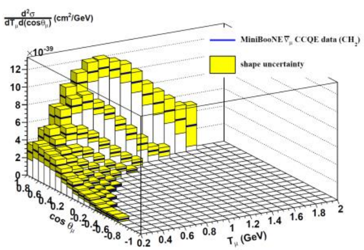
Martini, Ericson, Chanfray,
Phys. Rev. C 84 055502 (2011)

- Good agreement with data once multinucleon contributions are included
- Similar conclusions obtained by different theoretical calculations (see later)

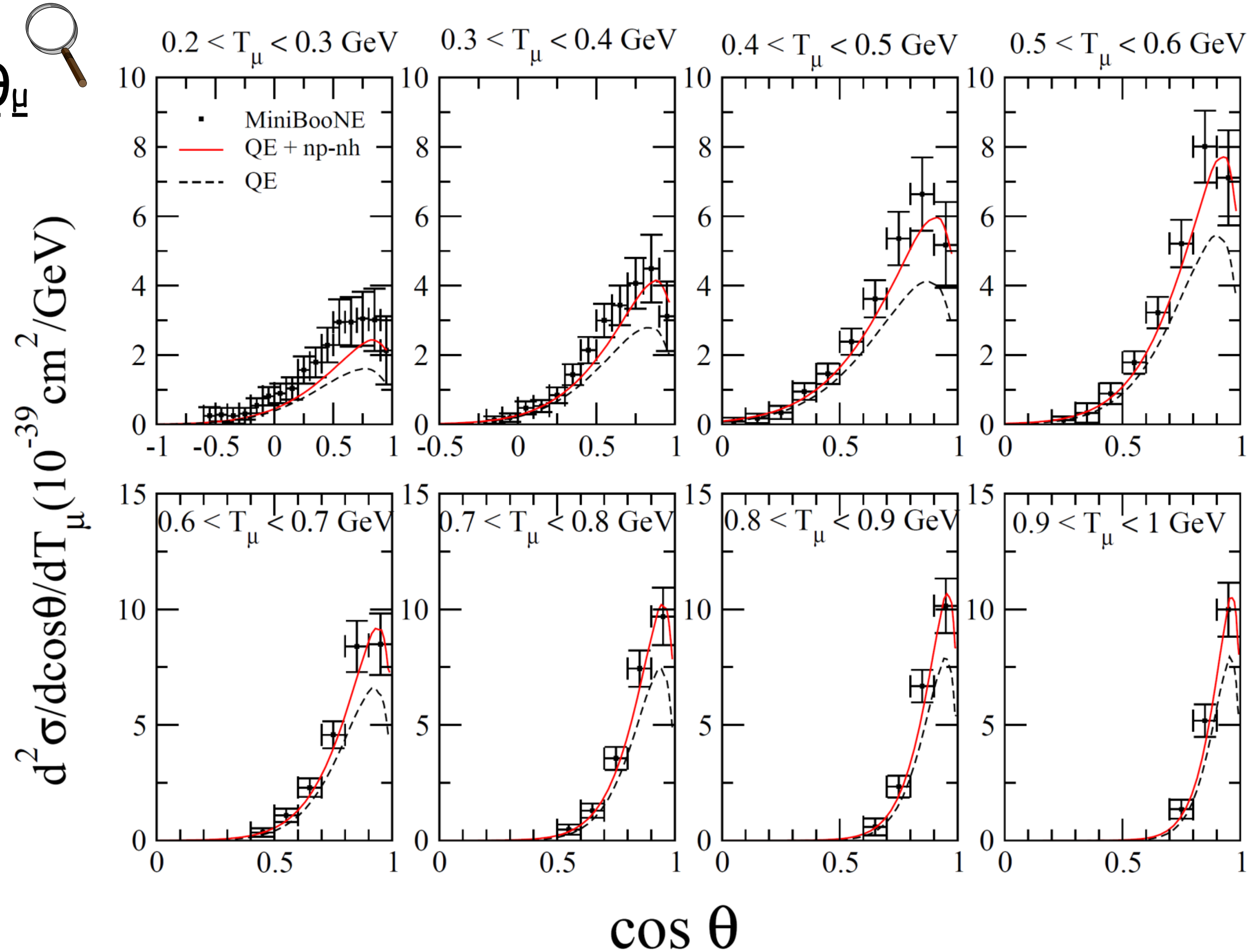
MiniBooNE CCQE-like flux-integrated double differential cross section



$\bar{\nu}$



MiniBooNE, Phys. Rev. D 88 032001 (2013)



Martini, Ericson, Phys. Rev. C 87 065501 (2013)

Similar conclusion also for the MiniBooNE CCQE-like antineutrino cross sections

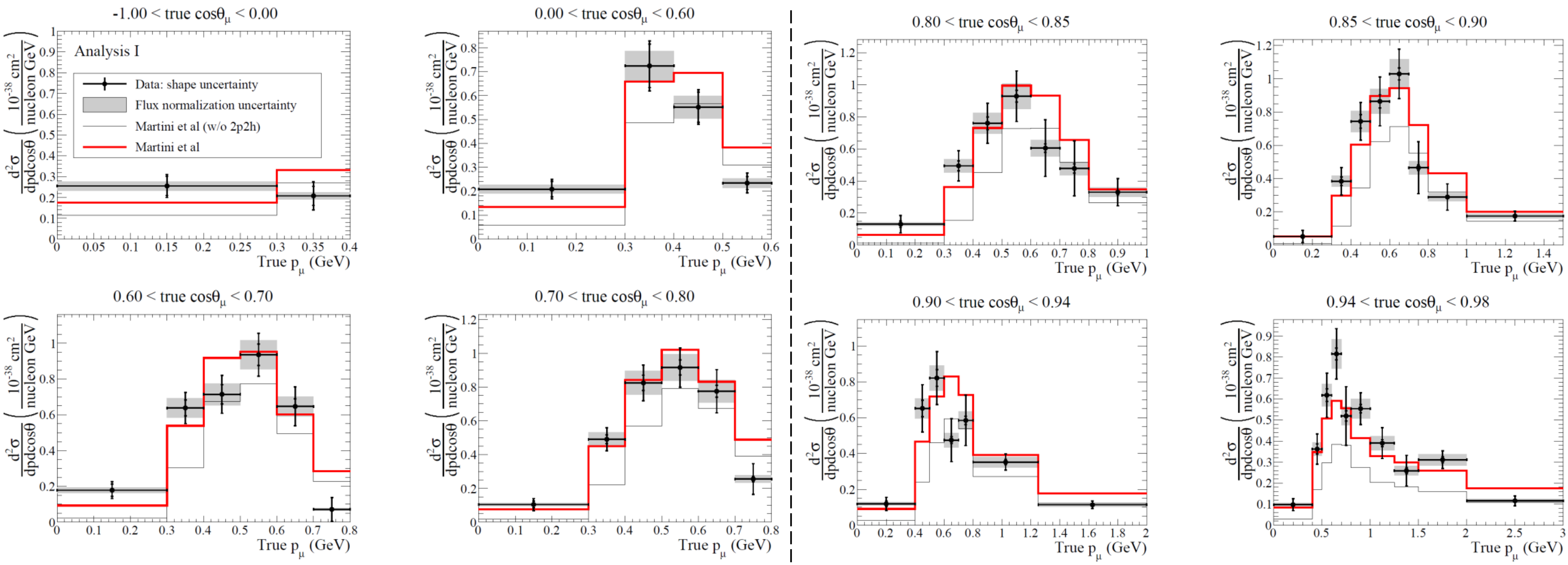
The **CC0π** measurement

After MiniBooNE, it has become more popular to present the data in terms of final state particles

CC0π = CCQE-like without subtraction of π absorption background (CC0π ≥ CCQE-like)

PHYSICAL REVIEW D **93,** 112012 (2016)

Measurement of double-differential muon neutrino charged-current interactions on C_8H_8 without pions in the final state using the T2K off-axis beam



Including np-nh

Without np-nh

Better agreement including np-nh

The **CC0π** measurement

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CC0π = CCQE-like without subtraction of π absorption background

PHYSICAL REVIEW D **93,** 112012 (2016)

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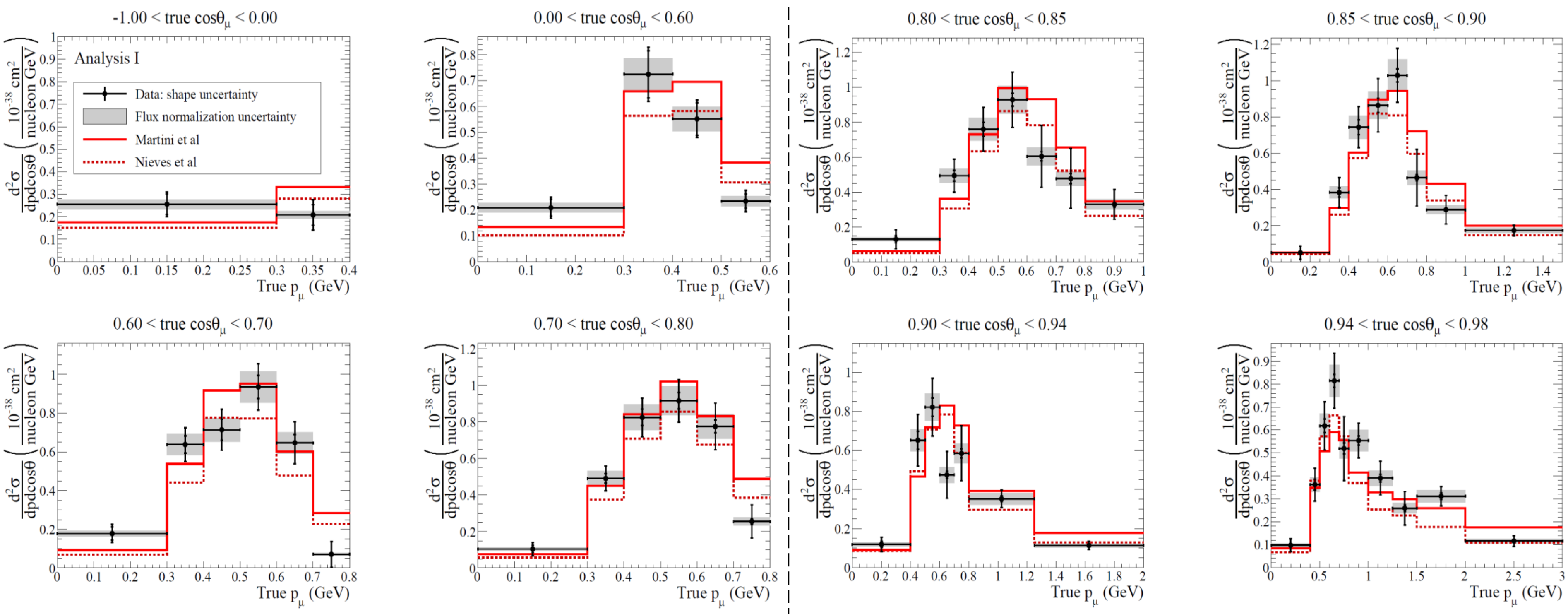


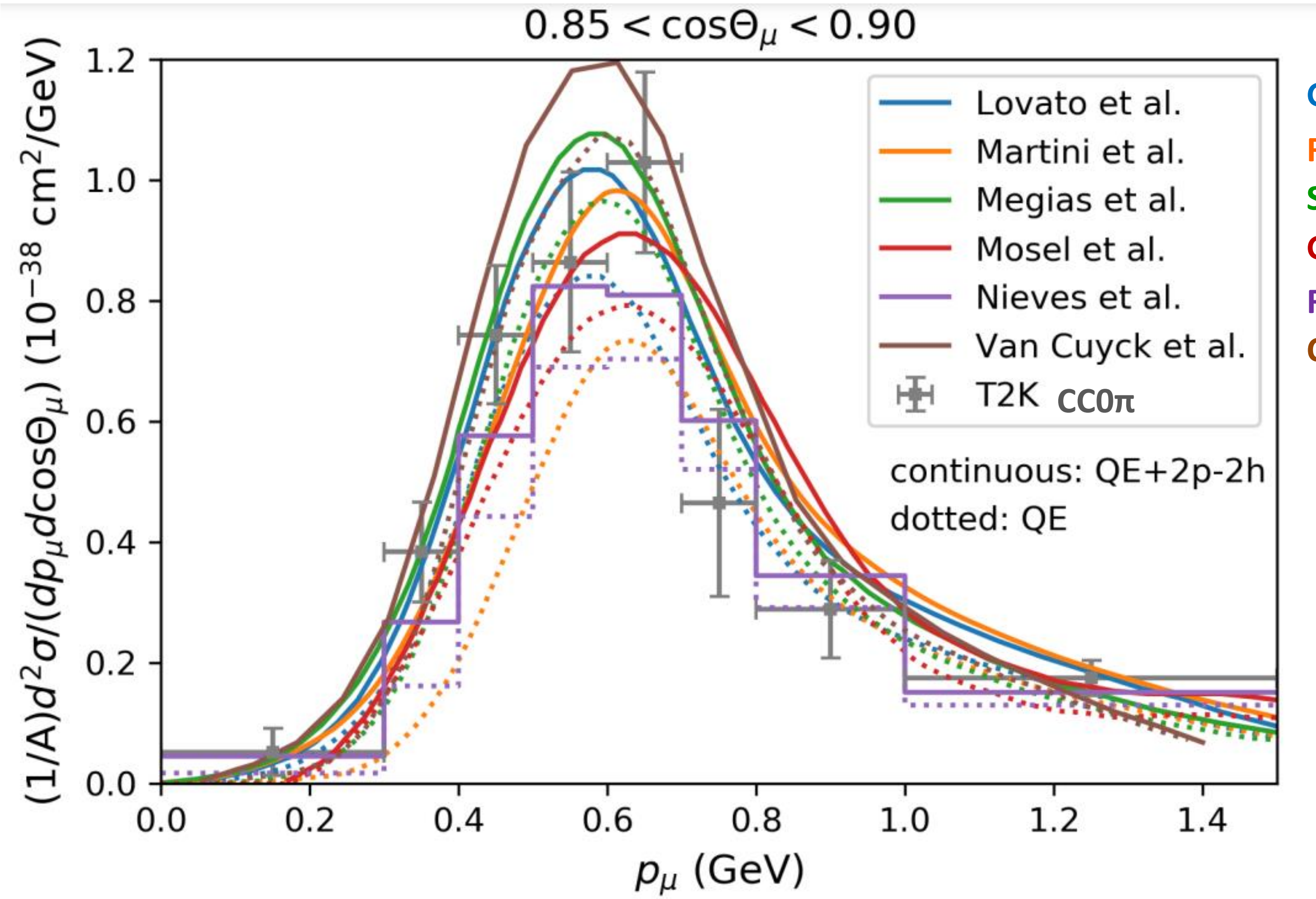
Martini et al.

Nieves et al.

Differences between models' predictions

Comparison between different CCQE+2p-2h theoretical predictions

A. Branca et al. Symmetry 13 (2021) 9, 1625



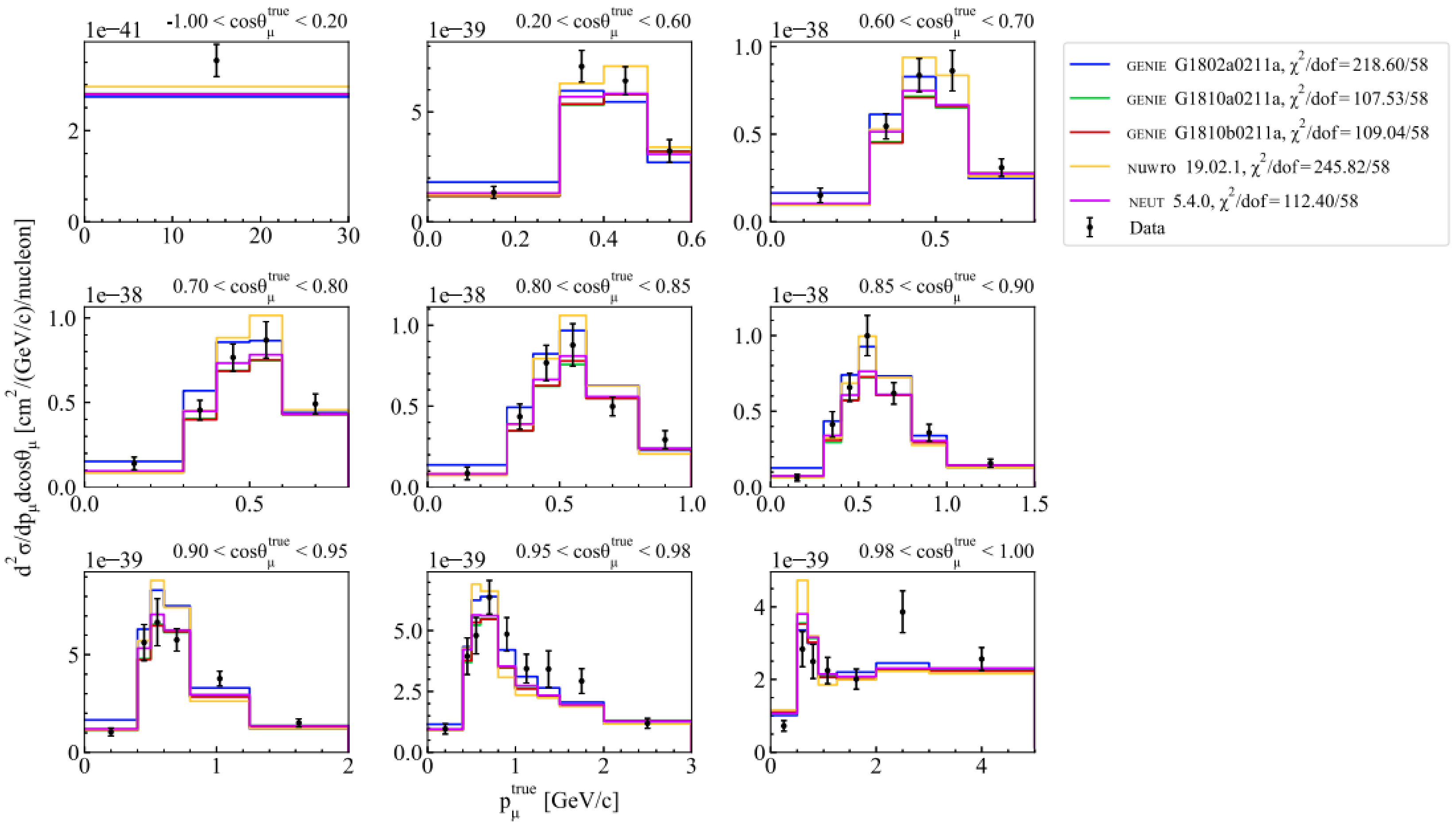
GFMC (Argonne, Los Alamos)
RPA (Lyon)
SuSAv2 (Granada, MIT,Sevilla, Torino)
GiBUU (Giessen)
RPA (Valencia)
CRPA (Ghent)

Several theoretical calculations agree on the crucial role of 2p-2h to reproduce data but there are discrepancies between the different models' predictions

⇨ 2p-2h are one of the most important source of the cross section uncertainties (systematic errors in oscillation experiments)

The T2K **CC0π** data and the Monte Carlo predictions

M. BUIZZA AVANZINI *et al.* PHYS. REV. D **105**, 092004 (2022)



Differences in the MC predictions (CCQE, 2p-2h and π absorption modeling)

p.s. The effort to implement different 2p-2h models in several Monte Carlo is still in progress

Monte Carlo Event Generators

Monte Carlo event generators connects theoretical models to experimental measurements

Main Event Generators for neutrino interactions:

Genie — UNIVERSAL NEUTRINO GENERATOR & GLOBAL FIT

GiBUU

NEUT

NuWro — Wroclaw Neutrino Event Generator

L. Alvarez-Ruso et al., EPJ Spec. Top. 230, 4449 (2021)

O. Buss et al., Phys.Rept. 512 1-124 (2012)

Y. Hayato and L. Pickering, EPJ Spec. Top. 230, 4469 (2021)

T. Golan et al., NPB 229–232, 499 (2012)

PHYSICAL REVIEW D **105,** 092004 (2022)

Comparisons and challenges of modern neutrino-scattering experiments

M. Buizza Avanzini,[1] M. Betancourt,[2] D. Cherdack,[3] M. Del Tutto,[2,4] S. Dytman,[5] A. P. Furmanski,[6,7] S. Gardiner,[2] Y. Hayato,[8] L. Koch,[9] K. Mahn,[10] A. Mastbaum,[11] B. Messerly,[5,7] C. Riccio,[12,13] D. Ruterbories,[14] J. Sobczyk,[15] C. Wilkinson,[16] and C. Wret[14]

Main models implemented for the quasielastic and 2p-2h:

- Relativistic global and local Fermi Gas
- RPA
- Spectral Function
- SuperScaling (SuSAv2)

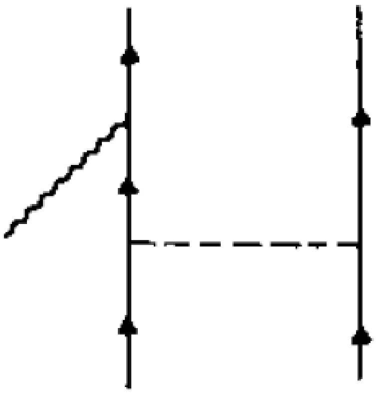
[For the illustration of the different models see for example the cross section lectures at the GIF school]

Some details on 2p-2h

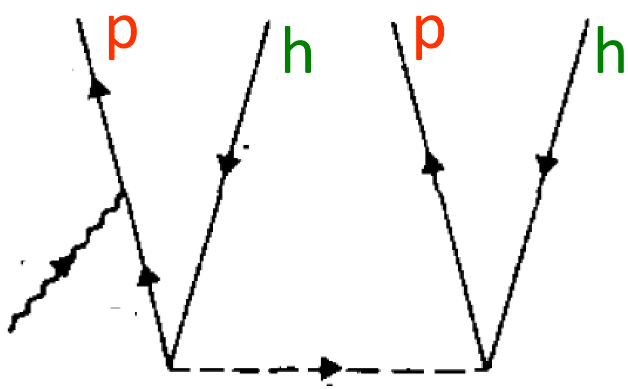
Two particle-two hole sector (2p-2h)

Three equivalent representations of the same process

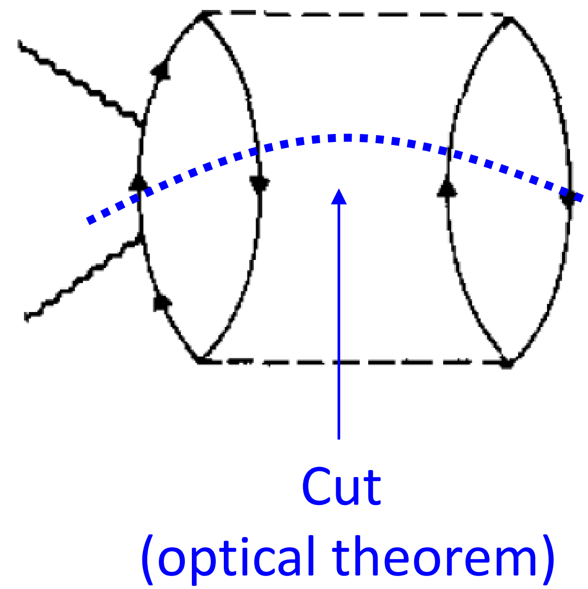
2 body current



2p-2h matrix element



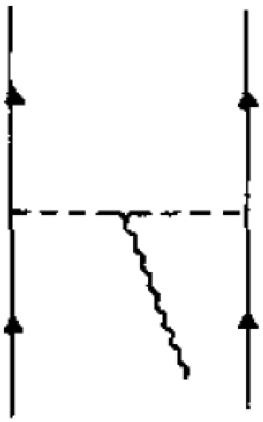
2p-2h response



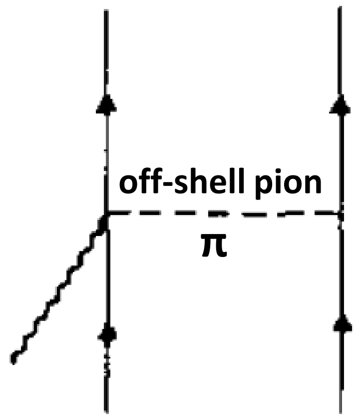
Final state: two particles-two holes

Diagrams for 2 body currents

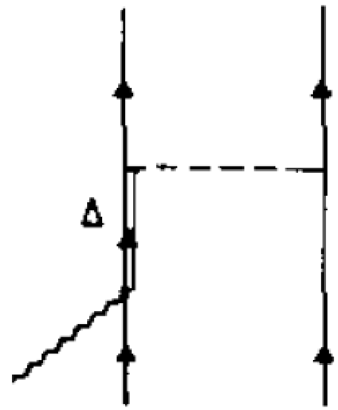
Meson Exchange Currents (MEC) J^{MEC}



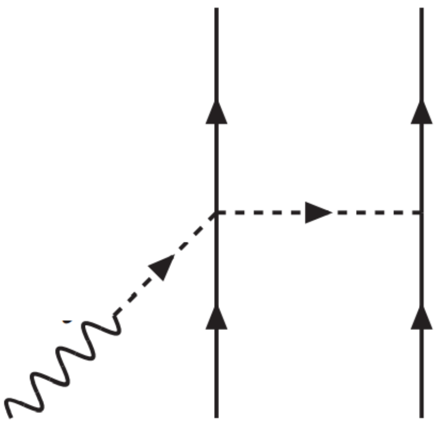
Pion in flight



Seagull or Contact

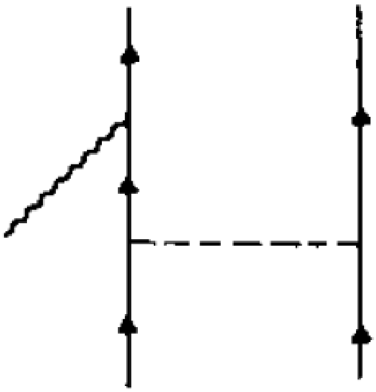


Delta



Pion pole (purely axial)

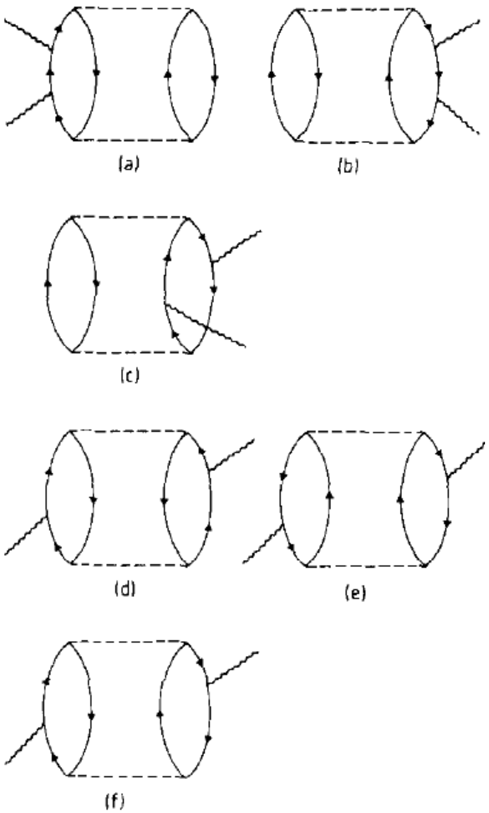
Nucleon-Nucleon Correlations (SRC) J^{corr}



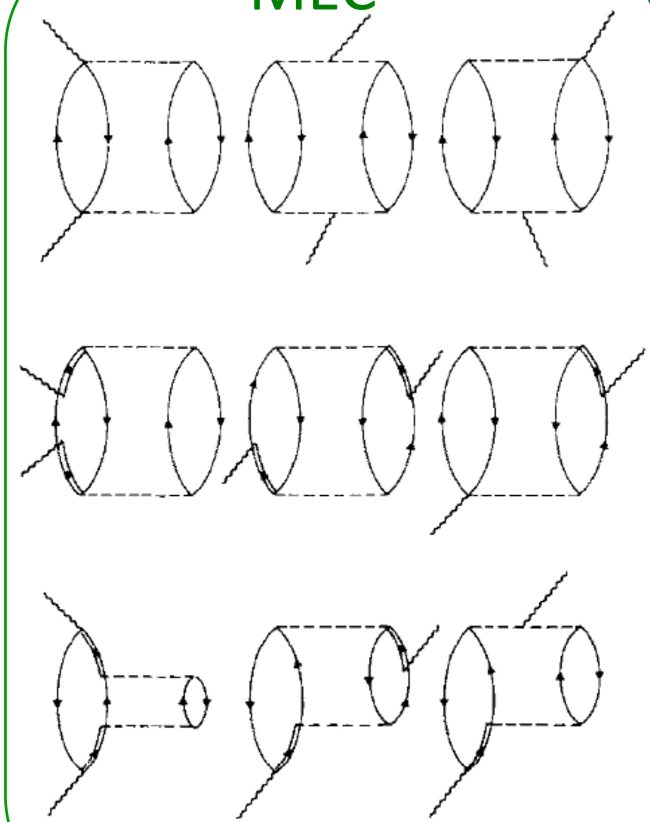
- An additional two-body current to be included in the framework of independent particle models for QE such as the Fermi Gas or Hartree-Fock.
- Absent in the approaches which start from the description of the nucleus in terms of correlated wave functions (such as CBF spectral function or GFMC) since the hadronic tensor of the one body current already includes this contribution.
- **There is a risk of a double counting of SRC in the Monte Carlo if different contributions to the neutrino cross sections are taken from different models.**

Some diagrams for 2p-2h responses

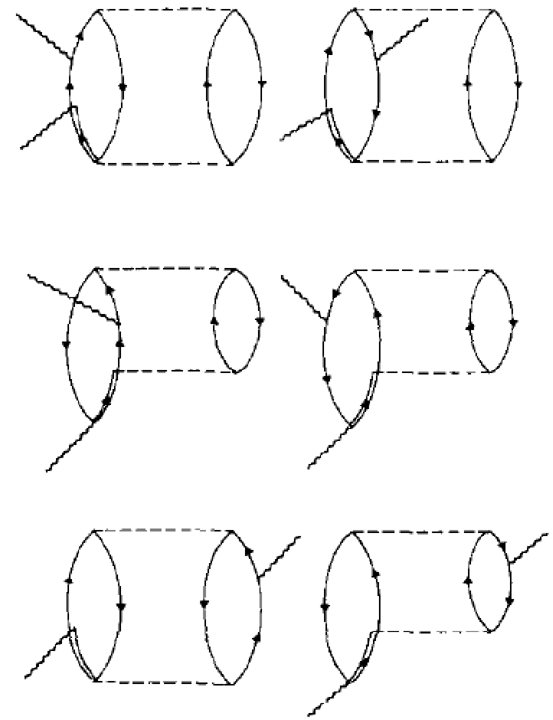
NN correlations



MEC



NN correlation-MEC interference



also called
1-body—2-body interference

Alberico, Ericson, Molinari, Ann. Phys. 154, 356 (1984)

Main difficulties in the np-nh sector

$$W^{\mu\nu}(\mathbf{q},\omega) = W^{\mu\nu}_{1p1h}(\mathbf{q},\omega) + W^{\mu\nu}_{2p2h}(\mathbf{q},\omega) + \cdots$$

$$W^{\mu\nu}_{2p-2h}(\mathbf{q},\omega) = \frac{V}{(2\pi)^9}\int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{m_N^4}{E_1E_2E'_1E'_2}\theta(p'_2-k_F)\theta(p'_1-k_F)\theta(k_F-h_1)\theta(k_F-h_2)$$
$$\underbrace{\langle 0|J^\mu|\mathbf{h}_1\mathbf{h}_2\mathbf{p}'_1\mathbf{p}'_2\rangle\langle\mathbf{h}_1\mathbf{h}_2\mathbf{p}'_1\mathbf{p}'_2|J^\nu|0\rangle}_{\text{matrix elements}}\delta(E'_1+E'_2-E_1-E_2-\omega)\delta(\mathbf{p}'_1+\mathbf{p}'_2-\mathbf{h}_1-\mathbf{h}_2-\mathbf{q})$$

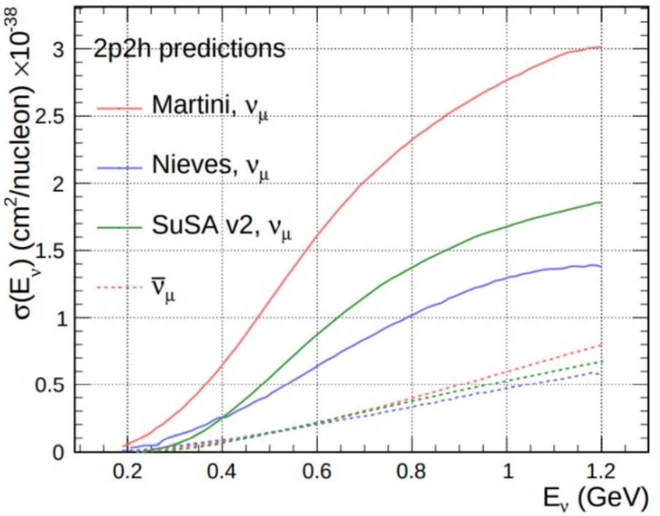
- 7-dimensional integrals $\int d^3h_1 d^3h_2\, d\theta'_1$ of thousands of terms
- Huge number of diagrams and terms
- Divergences (angular distribution; NN correlations contributions)
- Calculations for all the kinematics compatible with the experimental neutrino flux

Computing very demanding

Hence different approximations by different groups:

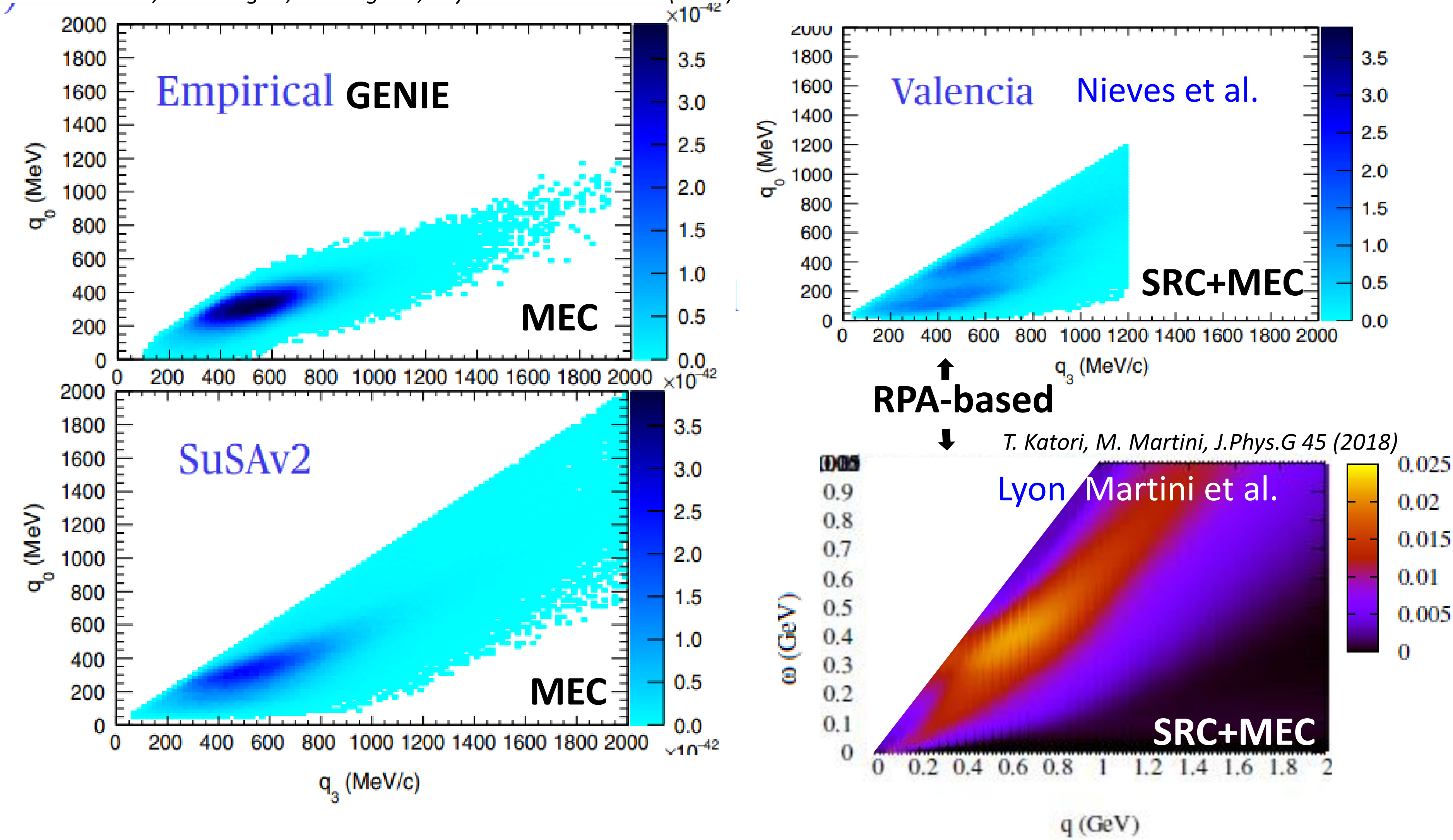
- choice of subset of diagrams and terms;
- different prescriptions to regularize the divergences;
- reduce the dimension of the integrals (7D --> 2D if non relativistic; 7D -->1D if h_1 = h_2 =0)

$\Rightarrow$ Different final results by different groups



Example of different results for 2p-2h in the (q,ω) or (q_0,q_3) plane

S. Dolan, G.D. Megias, S. Bolognesi, Phys.Rev.D 101 033003 (2020)

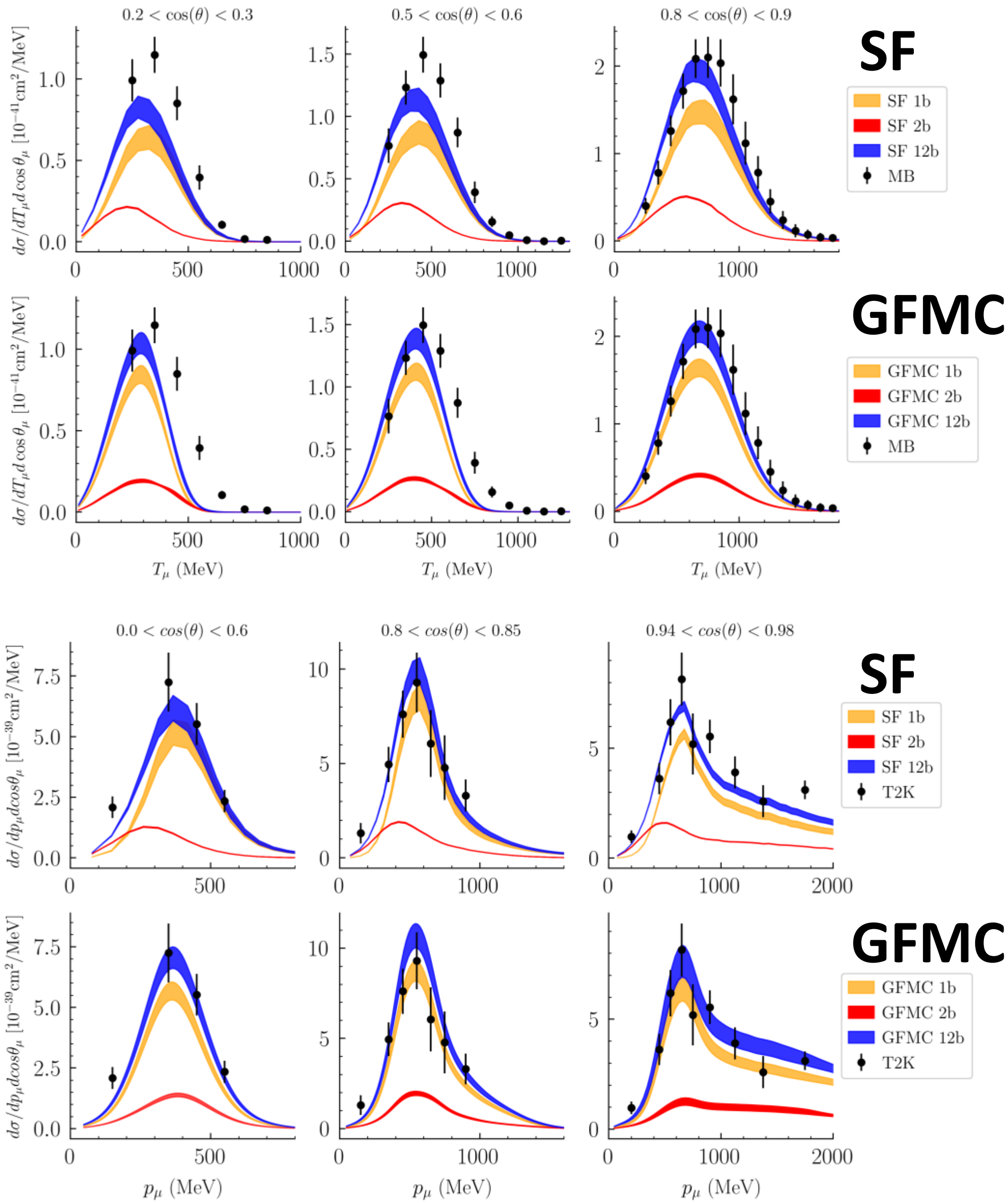


N.B. A one-to one correspondence between different exclusive channel's contributions can be misleading
[e.g. NN SRC contributions are part of the 2p-2h channel in RPA-based approaches while they are included in QE in SuSA.]

