Form factors in $b \to s$ transitions

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Méril Reboud

Based on Amhis, Bordone, MR 2208.08937

Technische Universität München

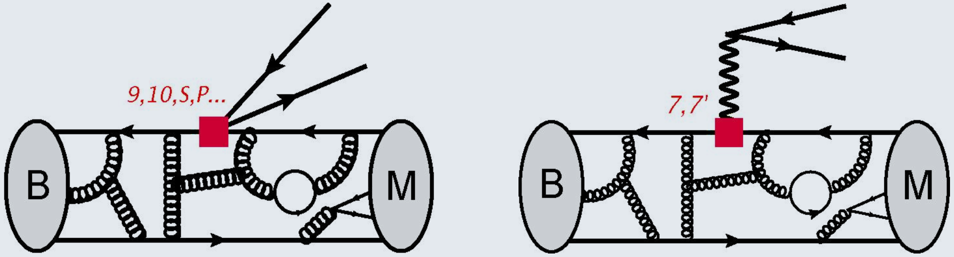
I. Introduction

Form factors in b → sℓℓ

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2}\left(\bar{s}_L\gamma_\mu b_L\right)\left(\bar{\ell}\gamma^\mu(\gamma_5)\ell\right)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\left(\bar{s}_L\sigma_{\mu\nu}b_R\right)F^{\mu\nu}$$



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda\ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7\mathcal{F}_\lambda^T(q^2) - 16\pi^2\frac{M_B}{m_b}\mathcal{H}_\lambda(q^2)\right]\right\}$$

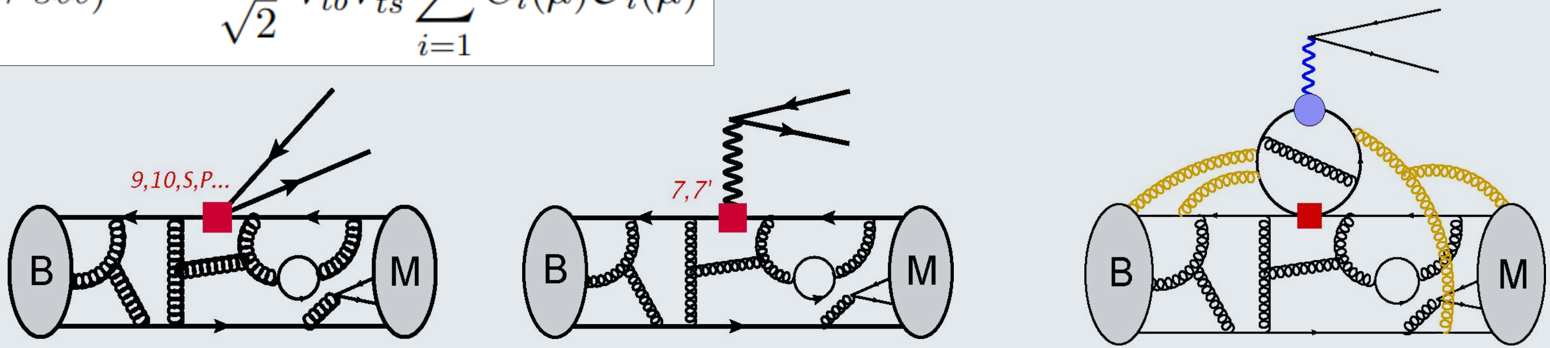
- B → $K^{(*)}$ μμ
- B_s → φ μμ
- Λ_b → $\Lambda^{(*)}$ μμ

Local form-factors,
involves e.g.

$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$$

Form factors in $b \to s\ell\ell$

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu)$$



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

$$\mathcal{H}_\mu(k,q) = i \int d^4x\, e^{iq\cdot x} \langle \bar{M}(k) | T\{ \mathcal{J}_\mu^{\text{em}}(x), \mathcal{C}_i \mathcal{O}_i \} | \bar{B}(q+k) \rangle$$

