On the effective lifetime of $B_s \to \mu\mu\gamma$

Carvunis, Dettori, Gangal, Guadagnoli, and CN,
arXiv: hep-ph/2102.13390v4 (hep-ph),
DOI: 10.1007/jhep12(2021)078.

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$B_s \to \mu\mu\gamma$ decay

- Sensitive to a larger set of Wilson coefficients (C_7, C_9, C_{10}) than $B_s \to \mu\mu$ (only C_{10})[1].
- On the one hand, the photon lifts the helicity suppression of $B_s \to \mu\mu$, partially compensing for the additional QED interaction,
- On the other hand, suffers from large theoretical uncertainties due to form factors of the $B_s \to \gamma$ transition[2].
- An upper limit was set recently by the LHCb experiment[3]:

$$\mathcal{B}(B_s \to \mu\mu\gamma)_{[4.9;6.0]\mathrm{GeV},ISR} < 2.0 \times 10^{-9} \quad (1)$$

[1] focusing on those related to $b \to s$ anomalies.
[2] Kruger and Melikhov, arXiv: hep-ph/0208256 (hep-ph), 2003.
[3] Aaij et al., arXiv: 2108.09284 (hep-ex), 2021; Aaij et al.,arXiv: 2108.09283 (hep-ex), 2021.

$B_s \to \mu\mu\gamma$ decay

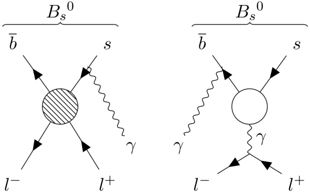

Figure: $B_s \to \mu\mu\gamma$ Initial State Radiation (ISR), hatched circle is C_9/C_{10}, empty circle is C_7.

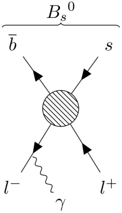

Figure: $B_s \to \mu\mu\gamma$ Final State Radiation (FSR), hatched circle is C_{10}.

$B_s \to \mu\mu\gamma$ decay

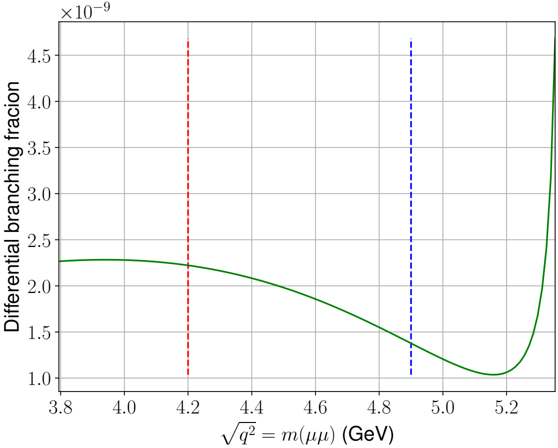

Figure: $B_s \to \mu\mu\gamma$ differential branching fraction with the dimuon mass $\sqrt{q^2} = M_B\sqrt{\hat{s}}$. Blue line: q^2_{min} from the recent $B_s \to \mu\mu$ analysis, red line: possible q^2_{min} for a dedicated $B_s \to \mu\mu\gamma$ analysis. Theoretical prediction from flavio.

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Untagged decay rate

The untagged decay rate is defined as

$$\langle \Gamma(B_s(t) \to f) \rangle \; \equiv \; \Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f) \; = \; \int_{\mathrm{PS}} \left(|\mathcal{A}_f(t)|^2 + |\bar{\mathcal{A}}_f(t)|^2 \right) , \tag{2}$$

and is related to the effective lifetime τ_{eff}^f :

$$\tau_{\mathrm{eff}}^f \; \equiv \; \frac{\int_0^\infty t \langle \Gamma(B_s(t) \to f) \rangle dt}{\int_0^\infty \langle \Gamma(B_s(t) \to f) \rangle dt} \; . \tag{3}$$

Phase-space integral:

$$\int_{\mathrm{PS}} \; \equiv \; \int \frac{(2\pi)^4}{2 M_{B_s}} d\Phi_f \, , \tag{4}$$