Example of different results in recent Spectral Function and Green's Function Monte Carlo (ab-initio) calculations

MiniBooNE

T2K



D. Simons et al. 2210.02455

N. Steinberg talk @ NUINT 2022

NEW

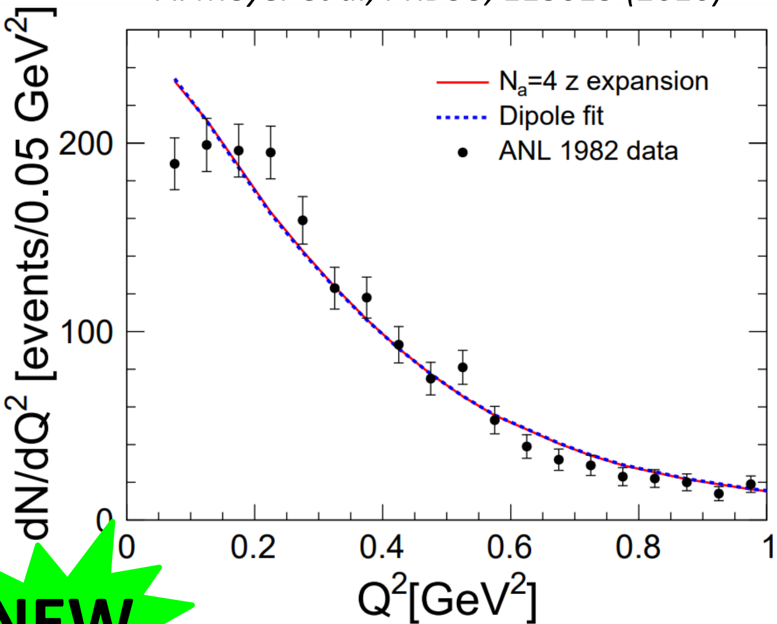
SF and GFMC 2-body contributions shifted

because of different

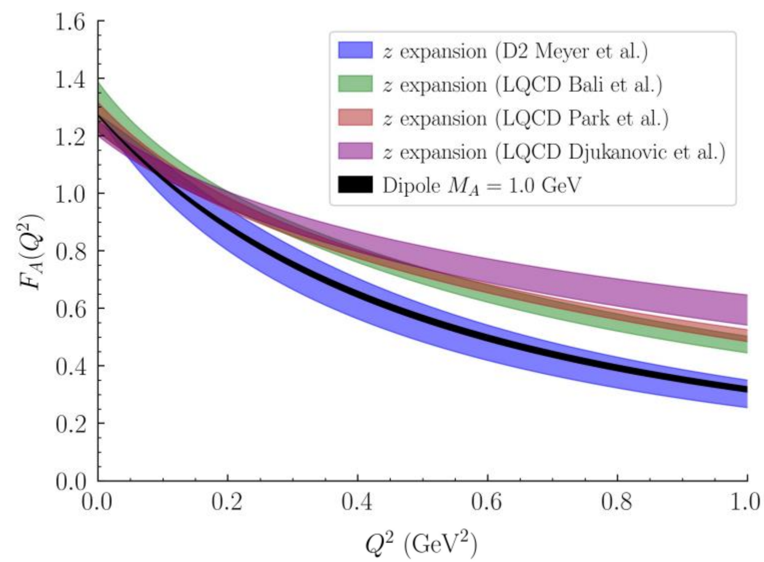
1 body – 2 body interference effects

Axial Form factor and Lattice QCD predictions

A. Meyer et al, PRD93, 113015 (2016)

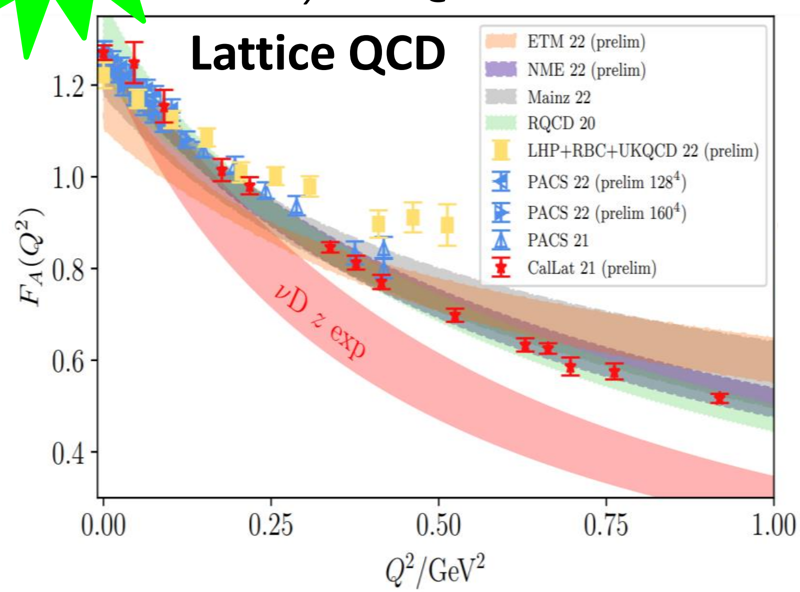


D. Simons et al. 2210.02455



NEW

A. Meyer talk @ NUINT 2022



- Dipole parameterization underestimates uncertainties
- Meyer et al. z-expansion: similar to dipole parameterization but larger errors
- Lattice QCD calculations show evidence of slow Q^2 falloff
- LQCD: much larger normalization at $Q^2 > 0.3$ GeV^2

Impact of enhanced axial form factor from LQCD

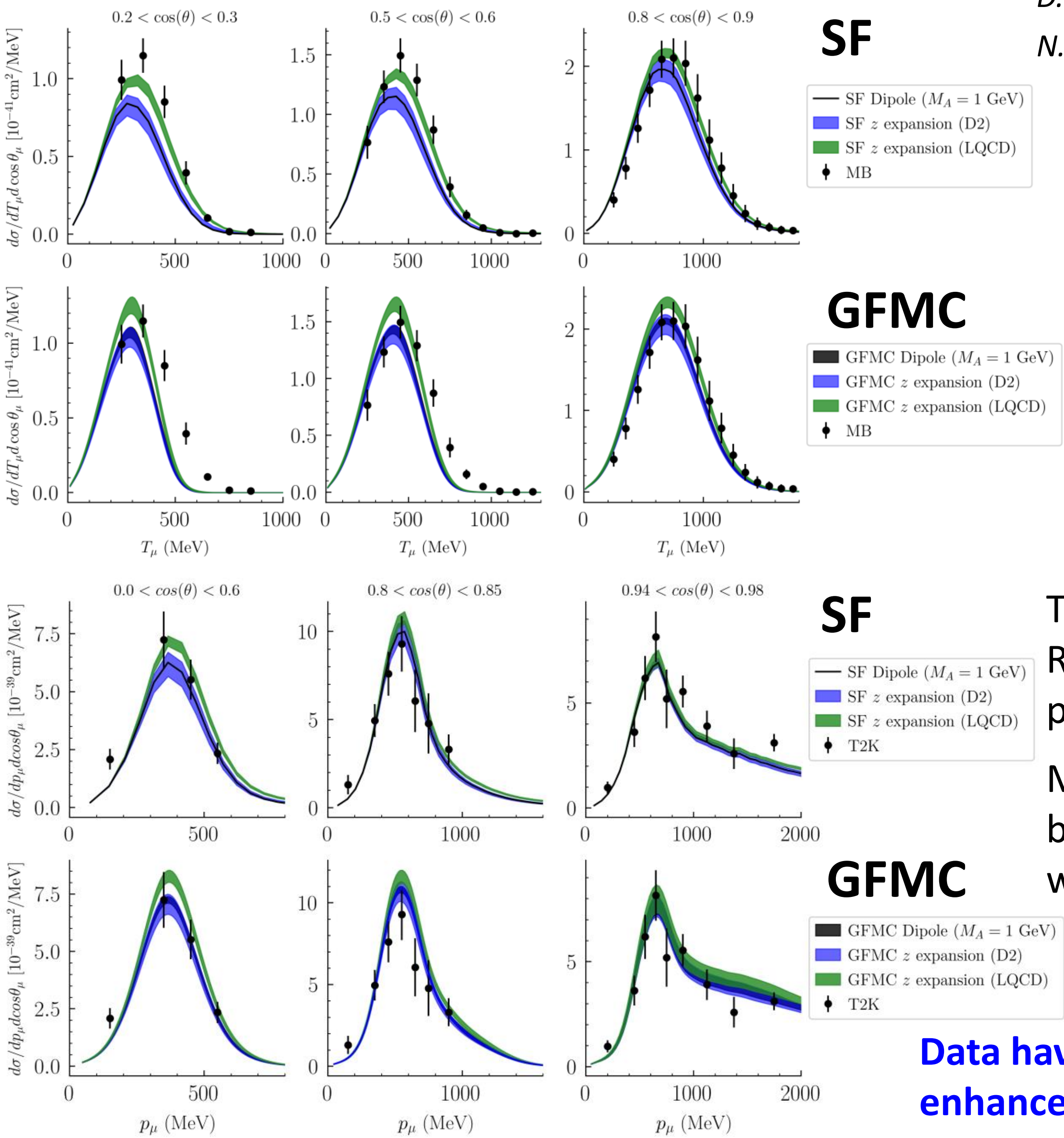
D. Simons et al. 2210.02455

N. Steinberg talk @ NUINT 2022

NEW

MiniBooNE

T2K



MiniBooNE:
Universal 10-20% increase in normalization with LQCD

T2K:
Results fairly independent of parameterization

Mostly due to T2K's lower beam energy hence lower Q^2 where form factors agree

Data have room for both 2p-2h and enhanced axial form factor for LQCD

Neutrino energy reconstruction

Energy reconstruction in neutrino oscillation experiments

Reconstructed ν energy

True ν energy

$$N_{\nu_\beta}(\overline{E_\nu}) \sim \int \Phi_{\nu_\alpha}(E_\nu) P_{\nu_\alpha \to \nu_\beta}(E_\nu, L, \{\Theta\}) \sigma_{\nu_\beta}(E_\nu) \epsilon_{\text{det.}} d(E_\nu, \overline{E_\nu}) dE_\nu$$

Number of detected events

ν flux	ν oscillation probability	ν cross section	Detector efficiency	Migration matrix

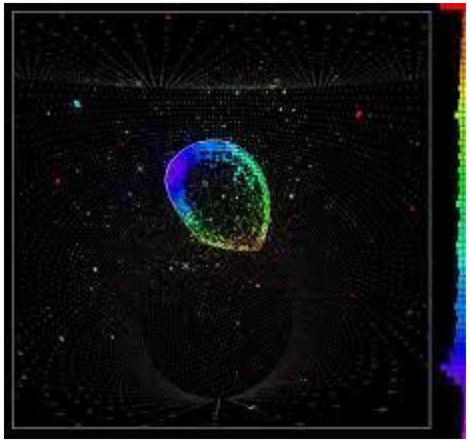
Two methods for ν energy reconstruction

Tracking detectors

- Use all the detected particles
- Calorimetric method

Cherenkov detectors

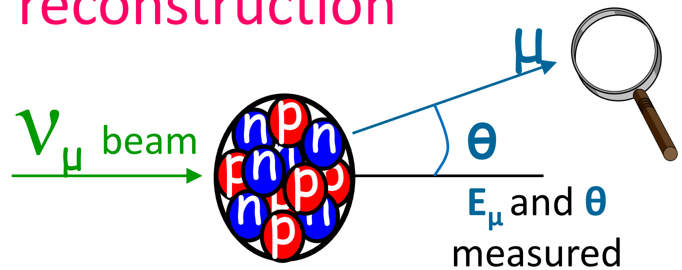
- Use only lepton (1 ring signal)
- Quasielastic-based method



Quasielastic-based neutrino energy reconstruction

Reconstructed neutrino energy

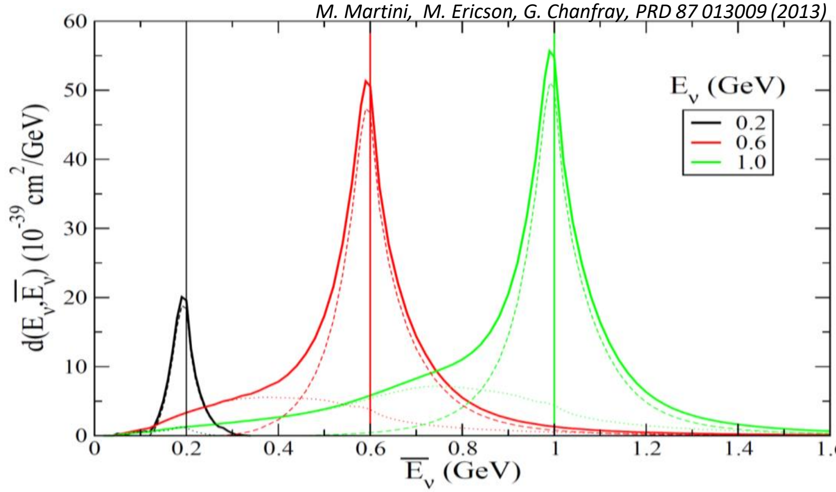
$$\overline{E_\nu} = \frac{m_p^2 - (m_n - E_b)^2 - m_\mu^2 + 2(m_n - E_b)E_\mu}{2\left(m_n - E_b - E_\mu + p_\mu \cos\theta_\mu\right)}$$



$\nu_\mu\, n \rightarrow \mu^- p$ via two-body kinematics

$\overline{E_\nu} = E_\nu$ exact only for CCQE with free nucleon

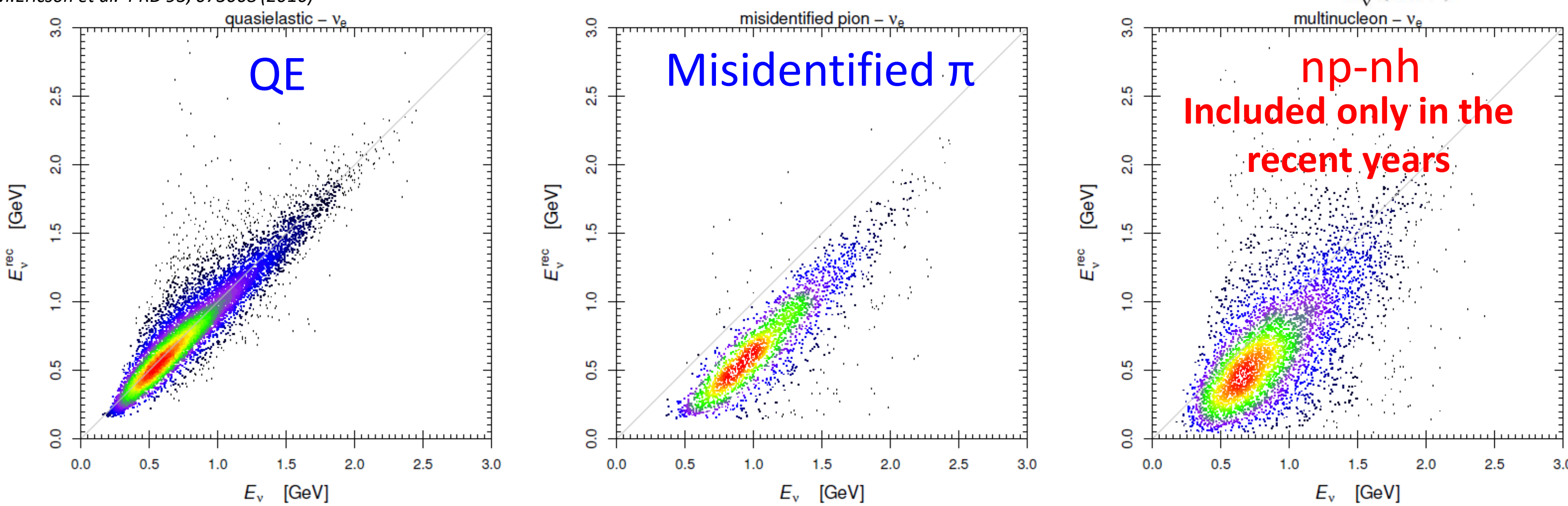
M. Martini, M. Ericson, G. Chanfray, PRD 87 013009 (2013)



$d(E_\nu, \overline{E_\nu})$

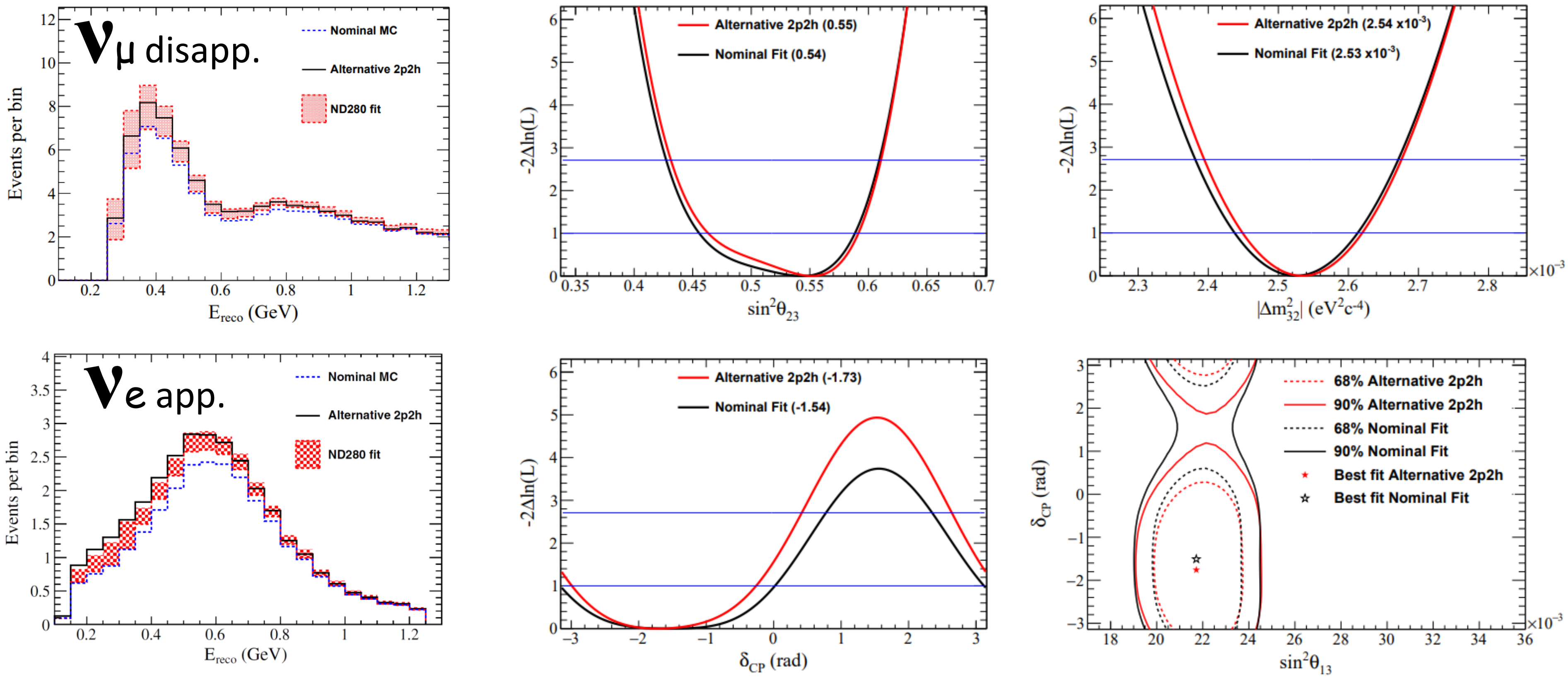
Migration matrix:
to take into account
nuclear effects

M.Ericson et al. PRD 93, 073008 (2016)

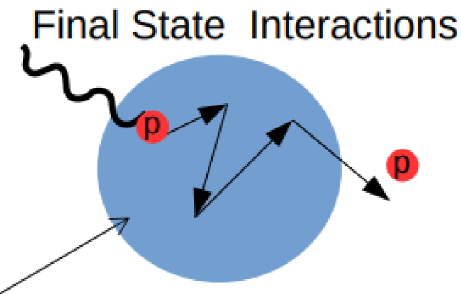


Impact of 2p-2h modeling on T2K oscillation analysis

T2K Phys.Rev.D 96 (2017) 9, 092006

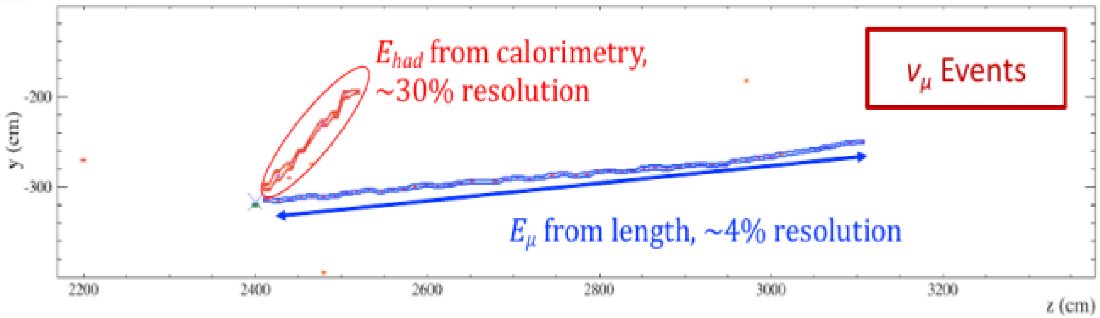


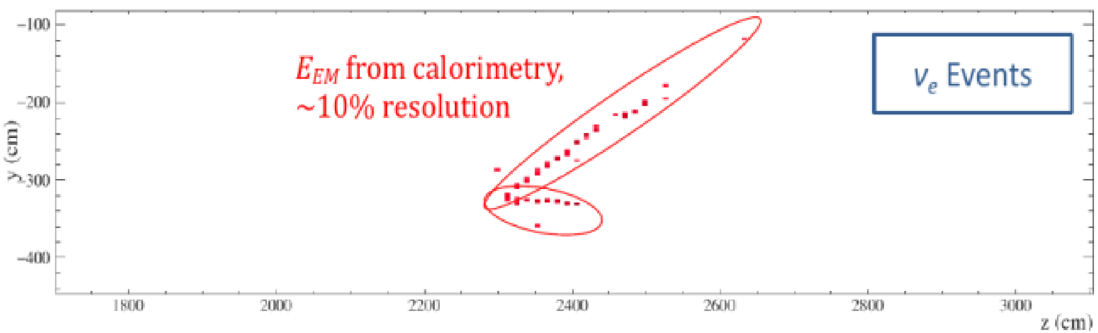
Eν reconstruction NOvA



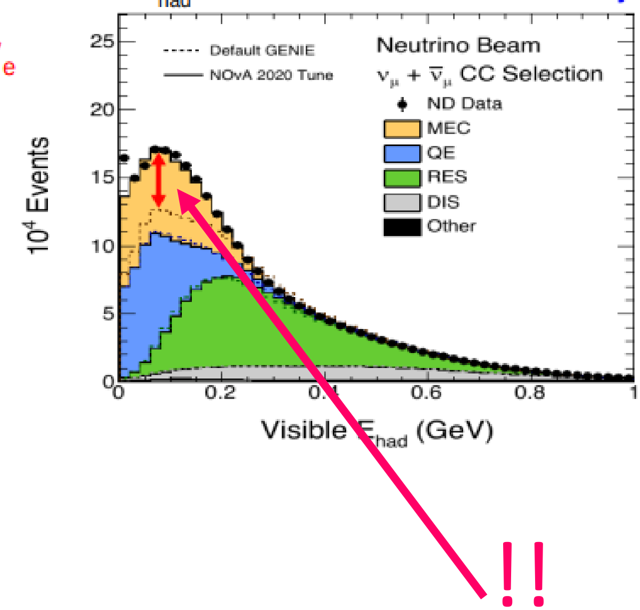
Calorimetric method

- **Eν reconstructed with hadronic deposits:**
  - important difference ν – $\bar{\nu}$: proton vs neutron (~undetected)
  - proton/pion energy smeared by Final State Interactions
- Different reconstruction and energy resolution for ν_μ and ν_e



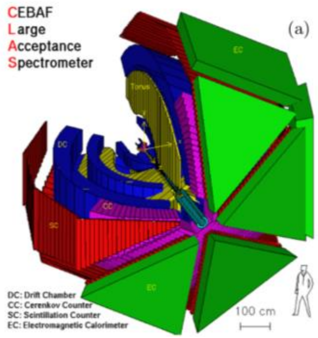


Important to tune model predictions for E_{had}

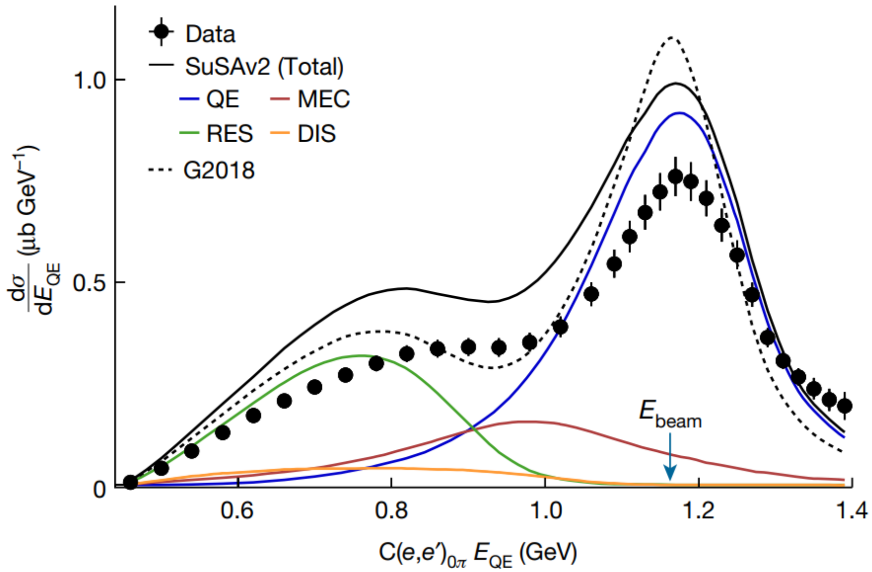


S. Bolognesi @ GIF school 119

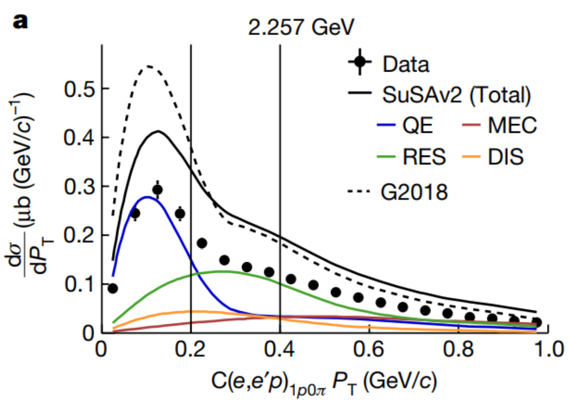
Electron-beam energy reconstruction for ν oscillation measurements



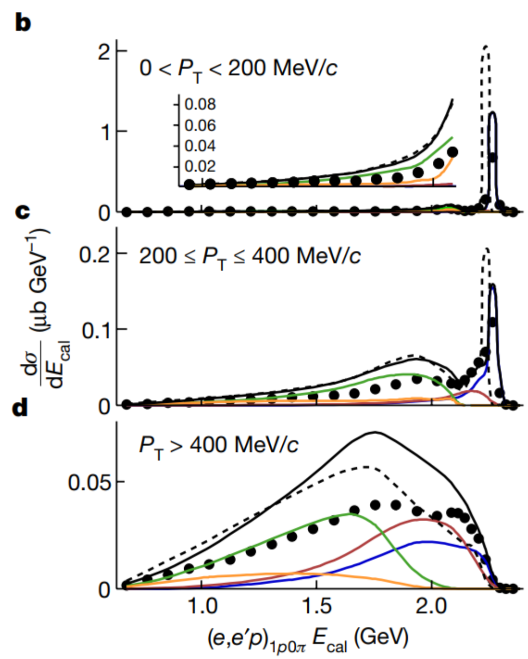
Nature 599 (2021) 7886, 565-570



QE-based (e,e')



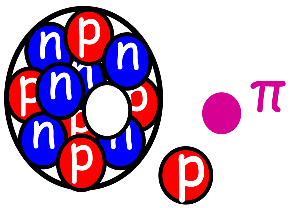
$$\mathbf{P}_T = \mathbf{P}_T^{e'} + \mathbf{P}_T^{p}$$



Calorimetric -based (e,e'p)

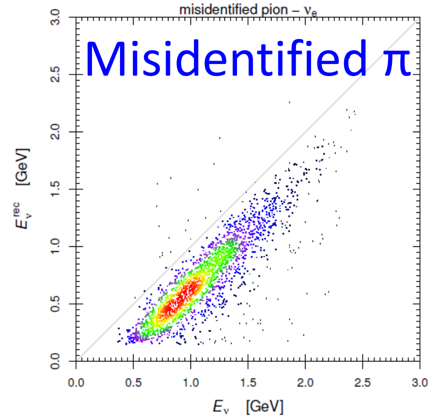
1π production

The one pion production channel

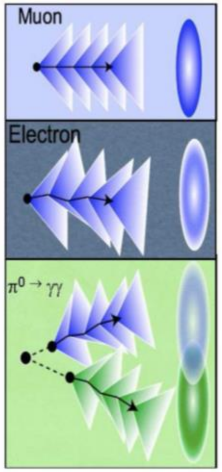


Important for several reasons:

- Misidentified π is part of the ν energy migration matrix



- In Cherenkov detectors NC1π^0 can mimic electron-like signal in $\nu_\mu \rightarrow \nu_e$ oscillation search



- There is an increasing interest on CC 2-ring signal (charged lepton and π) at Super-Kamiokande

$$E_{\nu_{reco}} = \frac{m_p^2 - (m_p - E_{bind} - E_\mu - E_\pi)^2 + |\vec{P}_\mu + \vec{P}_\pi|^2}{2\{m_p - E_{bind} - E_\mu - E_\pi + \hat{k}_\nu(\vec{P}_\mu + \vec{P}_\pi)\}}$$

Elementary vertices .vs. detection topologies

Elementary vertices (nucleon level) | Detection topologies (nucleus level)

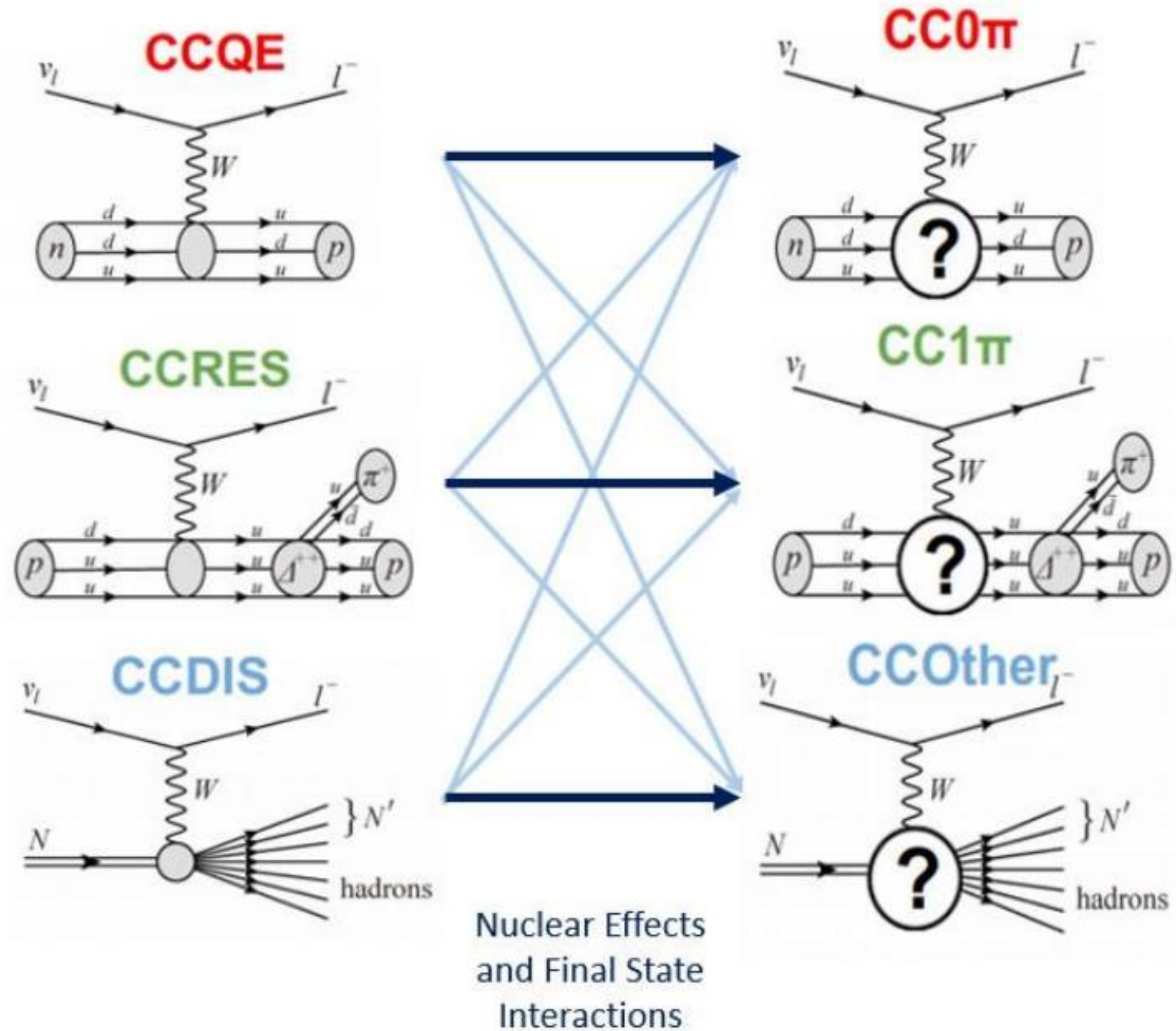


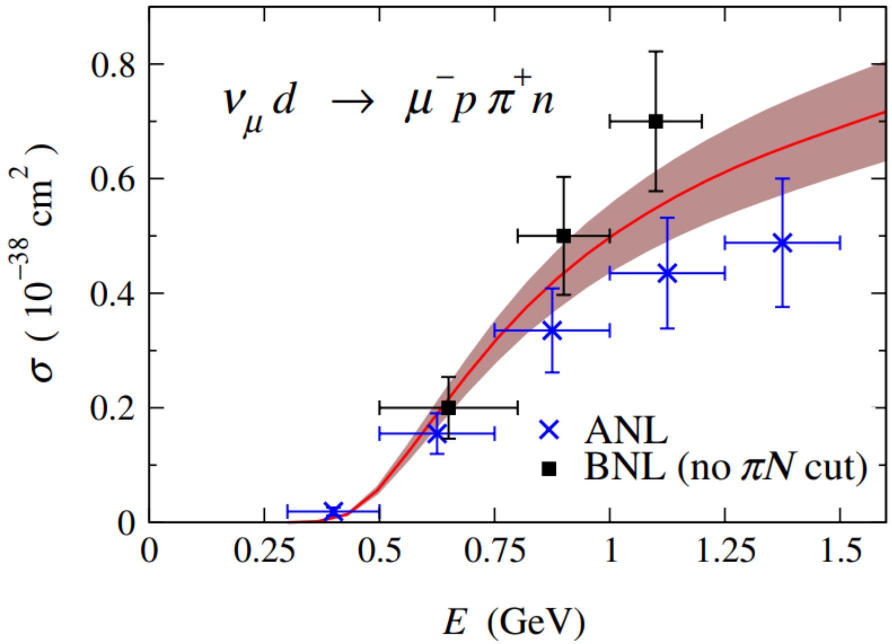
Figure from Dr. Stephen Dolan

Different interaction vertices can lead to the same final state due to nuclear effects and FSI

1π production in neutrino-deuteron scattering

- Discrepancies between "old" deuteron bubble-chamber data (Argonne ANL and Brookhaven BNL)
- Both ANL and BNL data suffer from a large flux-normalization error

E. Hernandez et al. Phys. Rev. D 87, 113009 (2013)

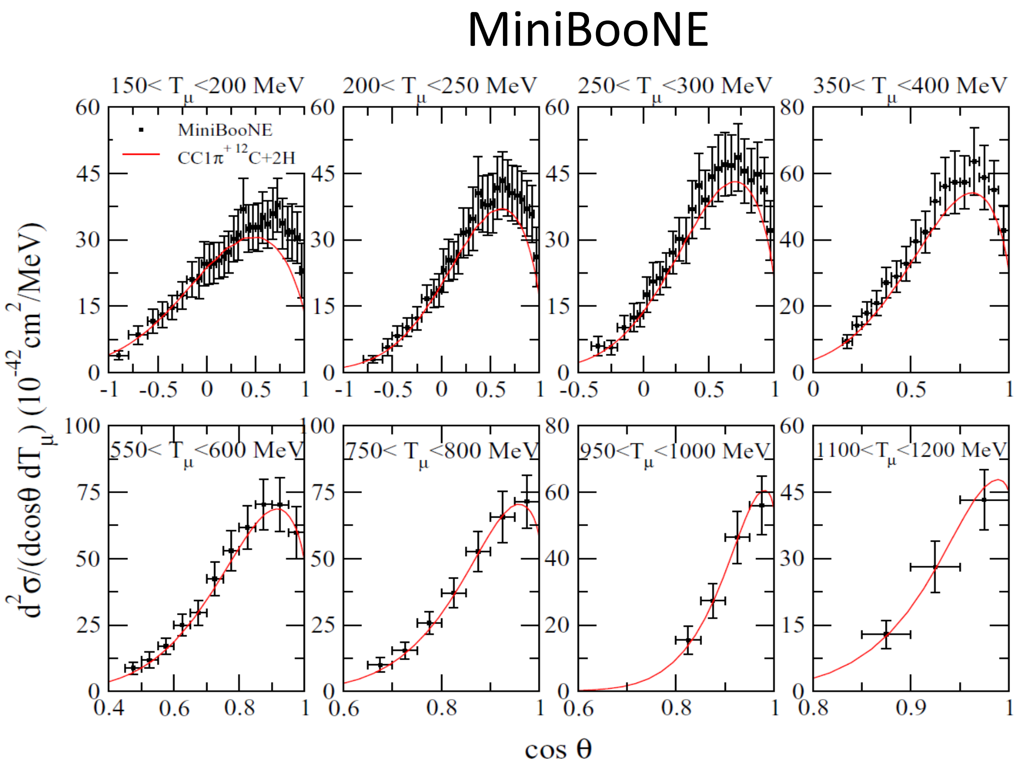


As for the CCQE, also for the 1π production there is a strong desire to repeat bubble-chamber experiments to better determine the axial form factors (in particular the C_5^A)

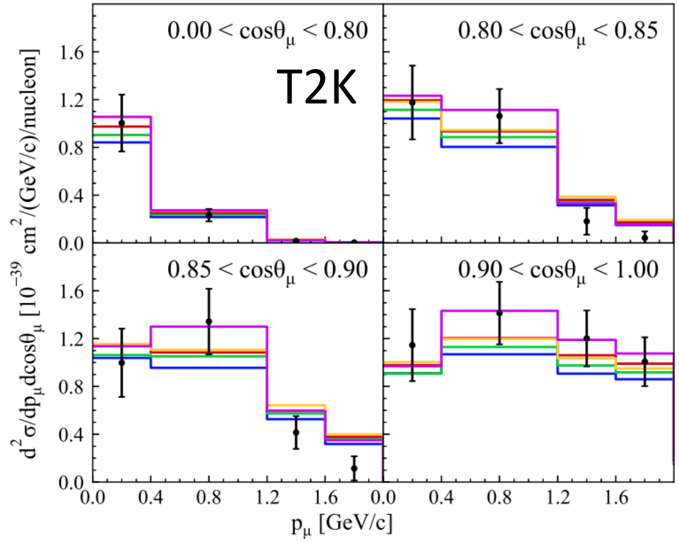
$$C_5^A(Q^2) = \frac{C_5^A(0)}{(1 + Q^2/M_{A\Delta}^2)^2}$$

CC1π+ flux-integrated differential cross sections on carbon

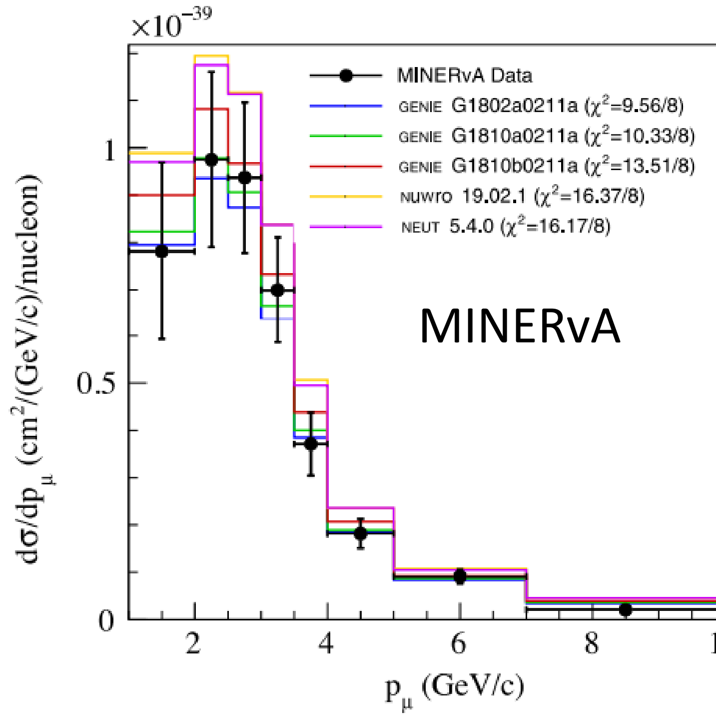
Results in terms of muon variables



M. Martini, M. Ericson, Phys. Rev. C 90 025501 (2014)



M. Buizza Avanzini et al. PRD 105, 092004 (2022)

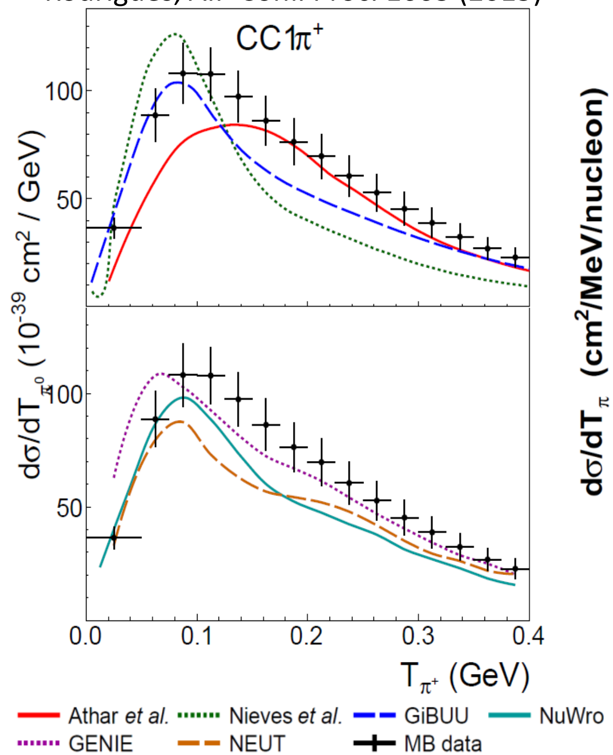


Reasonable agreement between models and data

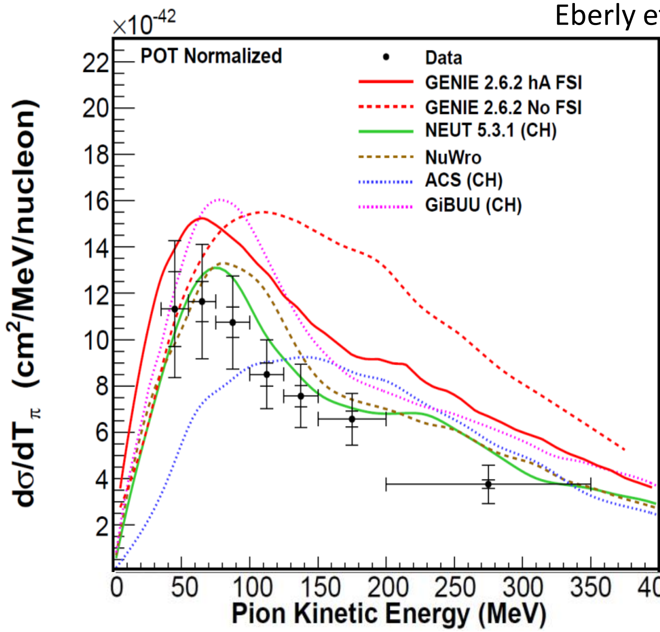
CC1π results in terms of pion variables

MiniBooNE

Rodrigues, AIP Conf. Proc. 1663 (2015)

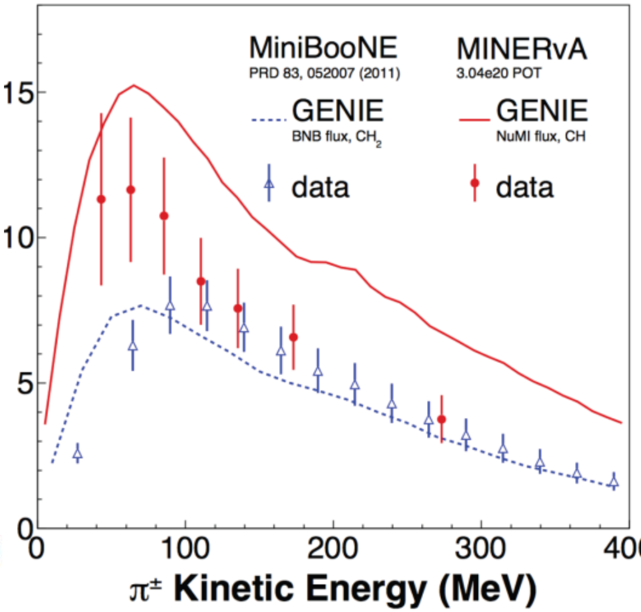


MINERvA

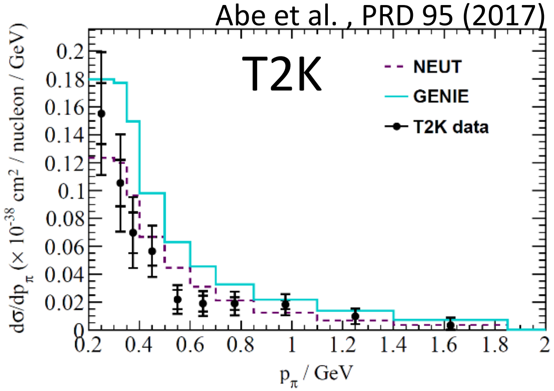


MiniBooNE - MINERvA

Eberly et al. , PRD 92 (2015)



Abe et al. , PRD 95 (2017)



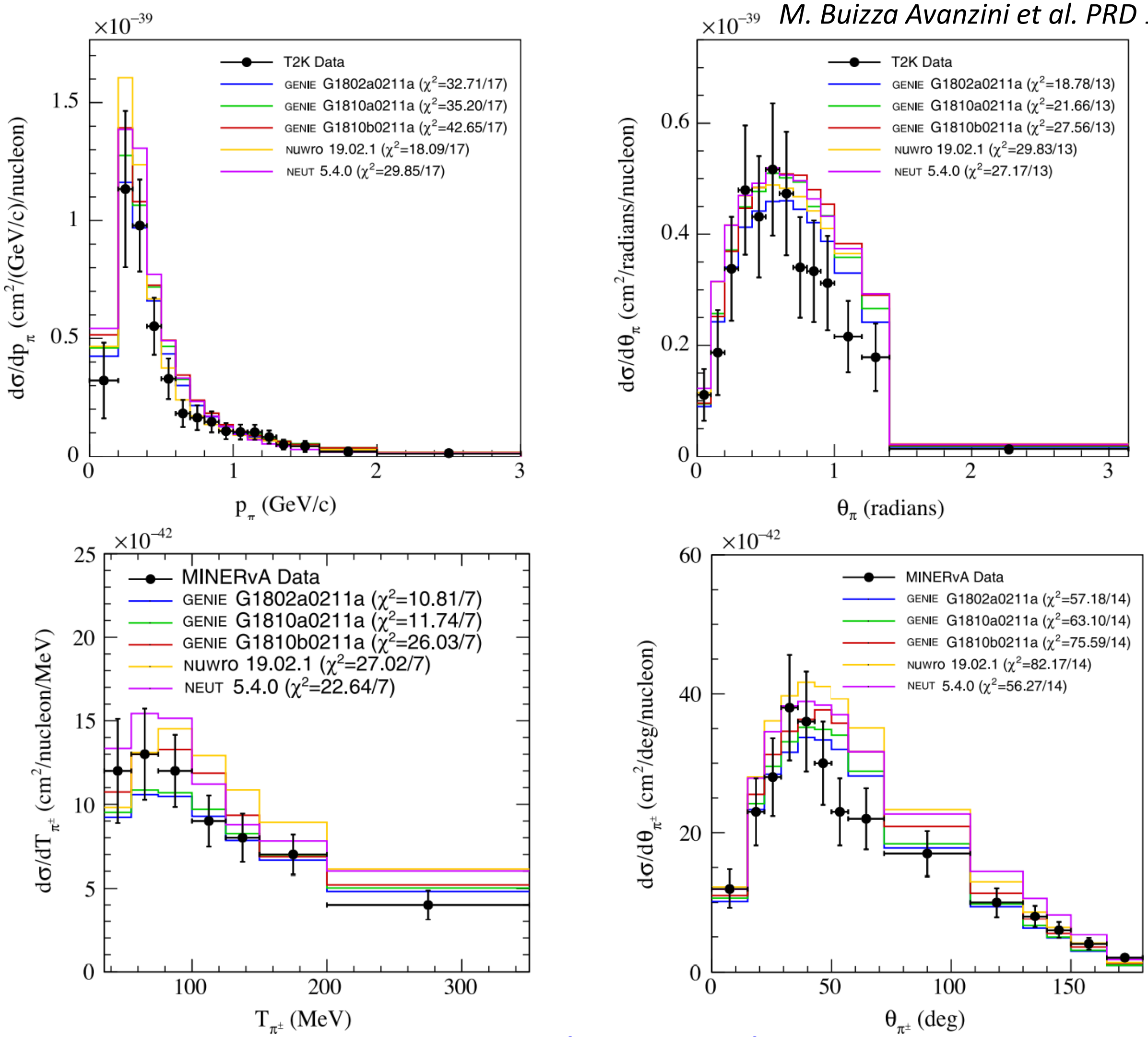
Historically many tensions

- models .vs. data ??
- models .vs. models??
- data .vs. data (through models)??

the 1π puzzle

Pion puzzle – T2K and MINERvA data .vs. Monte Carlo (2022)

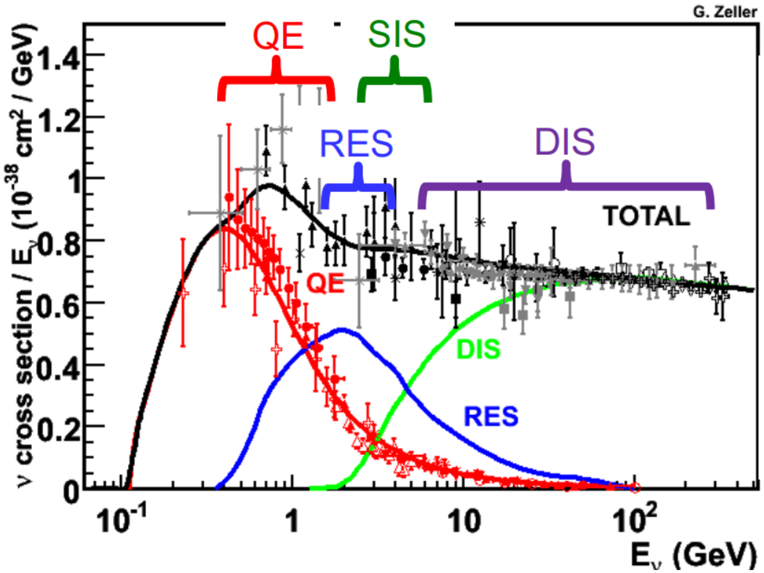
M. Buizza Avanzini et al. PRD 105, 092004 (2022)