Non-local form-factors

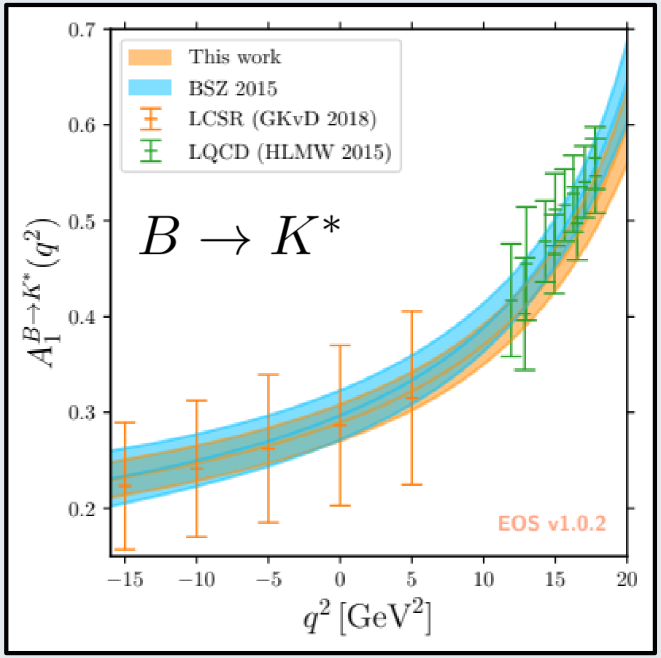
→ Not today, but same methods are used [Gubernari, MR, van Dyk, Virto '22]

Local form factors

- **2 main approaches**
  - Lattice QCD → most feasible at **large q²**
  - Light-cone sum rules → most feasible at **small q²**

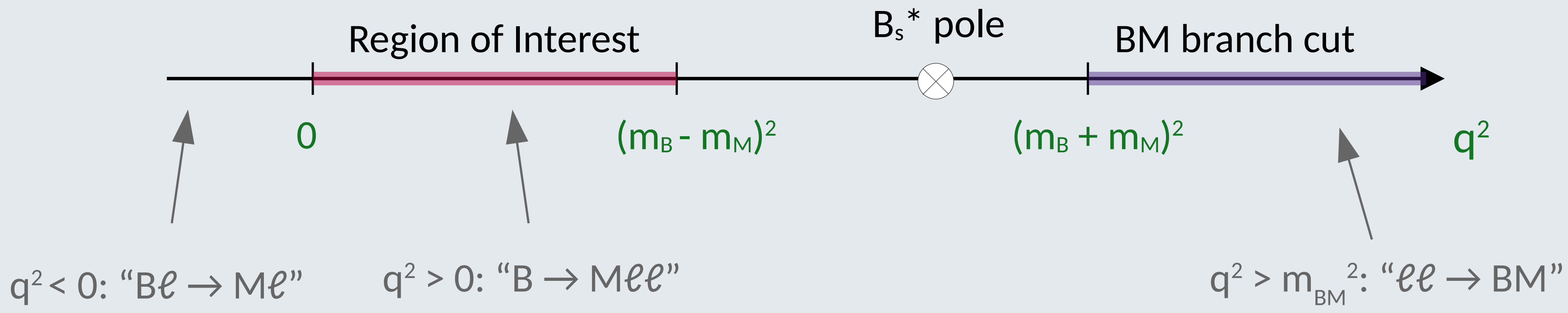
- 2 possible LCSRs:
  - Light meson LCDA [recent works: Bharrucha, Straub, Zwicky '15; Khodjamirian, Rusov '17]
  - B meson LCDA [recent works: Khodjamirian, Mannel, Pivovarov, Wang '10; Gubernari, Kokulu, van Dyk '18]

→ **Interpolation** in the physical range



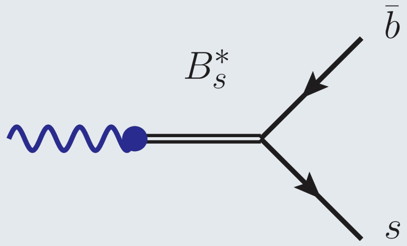
Form Factor Parametrization

$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k) | \bar{s}\gamma_\mu b_L | \bar{B}(q+k) \rangle$$

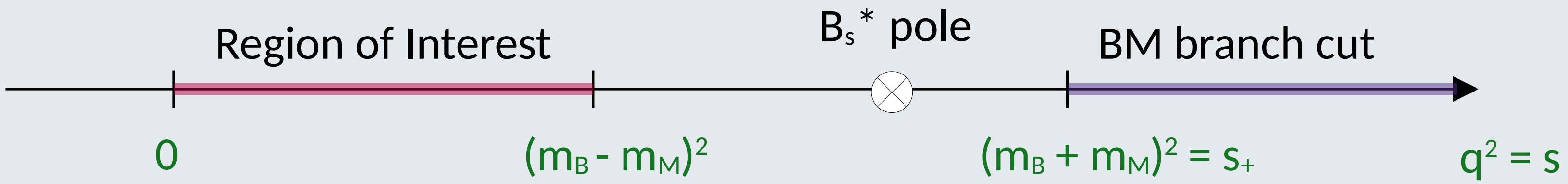


Analyticity properties of the form factors:

- Pole due to **$\bar{b}$s bound state**
- **Branch cut** due to on-shell pair production



Form Factor Parametrization



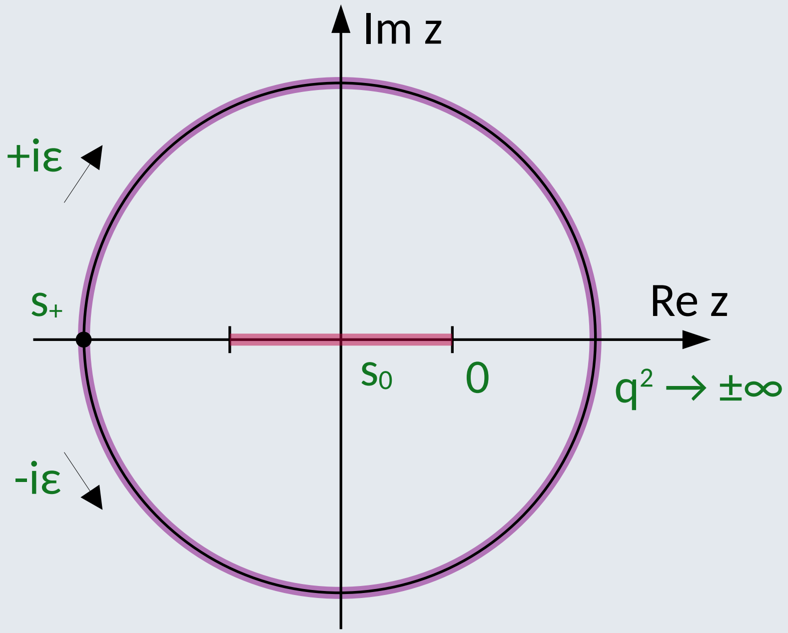
Conformal mapping [Boyd, Grinstein, Lebed '97]

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$

Simplified Series expansion [Bourrely, Caprini, Lellouch, '08; Bharucha, Feldmann, Wick '10]

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{q^2 - m_{B_s^*}^2} \sum_{k=0}^{N} \alpha_{\lambda,k} z^k$$

N = 2 is enough to provide an **excellent description of the data** (p-values > 70%)

