Classical vs. Quantum Corrections to Jet Broadening in a Weakly Coupled QGP

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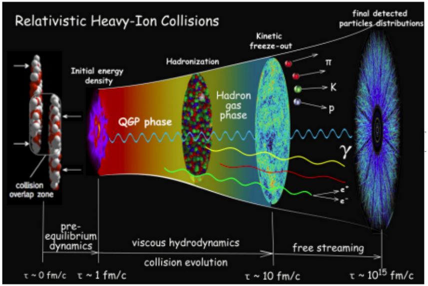
Outline

- Heavy nuclei are smashed together at the LHC and RHIC, liberating their constituents and forming the Quark Gluon Plasma (QGP)



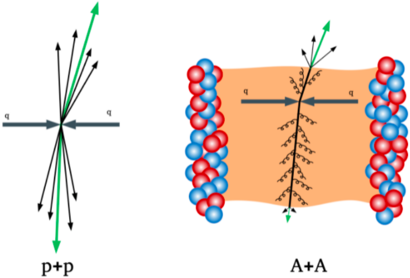
[Heinz, 2013]

- Short lifetime makes QGP extremely difficult to study, so what do we do?

Jet Quenching

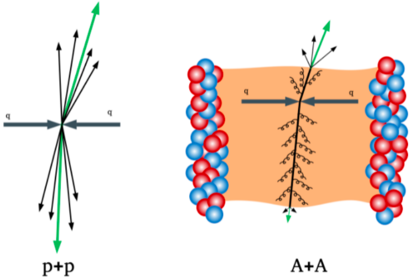
- A natural feature of gauge theories, jets, are structures of high enery, self-collimated final state particles

- Produced in both proton-proton collisions and heavy-ion collisions
  $\Rightarrow$ Nice probe for QGP
  $\Rightarrow$ Jet Quenching



M. Rybar/ATLAS Collaboration

- Transverse momentum broadening coefficient, $\hat{q}$ serves as key ingredient in characterising quenching
  - Relevant for computing in-medium splitting rates
  - Can be used for input to effective kinetic description of QGP



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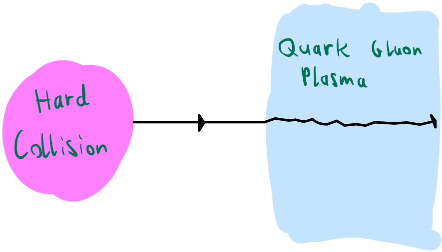
Jet Propagation

- Consider a nearly on shell, highly energetic (hard) parton with energy, E produced in a heavy ion collision

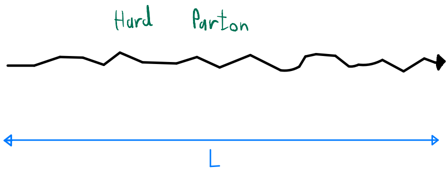
- Parton undergoes collisions with medium constituents while propagating through the plasma



Energy Loss from Elastic Collisions

- Hard parton picks up transverse momentum, $k_\perp \ll E$ from collisions with medium constituents

- View as diffusion process and define diffusion coefficient, $\hat{q}$ as $k_\perp^2 \equiv \hat{q}L$



Energy Loss from Bremsstrahlung

- The dominant mechanism for energy loss in the QGP is not the energy lost through these elastic collisions

- Instead, it comes from the bremsstrahlung induced from these elastic collisions

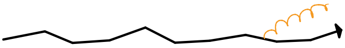
Outline

How exactly is bremsstrahlung triggered?

This depends on the quantum mechanical formation time, τ associated with the radiated gluon and can be crudely separated into two cases:

- Case 1: Radiated gluon with energy ω is triggered by just collision with medium constituent
  - Known as Bethe-Heitler or single-scattering regime

- Case 2: Many collisions with smaller momentum exchange add up to trigger gluon radiation with energy ω
  - Known as harmonic oscillator or multiple-scattering regime
  - Requires LPM resummation



Single Scattering

- Formation time of radiated particle is given parametrically as

$$\tau \sim \frac{\omega}{k_\perp^2} \tag{1}$$

- Let λ_{el} be the mean free path associated with collisions with the medium

- If $\tau \lesssim \lambda_{el}$, the so-called Bethe-Heitler spectrum turns out to be proportional to the number of elastic collisions, N

$$\omega \frac{dI}{d\omega} \simeq \frac{\alpha_s N_c}{\pi} N = \frac{\alpha_s N_c}{\pi} \frac{L}{\lambda_{el}} \tag{2}$$

