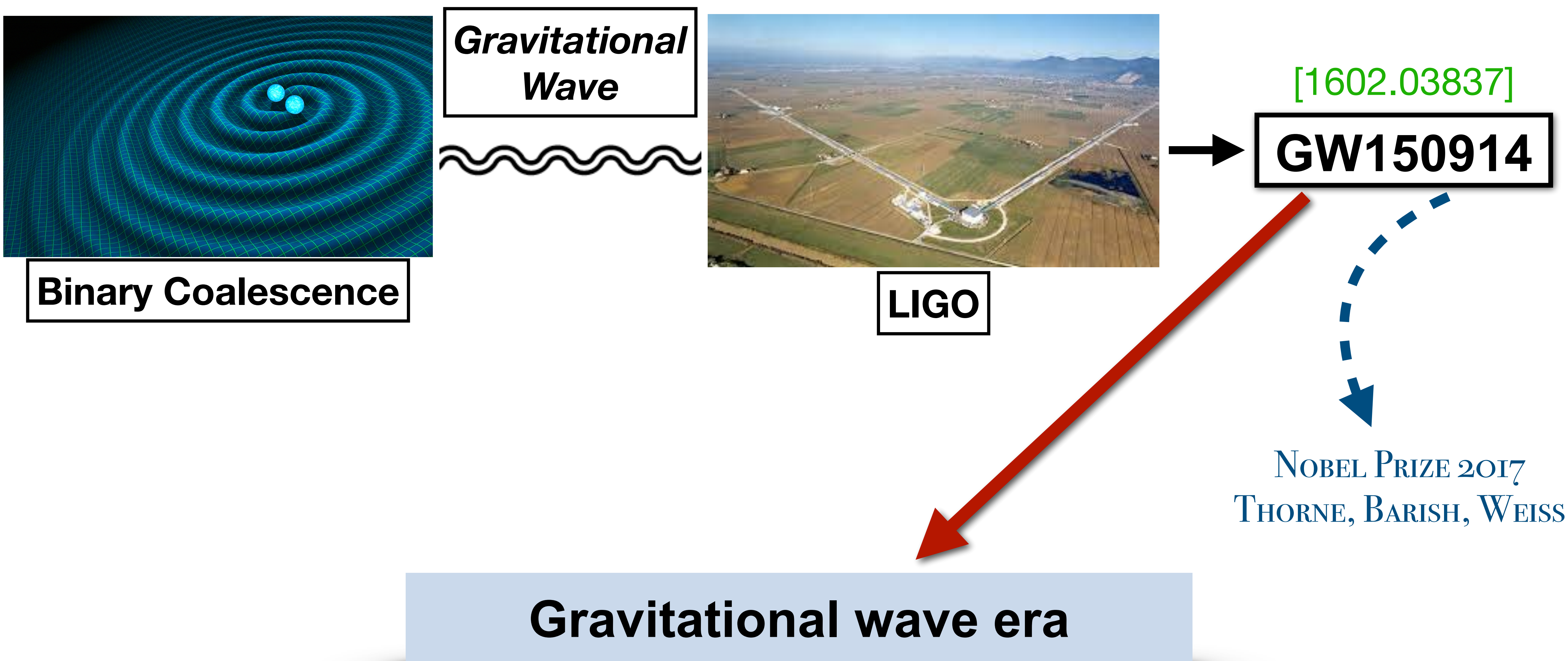
# QFT methods for GW Physics


***Stavros Mougiakakos***

LUTH l'Observatoire de Paris | PSL Université Paris Cité cnrs


Atelier API "Ondes gravitationnelles et objets compacts"


**[1912.06276]** M. Levi, **S.M.**, M. Vieira
**[2010.08882]** **S.M**, P. Vanhove
**[2102.08339]** **S.M**, M. M. Riva, F. Vernizzi
**[2204.06556]** **S.M**, M. M. Riva, F. Vernizzi


Binary Coalescence
Gravitational Wave
LIGO
[1602.03837]
GW150914
Nobel Prize 2017
Thorne, Barish, Weiss
Gravitational wave era

**Gravitational wave era**

1. Observational window on “strong” gravity
2. Multi-messenger Astronomy from the largest particle collider
3. Search for “new physics”

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**BUT**

**Weak signal (much noise)**

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**Weak signal (much noise)** → ***requires*** → **Accurate Prediction**

**(“new physics” highly suppressed)**

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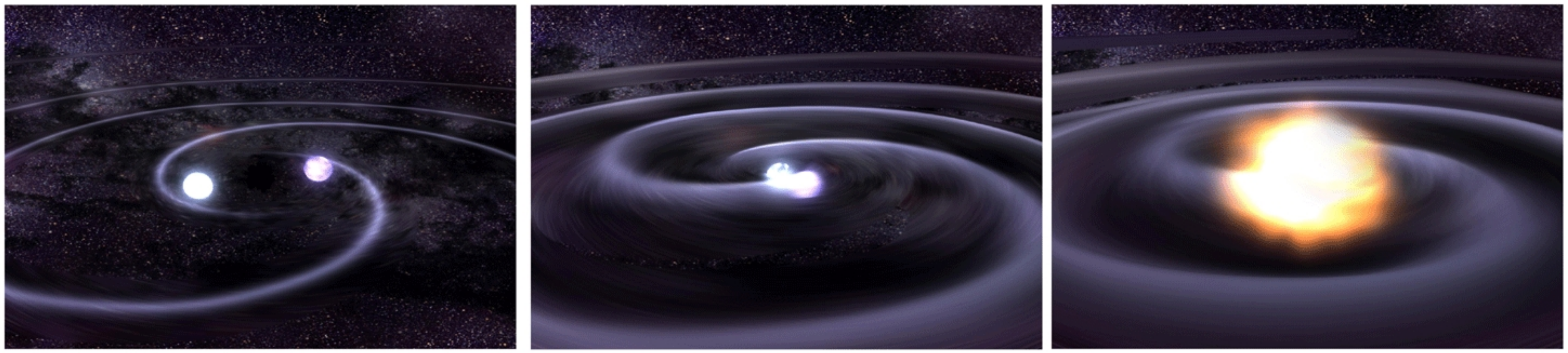
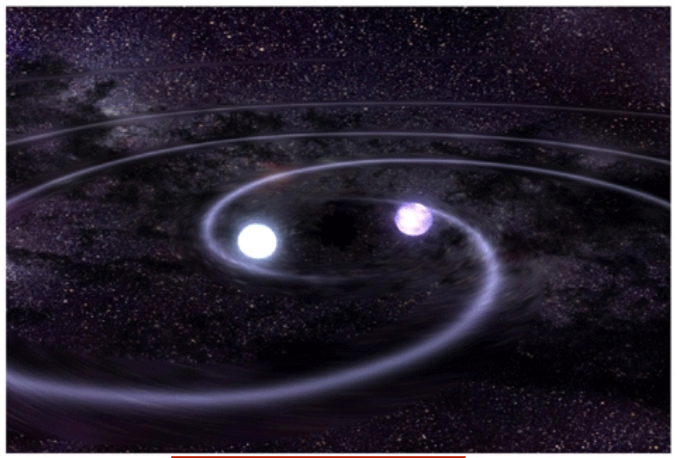
**Gravitational Binary Problem**