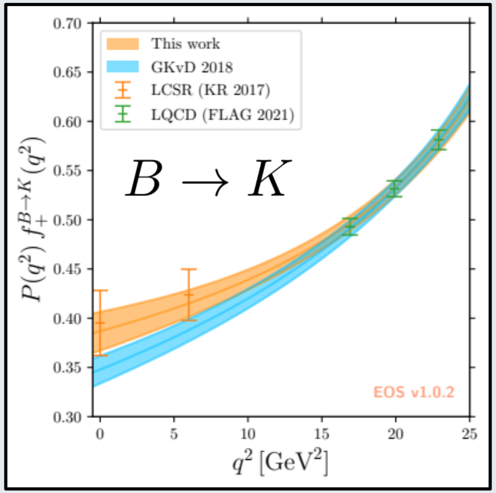


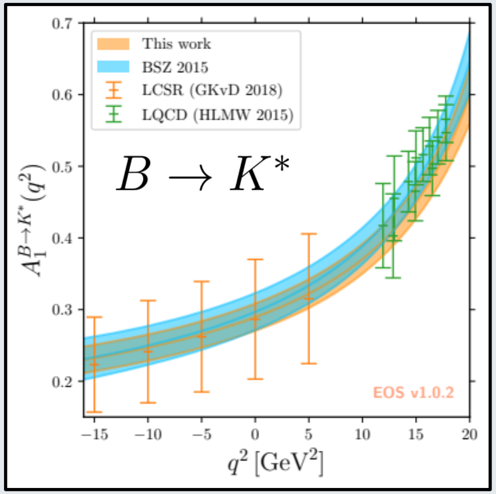
Local form factors

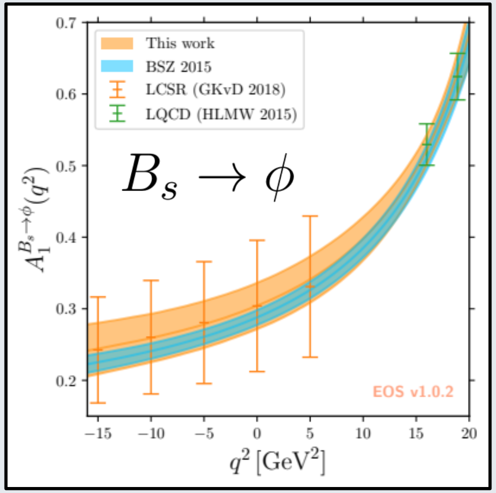
[Gubernari, MR, van Dyk, Virto ‘22]

Combined fit to LCSR and lattice QCD
Inputs:

- B → K:
  - [HPQCD’17; FNAL/MILC ’17]
  - [Khodjamiriam, Rusov ’17]
- B → K*:
  - [Horgan, Liu, Meinel, Wingate ’15]
  - [Gubernari, Kokulu, van Dyk ’18]
- B_s → φ:
  - [Horgan, Liu, Meinel, Wingate ’15]
  - [Gubernari, van Dyk, Virto ’20]







What about the **model uncertainties**? What if we only have LQCD?

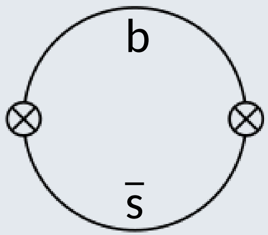
[BSZ = Bharucha, Straub, Zwicky ‘15]

II. Dispersive bound

Dispersive bound

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

Insertion of a scalar, vector or tensor current



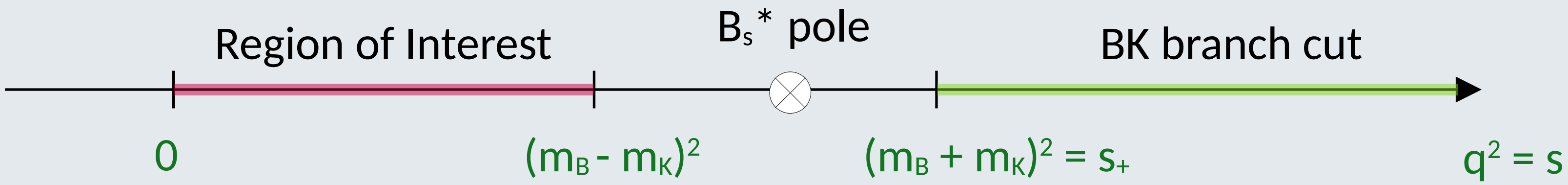
+ other diagrams: loops, quark and gluon condensates...

- Unitarity gives **shared bounds** for **all the b → s processes**: (schematically)

$$1 > 2\int_{(m_B+m_K)^2}^{\infty} \left|\hat{\mathcal{F}}_X^{B\to K}(t)\right|^2 dt \;+\; 2\int_{(m_B+m_{K^*})^2}^{\infty} \left|\hat{\mathcal{F}}_X^{B\to K^*}(t)\right|^2 dt \;+\; \ldots$$

