Dark Matter from Lorentz Invariance in 6 dimensions

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work in progress with
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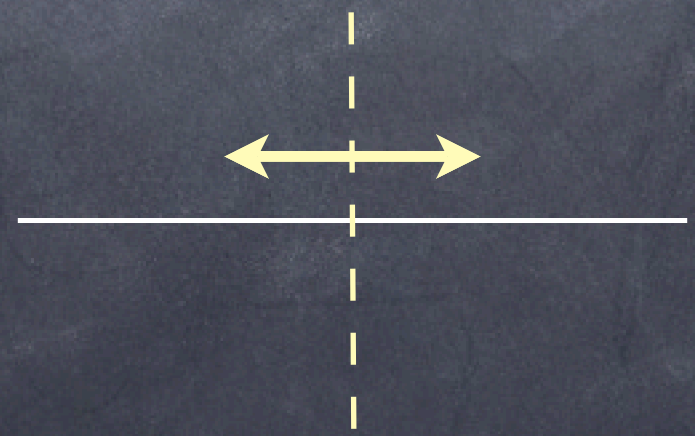
Theorie LHC France / GDR Terascale Tools / FCPPL Hadron Satellite

A "natural" Dark Matter candidate

- Dark Matter is a "necessary" ingredient in the Universe
- The presence of a Dark Matter candidate in models of New Physics is a desirable feature
- However, in most models, it follows from an ad-hoc symmetry: R-parity in supersymmetry, T-parity in little Higgs models, KK-parity in extra dimensions, etc...

KK-parity = reflection wrt centre of interval
good symmetry in the bulk, but...

5D case:



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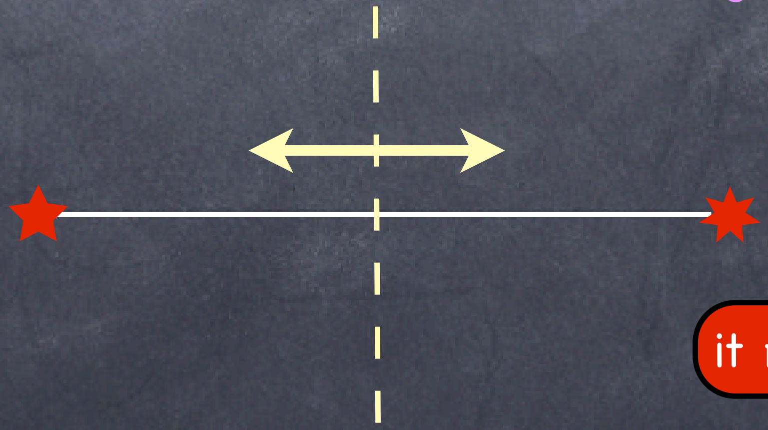


it requires identification of endpoints!

A “natural” Dark Matter candidate

Extra dimensions are a versatile tool:

- Many interesting models: Gauge-Higgs unification, Higgsless models, GUTs, composite Higgs, technicolour, QCD...
- Interesting models do not have a KK parity (i.e. incompatible with localisation, warping...): not generic and not “natural”!
- We found a unique “natural” scenario in 6 dimensions.
- I will briefly discuss the LHC phenomenology of such scenario.

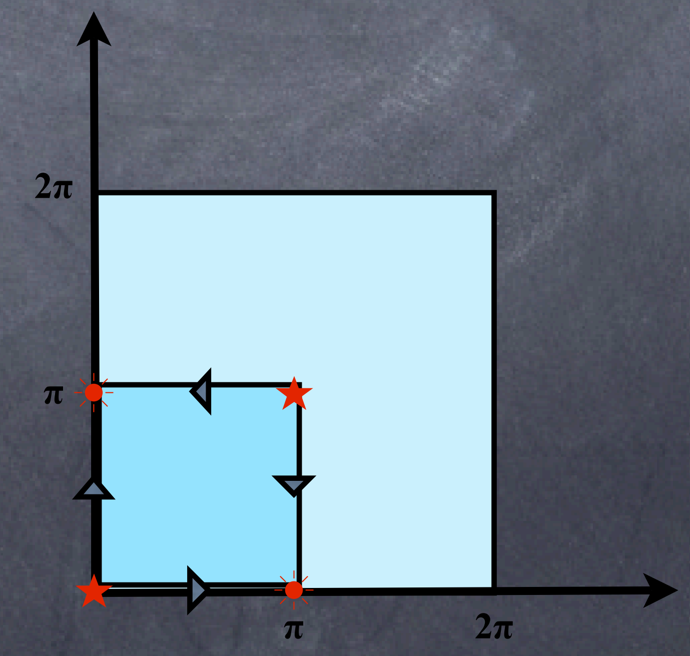
The real projective plane

$$\mathbf{pgg} = \langle r, g | r^2 = (g^2 r)^2 = \mathbf{1} \rangle$$

$$r : \begin{cases} x_5 \sim -x_5 \\ x_6 \sim -x_6 \end{cases} \qquad g : \begin{cases} x_5 \sim x_5 + \pi R_5 \\ x_6 \sim -x_6 + \pi R_6 \end{cases}$$

Translations defined as:

$$\begin{aligned} t_5 &= g^2 \\ t_6 &= (gr)^2 \end{aligned}$$



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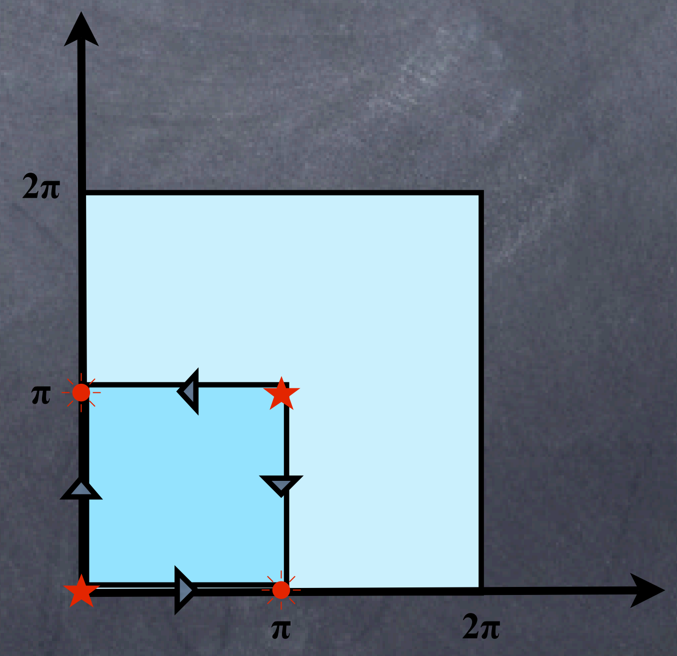
Two singular points:

$$(0, \pi) \sim (\pi, 0)$$

$$(0, 0) \sim (\pi, \pi)$$



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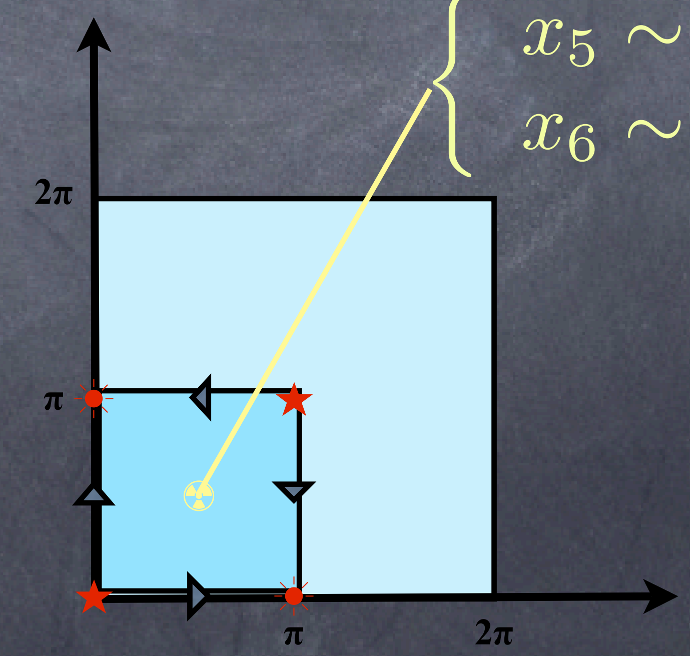
KK parity is an exact symmetry of the space!

$$p_{KK} : \begin{cases} x_5 \sim x_5 + \pi \\ x_6 \sim x_6 + \pi \end{cases}$$

$$\begin{cases} x_5 \sim -x_5 + \pi \\ x_6 \sim -x_6 + \pi \end{cases}$$



Example: a scalar field

Action for a massless scalar:

$$S = \int_0^{2\pi} dx_5 dx_6\, \partial_\mu \phi^\dagger \partial^\mu \phi - \partial_5 \phi^\dagger \partial_5 \phi - \partial_6 \phi^\dagger \partial_6 \phi$$

The equation of motion $[p^2 + \partial_5^2 + \partial_6^2]\phi(p, x_5, x_6) = 0$

is solved by $\phi(p, x_5, x_6) = \sum_{k,l} f_{(k,l)}(x_5, x_6)\, \phi_{(k,l)}(p)$

4D field!

with:

$$f_{(k,l)}(x_5, x_6) = \begin{cases} \cos(kx_5)\cos(lx_6) \\ \cos(kx_5)\sin(lx_6) \\ \sin(kx_5)\cos(lx_6) \\ \sin(kx_5)\sin(lx_6) \end{cases} \Rightarrow p^2 = k^2 + l^2$$

Example: a scalar field

The parity of the field selects the solutions!

$$f_{(k,l)}(x_5, x_6) = \begin{cases} \cos(kx_5)\ \cos(lx_6) \\ \cos(kx_5)\ \sin(lx_6) \\ \sin(kx_5)\ \cos(lx_6) \\ \sin(kx_5)\ \sin(lx_6) \end{cases}$$

Rot.	Glide	KK
+		
-		
-		
+		

$$\text{Rotation: } \begin{cases} x_5 \to -x_5 \\ x_6 \to -x_6 \end{cases}$$

Example: a scalar field

The parity of the field selects the solutions!

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Rot.	Glide	KK
+	$p_{k,l}$	
-	$-p_{k,l}$	
-	$p_{k,l}$	
+	$-p_{k,l}$	

$$p_{k,l} = (-1)^{k+l}$$

$$\text{Rotation: } \begin{cases} x_5 \to -x_5 \\ x_6 \to -x_6 \end{cases}$$

$$\text{Glide: } \begin{cases} x_5 \to x_5 + \pi \\ x_6 \to -x_6 + \pi \end{cases}$$

Example: a scalar field

The parity of the field selects the solutions!

$$f_{(k,l)}(x_5, x_6) = \begin{cases} \cos(kx_5)\ \cos(lx_6) \\ \cos(kx_5)\ \sin(lx_6) \\ \sin(kx_5)\ \cos(lx_6) \\ \sin(kx_5)\ \sin(lx_6) \end{cases}$$

Rot.	Glide	KK
+	$p_{k,l}$	$p_{k,l}$
-	$-p_{k,l}$	$p_{k,l}$
-	$p_{k,l}$	$p_{k,l}$
+	$-p_{k,l}$	$p_{k,l}$

$$p_{k,l} = (-1)^{k+l}$$

$$\text{Rotation:} \begin{cases} x_5 \to -x_5 \\ x_6 \to -x_6 \end{cases}$$

$$\text{KK parity:} \begin{cases} x_5 \to x_5 + \pi \\ x_6 \to x_6 + \pi \end{cases}$$

$$\text{Glide:} \begin{cases} x_5 \to x_5 + \pi \\ x_6 \to -x_6 + \pi \end{cases}$$