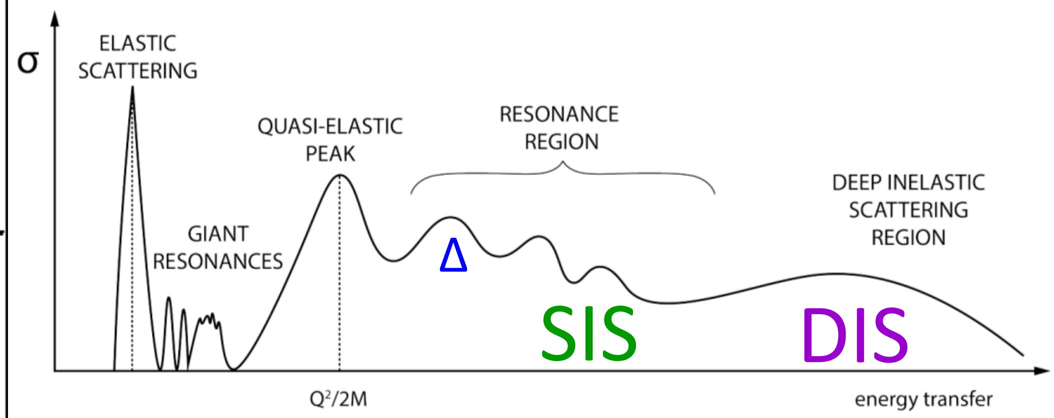


Tensions remain

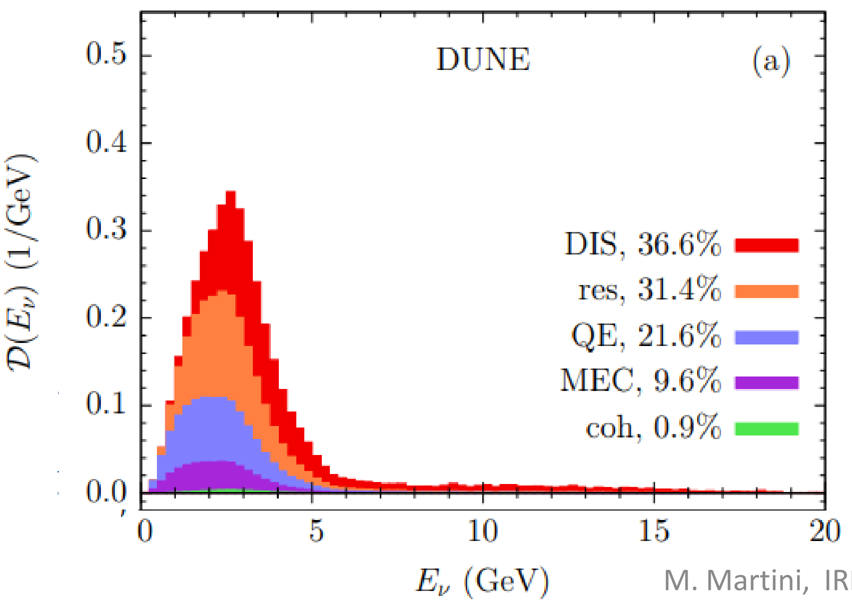
- The generators used to extract the cross section is often the one with the best description of the data
- Experimental collaborations have more advanced analyses in progress (T2K Vargas and MINERvA McFarland @ NuInt22)
- These Monte Carlo results are based on Δ dominated models
- None of the common event generators include nuclear medium effects for the Δ

Beyond Δ resonance





A. Ankowski



- The complications of pion data analyses lay not only on the modeling of primary production and pion FSI but also on the fact that all hadronic processes related to shallow inelastic scattering (SIS) and DIS regions must be modeled correctly
- **SIS and DIS have been minimally studied both experimentally and theoretically with neutrino scattering**
- **A major challenge, important in particular for DUNE**

T. Katori, M. Martini, J.Phys.G 45 1, 013001 (2018)
L. Alvarez-Ruso et al. Prog. Part. Nucl. Phys. 100, 1–68 (2018)
M. Sajjad Athar, J. G. Morfín, J.Phys. G 48, 034001 (2021)

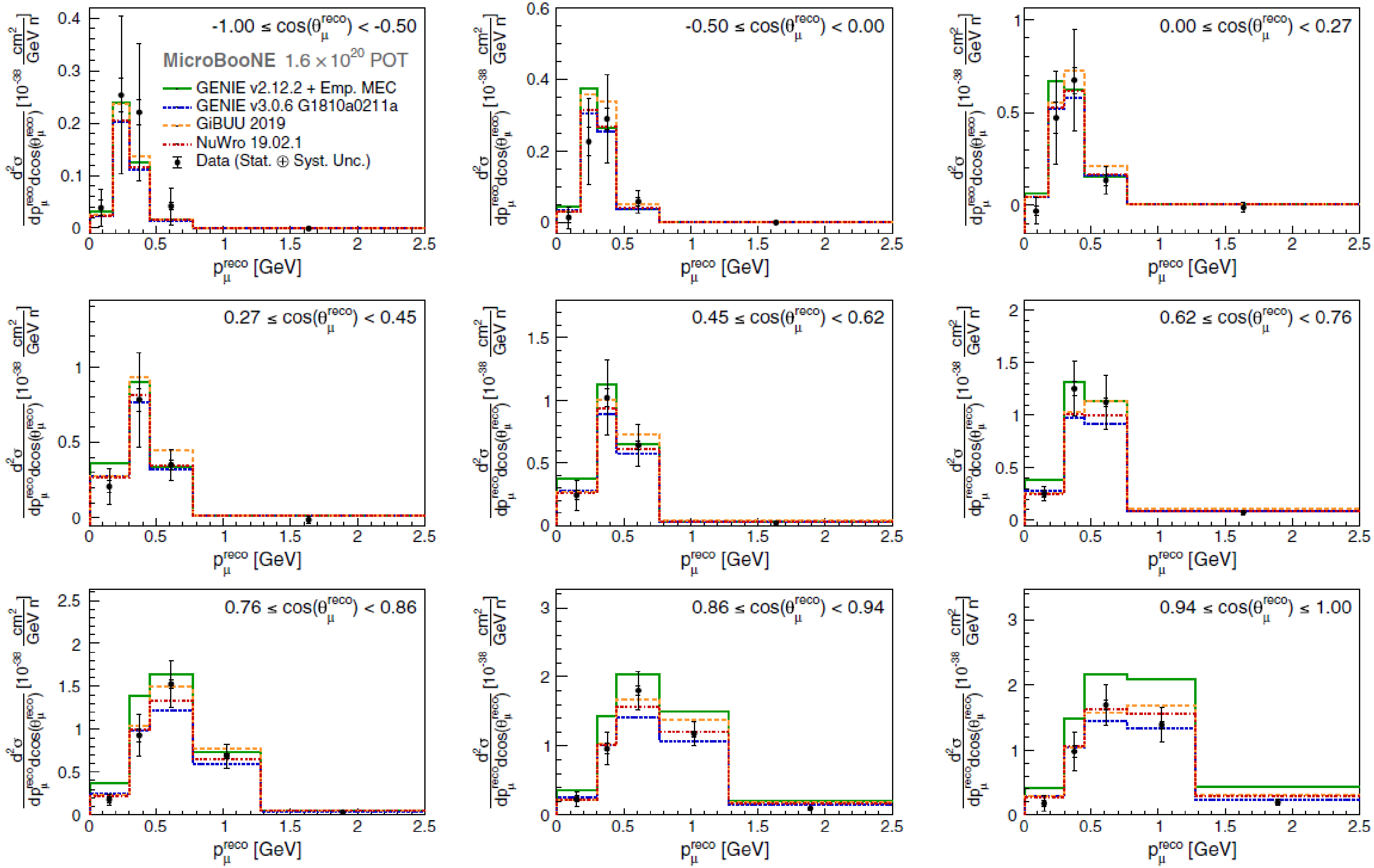
Recent hot topics:

- Argon cross sections (MicroBooNE)
- Semi-inclusive processes (proton detection)
- Single Transverse Kinematics Imbalance

First MicroBooNE measurement on Argon: inclusive $d^2\sigma/dp_\mu d\cos\theta_\mu$

- CC Inclusive: only the charged lepton is detected. All reaction mechanisms contribute

PHYSICAL REVIEW LETTERS **123,** 131801 (2019)

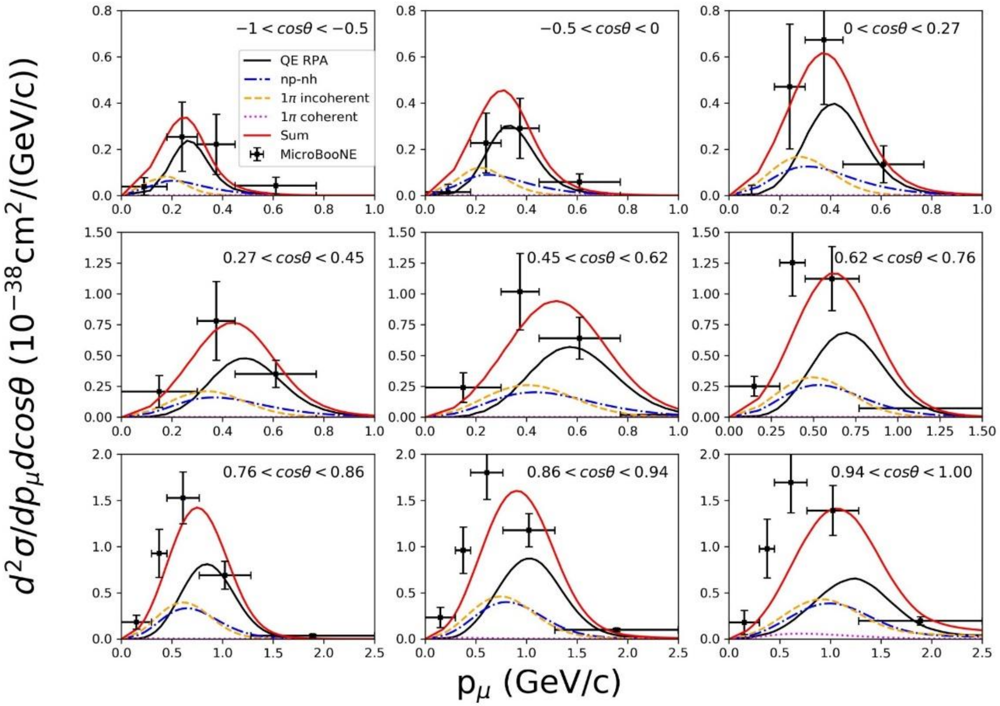


RPA and SuSAv2 calculations of MicroBooNE inclusive $d^2\sigma$ on argon

RPA

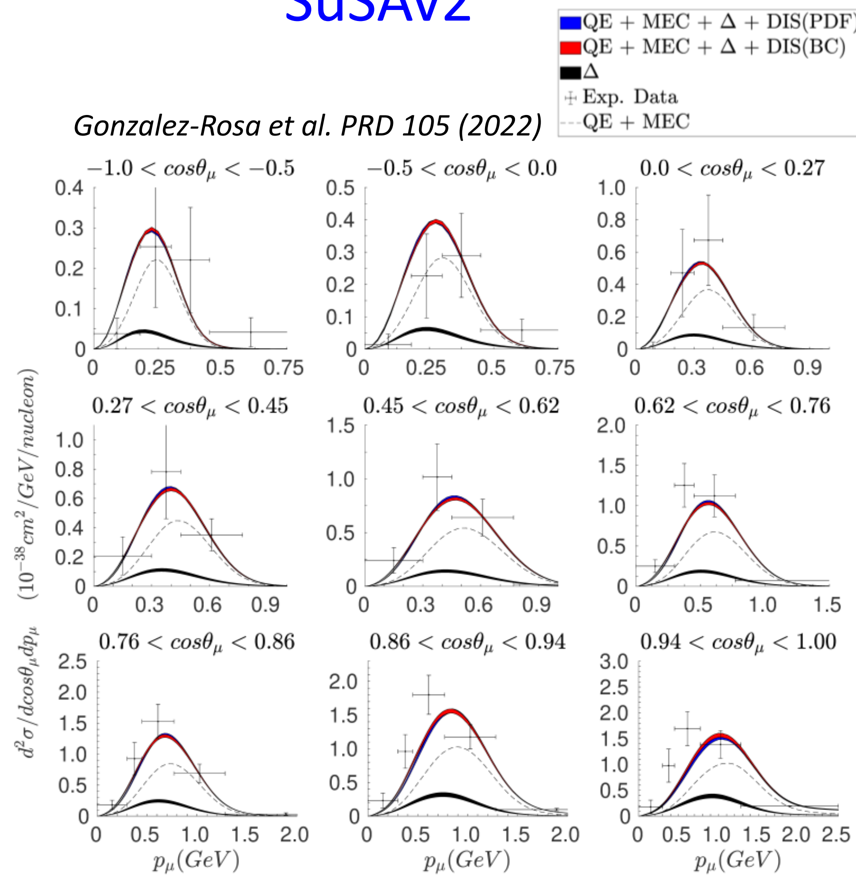
Total = QE + np-nh + 1π inc.+ 1π coh.

M. Martini, M. Ericson, G. Chanfray, PRC 106 (2022)



SuSAv2

Gonzalez-Rosa et al. PRD 105 (2022)



Results also with SuSA
Barbaro et al. Universe 7 (2021)

- Reasonable overall agreement, though not as good as in the ^{12}C T2K inclusive case (see next slide)
- At backward angles the predictions of the different models are slightly shifted to lower values of p_μ, whereas the reverse occurs at forward angles

RPA and Monte Carlos calculations of T2K inclusive $d^2\sigma$ on carbon

PHYSICAL REVIEW D **98,** 012004 (2018)

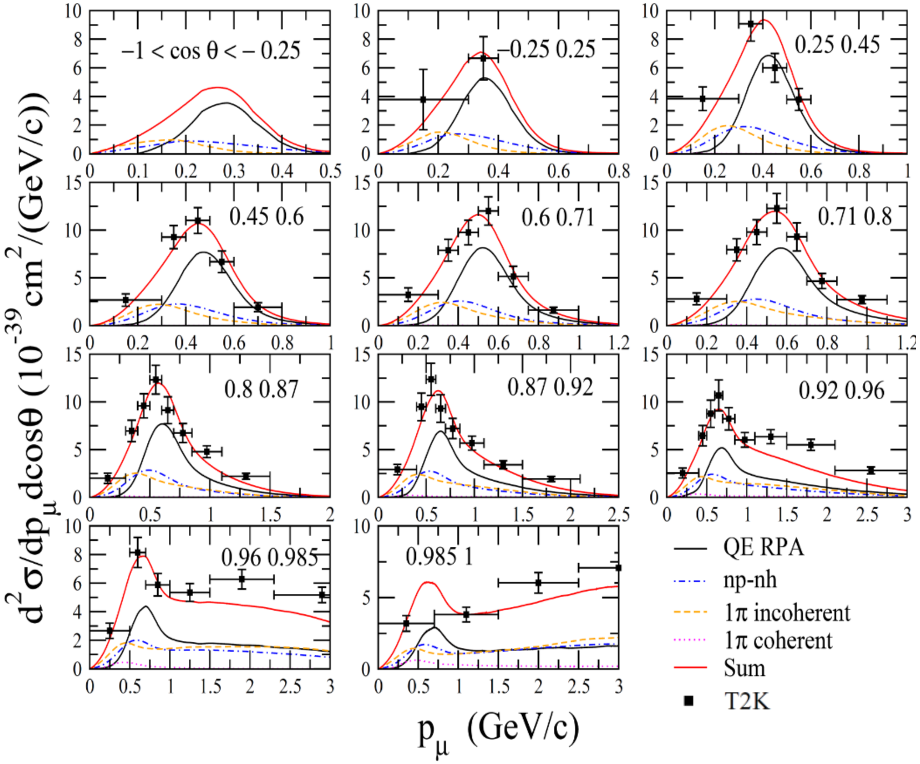
Measurement of inclusive double-differential ν_μ charged-current cross section with improved acceptance in the T2K off-axis near detector

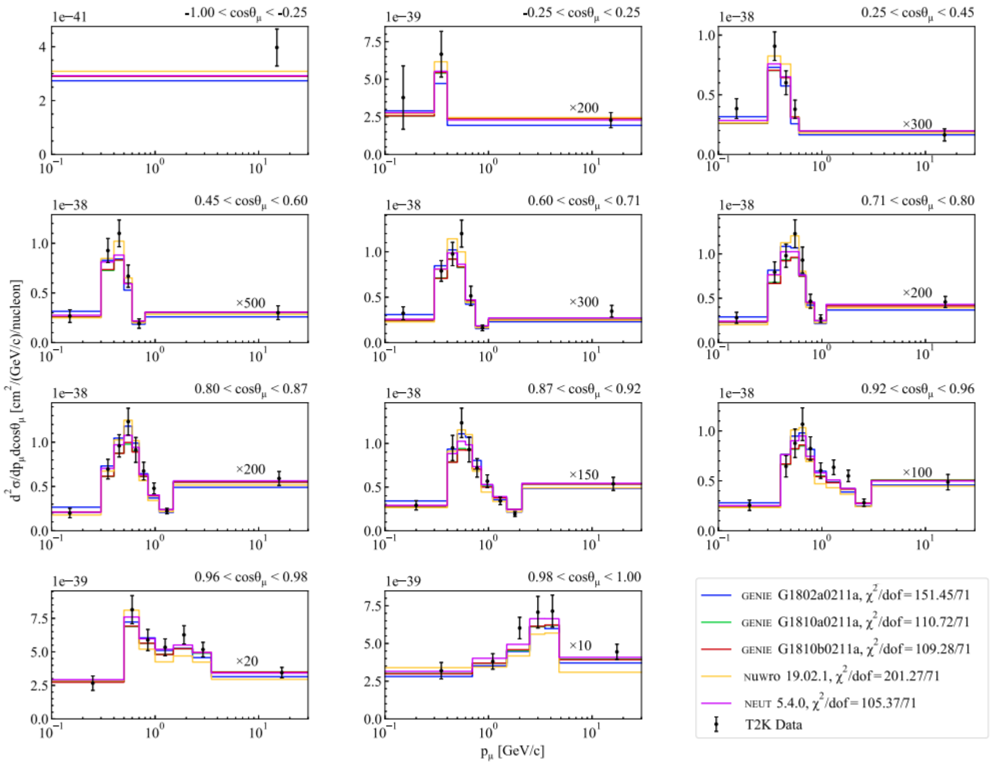
RPA

M. Martini, M. Ericson, G. Chanfray, PRC 106, 015503 (2022)

Monte Carlo

M. Buizza Avanzini et al. PRD 105, 092004 (2022)



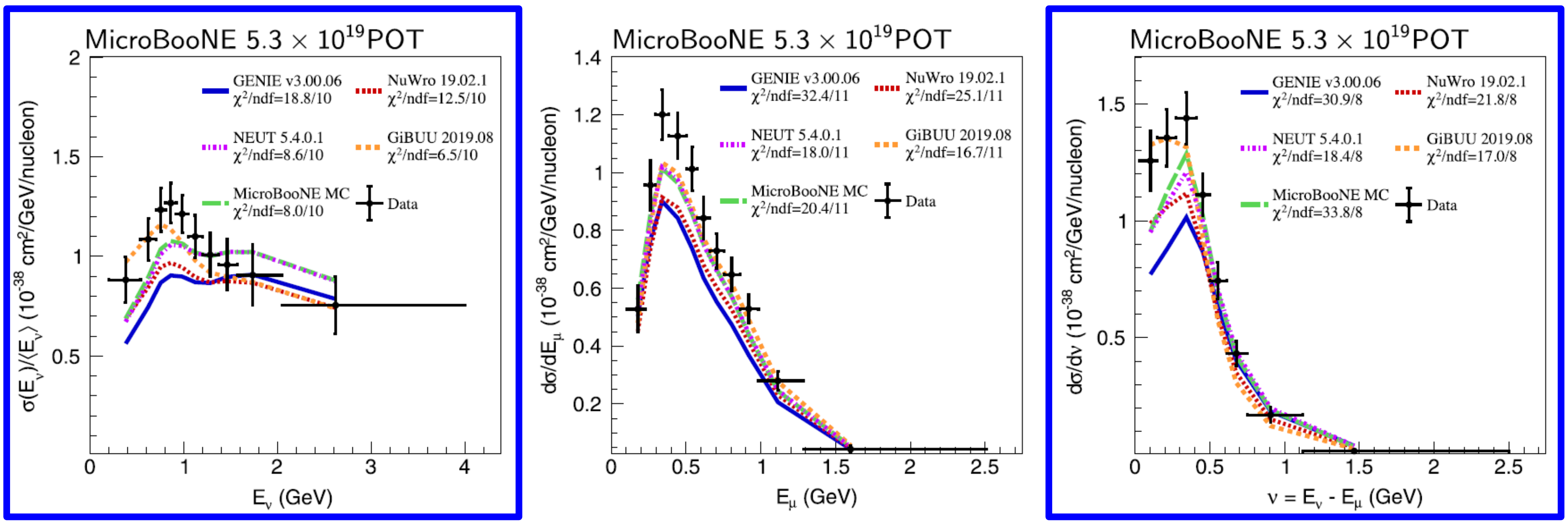


Remarkable agreement

Recent energy-dependent inclusive MicroBooNE cross sections on argon

PHYSICAL REVIEW LETTERS **128,** 151801 (2022)

First Measurement of Energy-Dependent Inclusive Muon Neutrino Charged-Current Cross Sections on Argon with the MicroBooNE Detector



Results presented for the first time as a function of true neutrino energy E_ν and transferred energy (ν or ω)

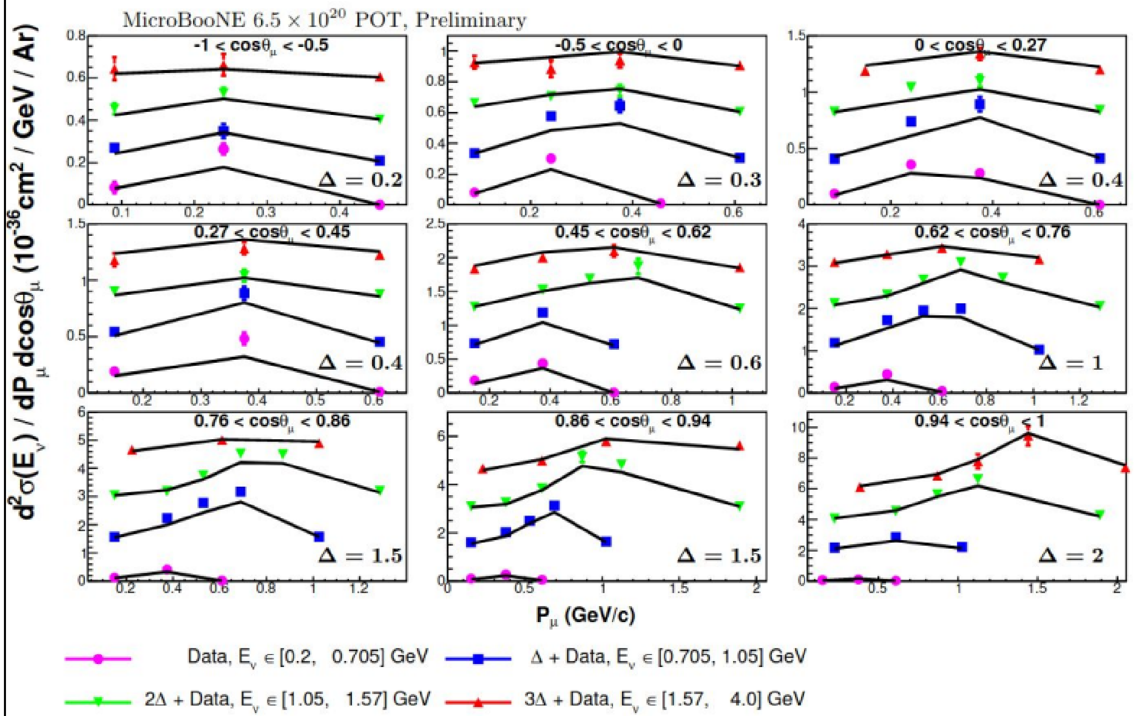
This has been made possible by a new procedure (based on the comparison between the data and the Monte Carlo predictions constrained on the lepton kinematics) allowing the mapping between the true E_ν and ω on one hand, and the reconstructed neutrino energy E_ν^{rec} and hadronic energy E_{had}^{rec} on the other hand

New (NuInt22) MicroBooNE result: inclusive triple differential X section

Results for different E_ν slices

NEW

Unfolded Measurement in 3D



Data plotted against NuWro prediction
E_ν slices overplot with offset NΔ for each angle slice
Δ in same units of $d^2\sigma(E_\nu)/dP_\mu d\cos(\theta_\mu)$($10^{-36}$cm^2/GeV/Ar)

Model Generator	χ^2/ndf
Genie v2.12.10	740.8/138
Genie v3.0.6 (MicroBooNE Tune)	313.9/138
Genie v3.0.6 (Untuned)	309.7/138
GIBUU 2021	265.6/138
NEUT v5.4.0.1	233.1/138
NuWro v19.02.01	200.9/138

Descending χ^2/ndf ↓

3D measurement contains wealth of information → all model central value predictions are now in tension with data

More powerful than 1D measurement, which was consistent with some models

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London Cooper-Troendle (MicroBooNE) talk @ NuInt22

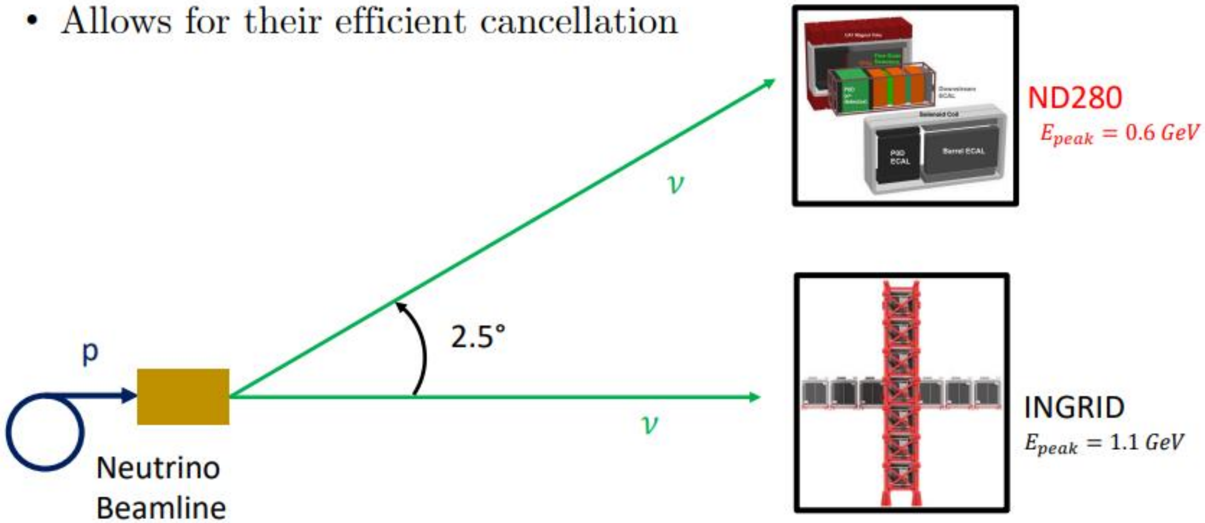
New (NuInt22) T2K result: Joint On-/Off-Axix CC0π X section

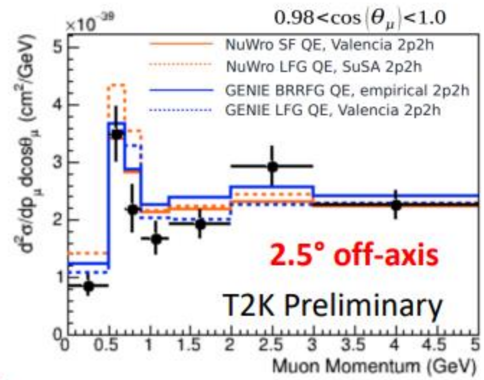
NEW

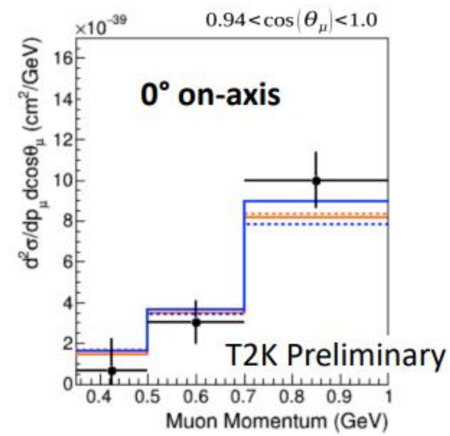
Direct probe of E_ν dependence

Sneak Peek: Joint On-/Off-Axis CC0π Cross Section

- First T2K multi-detector cross section measurement
- Direct probe of E_ν dependence
- Exploits correlations between the systematic uncertainties of the samples
  - Allows for their efficient cancellation







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Caspar Schloesser (T2K) talk @ NuInt22

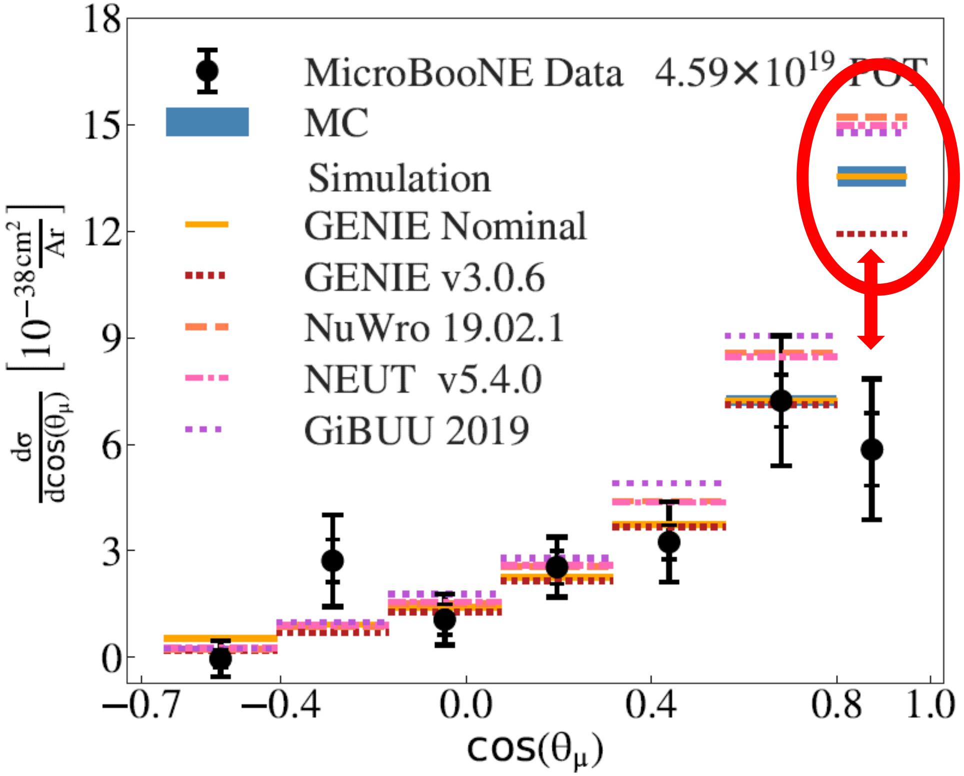
MicroBooNE **semi-inclusive CC0π1p** on argon

PHYSICAL REVIEW LETTERS **125,** 201803 (2020)

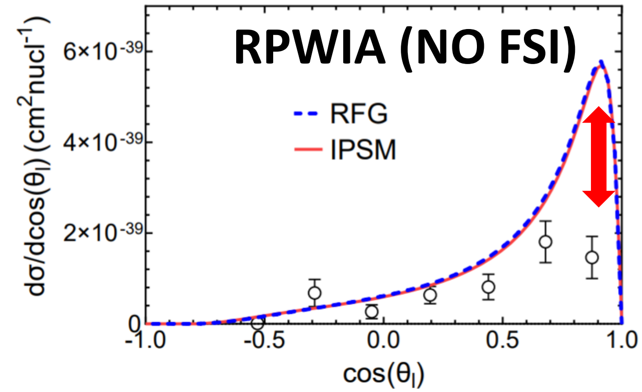
First Measurement of Differential Charged Current Quasielasticlike ν_μ-Argon Scattering Cross Sections with the MicroBooNE Detector

?! CCQE-like with another meaning than in the past

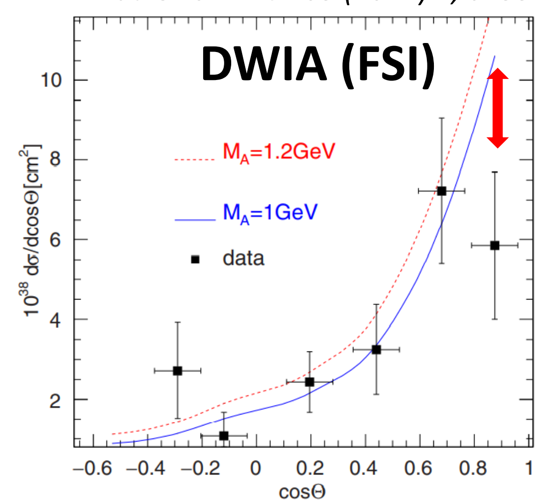
MicroBooNE PRL 125(2020)



J. M. Franco-Patino et al. PRD 104 (2021) 7, 073008



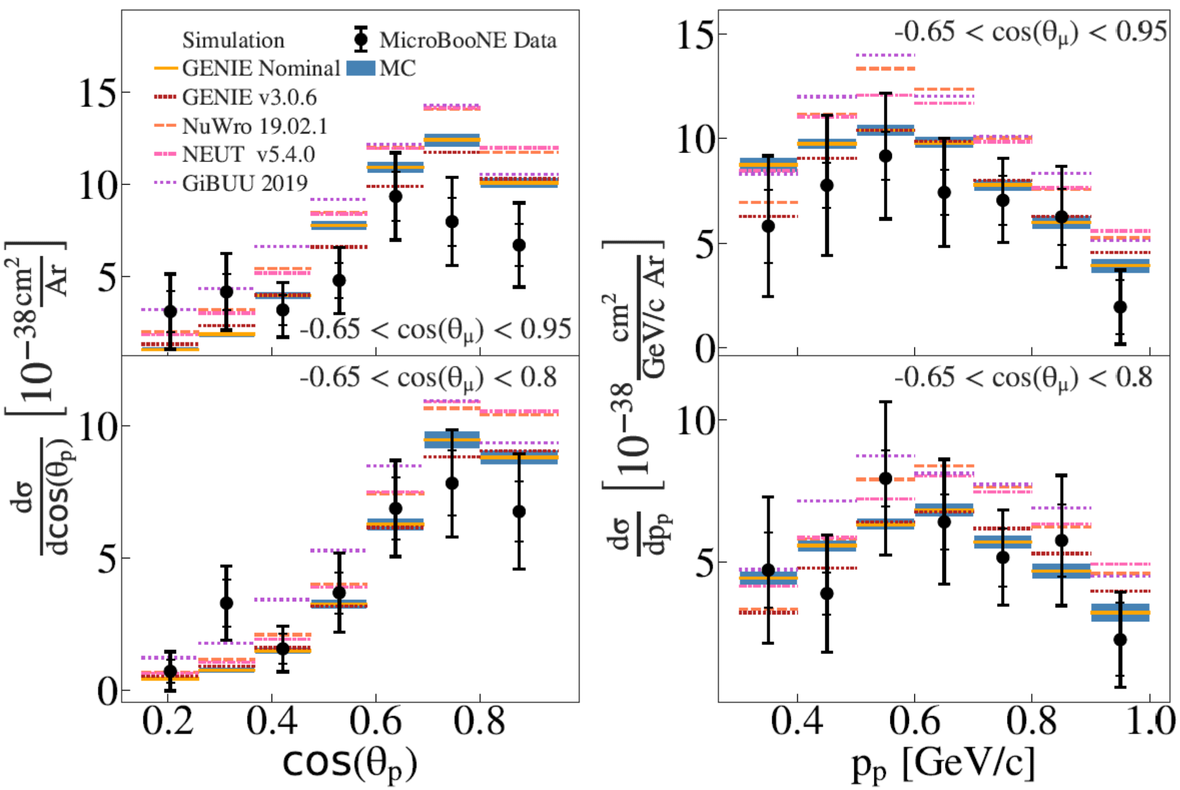
A.V. Butkevich PRC 105 (2022) 2, 025501



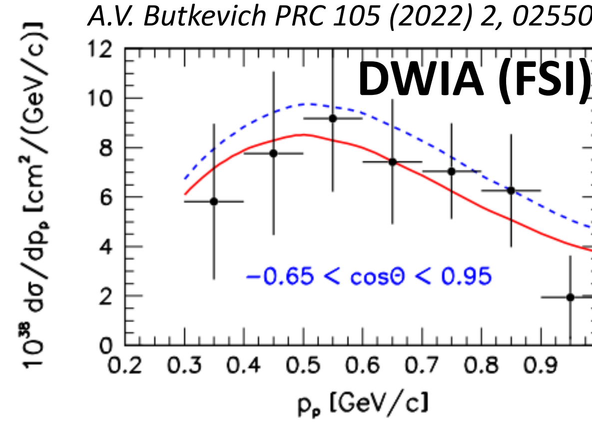
Overestimation in the muon forward direction

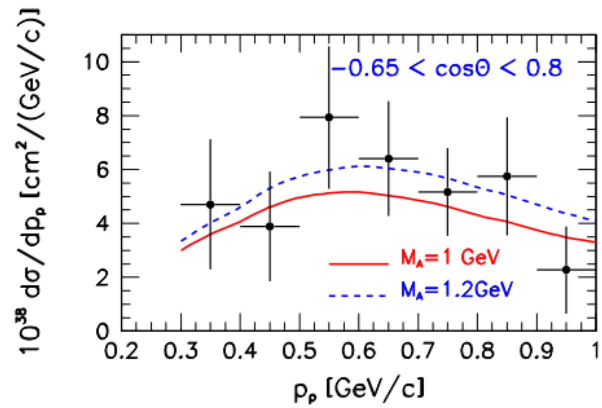
MicroBooNE semi-inclusive CC0π1p on argon versus proton variables

MicroBooNE PRL 125(2020)



A.V. Butkevich PRC 105 (2022) 2, 025501



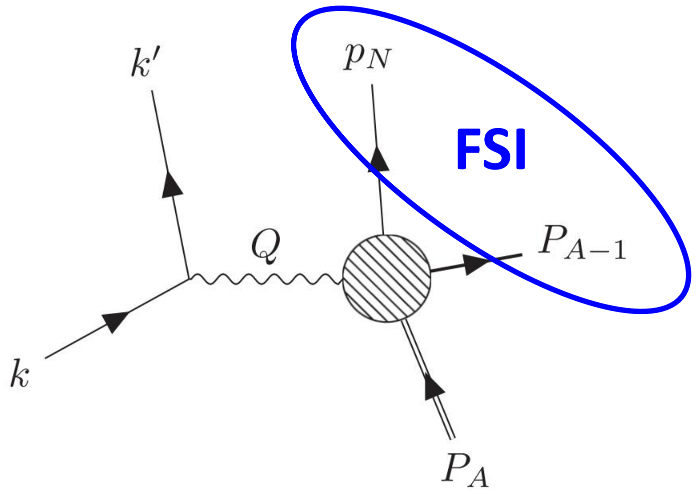


- Poor Monte Carlo – data agreement
- Spread of Monte Carlo predictions

How good are the current approximations (use “inclusive” models, factorization,…) of the Monte Carlos for the semi-inclusive processes?

Final State Interactions

FSI between the knocked-out particle(s) and the residual nucleus



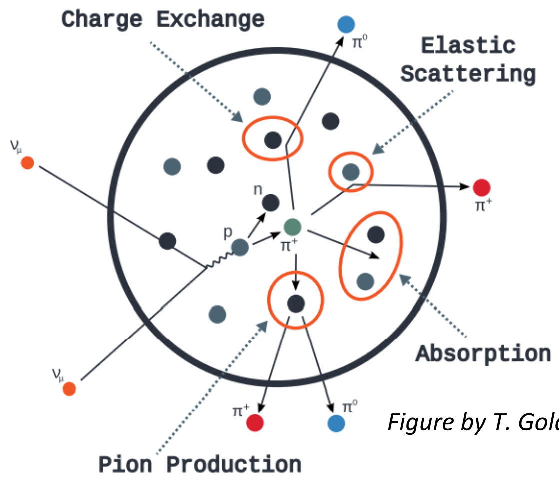


Figure by T. Golan

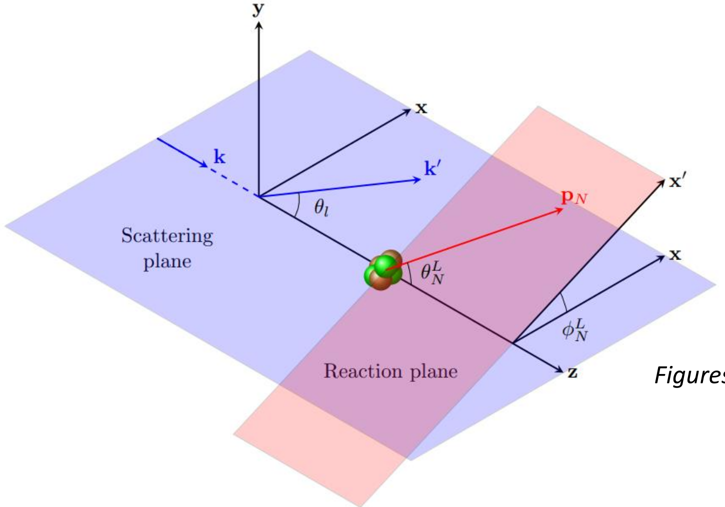
- Monte Carlo event generators includes different models of intra-nuclear cascades: particles are assumed to be classical and move along a straight line
- FSI between the knocked-out nucleon and the residual nucleus can be microscopically treated using different approaches: Optical Potential, RMF, Energy-Dependent RMF

The inclusion of FSI effects is extremely important for the description of semi-inclusive data

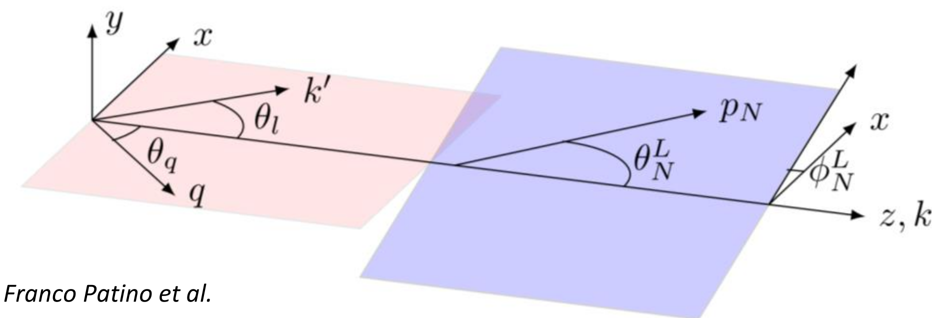
[Some recent references: *R. Gonzalez-Jimenez et al., PRC 101, 015503 (2020) ; J. Isaacson et al., PRC 103 015502 (2021); A. Nikolakopoulos et al. PRC 105, 054603 (2022); A. Ershova et al., PRD 106 032009 (2022)]*

The semi-inclusive neutrino cross section

There is a rapidly increasing interest on semi-inclusive cross sections



Figures by J. M. Franco Patino et al.



M. B. Barbaro talk @NUFACT 2021

$$\mathcal{F}_\chi^2 = L_{\mu\nu} W^{\mu\nu}$$
$$= V_{CC}R^{CC} + 2V_{CL}R^{CL} + V_{LL}R^{LL} + V_T R^T + V_{TT}R^{TT} + V_{TC}R^{TC} + V_{TL}R^{TL} + \chi\left(V_{T'}R^{T'} + V_{TC'}R^{TC'} + V_{TL'}R^{TL'}\right)$$

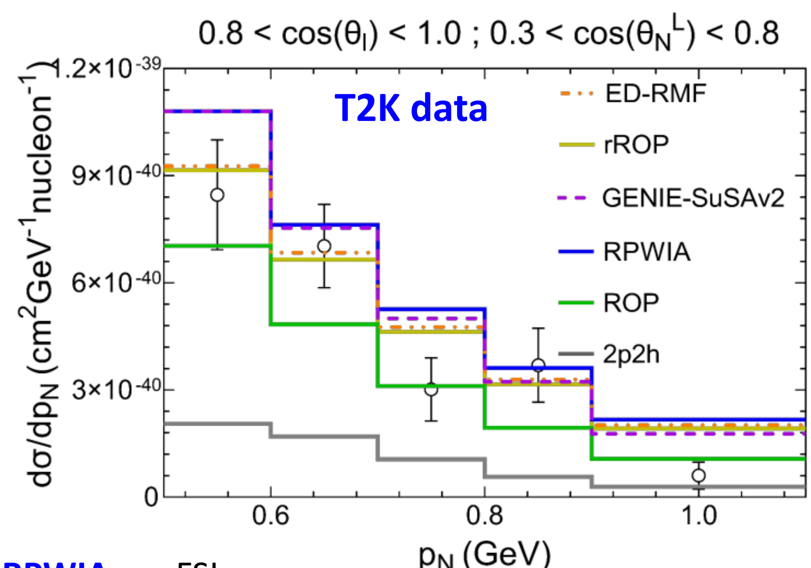
The $(\nu_\mu, \mu p)$ cross section is decomposed in **10 independent response functions** of **5 variables** $(\omega, q, \mathbf{p_N})$.
More complex structure than in the **inclusive** (ν_μ, μ) case: **5 new responses**, which vanish after integration over the final nucleon variables

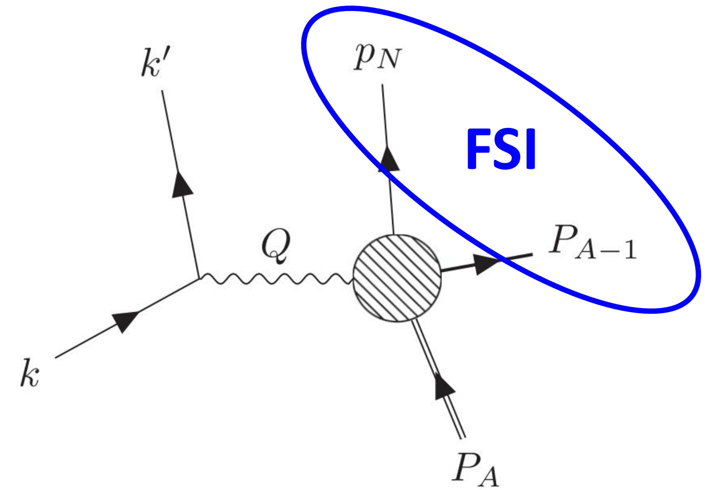
$R^{TT,TC,TL,TC',TL'} \propto \cos(\phi), \cos(2\phi)$ ϕ outgoing nucleon azimuthal angle

Semi-inclusive —> Inclusive (but not viceversa!)

