Testing likelihood accuracy for cluster count cosmology

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Talk based on arXiv:2210.11093 (11/2022)

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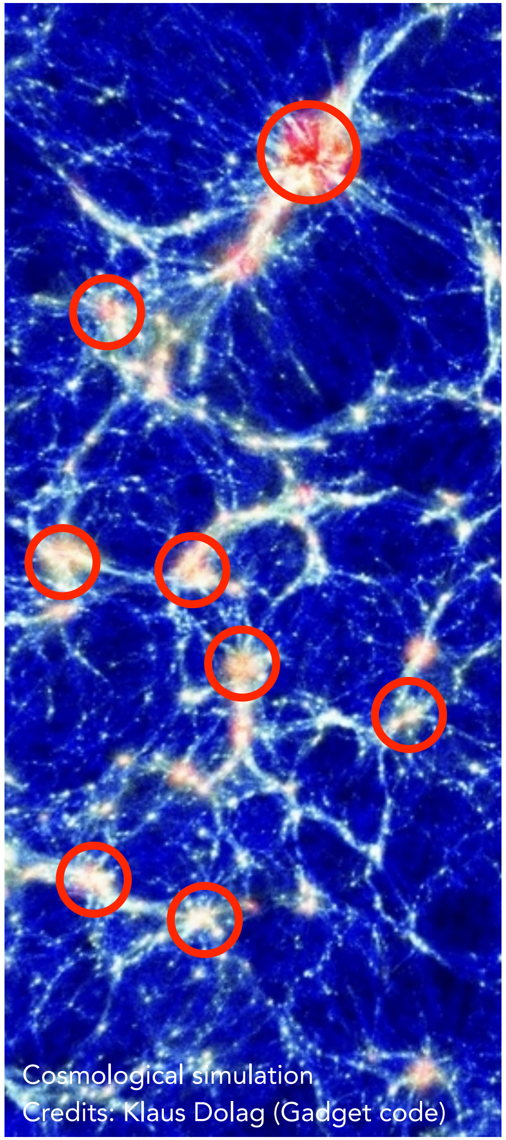
Euclid-France Theory and Likelihood Workshop - 28/11/2022

Outline

- **Describe the latest evolution of the Universe**
  - Most massive bound systems with $M \in 10^{13} - 10^{15}\ M_{\odot}$
  - $z < 2$, last step of hierarchical structure formation process
  - Formed from the growth of small density inhomogeneities
  - By the accretion and merging of smaller structures

- **Good candidates to trace the matter content in the universe**
  - Multi-component systems
    - Dark matter (~80%) and baryonic matter
    - Multi-wavelength objects (optical, near-IR, mm, X-ray)
    - Laboratories to study the co-evolution of the dark and the baryonic matter
  - At larger scales, lie at the intersections of the cosmic web filaments


Cosmological simulation
Credits: Klaus Dolag (Gadget code)

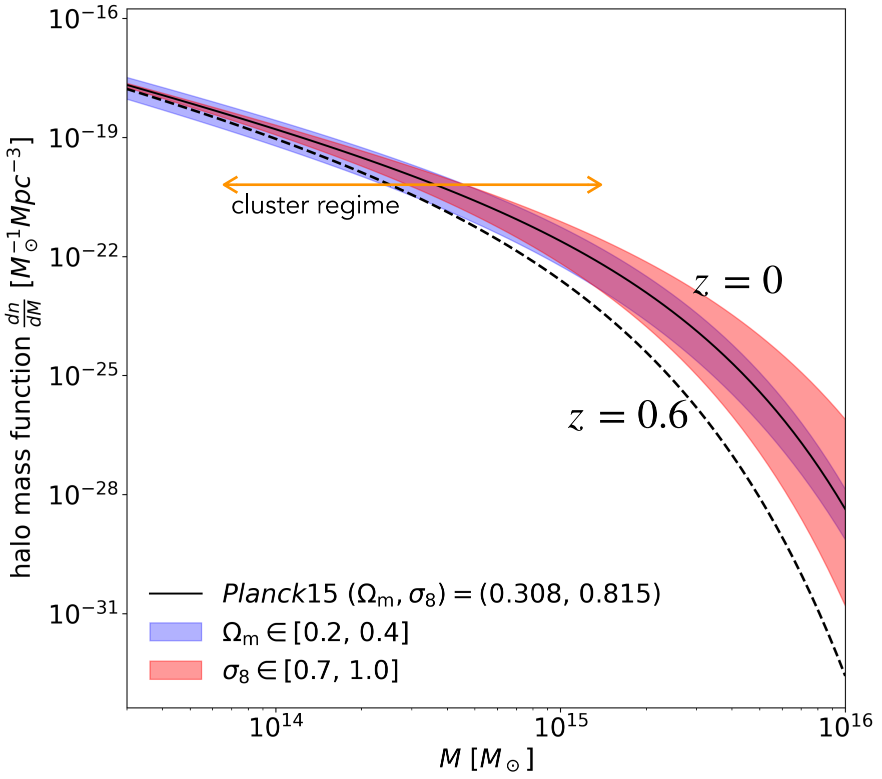
Cosmology with galaxy cluster abundance

The abundance of galaxy clusters

- Geometry + growth of structures in the Universe
- Count clusters a function of redshift and mass

$$N_{\rm th} = \Omega_s \int_{z_1}^{z_2} dz \int_{m_1}^{m_2} dm \; \frac{dn(m,z)}{dm} \frac{d^2V(z)}{dz d\Omega}$$

- Depends on:
  - **Halo Mass Function**
    - Matter content $\Omega_{\rm m}$
    - To the amplitude of matter density fluctuation σ_8
    - Formation history : growth rate over cosmological time $\sigma_8(z)$
  - **Volume**
    - Background cosmology
    - $z < 2$, sensitive to late time expansion history of the Universe, led by dark energy e.o.s



Cosmology with galaxy cluster abundance

- **Cluster abundance**
  - Probes ΛCDM (Ω_m, σ_8) as well as extensions
    - wCDM
    - Massive neutrinos $\sum m_\nu$
    - Primordial Non-Gaussianity
    - Testing gravity on large scales → modified gravity scenarios, …
- **An order of magnitude in observed clusters with next-generation surveys**
  - From 10^{3-4} to 10^5 clusters
  - Increase in size + in depth
  - Large statistical power + need of significant improvement in control of systematics (e.g. synergy space/ground experiments for WL mass calibration)