Toy model for phenomenology: the SM on the real projective plane

- To each SM field -> a 6D field

- For simplicity, from now on I'll set:

$$R_5 = R_6 = R = 1$$

- All masses in unit of:

$$m_{KK} = \frac{1}{R}$$

Spectrum of KK levels

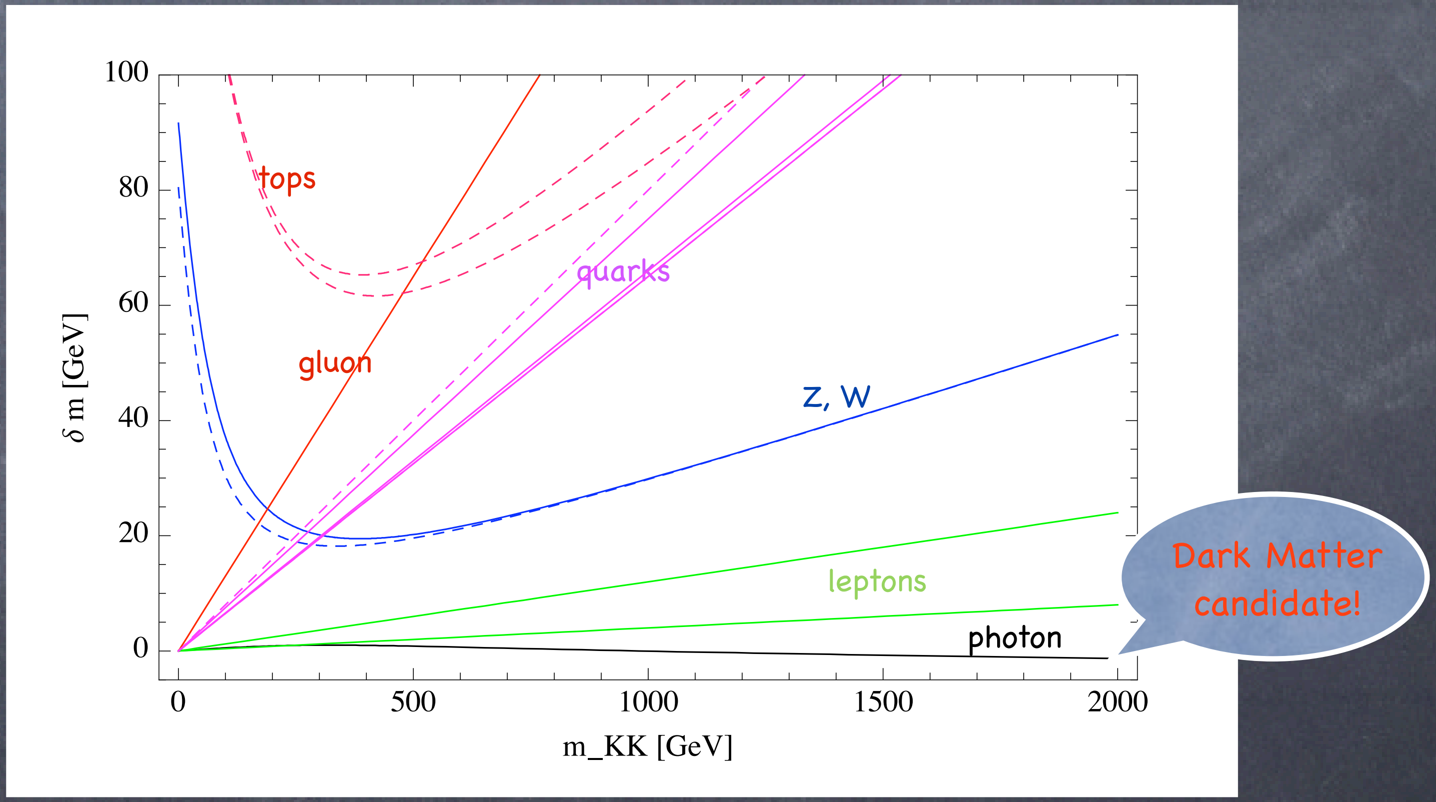
	+	–	+	+	–
$p_{KK} = (-1)^{k+l}$	(0,0) m = 0	(1,0) & (0,1) m = 1	(1,1) m = 1.41	(2,0) & (0,2) m = 2	(2,1) & (1,2) m = 2.24
Gauge bosons G, A, Z, W	✓		✓	✓	✓
Gauge scalars G, A, Z, W		✓	✓		✓
Higgs boson(s)	✓		✓	✓	✓
Fermions	✓	✓	✓ (x2)	✓	✓ (x2)

Spectrum of KK levels

- Small splittings inside the KK tier are generated by loop corrections, the Higgs VEV and localised operators (counterterms)

Level (1,0) and (0,1)