Untagged decay rate

By working out the previous expression:

$$\langle \Gamma(B_s(t) \to f) \rangle \;\propto\; \left[\cosh\left(\frac{y_s t}{\tau_s}\right) + A^f_{\Delta\Gamma_s} \sinh\left(\frac{y_s t}{\tau_s}\right)\right] , \tag{5}$$

with $A^f_{\Delta\Gamma_s}$ defined as:

$$A^f_{\Delta\Gamma_s} = -\frac{\int_{\mathrm{PS}} \mathrm{Re}\left(q/p\, \bar{\mathcal{A}}_f \mathcal{A}^*_f\right)}{\int_{\mathrm{PS}} |\mathcal{A}_f|^2} \; . \tag{6}$$

and $y_s \equiv \Delta\Gamma_s/(2\Gamma_s) \approx 0.06$, $\overset{(-)}{\mathcal{A}}_f$ instantaneous amplitudes of decay of $\overset{(-)}{B}{}^0_s$.

Finally, $A^f_{\Delta\Gamma_s}$ is related to the effective lifetime by:

$$\tau^f_{\mathrm{eff}} \;=\; \frac{\tau_s}{1 - y_s^2}\left(\frac{1 + 2A^f_{\Delta\Gamma_s} y_s + y_s^2}{1 + A^f_{\Delta\Gamma_s} y_s}\right) \; . \tag{7}$$

Interest of $A^f_{\Delta\Gamma_s}$

Let's look at this expression:

$$A^f_{\Delta\Gamma_s} = -\frac{\int_{\mathrm{PS}} \mathrm{Re}\left(q/p\,\bar{\mathcal{A}}_f \mathcal{A}^*_f\right)}{\int_{\mathrm{PS}} |\mathcal{A}_f|^2} \ . \tag{8}$$

- The Wilson coefficients enter as

$$|\mathcal{A}_f|^2 \quad \propto |C_9|^2, |C_{10}|^2 \ , \tag{9}$$

$$\mathcal{A}_f \bar{\mathcal{A}}^*_f \quad \propto (C_9)^2, (C_{10})^2 \ , \tag{10}$$

  such that if the Wilson coefficients have complex phases, they will appear in the latter, and not in the former.
  $\rightarrow$ Sensitive to new physics in phase of Wilson coefficients.

- q/p exactly cancels the CKM phases dependence of $\bar{\mathcal{A}}_f \mathcal{A}^*_f$.

- Ratio observable: might expect a cancellation of uncertainties.

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Choice of NP scenario

We want to study the "ability" of $A_{\Delta\Gamma_s}^{\mu\mu\gamma}$ to disentangle different NP scenarios.

Scenario	C_9^{NP}	C_{10}^{NP}
SM	0	0
C_9	$-1.0-0.9i$	0
C_{10}	0	$1.0+1.4i$
C_{LL}	$-0.7-1.4i$	$0.7+1.4i$
SM values	$C_9 = 4.327$	$C_{10} = -4.262$

(11)

$A^{\mu\mu\gamma}_{\Delta\Gamma_s}$ at high-q^2

Then, $A^{\mu\mu\gamma}_{\Delta\Gamma_s}$ depends on the phase-space integration range that is used (set by the experimentalists). In the high-q^2 region, we define

$$\int_{PS} = \int_{\hat{s}_{min}}^{1} \int_{\theta} \ , \tag{12}$$

and we compute $A^{\mu\mu\gamma}_{\Delta\Gamma_s}$ with varying $\hat{s}_{min}$, the lower bound of integration.

Note: $\hat{s} = \frac{s}{M_B^2}$.

Sources of theoretical uncertainties

There are two main sources of uncertainty:

- Broad charmonium resonances:

$$C_9 \;\to\; C_9 \;-\; \frac{9\pi}{\alpha^2}\,\bar{C}\sum_V |\eta_V| e^{i\delta_V}\,\frac{\hat{m}_V\,\mathcal{B}(V\to\mu^+\mu^-)\,\hat{\Gamma}^V_{\rm tot}}{\hat{q}^2-\hat{m}_V^2+i\hat{m}_V\hat{\Gamma}^V_{\rm tot}}\;, \tag{13}$$

  with $|\eta_V| \in [1,3]$ and $\delta_V \in [0, 2\pi]$.

  $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$
  $\hat{s} = \{0.47, 0.49, 0.57, 0.61, 0.68\}$

[4] Janowski, Pullin, and Zwicky, arXiv: 2106.13616 (hep-ph), 2021.