$$\text{known functions} \times \mathcal{F}_X^{B\to K}(t)$$

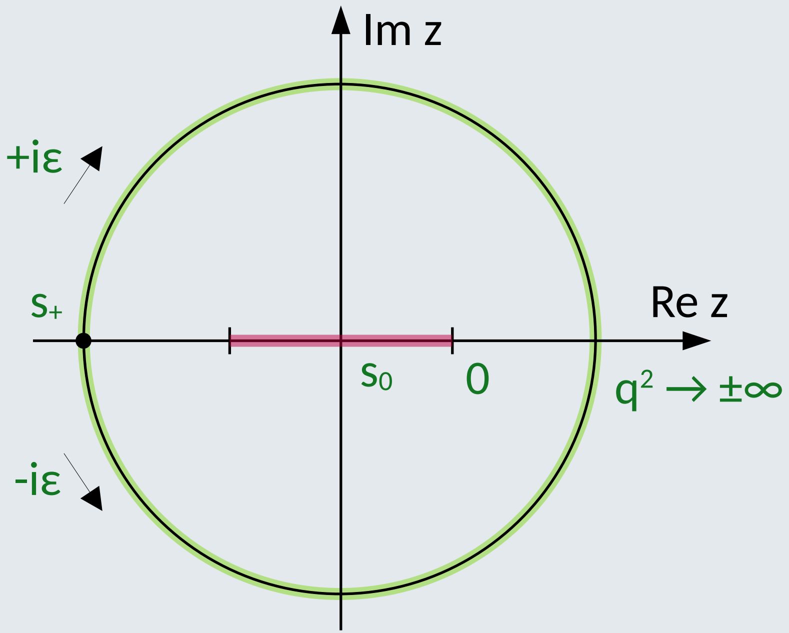
Simple case: B → K



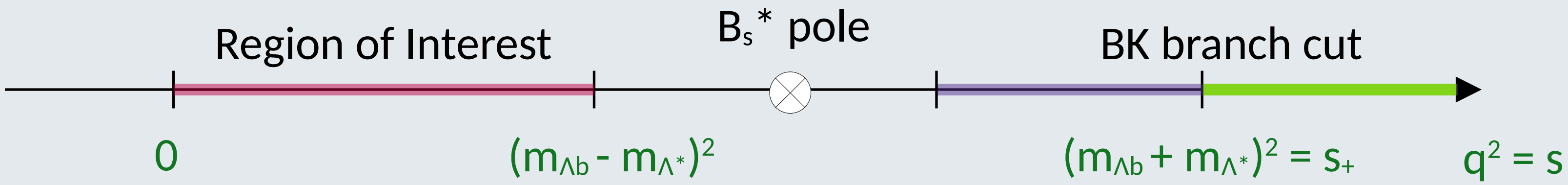
- The branch cut starts **at** the pair production threshold
- The monomial z^k are **orthogonal** on the unit circle

$$\hat{\mathcal{F}}_X^{B\to K} = \sum_{k=0}^{N} a_{X,k}\, z^k$$

$$\int_{(m_B+m_K)^2}^{\infty} \left|\hat{\mathcal{F}}_X^{B\to K}(t)\right|^2 dt = \sum_{k=0}^{N} |a_{X,k}|^2$$



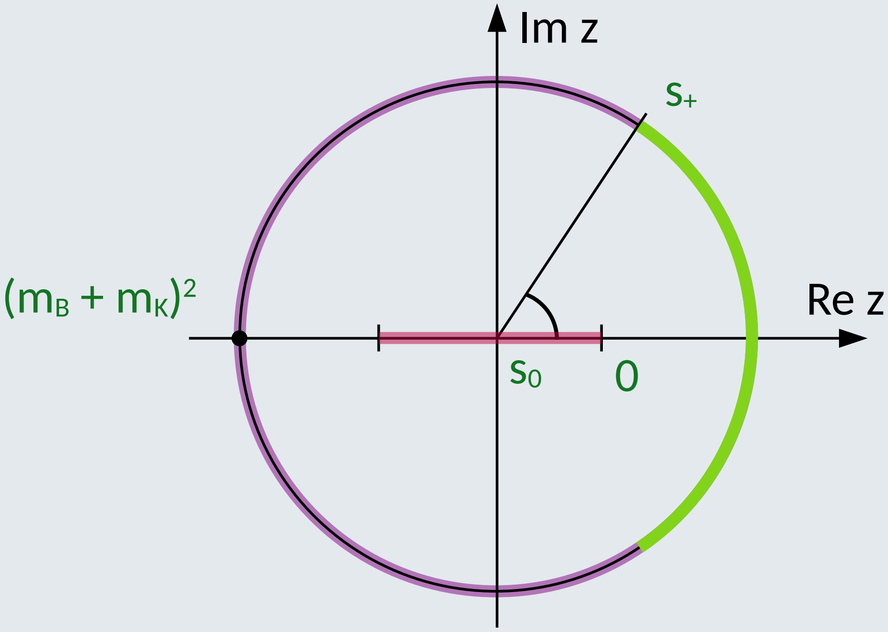
Less simple case, e.g. $\Lambda_b \to \Lambda^*$



- The first branch cut (BK) starts **before** the pair production threshold
- Introduce orthonormal polynomials of the **arc of the unit circle**

$$\hat{\mathcal{F}}_X^{B\to K} = \sum_{k=0}^{N} a_{X,k}\, p_k(z)$$

$$\int_{(m_B+m_K)^2}^{\infty} \left|\hat{\mathcal{F}}_X^{B\to K}(t)\right|^2 dt = \sum_{k=0}^{N} |a_{X,k}|^2$$