$$m = m_{KK} + \delta m$$



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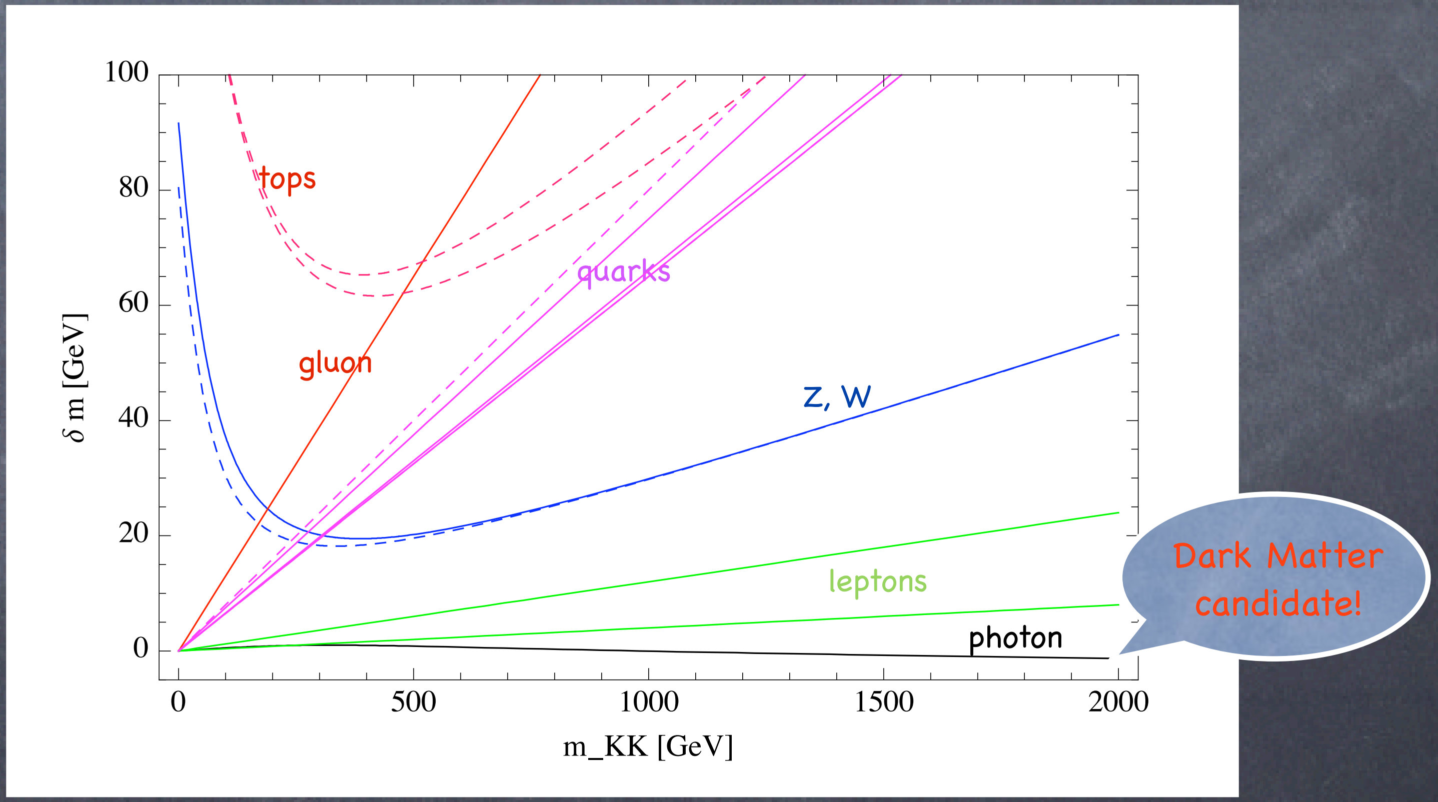
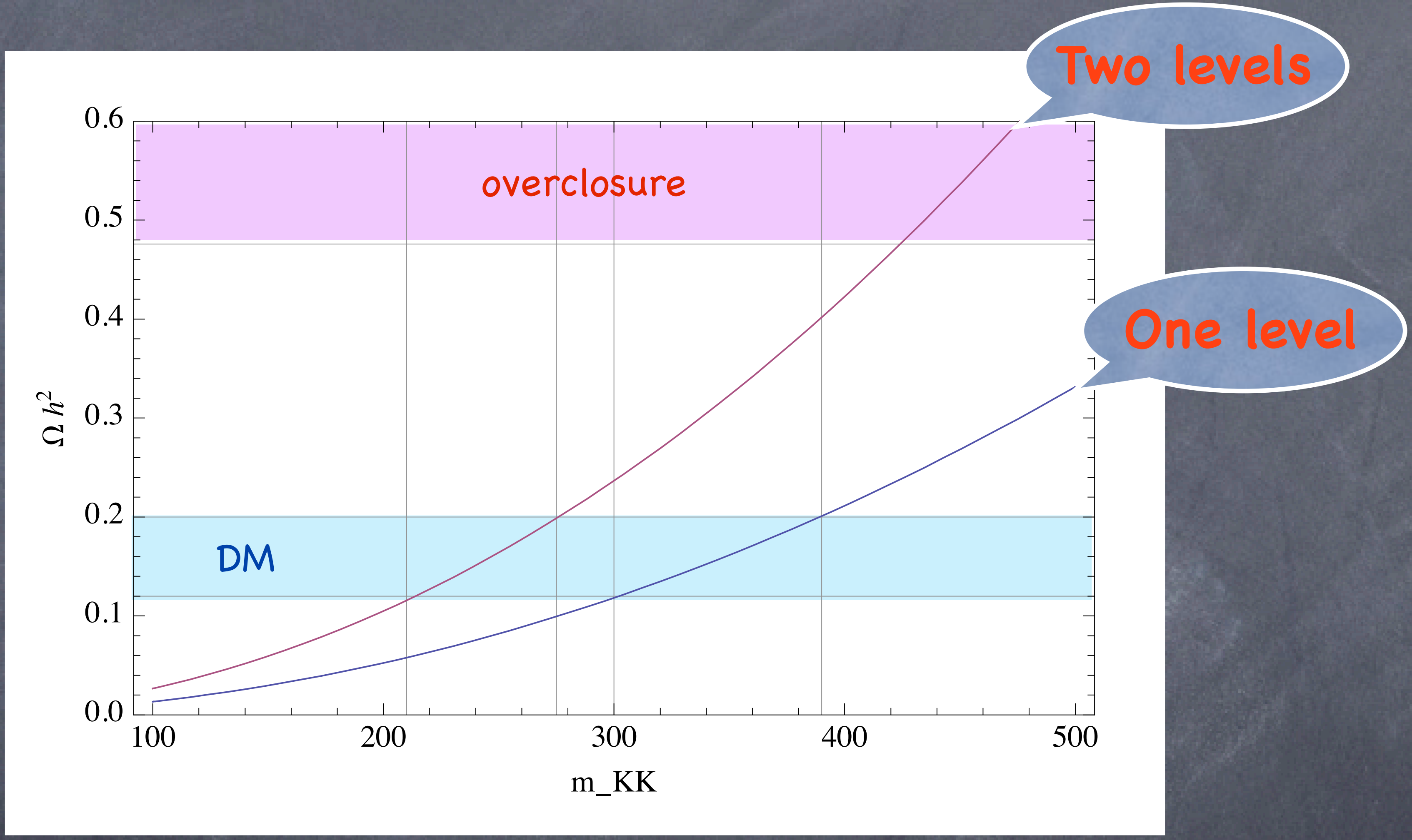