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- Form factors, which describe the $B_s \to \gamma$ transition. In this work, we use a recent parametrization[4] from Light Cone Sume Rules (LCSR), with estimated errors and correlations between the form factors.

[4] Janowski, Pullin, and Zwicky, arXiv: 2106.13616 (hep-ph), 2021.

Results for $A_{\Delta\Gamma_s}^{\mu\mu\gamma}$

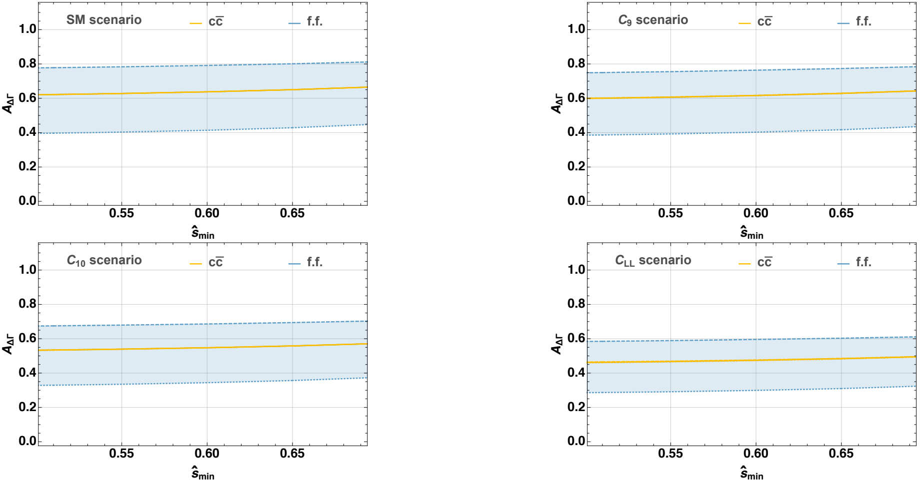

Figure: $A_{\Delta\Gamma_s}^{\mu\mu\gamma}$ prediction in the kinematic interval $[\hat{s}_{min}, 1]$. The blue vs. yellow bands refer to the f.f. error, and respectively on the uncertainty associated to the modelling of broad-charmonium resonances.

Near-cancellation of broad-charmonium

$$C_9 \;\to\; C_9 \;-\; \frac{9\pi}{\alpha^2}\,\bar{C}\sum_V |\eta_V| e^{i\delta_V}\frac{\hat{m}_V\,\mathcal{B}(V\to\mu^+\mu^-)\,\hat{\Gamma}^V_{\mathrm{tot}}}{\hat{q}^2-\hat{m}_V^2+i\hat{m}_V\hat{\Gamma}^V_{\mathrm{tot}}}\;. \qquad (14)$$

Shift to $C_9 = O(4\% \times C_9)$; yet, error from broad-$c\bar{c} < 1\%$
$\to$ Two main factors:

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Estimation of these resonant contributions would otherwise be very challenging.

Conclusion

- New observable, reachable in a medium term (Run 3), with a sensitivity to complex phases in the Wilson coefficients suggested by $b \to s$ transitions data.

- Very low (sub percent) pollution from broad-charmonium uncertainties.

- Cancellation of form factor uncertainties is not as efficient,

- but, these uncertainties can be hoped to be reduced with improvements on form factors estimation.

Question time !

Thank you for your attention !

[1] R. Aaij et al. "Analysis of neutral B-meson decays into two muons". In: (Aug. 2021). arXiv: 2108.09284 [hep-ex].

[2] R. Aaij et al. "Measurement of the $B_s^0 \to \mu^+\mu^-$ decay properties and search for the $B^0 \to \mu^+\mu^-$ and $B_s^0 \to \mu^+\mu^-\gamma$ decays". In: (Aug. 2021). arXiv: 2108.09283 [hep-ex].

[3] A. Carvunis, F. Dettori, S. Gangal, D. Guadagnoli, and CN. "On the effective lifetime of $B_s \to \mu\mu\gamma$". In: *Journal of High Energy Physics* 2021.12 (Dec. 2021). DOI: 10.1007/jhep12(2021)078. arXiv: hep-ph/2102.13390v4 [hep-ph].