Cluster abundance cosmology overview

Surveys	Start	Wavelengths	CL Analysis	Number of clusters
ACT	2007	mm	2013	68 (2020: > 4000)
WtG (ROSAT)	2000	X-rays	2014	224
Planck	2009	mm	2015	439 (all: 1653)
SPT	2007	mm	2016	343
SDSS	2000	Visible	2019	25 000
KiDs	2011	Visible	2020	3 652
DES	2013	Visible	2020	7 000
eROSITA	2019	X-rays	2022	455 (all: 100 000)
Rubin LSST	2023	Visible		> 100 000
Euclid	2023	Visible, near IR		> 100 000
S0	2023	mm		16 000
WFIRST	2026	Visible, near IR		40 000
CMB-S4	2029	mm		100 000
Roman	2027	Blue, near IR		23 000

Cosmology with galaxy cluster abundance

Basic recipe for cluster abundance cosmology

- From a galaxy cluster survey with known redshifts, masses
- Count the number $\vec{N}_{\rm obs}$ of galaxy clusters within bins of redshift and mass
- Posterior of cosmological parameter

$$p(\vec{\theta}\,|\,\vec{N}_{\rm obs}) = \pi(\vec{\theta})\,\mathscr{L}(\vec{N}_{\rm obs}\,|\,\vec{\theta})$$

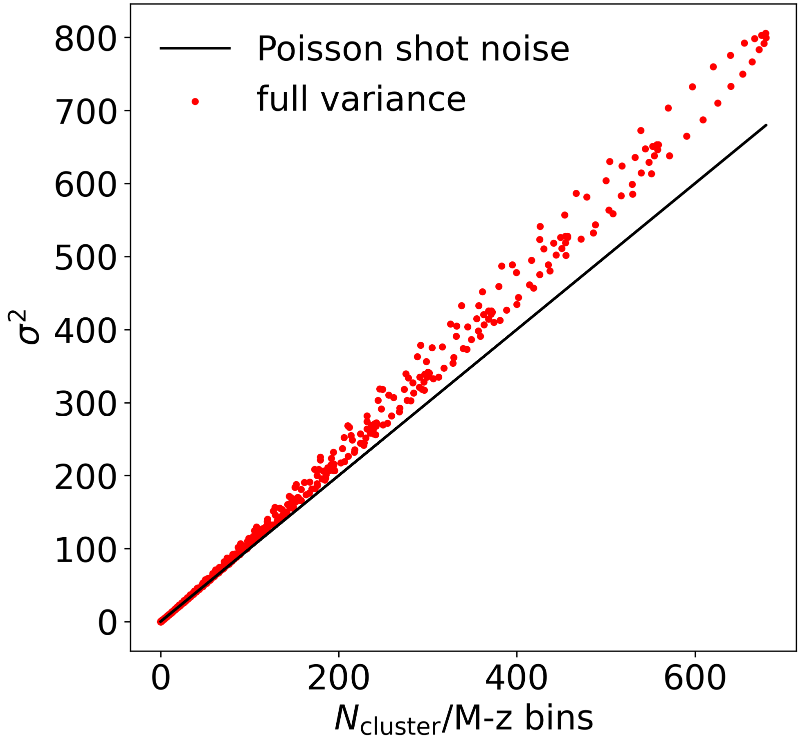
Likelihood = $\mathscr{L}(\vec{N}_{\rm obs}\,|\,\vec{\theta})$

- $\vec{N}_{\rm th}$ at arbitrary cosmology
- Statistics
  - Count of discrete objects in bins → Poisson sampling
  - Fluctuation + clustering of the matter density field → Gaussian contributions
  - Non-linear physics of halo formation → More complications

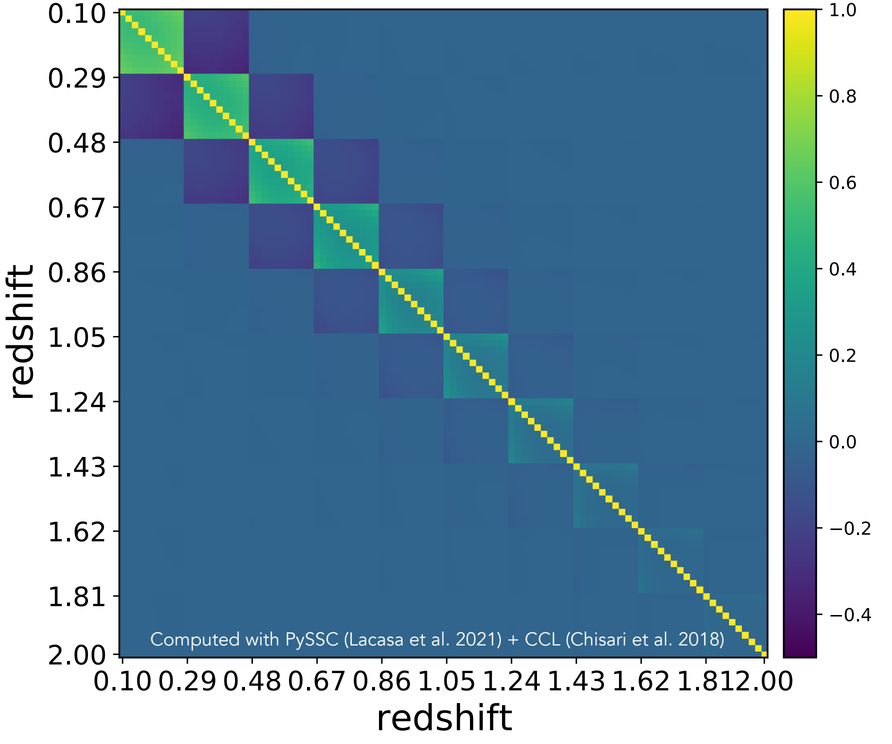
Clustering + fluctuation of matter density field (within/beyond survey volume) = sample covariance