# Gravitational Binary Problem



**Inspiral** **Merger** **Ringdown**

# Gravitational Binary Problem

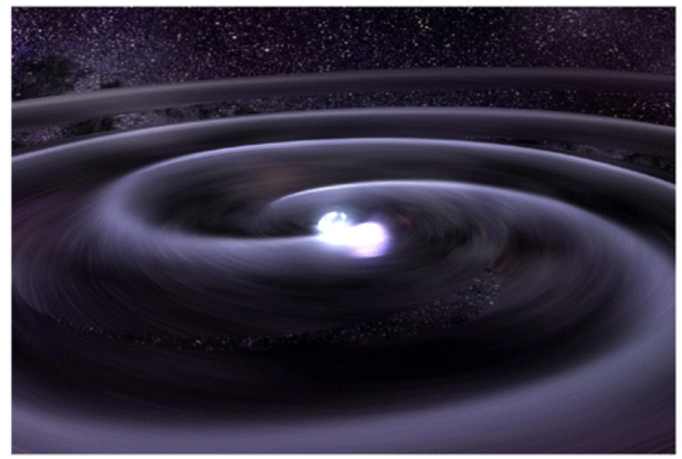


**Inspiral**

*Analytic treatment*

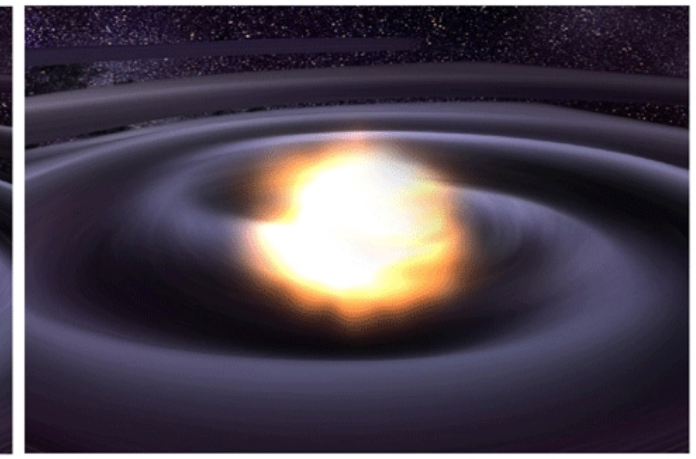
$$\frac{v}{c} \ll 1, \frac{R_{Schw}}{r_{orb}} \ll 1$$