Relic abundance

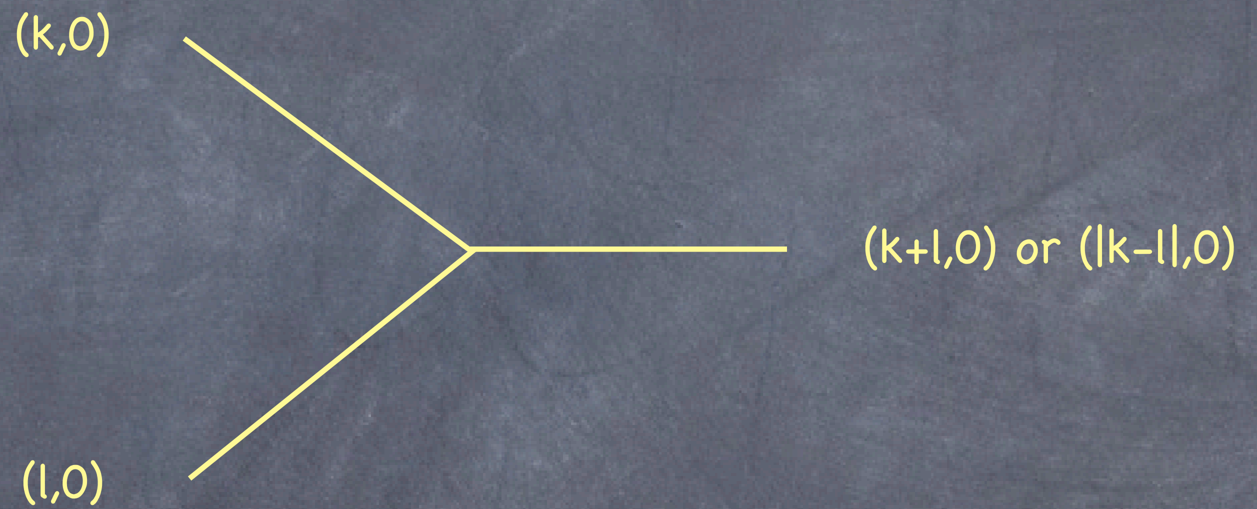


200 < mKK < 400 GeV

5D: 600-1200 GeV, 6D: 200 GeV

Phenomenology: interactions

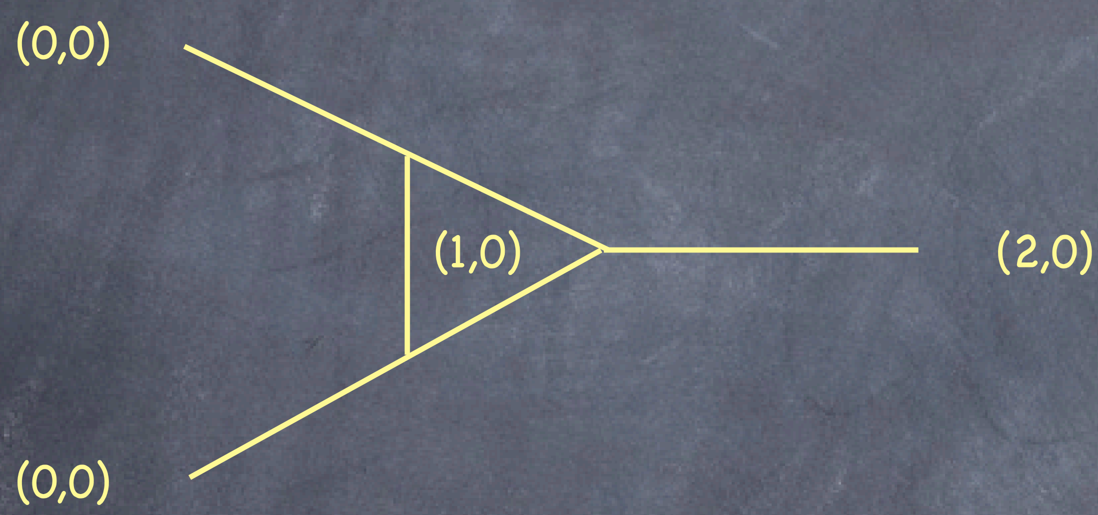
- Bulk interactions: conservation of XD momentum!

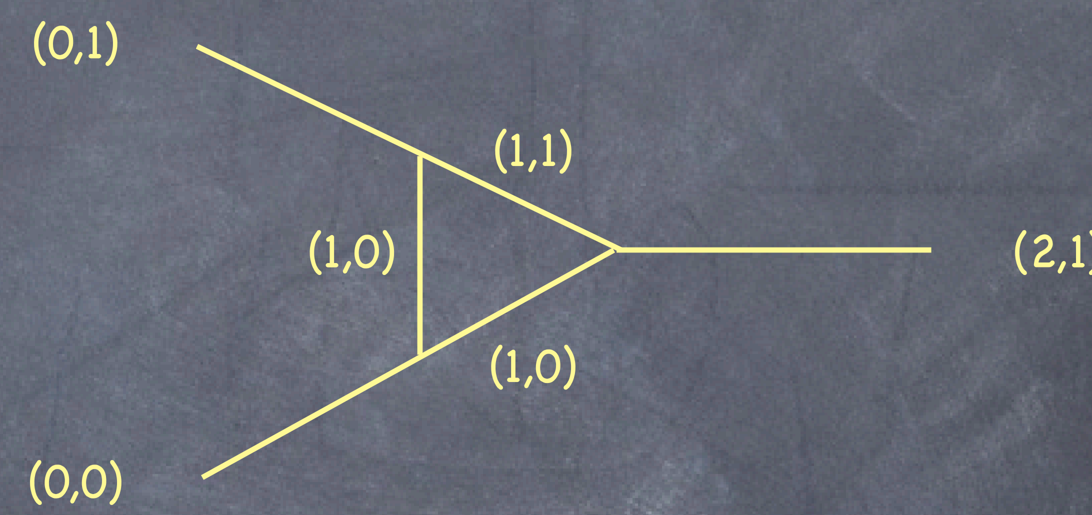


- Only pair production off SM states is allowed!
- Same as SM couplings (up to normalization factors)!
- Large production cross sections!

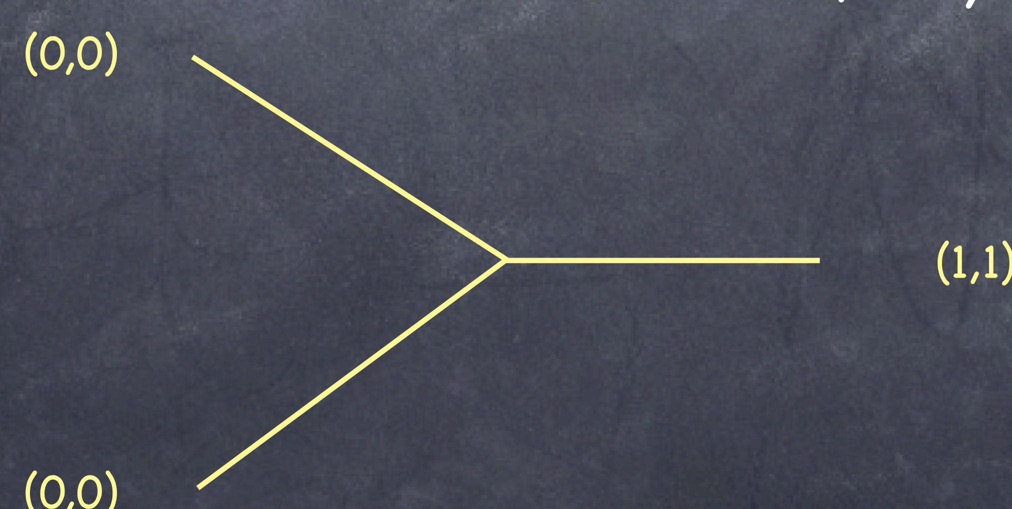
Phenomenology: interactions

- Loop interactions: suppressed, but less constrained.
- Single production and decays