Additive variance

$$\sigma^2(\widehat{N}) = N + \sigma^2_{\text{sample}}(N)$$



correlation between M-z bins



Likelihoods for cluster count cosmology

Likelihoods

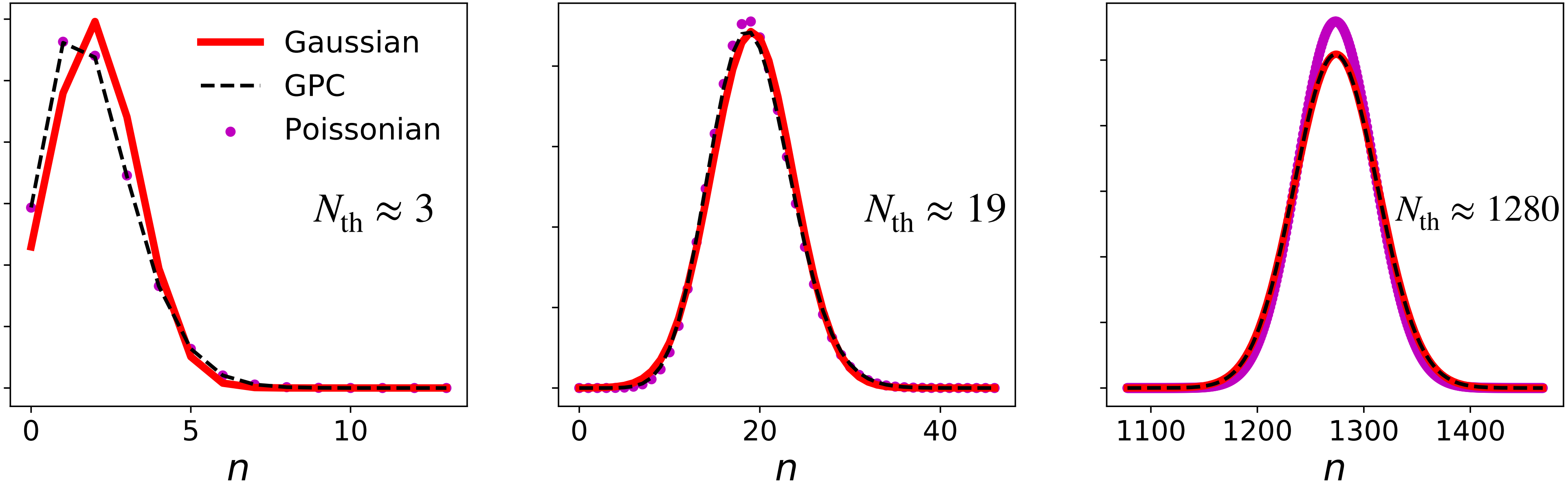
- Ideally should describe completely abundance statistics
- There exist approximations

  - **Poisson likelihood** (Planck, 2015 ~ 500 clusters)
    - Accounts for Poisson sampling
    - Does not account for sample covariance
    - Valid for low number of clusters, Shot Noise > Sample variance

  - **Gaussian likelihood** (DES, 2021 ~ 7000 clusters)
    - Sample covariance
    - Limited to continuous approximation
    - Valid for high number of clusters, Shot Noise ~ Sample variance

  - **Gauss-Poisson Compound** (GPC) (KiDS, 2021 ~ 4000 clusters)
    - Takes into account both Poisson sampling and sample covariance (Hu & Kravtsov, 2003)
    - Computationally expansive to compute
    - Multidimensional integral $\mathscr{L}(\widehat{N} \mid \vec{\theta}) \propto \int d\vec{x}\ \mathscr{N}[\vec{x} \mid \vec{N}(\theta)] \times \prod_{k=1}^{n} \mathscr{P}[\widehat{N}_k \mid x_k]$
    - More precise, can we gain cosmological information?

Single variate likelihood $\mathscr{L}(n\,|\,N_{\rm th})$



Accuracy of likelihoods for cluster abundance cosmology

*Dodelson, Schneider 2013, Percival et al. 2022
**Sellentin, Heavens 2018 for cosmic shear

Bias on parameter inference

- Deviation from the latent likelihood may bias results
  - Data covariance matrix is incorrect*
    - Inferred posteriors will be incorrect
  - The latent likelihood is not Gaussian**
    - Can shift posteriors

- In our case:
  - Latent likelihood is not Poisson, Gaussian, or Gauss-Poisson Compound
  - Halo model is an approximation

- Most robust constraints with analysis likelihood closest to latent one

Using simulations to test cluster abundance likelihoods

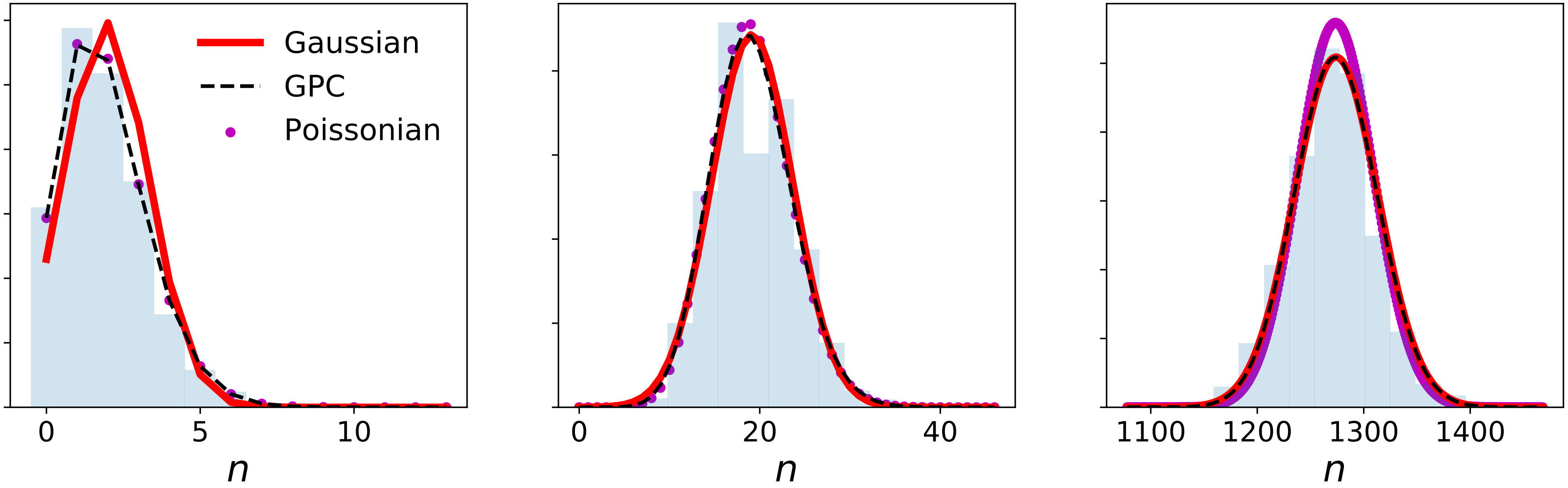
- Likelihood: statistical properties of the data at input cosmology
- With multiples simulations, can have access to "true" statistics of abundance