III. Numerics

Note: I will continue with $\Lambda_b \to \Lambda^*$, see [Blake, Meinel, Rahimi, van Dyk 2205.06041] for $\Lambda_b \to \Lambda$

Fit results

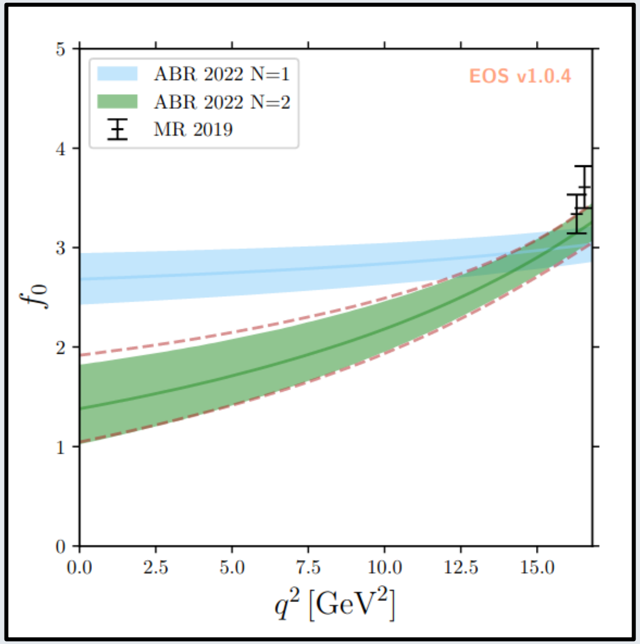
- Inputs:
  - **LQCD** [Meinel, Rendon '21]
  - no LCSR → use **SCET relations** [Descotes-Genon, M. Novoa-Brunet '19]

$$f_{\perp'}(0) = 0 \pm 0.2\,, \quad g_{\perp'}(0) = 0 \pm 0.2\,, \quad h_{\perp'}(0) = 0 \pm 0.2\,,$$
$$\tilde{h}_{\perp'}(0) = 0 \pm 0.2\,, \quad f_+(0)/f_\perp(0) = 1 \pm 0.2\,, \quad f_\perp(0)/g_0(0) = 1 \pm 0.2\,,$$
$$g_\perp(0)/g_+(0) = 1 \pm 0.2\,, \quad h_+(0)/h_\perp(0) = 1 \pm 0.2\,, \quad f_+(0)/h_+(0) = 1 \pm 0.2\,,$$

$O(\alpha_s/\pi, \Lambda_{QCD}/m_b)$

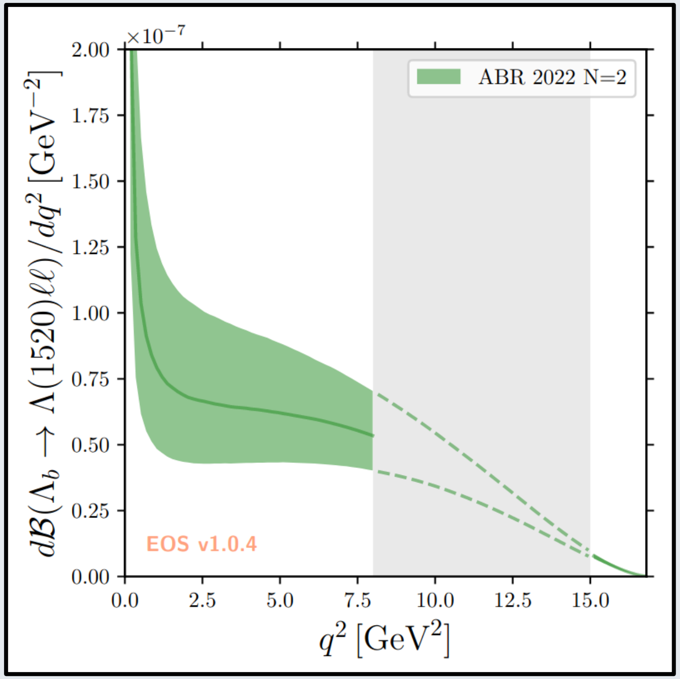
- Use an **under-constrained fit** (N>1) and allows for saturation of the dispersive bound