**weak field**



**Merger**

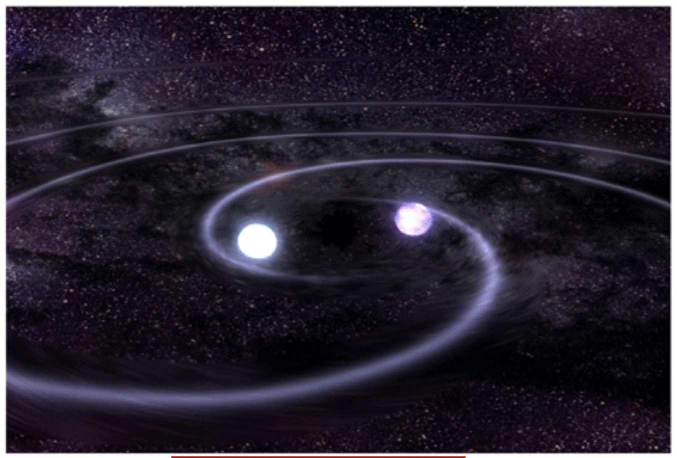
Numerical Relativity



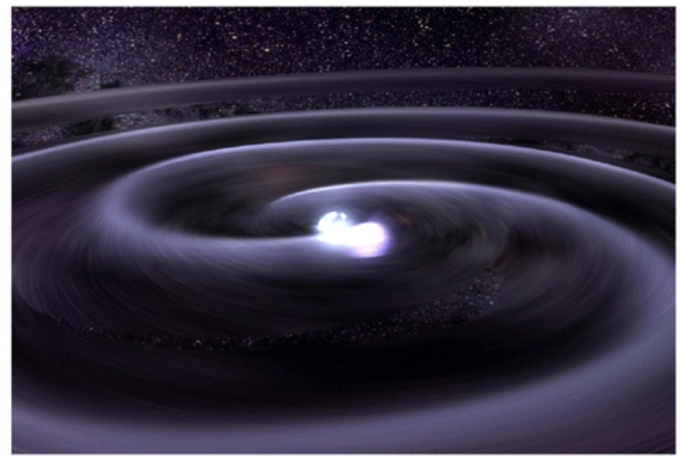
**Ringdown**

BH perturbation theory

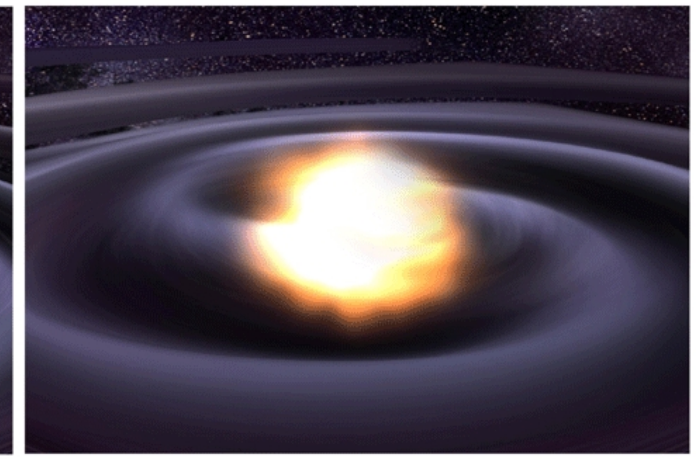
# Gravitational Binary Problem



Inspiral



Merger



Ringdown

*Analytic treatment*

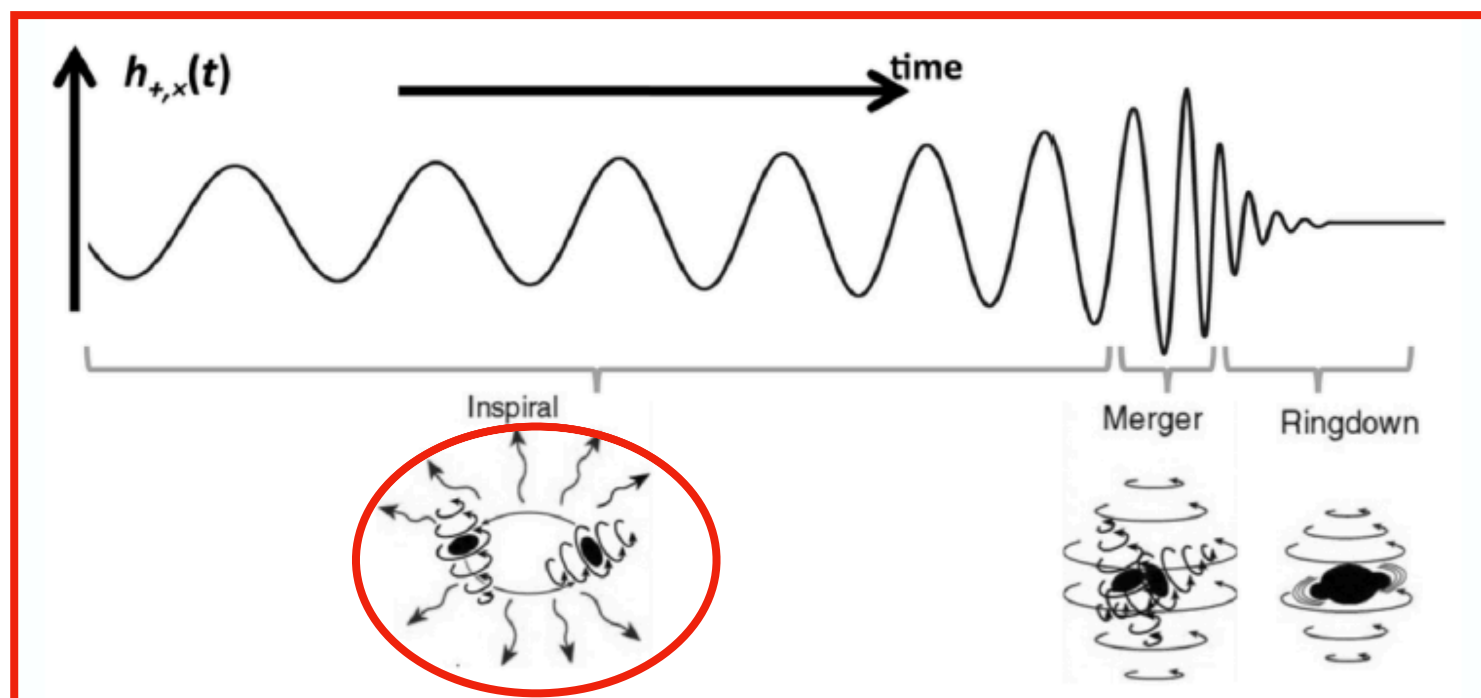
Numerical Relativity

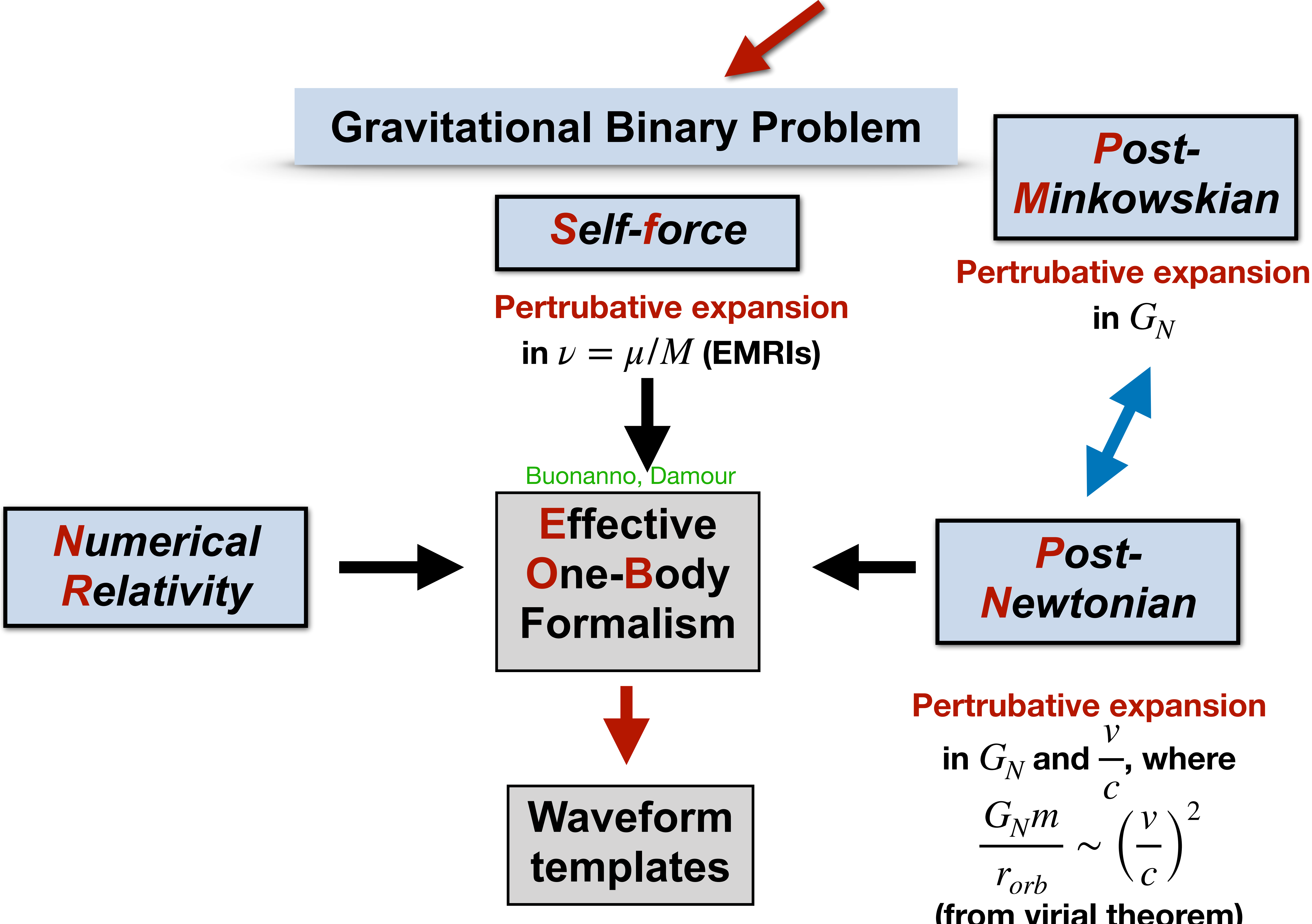
BH perturbation theory

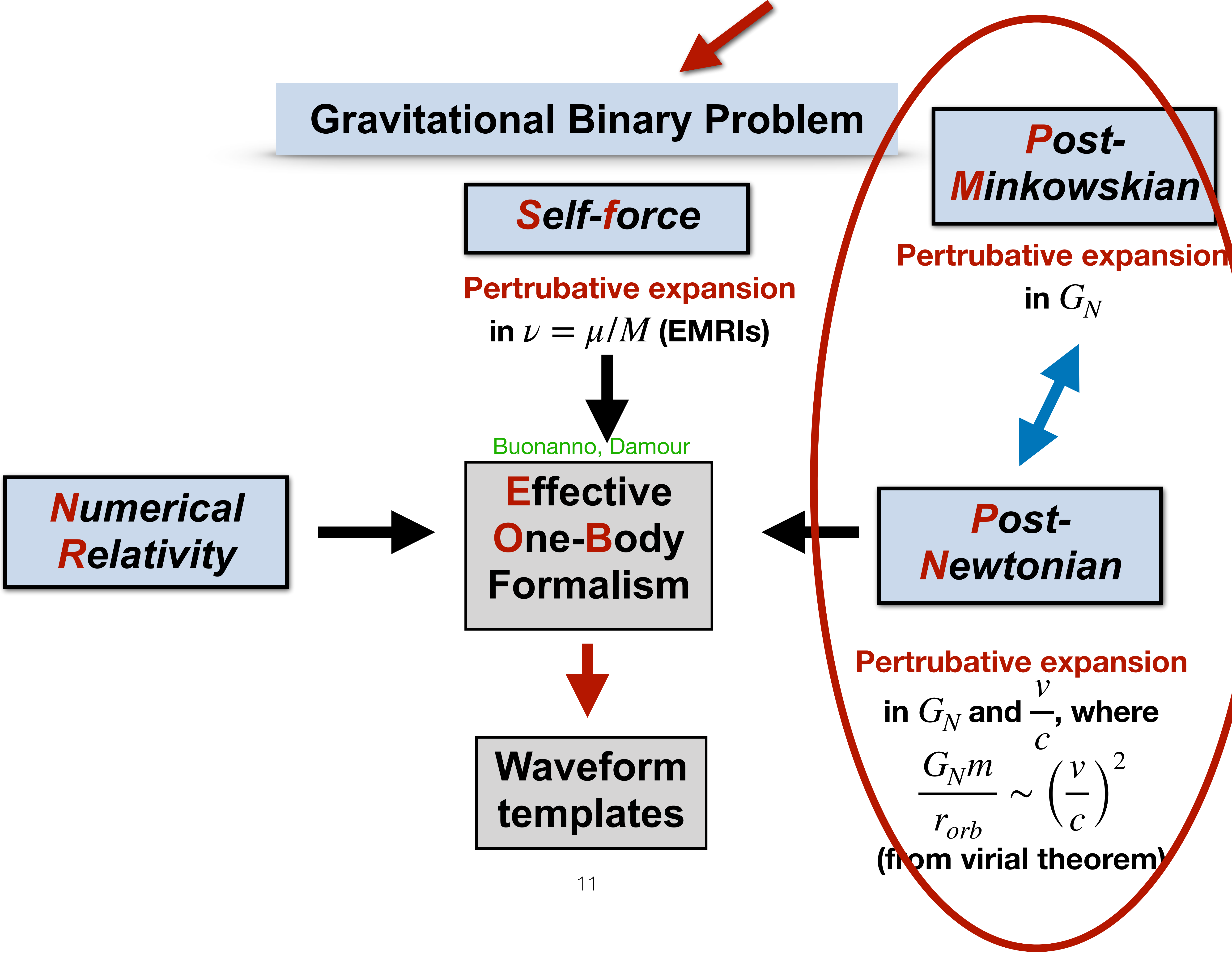
$$\frac{v}{c} \ll 1, \frac{R_{Schw}}{r_{orb}} \ll 1$$