Theoretical situation:
- few models and papers for genuine CCQE [*J. M. Franco Patino et al, PRC 102 (2020); PRD 104 (2021), 2207.02086;* A. V. Butkevich *PRC 105 (2022)]*
- one (incomplete due to the absence of Δ-MEC) model for 2p-2h [T. Van Cuyck et al. PRC 94 (2016); PRC 95 (2017)]

Semi-inclusive CC0π cross section on carbon: role of proton FSI

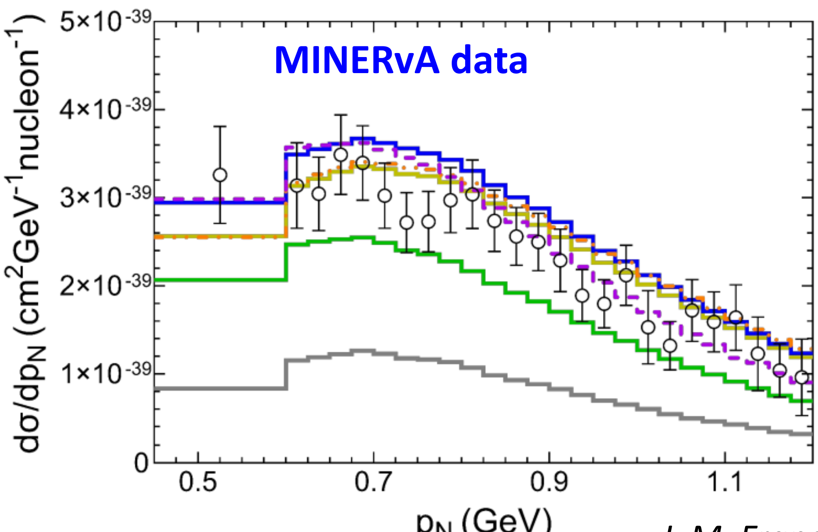




RPWIA: no FSI

GENIE-SuSAv2: include FSI but from inclusive model (factorization)

ED-RMF, **rROP**, **ROP**: different theoretical approaches for FSI



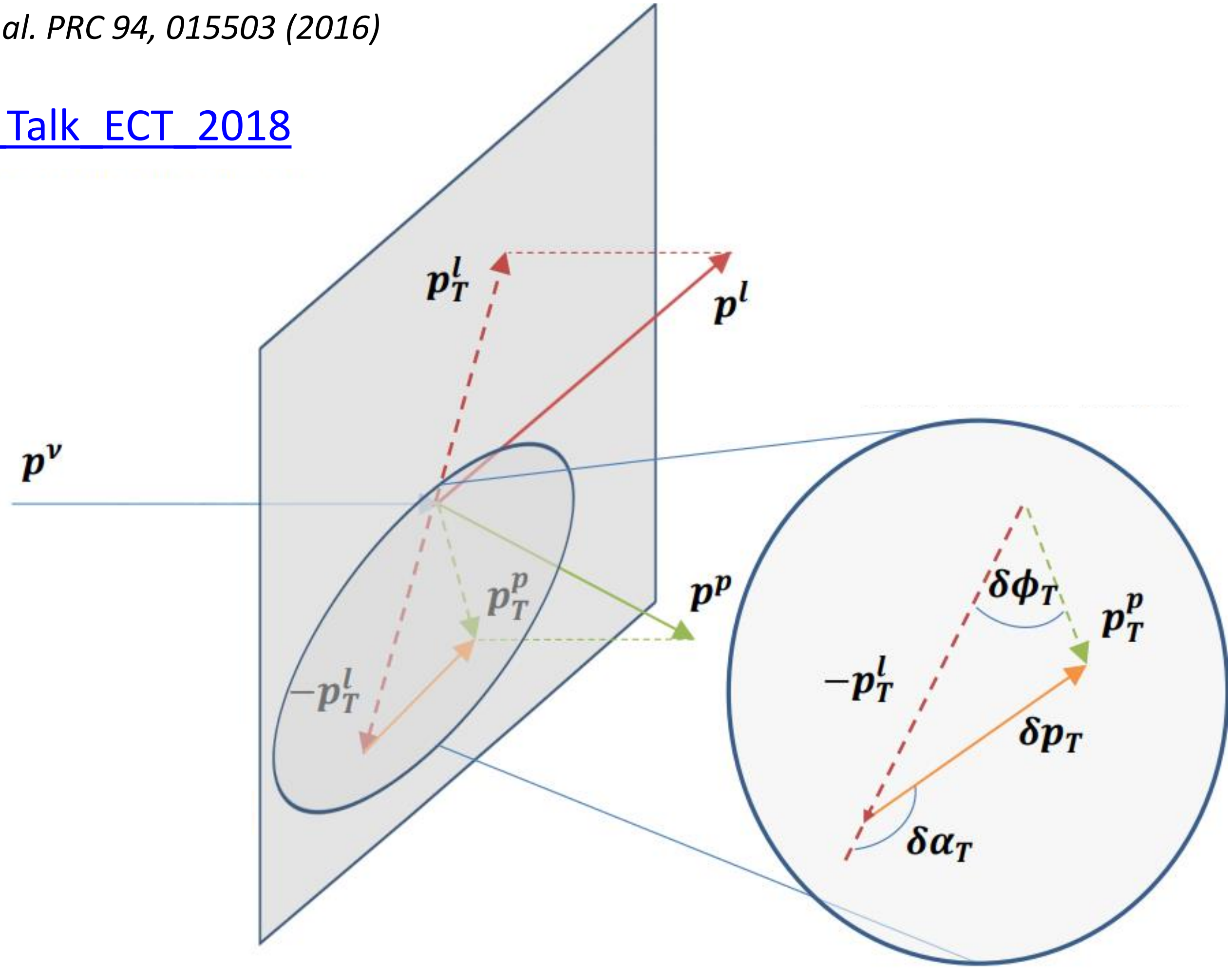
- FSI improve the agreement with data with respect to the RPWIA (no FSI) prediction
- Too large data error bars to discriminate between different FSI models

J. M. Franco-Patino et al. , arXiv 2207.02086

Single Transverse Kinematic Imbalance (STKI) – 3 variables (STV)

X. –G. Lu et al. PRC 94, 015503 (2016)

S_Dolan_Talk_ECT_2018



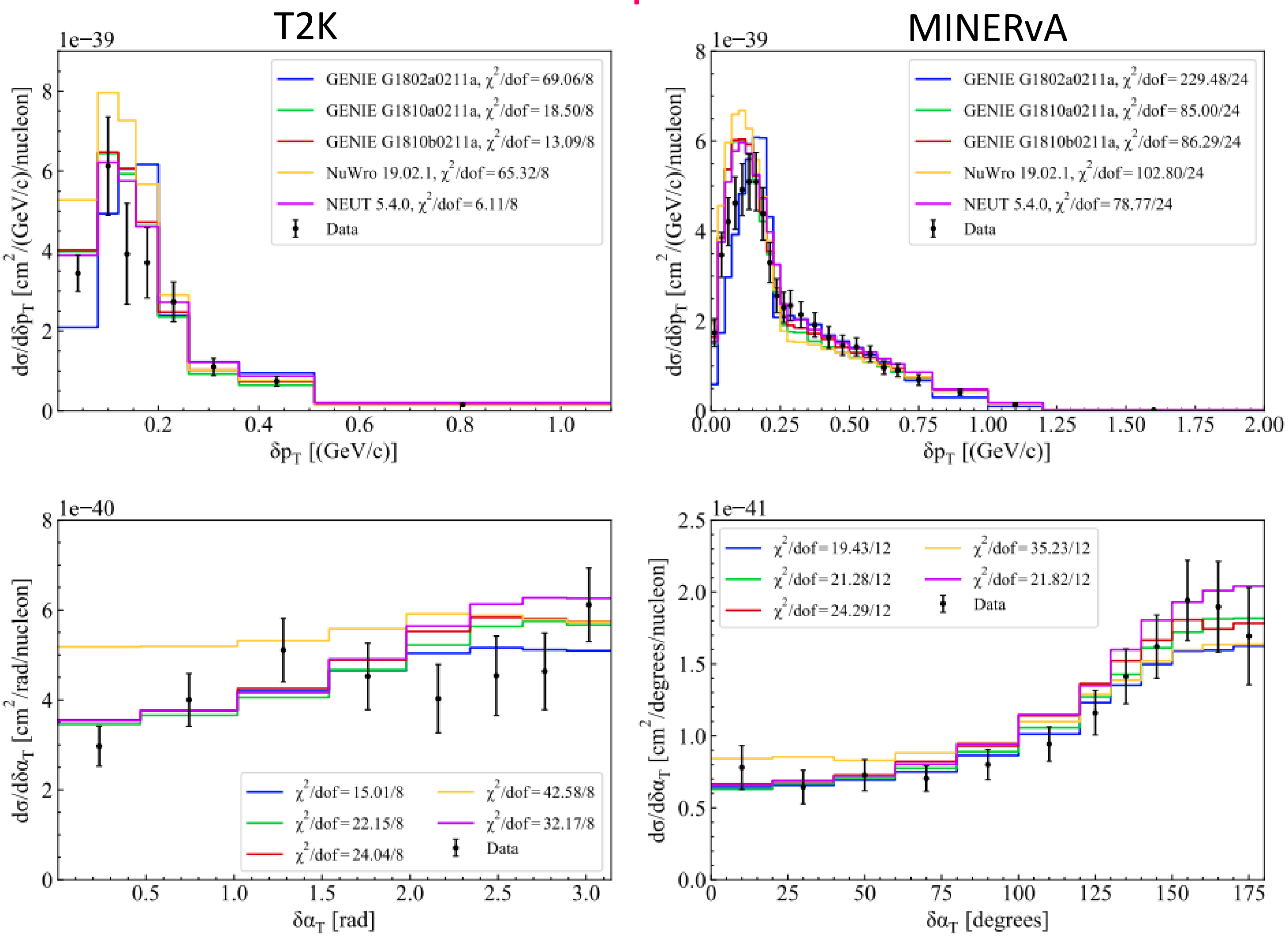
Scattering on a free nucleon at rest: $\vec{p}_T^{\,p} = -\vec{p}_T^{\,l}$

$\delta p_T = 0$; $\delta\phi_T = 0$ ⟶ peaked distributions

$\delta\alpha_T$ undefined ⟶ flat distribution

Deviations (imbalance) from these behaviors "measure" nuclear effects

Semi-inclusive CC0π dσ on carbon versus STKI Variables: Monte Carlo predictions



M. Buizza Avanzini et al. PRD 105, 092004 (2022)

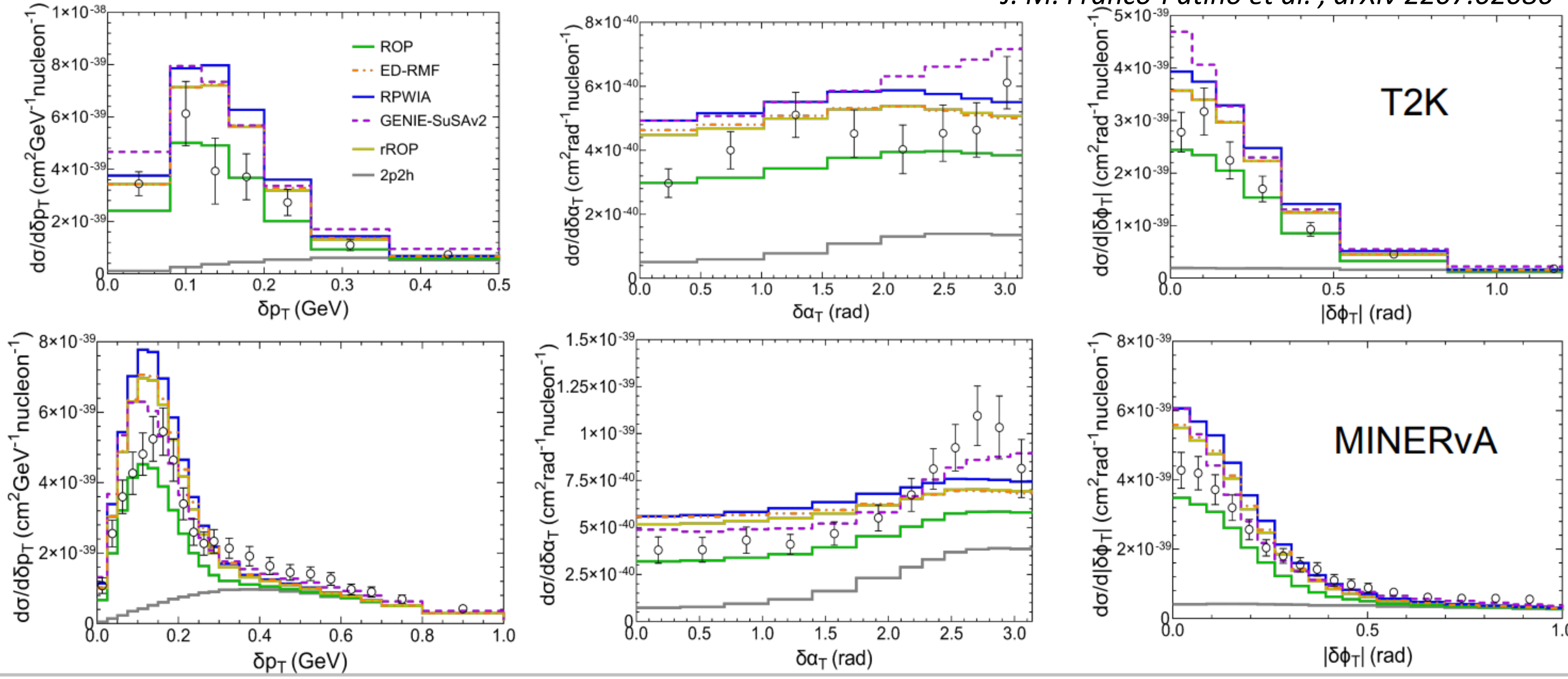
None of the generators correctly reproduces all the data in the STKI variables without tuning

This is not a surprise since these generators implement "inclusive" microscopic models

Semi-inclusive CC0π dσ on carbon versus STKI Variables: discrimination of FSI microscopic modeling

A recent theoretical study

J. M. Franco-Patino et al. , arXiv 2207.02086



RPWIA: no FSI

GENIE-SuSAv2: include FSI but from inclusive model (factorization)

ED-RMF, **rROP**, **ROP**: different theoretical approaches for FSI

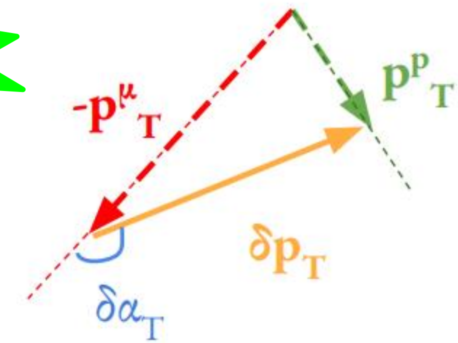
- FSI improve the agreement with data respect to the RPWIA prediction
- STKI Variables helps to discriminate between different FSI models: data (at least T2K) seem to prefer **ROP**
- 2p2h (from an inclusive-based model) give non-negligible contribution

New (NuInt22) Semi-inclusive CC0π dσ on Argon versus STKI Variables

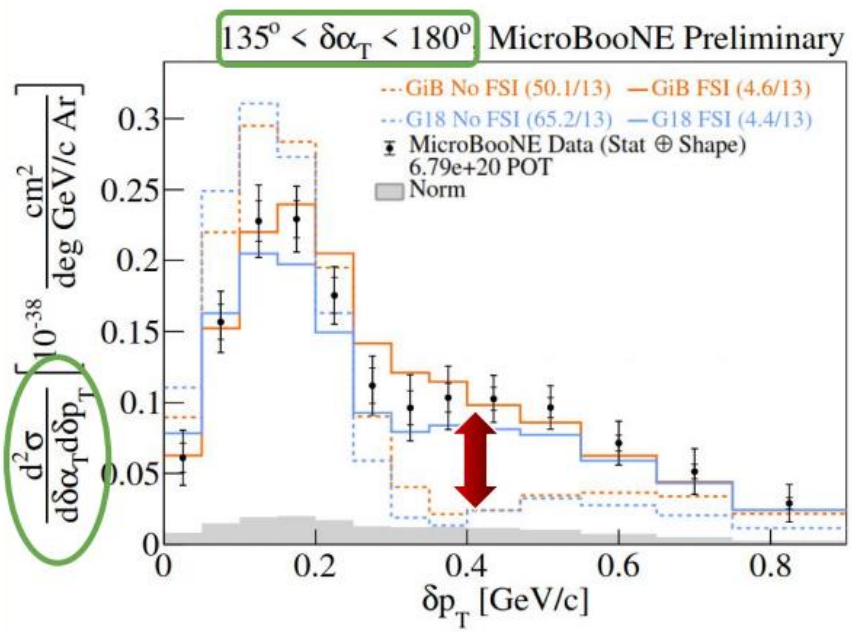
2D results for the first time on any neutrino target

High Statistics→Into the Multiverse!

NEW



- **Extension to 2D** for the first time on any neutrino target
- Probe regions with greater model discrimination power



- FSI predictions in good agreement with data
- Minimal no-FSI contributions at high δp_T
- High $\delta\alpha_T$ & high δp_T part of phase-space ideal to test FSI / multinucleon effect sensitivity

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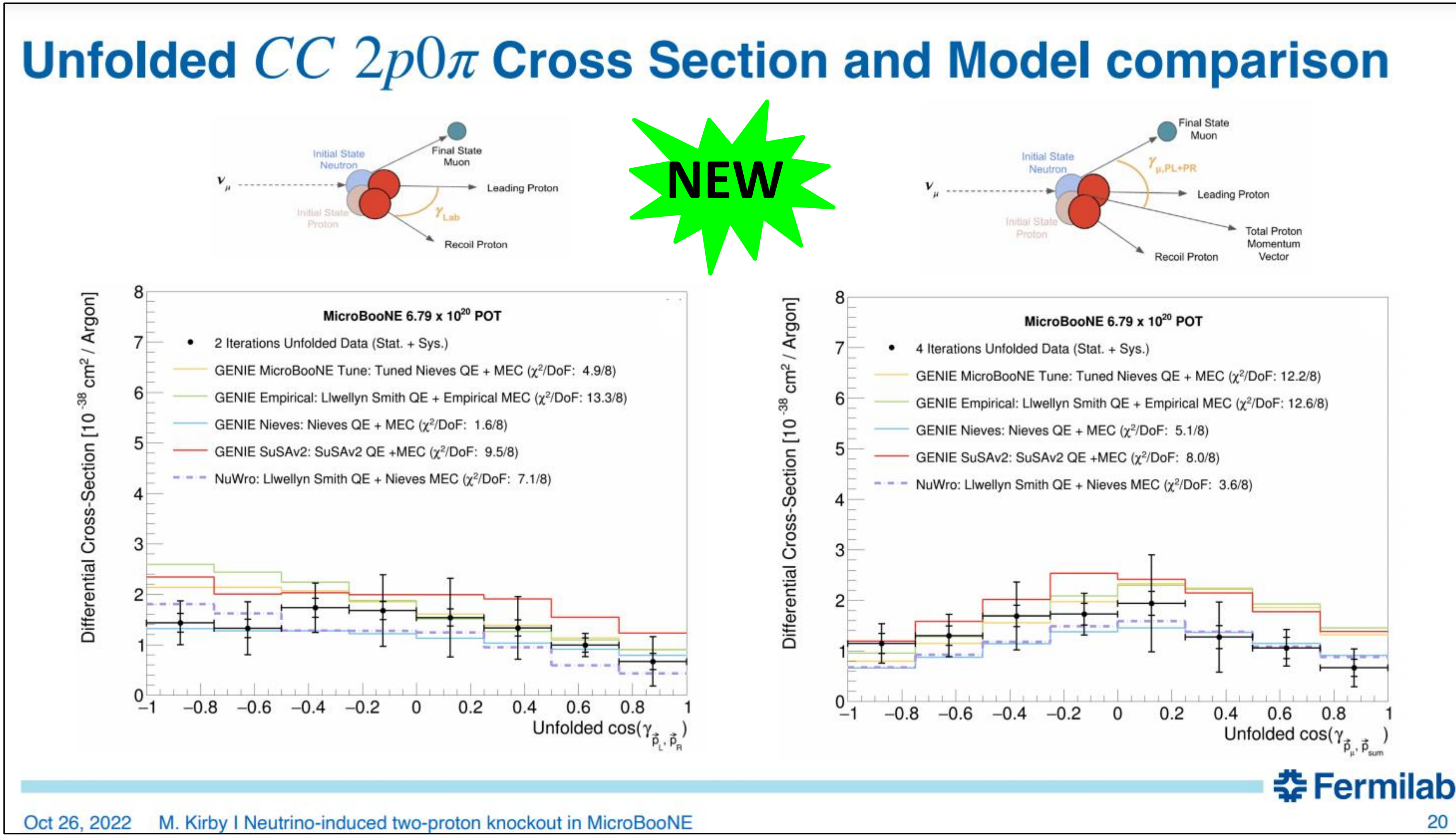
MICROBOONE-NOTE-1108-PUB G18 = GENIE v3.0.6 G18_10a_02_11b + T2K Tune, GiB = GiBUU 2021

Afroditi Papadopoulou (MicroBooNE) talk at NuInt 22

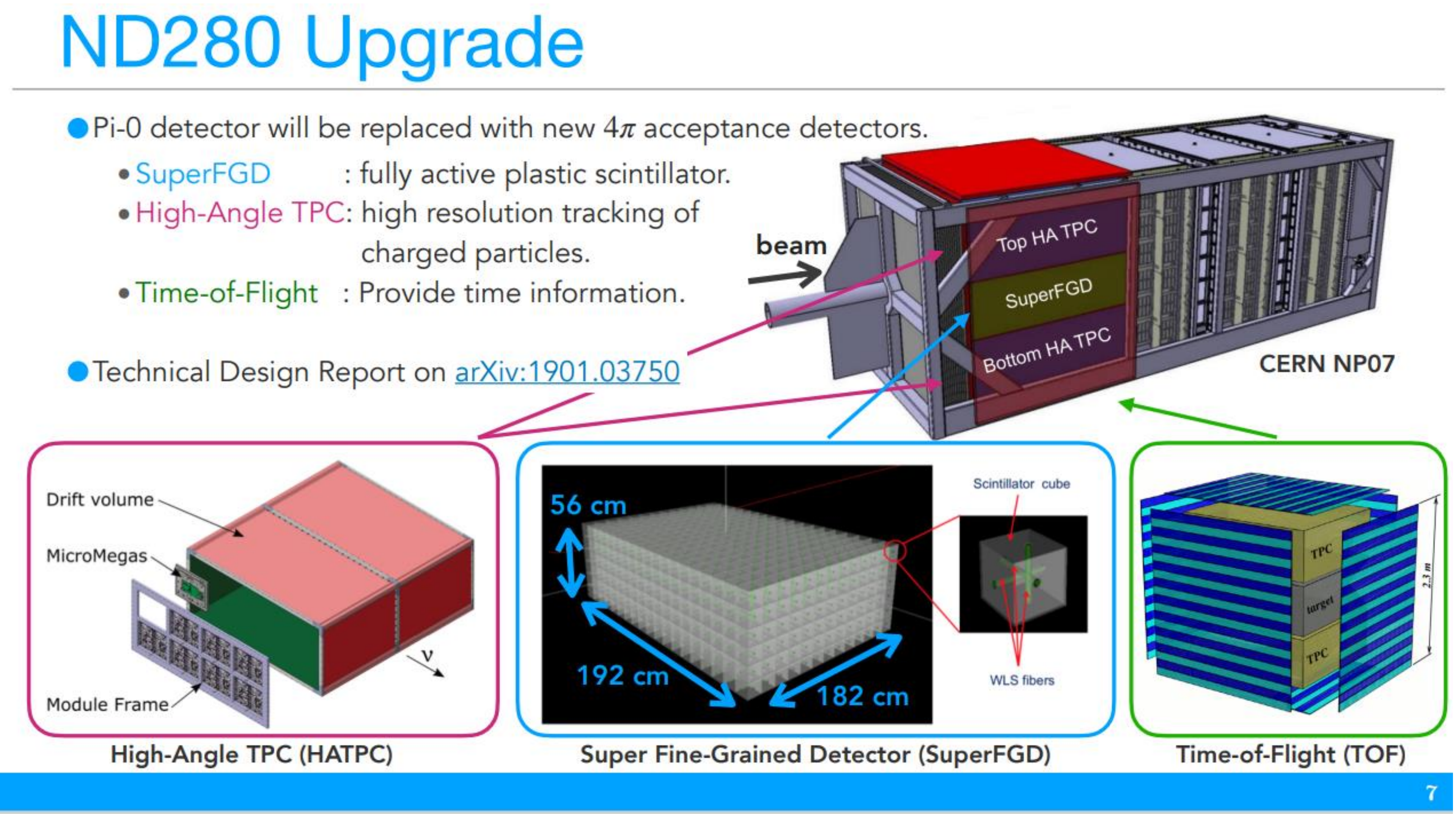
New (NuInt22) CC0π**2p** dσ on Argon



Michael Kirby (MicroBooNE) talk at NuInt 22

P. Abratenko et al. 2211.03734

The T2K ND280 Upgrade



[A_Eguchi_T2KND280upgrade_NuFACT2022 (fnal.gov)]

The T2K ND280 Upgrade – Physics sensitivity studies

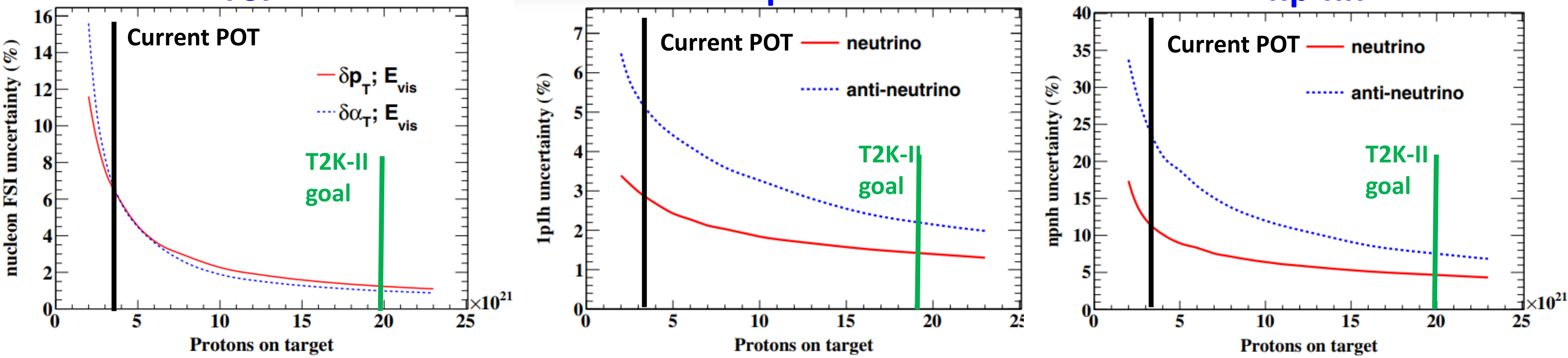
- More mass, more data, better acceptance
- Improved reconstruction at high and backward lepton angles
- Better reconstruction of outgoing nucleons

The Upgrade opens the door to new multi-dimensional analyses (e.g. δp_T in bins of $\delta\alpha_T$)

PHYSICAL REVIEW D **105**, 032010 (2022)

Sensitivity of the upgraded T2K Near Detector to constrain neutrino and antineutrino interactions with no mesons in the final state by exploiting nucleon-lepton correlations

FSI **1p-1h** **np-nh**



Significant decrease of the nuclear effects uncertainties

Neutrino cross sections: summary of status and challenges

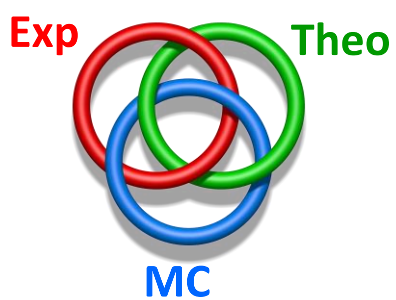
A) Cross sections in terms of muon variables (CC inclusive, CC0π)

- **Significant progress in the last 15 years**
- Many experimental and theoretical results
- Still we have to tackle currently existing degeneracies:
    1. between cross sections and flux uncertainties
    2. between nucleon uncertainties and nuclear effects
    3. between different nuclear models and approximations

B) Cross sections in terms of hadronic variables (CC1π, CC0π1p, CC0πNp, CCOther)

We are only at the beginning!

- Few experimental and theoretical results
- The one pion puzzle is still there
- SIS and DIS have been minimally studied
- Theoretical models and Monte Carlo implementation of semi-inclusive processes are needed



Needs:

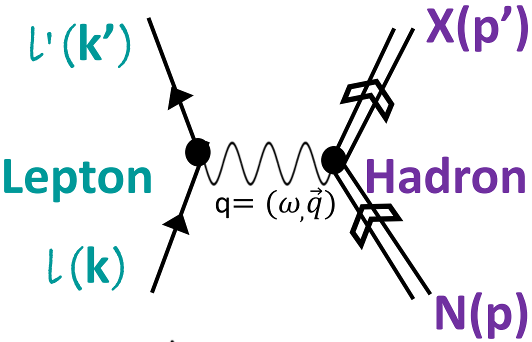
- More neutrino data on nuclei and nucleons
- Further theory efforts
- Generators equipped with more detailed (/consistent/adapted) models

Close collaboration between theorists, experimentalists and generator developers is crucial

In the precision era of neutrino physics new intriguing results, like CP violation, necessary passes through a precise knowledge of neutrino-nucleus cross sections

BACKUP

Electroweak transition matrix elements



Electromagnetic transition $\ell^- N \rightarrow \ell^- X$

$$-i\mathcal{M} = -(ie)^2 \underbrace{\bar{u}(k')\gamma_\mu u(k)}_{\text{e.m. lepton current}} \frac{-ig^{\mu\nu}}{q^2} \underbrace{\left\langle X(p'_f) \middle| J_\nu(0) \middle| N(p) \right\rangle}_{\text{hadronic current (Vector)}}$$

Charged current transition $\nu N \rightarrow \ell^- X$

$$-i\mathcal{M} = \left(\frac{-ig}{2\sqrt{2}}\right)^2 \cos\theta_C \underbrace{\bar{u}(k')\gamma_\mu(1-\gamma^5)u(k)}_{\text{weak lepton current}} \frac{ig^{\mu\nu}}{M_W^2} \underbrace{\left\langle X(p'_f) \middle| J_\nu(0) \middle| N(p) \right\rangle}_{\text{hadronic current (Vector-Axial)}}$$

θ_C Cabibbo angle

$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$

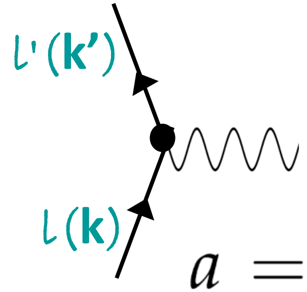
Fermi coupling constant

The leptonic tensor

$$L_{\mu\nu} = \frac{1+|a|}{2}\sum_{s_i}\sum_{s_f} j_\mu{}^\dagger j_\nu = \frac{1+|a|}{2}\sum_{s_i}\sum_{s_f} \bar{u}(k)\tilde{l}_\mu u(k')\bar{u}(k')l_\nu u(k)$$

$$= \frac{1+|a|}{2}\mathrm{Tr}\left[(\not{k}+m_\ell)\tilde{l}_\mu(\not{k}'+m_{\ell'})l_\nu\right]$$

Leptonic component of the electroweak current



$$j_\mu = \bar{u}(k')l_\mu u(k)$$

$$\not{p} = \gamma_\mu p^\mu$$

$$l_\mu = \gamma_\mu(1-a\gamma^5) \quad \tilde{l}_\mu = \gamma_0 l_\mu^\dagger \gamma_0$$

$a = 0$ EM

$a = 1\ (-1)$ CC and NC

Electron scattering

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k\,k' - m_e^2)$$

$$g_{\mu\nu} = (+,-,-,-)$$
$$\varepsilon_{0123} = +1$$

Neutrino scattering

$$L_{\mu\nu} = k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k\cdot k' \mp i\varepsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta$$

($-$: ν; $+$: $\bar{\nu}$)

p.s. In literature L is defined with different multiplicative and normalization factors

The hadronic tensor

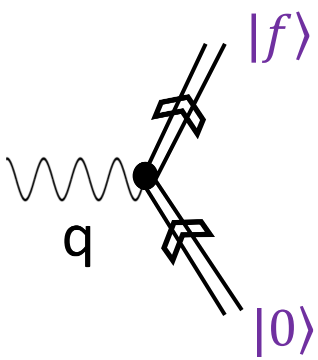
The hadronic tensor contains all the information on the target response

$$W^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^\nu(q)|0\rangle\delta^{(4)}(p_0+q-p_f)$$

$|0\rangle$ hadronic initial state

$|f\rangle$ hadronic final state

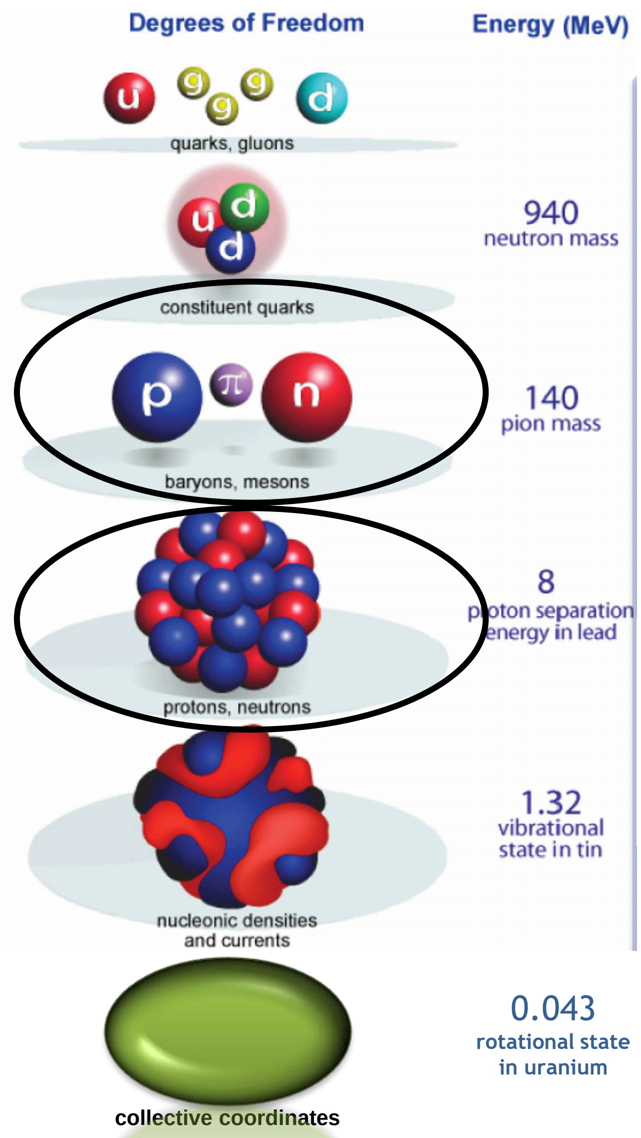
J^ν Hadronic component of the electroweak current

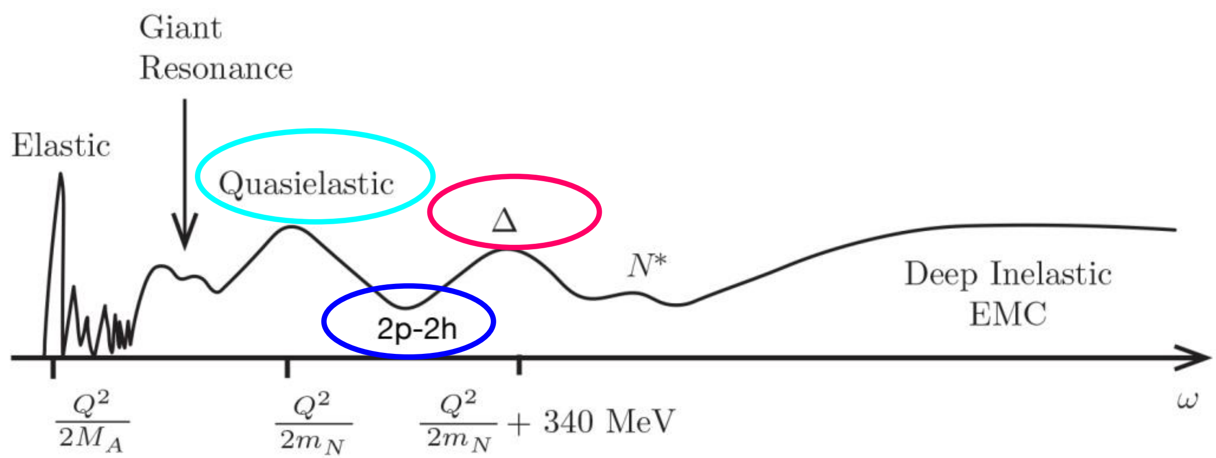


A general expression

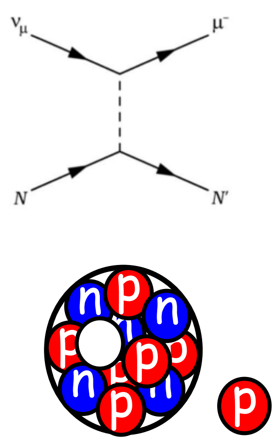
- valid for different degrees of freedom (quark, nucleon, nucleon resonances, nucleus)
- valid for different currents (electromagnetic, weak; one-body, two-body,...)

In the following:

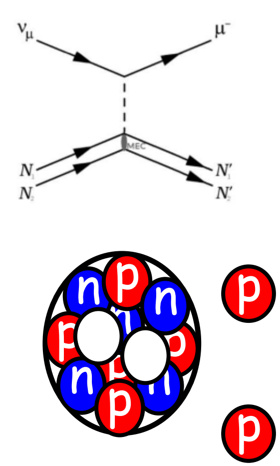




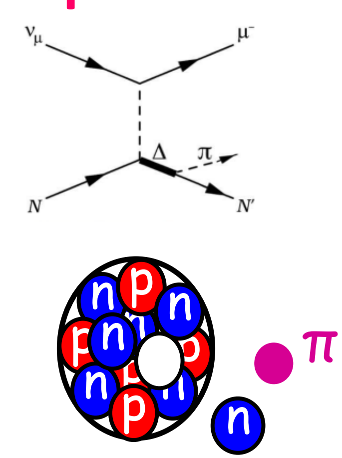
Quasielastic



2p-2h



RES π production



A simplified expressions particularly useful for illustration

- Final lepton mass contributions ignored (m_l=0)
- Obtained by keeping only the leading terms for the hadronic tensor in the development of the hadronic current in p/M_N

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2\cos^2\theta_c}{\pi}|\mathbf{k}'|E_l'\cos^2\frac{\theta}{2}\Bigg[\frac{(\mathbf{q}^2-\omega^2)^2}{\mathbf{q}^4}G_E^2 R_\tau(\mathbf{q},\omega)+\frac{\omega^2}{\mathbf{q}^2}G_A^2 R_{\sigma\tau(L)}(\mathbf{q},\omega)$$

$$+2\left(\tan^2\frac{\theta}{2}+\frac{\mathbf{q}^2-\omega^2}{2\mathbf{q}^2}\right)\left(G_M^2\frac{\mathbf{q}^2}{4M_N^2}+G_A^2\right)R_{\sigma\tau(T)}(\mathbf{q},\omega) \pm 2\frac{E_\nu+E_l'}{M_N}\tan^2\frac{\theta}{2}G_A G_M R_{\sigma\tau(T)}(\mathbf{q},\omega)\Bigg]$$

Explicitly appear:

1. The different **kinematic variables** (related to the leptonic tensor)
2. The nucleon Electric, Magnetic, and Axial **form factors** (↔ nucleon properties)
3. The **nuclear response functions** (↔ nuclear dynamics)

Nuclear response functions R(q,ω):

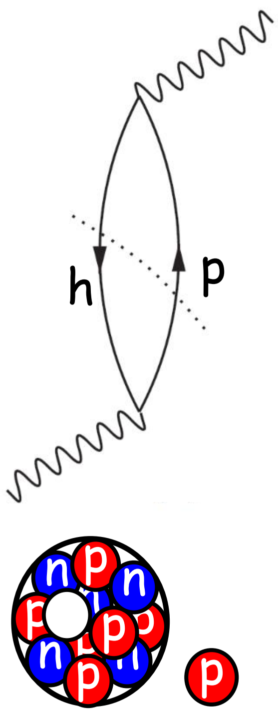
$$R_\alpha^{PP'}(\mathbf{q},\omega)=\sum_n \langle n|\sum_{j=1}^{A} O_\alpha^P(j)\,e^{i\,\mathbf{q}\cdot\mathbf{x}_j}|0\rangle\langle n|\sum_{k=1}^{A} O_\alpha^{P'}(k)\,e^{i\,\mathbf{q}\cdot\mathbf{x}_k}|0\rangle^*\,\delta(\omega-E_n+E_0),$$

Isovector R_τ	Isospin Spin-Longitudinal $R_{\sigma\tau(L)}$	Isospin Spin-Transverse $R_{\sigma\tau(T)}$
$O_\alpha^N(j)=\tau_j^\pm$	$(\boldsymbol{\sigma}_j\cdot\widehat{q})\,\tau_j^\pm$	$(\boldsymbol{\sigma}_j\times\widehat{q})^i\,\tau_j^\pm$

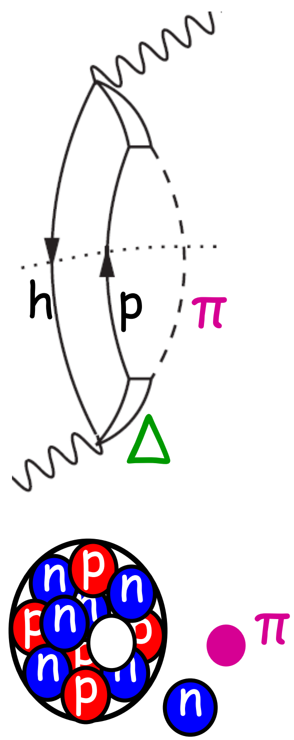
Nuclear Responses for different excitations

$$R_\alpha = \sum_{n \neq 0} |\langle n | \hat{O}_{(\alpha)} | 0 \rangle|^2 \; \delta[\omega - (E_n - E_0)]$$

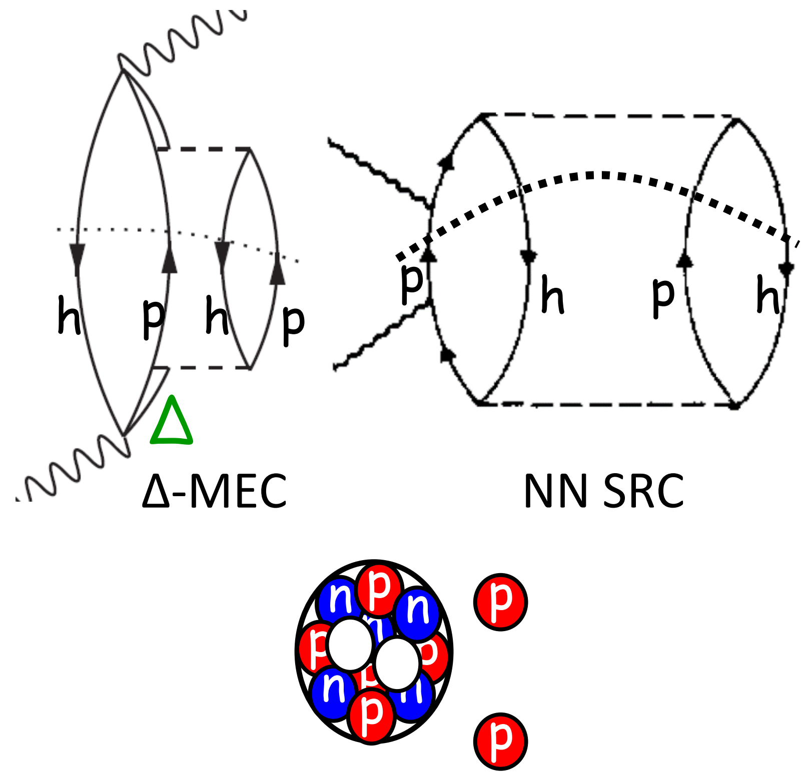
1p-1h
Quasielastic



1p-1h
(Δ→πN) 1π production



2p-2h:
two examples



The single nucleon electroweak current

Electromagnetic current - Electron scattering

$$J^{\mu}_{s' s}(\mathbf{p}',\mathbf{p}) = \overline{u}_{s'}(\mathbf{p}')\left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2) i\sigma^{\mu\nu}\frac{q_{\nu}}{2m_N}\right]u_s(\mathbf{p})$$

$$Q^2 = -q^2 \qquad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu},\gamma^{\nu}]$$

Weak current – CC neutrino scattering

$$\boxed{J^{\mu} = V^{\mu} - A^{\mu}}$$ Vector – Axial

Vector $$V^{\mu}_{s' s}(\mathbf{p}',\mathbf{p}) = \overline{u}_{s'}(\mathbf{p}')\left[2F_1^V\gamma^{\mu} + 2F_2^V i\sigma^{\mu\nu}\frac{q_{\nu}}{2m_N}\right]u_s(\mathbf{p})$$

Conserved Vector Current (CVC) $q_{\alpha}V^{\alpha} = 0$ and isospin symmetry $\Rightarrow$ $F_i^V = F_i^p - F_i^n$

Axial $$A^{\mu}_{s' s}(\mathbf{p}',\mathbf{p}) = \overline{u}_{s'}(\mathbf{p}')\left[G_A\gamma^{\mu}\gamma_5 + G_P\frac{q^{\mu}}{2m_N}\gamma_5\right]u_s(\mathbf{p})$$

