Primordial non-gaussianities or relativistic effects in Large Scale Structures?

Clément Stahl

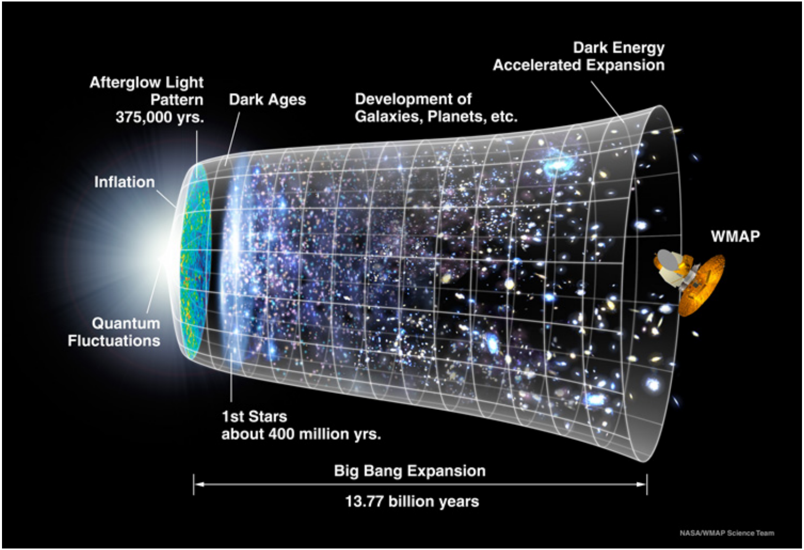
Observatoire Astronomique de Strasbourg, Université de Strasbourg

28 November 2022

Based on 1811.05452, 2110.11249, 2212,xxxxx

Current collaborators: J. Adamek, O. Hahn, T. Montandon, C. Rampf, B. van Tent.

Large Scale Structures (LSS) formation



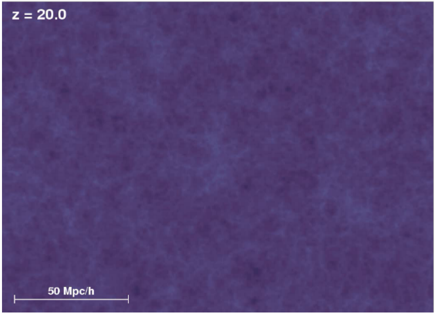
Cosmological structures formation

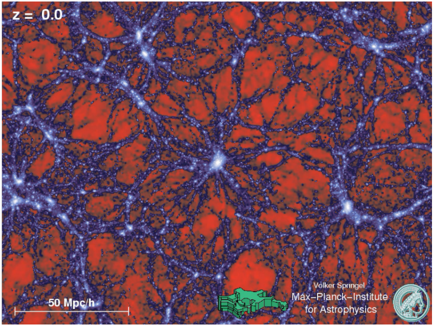
Fluids mechanics in an expanding universe.

Large Scale Structures (LSS) formation

In LSS, split between large scales *background* (expanding universe, well defined mean density) and intermediate scales *perturbations* (density differs little from background).

Cosmic structures grow out of tiny initial fluctuations.





Newtonian structure formation

- Study of LSS on scales smaller than the Hubble scale (3000 h^{-1} Mpc).
- typically $v \sim 10^{-2}$, $\phi \sim 10^{-5}$.
- Linear fluids mechanics in an expanding universe: success story (cf. CMB).

Epic Battle: Newton vs Einstein

For CDM (non-relativistic matter):

- On background level (FLRW): Newton and Einstein agree.
- For linear (scalar) perturbations: Newton and Einstein agree.
- In the non-linear regime: small scales: *Newton and Einstein agree*.

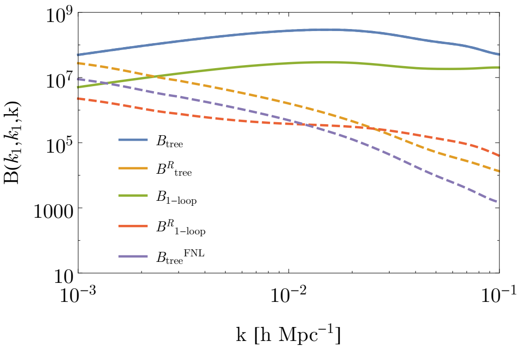
A case for Einstein

Relativistic structure formation

- Relativistic matter content of the universe (neutrinos, cosmic strings, DDE).
- Gravity has 6 degrees of freedom (2 scalars, 2 vectors and 2 tensors)
- Backreaction: how non-linear evolution impacts means quantities.
- Observations are made on the relativistic perturbed light cone.

Analytical result: I argue (**Castiblanco** 1811.05452)

The bispectrum in the squeezed limit at 1-loop receives relativistic corrections due to the dynamics of the CDM field of the same order than Newtonian results.



Bispectrum: generalities

Analytical result: I argue (**Castiblanco** 1811.05452)

The bispectrum in the squeezed limit at 1-loop receives relativistic corrections due to the dynamics of the CDM field of the same order than Newtonian results.