**weak field**

**Most of the signal during the inspiral phase**




Gravitational Binary Problem
Self-force
Pertrubative expansion
in $\nu = \mu/M$ (EMRIs)
Post-Minkowskian
Pertrubative expansion
in $G_N$
Buonanno, Damour
Effective One-Body Formalism
Numerical Relativity
Post-Newtonian
Pertrubative expansion
in $G_N$ and $\frac{v}{c}$, where
$\frac{G_N m}{r_{orb}} \sim \left(\frac{v}{c}\right)^2$
(from virial theorem)
Waveform templates



Gravitational Binary Problem
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# Gravitational Binary Problem

**Post-Minkowskian** ⟷ **Post-Newtonian**

**Pertrubative expansion**

**in $G_N$**

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**in $G_N$ and $\frac{v}{c}$, where**

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# Gravitational Binary Problem

**Post-Minkowskian** ⟷ **Post-Newtonian**

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(from virial theorem)

- Traditional approaches within GR [Damour, Blanchet, Buonanno, Bernard et al.]
- Do we have existing toolbox that can be exploited?
- Alternative way to reformulate the problem using QFT language and tools
- EFT + Scattering Amplitudes + Feynman Integrals
- Theoretically interesting and computationally efficient

# Outline

1. **Post-Newtonian (PN)**
2. **Post-Minkowskian (PM) vs Post-Newtonian (PN)**
3. **Outlook**

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# Post-Newtonian

***Post-Newtonian***

**Particle Physicist's point of view**

**Traditional GR**

***NRGR/PNEFT***

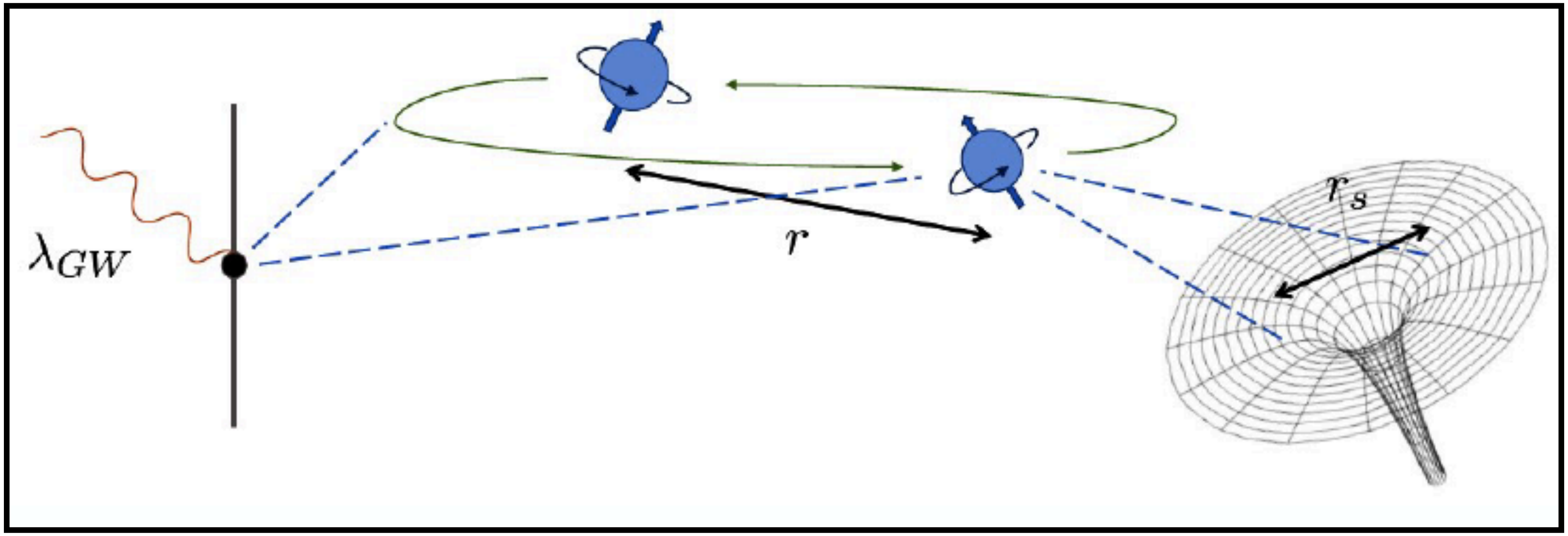
**Damour, Blanchet, Buonanno, Bernard et al.**

**[0409156] Rothstein, Goldberger, Porto, Foffa, Sturani, Levi, Steinhoff et al.**

# PNEFT / NRGR

**_Tower of EFTs_**

[1601.04914] Porto
[1807.01699] Levi

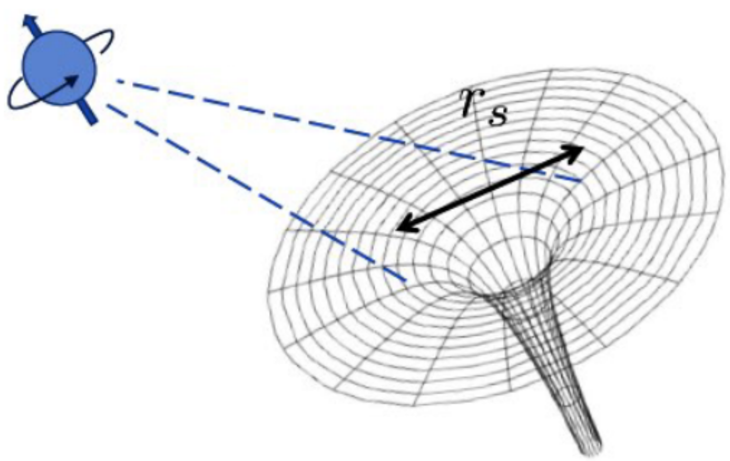



**<u>Hierarchy of scales:</u>** $r_s \ll r \ll \lambda_{rad}$

$$\frac{r_S}{r} \approx v^2, \ \frac{r}{\lambda_{rad}} \approx v$$

# PNEFT / NRGR

**Internal zone**




$$\mathcal{S} = -\frac{1}{16\pi G_N}\int d^4x\sqrt{g}R[g_{\mu\nu}] + \ldots$$

$$g_{\mu\nu} \equiv g^s_{\mu\nu} + \tilde{g}_{\mu\nu}$$

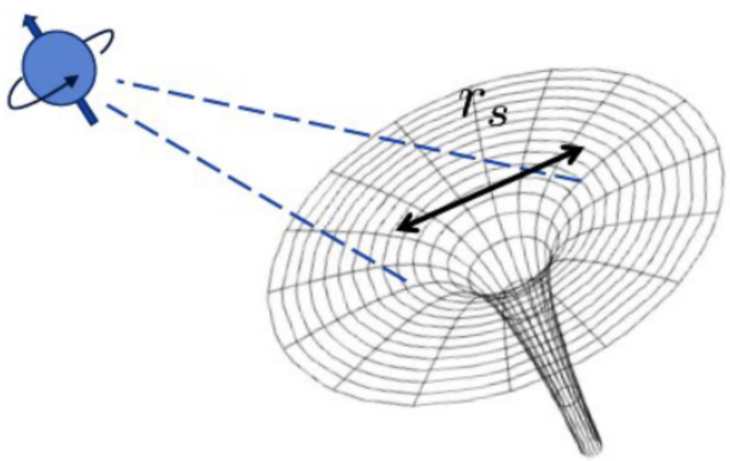
**(Bottom-Up)**

**(pure GR)** →

**integrate out** $g^s_{\mu\nu}$

# PNEFT / NRGR

## *Internal zone*

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**(Bottom-Up)**

**(pure GR)** → **integrate out** $g^s_{\mu\nu}$

$$\mathcal{S}_{eff}[x(\sigma),\tilde{g}] = -\frac{1}{16\pi G_N}\int d^4x\sqrt{\tilde{g}}\tilde{R}[\tilde{g}_{\mu\nu}] + \mathcal{S}_{p.p.}$$

$$\mathcal{S}_{p.p.} = -\int d\sigma\Big[m\sqrt{u^2} + \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu}\Big] + c_R\int d\sigma\,\tilde{R}\sqrt{u^2} + c_V\int d\sigma\,\tilde{R}_{\mu\nu}\frac{u^\mu u^\nu}{\sqrt{u^2}} + \ldots$$