Outline

We use a set 1000 simulated dark matter halo catalogs

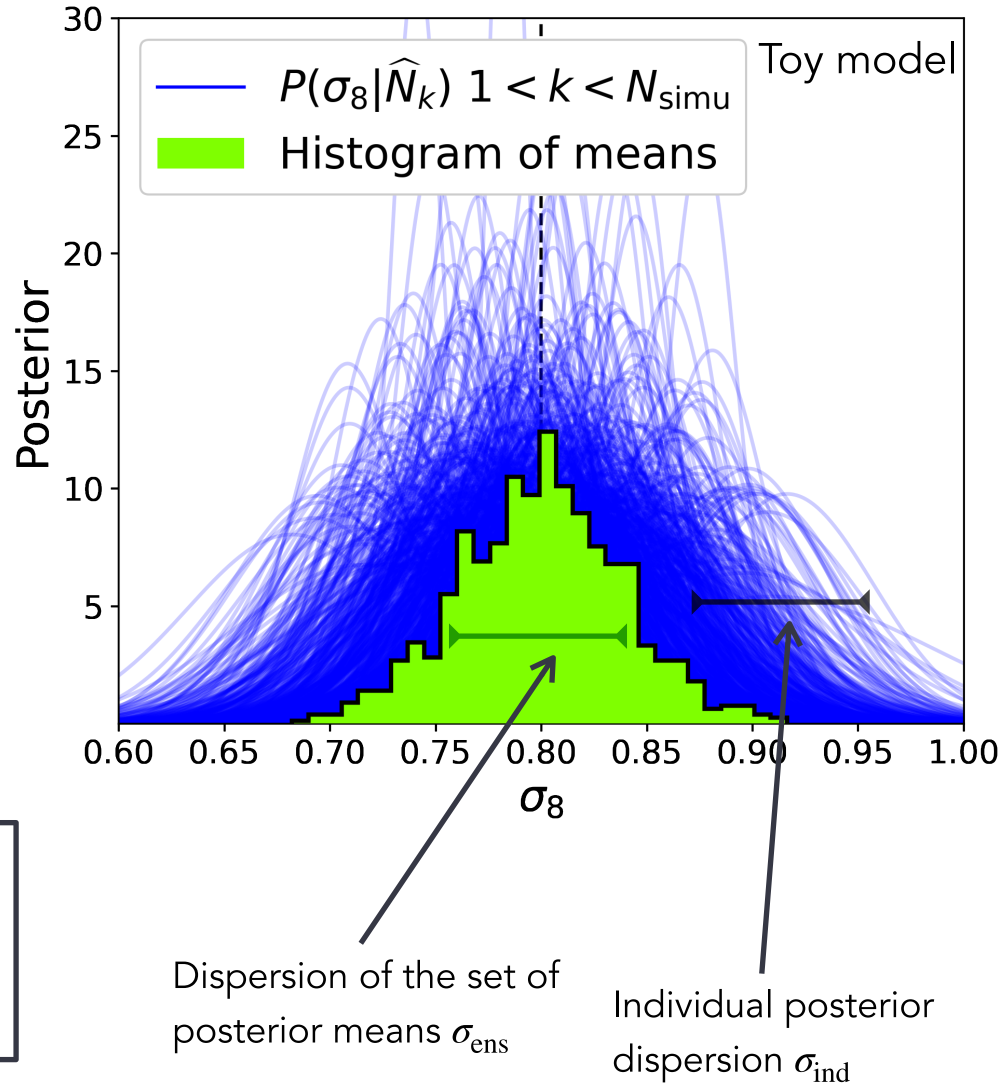
- PINOCCHIO algorithm (Monaco et al., 2013)
- Planck cosmology
- Masses calibrated on known halo mass function (Despali et al., 2015)
- Euclid-like sky area $\sim$ ¼ of full-sky
- $\sim 10^5$ halos per simulation
- $M > 10^{14}\ M_\odot$

Abundance likelihood can be estimated from counts over the 1000 cosmological simulations



Methodology

1. Estimate the posterior for each of the 1000 Pinocchio mocks



More than 1 parameter: compare covariances

$\sigma^2_{\rm ind} \rightarrow C^{\rm ind}$ Individual parameter covariance

$\sigma^2_{\rm ens} \rightarrow C^{\rm ens}$ Ensemble parameter covariance

Why comparing individual errors to the spread of means ?

- **Robust constraints ?**
    - Reasonable request: For each simulation, the recovered error should be representative of the spread of recovered parameters over many realisations of the "Universe"

Gaussian likelihood

- Latent likelihood $\mathscr{L}_X$ and analysis likelihood $\mathscr{L}_Y$
- Two data covariance matrices Σ_X and Σ_Y