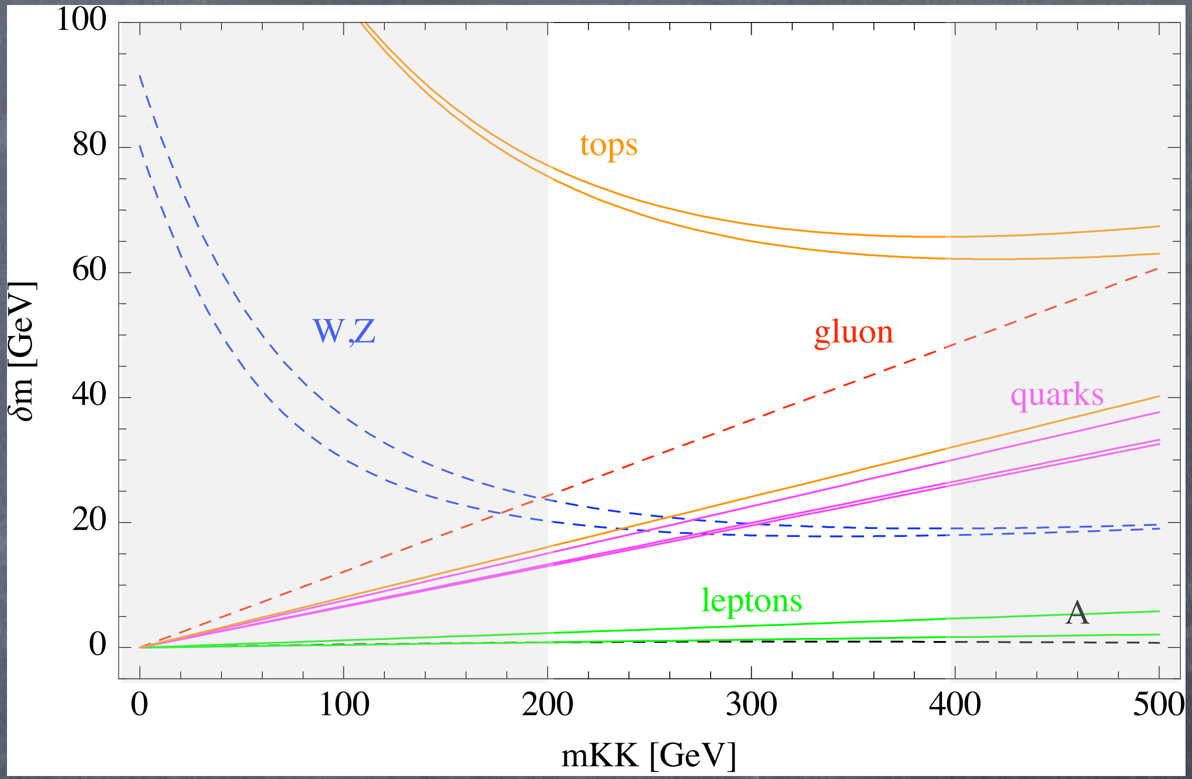
- Localized interactions: even less constrained, only preserve KK parity



Phenomenology at the LHC: tiers (1,0) and (0,1)

- Small splittings make detection of lightest tier challenging:

	$m_X - m_{LLP}$ in GeV	decay mode	final state + MET
$t^{(1,0)}$	70	$bW^{(1,0)}$	bjj $bl\nu$
$G^{(1,0)}$	40-70	$qq^{(1,0)}$	jj
$q^{(1,0)}$	20-40	$qA^{(1,0)}$	j
$W^{(1,0)}$	20	$l\nu^{(1,0)}$, $\nu l^{(1,0)}$	$l\nu$
$Z^{(1,0)}$	20	$ll^{(1,0)}$	ll
$l^{(1,0)}$	< 5	$lA^{(1,0)}$	l
$A^{(1,0)}$	0	-	



Phenomenology at the LHC: tiers (2,0) and (0,2)

- Decay in pair of SM particles (via vertices at 1-loop)
- Small splittings: suppressed or forbidden decays in pair of (1,0)

exceptions:

$W_{(2,0)}$, $Z_{(2,0)}$ -> $l_{(1,0)}$ $l_{(1,0)}$

$top_{(2,0)}$ -> $W_{(1,0)}$ $b_{(1,0)}$

$g_{(2,0)}$ -> $q_{(1,0)}$ $q_{(1,0)}$

...