**point particle** **redundant on-shell**

$$+\int d\tau\, Q_E^{ij}(\tau)E_{ij}(x) + \ldots + (E \to B)$$

**finite size**

$$\left(Q_E^{ij}\right)_R = c_E E^{ij} + \ldots$$

**dissipative (6.5PN)** **tidal (5PN)**

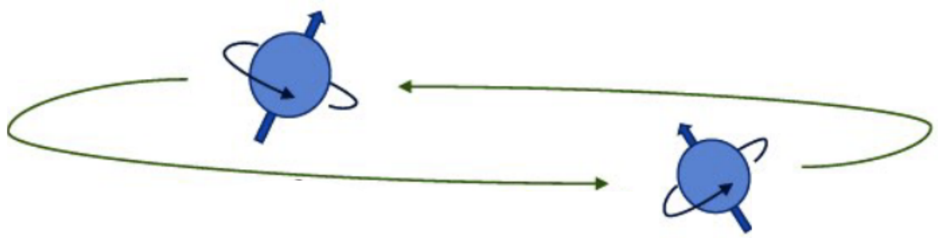
$$+\sum_{n=1}^{\infty}\int d\sigma \frac{(-1)^n C_{ES^{2n}}}{(2n)!m^{2n-1}} D_{\mu_{2n}}\ldots D_{\mu_3}E_{\mu_1\mu_2}\frac{S^{\mu_1}\ldots S^{\mu_{2n}}}{\sqrt{u^2}}$$

$$+\sum_{n=1}^{\infty}\int d\sigma \frac{(-1)^n C_{BS^{2n}}}{(2n+1)!m^{2n}} D_{\mu_{2n+1}}\ldots D_{\mu_3}B_{\mu_1\mu_2}\frac{S^{\mu_1}\ldots S^{\mu_{2n+1}}}{\sqrt{u^2}}$$

**non-minimal spin couplings**

# PNEFT / NRGR

## Potential zone



$$\mathcal{S}_{cons.} = \mathcal{S}_{EH} + \mathcal{S}_{GF} + \mathcal{S}_{p.p.1} + \mathcal{S}_{p.p.2}$$

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu} + h_{\mu\nu}$$

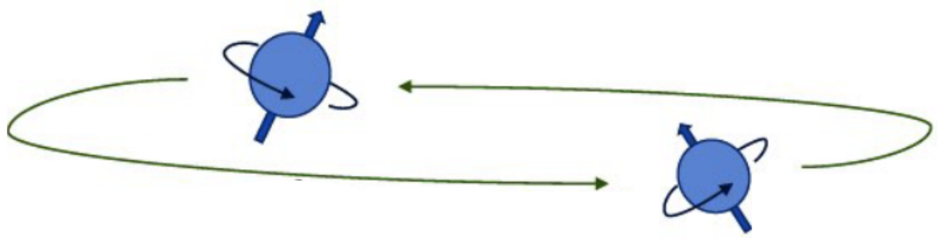
**(Top-Down)**

→

**integrate out** $H_{\mu\nu}$

# PNEFT / NRGR

## *Potential zone*



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$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu} + h_{\mu\nu}$$

**(Top-Down)**

→ **integrate out** $H_{\mu\nu}$

**($h_{\mu\nu} = 0$)** → $H_{\mu\nu}$ **instantaneous propagators** $k_0 \sim v/r,\ |\vec{k}| \sim 1/r$

$$\frac{1}{k_0^2 - \vec{k}^2} = -\frac{1}{\vec{k}^2}\left(1 + \frac{k_0^2}{\vec{k}^2} + \dots\right) = -\frac{1}{\vec{k}^2}\left(1 + \mathcal{O}(v^2)\right)$$

**Feynman rules** → **QFT diagrammatics**

→ $\mathcal{S}_{cons.}$

- → **UV divergencies (*renormalization*)**
- → **IR divergencies (*zero-bin*)**

# PNEFT / NRGR

**Radiation zone**

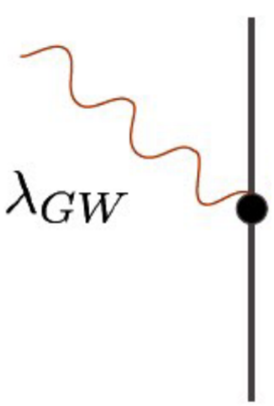
**(Bottom-Up)**

$$\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{S}_{eff} = -\frac{1}{16\pi G_N}\int d^4x\sqrt{\bar{g}}\bar{R} + \mathcal{S}_{GF}[h] + \mathcal{S}_{p.p.(comp.)}$$

$$\mathcal{S}_{p.p.(comp.)} = -\int dt\sqrt{\bar{g}}\Big(M(t) + \frac{1}{2}\epsilon_{ijk}L^k(t)(\Omega^{ij}_{LF} + \omega^{ij}_{\mu}u^{\mu})$$

$$-\sum_{l=2}^{\infty}\big(\frac{1}{l!}I^L(t)\nabla_{L-2}E_{i_{l-1}i_l} - \frac{2l}{(l+1)!}J^L(t)\nabla_{L-2}B_{i_{l-1}i_l}\big)\Big)$$



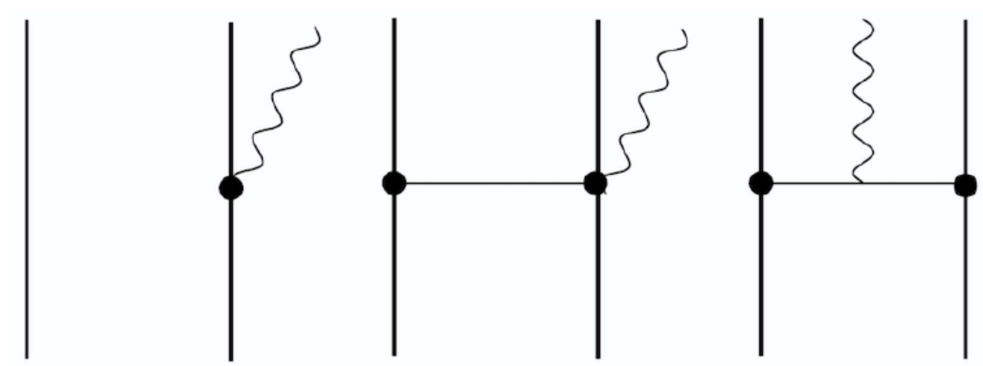

$$\mathcal{A}_h(\omega,\mathbf{k}) = \frac{\epsilon^*_{ij}(\mathbf{k},h)}{4M_{Pl}}[\omega^2 I^{ij}(\omega) + \ldots]$$

$\mathcal{A}_h(\omega,\mathbf{k})$

***MATCHING***

**(Top-Down)**

**Potential zone** $+h_{\mu\nu}$



$$\mathcal{A}_h(\omega,\mathbf{k}) = -\frac{\epsilon^*_{ij}(\mathbf{k},h)}{2M_{Pl}}T^{ij}(\omega,\mathbf{k})$$