Partially Conserved Axial Current (PCAC) and pion-pole dominance $\Rightarrow$ $G_P = \dfrac{4m_N^2}{m_{\pi}^2 + Q^2}G_A$

$$q_{\alpha}A^{\alpha} = i(m_u + m_d)\bar{q}_u\gamma_5 q_d \rightarrow 0$$

The nucleon form factors

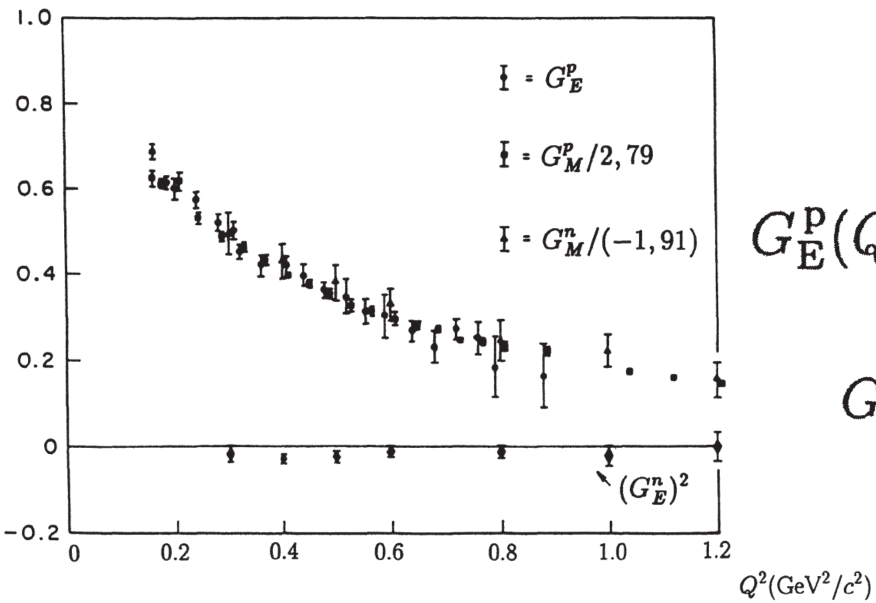
The form factors are corrections to "point-like coupling''
They reflect the fact that the nucleon has an internal structure and a finite size

F_1 and F_2 can be written as a combination of the Electric and Magnetic form factors G_E and G_M

$$F_1^{p,n} = \left[G_E^{p,n} + \frac{Q^2}{4m_N^2} G_M^{p,n}\right]\left[1 + \frac{Q^2}{4m_N^2}\right]^{-1} \qquad F_2^{p,n} = \left[G_M^{p,n} - G_E^{p,n}\right]\left[1 + \underbrace{\frac{Q^2}{4m_N^2}}_{\tau}\right]^{-1}$$

Electron-nucleon cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}\right]$$



Global **dipole**-like behavior

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{2.79} = \frac{G_M^n(Q^2)}{-1.91} = G^{\text{dipole}}(Q^2)$$

$$G^{\text{dipole}}(Q^2) = \left(1 + \frac{Q^2}{0.71\ (\text{GeV}/c)^2}\right)^{-2}$$

Several models to calculate the responses and the ν cross sections

- Local Fermi Gas + Random Phase Approximation

Lyon *M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C 80 065501 (2009)*

Valencia *J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas, Phys. Rev. C 83 045501 (2011)*

- Hartree-Fock + (Continuum) Random Phase Approximation

Ghent *V. Pandey, N. Jachowicz, T. Van Cuyck, J. Ryckebusch, M. Martini, Phys. Rev. C 92 024606 (2015)*

Other groups focused on giant resonances and below *Kolbe et al. ; Volpe et al.; Co' et al.; …*

- SuSAv2 superscaling/relativistic mean field

Granada, Madrid, MIT, Sevilla, Torino

G.D. Megias, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, I. Ruiz Simo, PRD 94 093004 (2016)

- Spectral function approach

Roma *N. Rocco, C. Barbieri, O. Benhar, A. De Pace, A. Lovato, Phys. Rev. C 99 025502 (2019)*

- Relativistic Green's function

Pavia *A. Meucci, C. Giusti, F. D. Pacati, Nucl.Phys.A 739 277-290 (2004)*

- Green's function Monte Carlo ("ab initio")

Argonne, Los Alamos *A. Lovato, J. Carlson, S. Gandolfi, N. Rocco, R. Schiavilla, PRX 10 031068 (2020)*

- GiBUU transport theory

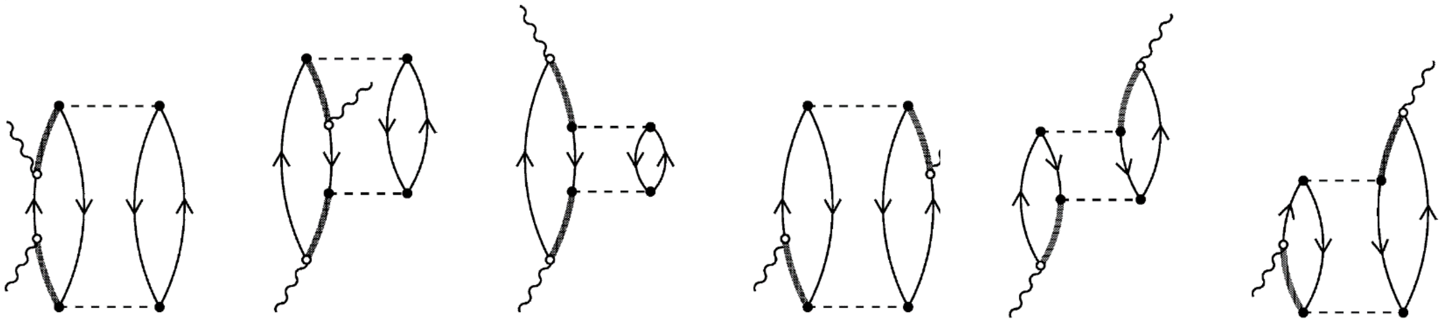
Giessen *O. Buss, T. Gaitanos, K. Gallmeister, H. van Hees, M. Kaskulov, O. Lalakulich, A.B. Larionov, T. Leitner, J. Weil, U. Mosel, Phys.Rept. 512 1-124 (2012)*

p.s. only one representative reference for each approach (not necessarily the founding paper)

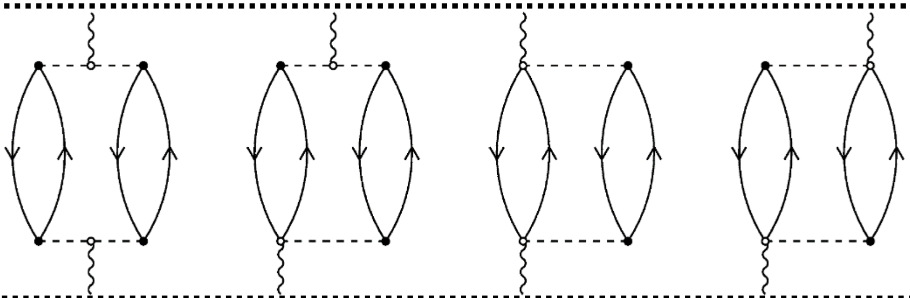
For discussions and comparisons of different models see for example:

- *G.T. Garvey, D.A. Harris, H.A. Tanaka, R. Tayloe, G.P. Zeller, Phys.Rept. 580 (2015) 1-45*
- *T. Katori, M. Martini, J.Phys.G 45 (2018) 1, 013001*
- *M. Sajjad Athar, A. Fatima, S. K. Singh arxiv. 2206.13792*

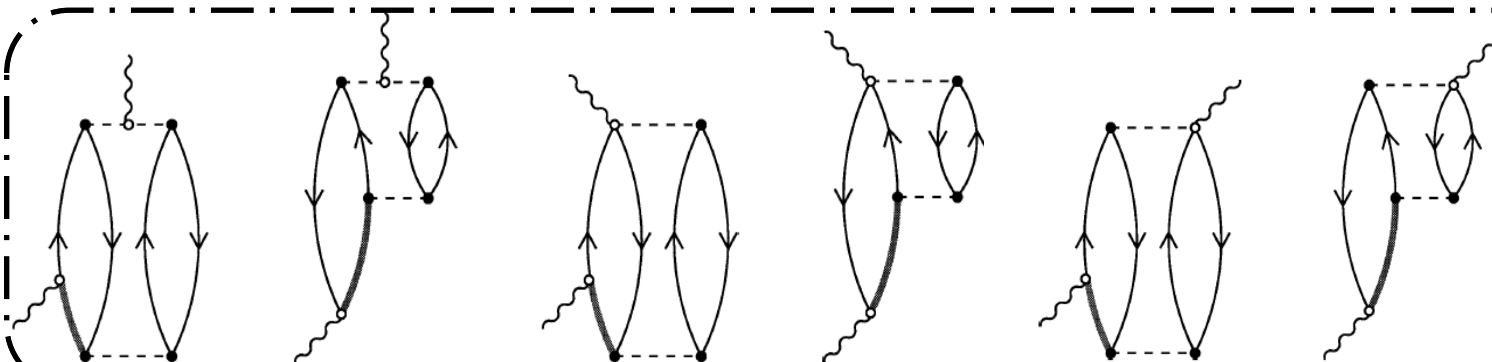
MEC contributions



Δ-MEC



Pionic

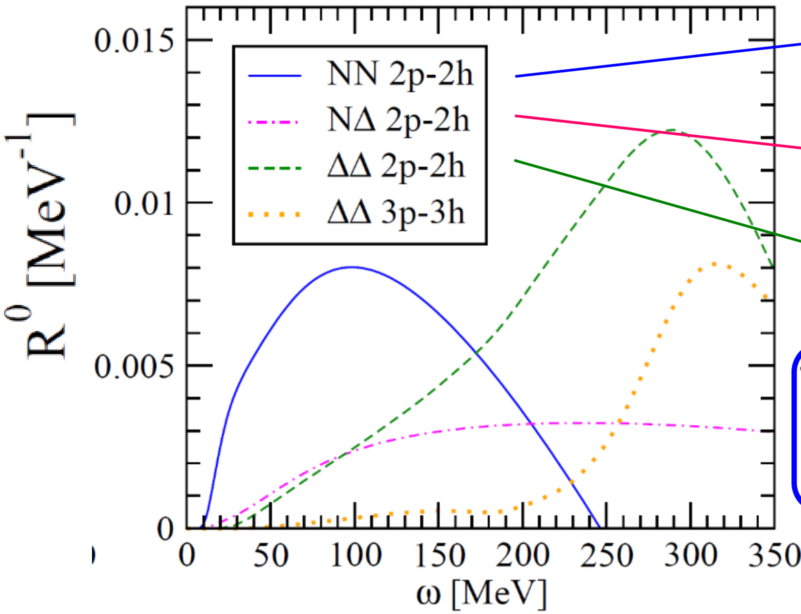


π-Δ interference

De Pace, Nardi, Alberico, Donnelly, Molinari, NPA741 (2004)

Separation of np-nh contributions in the nuclear responses

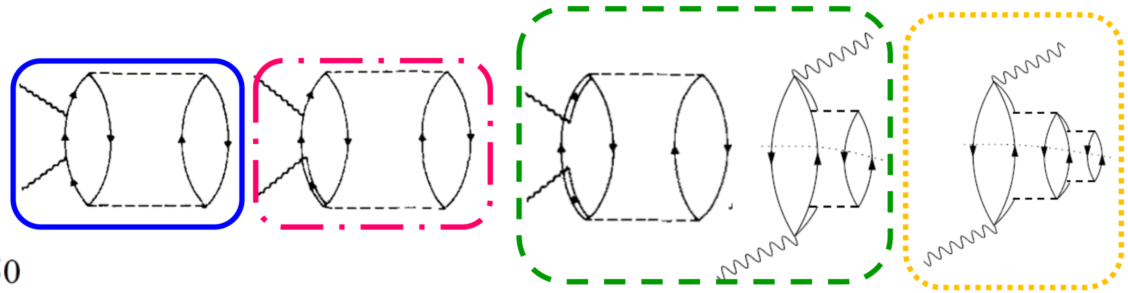
M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)



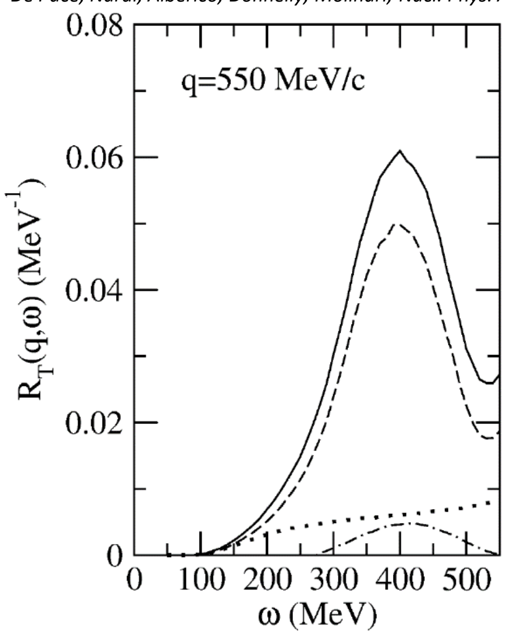
also called NN SRC; part of 1-body current contribution in correlated nuclear wave functions approaches, like SF or GFMC

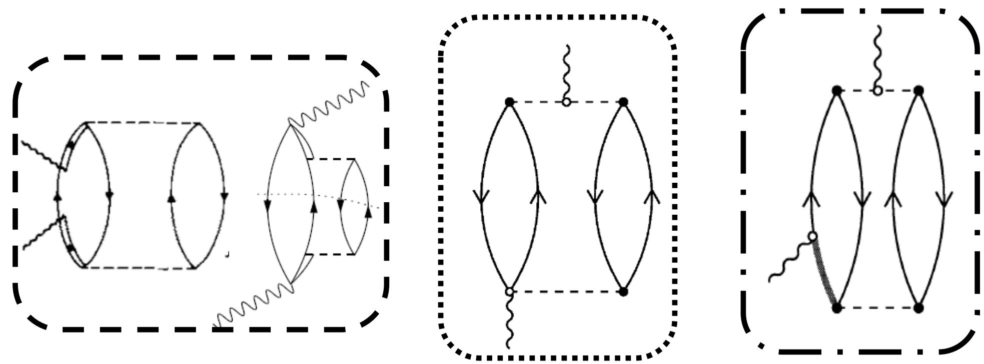
NΔ interference, also called NN correlation-ΔMEC interference or 1-body—2-body interference

Δ mediated MEC



De Pace, Nardi, Alberico, Donnelly, Molinari, Nucl. Phys. A741, 249 (2004)

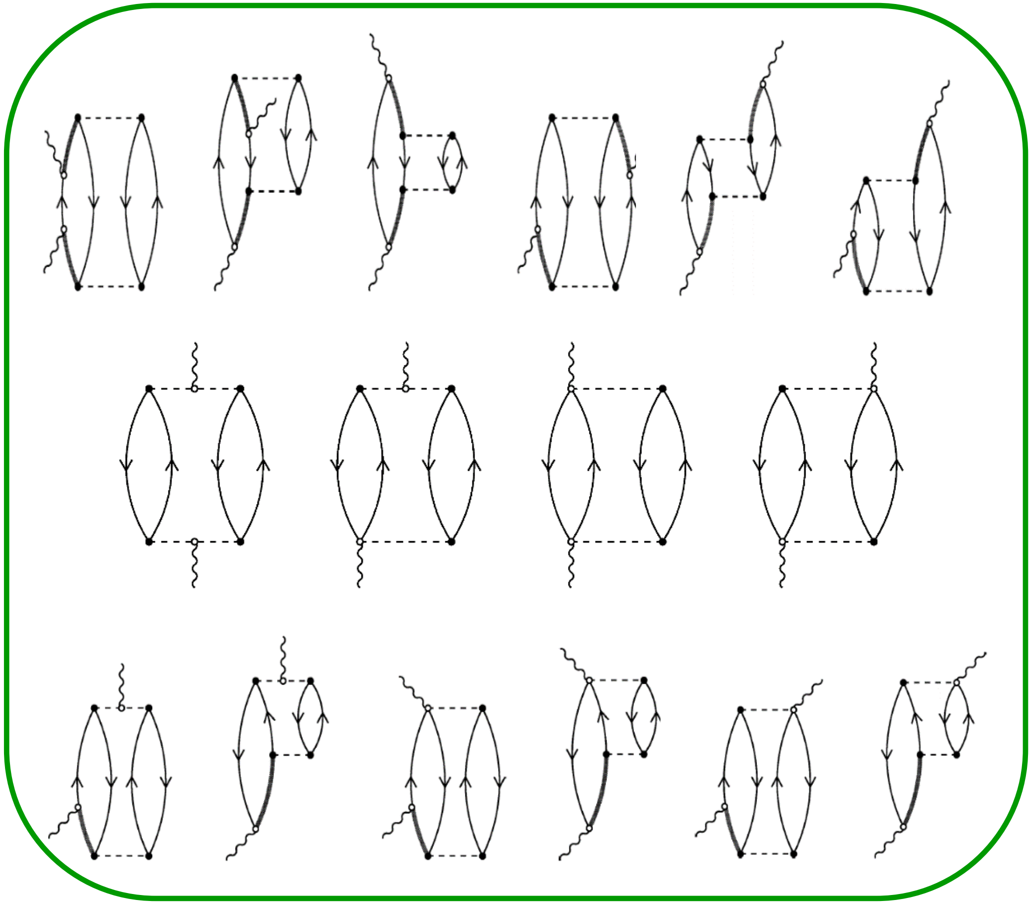




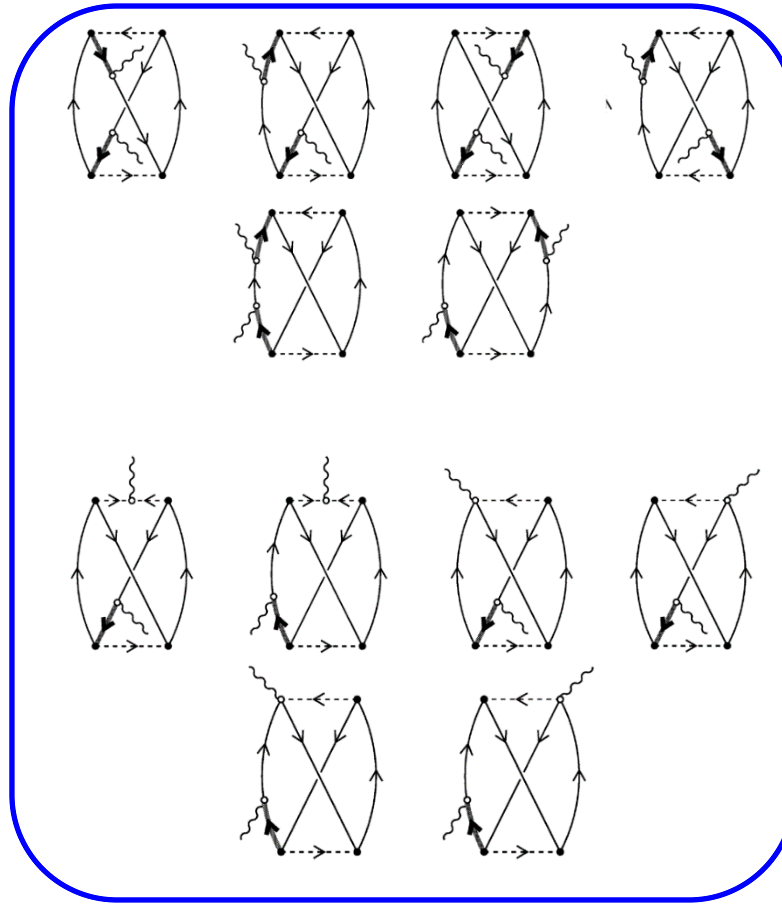
Δ **π** **π-Δ intf.**

Direct and exchange MEC contributions

Direct



Exchange



Fully relativistic calculation of *De Pace, Nardi, Alberico, Donnelly, Molinari, NPA741 (2004):*

3000 direct terms

More than **100 000** exchange terms

Electromagnetic 2p-2h MEC response

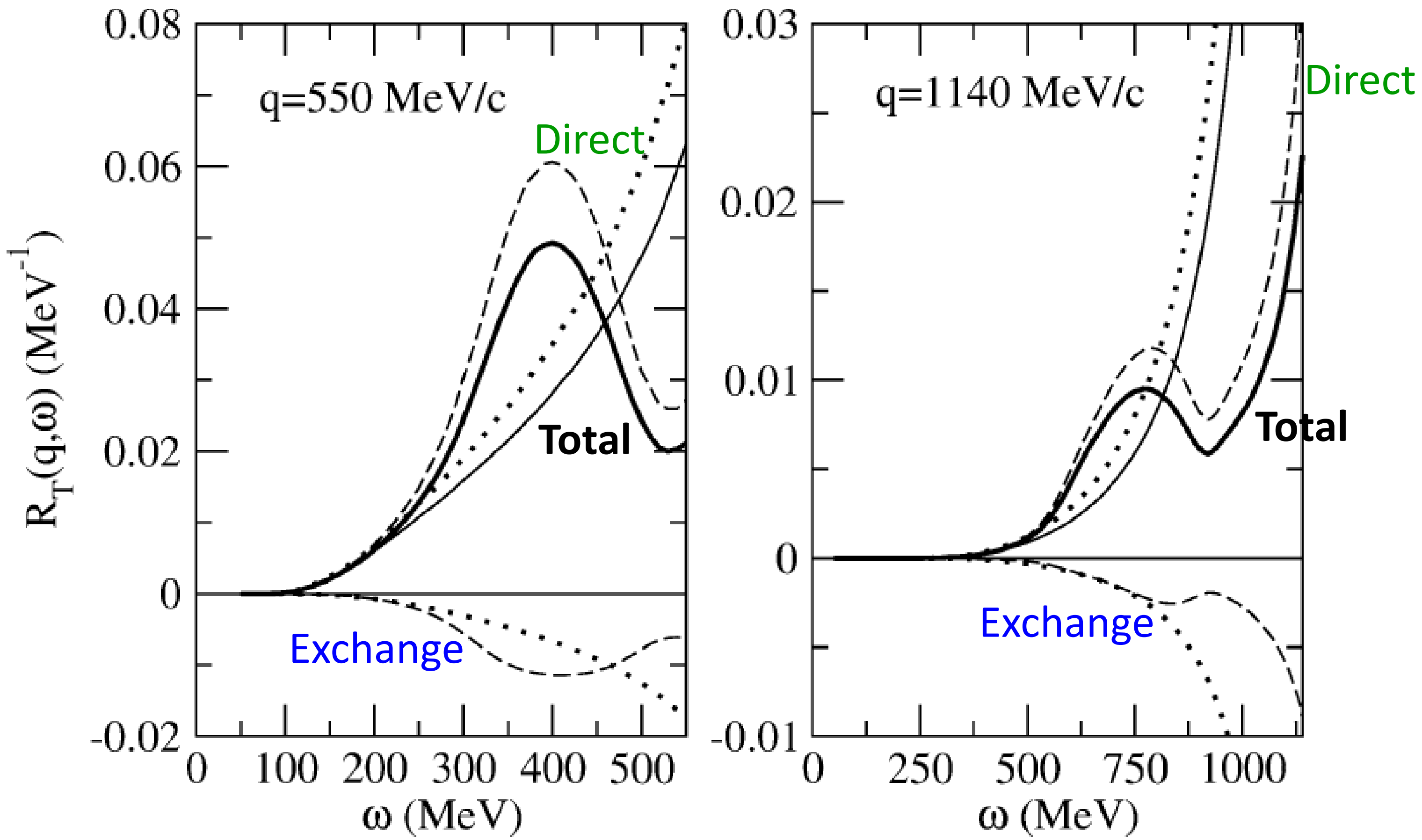
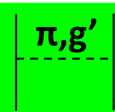
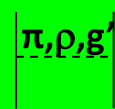
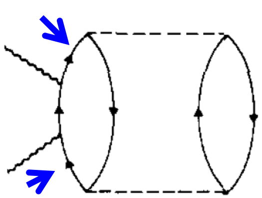


Fig. 12. The transverse response function $R_T(q, \omega)$ at $q = 550$ MeV/c and $q = 1140$ MeV/c including the exchange contributions: non-relativistic direct (positive dotted), non-relativistic exchange (negative dotted), non-relativistic total (light solid), relativistic direct (positive dashed), relativistic exchange (negative dashed) and relativistic total (heavy solid). In all instances $\bar{\epsilon}_2 = 70$ MeV and $k_F = 1.3$ fm^{-1}.

De Pace, Nardi, Alberico, Donnelly, Molinari, Nucl. Phys. A741, 249 (2004)

Different approximations for the 2p-2h calculations

Approach	Vector	Axial	NN correlations	MEC	NN-MEC interference	Relativistic	
RPA Lyon Martini et al.	Yes	Yes	 π,g'	Yes (Only Δ MEC)	Yes	Some ingredients	No
RPA Valencia Nieves et al.	Yes	Yes	 π,ρ,g'	Yes	Yes	Approximations in the WNNπ vertex	No
SuSAv2	Yes	Yes	Already in Superscaling function (1p-1h part)	Yes	No	Fully Relativistic	Yes



$$(p_0 - E_{\mathbf{p}} + i\epsilon)^{-2}$$

- **Divergences in NN correlations, prescriptions:**

-nucleon propagator only off the mass shell (*Alberico et al. Ann. Phys. 1984*)

-kinematical constraints + nucleon self energy in the medium (*Nieves et al PRC 83*)

- regularization parameter taking into account the finite size of the nucleus to be fitted to data (*Amaro et al. PRC 82 044601 2010*)

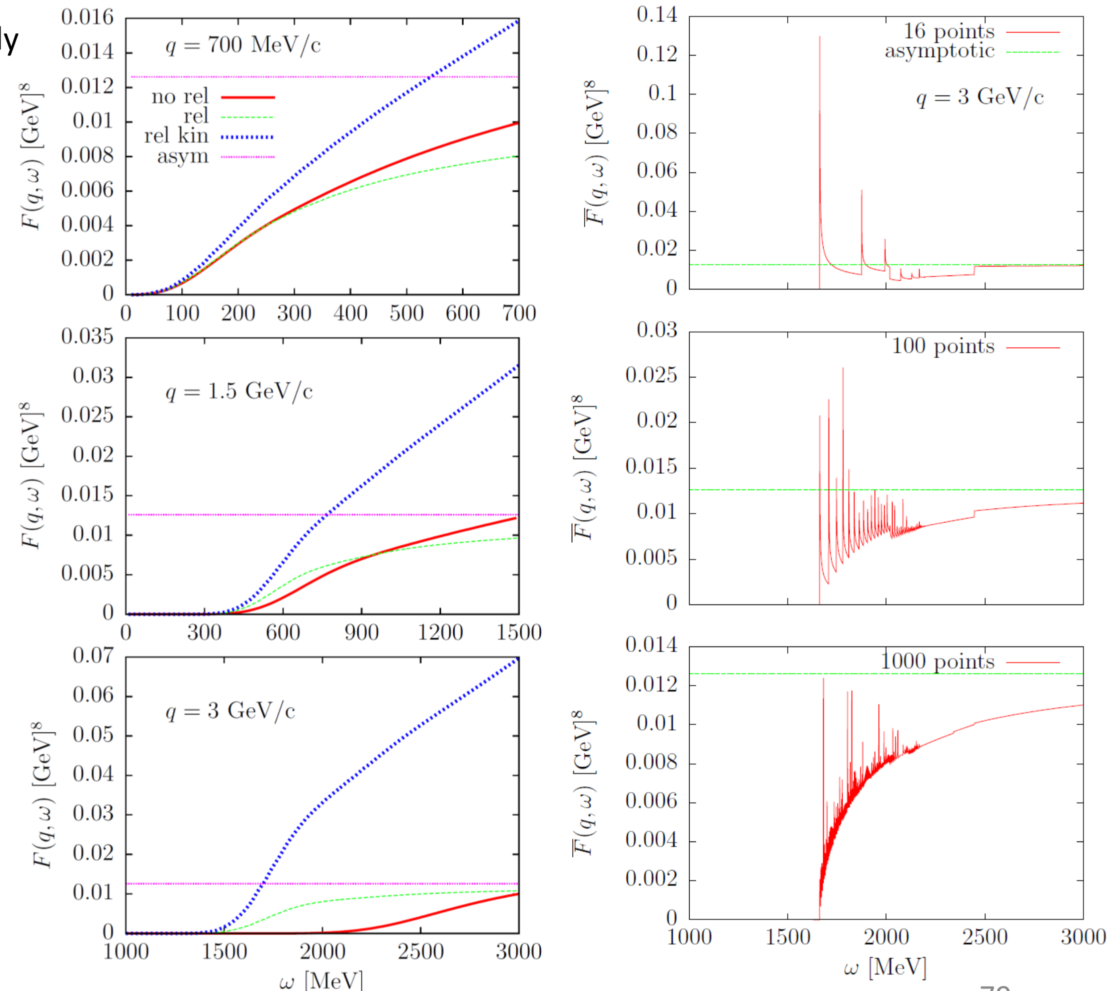
T. Katori, M. Martini, J.Phys.G 45 (2018) 1, 013001

2p-2h phase space integral

$$F(\omega,q) \equiv \int d^3h_1\, d^3h_2\, d^3p'_1\, \frac{m_N^4}{E_1E_2E'_1E'_2}\Theta(p'_1,p'_2,h_1,h_2)\delta(E'_1+E'_2-E_1-E_2-\omega)$$

$$\overline{F}(\omega,q) = \left(\frac{4}{3}\pi k_F^3\right)^2 \int d^3p'_1\, \delta(E'_1+E'_2-\omega-2m_N)\,\Theta(p'_1,p'_2,0,0)\frac{m_N^2}{E'_1E'_2}$$

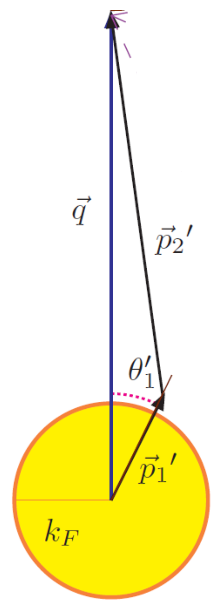
Ruiz Simo, Albertus, Amaro, Barbaro, Caballero, Donnelly
Phys. Rev. D 90 033012 (2014)
Phys. Rev. D 90 053010 (2014)

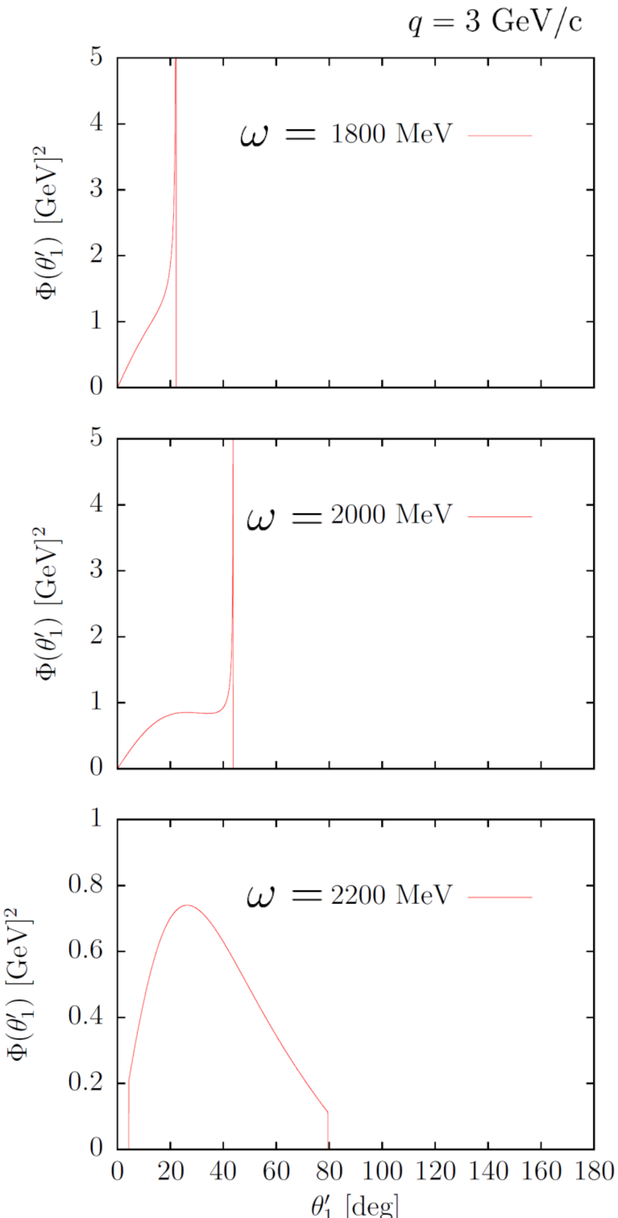


Angular distribution of ejected nucleons

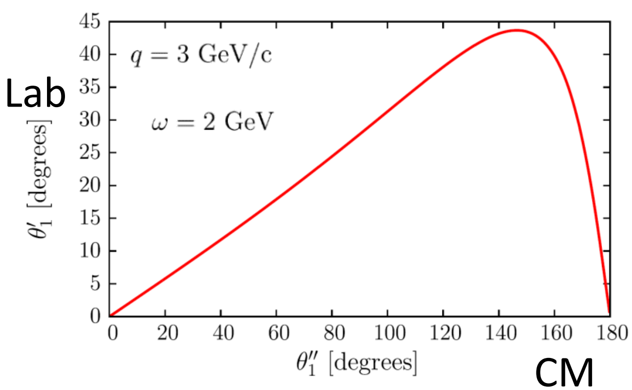
$$\overline{F}(\omega,q) = \left(\frac{4}{3}\pi k_F^3\right)^2 2\pi \int_0^{\pi} d\theta'_1\,\Phi(\theta'_1)$$

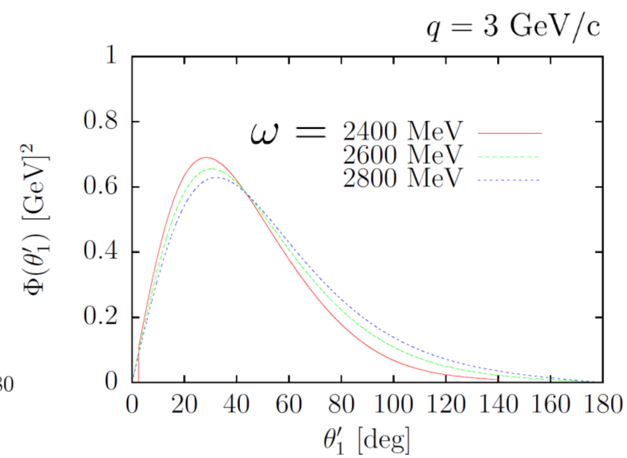
$$\Phi(\theta'_1) = \sin\theta'_1 \int p_1'^2 dp'_1 \delta(E_1+E_2+\omega-E'_1-E'_2) \times \Theta(p'_1,p'_2,h_1,h_2)\frac{m_N^4}{E_1E_2E'_1E'_2} = \sum_{\alpha=\pm} \frac{m_N^4 \sin\theta'_1 p_1'^2 \Theta(p'_1,p'_2,h_1,h_2)}{E_1E_2E'_1E'_2\left|\frac{p'_1}{E'_1}-\frac{\mathbf{p}'_2\cdot\hat{\mathbf{p}}'_1}{E'_2}\right|}\Bigg|_{p'_1=p_1'^{(\alpha)}}$$





Ruiz Simo, Albertus, Amaro, Barbaro, Caballero, Donnelly
Phys. Rev. D 90 033012 (2014)
Phys. Rev. D 90 053010 (2014)

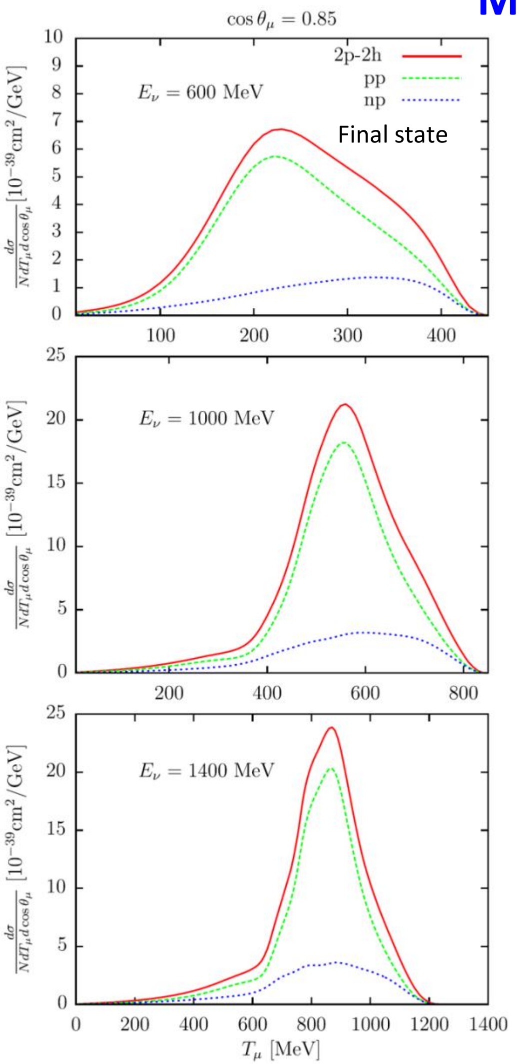


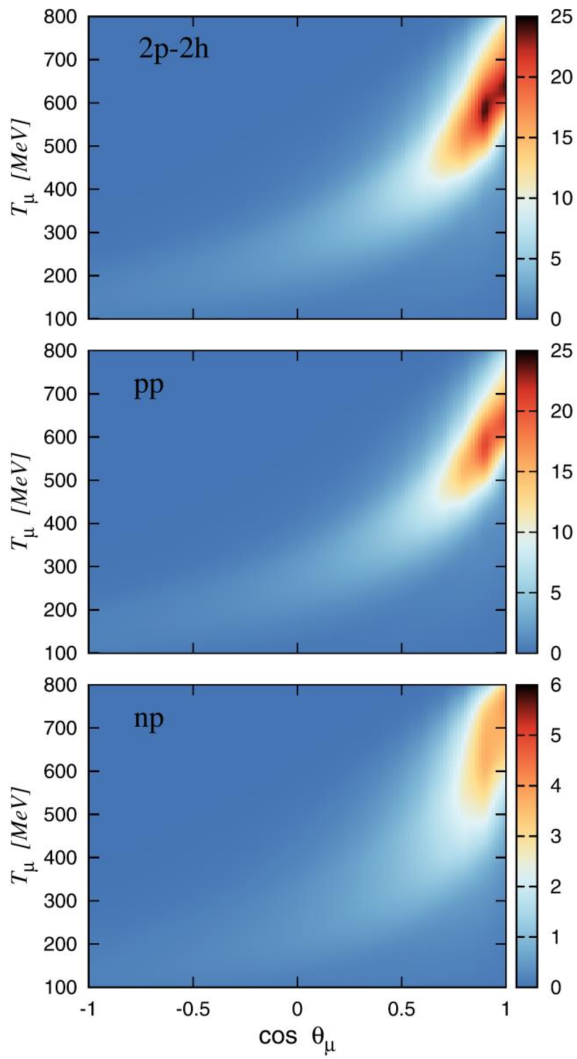


Theoretical studies on hadron information – Isospin content

I. Ruiz Simo et al. Phys. Lett.B762, 124 (2016)

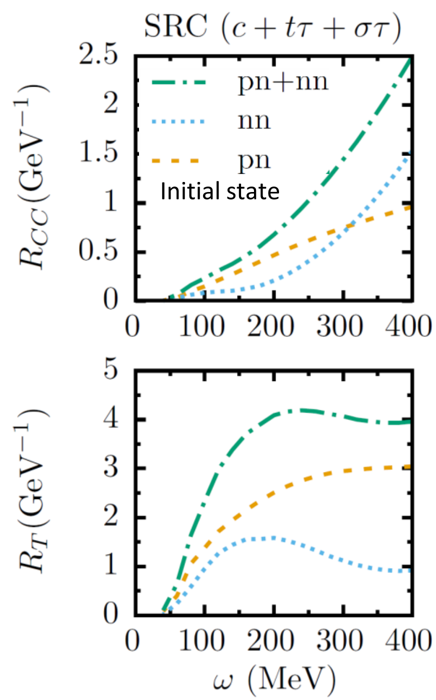
MEC





T. Van Cuyck et al. PRC 94, 024611(2016)

NN SRC

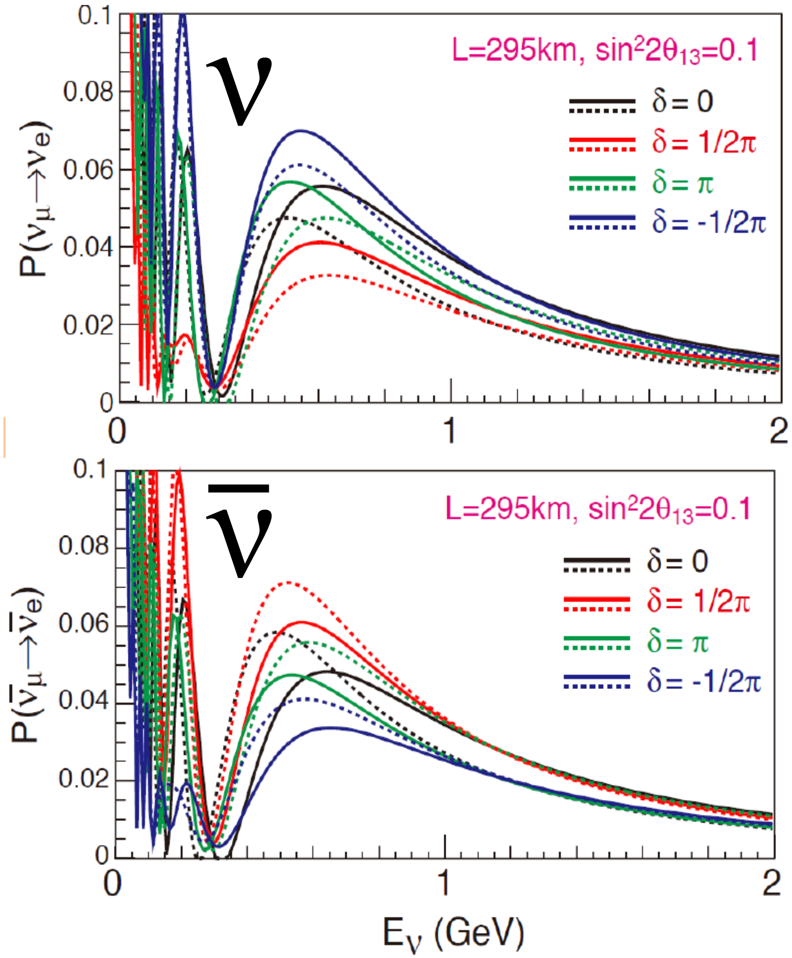


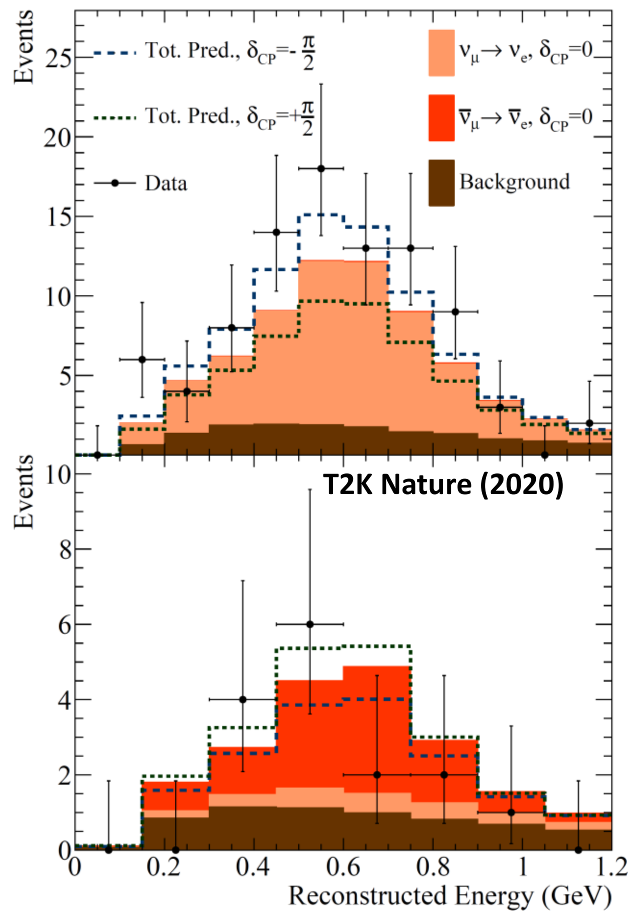
- The **pp** channel final state (**np** in the initial state) dominates in MEC and SRC
- The **pp**/**np** ratio depends on the kinematics

ν .vs. $\bar{\nu}$ and ν_μ .vs. ν_e

ν oscillation and CP violation

$$P(\nu_\mu \rightarrow \nu_e) \overset{?}{\neq} P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \Longleftrightarrow \delta_{CP}$$





$$\nu_\mu \rightarrow \nu_e$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

A precise and simultaneous knowledge of the four cross sections is important in connection to the oscillation experiments aiming at the search for CP violation in the lepton sector.