Power spectrum vs Bispectrum

$$\langle \delta(\boldsymbol{k}_1, t)\delta(\boldsymbol{k}_2, t)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_1 + \boldsymbol{k}_2) P(k_1, t)\,, \tag{1}$$

$$\langle \delta(\boldsymbol{k}_1, t)\delta(\boldsymbol{k}_2, t)\delta(\boldsymbol{k}_3, t)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B(k_1, k_2, k_3, t)\,. \tag{2}$$

Note that the bispectrum couples the scales !!

Constraints

$f_{\mathrm{NL}} = 37 \pm 20$ (WMAP 1212.5225),
$f_{\mathrm{NL}} = -0.9 \pm 5.1$ (Planck 1905.05697). Compatible with zero at 2σ.
Could LSS improve those constraints?

SPOILER ALERT: yes... Will reach $\sigma(f_{\mathrm{NL}}) = \mathcal{O}(1)$

LSS: $N^{\mathrm{LSS}}_{\mathrm{modes}} \sim V k_{max}^3 \sim 10^{10}$: $V = (10^4 \mathrm{Mpc}/h)^3$; $k_{max} = 0.5h.\mathrm{Mpc}^{-1}$.
CMB: $N^{\mathrm{CMB}}_{\mathrm{modes}} \sim S k_{max}^2 \sim 10^7$.

Bispectrum: generalities

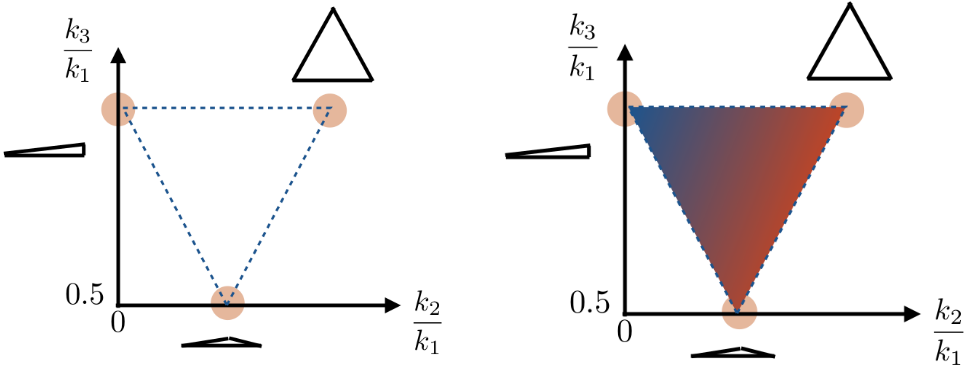


Image credit: J. Noreña

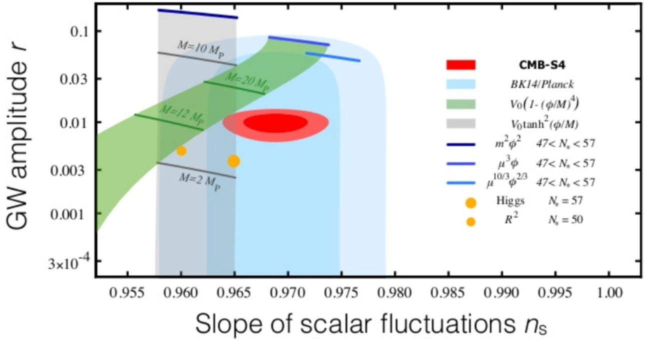
The red zone is degenerated with non-linear growth, biasing and astrophysics.

The blue zone, the the squeezed limit is believed to be much more solid.

Bispectrum for Fundamental physics: Inflation

The squeezed limit contains model independent information about the physics during inflation.

Energy scale at which inflation occurs is unknown and can range across 10 orders of magnitude. Quantum fluctuations imprint into the *full* gravitational fields of the universe $\rightarrow$ Production of gravitational waves! Potential observation for highest energy model of inflation ($>10^{16}$ Gev) through interaction with polarization of CMB photons (B-modes).



CMS-S4 Science book 1610.02743

$$\left(\frac{r}{0.01}\right) \simeq \frac{V^{1/4}}{10^{16}\text{Gev}}$$

Bispectrum for Fundamental physics

Models with energy scale below 10^{16} Gev have no observable primordial gravitational waves. Class these models using **primordial non-gaussianities**: complements GW searches (Meerburg 1903.04409).

Theorem: (Consistency relations), Maldacena 0210603

If only one light scalar field is active during inflation, the behavior of the three-point correlation function, in the squeezed limit, is entirely fixed by the two-point correlation function.