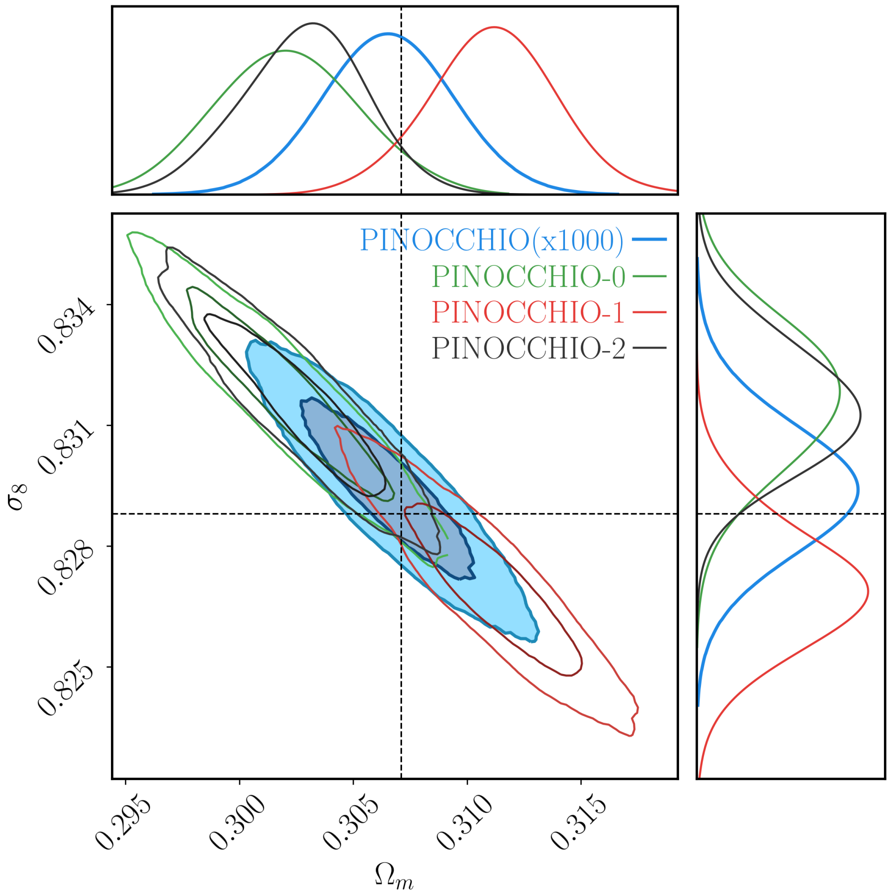
- If $\Sigma_X = \Sigma_Y$
    - Then $C^{\rm ens} = C^{\rm ind}$
    - Likelihood accuracy can be forecasted (Fisher formalism) $\rightarrow C^{\rm Fisher}$

- If $\Sigma_X \neq \Sigma_Y$
    - Then $C^{\rm ens} \neq C^{\rm ind}$
    - $C^{\rm Fisher}$ is not sufficient to forecast likelihood accuracy
    - $C^{\rm ens}$ can be forecasted $C^{\rm ens}_{\alpha\beta} = (C^{\rm Fisher}N_{,})^T_\alpha \Sigma_Y^{-1}\Sigma_X\Sigma_Y^{-1}(C^{\rm Fisher}N_{,})_\beta \neq C^{\rm Fisher}_{\alpha\beta}$
    - Example: $\Sigma_{Y_{ii}} < \Sigma_{X_{ii}}$ then we have ${C^{\rm ind}}_{\alpha\alpha} < {C^{\rm correct}}_{\alpha\alpha} < {C^{\rm ens}}_{\alpha\alpha}$

Using correct likelihood $C^{\rm ens} = C^{\rm ind}$

- Likelihood and posterior are not always gaussians
- Rather closeness between individual errors and ensemble error
- Used as a metric to test likelihood accuracy

Framework for testing the accuracy of likelihoods



Study limited to idealised halos

- Ideal setup: individual "true" masses
- Real data: proxy-selected clusters
- Limited by knowledge of mass-proxy relations + detection efficiency
- Increase error of cosmological parameters

Cosmological inference setup

- The Poisson, Gaussian and GPC likelihood are approximations
- Valid:
    - At linear scales (clusters are biased tracers of the density field)
    - For given shot noise/sample (co)variance relative importance
        - binning scheme of the mass-redshift plane
        - Sky survey area (Shot noise, sample (co)variance $\sim \Omega$)

Methodology: Test accuracy of likelihoods for various regimes

For each likelihood

1. $P(\Omega_m, \sigma_8 \,|\, \vec{N}_{\rm obs})$ for each PINOCCHIO simulation
2. For 3 binning schemes

	Redshift bins	Mass bins	# of bins	Average # N/bin
#1	4	4	16	5000
#2	20	30	600	150
#3	100	100	10 000	10

Poisson sampling ↓ Sample Variance ↑

$\sim 10^4$ cosmological constraints ! **Importance sampling (efficient for 2 parameters)**