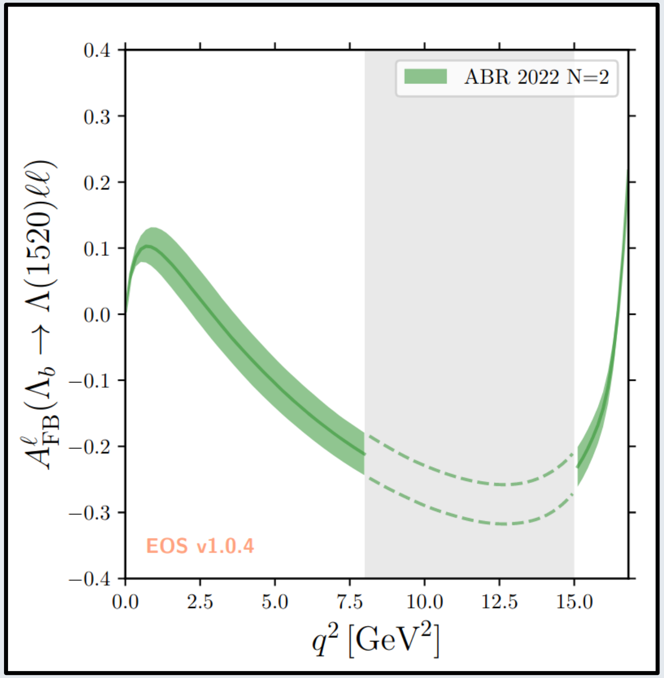
  → The uncertainties are model-independent, increasing the expansion order does not change their size



Phenomenology

- Uncertainties are large but **under control** and **systematically improvable**
- LHCb analysis is **ongoing**



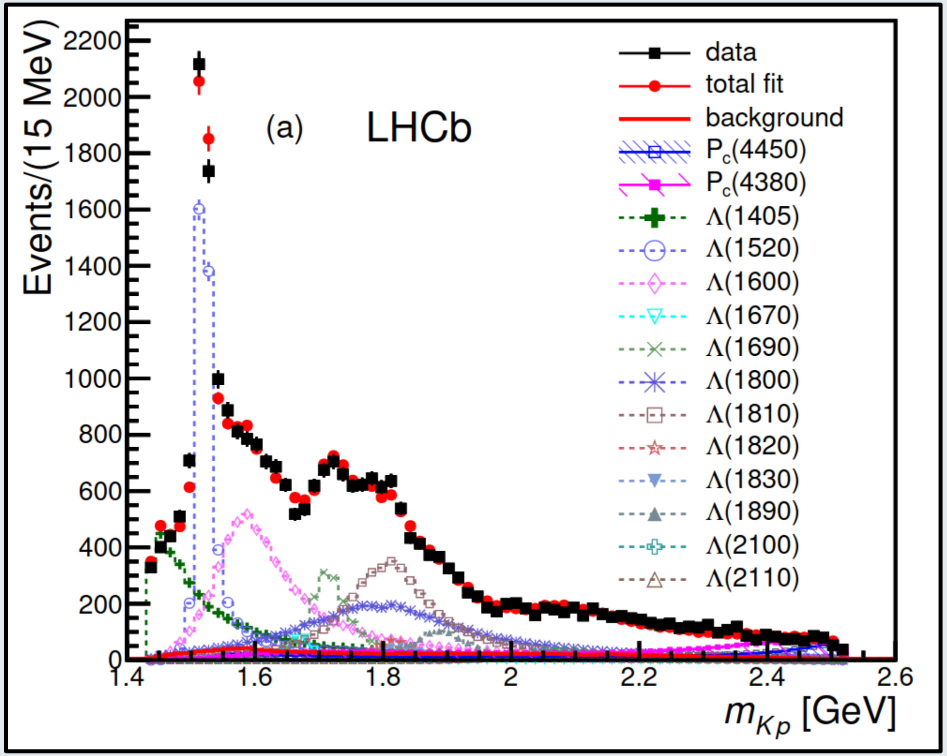


Conclusion

- Local form factors are the **main source of uncertainties** in $b \to s$ observables
- Dispersive bounds allow:
  - A control of the uncertainties in the z-expansions
  - **Systematically reducible** uncertainties, even in the absence of new LQCD/LCSR results!