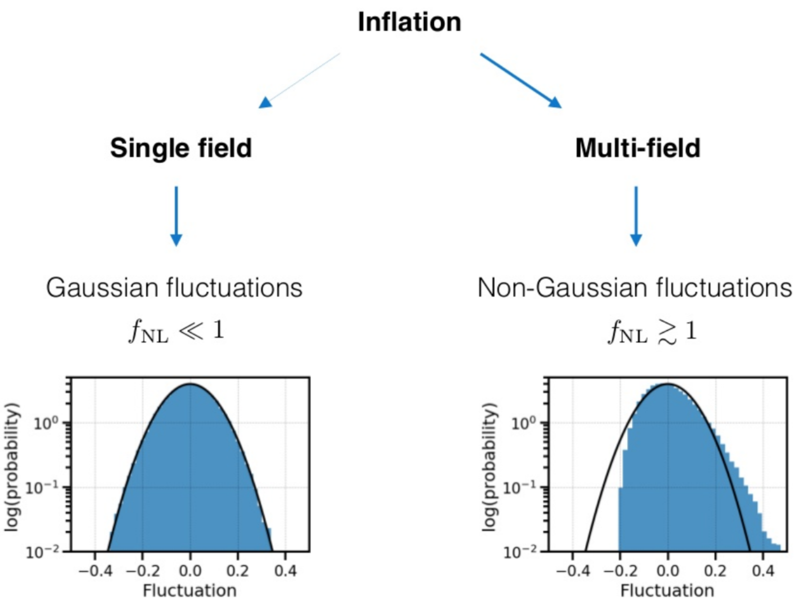
Single field predicts $f_{\rm NL} \simeq \frac{5}{12}(1 - n_S) \simeq 0.02$. A detection of $f_{\rm NL} \gg 0.02$ rules out all single inflation.

Way out of the theorem:

- Several fields active during inflation Sugiyama 1101.3636
- higher spin Arkani-Hamed 1503.08043
- 'modified' gravity Tahara 1805.00186
- anisotropic inflation Emaml 1511.01683
- electromagnetic field Chua 1810.09815 **Stahl** 1507.01686

These theorems also apply to the late universe (Creminelli 1309.3557)
$\rightarrow$ probe the early universe with LSS observables.

Conclusions



Slide from M. Schmittfull.

Conclusions

Motivations

- While most of LSS do not need relativity, the bispectrum couples scales, its non-linear evolution has to be calculated within GR.
- The bispectrum in the squeezed limit is 'protected' from astrophysical effects (equivalence principle)
- In LSS, the bispectrum can be used to probe early universe physics.
- The next generation of LSS experiments should be able to measure $f_{\mathrm{NL}} = \mathcal{O}(1)$.

State of the art

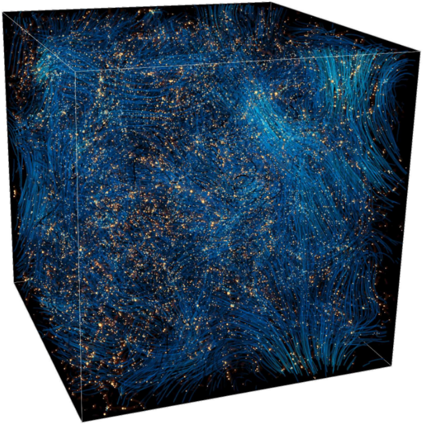
- Several groups, from an analytical point of view, have tried and computed relativistic effects for LSS: Matarrese 9707278, Boubekeur 0806.1016, Fitzpatrick 0902.2814, Bonvin 1105.5280, Jeong 1107.5427, Pajer 1305.0824, Yoo 1406.4140, Gallagher 1807.01655, **Castiblanco** 1811.05452, Erschfeld 2005.12923, Maartens 2011.13660, Castorina 2106.08857.
- Difficult to compare the intermediate steps because of gauge freedom beyond linear order. Need to compute a (gauge-invariant) observable, eg. number count.
- State of the art in LSS often relies on a numerical approach (N-body + emulators)

1. Introduction and Motivations
   - Relativistic structure formation
   - What is the bispectrum?
   - Why the bispectrum?

2. Relativistic corrections in N-body simulations
   - `Gevolution`
   - `RELIC`
   - Toward a fully relativistic pipeline

Gevolution: a general relativistic N-body code.

Adamek 1604.06065 ; https://github.com/gevolution-code/gevolution-1.2



Spin one metric perturbation
Image credit: https://youtu.be/9y6T5CoZgi4

Based on weak field expansion of general relativity.

For any given $T_{\mu\nu}$ computes the six degree of freedom of GR: ϕ, ψ, ω_i, h_{ij}.

N-body particles ensemble evolved using relativistic geodesic equation

Successfully applied to dark energy, backreaction, ray tracing, radiation and neutrinos.

Gevolution in its vanilla form uses linear initial condition

Not enough for non-gaussianities and relativistic corrections.
Non-linear initial conditions adds several layer of complication: aliasing of fields, computationally expensive, transiants...

RELIC (RElativistic second-order Initial Conditions)

Goal

We want to plug SONG[a] (the second order generalization of CLASS[b]) to gevolution. SONG agrees with analytic estimates *eg.* **Castiblanco** 1811.05452.

[a] Second Order Non-Gaussianity
[b] the Cosmic Linear Anisotropy Solving System

Generating the most general non-gaussian field is costly!