# PNEFT / NRGR

**Radiation zone**

**(Bottom-Up)**

$\mathcal{A}_h(\omega, \mathbf{k})$

**MATCHING**

**(Top-Down)**

**Potential zone** $+h_{\mu\nu}$

$$\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$$

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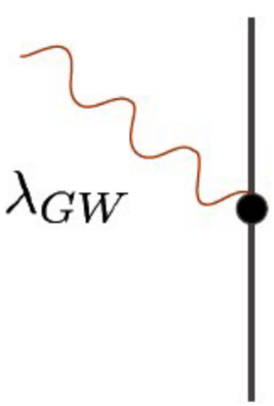
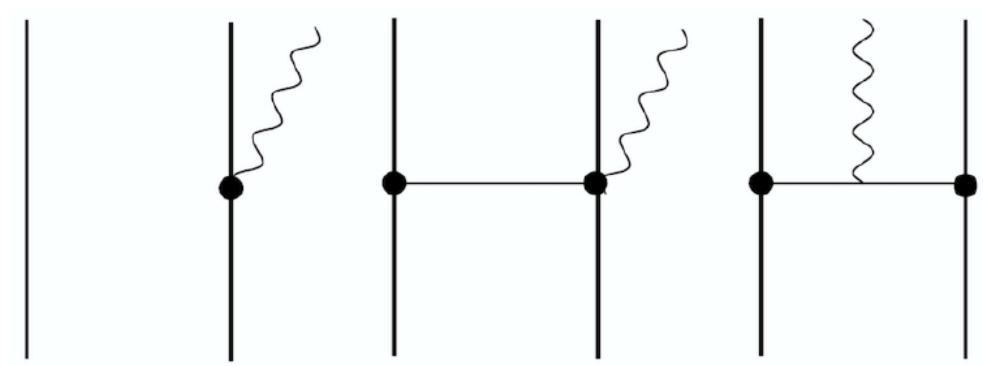
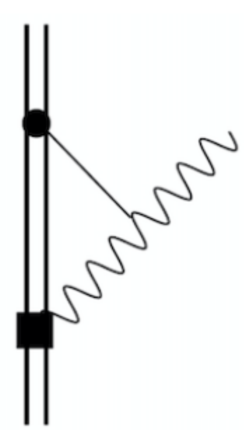


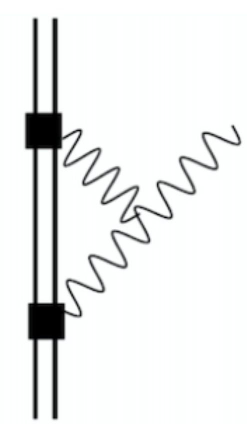



$$\mathcal{A}_h(\omega, \mathbf{k}) = \frac{\epsilon^*_{ij}(\mathbf{k}, h)}{4M_{Pl}}[\omega^2 I^{ij}(\omega) + \dots]$$

$$\mathcal{A}_h(\omega, \mathbf{k}) = -\frac{\epsilon^*_{ij}(\mathbf{k}, h)}{2M_{Pl}}T^{ij}(\omega, \mathbf{k})$$



**Tail**



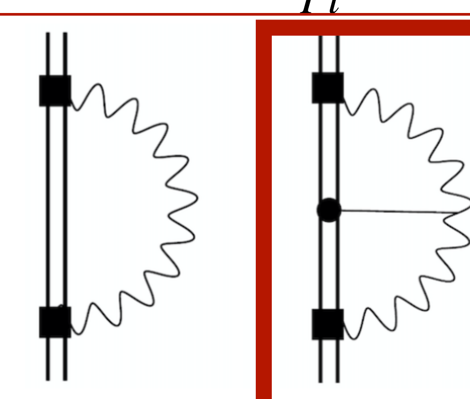
**Memory**

[1703.06433] Porto, Rothstein

**zero-bin subtraction**

**IR(potential)/UV(radiation)**

**conservative contribution**



**Radiation Reaction**

**in-in formalism**

# Outline

1. Post-Newtonian (PN)
2. **Post-Minkowskian (PM) vs Post-Newtonian (PN)**
3. Outlook

# Quantum Amplitudes for Classical Gravity

## *Gravity as an Effective Field Theory*

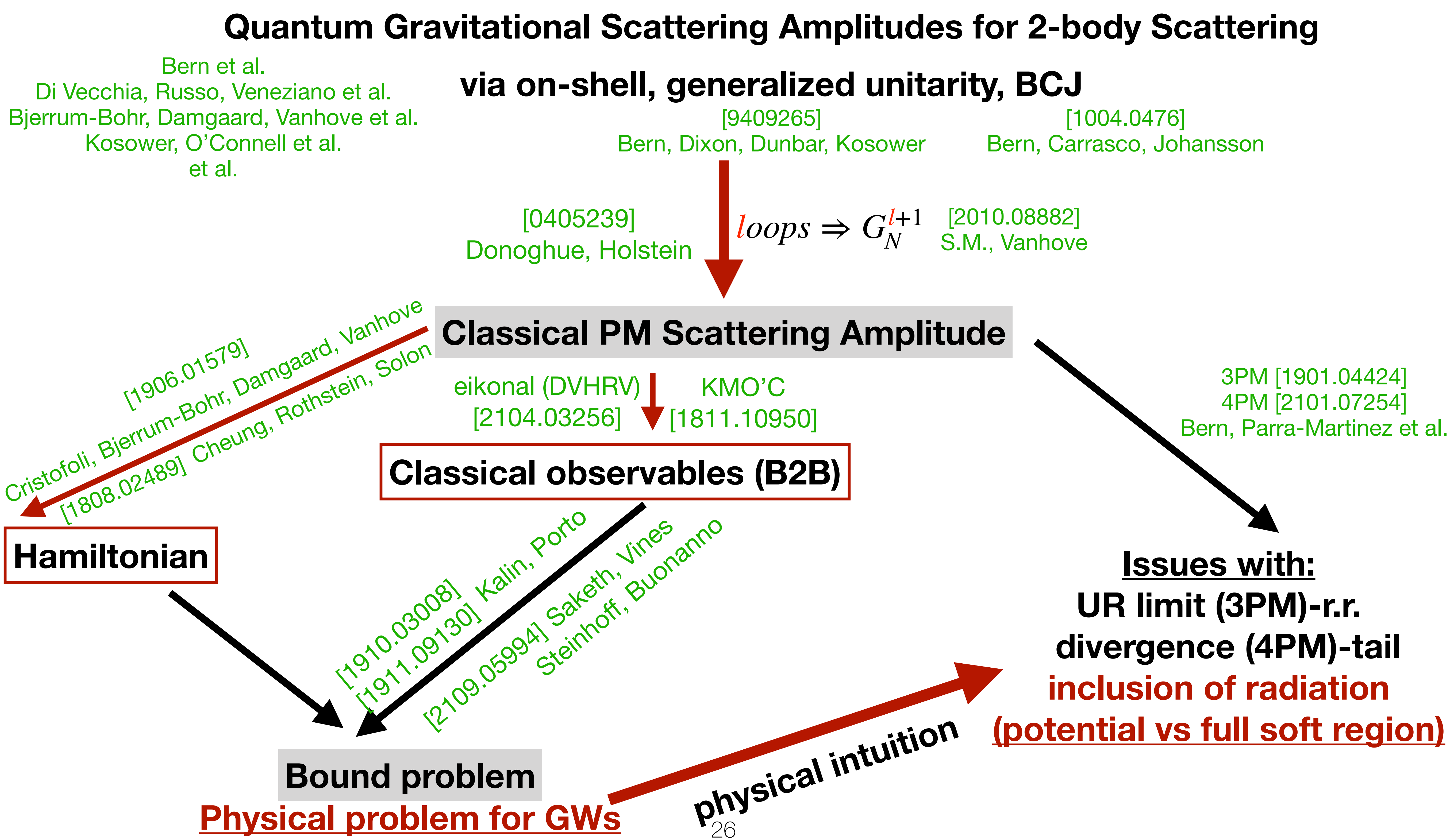
DeWitt
t'Hooft, Veltman
Donoghue et al.

$$\mathcal{S}_{eff} = \frac{1}{16\pi G_N}\int d^dx\sqrt{g}R + \mathcal{O}(R^2, R_{\mu\nu}R^{\mu\nu},\ldots) + \mathcal{S}_{matter}$$

- **Non-Renormalizable QFT**: (local, unitary, Lorentz invariant)
- **GR as a first order approximation**
- **Standard symmetries** of GR
- **Low energy DOF's**: graviton + matter fields
- **Weak field approximation:** $g_{\mu\nu} = \eta_{\mu\nu} + \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}$