- Now, assume that $\tau \gg \lambda_{el}$ and that radiated gluon undergoes transverse momentum kicks during formation time and picks up $k_\perp^2 \sim \hat{q}\tau$

$$\implies \tau = \sqrt{\frac{\omega}{\hat{q}}} \tag{3}$$

- Then, we can crudely say that if $N_{coh} = \tau/\lambda_{el}$ is the number of coherent collisions that

$$\omega \frac{dI}{d\omega} \simeq \frac{\alpha_s N_c}{\pi} \frac{N}{N_{coh}} = \frac{\alpha_s N_c}{\pi} L \sqrt{\frac{\hat{q}}{\omega}} \tag{4}$$

$\Rightarrow$ As ω increases, spectrum suppressed $\Rightarrow$ LPM Effect

Things to Note

- Physics of these two regimes is very different

- In multiple-scattering regime, many collisions need to be resummed via BDMPS-Z/AMY formalisms [Baier et al., 1995, Zakharov, 1997, Arnold et al., 2003]

- Within this formalism, analytical solutions can be found if Harmonic Oscillator Approximation (HOA) is made, where potential describing soft interactions of hard partons with the medium $\propto \hat{q}_0 x_\perp^2$

Outline

LO and NLO contributions to $\hat{q}$

- Leading order contributions $\hat{q}_0 \sim g^4 T^3$ calculated by [Aurenche et al., 2002] and [Arnold and Xiao, 2008] coming from soft scale $k_\perp \sim gT$ and hard scale $k_\perp \sim T$ respectively

- $\mathcal{O}(g)$ classical contributions from soft scale calculated perturbatively by [Caron-Huot, 2009] and later on the lattice by [Panero et al., 2014, Moore and Schlusser, 2020, Moore et al., 2021]

Logarithmically Enhanced Corrections

- $\mathcal{O}(g^2)$ correction found to have double logarithmic $\sim \ln^2(L/\tau_{\text{min}})$ and single logarithmic enhancements by [Liou et al., 2013](LMW) and separately by [Blaizot et al., 2014](BDIM)

- These are radiative, quantum corrections, argued to come from the single-scattering regime

- Both of these calculations were done
  - in a static-scatterer/random-colour field picture $\Rightarrow$ justifies Instantaneous Approximation
  - using HOA $\Rightarrow$ not well-suited to single-scattering regime

Classical vs. Quantum

Which is larger: $K\mathcal{O}(g)$ or $\ln^2(\#)\mathcal{O}(g^2)$?

Hard to say... But can definitey make a start by revisiting computation of quantum corrections

Outline

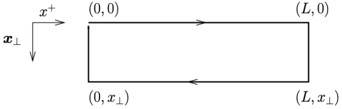
How do we calculate it?

- Assume an infinitely long medium and send $E \to \infty$ so that the parton's behaviour eikonalizes

- $\hat{q}$ can be related to the transverse scattering rate, $\mathcal{C}(k_\perp)$

$$\hat{q}(\mu) = \int^\mu \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp)$$

$$\lim_{L\to\infty} \langle W(x_\perp) \rangle = \exp(-\mathcal{C}(x_\perp) L)$$



[Ghiglieri and Teaney, 2015]

- $W(x_\perp)$ is a Wilson loop defined in the $(x^+, x_\perp)$ plane [Casalderrey-Solana and Teaney, 2007, D'Eramo et al., 2011, Benzke et al., 2013]

Thermal Scales in a Weakly Coupled QGP

- T, hard scale associated with energy of individual particles
  $\Rightarrow$ hard-hard interactions can be described perturbatively

- gT, soft scale associated with energy of collective excitations
  $\Rightarrow$ soft-soft interactions can also be described perturbatively

- g^2T, ultrasoft scale is associated with nonperturbative physics
  $\Rightarrow$ loops can be added at no extra cost
  $\Rightarrow$ cannot use perturbation theory

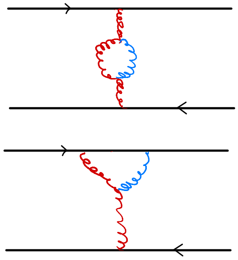
- For hard-soft interactions, we are not so lucky either...
- Turns out that one can add loops for free
  $\implies$ perturbative expansion breaks down
- Hard Thermal Loop (HTL) effective theory comes to the rescue, allowing us to resum these loops