Beyond local Anzatz such as $\delta^{(2)} \sim \delta^{(1)} + f_{NL}\delta^{(1)}\delta^{(1)}$, we want:

$$\delta^{(2)}(\mathbf{k}, t) = \int \frac{d\mathbf{k_1}}{(2\pi)^3} F_2(k_1, |\mathbf{k} - \mathbf{k_1}|, t)\delta^{(1)}(\mathbf{k_1})\delta^{(1)}(\mathbf{k} - \mathbf{k_1}), \tag{3}$$

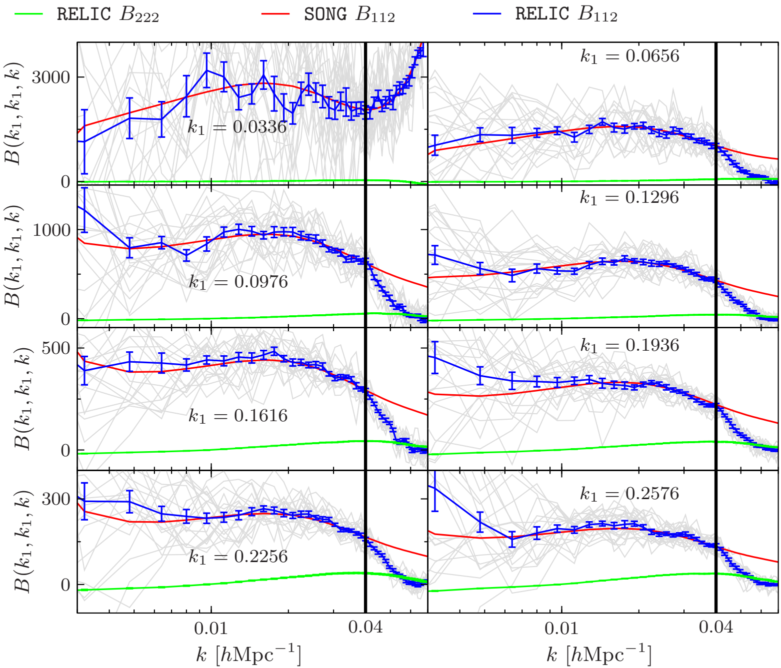
for any F_2. Need for each **k** to know the value of $\delta^{(1)}(\mathbf{k_1})$ and $\delta^{(1)}(\mathbf{k} - \mathbf{k_1})$. If we work on a grid with N points $\rightarrow$ scales as N^6, a lot!

Adamek 2110.11249: We ignore small scales correlations

Introduce a cutoff scale k_Λ which splits small and large scales: $\delta^{(1)}(\mathbf{k}) = \delta^{(1)}_{\text{small}}(\mathbf{k}) + \delta^{(1)}_{\text{large}}(\mathbf{k})$, plug into (3) and ignore the small $\times$ small contribution. $\rightarrow$ will give the right correlations at larges scales and in the squeezed limit for the bispectrum but cannot be trusted in the general case. Scales as N^3.

Results

$N = 512$, z=100, Planck cosmology, BoxSize=4 Gpc/h, $k_\Lambda = 0.04\ h$/Mpc

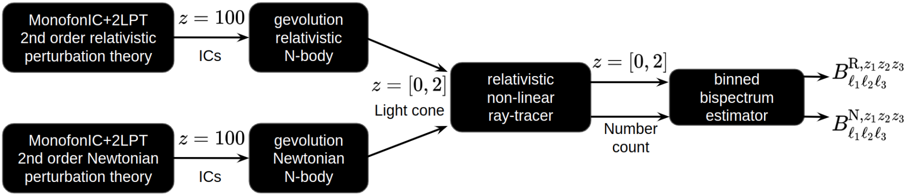


Merging RELIC with Monofonic

Montandon 2212.xxxxx

- Monofonic[a] (evolution of MUSIC) is a popular IC generator. Several attractive in-build features, such as 3LPT and a de-aliasing module (based on Orszag's $3/2$ rule), Michaux 2008.09588.
- Instead of SONG, use analytic estimate (1 % accurate) from Tram 1602.05933.

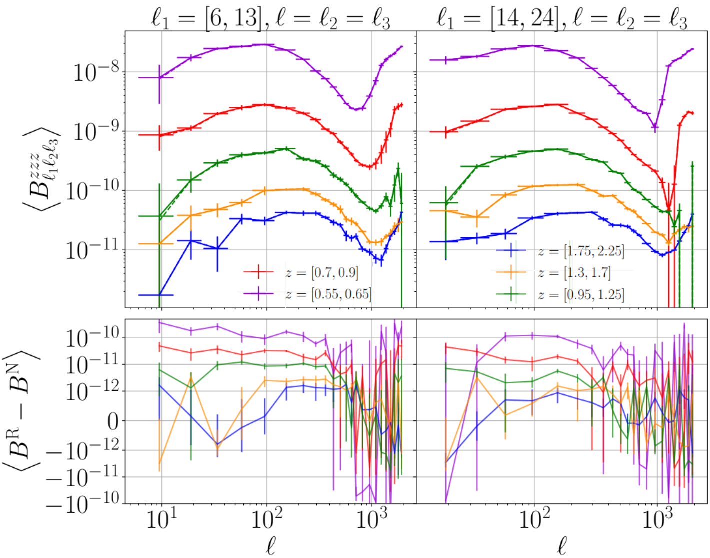
[a] https://bitbucket.org/ohahn/monofonic



Modified the binned bispectrum estimator (Bucher 0911.1642) used for Planck. Currently only two redshift bins are supported (equivalent of polarization and temperature).