Back-up

Resonance structure in $\Lambda_b \to J/\psi p K$

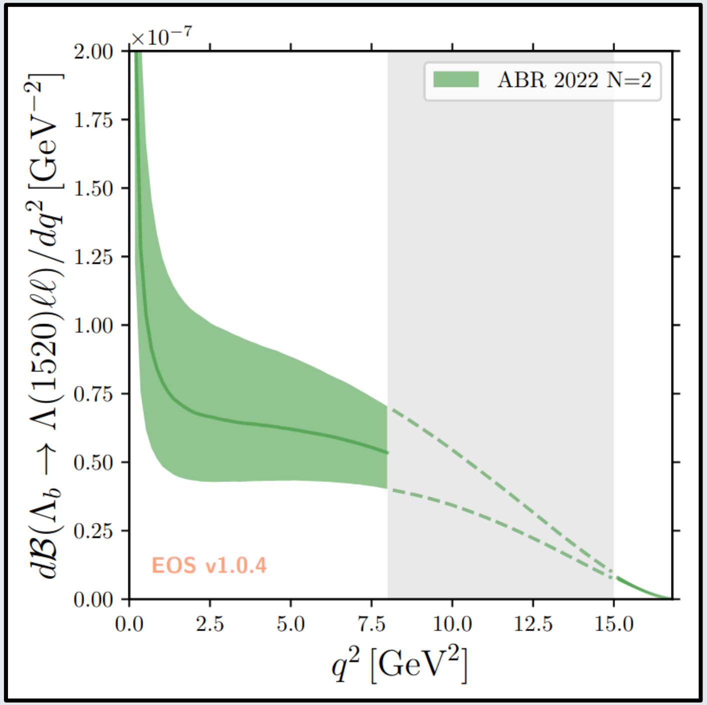


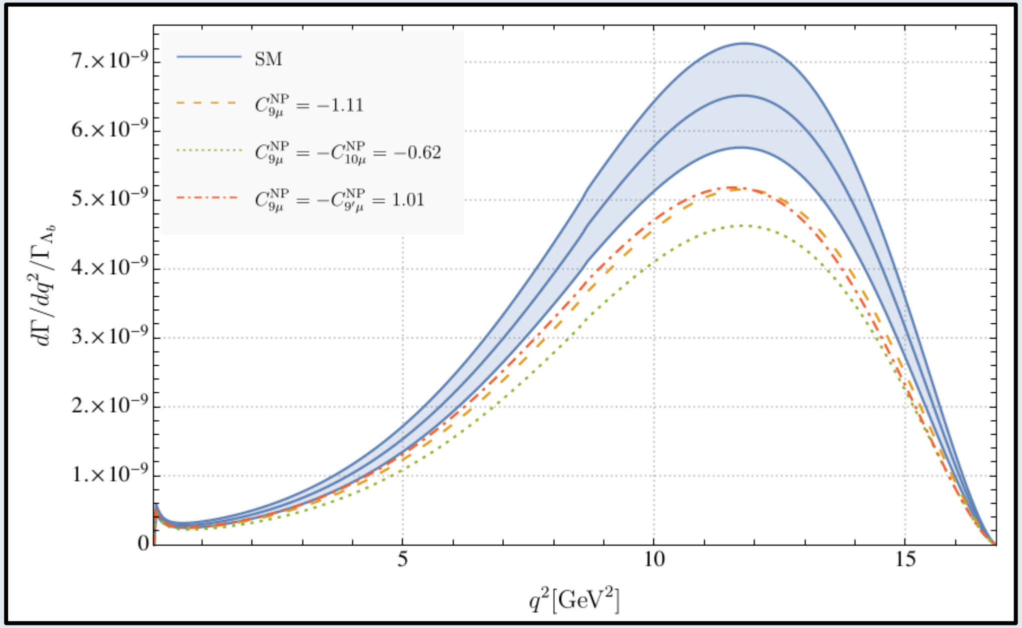
[LHCb 1507.03414]

- The resonance structure way richer than the one in $B \to J/\psi \pi K$
- Λ(1520) is the narrowest observed resonance
- Narrow-width approximation can be applied

Comparison with the literature: BR

- Branching ratio changes with respect to [Descotes-Genon, M. Novoa-Brunet ‘19] → this is due to the Quark Model they used for the form factors, which is in large tension both with SCET relations and LQCD results





Comparison with the literature: A_FB

- The forward-backward asymmetry is not impacted