Analytical representations of unified and modern Equations of State

Atelier API "Ondes gravitationnelles et objets compacts"

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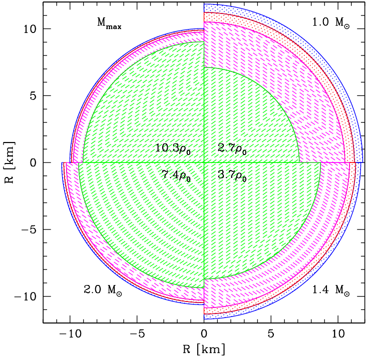
Neutron Stars main characteristics

- Mass $\sim [1-2]\ M_\odot$
- Radius ~ 10 km
- Density $\rightarrow 10^{15}$ g/cm^3

Structure:

- Crust: lattice state of matter
  - outer crust: up to $\sim 10^{11}$ g/cm^3
  - inner crust: free neutrons outside nuclei
- Core: soup of particles
  - outer core: *npeμ* gaz
  - inner core: ? hyperons, deconfined quarks, strange quarks etc.

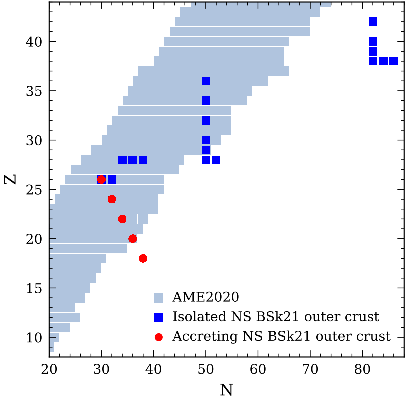
Multitude of equations of state for dense matter



Prof. J.L. Zdunik

Common assumption that the crust is constrained...

BSk21 = Brussels Skyrme 21 [Fantina et al. 2022], [Chamel et al. 2009]



... but only part of the outer crust is constrained by nuclear experiment!

A crust only partly constrained

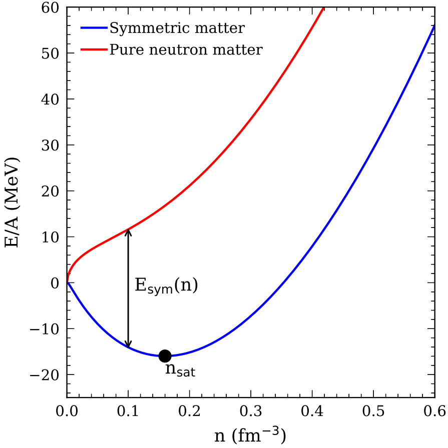
Common assumption that the crust is constrained...

Establishing the equation of state (EoS) defines a number of nuclear parameters: J, L, K...

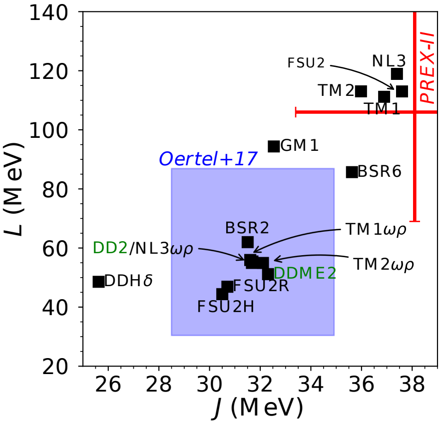
$$u(n) = \frac{n - n_{\rm sat}}{3n_{\rm sat}} ,$$

$$E(n, \delta = 0) = \mathscr{E}_{\rm sat} + \mathscr{K}_{\rm sat} \frac{u^2}{2!} + \dots ,$$

$$E_{\rm sym} = J + Lu + K_{\rm sym} \frac{u^2}{2!} + \dots ,$$



Common assumption that the crust is constrained...



... but the constraints on J and L are large, if not in tension with one another !

Non-unified equations of state = core and crust are not calculated with the same nuclear model.

Why ?

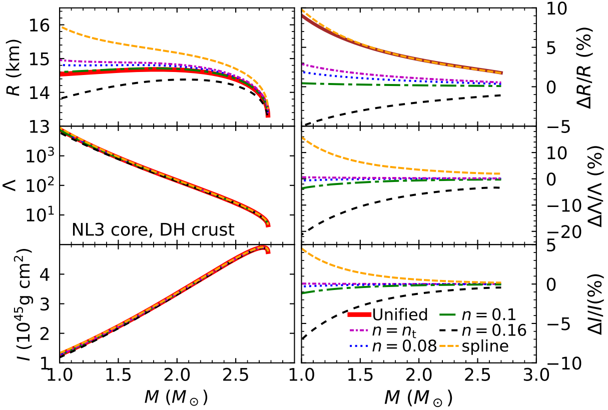
- based on the belief of a fixed crust,
- easier to compute the core than the crust.