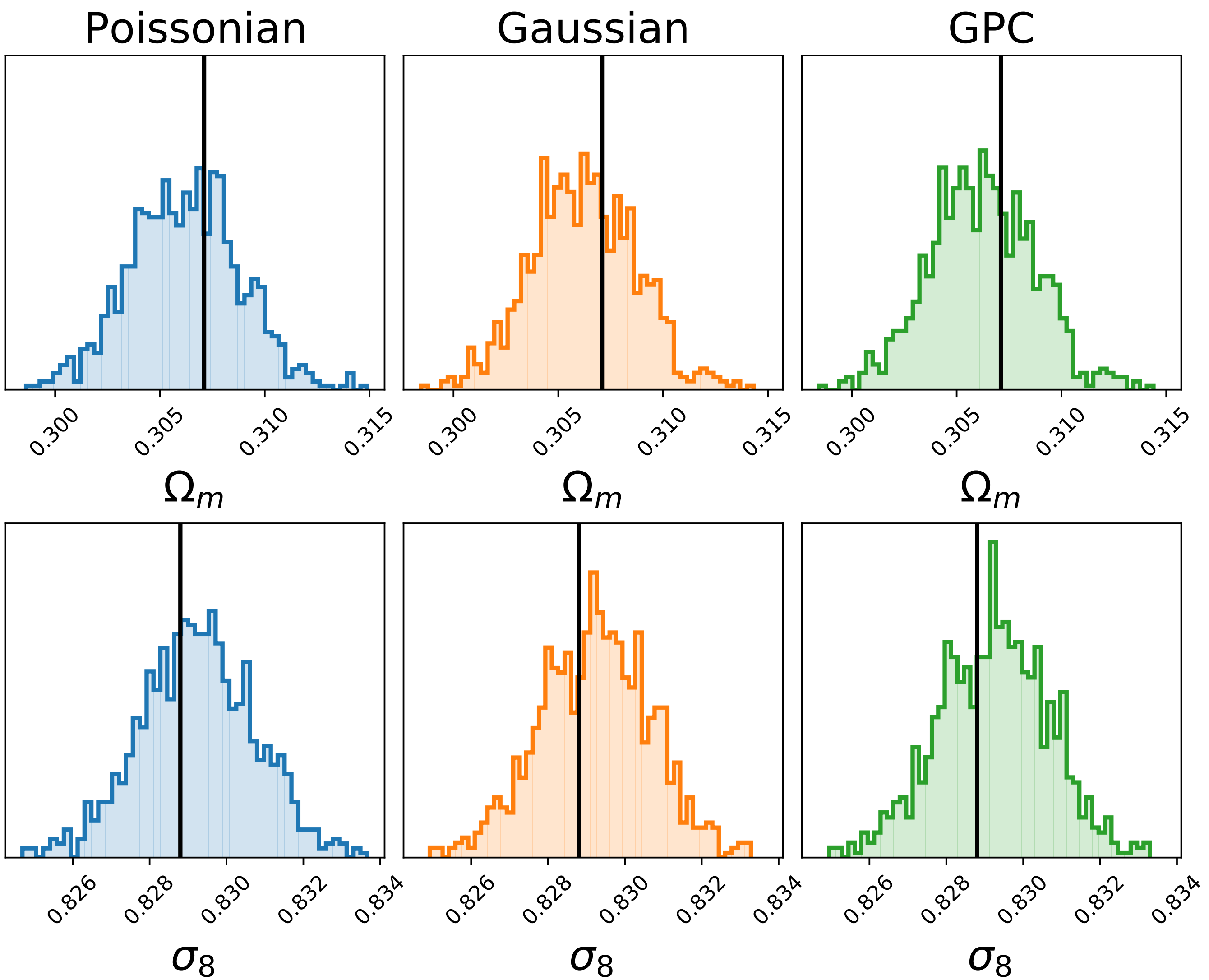
Outline

1. **Introduction**
    1. Cosmology with galaxy cluster abundance
    2. Likelihoods for cluster count cosmology
2. **Framework for testing likelihood accuracy**
    1. The 1000 PINOCCHIO simulations
    2. The methodology
3. **Results**
4. **Conclusions**

Results: (4 redshift bins)x(4 mass bins) case

Histograms of 1000 means

- Scatter around input cosmology
- Validate the modelling input

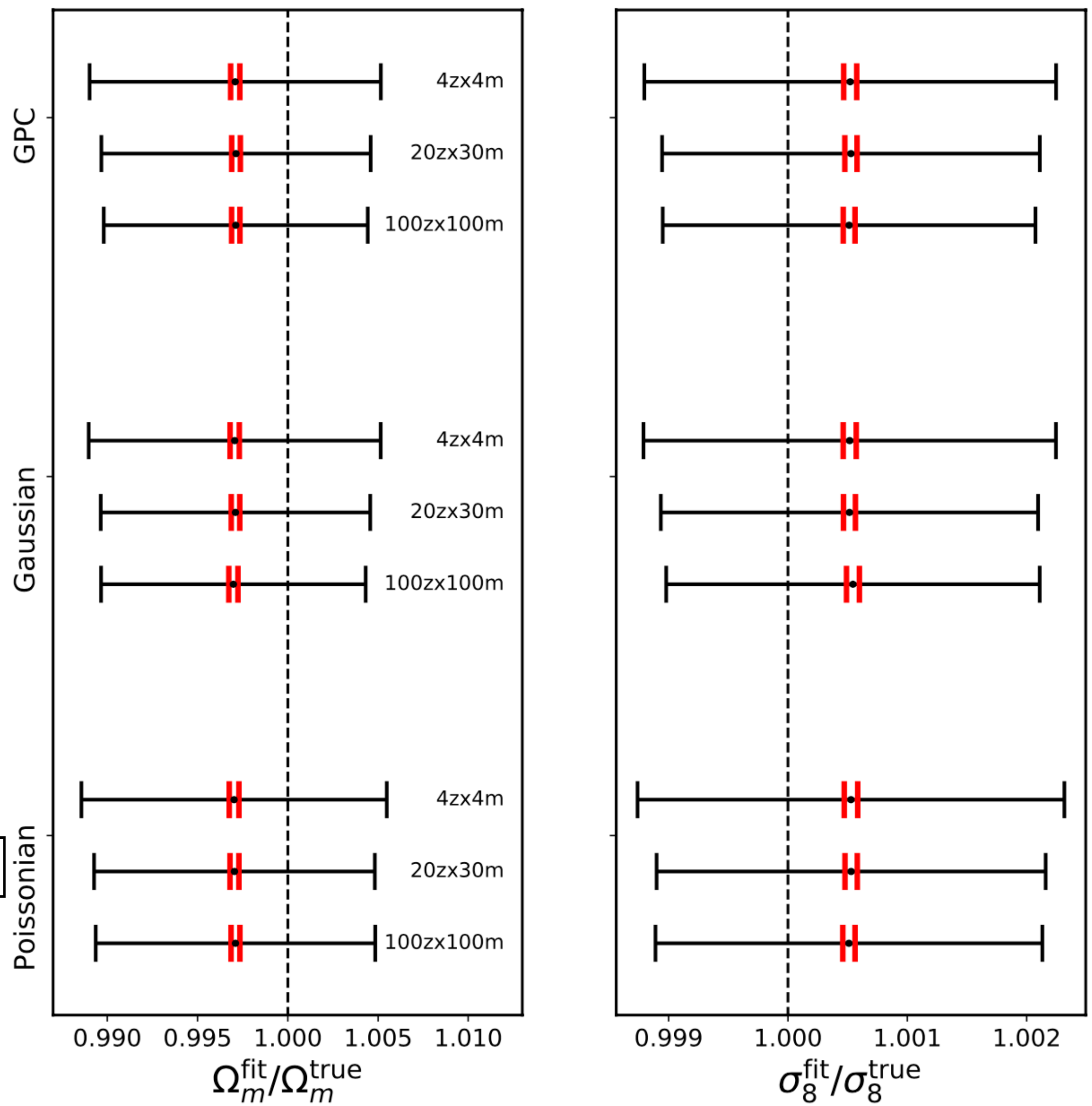


Bias on the posterior mean

(black) Spread of posterior means
(red) Error on the mean ($\times 1/\sqrt{1000}$)