[4] T. Janowski, B. Pullin, and R. Zwicky. "Charged and Neutral $\bar{B}_{u,d,s} \to \gamma$ Form Factors from Light Cone Sum Rules at NLO". In: (June 2021). arXiv: 2106.13616 [hep-ph].

[5] F. Kruger and D. Melikhov. "Gauge invariance and form-factors for the decay $B \to \gamma l^+ l^-$ ". In: *Phys. Rev.* D67 (2003), p. 034002. DOI: 10.1103/PhysRevD.67.034002. arXiv: hep-ph/0208256 [hep-ph].

B_s mixing

$$
\begin{aligned}
|B_{L,H}\rangle &= p|B_s^0\rangle \pm q|\bar{B}_s^0\rangle\,, \\
\left(\frac{q}{p}\right)^2 &= e^{-2i\phi_M}\,(1+a)\,,
\end{aligned}
$$

$$
\Delta M_s = M_H - M_L\,, \quad \Gamma_s = \frac{\Gamma_H + \Gamma_L}{2}\,, \quad \Delta\Gamma_s = \Gamma_L - \Gamma_H\,, \tag{15}
$$

$a \approx A_{\mathrm{SL}} \simeq -3.5 \times 10^{-3}$

Global fits

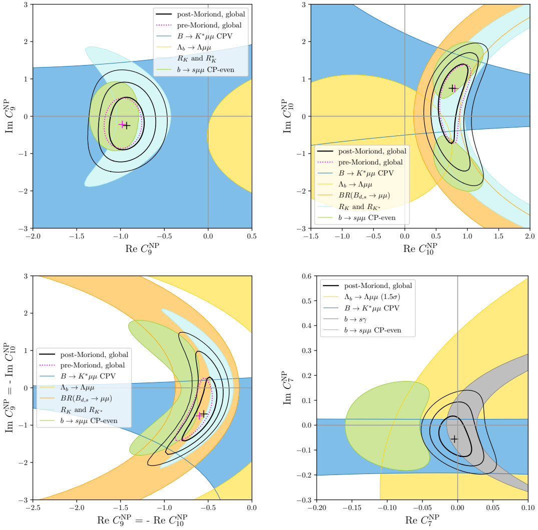

Figure: 1σ constraints on the NP Wilson coefficients.

Shift to C_9

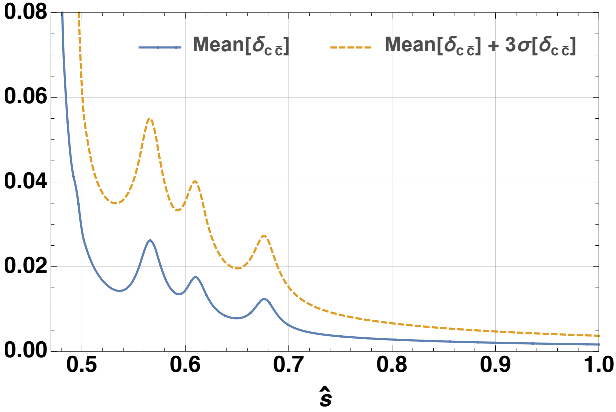

Figure: Blue solid line: mean of $|\delta_{c\bar{c}}|/C_{9,\mathrm{SD}}$ as a function of $\hat{s}$, obtained from a uniform scan to $|\eta_V| \in [1,3]$ and $\delta_V \in [0, 2\pi)$. Dashed orange line: mean plus 3 times the standard deviation.

$B_s \to \mu\mu\gamma$ Lagrangian

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \left(\sum_{i=1}^{2} (\lambda_u C_i \mathcal{O}_i^u + \lambda_c C_i O_i^c) - \lambda_t \sum_{i=3}^{6} C_i \mathcal{O}_i - \lambda_t \sum_{i=7}^{10} (C_i \mathcal{O}_i + C_i' \mathcal{O}_i') \right) , \tag{16}$$

where $\lambda_i \equiv V_{is}^* V_{ib}$ and V the CKM matrix. The operators relevant to our discussion read