Results

$N = 1024$, Planck cosmology, BoxSize=7.9 Gpc/h



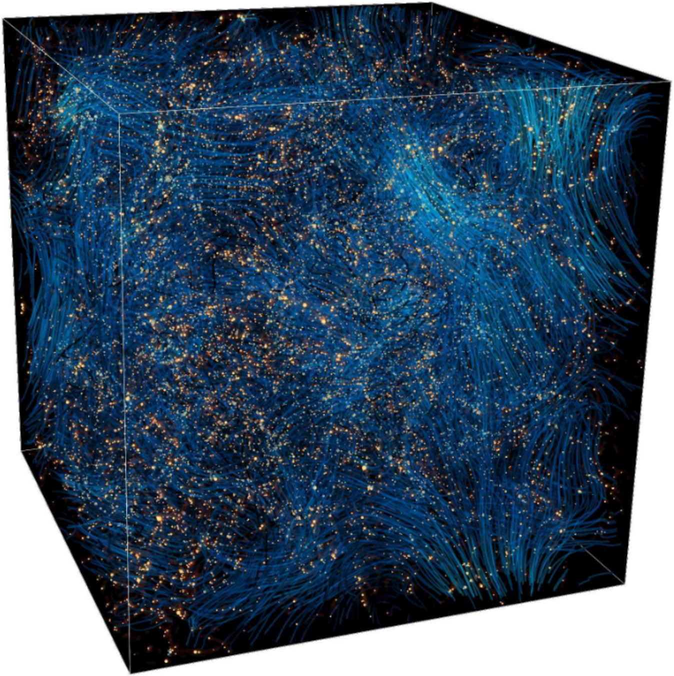
- For $\ell \in [100; 600]$, detection of the relativistic bispectrum from 2 to 4 σ.
- Relative contribution of relativistic wrt to Newtonian bispectrum: 1 to 6 %.

Conclusions and next steps

- The bispectrum couples scales: GR+non-linear physics required to model it.
- The bispectrum in the squeezed limit is protected from astrophysics and allow to probe early universe physics (primordial non-gaussianities)
- The relativistic contributions are of the same order than a primordial non-gaussianity of the local type.

- We used state of the art tools to develop a fully relativistic pipeline to predict the bispectrum in redshift space.
- 'Detection' of relativistic effects from 2 to 4 σ.
- Next steps: include also PNG and try to detect them.
- Include baryons, or at least biased tracers (**Calles** 1912.13034)
- More correlation of redshift bins
- Is any of these relevant for Euclid consortium?

Thank you for your attention



General Relativity: diffeomorphism invariance

Perturbations around a FLRW universe

$$ds^2 = -(1+2\phi)dt^2 + 2\omega_i dx^i dt + a(t)^2\left[(1-2\psi)\delta_{ij} + h_{ij}\right]dx^i dx^j . \qquad (4)$$

Poisson gauge

- $\delta^{ij}\omega_{i,j} = \delta^{ij}h_{ij} = \delta^{jk}h_{ij,k} = 0.$
- Velocity of the fluid:

$$u^\mu = \left(1 - \phi + \frac{a^2v^2}{2}, v^i\right).$$

- Physical interpretation simple.
- Gauge used for relativistic N-body simulations gevolution (Adamek 1604.06065).

Synchronous-Comoving gauge

- $\delta^{ij}h_{ij} = \delta^{jk}h_{ij,k} = 0$ and $u^0 = 1$.
- Velocity of the fluid:

$$u^\mu = \left(1, -\frac{(1+2\psi)\partial_i\omega + w_i}{a^2(t)}\right),$$

where $\omega_i \equiv \partial_i\omega + w_i$.

- Gauge relevant when it comes for observation: use the time measured by a local observer.

Weak Field Approximation

Typically $v \sim 10^{-2}$, $\phi \sim 10^{-5}$, but: $\delta = \frac{2}{3(aH)^2}k^2\phi \sim \frac{0.1\text{Mpc}^{-1}}{10^{-6}\text{Mpc}^{-1}}\phi \sim 1$.
$\rightarrow$ Work perturbatively in v and ϕ but full non-linear in δ.