	$m_X - m_{LLP}$ in GeV	decay mode	final state + MET
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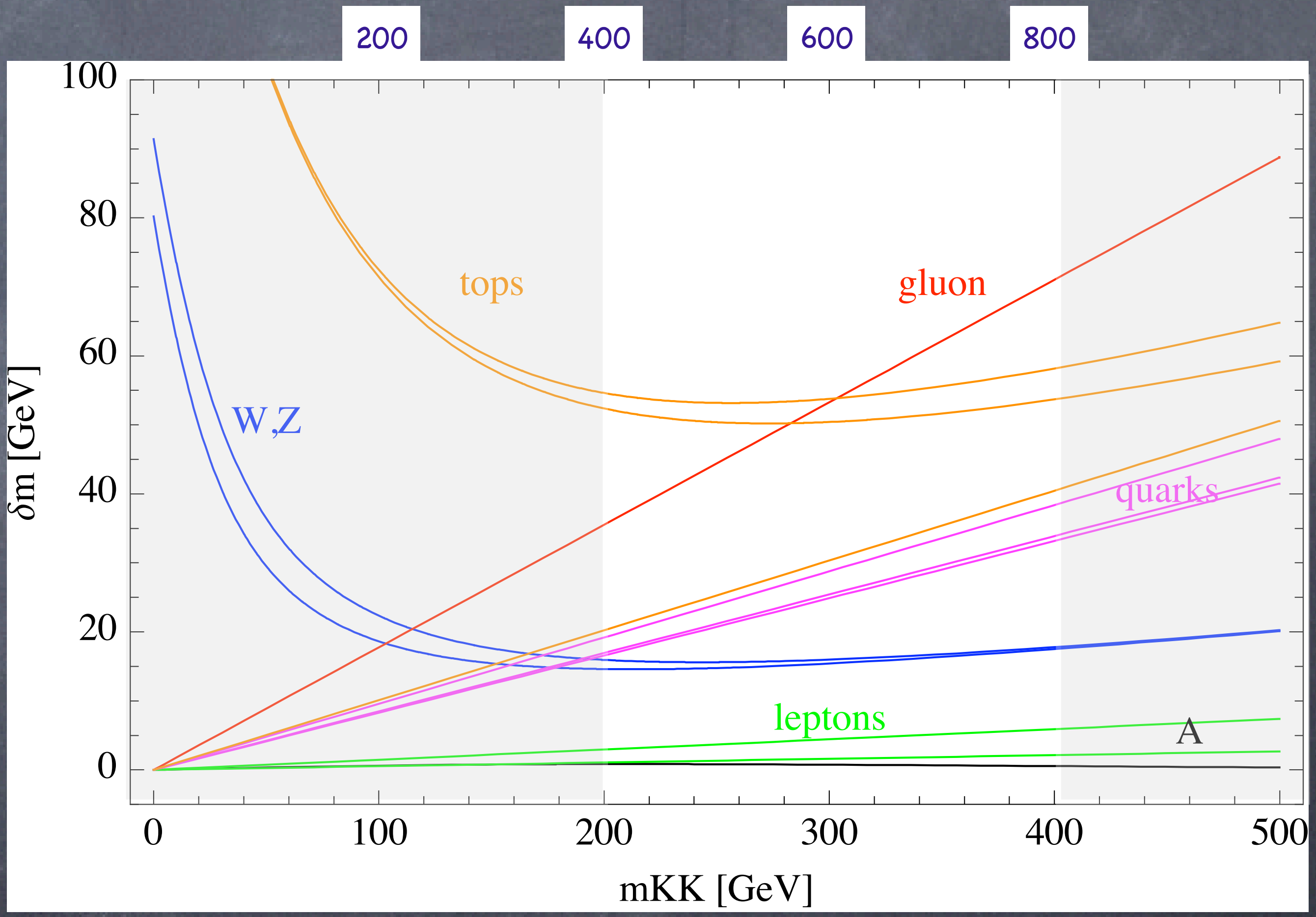
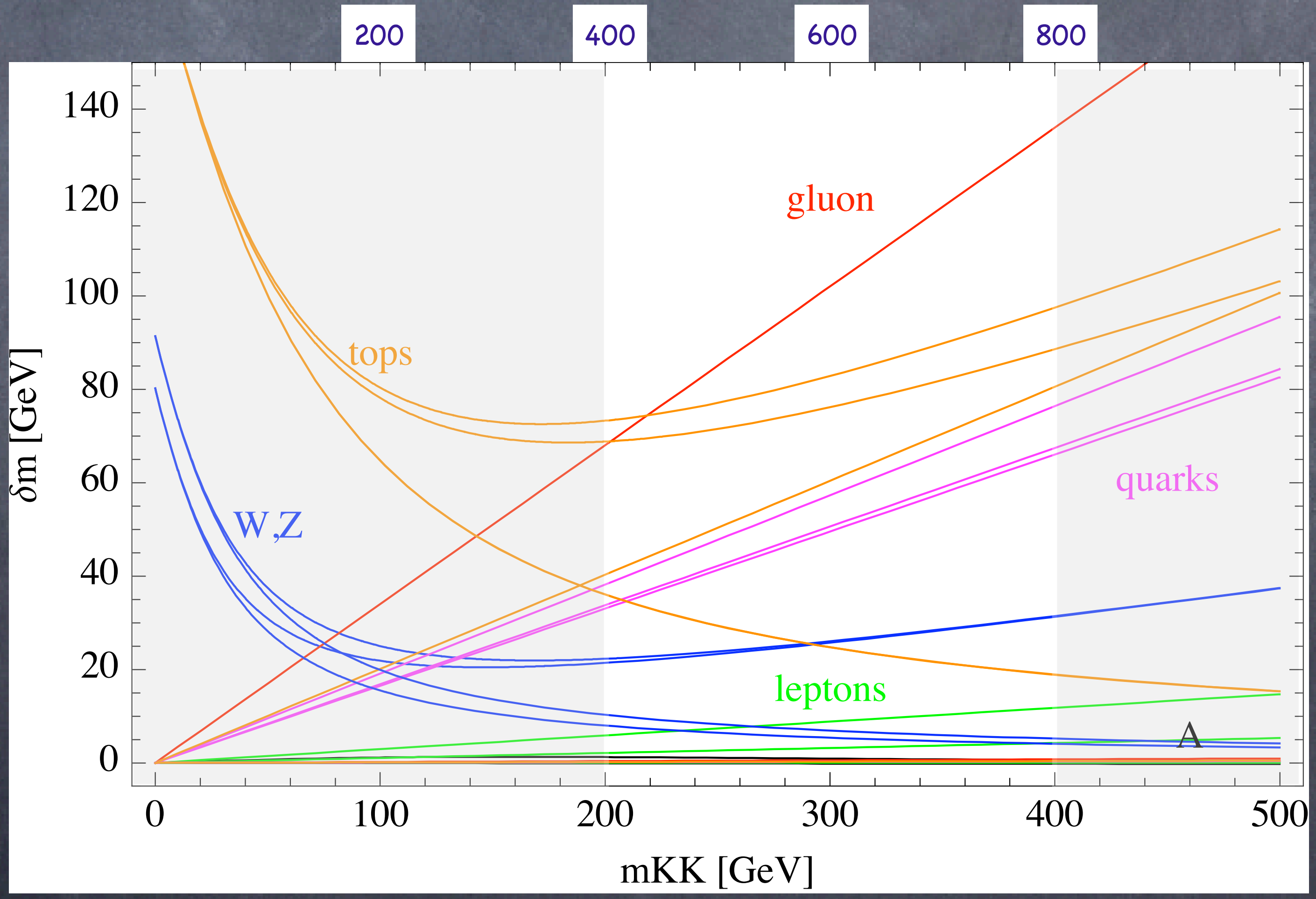