# Quantum Amplitudes for Classical Gravity

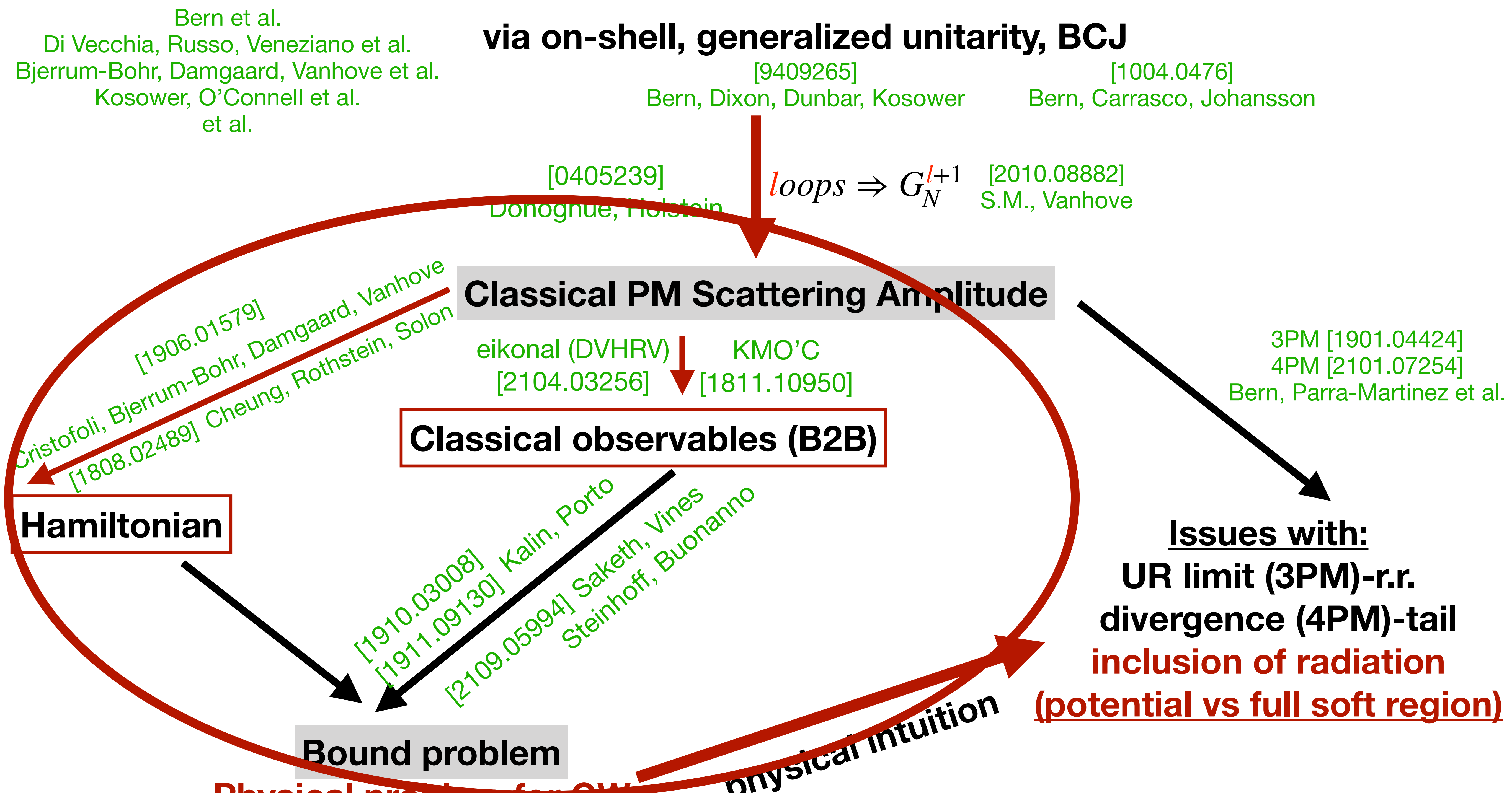
## Experience from particle physics



# Quantum Amplitudes for Classical Gravity

## Experience from particle physics

**Quantum Gravitational Scattering Amplitudes for 2-body Scattering**



# PM vs(?) PN

**Post-Minkowskian**

(Scattering)

**Post-Newtonian**

(Bound orbits)

| | 0PN | | 1PN | | 2PN | | 3PN | | 4PN | | 5PN | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1PM | [1 | + | $v^2$ | + | $v^4$ | + | $v^6$ | + | $v^8$ | + | $v^{10}$ | + | ...] | x | $G^1$ |
| 2PM | | | [1 | + | $v^2$ | + | $v^4$ | + | $v^6$ | + | $v^8$ | + | ...] | x | $G^2$ |
| 3PM | | | | | [1 | + | $v^2$ | + | $v^4$ | + | $v^6$ | + | ...] | x | $G^3$ |
| 4PM | | | | | | | [1 | + | $v^2$ | + | $v^4$ | + | ...] | x | $G^4$ |
| 5PM | | | | | | | | | [1 | + | $v^2$ | + | ...] | x | $G^5$ |
| 6PM | | | | | | | | | | | [1 | + | ...] | x | $G^6$ |

[1908.01493] Bern et al

# PM ~~vs~~ PN

**EFT matching/ Lippmann-Schwinger**

[1808.02489] Cheung, Rothstein, Solon
[1906.01579] Cristofoli, Bjerrum-Bohr, Damgaard, Vanhove

***P*ost-*M*inkowskian**

(Scattering)

**(Hamiltonian)**

⟷

**Boundary to Bound (B2B) map**

**(Observables)**

[1910.03008]
[1911.09130]
Kalin, Porto

***P*ost-*N*ewtonian**

(Bound orbits)

[2109.05994] Saketh, Vines
Steinhoff, Buonanno

| | 0PN | | 1PN | | 2PN | | 3PN | | 4PN | | 5PN | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
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| 3PM | | | | | [1 | + | $v^2$ | + | $v^4$ | + | $v^6$ | + | ...] | x | $G^3$ |
| 4PM | | | | | | | [1 | + | $v^2$ | + | $v^4$ | + | ...] | x | $G^4$ |
| 5PM | | | | | | | | | [1 | + | $v^2$ | + | ...] | x | $G^5$ |
| 6PM | | | | | | | | | | | [1 | + | ...] | x | $G^6$ |

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# PM ~~vs~~ PN

**EFT matching/ Lippmann-Schwinger**

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***P**ost-**M**inkowskian* (Scattering)

**(Hamiltonian)**

***P**ost-**N**ewtonian* (Bound orbits)

**Boundary to Bound (B2B) map**

**(Observables)**

[1910.03008]
[1911.09130]
Kalin, Porto

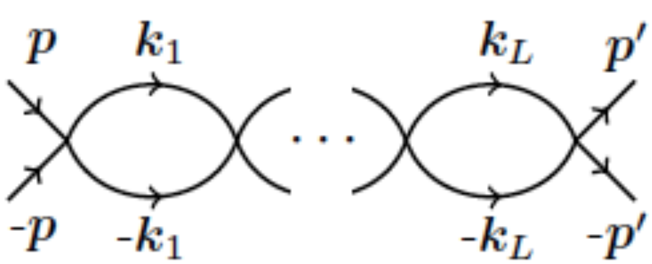
[2109.05994] Saketh, Vines
Steinhoff, Buonanno