Equation of motion

Conservation of the energy momentum tensor + Einstein equation

$$\nabla_\mu(\rho u^\mu) = 0, u^\mu \nabla_\mu u^\nu = 0, G_{\mu\nu} = T_{\mu\nu}. \tag{5}$$

Full non-linear equations: Euler + conservation of mass

$$\dot{\delta} + \theta = -\int_{\boldsymbol{k}_1,\boldsymbol{k}_2} (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}_{12}) \alpha(\boldsymbol{k}_1, \boldsymbol{k}_2) \theta(\boldsymbol{k}_1) \delta(\boldsymbol{k}_2) + \mathcal{S}_\delta[\delta, \theta],$$

$$\dot{\theta} + 2H\theta + \frac{3H^2}{2}\delta = -2\int_{\boldsymbol{k}_1,\boldsymbol{k}_2} (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}_{12}) \beta(\boldsymbol{k}_1, \boldsymbol{k}_2) \theta(\boldsymbol{k}_1) \theta(\boldsymbol{k}_2) + \mathcal{S}_\theta[\delta, \theta].$$

$\theta \equiv \partial_i v^i$. Use G_i^0 to include frame dragging effects (ω_i) and G_0^0 for potentials ϕ, ψ. $S_{\delta/\theta}$ are the relativistic corrections: eg. $\sim \dot{\delta}\delta/k^2$.

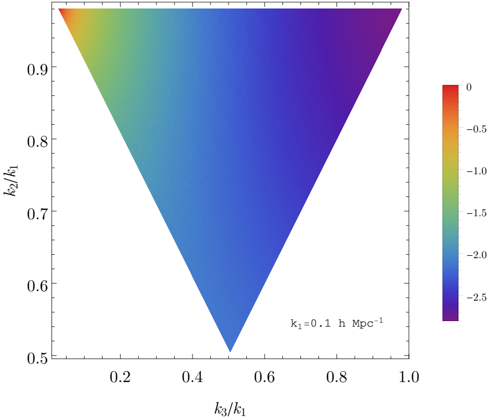
Perturbation theory: take $\delta \ll 1$

$$\delta(\boldsymbol{k}, t) = \sum_{n=1}^{\infty} a^n(t) \int_{\boldsymbol{k}_1..\boldsymbol{k}_n} \Big[F_n(\boldsymbol{k}_1, .., \boldsymbol{k}_n) + a^2(t) H^2(t) F_n^R(\boldsymbol{k}_1, .., \boldsymbol{k}_n) \Big] \delta_l(\boldsymbol{k}_1)..\delta_l(\boldsymbol{k}_n) .$$

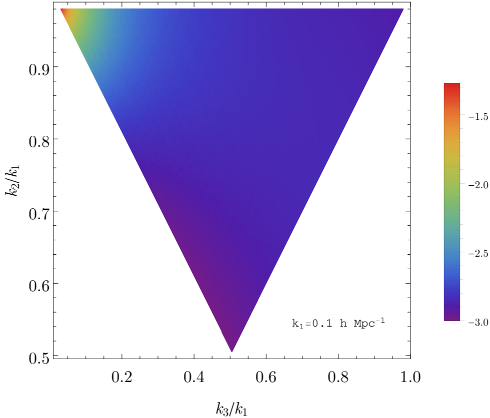
The end !

We plot $\frac{B^R(k_1,k_2,k_3)}{B(k_1,k_2,k_3)}$ (1-loop=stopping at $n = 4$ in perturbation theory).