$$\begin{aligned}
\mathcal{O}_1^q &= (\bar{s}^\alpha \gamma_\mu q_L^\beta)(\bar{q}^\beta \gamma^\mu b_L^\alpha) , & \mathcal{O}_2^q &= (\bar{s}^\alpha \gamma_\mu q_L^\alpha)(\bar{q}^\beta \gamma^\mu b_L^\beta) , \\
\mathcal{O}_7 &= \frac{e}{16\pi^2} m_b \bar{s} \sigma_{\mu\nu} F^{\mu\nu} b_R , & \mathcal{O}_8 &= \frac{g_s}{16\pi^2} m_b \bar{s} \sigma_{\mu\nu} G^{\mu\nu} b_R , \\
\mathcal{O}_9 &= \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell) , & \mathcal{O}_{10} &= \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell) ,
\end{aligned} \tag{17}$$

$B_s \to \mu\mu\gamma$ amplitudes

$$\begin{aligned}\mathcal{A}_{\rm DE}(\bar{B}_s^0 \to \mu\mu\gamma) \; = \; & \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{2\pi} \times \\ & \Big\{ -\frac{2 i m_b C_7}{q^2} \langle \gamma(k,\epsilon) | \bar{s} \sigma_{\mu\nu} (1+\gamma_5) q^\nu b | \bar{B}_s^0(p) \rangle \, \bar{u}(p_2) \gamma^\mu v(p_1) \\ & + C_9 \langle \gamma(k,\epsilon) | \bar{s} \gamma_\mu (1-\gamma_5) b | \bar{B}_s^0(p) \rangle \, \bar{u}(p_2) \gamma^\mu v(p_1) \\ & + C_{10} \langle \gamma(k,\epsilon) | \bar{s} \gamma_\mu (1-\gamma_5) b | \bar{B}_s^0(p) \rangle \, \bar{u}(p_2) \gamma^\mu \gamma_5 v(p_1) \Big\} , \quad (18)\end{aligned}$$

$$\begin{aligned}\mathcal{A}_{\rm Brems} \; = \; & + i \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{2\pi} e \, X_f f_{B_s} \, 2 m_\mu \, C_{10} \\ & \Big\{ \bar{u}(p_2) \left(\frac{\not{\epsilon}^* \not{p}}{t - m_\mu^2} - \frac{\not{p} \not{\epsilon}^*}{u - m_\mu^2} \right) v(p_1) \Big\} . \quad (19)\end{aligned}$$

Form factors definition

$$
\begin{aligned}
\langle \gamma(k,\lambda)|\bar{s}\gamma^{\mu} b|\bar{B}_s^0(q+k)\rangle &= e\,\epsilon^{\mu\lambda^* qk}\,\frac{F_V(q^2)}{M_{B_s}}\,, \\
\langle \gamma(k,\lambda)|\bar{s}\gamma^{\mu}\gamma_5 b|\bar{B}_s^0(q+k)\rangle &= ie\,(\lambda^{*\mu}\,qk - k^{\mu}\,\lambda^* q)\,\frac{F_A(q^2)}{M_{B_s}}\,, \\
\langle \gamma(k,\lambda)|\bar{s}\sigma^{\mu\nu} b q_{\nu}|\bar{B}_s^0(q+k)\rangle &= ie\,\epsilon^{\mu\lambda^* qk}\,F_{TV}(q^2,0)\,, \\
\langle \gamma(k,\lambda)|\bar{s}\sigma^{\mu\nu}\gamma_5 b q_{\nu}|\bar{B}_s^0(q+k)\rangle &= e\,(\lambda^{*\mu}\,qk - k^{\mu}\,\lambda^* q) F_{TA}(q^2,0)\,, \qquad (20)
\end{aligned}
$$

Explicit formula for $A_{\Delta\Gamma_s}^{\mu\mu\gamma}$

$$A_{\Delta\Gamma_s}^{\mu\mu\gamma} \;=\; -\,\frac{\int d\hat{s}\, d\cos\theta \;\, f(\hat{s}, \hat{m}_\mu^2)\; \mathrm{Re}\left(q/p\, \bar{\mathcal{A}}_{\mu\mu\gamma} \mathcal{A}^*_{\mu\mu\gamma}\right)}{\int d\hat{s}\, d\cos\theta \;\, f(\hat{s}, \hat{m}_\mu^2)\;\, \left|\mathcal{A}_{\mu\mu\gamma}\right|^2}\,, \tag{21}$$