Some Wilson loop diagrams

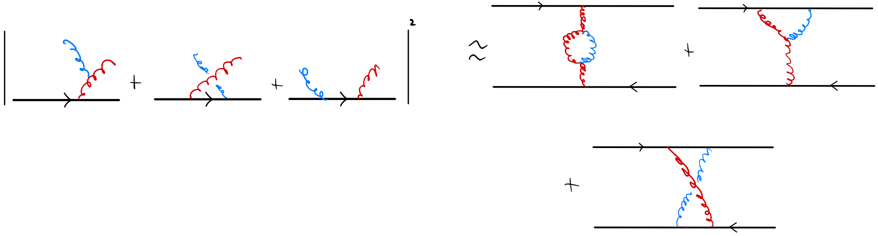
- Can think of sticking together amplitude and conjugate amplitude to get diagrams on the right

- **Black lines** represent hard parton in the amplitude and conjugate amplitude
- Red gluons are bremsstrahlung, represented by thermal propagators

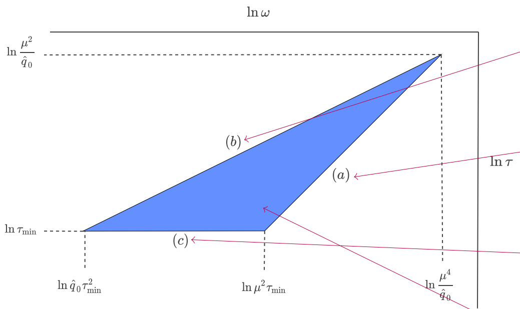
- Blue gluons are those that are exchanged with the medium and are represented by Hard Thermal Loop propagators



Where do these diagrams come from?



Double Logs from BDIM/LMW



$$\tau < \sqrt{\frac{\omega}{\hat{q}_0}}$$

Keeps us away from multiple-scattering

$$\tau > \frac{\omega}{\mu^2}$$

Comes from enforcing UV cutoff, μ on $k_\perp$

$$\tau > \tau_{\rm min} \sim \frac{1}{T}$$

IR cutoff on τ

$$\delta\hat{q}_0 = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\rm min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau}\int_{\hat{q}_0\tau^2}^{\mu^2\tau}\frac{d\omega}{\omega} = \frac{1}{2}\ln^2\frac{\mu^2}{\hat{q}_0\tau_{\rm min}} \propto \text{Area}$$

Need to adapt BDIM/LMW result to weakly coupled QGP setting

$$\delta\hat{q}_0 = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}$$

$\hat{q}_0 \sim g^4 T^3$

$1/g^2T$ timescale
for multiple soft scatterings

$$\downarrow$$

$$\delta\hat{q}_{1+2} = \frac{\alpha_s C_R}{\pi} \int_{\tau_{\text{int}}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}(1 + 2n_B(\omega))$$

$$= \frac{\alpha_s C_R}{2\pi}\hat{q}_0 \left\{ \ln^2 \frac{\mu^2}{\hat{q}_0\tau_{\text{int}}} - \frac{1}{2}\ln^2 \frac{\omega_T}{\hat{q}_0\tau_{\text{int}}^2} \right\}$$

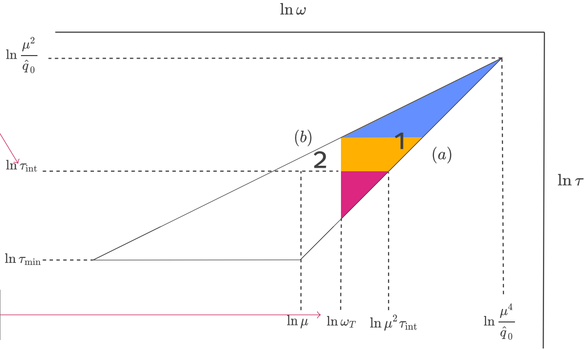
Consider $\mu < T$

$n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$

Introduce intermediate regulator
$\tau_{\text{int}} \ll 1/g^2T$

Double Logs in a Weakly Coupled QGP

Intermediate regulator τ_{int}



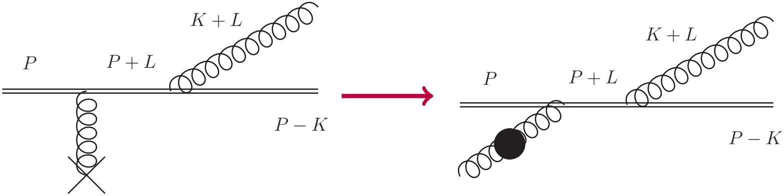
$\omega_T \equiv 2\pi T e^{-\gamma_E}$

$$\delta\hat{q}_{1+2} = \frac{\alpha_s C_R}{2\pi}\hat{q}_0\left\{\ln^2\frac{\mu^2}{\hat{q}_0\tau_{\text{int}}} - \frac{1}{2}\ln^2\frac{\omega_T}{\hat{q}_0\tau_{\text{int}}^2}\right\}$$