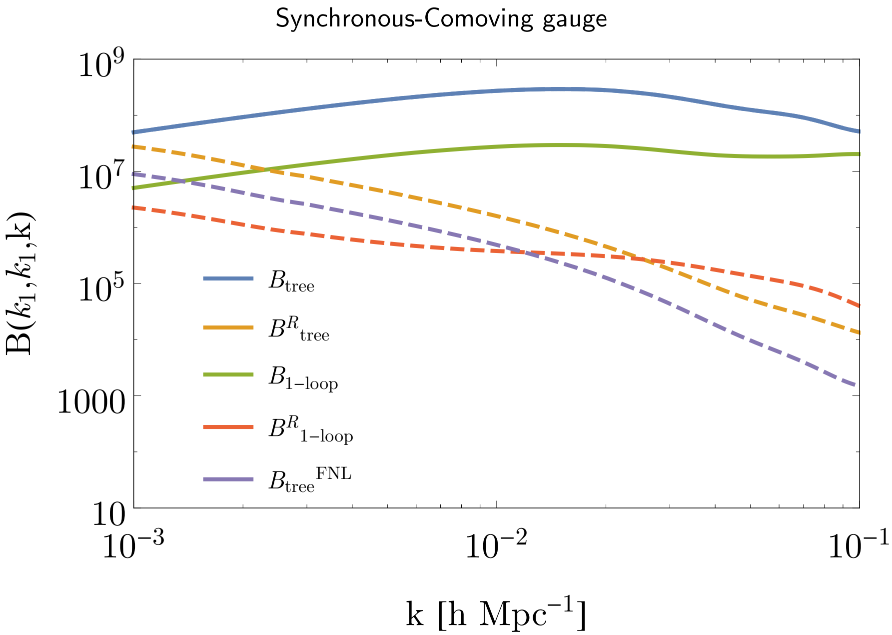
Poisson gauge



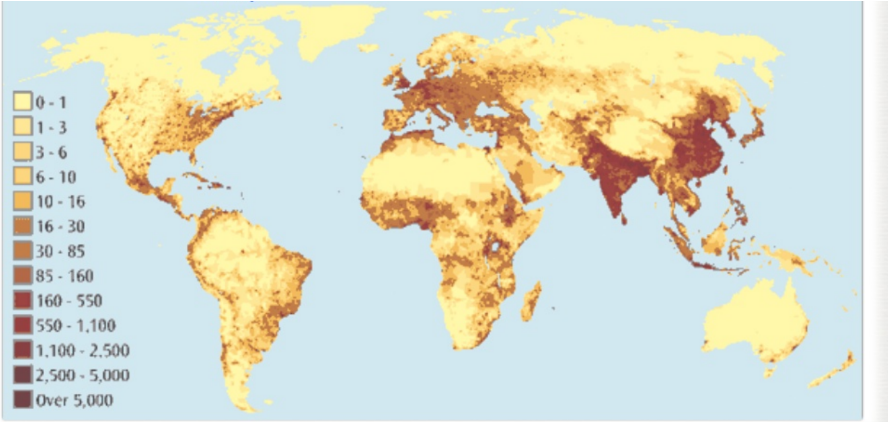
Synchronous-Comoving gauge



Results


Synchronous-Comoving gauge
10^9
10^7
10^5
1000
10
10^{-3}
10^{-2}
10^{-1}
$B(k_1,k_1,k)$
k [h Mpc^{-1}]
B_{tree}
${B^R}_{\text{tree}}$
$B_{1\text{-loop}}$
${B^R}_{1\text{-loop}}$
${B_{\text{tree}}}^{\text{FNL}}$

Human population density



Adopted from Tobias Baldauf

At night

Adopted from Tobias Baldauf

On which geometric quantities the formation of galaxies depends?

Working frame

- Approach à la Effective field Theory: smaller scales are smoothed out and the astrophysical processed are encoded in a handful of *bias* coefficients $b_{\mathcal{O}}$ to be determined (Desjacques 1611.09787).
- Frame of reference of an observer moving with the halo's center of mass ($\rightarrow$ Synchronous-comoving gauge).
- Velocity of dark matter = velocity of halos/galaxies.
- No creation of galaxies.

Build on Umeh 1901.07460, generalized to 4th order:

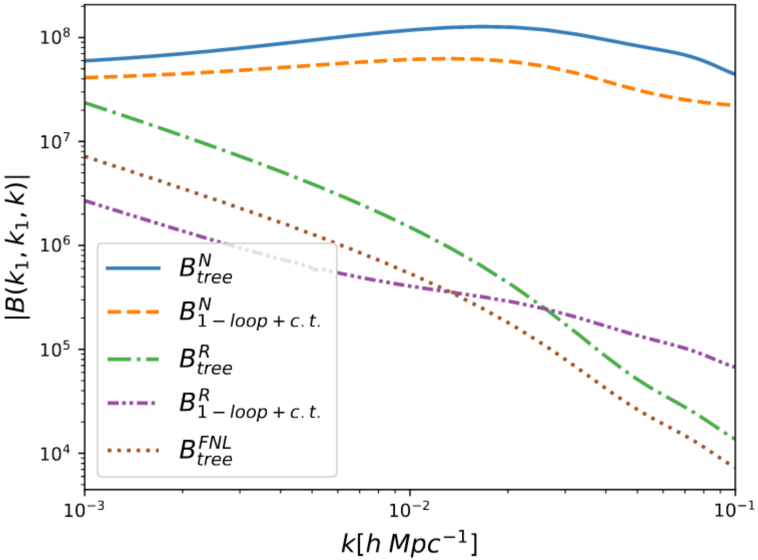
$$\delta_g^{(n)} = a^n \left(F_n^T + \sum b_{\mathcal{O}}^{\mathcal{L}} M_n^{\mathcal{O}} \right) \delta_\ell^n \, , \tag{6}$$

where $F_n^T \equiv F_n + a^2(t)H^2(t)F_n^R$ and $M_n^{\mathcal{O}} \equiv M_n^{\mathcal{O}} + a^2(t)H^2(t)M_n^{\mathcal{O},R}$