- Small constant bias between input and recovered cosmology
    - Accuracy of the underlying halo model
    - Not due to 2-point statistics (Poisson does not depends on it)
    - Numerical error

Small bias on recovered cosmology (sub-percent)



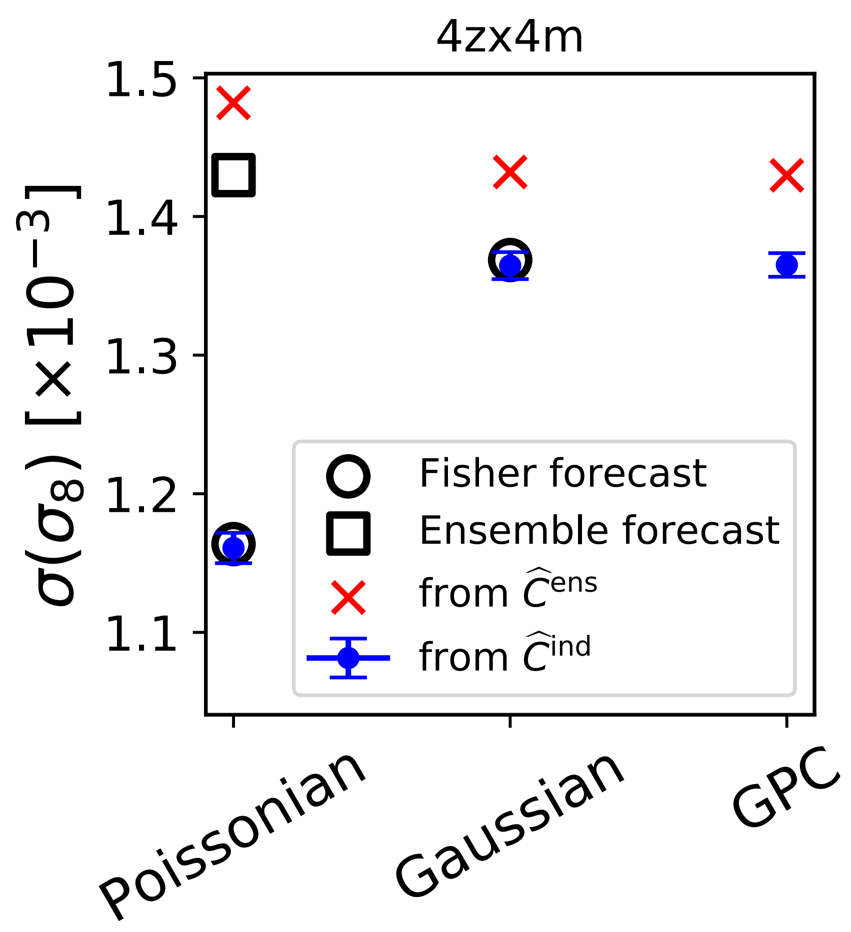
Results: (4 redshift bins)x(4 mass bins) case

- Individual errors on each simulation (blue)
- Spread of best fits (red)

Parameter error

- Poisson underestimates the errors, since it not take account of sample variance
- Gaussian = Gauss-Poisson Compound
  - Slightly underestimate errors, likely due to approximations made for the 2-pt statistics
  - The same level of constraints

- Fisher forecasts (circle) in agreement with individual errors
- Ensemble forecast (square) for the spread of posterior means