Model	n_{sat} (fm^{-3})	$\mathscr{E}_{sat}$ (MeV)	K_{sat} (MeV)	J (MeV)	L (MeV)	n_t (fm^{-3})
NL3 [Horowitz et al. 2020]	0.149	-16.2	271.6	37.4	118.9	0.057
DD2 [Typel et al. 2010]	0.149	-16.0	242.6	31.7	55.0	0.067
DH [Douchin & Haensel 2001]	0.159	-16.0	230.0	32.0	46.0	0.076

Where can we find non-unified EoS ?

Analytical representations used for gravitational wave analysis, finite temperature simulations, modelisation of neutron star's parameters in modified gravity, magneto-hydrodynamics, universal relations etc...

Common assumption that the crust does not matter for NS modeling ...



... but it does!

Analytical representation of 1D EoSs:
Piecewise Polytropic fits in [Read et al. 2009]

A polytrope:

$$P = \kappa \rho^{\Gamma} , \quad (1)$$

with Γ the adiabatic index, and κ the polytropic constant.

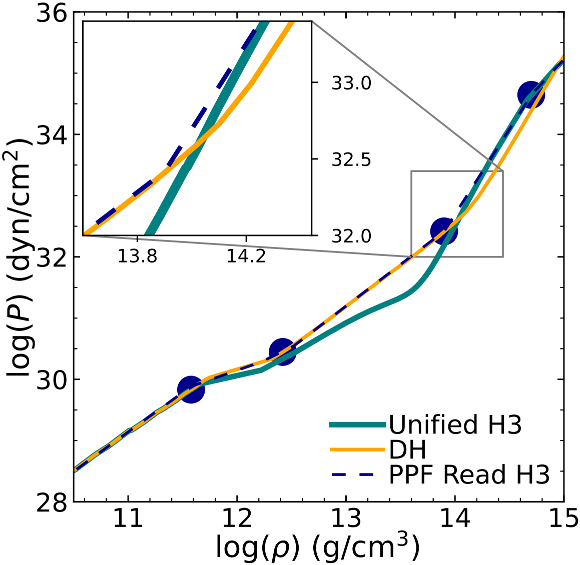
Realistic EoS = several polytropes
with parameters defined from the pressure continuity

$$\kappa_i \rho_t^{\Gamma_i} = \kappa_{i+1} \rho_t^{\Gamma_{i+1}} ; \quad (2)$$

first law of thermodynamics defines the energy density

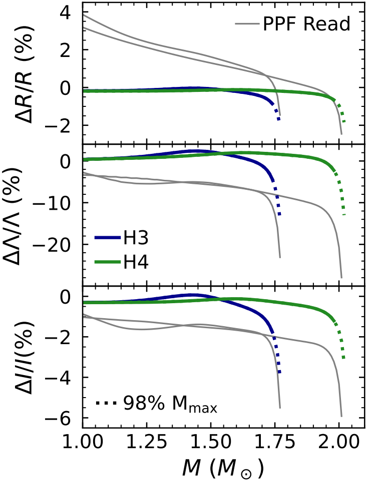
$$\epsilon(\rho) = (1 + a_i)\rho + \frac{\kappa_i}{\Gamma_i - 1} \rho^{\Gamma_i} . \quad (3)$$

[Read et al. 2009] : 7 polytropes, ~ 30 EoSs fitted ⚠ 30 cores + DH crust


36
34
32
30
28
11
12
13
14
15
log(P) (dyn/cm²)
log(ρ) (g/cm³)
33.0
32.5
32.0
13.8
14.2
Unified H3
DH
PPF Read H3

Many groups work on unified equations of state

- DH and Brussels-Skyrme models are unified by design,
- [Fortin et al 2016]: inner crust established with the same Lagrangian density as established in the core with RMF models;
- Meta-modeling + [Carreau et al. 2019]: a routine provided to establish a unified crust from EoS parameters J, L, K, etc.
- CompOSE database: large number of unified EoSs [Gulminelli & Raduta 2015],
- [Suleiman et al. 2022]: piecewise polytropic fits of unified and modern EoSs.



Conclusion

Non-unified EoSs leads to error on the macroscopic parameters

- the crust of neutron stars is only partly constrained,
- gluing two models with minimal error requires knowledge of nuclear parameters,
- unified models are necessary to correctly infer dense matter knowledge from astrophysical data.

Prospects and future work on NS crust physics

- Ligo-Virgo-Kagra *LalSimulation*: a lot of outdated EoSs
  - provide modern and unified models for O4,
  - linking the CompOSE data base with *LalSimulation*,
  - include a routine [Carreau et al. 2019] to compute a unified crust for available cores.
- Revising bayesian analysis for neutron stars, with a fixed crust.

Any questions ?