Strict Single-Scattering in a Weakly Coupled QGP

- Compute $\mathcal{C}(k_\perp)$ using HTL resummation instead of Random Colour Approximation

- Investigate which logs are produced by *soft, collinear* modes through a *semi-collinear* process associated with formation time $\tau_{\text{semi}} \sim 1/gT$ [Ghiglieri et al., 2013, Ghiglieri et al., 2016]

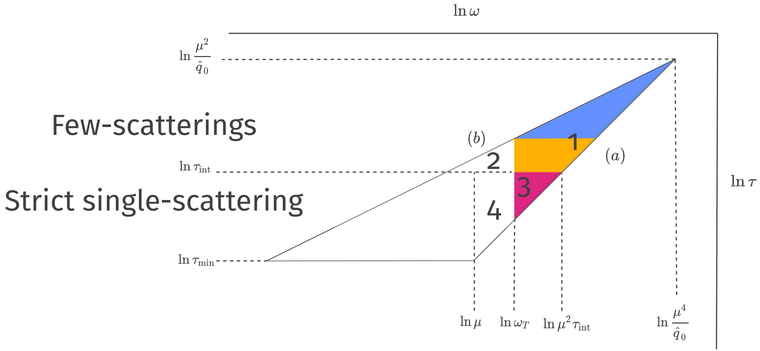


Only spacelike interactions with medium

Now timelike interactions are allowed too

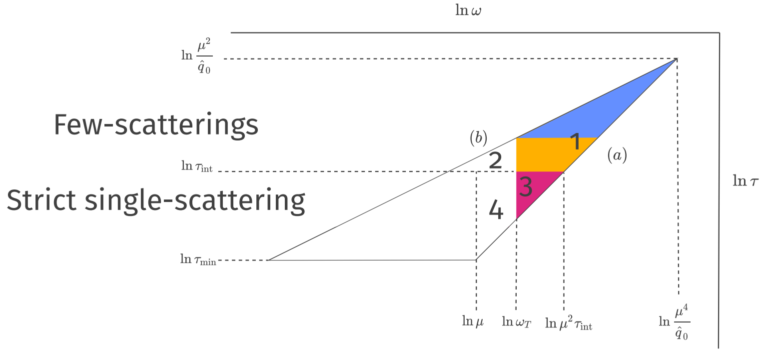
$\Rightarrow$ Going beyond instantaneous approximation

Strict Single-Scattering Contribution



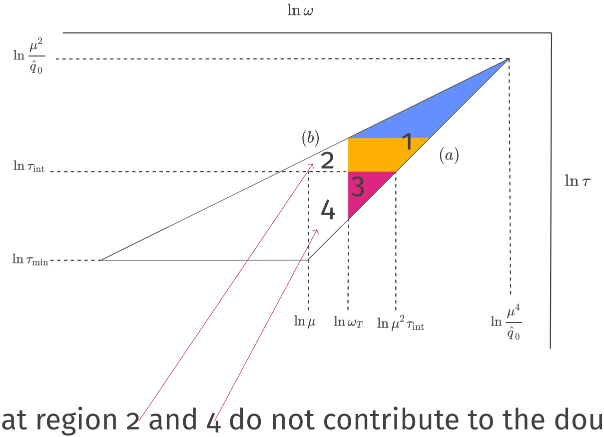
$$\delta \hat{q}_{3+4} = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \ln^2 \frac{\mu^2 \tau_{\rm int}}{\omega_T} + \text{subleading logs}$$

Total Double Log Contribution



$$\delta\hat{q}_{\rm DLog} = \delta\hat{q}_{1+2} + \delta\hat{q}_{3+4} = \frac{\alpha_s C_R}{4\pi}\hat{q}_0 \ln^2\frac{\mu^4}{\hat{q}_0\omega_T} \propto \mathrm{Area}_{1+3}$$

Double Logs in a Weakly Coupled QGP



Why is it that region 2 and 4 do not contribute to the double Logs?