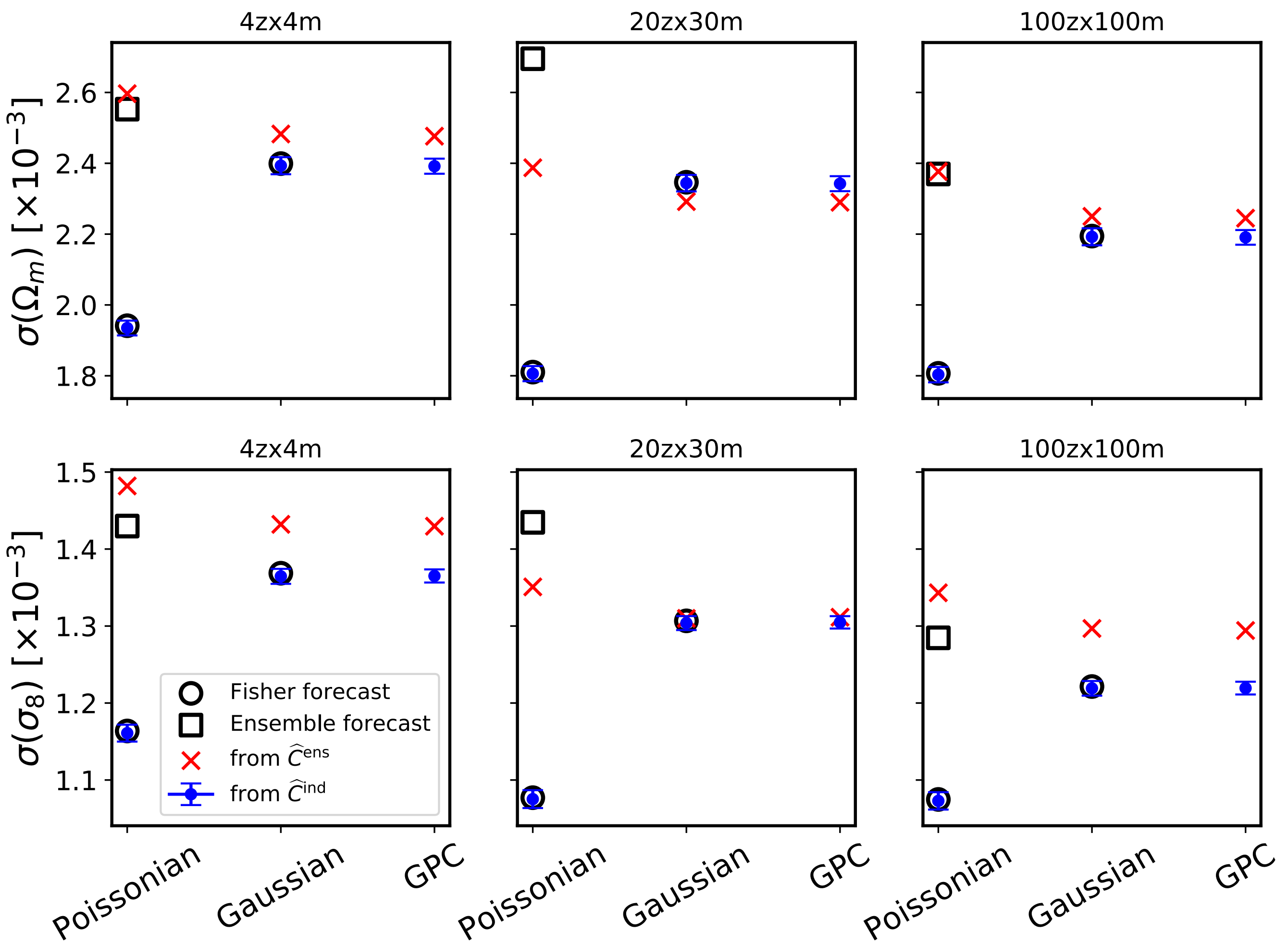
Results: all binning schemes

Parameter error

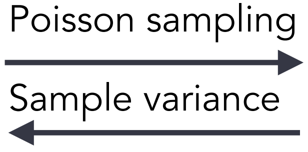
- Errors decreases with the number of bins (10% improvement from 16 to 10^4 bins)

- **Poisson**
  - Underestimates the error, even for fine binning, does not account for sample variance

- **Gaussian = Gauss-Poisson Comp.**
  - Over/under estimate constraints (approximation for computing the covariance matrix)
  - The same level of constraints



Gaussian likelihood remains an accurate description of the data



Pushing toward the Poisson regime?

For all binning setups
- Gaussian is more accurate than Poisson
- Gaussian misses Poisson sampling

Find where Poisson and Gauss-Poisson Compound are valid and the Gaussian is not valid ?

- Force shot noise dominant regime
  1. Use only high mass halos $M > 5.10^{14}\ M_\odot$
  2. Reducing survey sky area $\times\, 1/10$

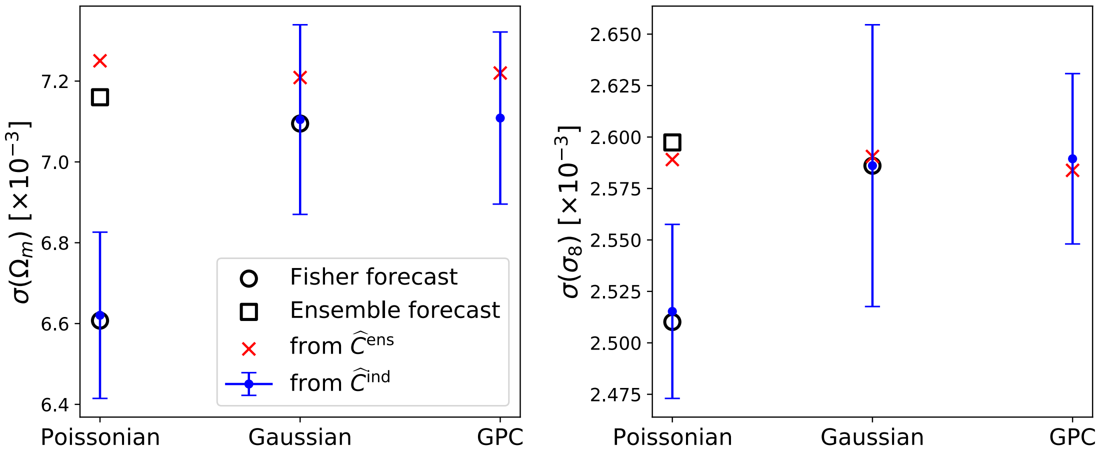
1. High mass sample

Poisson underestimates error by 20 - 30 %
GPC and Gaussian: closer to ensemble error

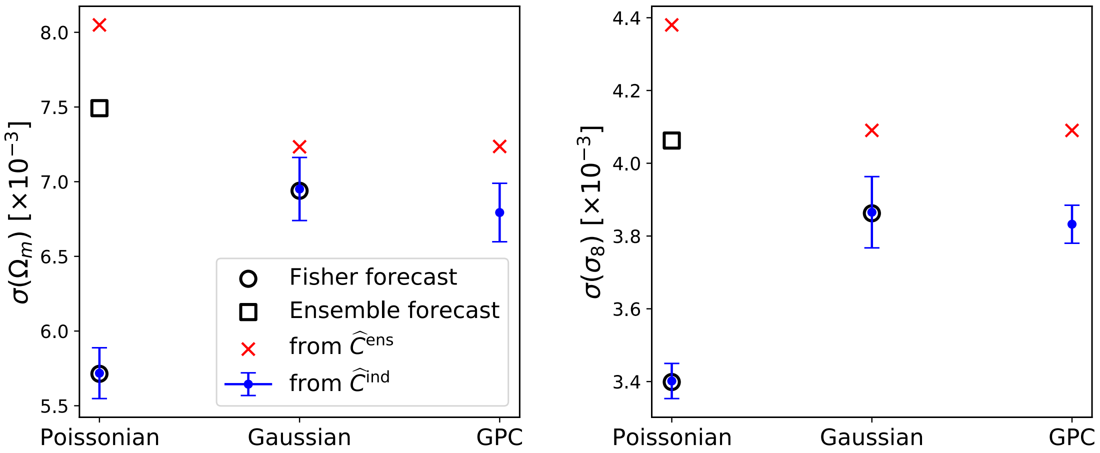
2. Reduced volume sample

Poisson underestimates error by 5 - 20 %

1. High mass sample



2. Reduced volume sample



Conclusions

Recap

- We tested the accuracy of cluster likelihoods with
    - 1000 simulated dark matter halo catalogs
    - By comparing posterior variances to spread of means over the 1000 simulations
    - Sensitive to analysis likelihood and latent likelihood properties

Conclusions: For future Euclid or Rubin-like surveys

- Gaussian gives robust constraints over a wide range of inference setup
- No gain in using Gauss-Poisson Compound (same level of constraints but computationally expansive)
- Gauss-Poisson Compound = Gaussian (under/overestimating errors at most 5%)
- Poisson likelihood always underestimates errors