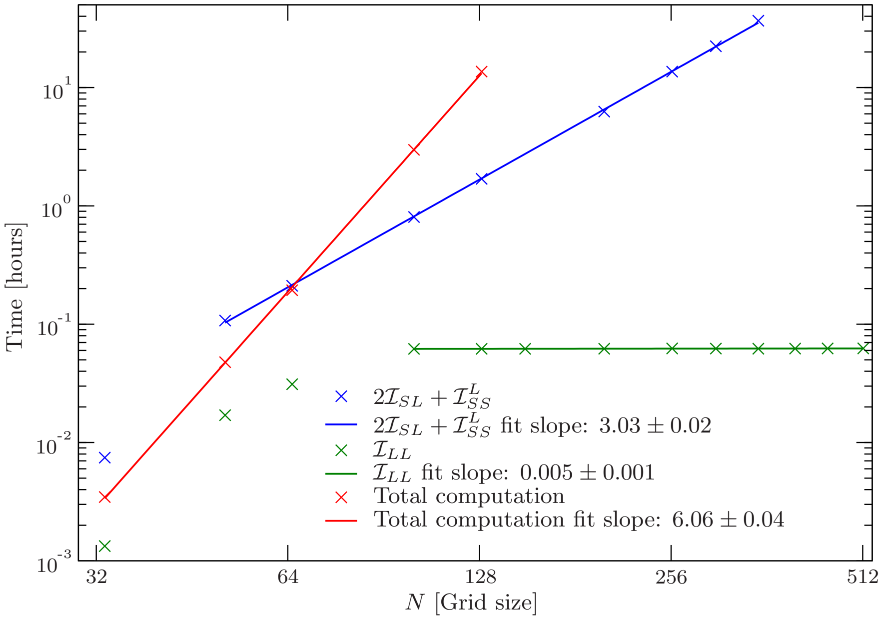
at second order ($n = 2$), we find $\mathcal{O} = \{\delta; \delta^2; s^2\}$ such that: (**Calles** 1912.13034)

$$\delta_g^{(2)} = a^2 \left[\left(1 + \frac{b_1}{a}\right) F_2^T + \frac{1}{2}\left(\frac{b_2}{a^2} - \frac{4}{21}\frac{b_1}{a}\right) + \left(\frac{b_{s^2}}{a^2} - \frac{2}{7}\frac{b_1}{a}\right) s^2 \right] \delta_\ell^2 \, , \tag{7}$$

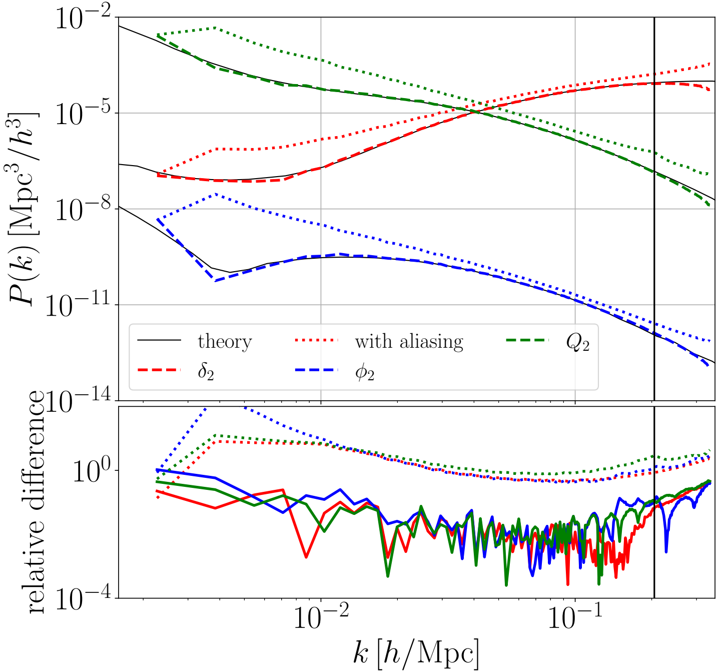
Results


$|B(k_1, k_1, k)|$
10^8
10^7
10^6
10^5
10^4
10^{-3}
10^{-2}
10^{-1}
$k[h\ Mpc^{-1}]$
B^N_{tree}
$B^N_{1-loop+c.t.}$
B^R_{tree}
$B^R_{1-loop+c.t.}$
B^{FNL}_{tree}

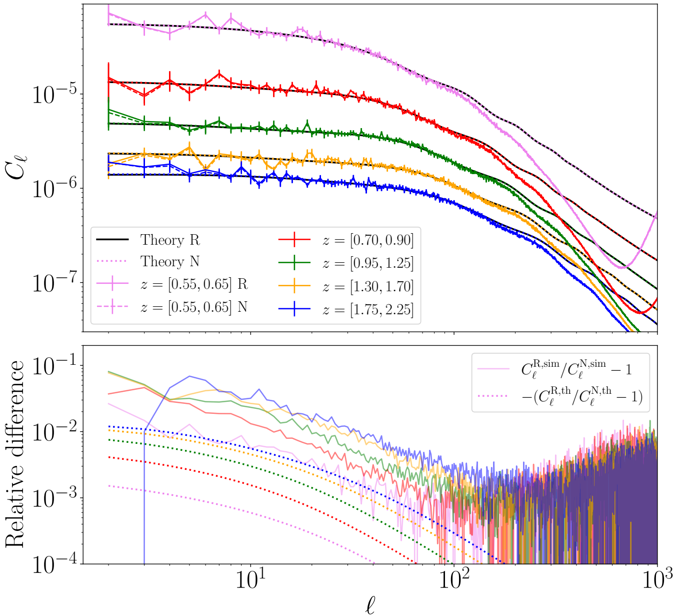
Time complexity of RELIC



De-aliaising



Angular power spectrum



for $\ell < 10$, up to an error of 10%, relativistic effects dominate with a scaling of ℓ^{-1}.