Neutrino *vs* Antineutrino interactions

The ν and anti ν cross sections differ by the sign of the V-A interference term

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2\cos^2\theta_c}{\pi}|\mathbf{k}'|E_l'\cos^2\frac{\theta}{2}\left[\frac{(\mathbf{q}^2-\omega^2)^2}{\mathbf{q}^4}G_E^2 R_\tau(\mathbf{q},\omega) + \frac{\omega^2}{\mathbf{q}^2}G_A^2 R_{\sigma\tau(L)}(\mathbf{q},\omega)\right.$$

$$\left. + 2\left(\tan^2\frac{\theta}{2}+\frac{\mathbf{q}^2-\omega^2}{2\mathbf{q}^2}\right)\left(G_M^2\frac{\mathbf{q}^2}{4M_N^2}+G_A^2\right)R_{\sigma\tau(T)}(\mathbf{q},\omega) \pm 2\frac{E_\nu+E_l'}{M_N}\tan^2\frac{\theta}{2}G_A G_M R_{\sigma\tau(T)}(\mathbf{q},\omega)\right]$$

Vector-Axial interference

$$\begin{cases} + & (\nu) \\ - & (\bar{\nu}) \end{cases}$$

Vector-Axial interference:

basic asymmetry from weak interaction theory

different sign in the Leptonic tensor

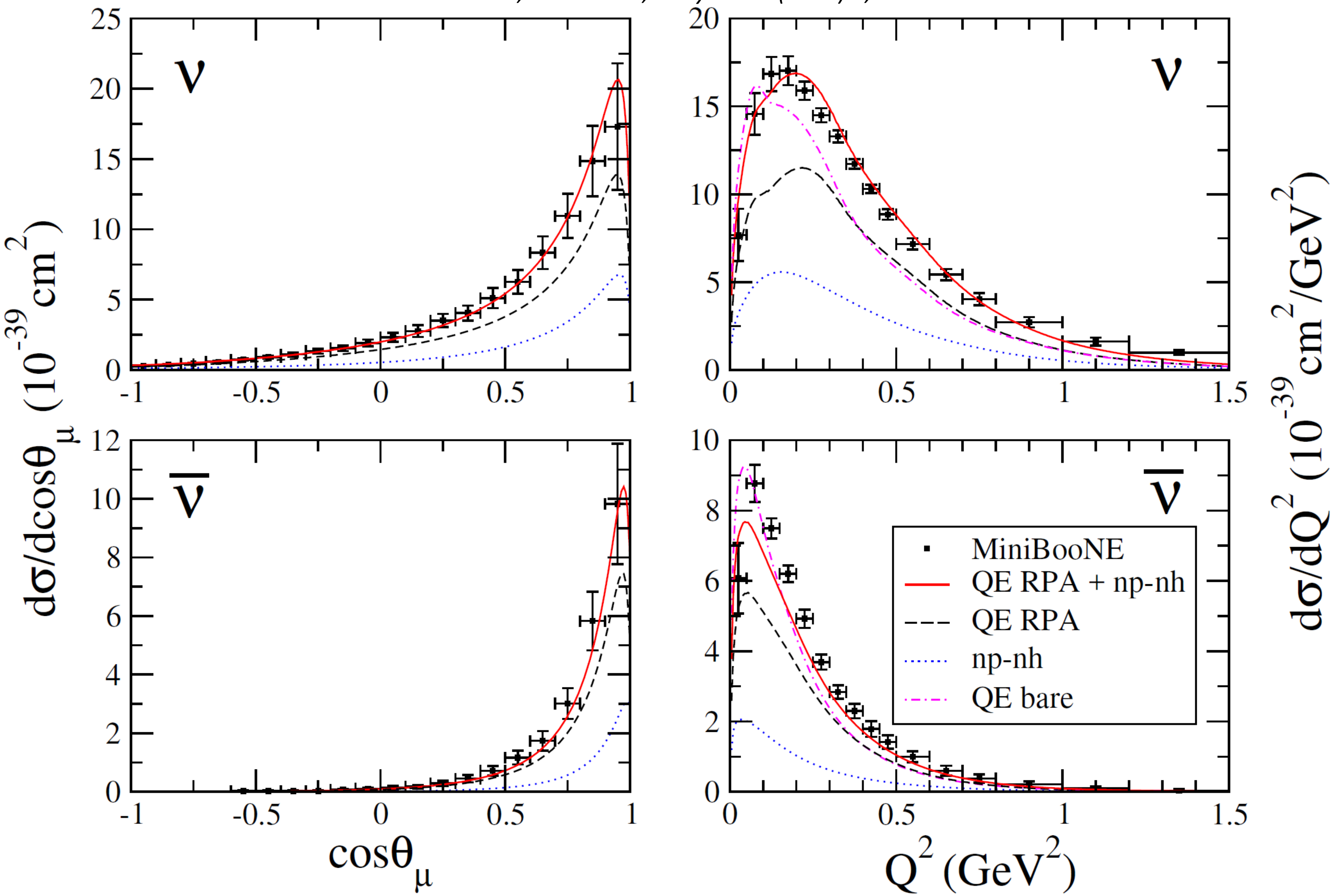
$$L_{\mu\nu} = k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k\cdot k' \mp i\varepsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta$$

(upper sign: ν; lower sign: $\bar{\nu}$)

⇒ Even neglecting nuclear effects, the absolute value and the kinematic behavior of neutrino and antineutrino cross sections are different

dσ/dcosθ Q^2 distribution

T. Katori, M. Martini, J.Phys.G 45 (2018) 1, 013001



- Antineutrino cross section falls more rapidly than the neutrino one
- Antineutrino Q^2 distribution peaks at smaller Q^2 values than the neutrino one

Neutrino *vs* Antineutrino interactions and nuclear effects

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2\cos^2\theta_c}{\pi}|\mathbf{k}'|E_l'\cos^2\frac{\theta}{2}\left[\frac{(\mathbf{q}^2-\omega^2)^2}{\mathbf{q}^4}G_E^2 R_\tau(\mathbf{q},\omega) + \frac{\omega^2}{\mathbf{q}^2}G_A^2 R_{\sigma\tau(L)}(\mathbf{q},\omega)\right.$$

$$\left. + 2\left(\tan^2\frac{\theta}{2}+\frac{\mathbf{q}^2-\omega^2}{2\mathbf{q}^2}\right)\left(G_M^2\frac{\mathbf{q}^2}{4M_N^2}+G_A^2\right)R_{\sigma\tau(T)}(\mathbf{q},\omega) \pm 2\frac{E_\nu+E_l'}{M_N}\tan^2\frac{\theta}{2}G_A G_M R_{\sigma\tau(T)}(\mathbf{q},\omega)\right]$$

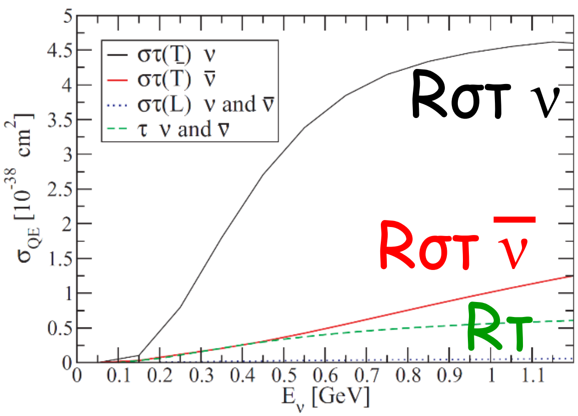
Vector-Axial interference

The ν and anti ν interactions differ by the sign of the V-A interference term

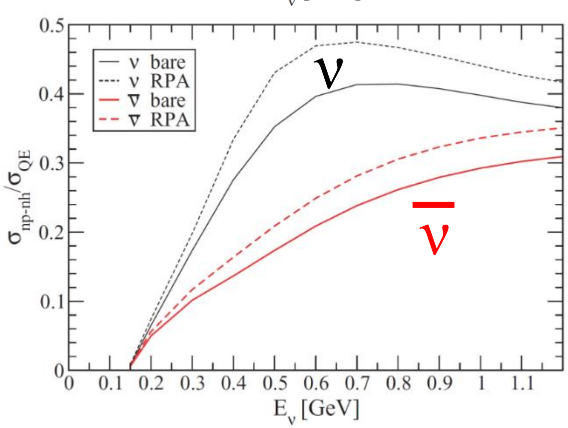
→the relative weight of the different nuclear responses is different for neutrinos and antineutrinos

→the relative role of np-nh contributions is different for neutrinos and antineutrinos

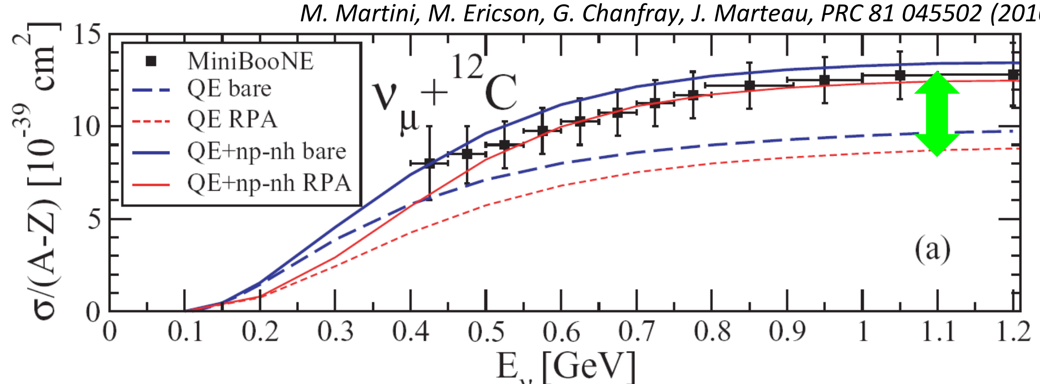
QE

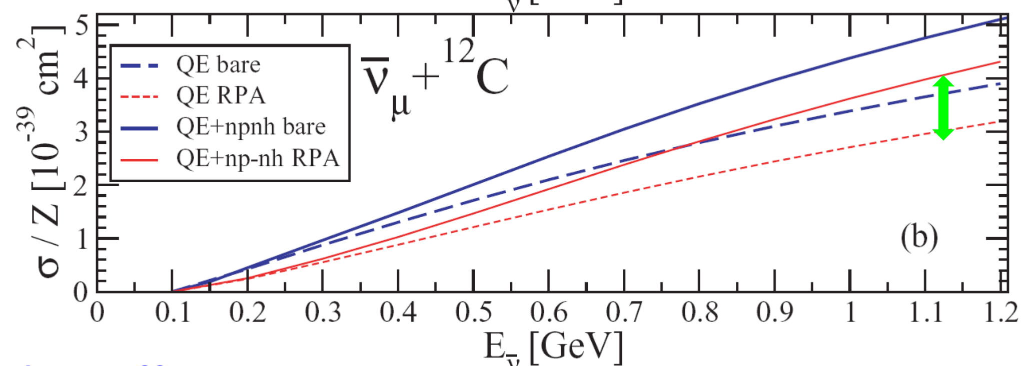


$\frac{np-nh}{QE}$



M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 81 045502 (2010)





Nuclear effects generate an asymmetry unrelated to CP violation

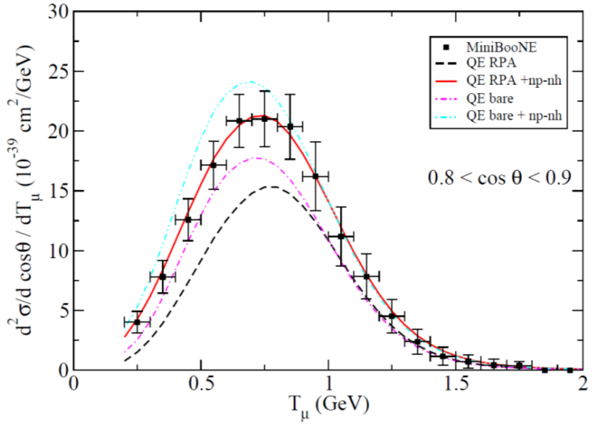
ν

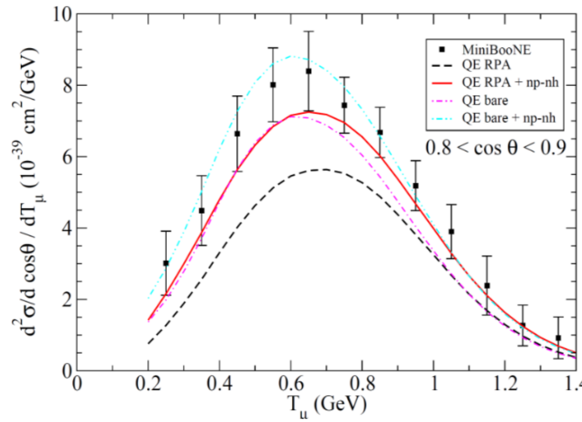
The relative role of np-nh for neutrinos and antineutrinos is different in different approaches

$\bar{\nu}$

Lyon RPA
Martini et al.

PRC 84 (2011)

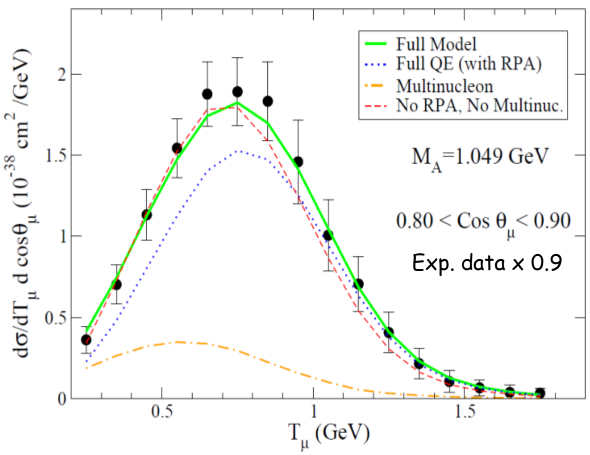


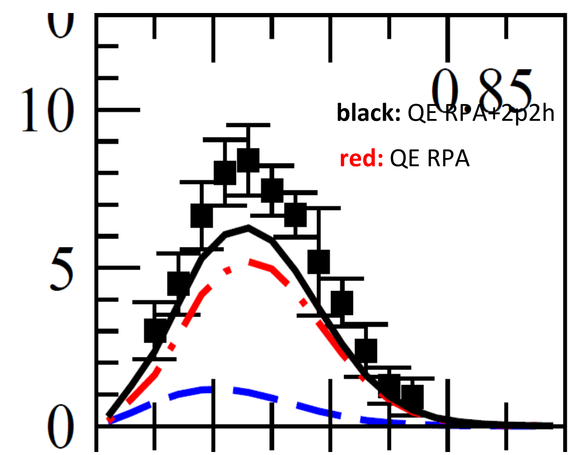


PRC 87 (2013)

Valencia RPA
Nieves et al.

PLB 707 (2012)

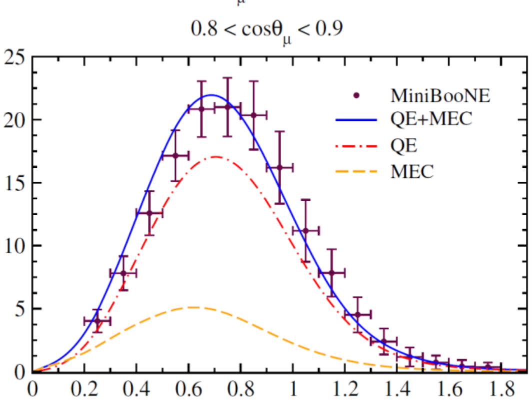


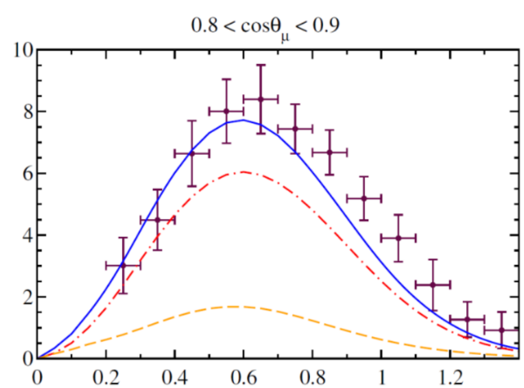


PLB 721 (2013)

SuSAv2

PRD 94 (2016)





PRD 94 (2016)

T. Katori, M. Martini, J.Phys.G 45 (2018) 1, 013001

Difference of ν and antiν cross sections and the VA interference term

$$d\sigma \sim d\sigma_L + d\sigma_T \pm d\sigma_{VA}$$

$$d\sigma_\nu - d\sigma_{\bar{\nu}} \overset{?}{\leftrightarrow} 2d\sigma_{VA}$$

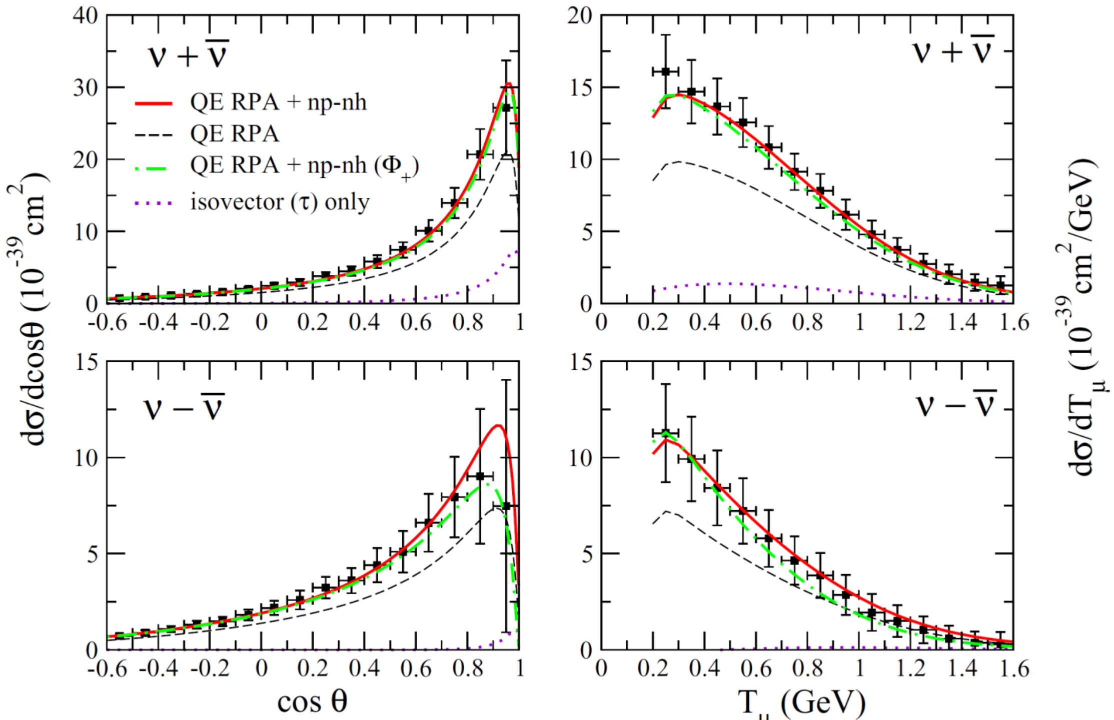
Difference gives only the VA term for identical ν and antiν flux

Problem: flux dependence of dσ $\frac{d^2\sigma}{dE_\mu d\cos\theta} = \int dE_\nu \left[\frac{d^2\sigma}{d\omega d\cos\theta}\right]_{\omega=E_\nu - E_\mu} \Phi(E_\nu)$

We introduce the **mean flux** $\Phi_+ = 1/2[\Phi_\nu + \Phi_{\bar{\nu}}]$

We calculate the sum and the difference using **real** and **mean** MiniBooNE fluxes results

M. Ericson, M. Martini Phys. Rev. C 91 035501 (2015)



The mean flux contribution is dominant

⇓

The VA interference term is experimentally accessible in MBdata

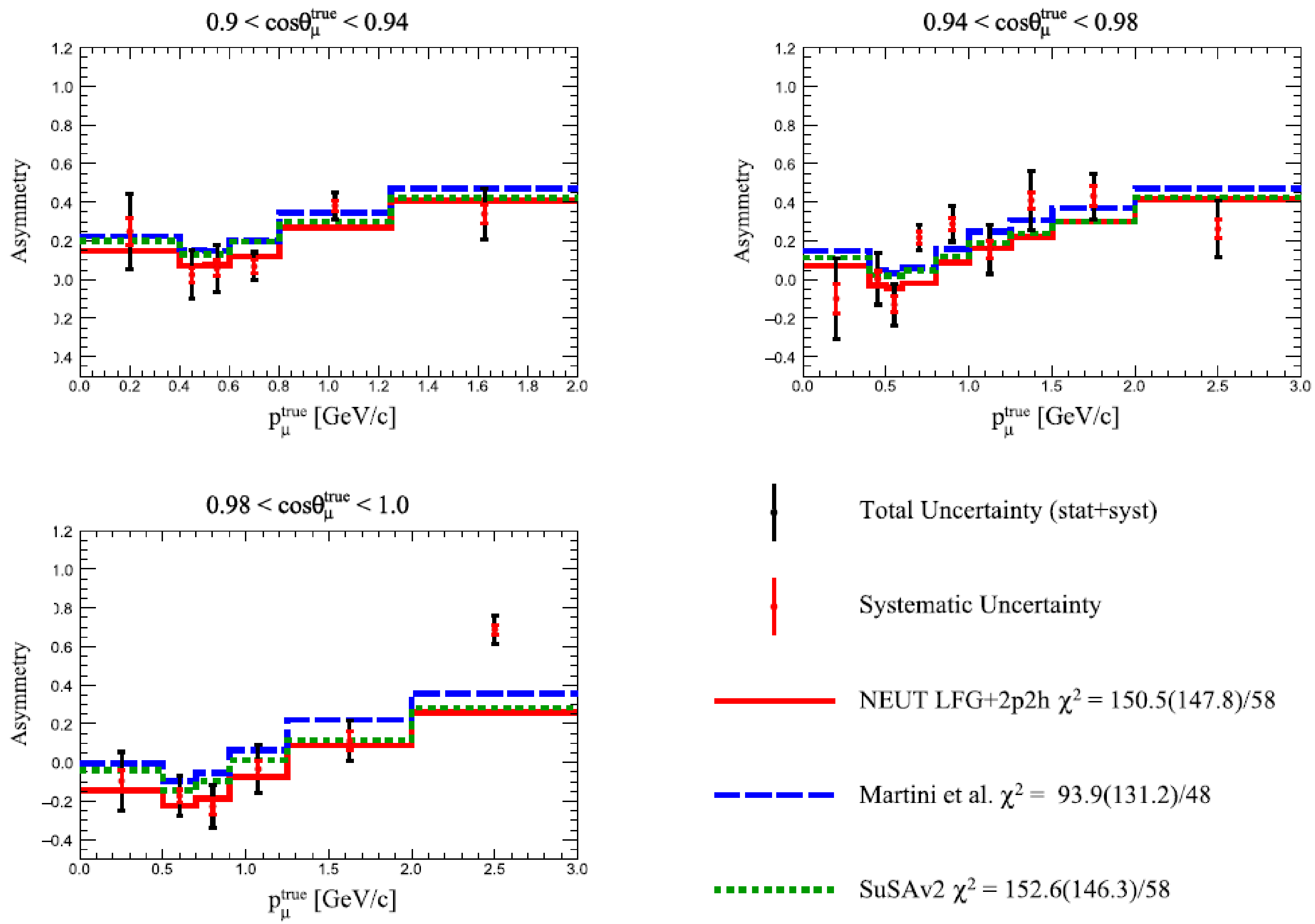
⇓

Need for the **multinucleon** component in the **VA interference**

PHYSICAL REVIEW D **101,** 112001 (2020)

First combined measurement of the muon neutrino and antineutrino charged-current cross section without pions in the final state at T2K

$$\frac{\nu-\bar{\nu}}{\nu+\bar{\nu}}$$

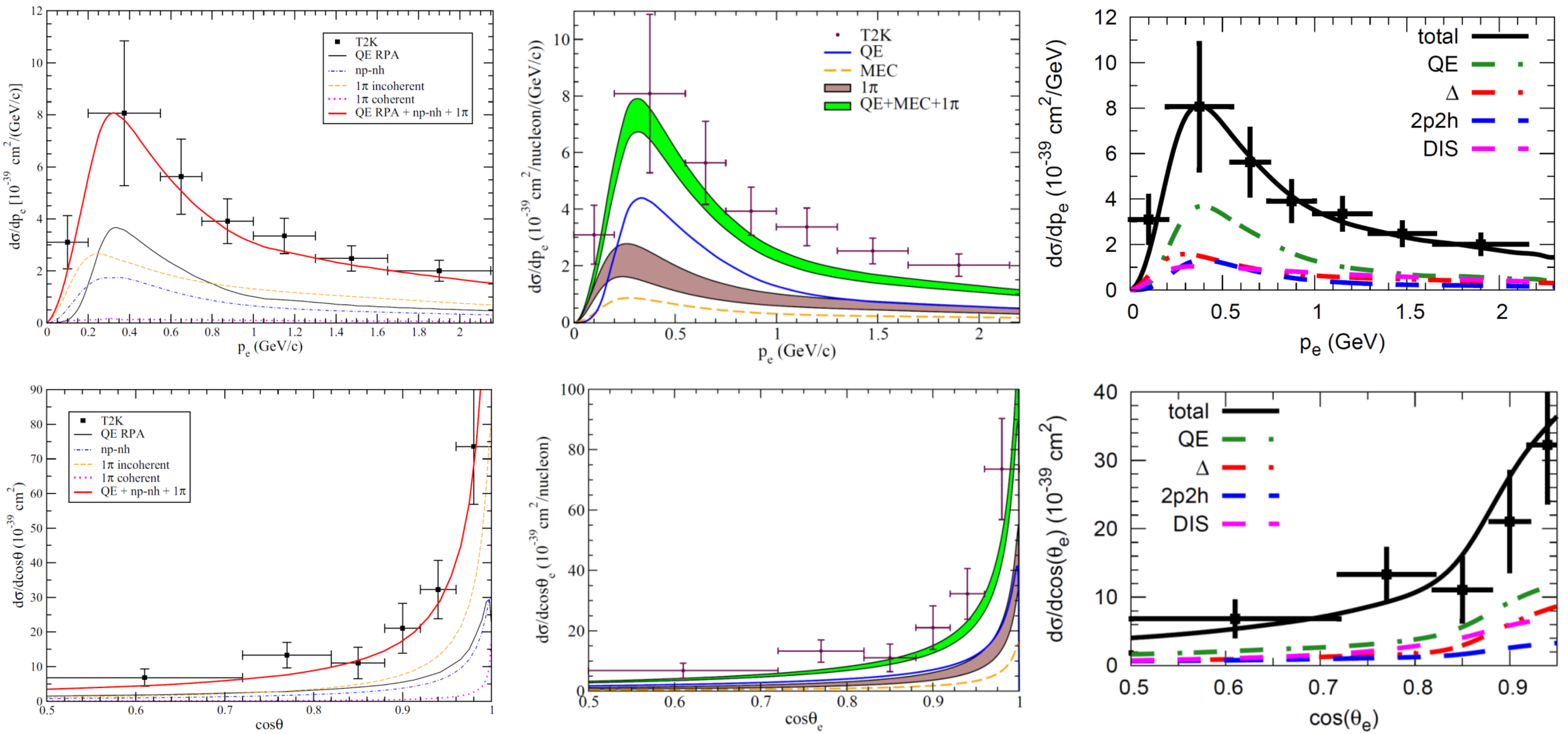


ν_e cross sections

- There are few published results on ν_e cross sections. This is essentially due the relatively small component of ν_e fluxes with respect to the ν_μ ones hence to small statistics.
- The ν_e experimental published results essentially concern inclusive cross sections

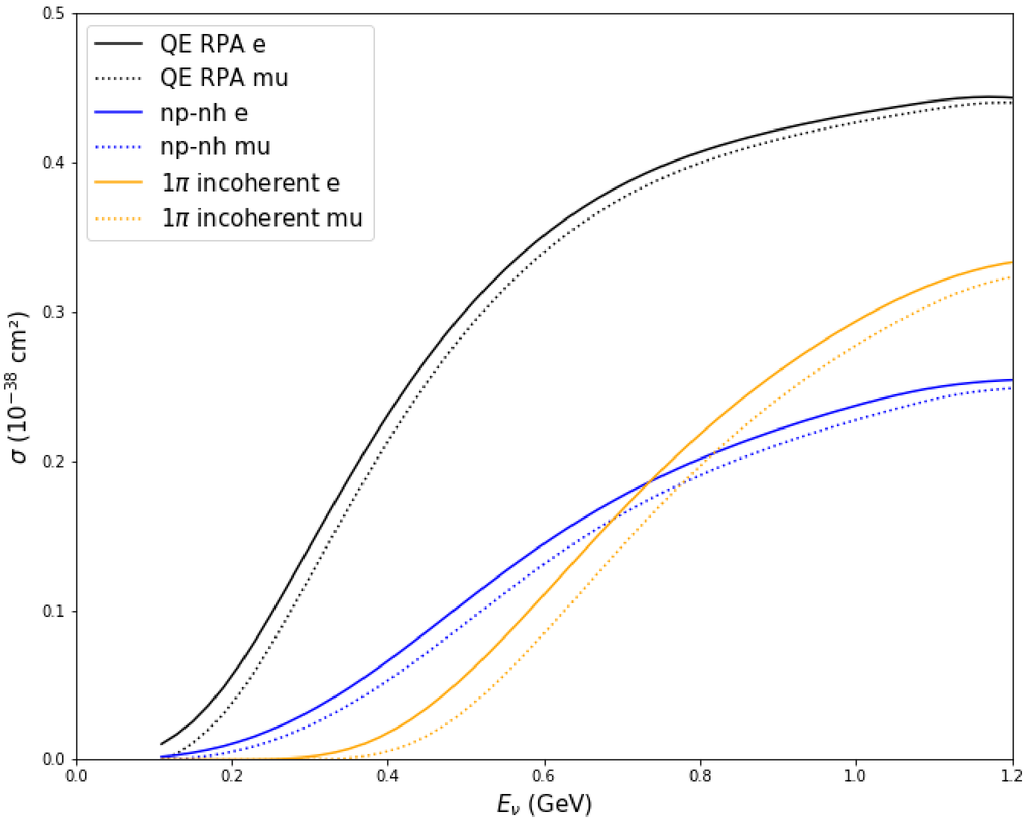
T2K flux-integrated ν_e CC inclusive differential cross sections on carbon

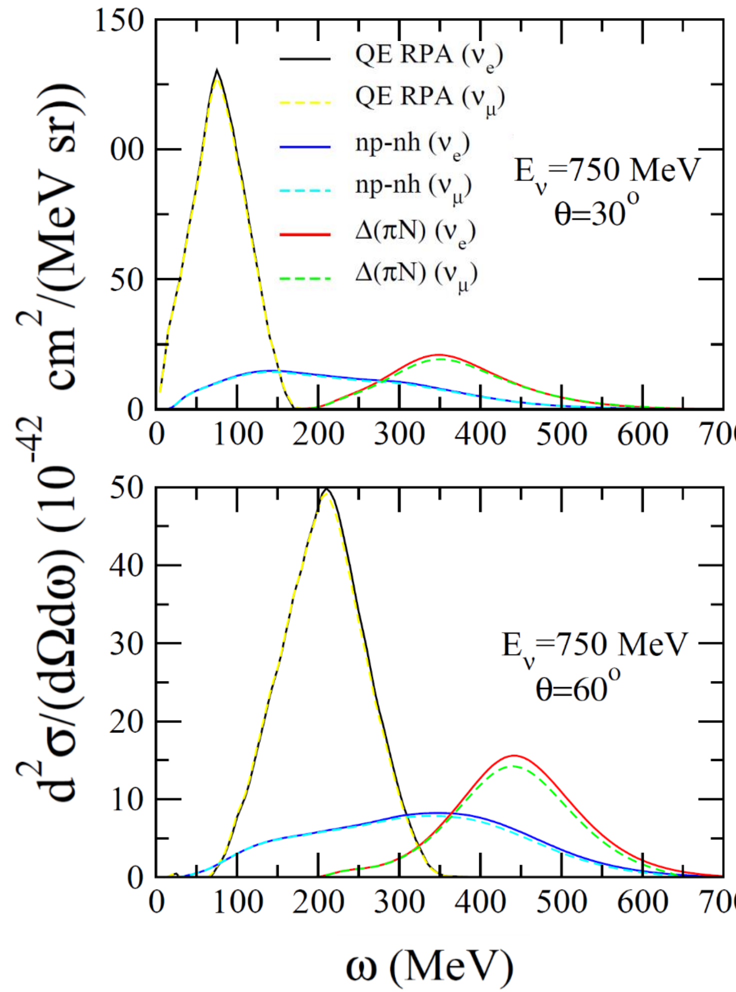
Martini et al., PRC 94 (2016) ***Megias et al., PRD 94 (2016)*** ***Gallmeister et al.PRC 94(2016)***



- Theoretical results agree with data
- Similarity of the theoretical results for the inclusive dσ

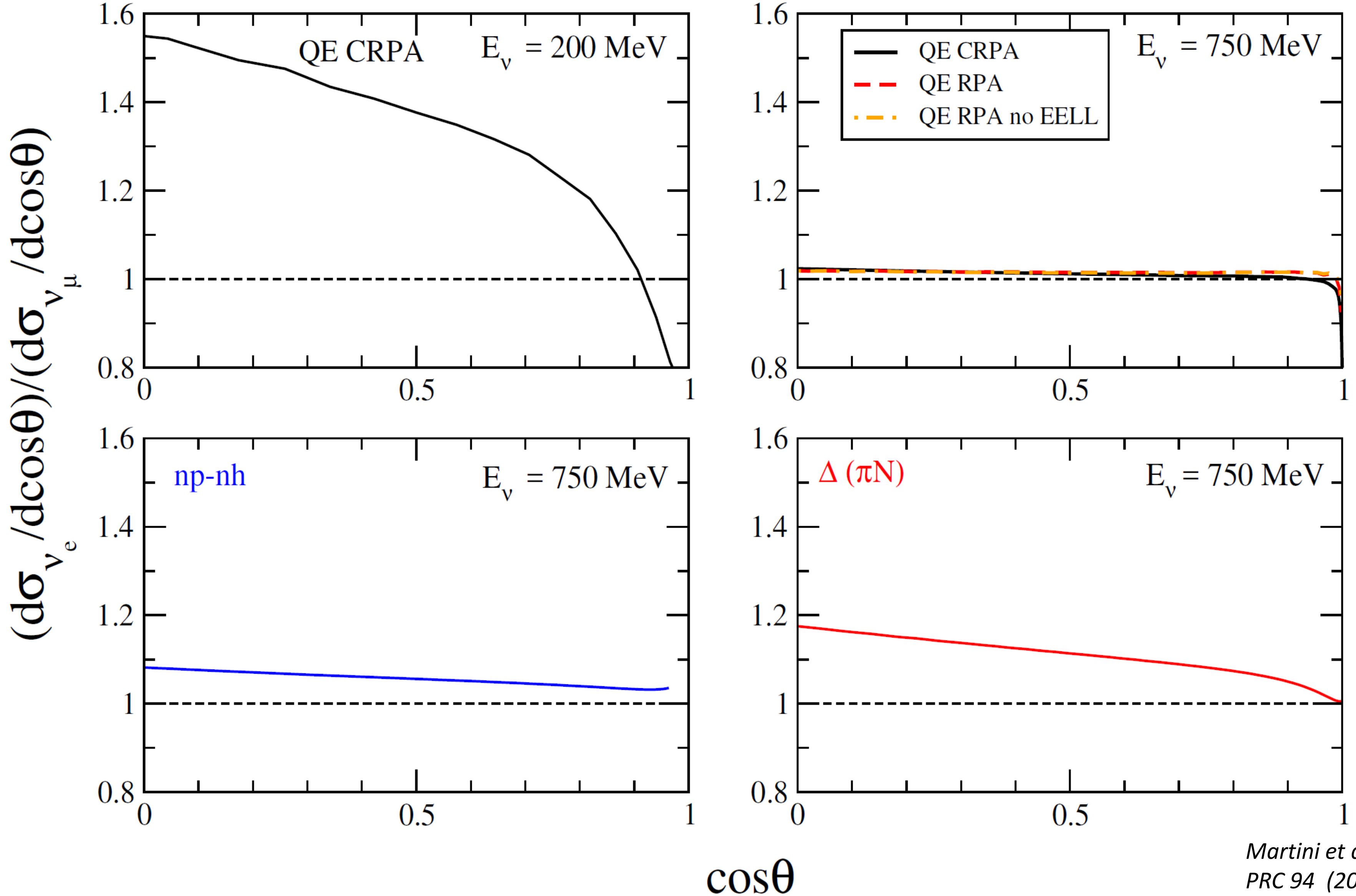
ν_e and ν_μ total and double differential cross sections





Due to the different kinematic limits, the ν_e cross sections are expected to be larger than the ν_μ ones

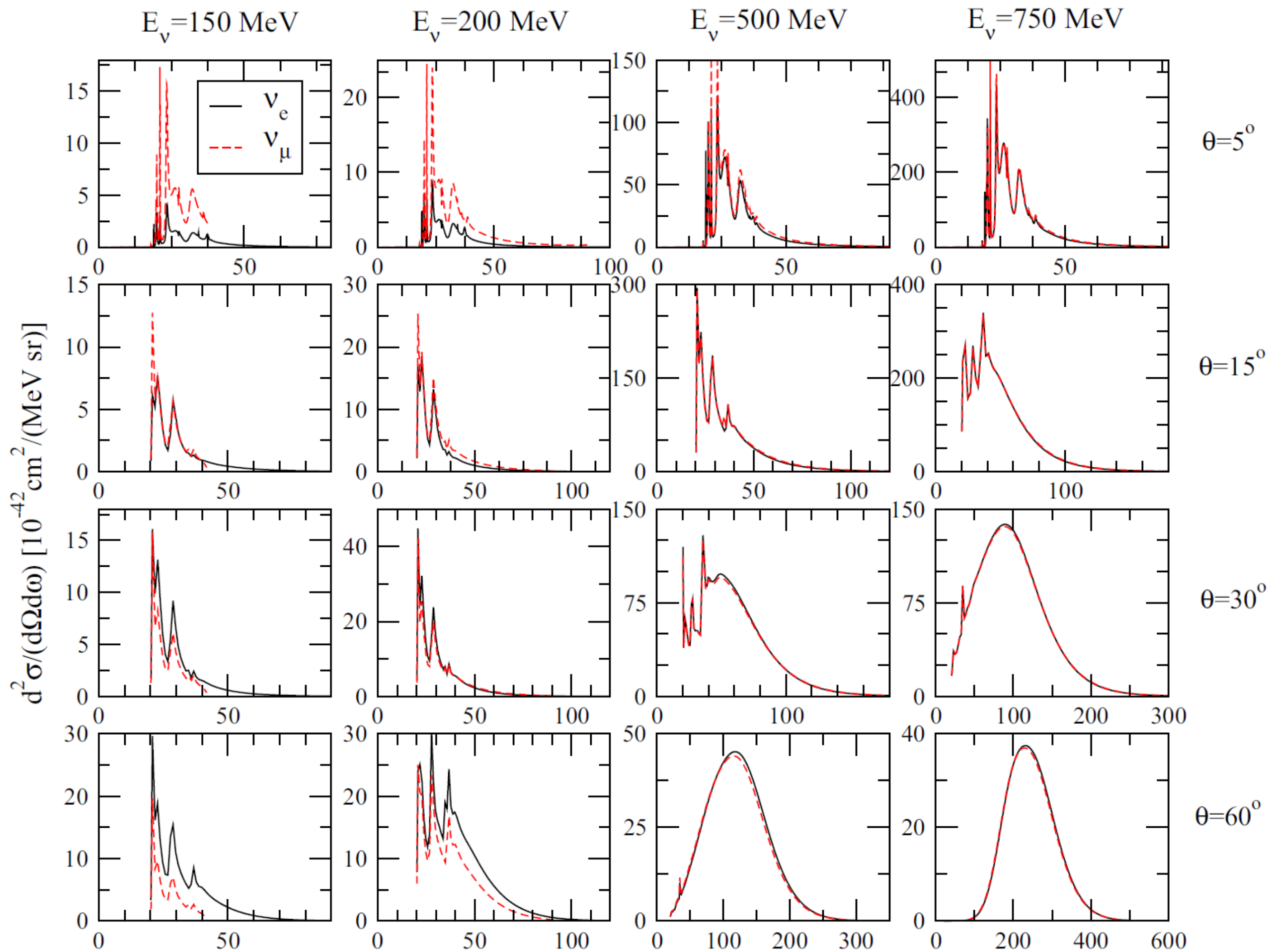
Ratio ν_e/ν_μ for $d\sigma/d\cos\theta$ in different channels



Martini et al., PRC 94 (2016)

Due to the different kinematic limits, the ν_e cross sections are expected to be larger than the ν_μ ones.
However for forward scattering angles this hierarchy is opposite in the QE channel.

A theoretical study (HF+CRPA Ghent) of the ν_μ and ν_e $d^2\sigma$



M. Martini et al., Phys. Rev. C 94 015501 (2016)

Due to the different kinematic limits, the ν_e cross sections are expected to be larger than the ν_μ ones. However for forward scattering angles this hierarchy is opposite.

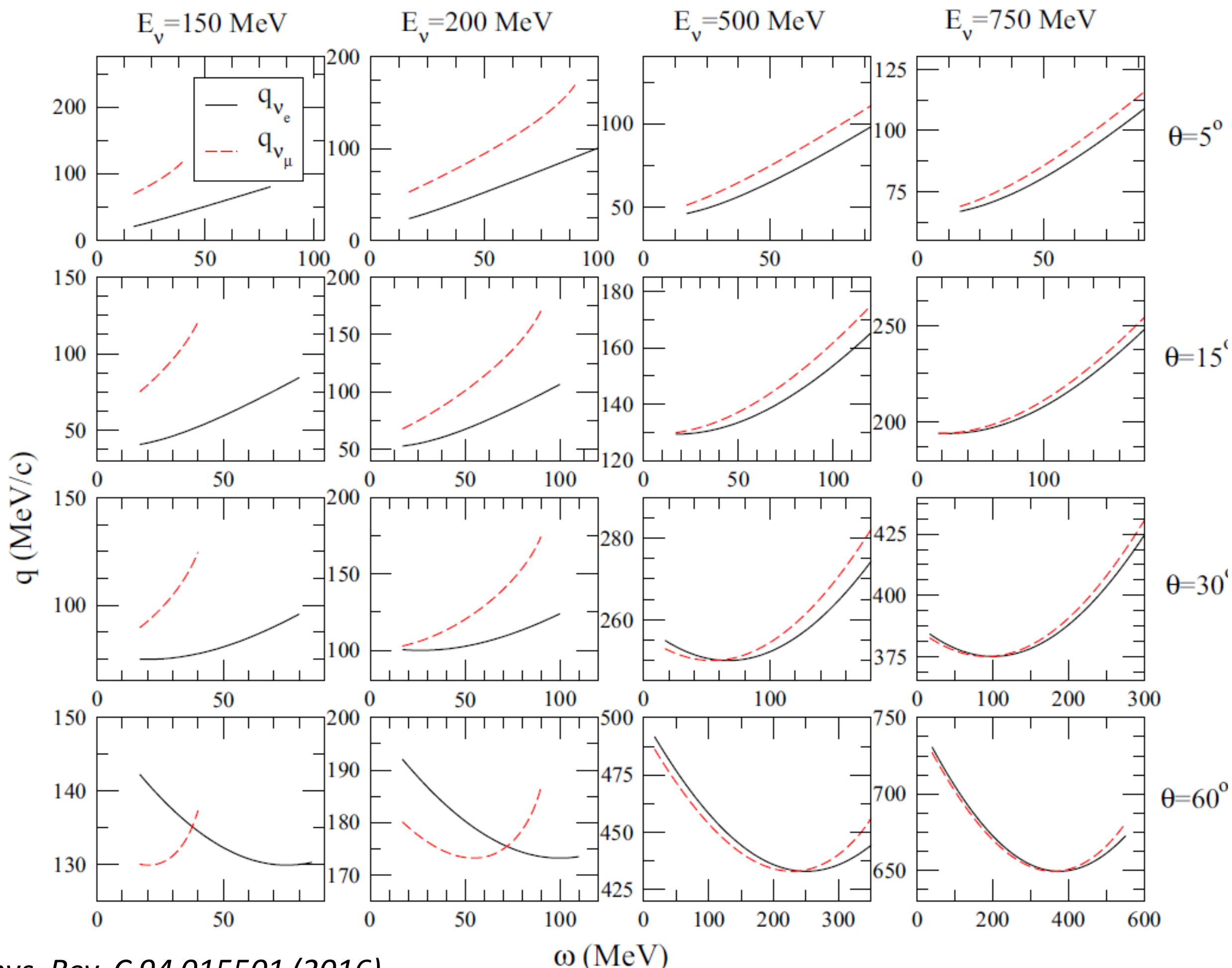
The only difference between ν_μ and ν_e cross sections is the mass of the outgoing lepton.