we can perform the integration $\int_{-1}^{1} d\cos\theta$. We obtain

$$\int d\cos\theta \; \mathrm{Re}\left(q/p\, \bar{\mathcal{A}}_{\mu\mu\gamma} \mathcal{A}^*_{\mu\mu\gamma}\right) \;=\; |\mathcal{N}|^2 \sum_{i,j}^{7,9,10} \mathrm{Re}(f_{ij}\, C_i C_j) \tag{22}$$

and

$$\int d\cos\theta \;\, \left|\mathcal{A}_{\mu\mu\gamma}\right|^2 \;=\; |\mathcal{N}|^2 \sum_{i,j}^{7,9,10} \mathrm{Re}(\bar{f}_{ij}\, C_i C_j^*)\,, \tag{23}$$

Explicit formula for $A_{\Delta\Gamma_s}^{\mu\mu\gamma}$

$$f_{77} = \frac{16}{3} M_{B_s}^4 \hat{m}_b^2 x^2 \frac{2\hat{m}_\mu^2 + \hat{s}}{\hat{s}^2} (\bar{F}_{TA}^2 - \bar{F}_{TV}^2), \quad (24)$$

$$\bar{f}_{77} = \frac{16}{3} M_{B_s}^4 \hat{m}_b^2 x^2 \frac{2\hat{m}_\mu^2 + \hat{s}}{\hat{s}^2} (|\bar{F}_{TA}|^2 + |\bar{F}_{TV}|^2), \quad (25)$$

$$f_{99} = \frac{4}{3} M_{B_s}^4 x^2 (2\hat{m}_\mu^2 + \hat{s})(F_{A,9}^2 - F_{V,9}^2), \quad (26)$$

$$\bar{f}_{99} = \frac{4}{3} M_{B_s}^4 x^2 (2\hat{m}_\mu^2 + \hat{s})(|F_{A,9}|^2 + |F_{V,9}|^2), \quad (27)$$

$$f_{10\,10} = -\frac{4 M_{B_s}^4 x^2}{3} (4\hat{m}_\mu^2 - \hat{s})(F_{A,10}^2 - F_{V,10}^2), \quad (28)$$

$$\bar{f}_{10\,10} = -\frac{4 M_{B_s}^4 x^2}{3} (4\hat{m}_\mu^2 - \hat{s})(|F_{A,10}|^2 + |F_{V,10}|^2), \quad (29)$$

Explicit formula for $A_{\Delta\Gamma_s}^{\mu\mu\gamma}$

$$f_{7\,9} + f_{9\,7} \;=\; \frac{16}{3} M_{B_s}^4\, \hat{m}_b\, x^2\, \frac{2\hat{m}_\mu^2 + \hat{s}}{\hat{s}} \left(\bar{F}_{TA} F_{A,9} - \bar{F}_{TV} F_{V,9} \right), \tag{30}$$

$$\bar{f}_{7\,9}\, C_7 C_9^* + \bar{f}_{9\,7}\, C_9 C_7^* \;=\; \frac{16}{3} M_{B_s}^4\, \hat{m}_b\, x^2\, \frac{2\hat{m}_\mu^2 + \hat{s}}{\hat{s}} \times \mathrm{Re}\left[\bar{F}_{TA} C_7 \left(F_{A,9} C_9 \right)^* + \bar{F}_{TV} C_7 \left(F_{V,9} C_9 \right)^* \right], \tag{31}$$

$$\bar{f}_{7\,10}\, C_7 C_{10}^* + \bar{f}_{10\,7}\, C_{10} C_7^* \;=\; 128\, f_{B_s} M_{B_s}^3\, \hat{m}_b \hat{m}_\mu^2\, \frac{x}{\hat{s}}\, \frac{\mathrm{atan}(z)}{z} \mathrm{Re}\left[\bar{F}_{TV} C_7\, C_{10}^* \right], \tag{32}$$

$$\bar{f}_{9\,10}\, C_9 C_{10}^* + \bar{f}_{10\,9}\, C_{10} C_9^* \;=\; 64\, f_{B_s} M_{B_s}^3\, \hat{m}_\mu^2\, x \frac{\mathrm{atan}(z)}{z} \mathrm{Re}\left[F_{V,9} C_9\, C_{10}^* \right], \tag{33}$$