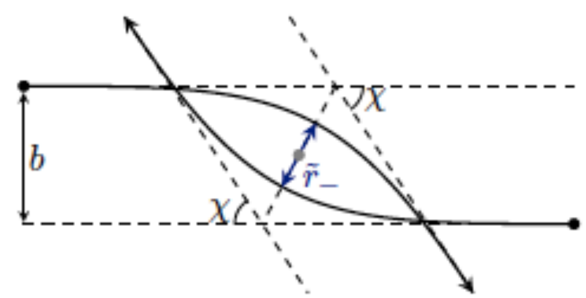
**EFT for NR scalars**

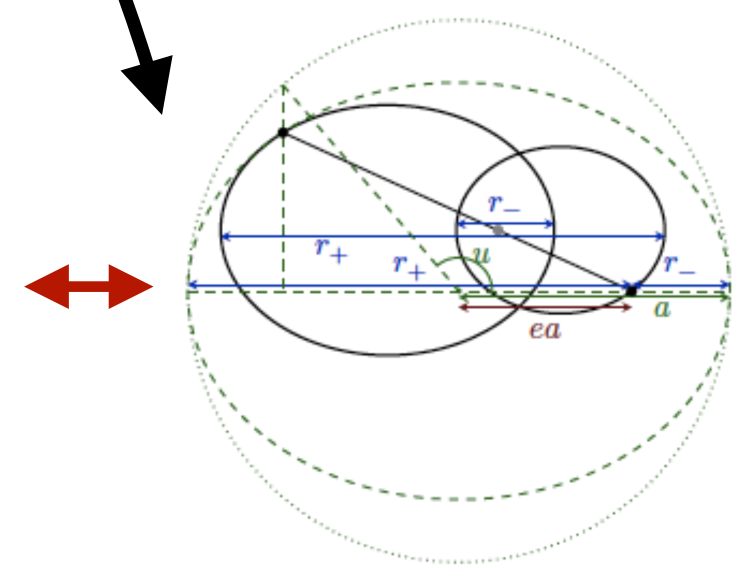
$$L_{\rm int} = -\int_{\boldsymbol{k},\boldsymbol{k}'} V(\boldsymbol{k},\boldsymbol{k}')\, A^\dagger(\boldsymbol{k}')A(\boldsymbol{k})B^\dagger(-\boldsymbol{k}')B(-\boldsymbol{k})$$

**Ansatz:** $V(\boldsymbol{p},\boldsymbol{r}) = \dfrac{Gc_1(\boldsymbol{p}^2)}{|\boldsymbol{r}|} + \dfrac{8G^2c_2(\boldsymbol{p}^2)}{\boldsymbol{r}^2} + \cdots$

$M_{\rm EFT}^{L\text{-loop}} =$




$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})\,, \qquad \mathcal{E}<0\,,$$

$$\Delta E_{\rm ell}(J) = \Delta E_{\rm hyp}(J) - \Delta E_{\rm hyp}(-J)$$

$$\Delta J_{\rm ell}(J) = \Delta J_{\rm hyp}(J) + \Delta J_{\rm hyp}(-J)$$

**<u>Matching with full theory Amplitude fixes coeffs.</u>**

# PM ~~vs~~ PN

**Post-Minkowskian**
(Scattering)

**EFT matching/ Lippmann-Schwinger**
[1808.02489] Cheung, Rothstein, Solon
[1906.01579] Cristofoli, Bjerrum-Bohr, Damgaard, Vanhove

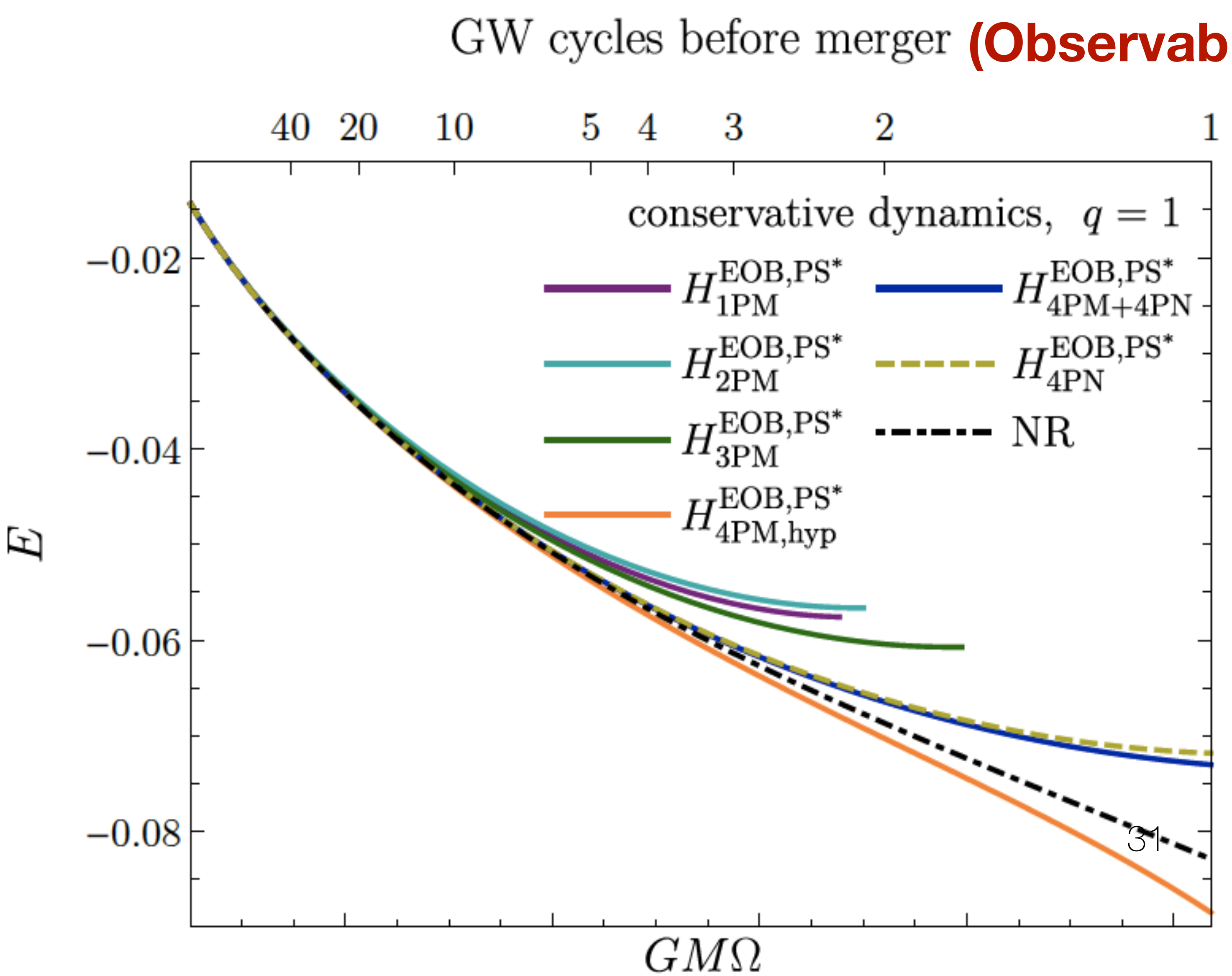
**(Hamiltonian)**

**Boundary to Bound (B2B) map**
[1910.03008]
[1911.09130]
Kalin, Porto

**Post-Newtonian**
(Bound orbits)

[2109.05994] Saketh, Vines
Steinhoff, Buonanno

**(Observables)**




[2204.05047] Buonanno et al

# PM ~~vs~~ PN

**EFT matching/ Lippmann-Schwinger**

[1808.02489] Cheung, Rothstein, Solon
[1906.01579] Cristofoli, Bjerrum-Bohr, Damgaard, Vanhove

***Post-Minkowskian***

(Scattering)

**(Hamiltonian)** ⟷

***Post-Newtonian***

(Bound orbits)

**Boundary to Bound (B2B) map**

**(Observables)**

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1. **Scattering to Bound with radiation**
2. **Higher orders**
3. **Radiation effects**
4. **Spin, finite size**

→

**High precision**

1. **NS (EoS)**
2. **Exotic objects**
3. **GR modifications**
4. **Quantum gravity(?)**

# Outline

1. Post-Newtonian (PN)
2. Post-Minkowskian (PM) vs Post-Newtonian (PN)
3. Post-Minkowskian Effective Field Theory (PMEFT)
4. **Outlook**

# Outlook

***WHAT WE HAVE LEARNED SO FAR***

- **QFT methods are competitive/complementary to traditional**
- **PN & PM complementarity**
- **NRGR self consistent + physical intuition**
- **Radiation effects are crucial**
- **Integration techniques are a bottleneck**
- **Each higher order exhibits new difficulties**

***WHAT WE ARE LOOKING FOR***

- **Higher orders both in PN & PM**
- **Radiation, spin, finite size effects**
- **Extension of Scattering to Bound maps**
- **GR modifications & (?) Quantum signatures**

*Thank you very much for your attention!*