But the mass affects the three momentum transfer which enters into the kinematics as well as the dynamics of the nuclear model

Further studies: A Nikolakopoulos et al. , PRL 123, 052501 (2019); R. González-Jiménez, PRC, 100, 045501 (2019)

Momentum transfer q versus transferred energy ω for ν_μ and ν_e $d^2\sigma$

Kinematical conditions of the previous slide



M. Martini et al., Phys. Rev. C 94 015501 (2016)

$$q^2 = E_\nu^2 + p_l^2 - 2E_\nu p_l \cos\theta \qquad\qquad p_l^2 = E_l^2 - m_l^2 = (E_\nu - \omega)^2 - m_l^2$$

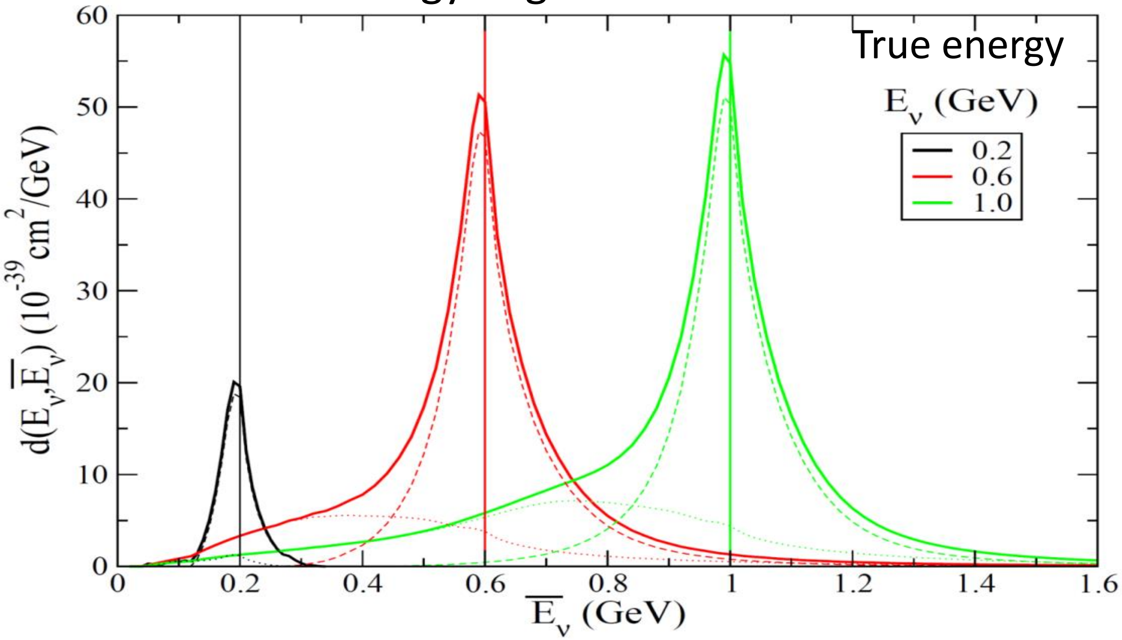
The only difference between ν_μ and ν_e cross sections is the mass of the outgoing lepton.
But the mass affects the three-momentum transfer which enters into the kinematics as well as the dynamics of the nuclear model

From true neutrino energy to reconstructed neutrino energy

$$D_{rec}(\overline{E_\nu}) = \int dE_\nu \Phi(E_\nu) \underbrace{\int_{E_l^{min}}^{E_l^{max}} dE_l \frac{ME_l - m_l^2/2}{\overline{E}_\nu^2 P_l} \left[\frac{d^2\sigma}{d\omega\ d\cos\theta}\right]_{\omega=E_\nu-E_l,\ \cos\theta=\cos\theta(E_l,\overline{E_\nu})}}_{\nu \text{ energy migration matrix}}$$

The quantity $D_{rec}(\overline{E_\nu})$ corresponds to the product $\sigma(E_\nu)\Phi(E_\nu)$ but in terms of reconstructed neutrino energy

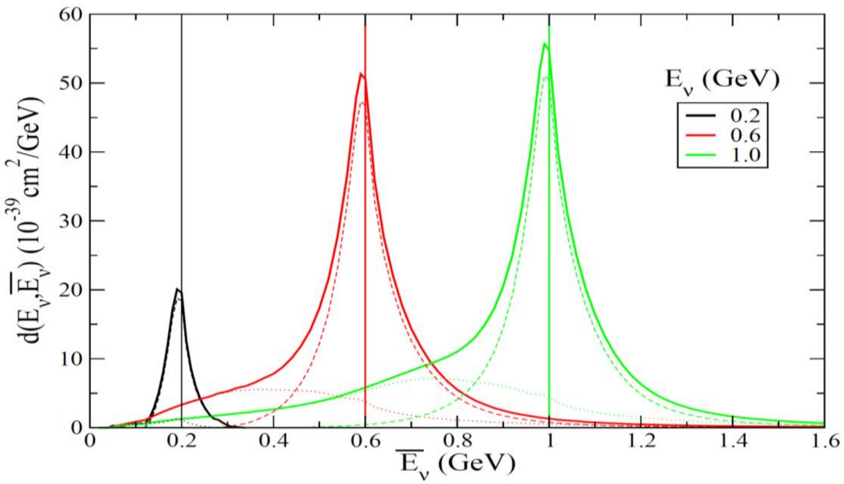
M. Martini, M. Ericson, G. Chanfray
- Phys. Rev. D 85 093012 (2012)
- Phys. Rev. D 87 013009 (2013)



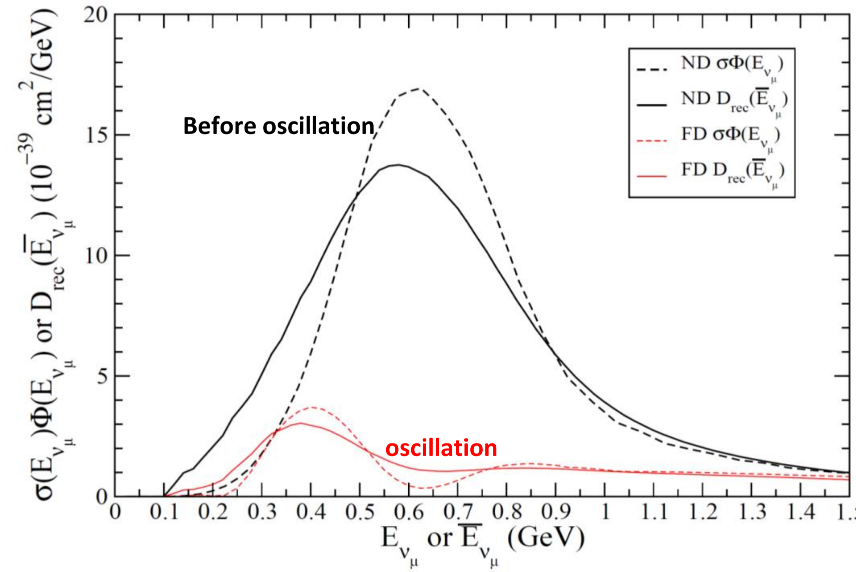
- Distributions not symmetrical around Eν
- Crucial role of np-nh: low energy tail

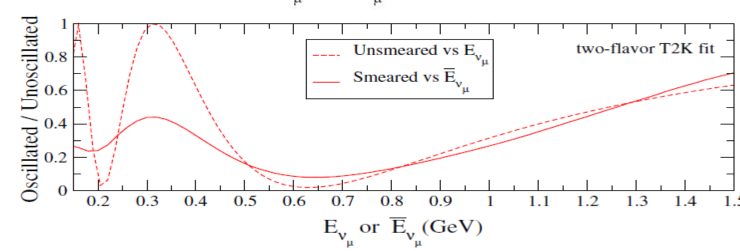
QE-based neutrino energy reconstruction and neutrino oscillations

ν energy migration matrix

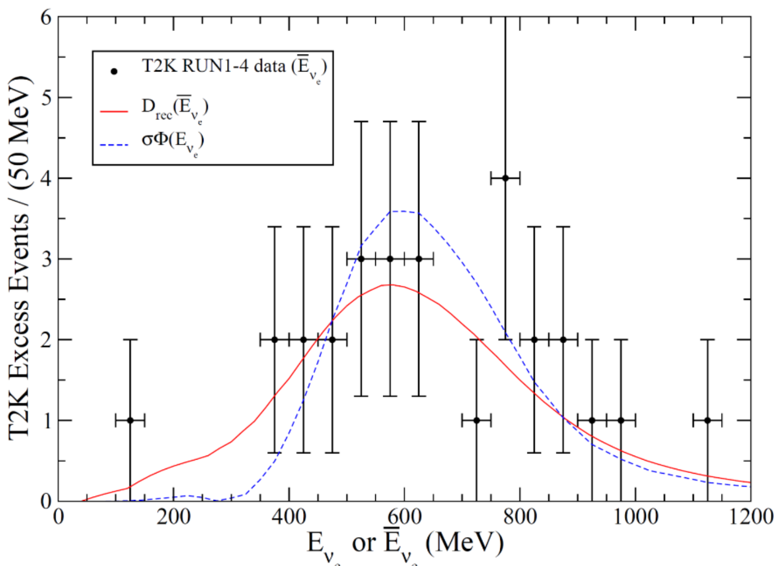


ν_μ disappearance T2K





ν_e appearance T2K

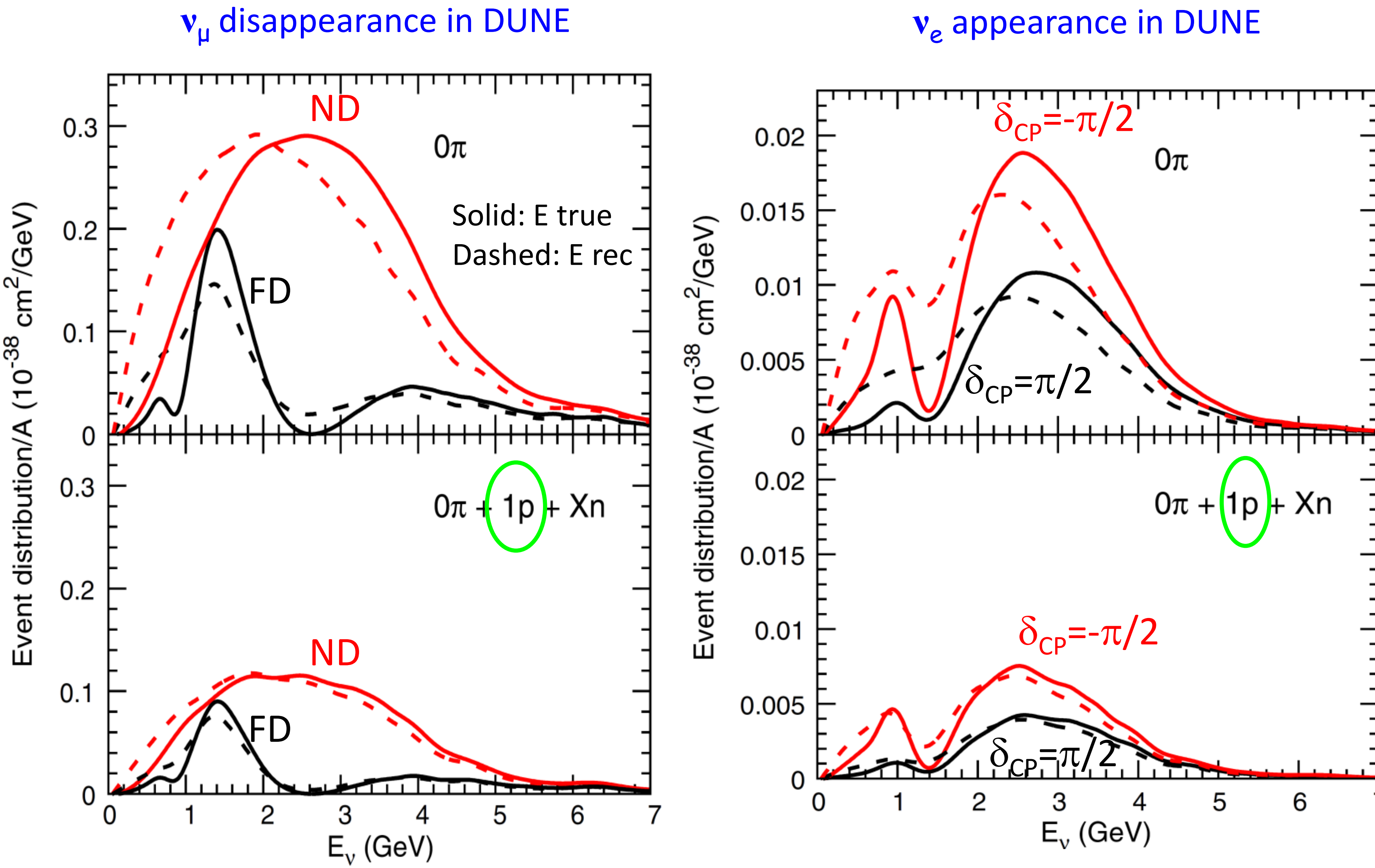


M. Martini, M. Ericson, G. Chanfray
Phys. Rev. D 85 093012 (2012); Phys. Rev. D 87 013009 (2013)

Similar results in:
Nieves et al. PRD 85 113008 (2012);
Lalakulich et al. PRC 86 054606 (2012)

Neutrino energy reconstruction and neutrino oscillation analysis are affected by np-nh

QE-based E_ν reconstruction using proton information

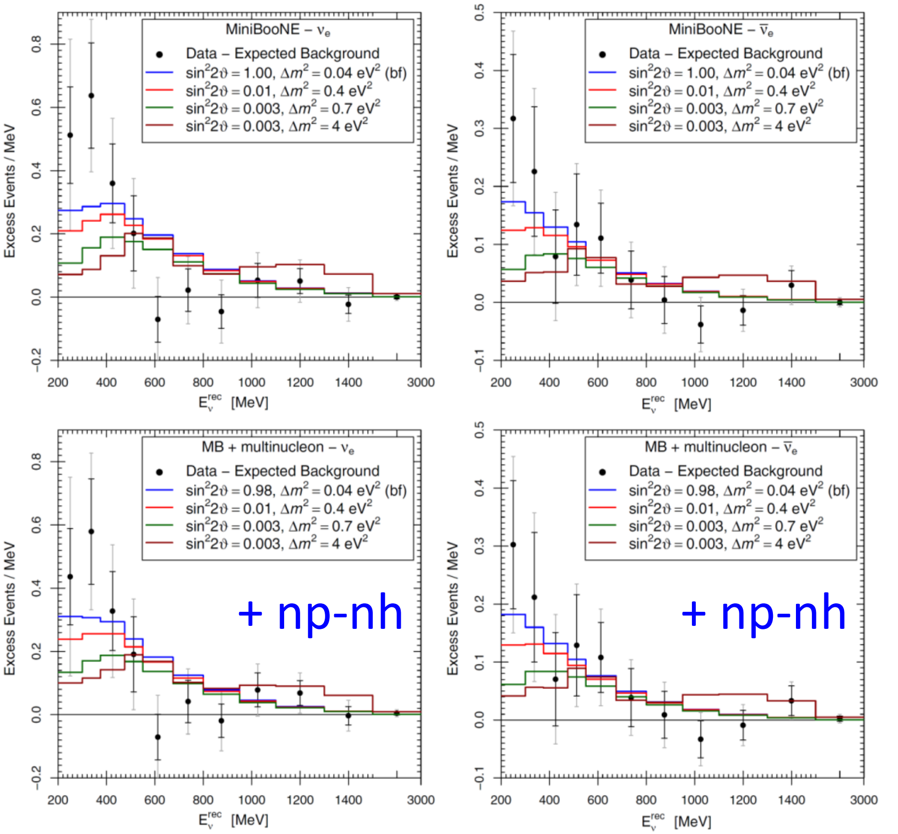


Major improvement in 0π **+ 1p** + Xn sample, events down by only factor 3

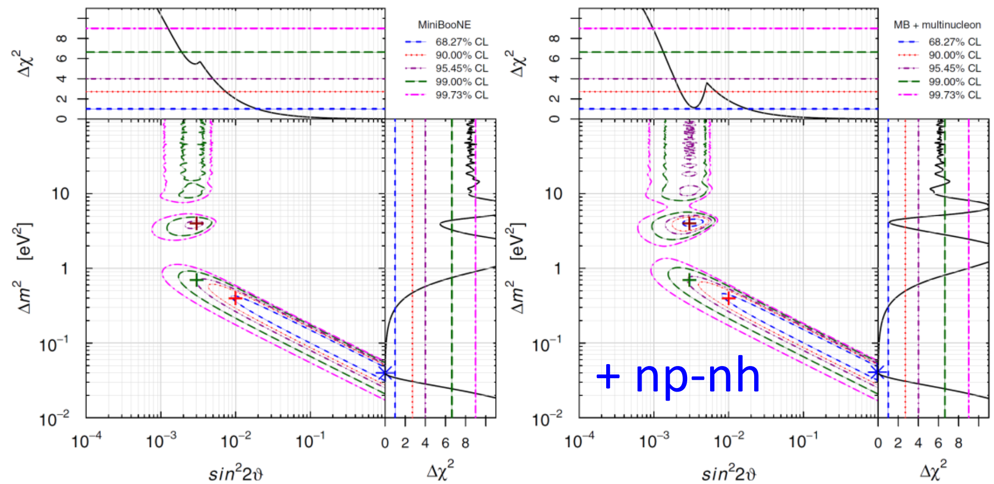
Mosel et al. Phys. Rev. Lett. 112 151802 (2014)

The role of np-nh in the $\nu_\mu \rightarrow \nu_e$ MiniBooNE low-energy anomaly

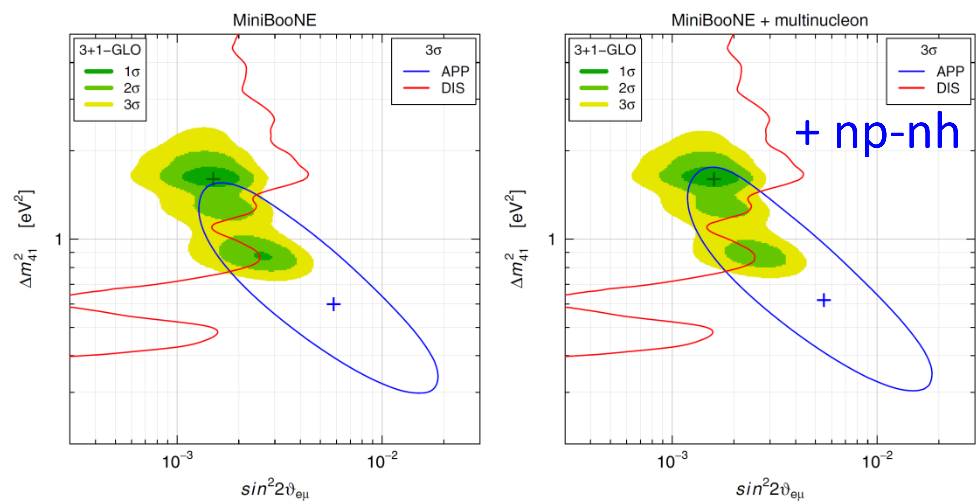
Quantitative analysis



Taking into account np-nh allows a slightly better fit of the MiniBooNE low-energy excess



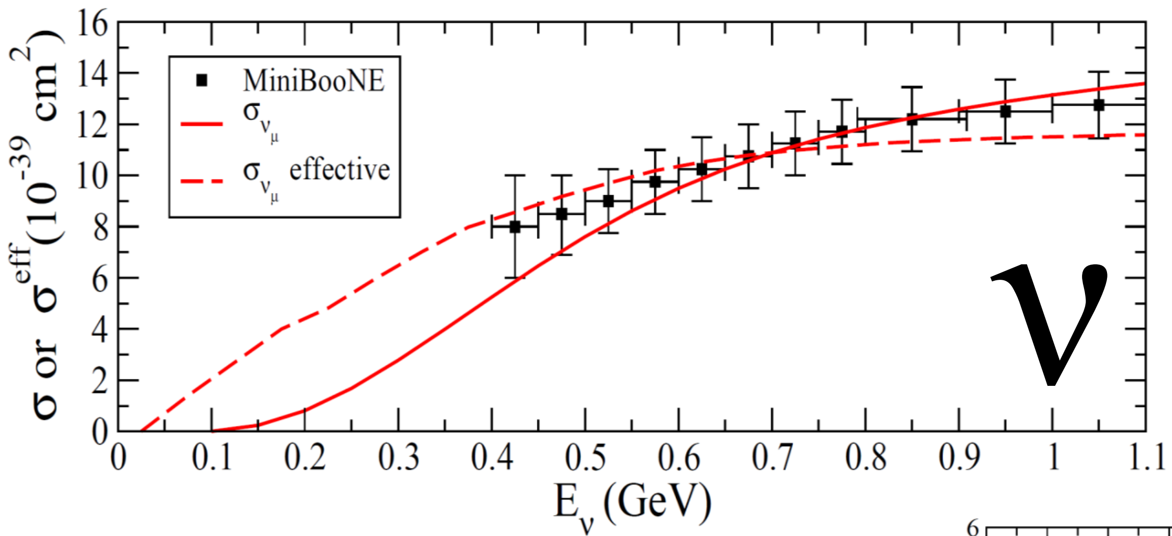
Taking into account np-nh induces a shift of the allowed region towards smaller values of $\sin^2 2\vartheta$ and larger values of Δm^2 in the framework of 2ν oscillations



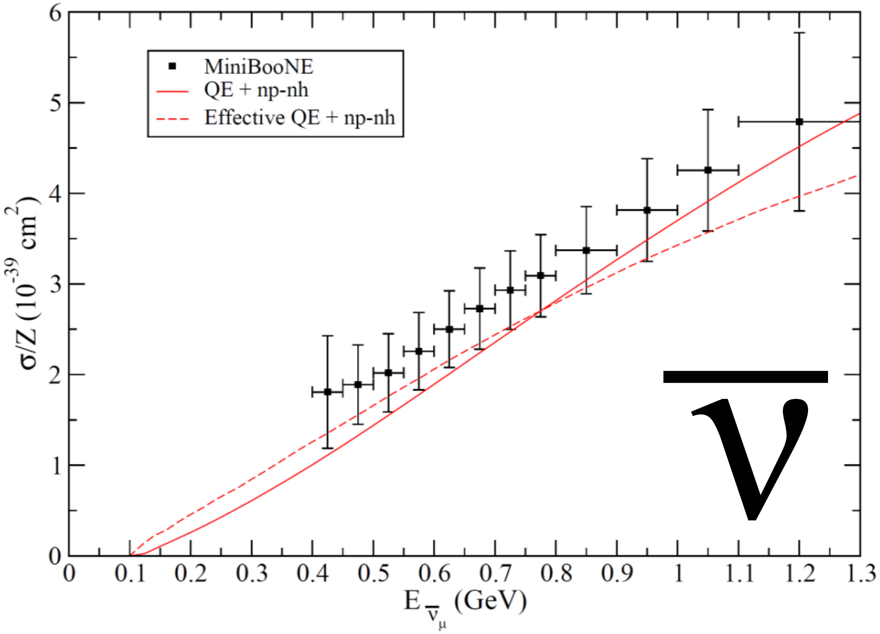
Taking into account np-nh leads to a decrease of the appearance-disappearance tension but not enough to solve the problem in the global fit of short-baseline ν oscillation data

M.Ericson, M.V.Garzelli, C.Giunti, M.Martini, Phys. Rev. D 93, 073008 (2016)

CCQE-like cross sections as a function of real (continuous line) and reconstructed (dashed line) neutrino energy

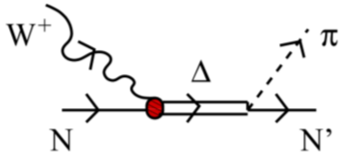


M. Martini, M. Ericson, G. Chanfray, Phys. Rev. D 87 013009 (2013)



Martini, Ericson, Phys. Rev. C 87 065501 (2013)

The 1π production via Δ(1232) resonance excitation and decay



At energies of our interest, it is the dominant mechanism of the reaction $W N \to N' \pi$

E. Hernandez et al. Phys. Rev. D 76, 033005 (2007)

$W^+ n \to \Delta^+$ **Hadron matrix element** $\langle \Delta^+; p_\Delta = p + q \,|j^\mu_{cc+}(0)|n; p\rangle = \bar{u}_\alpha(\vec{p}_\Delta)\Gamma^{\alpha\mu}(p,q)\,u(\vec{p})\cos\theta_C$

Electroweak vertex

$$\Gamma^{\alpha\mu}(p,q) = \left[\frac{C_3^V}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^V}{M^2}(g^{\alpha\mu}q\cdot p_\Delta - q^\alpha p_\Delta^\mu) + \frac{C_5^V}{M^2}(g^{\alpha\mu}q\cdot p - q^\alpha p^\mu) + C_6^V g^{\mu\alpha}\right]\gamma_5$$
$$+ \left[\frac{C_3^A}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^A}{M^2}(g^{\alpha\mu}q\cdot p_\Delta - q^\alpha p_\Delta^\mu) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2}q^\mu q^\alpha\right], \quad p_\Delta = p + q$$

Vector form factors $C^V_{3,4,5,6}$ can be extracted from single-pion electro-production data

Axial form factors $C^A_{3,4,5,6}$ $C_5^A(Q^2) = \frac{C_5^A(0)}{(1+Q^2/M_{A\Delta}^2)^2}$ $C_6^A = M^2/(m_\pi^2 + Q^2)\cdot C_5^A$ PCAC $C_4^A \underset{\text{Adler}}{=} -1/4\cdot C_5^A$ C_3^A Small, usually neglected

Δ propagator $$G^{\mu\nu}(p_\Delta) = \frac{P^{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta}$$

Spin 3/2 projection operator $$P^{\mu\nu}(p_\Delta) = -(\not{p}_\Delta + M_\Delta)\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\mu\gamma^\nu - p_\Delta^\nu\gamma^\mu}{M_\Delta}\right]$$

NΔπ coupling $$\mathcal{L}_{\pi N\Delta} = \frac{f^*}{m_\pi}\bar{\Psi}_\mu\vec{T}^\dagger(\partial^\mu\vec{\phi})\Psi + \text{h.c.}$$

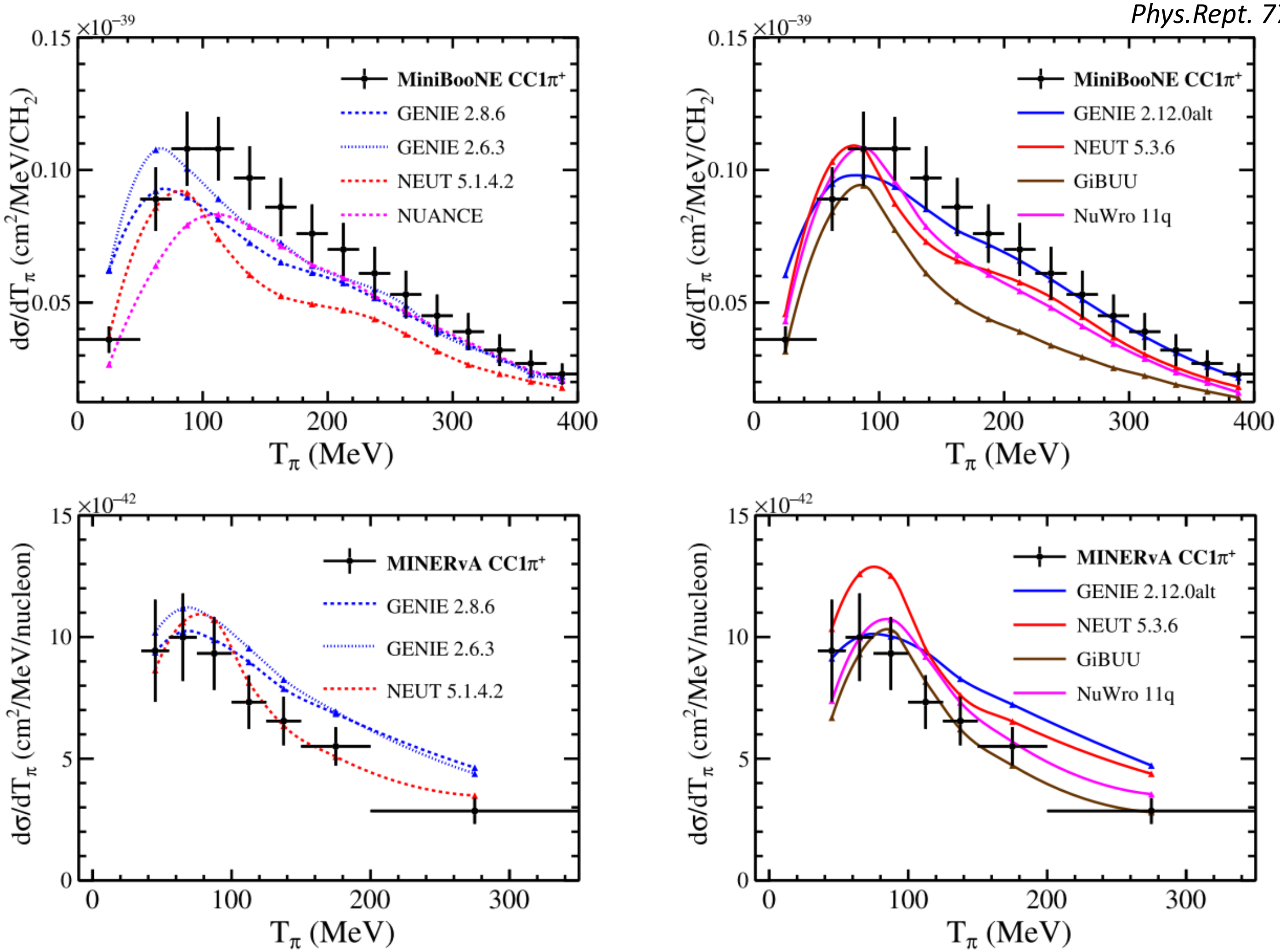
Pion puzzle – Tension 2016 Workshop

Old | **New (after Tension)**

M. Betancourt et al. Phys.Rept. 773-774 (2018) 1-28

MiniBooNE

MINERvA



- Same models, correct signal definition, proper flux averaging
- Updated flux prediction from MINERvA

Better normalization agreement but shape discrepancies remain

Pion puzzle – Tuning GENIE with MINERvA data (2019)

P. Stowell et al. PRD 100 (2019) 7, 072005

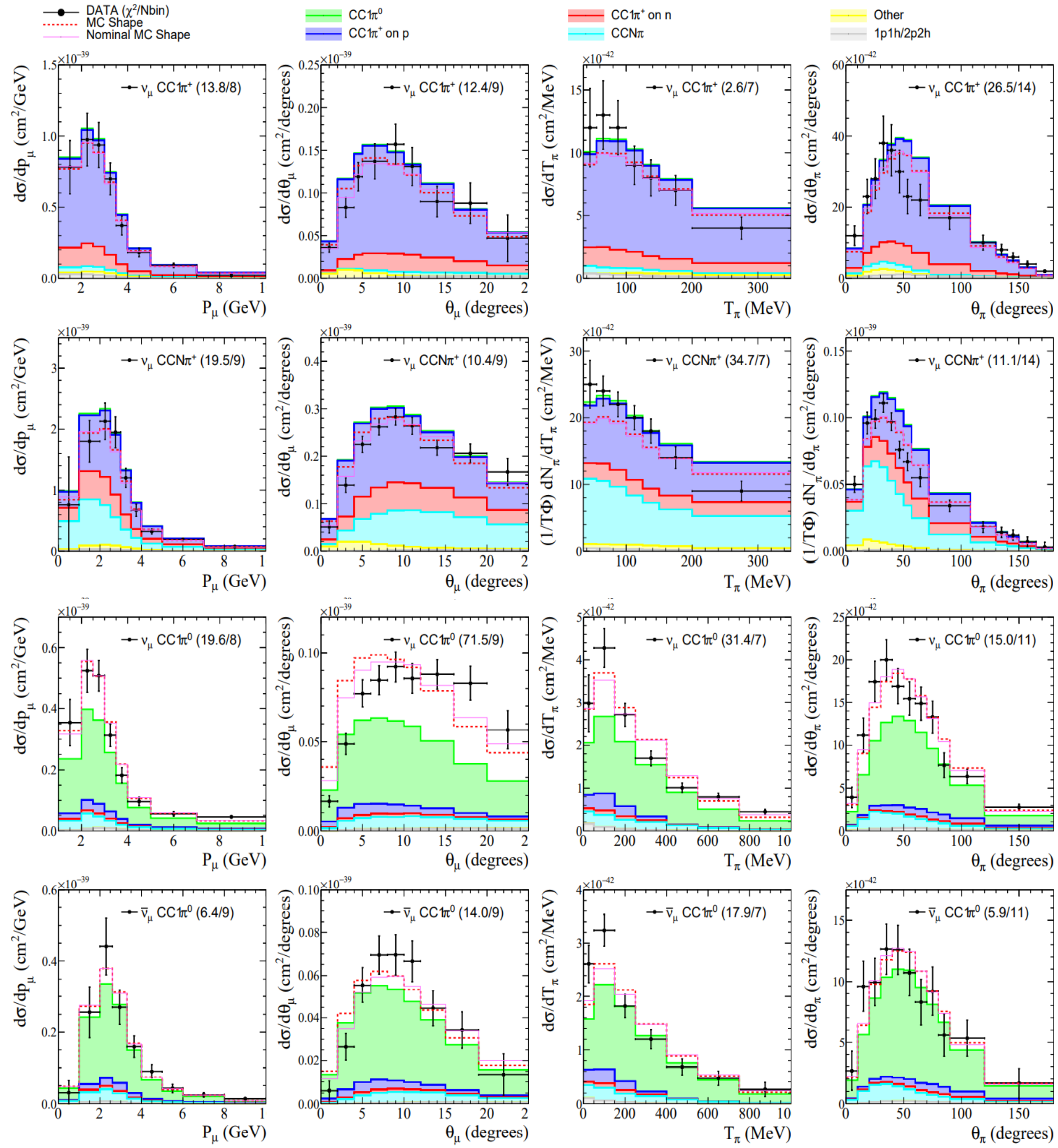
Tune Monte Carlo to simultaneously fit 4 datasets

$\nu_\mu CC1\pi^+$

$\nu_\mu CCN\pi^+$

$\nu_\mu CC1\pi^0$

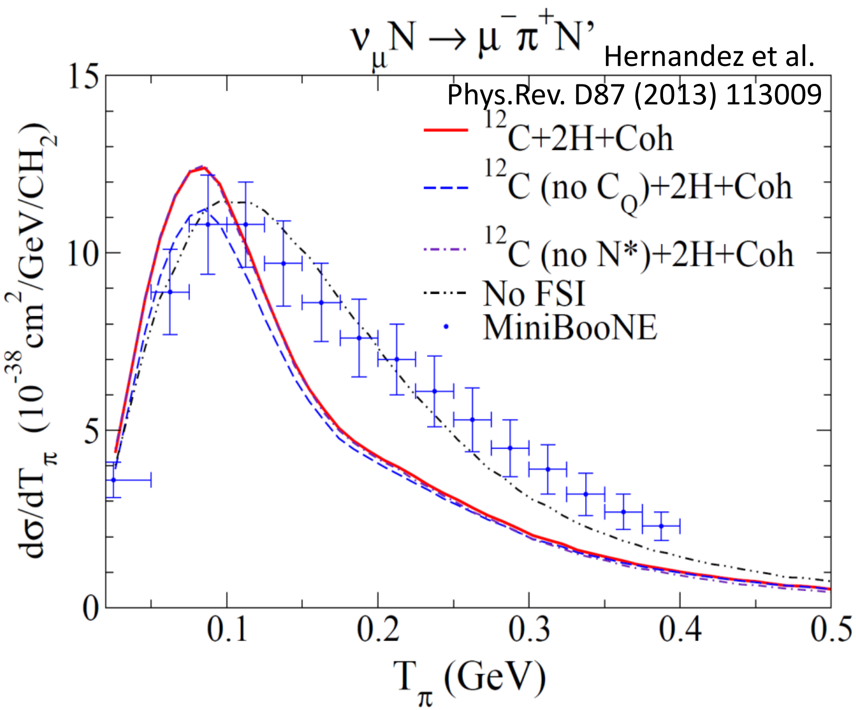
$\overline{\nu_\mu} CC1\pi^0$



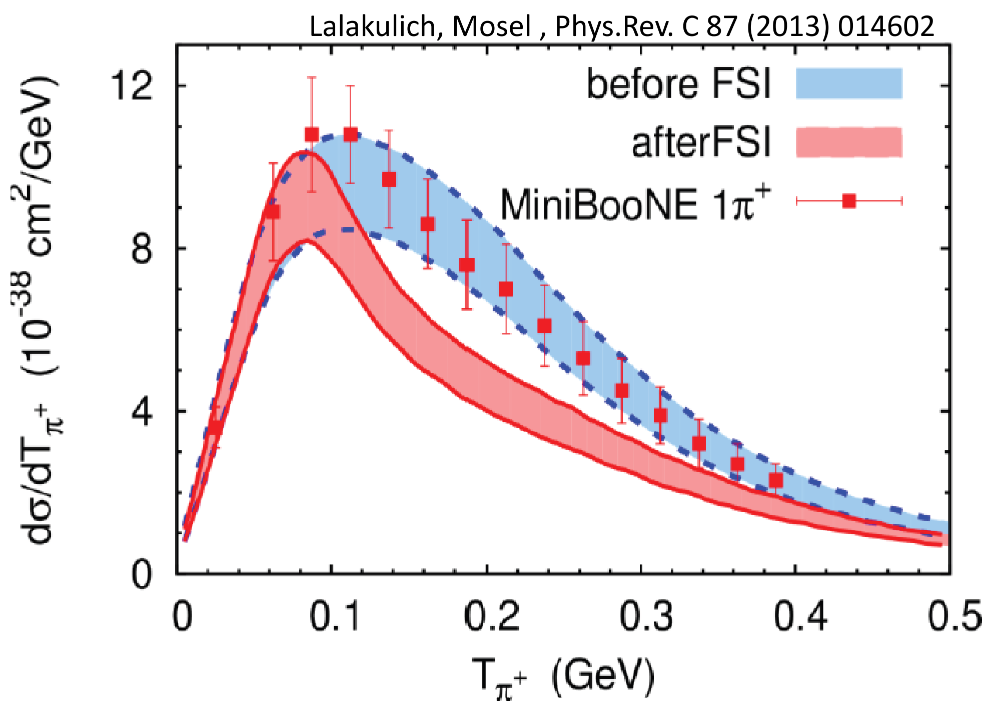
The tuning improves the model, but tensions remain

1 Pion production controversy

Best theories (with Δ medium effects and pion rescattering) do not agree with pion KE spectrum



Valencia



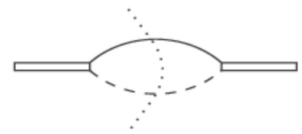
GiBUU

Data prefer calculations with no Final State Interaction for the pion

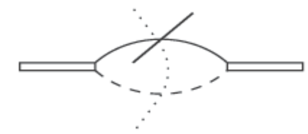
Delta in the nuclear medium

Mass

$$\tilde{M}_\Delta = M_\Delta + 40(MeV)\frac{\rho}{\rho_0}$$



Δ→π N

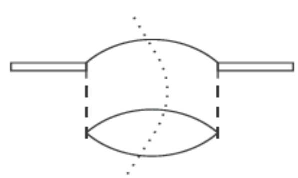


Pauli correction (F_P)

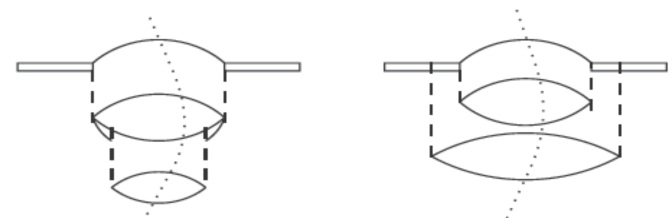
Width

$$\widetilde{\Gamma_\Delta} = \Gamma_\Delta \; F_P - 2\mathrm{Im}(\Sigma_\Delta)$$

Pion distortion (C_Q)



2p-2h



3p-3h

Self energy

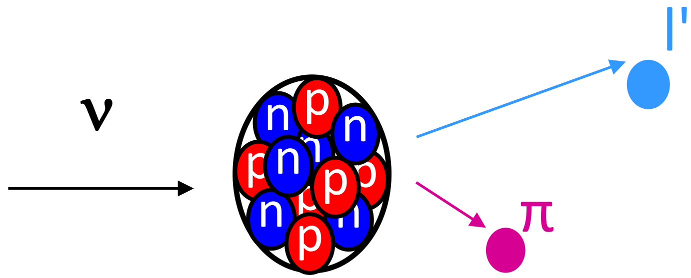
$$\mathrm{Im}(\Sigma_\Delta(\omega)) = -\left[C_Q(\frac{\rho}{\rho_0})^\alpha + C_{2p2h}(\frac{\rho}{\rho_0})^\beta + C_{3p3h}(\frac{\rho}{\rho_0})^\gamma\right]$$

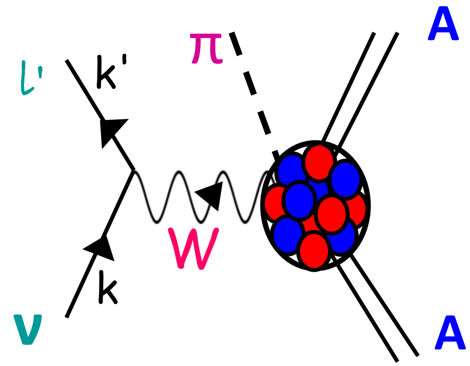
E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 631 (1987)

The coherent 1π production

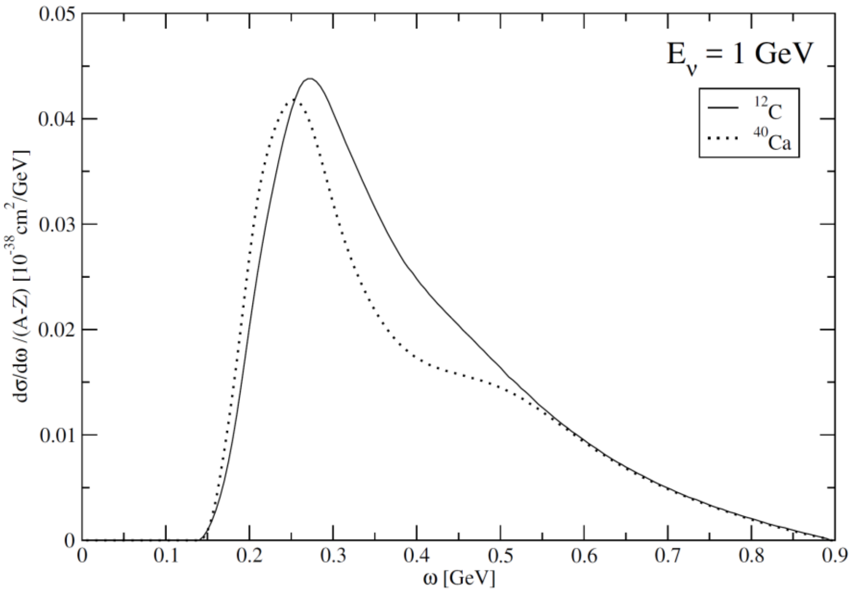
Production of 1 pion with the nucleus remaining in its ground state

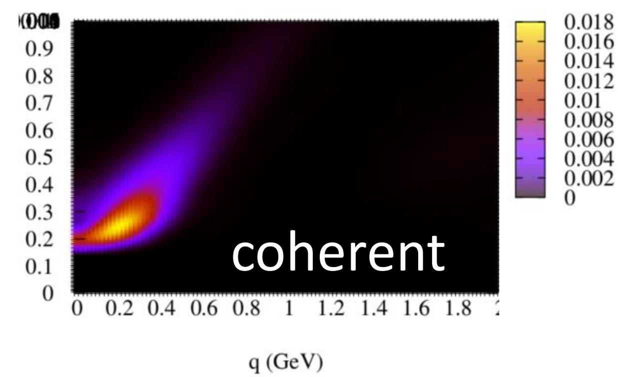
Relatively rare interaction channel, but can mimic oscillation signals

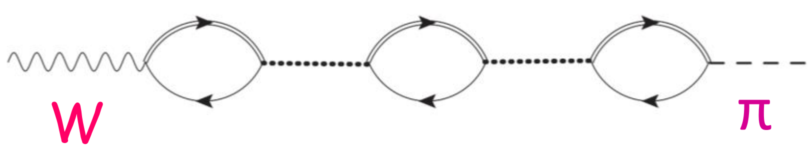




M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)







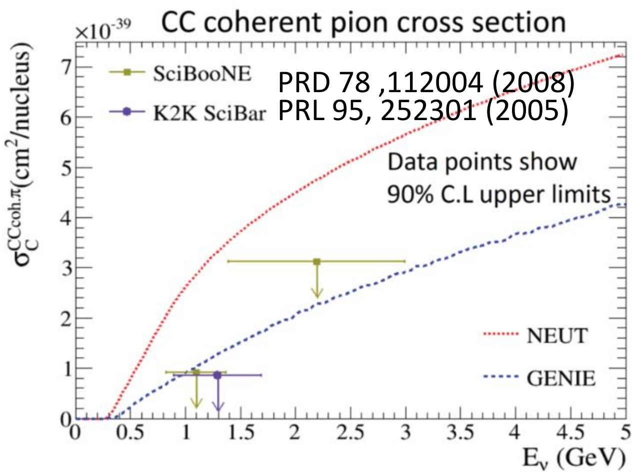
Cross sections reshaped by nuclear collective effects

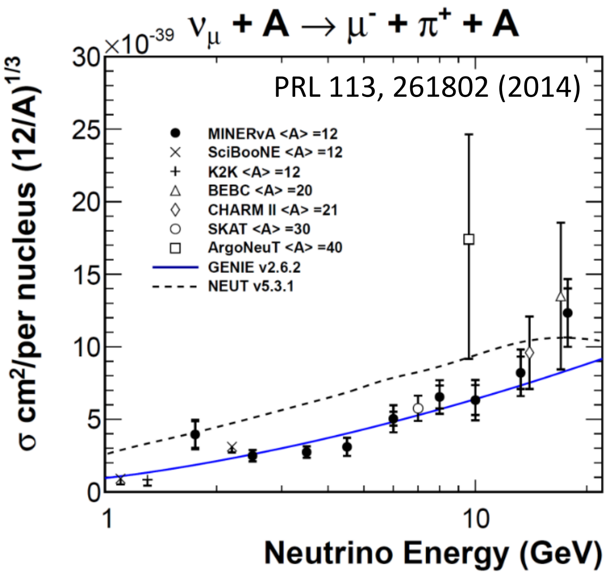
Coherent 1π production experimental results

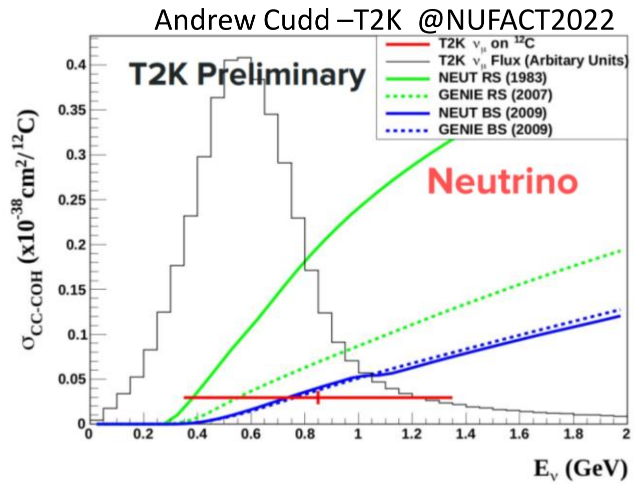
K2K and SciBooNE did not observe coherent π^+ production at neutrino energies ~1GeV

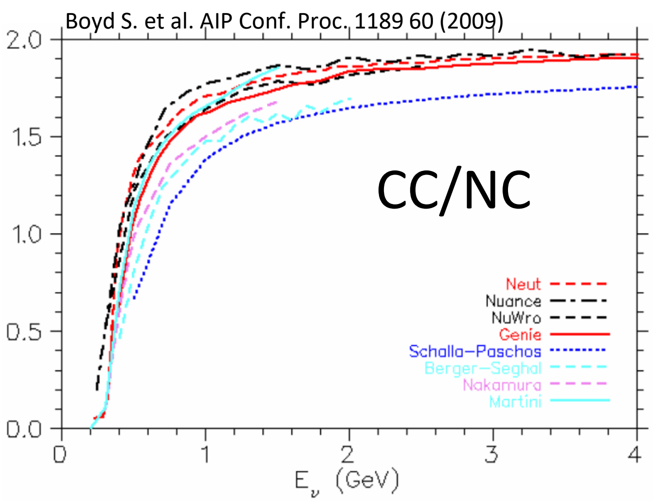
MINERνA and ArgoNeut see evidence for CC coherent pion production

Preliminary T2K cross section measurement: coherent π+ production at neutrino energies ~1GeV









Coherent puzzle at $E_\nu \sim 1$ GeV

Theoretical models:

$$\frac{\pi+ \text{ coh. CC}}{\pi 0 \text{ coh. NC}} = 1.5 \sim 2$$

SciBooNE:

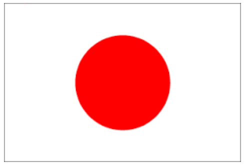
$$\frac{\pi+ \text{ coh. CC}}{\pi 0 \text{ coh. NC}} = 0.14^{+0.30}_{-0.28}$$

Kurimoto et al, PRD 81 (2010)