Vacuum and Quantum Correction Cancellation

First, note that

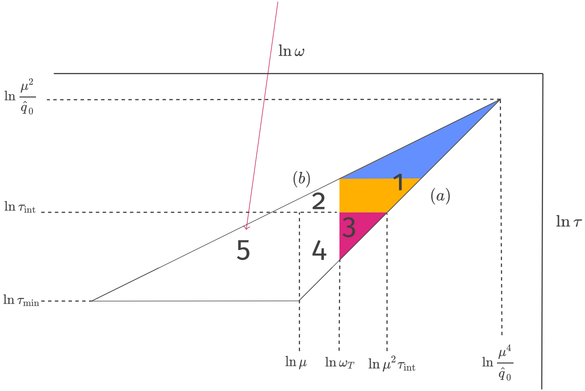
$$\lim_{\frac{\omega}{T}->0}\Big(1+2n_B(\omega)\Big)=1+\frac{2T}{\omega}-1 \tag{5}$$

The absence of the IR scale in any logarithms can then be seen by looking at the following integral, with $\nu_{IR} \ll T \ll \nu_{UV}$

$$\begin{aligned}\int_{\nu_{IR}}^{\nu_{UV}}\frac{d\omega}{\omega}\Big(\underbrace{1}_{\text{vacuum}}+\underbrace{2n_B(\omega)}_{\text{thermal}}\Big) &= \underbrace{\ln\frac{\nu_{uv}}{\nu_{IR}}}_{\text{vacuum}}+\underbrace{\frac{2T}{\nu_{IR}}-\ln\frac{2\pi T}{\nu_{IR}e^{\gamma_E}}+\ldots}_{\text{thermal}}\\ &= \frac{2T}{\nu_{IR}}+\ln\frac{\nu_{UV}e^{\gamma_E}}{2\pi T}+\ldots\end{aligned} \tag{6}$$

Relation to Soft Corrections

Region of phase space from which $\mathcal{O}(g)$ corrections emerge



Classical $\dfrac{T}{\nu_{\rm IR}}$ corrections cancel against those extracted from [Caron-Huot, 2009]!

More on our Corrections

- Take HOA so as to postpone dealing with neighbouring region where single scattering and multiple scattering simultaneously become important
- We find that all parts of our final expression, which come from relaxing the instantaneous approximation are subleading
- If we consider $\mu > T$, we can show that our results become closer to the BDIM/LMW double logs

Summary of Results/Conclusions

- Double and single logarithmic corrections to $\hat{q}$ computed within the setting of a weakly coupled QGP
- Emended BDIM/LMW result so that it includes thermal corrections
- Can show how our result fits with respect to these emended corrections as well as the soft corrections coming in at $\mathcal{O}(g)$
- Still would like to better understand the phase space boundary between single-scattering and multiple-scattering

Outlook – Going Beyond Harmonic Oscillator Approximation

HOA not well-suited to single-scattering

⇒So how can we go beyond it?

$$\delta\hat{q}_{\mathrm{DLog}} = \frac{\alpha_s C_R}{4\pi}\hat{q}_0 \ln^2 \# \longrightarrow \frac{\alpha_s C_R}{4\pi}\hat{q}(\rho) \ln^2 \#$$

where $\hat{q}(\rho) \propto \ln \frac{\rho^2}{m_D^2}$

ρ separates us from neighbouring region
with simultaneously single-scattering and multiple scatterings

Need to solve transverse momentum-dependent LPM equaition
without HOA in order to shed light on how this
region could be dealt with

Thanks for listening!

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Relating $\mathcal{C}(k_\perp)$ to the Wilson Loop

Wilson loop defined, in the $x^- = 0$ plane in as

$$\langle W(x_\perp, 0)\rangle = \frac{1}{N_c}\,\mathrm{Tr}\langle [0, x_\perp]_- \mathcal{W}^\dagger(x_\perp)[x_\perp, 0]_+ \mathcal{W}(0)\rangle, \tag{7}$$

where

$$\mathcal{W}(x_\perp) = \mathcal{P}\exp\left(ig\int_{-\frac{L}{2}}^{\frac{L}{2}} dx^+ A^-(x^+, x_\perp)\right) \tag{8}$$

One can show that [D'Eramo et al., 2011, Benzke et al., 2013]

$$\lim_{L\to\infty}\langle W(x_\perp)\rangle = \exp(-\mathcal{C}(x_\perp)L) \tag{9}$$

where

$$\mathcal{C}(x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}(1 - e^{i\mathbf{x}_\perp\cdot\mathbf{k}_\perp})\mathcal{C}(k_\perp) \tag{10}$$

Parametric form of $\hat{q}$

$$\hat{q} \sim \alpha_s^2 T^3 \{C_1 \ln \frac{T}{m_D} + C_2 \ln \frac{\mu}{T} + C_3 + Kg + \alpha_s (C_5 \ln^2(\#) + C_6 \ln \#' + \ldots)\} \tag{11}$$