Nuclear targets of present and future LBL oscillation experiments

Present

Future



Carbon: T2K(ND) and NOvA

Oxygen (water): T2K (SuperK) and Hyper-K

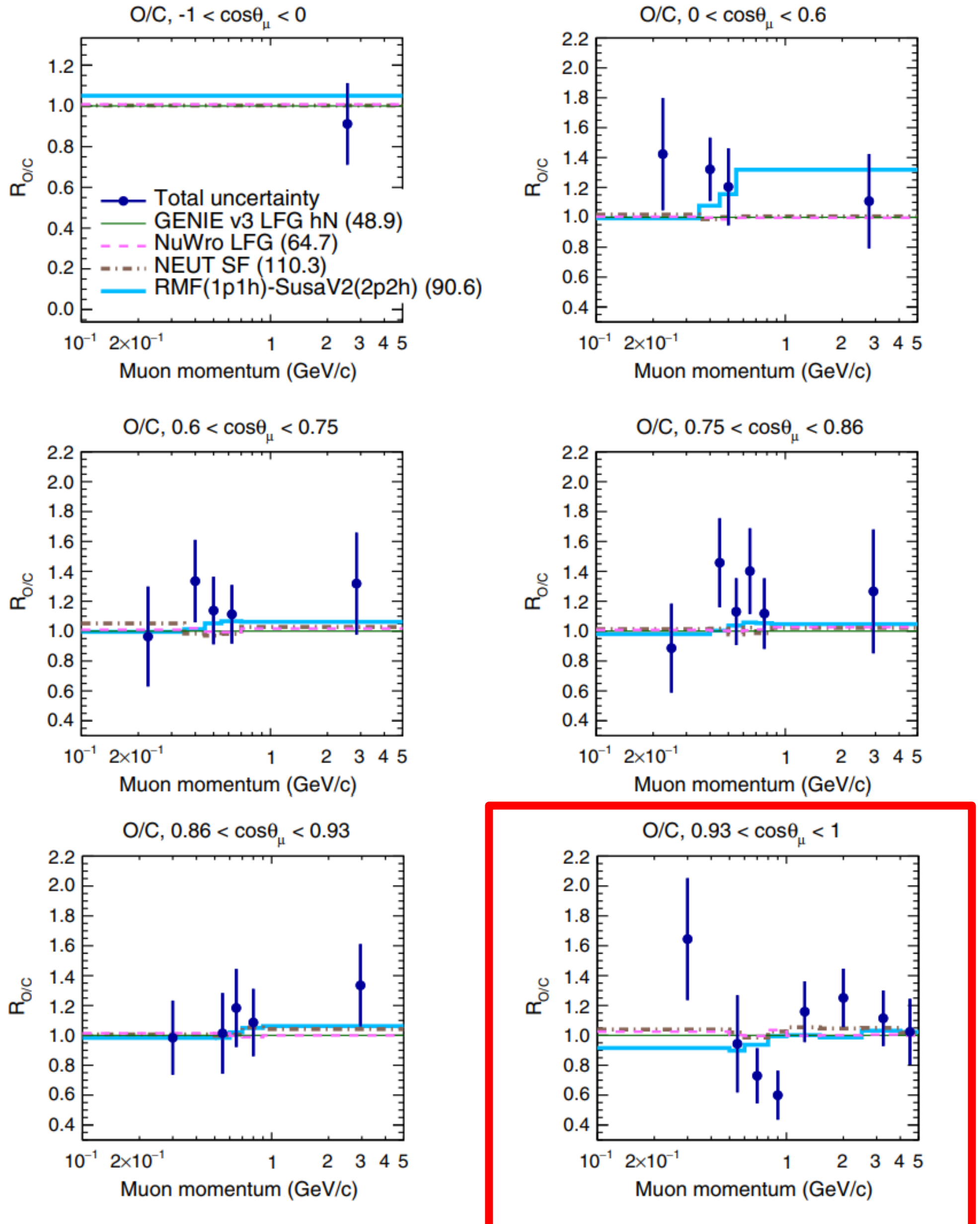
Argon: DUNE

In the last 15 years many cross sections measurements and theoretical studies have been performed for Carbon (^{12}C). Less for Oxygen (^{16}O) and Argon (^{40}Ar)

T2K CC0π $d^2\sigma$ cross sections on oxygen and carbon

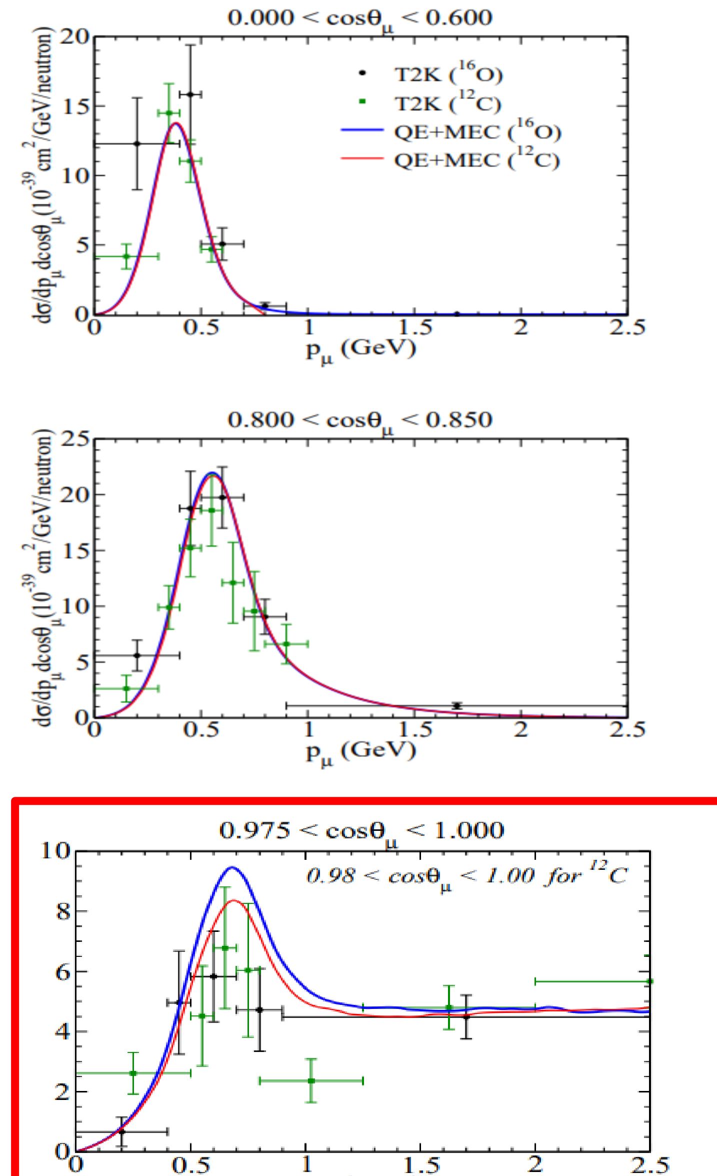
Ratio $^{16}O/^{12}C$ per nucleon

T2K PRD 101 (2020)

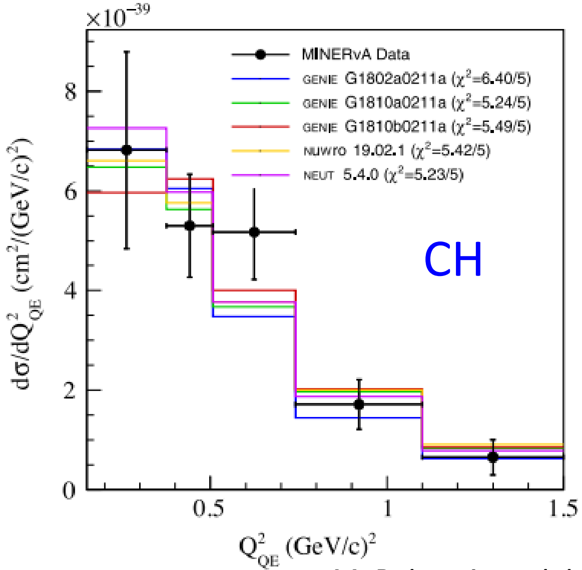


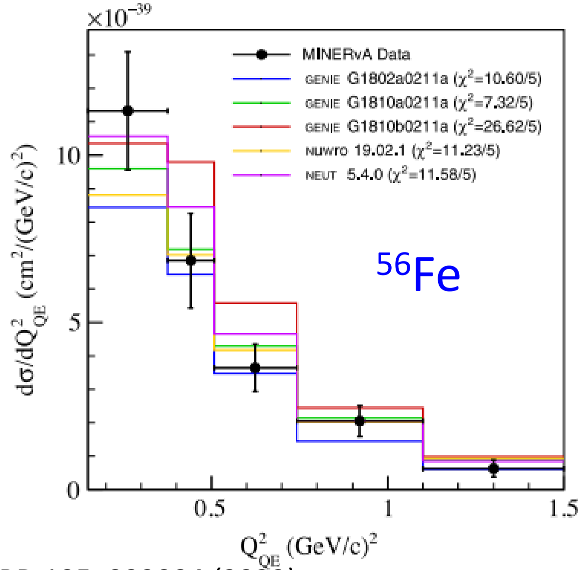
SuSAv2+MEC

Megias et al., JPG46 (2019)

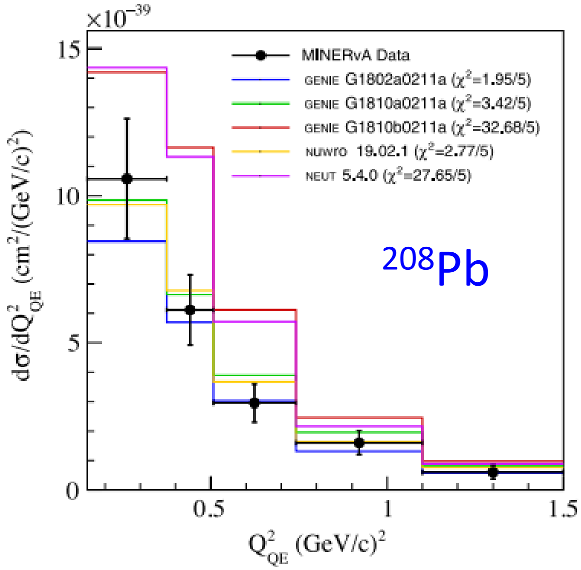


MINERvA CC0π1p(at least) Q^2distributions for carbon, iron, lead





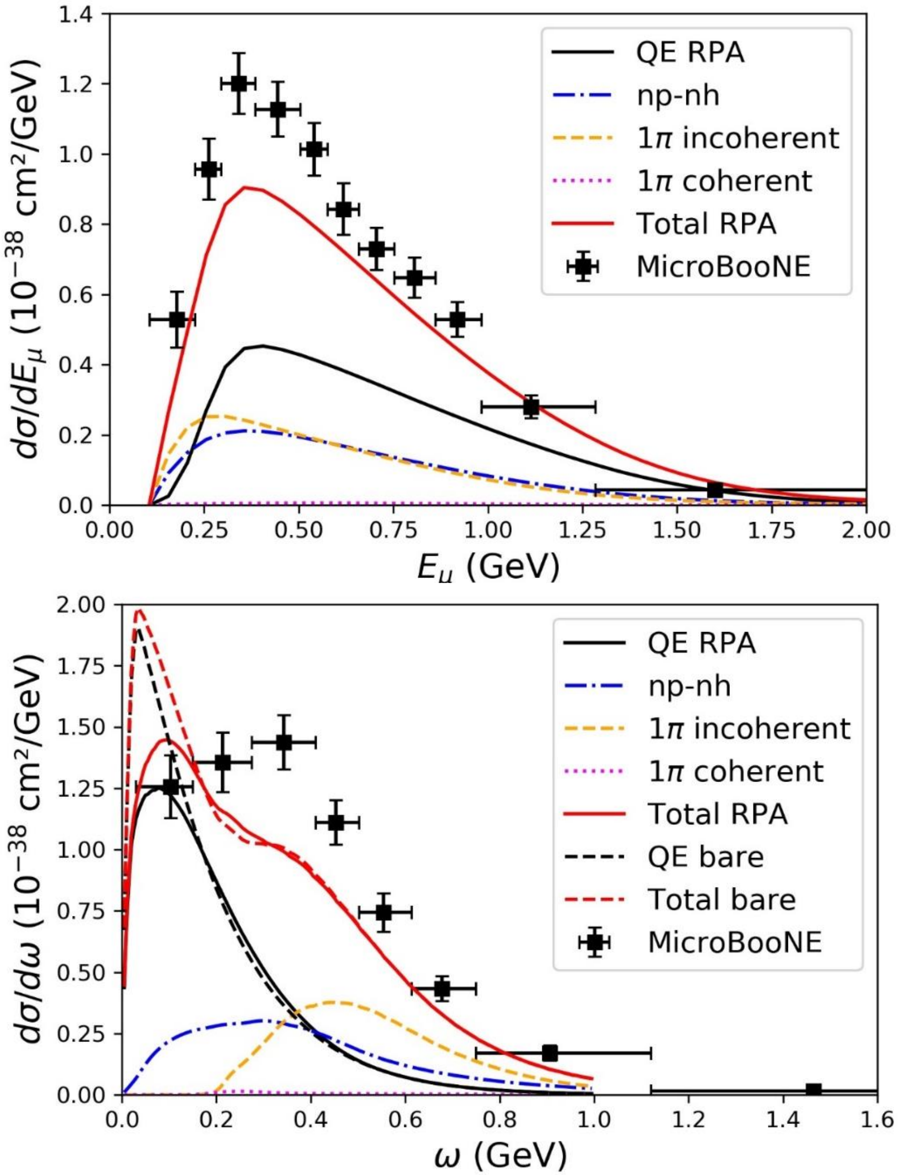
M. Buizza Avanzini et al. PRD 105, 092004 (2022)



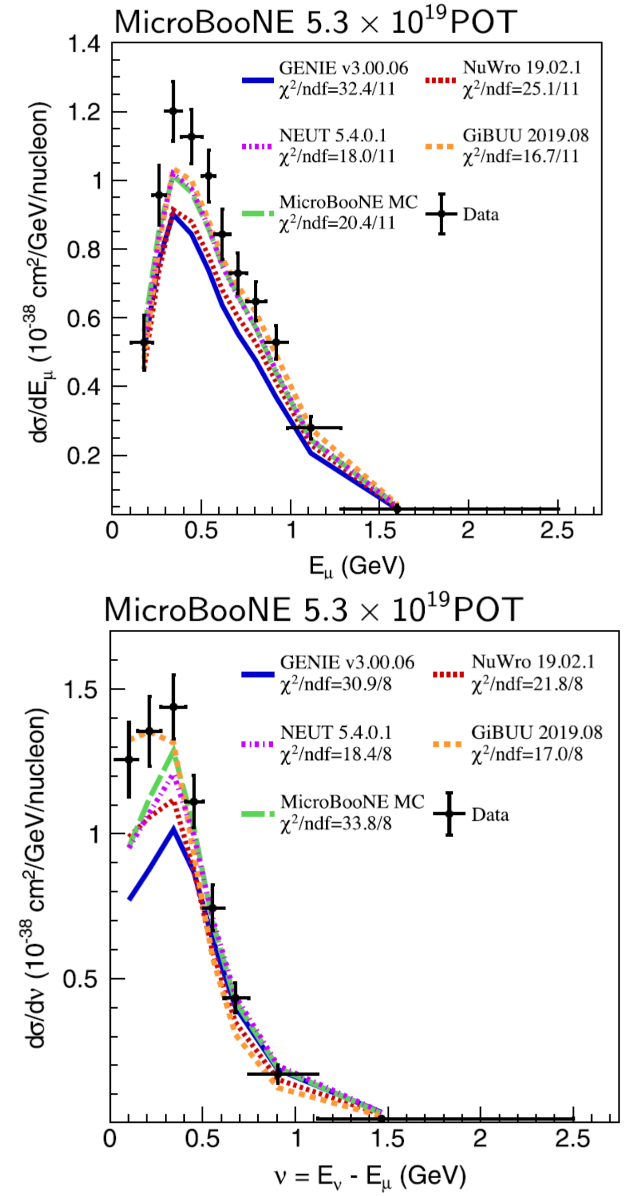
- The spread of distributions predicted by generators increases from carbon to lead
- Most significant deviations are at low Q^2 where nuclear effects are more important

MicroBooNE flux-averaged inclusive $d\sigma/dE_\mu$ and $d\sigma/d\omega$ on argon

M. Martini, M. Ericson, G. Chanfray, Phys. Rev. C 106, 015503 (2022)



PRL 128, 151801 (2022)

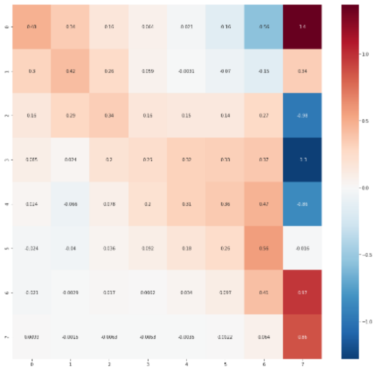


$d\sigma/d\omega$ allows a better separation of the different channels

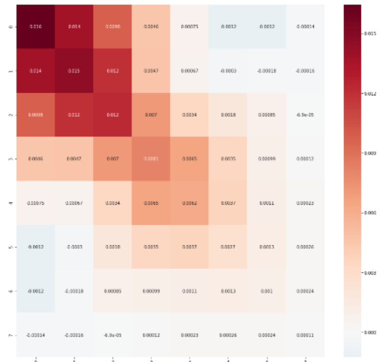
Quantitative analysis of MicroBooNE inclusive dσ/dω on argon

MicroBooNE shared additional smearing and covariant matrices for quantitative analysis

Additional Smearing Matrix



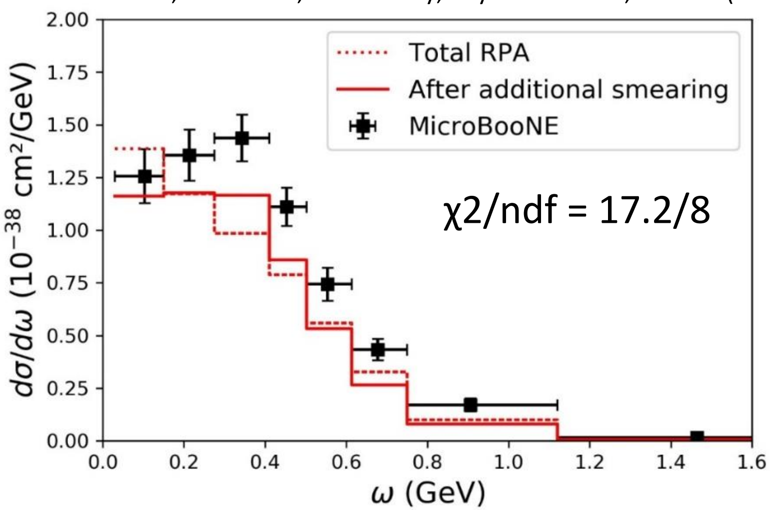
Covariant Matrix



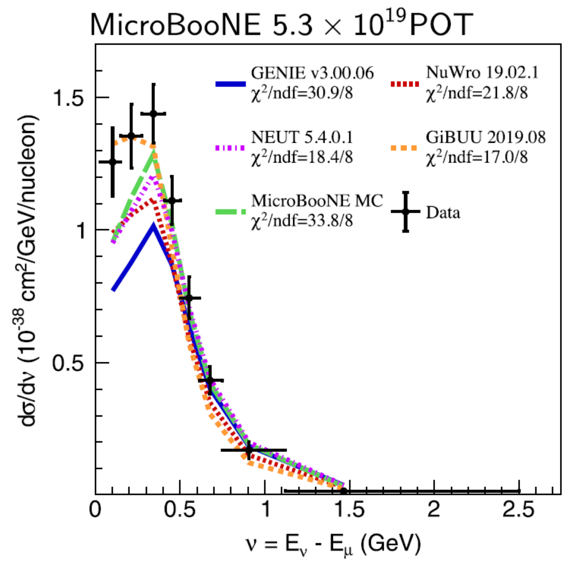
$$\sigma_{smeared} = M_{add\_smr} \times \sigma_{model}$$

$$\chi^2 = (M - P)^T \times Cov^{-1}_{full}(M, P) \times (M - P)$$

M. Martini, M. Ericson, G. Chanfray, Phys. Rev. C 106, 015503 (2022)

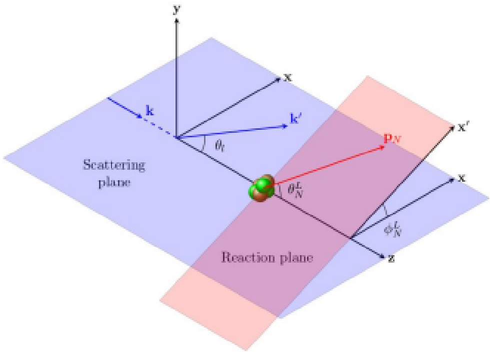


PRL 128, 151801 (2022)



- RPA χ2/ndf=17.2/8. Comparable with the one of GiBUU and better than all the Monte Carlo predictions
- A possible reason is that GENIEv3, MicroBooNE MC, NEUT and NuWro implement np-nh contribution deduced by Nieves et al. model. This contribution is smaller than the one of other evaluations (GiBUU, Martini et al,…)

Semi-inclusive neutrino-nucleus formalism in the IA



$$\left\langle \frac{d^6\sigma}{dk_l d\Omega_l dp_N d\Omega_N} \right\rangle = \int dk\, \Phi(k) \times K \times L_{\mu\nu} H^{\mu\nu}$$

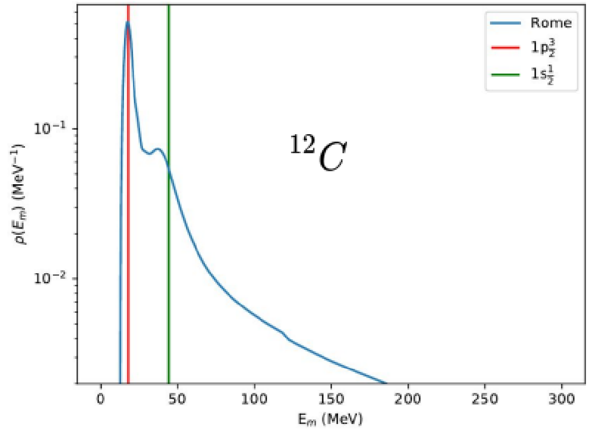
$$H_\kappa^{\mu\nu} = \rho_\kappa(E_m) \times \sum_{m_j,\, s_N} \left[J_{\kappa,\, m_j,\, s_N}(Q, P_N)\right]^\star J_{k,\, m_j,\, s_N}(Q,\, P_N) \Rightarrow$$ **Factorization NOT assumed!!**

$$J^\mu_{\kappa,\, m_j,\, s_N} = \int d\mathbf{r}\, e^{i\mathbf{r}\cdot\mathbf{q}}\, \overline{\Psi}_{s_N}(\mathbf{p}_N,\, \mathbf{r}) \left(F_1 \gamma^\mu + \frac{iF_2}{2m_N} \sigma^{\mu\nu} Q_\nu + G_A \gamma^\mu \gamma^5 + \frac{G_P}{2m_N} Q^\mu \gamma^5 \right) \Psi_\kappa^{m_j}(\mathbf{r}) \Rightarrow$$

- W.F. scattered nucleon
- CC2 operator
- W.F. bound nucleon

Description of the <u>initial state</u>:

- Pure shell model (first approximation): energy density is given by a Dirac delta per shell
- Realistic model (i.e. Rome spectral function used in electron exclusive processes): short- and long-range correlations included



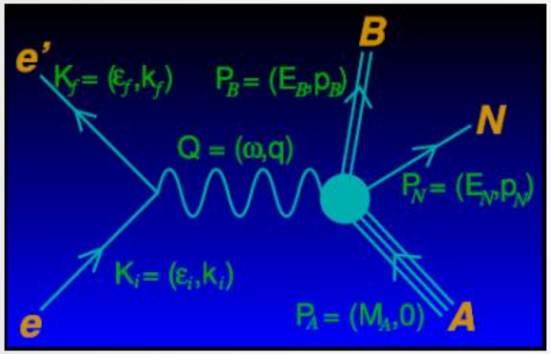
4

Relativistic scattering: notations and formalism

Four-vectors $A^\mu = (A_0, \vec{A})$

Bjorken&Drell conventions $g^{00} = 1, g^{kk} = -1 \ (k = 1, 2, 3)$

First Born approximation: one virtual boson exchange



$Q^2 = \omega^2 - q^2 < 0$ space-like virtual boson

$Q^\mu = K^\mu - K'^\mu = P_f^\mu - P_i^\mu$ 4-momentum conservation

$P^2 = M^2$ on-shell condition

$\not{\partial} \equiv \gamma_\mu \partial^\mu$ $\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\mathbb{1}$

$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$

<u>Dirac equation and Dirac spinors:</u>

$(i\not{\partial} - M)\Psi = 0$ free particles

$(i\not{\partial} - e\not{A} - M)\Psi = 0$ in presence of e.m. field

$\Psi_{\mathbf{p}}^{(+)}(\mathbf{x}, t) = \sqrt{\frac{M}{EV}} u(\mathbf{p}, s) e^{-iP_\mu X^\mu}$ positive energy

$\Psi_{\mathbf{p}}^{(-)}(\mathbf{x}, t) = \sqrt{\frac{M}{EV}} v(\mathbf{p}, s) e^{iP_\mu X^\mu}$ negative energy

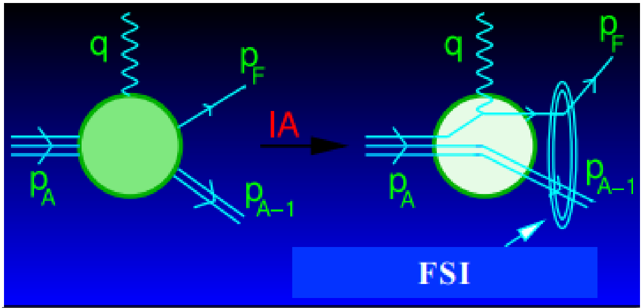
$(\not{P} - M) u(\mathbf{p}, s) = 0$

$(\not{P} + M) v(\mathbf{p}, s) = 0$

$$u(\mathbf{p}, s) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M}\chi_s \end{pmatrix}, \quad v(\mathbf{p}, s) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M}\xi_s \\ \xi_s \end{pmatrix}$$

Realtivistic Mean Field Model

The RMF model is based on the **impulse approximation (IA)**:
scattering off a nucleus = incoherent sum of single nucleon scattering processes.

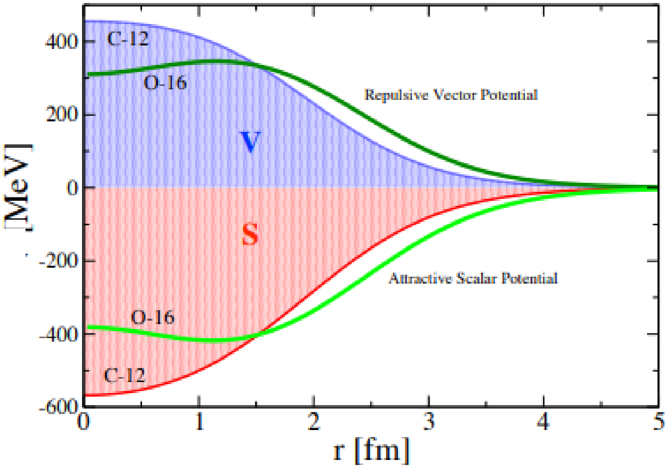


Nuclear Current ⟹ One-body operator

$$J^\mu_N(\omega,\vec{q}) = \int d\vec{p}\,\overline{\Psi}_F(\vec{p}+\vec{q})\hat{J}^\mu_N\Psi_B(\vec{p})$$

The nucleon wave functions are finite nucleus solutions of the Dirac equation with phenomenological **relativistic scalar and vector potentials** obtained from a Walecka-type Lagrangian fitted to properties of nuclear radii and masses:

$$(i\gamma^\mu\partial_\mu - M - S + V)\,\psi(\vec{r},t) = 0$$



Bound wave function

$$\Psi_B = \begin{pmatrix}\phi^{up}\\ \phi^{down}\end{pmatrix} = \begin{pmatrix}\phi^{up}\\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M+S-V}\phi^{up}\end{pmatrix} = \alpha u + \beta v$$

Scattered wave function

The ejected nucleon wave function is distorted by **Final State Interactions** (FSI) with the residual nucleus. In the RMF model it is a scattering solution of the same Dirac equation used to describe the bound state.
Orthogonality is preserved: the initial and final nucleon wave functions are eigenstates of the same Hamiltonian.

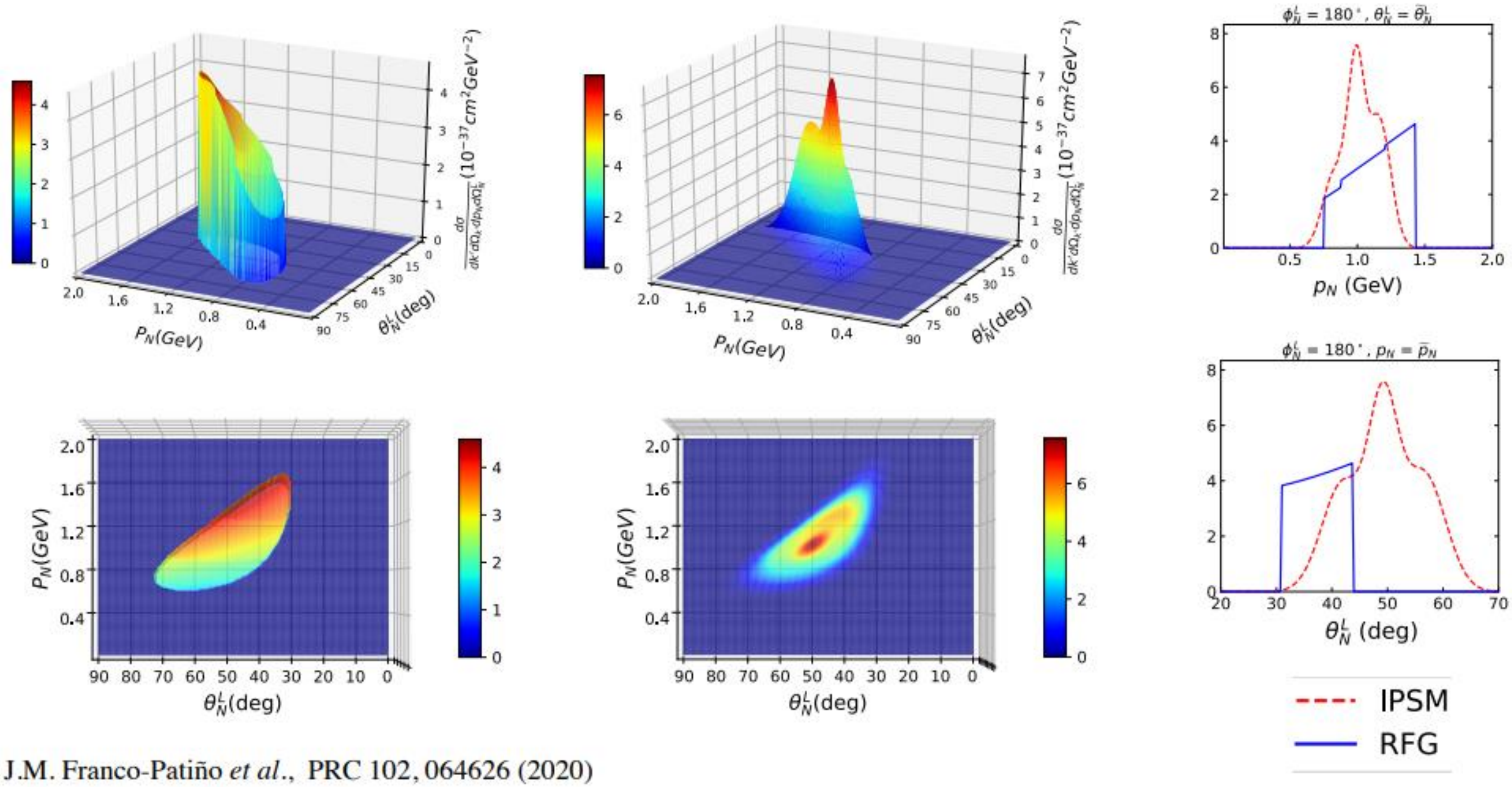
Semi-inclusive cross section: impact of different initial state modeling

M. Barbaro talk @IPSA 2022

6-differential semi-inclusive cross section $(\nu_\mu, \mu p)$ nucleon knock-out

$^{40}Ar(\nu_\mu, \mu^- p)^{39}Cl$ DUNE flux $k' = 1.5\,\text{GeV}, \theta_\mu = 30^0, \phi_N^L = \pi$

Relativistic Fermi Gas **Independent Particle Shell Model**



J.M. Franco-Patiño *et al.*, PRC 102, 064626 (2020)

Relativistic Plane Wave Impulse Approximation (no FSI included)

Striking differences in the cross section due to initial state physics described by different spectral functions.
The precise knowledge of the SF is crucial for a reliable modelling of semi-inclusive reactions.

Scattered Nucleon Description

Regarding the scattered nucleon, we can consider several situations:

- Relativistic Plane-Wave Impulse Approximation (RPWIA): the ejected nucleon is considered a plane-wave (i.e, there are not final state interactions)

- Energy-Dependent Relativistic Mean Field (ED-RMF): W.F. solution of the Dirac equation in the continuum using the same RMF potential that describes the initial state times a phenomenological function that weakens the potentials at high energies

- Relativistic Optical Potential (ROP): The scattered nucleon travels under the influence of a phenomenological relativistic optical potential fitted to reproduce elastic proton scattering data. Keeping only the real part of the OP (rROP) is an effective way to take into account all the channels (elastic and inelastic)

$$\Psi_{s_N}(\mathbf{r},\,\mathbf{p_N}) = 4\pi\sqrt{\frac{E_N+m_N}{2E_N}}\sum_{\kappa,\,m_l,\,m_j} e^{-i\delta^*_\kappa} i^l \langle l\, m_l\, 1/2\, s_N |\, j\, m_j\rangle Y^*_{l\,m_l}(\Omega_N)\psi^{m_j}_\kappa(\mathbf{r})$$

$$\psi^{m_j}_\kappa(\mathbf{r}) = \begin{pmatrix} g_\kappa(r)\phi^{m_j}_\kappa(\Omega_r) \\ i f_\kappa(r)\phi^{m_j}_{-\kappa}(\Omega_r) \end{pmatrix}$$

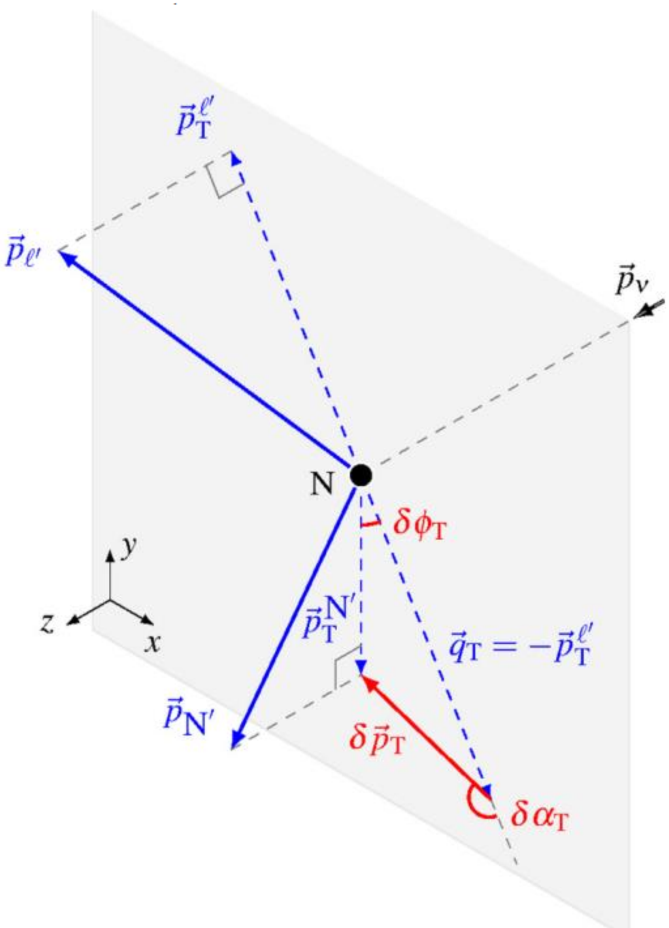
$$\frac{dg_\kappa}{dr} = -\frac{\kappa}{r}g_\kappa + [E_N + S(r,\,E_N) - V(r,\,E_N)]f_\kappa$$

$$\frac{df_\kappa}{dr} = -\frac{\kappa}{r}f_\kappa + [E_N - S(r,\,E_N) - V(r,\,E_N)]g_\kappa$$

PHYSICAL REVIEW C **94**, 015503 (2016)

Measurement of nuclear effects in neutrino interactions with minimal dependence on neutrino energy

X.-G. Lu,[1,*] L. Pickering,[2] S. Dolan,[1] G. Barr,[1] D. Coplowe,[1] Y. Uchida,[2] D. Wark,[1,3] M. O. Wascko,[2] A. Weber,[1,3] and T. Yuan[4]



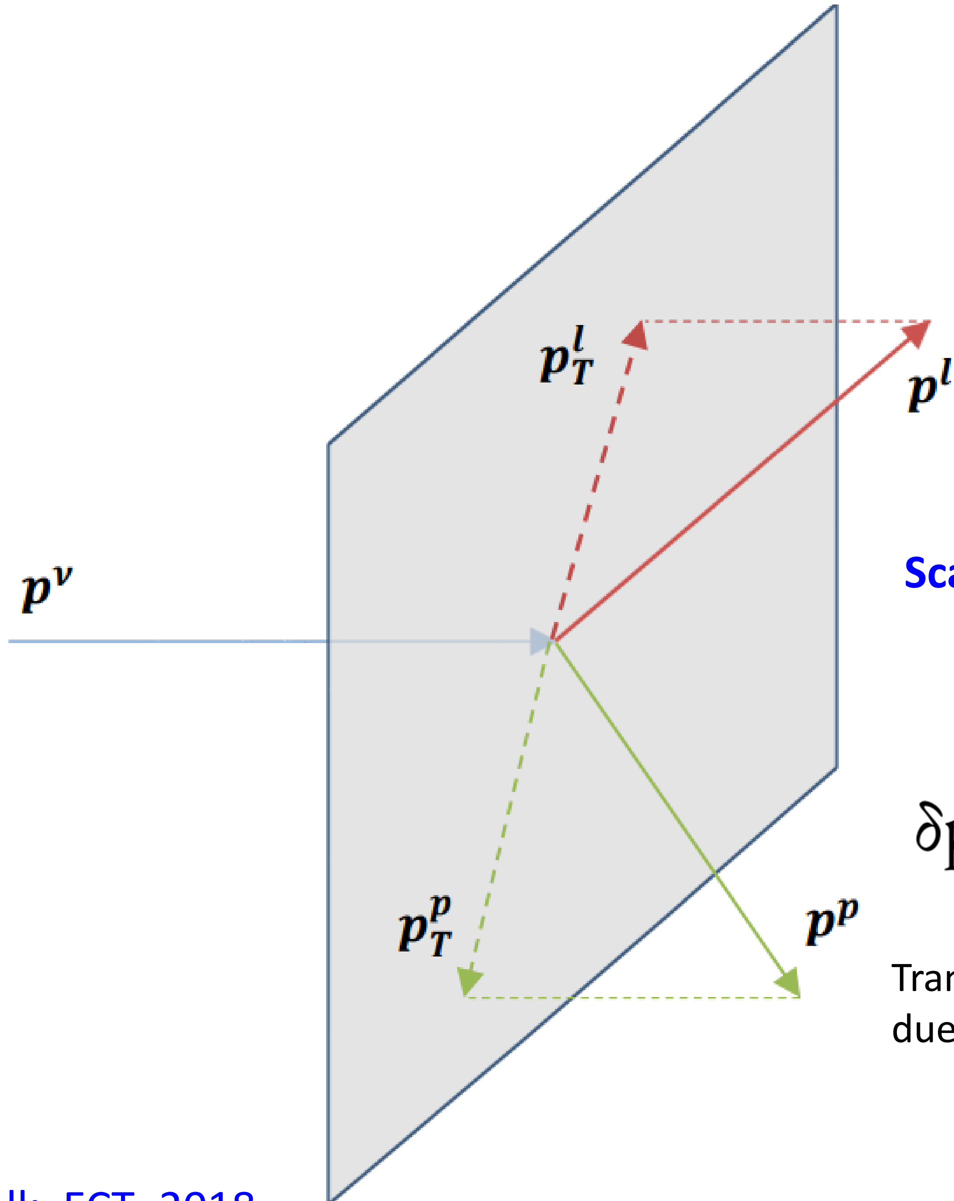
Single Transverse Variables (STV)

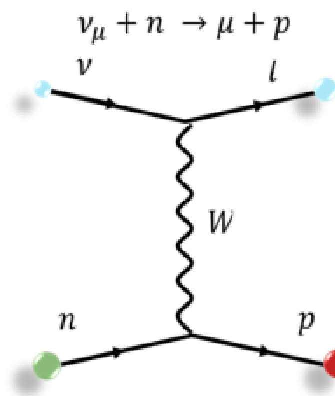
$$\delta\vec{p}_T \equiv \vec{p}_T^{\,\ell'} + \vec{p}_T^{\,N'}$$

$$\delta\alpha_T \equiv \arccos\frac{-\vec{p}_T^{\,\ell'}\cdot\delta\vec{p}_T}{p_T^{\ell'}\,\delta p_T}$$

$$\delta\phi_T \equiv \arccos\frac{-\vec{p}_T^{\,\ell'}\cdot\vec{p}_T^{\,N'}}{p_T^{\ell'}\,p_T^{N'}},$$

Single Transverse Kinematic Variables





No nuclear Effects

Scattering on a free nucleon at rest

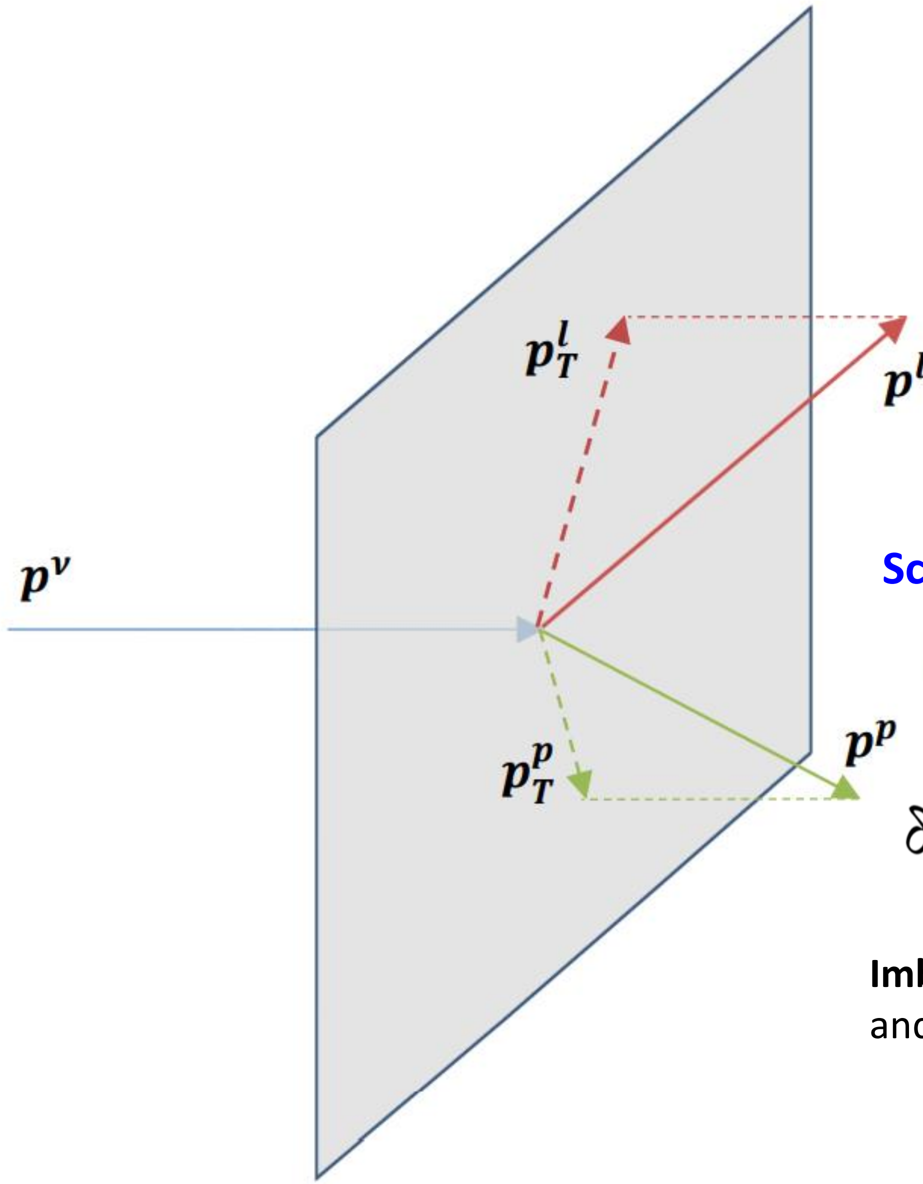
$$\boldsymbol{p}_T^l = -\boldsymbol{p}_T^p$$

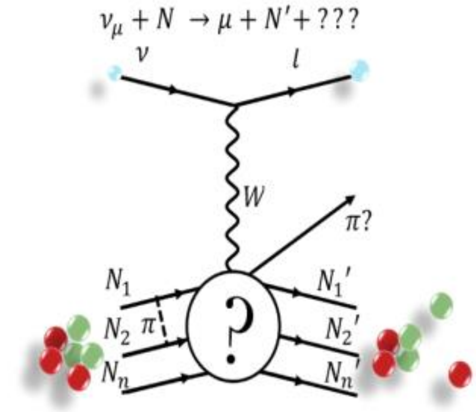
$$\delta p_T = |\, \mathbf{p_T}^{\mu} + \mathbf{p_T}^{\mathbf{p}} \,| = 0$$

Transverse projections equal and opposite due to momentum conservation

S_Dolan_Talk_ECT_2018

Single Transverse Kinematic imbalance (STKI)





With Nuclear Effects

Scattering on nucleus

$$p_T^l \neq -p_T^p$$

$$\delta p_T = |\,\mathbf{p_T}^{\mu} + \mathbf{p_T}^{p}\,| > 0$$

Imbalance due to initial nucleon motion and other nuclear effects

S_Dolan_Talk_ECT_2018

Unfolded $CC\ 2p0\pi$ Cross Section and Model comparison