Phenomenology at the LHC: (2,0)-(0,2) degenerate case

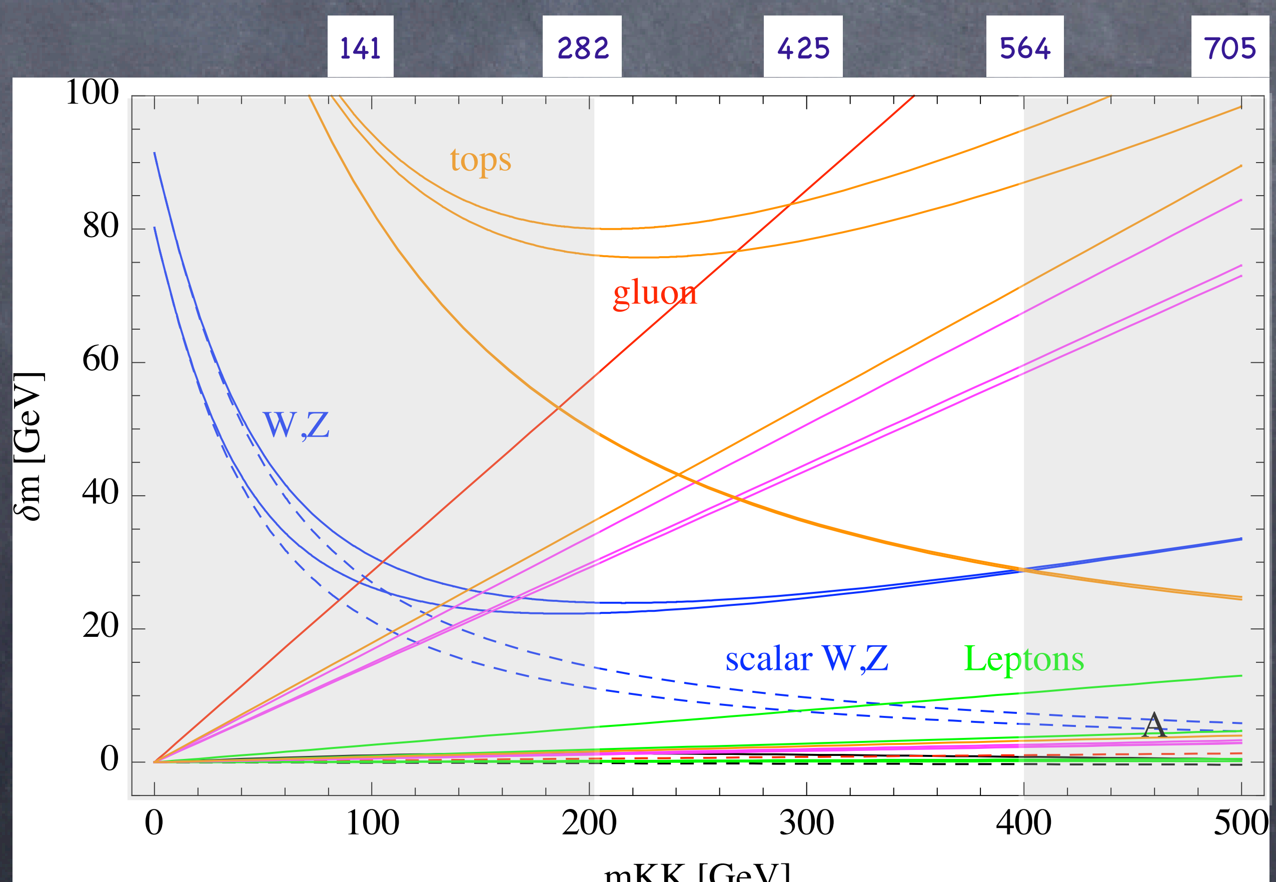
- loop induced mixing cannot be neglected: one heavier state, and a lighter (cut-off independent) one
- More (1,0)-(1,0) channels are open



Phenomenology at the LHC: tier (1,1) - 4 tops signal

In collab. with Lyon CMS group

- Large corrections for vectors, small (finite) for scalars.
- vector gluon largely produced (few to 60 pb!) and chain decay to vector photon.
- vector photon may decay to pair of tops: 4 tops + 4 soft jets!
- 2 x ($G_{(1,1)} \to \bar{q}\, q\, A_{(1,1)} \to \bar{q}\, q\, t\, t$)



Phenomenology at the LHC

- Small splittings make detection of lightest tier challenging: need boost to see!
- Tiers (1,1) and (2,0) decay to SM particles: nice resonances, but no MET! Interesting degenerate case.
- Tier (2,1) decays in (1,0) + (0,0): SM + MET!

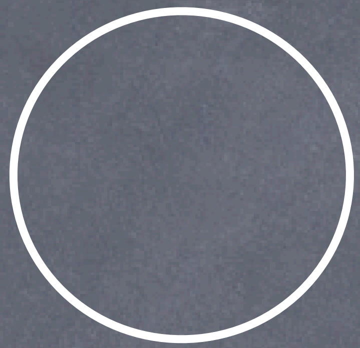
Conclusions and outlook

- KK parity can be a "natural" (not ad-hoc) symmetry - relic of Lorentz invariance
- Interesting models can be implemented: Gauge-Higgs unification, fermion masses, etc.
- We studied the UNIQUE 6D geometry where this happens
- New Phenomenology from other models in the literature: light resonances, small splittings...
- We implemented the model in FeynRules: easy interface with calcHep, Madgraph, FeynArt...
- Rich phenomenology: challenging MET signals, 4 tops, resonances...

Bonus track

Intro to XD: a scalar field

Action for a massless scalar:



$$S = \int_0^{2\pi} dx_5 \, \partial_\mu \phi^\dagger \partial^\mu \phi - \partial_5 \phi^\dagger \partial_5 \phi$$

The equation of motion $$[p^2 + \partial_5^2] \, \phi(p, x_5) = 0$$

is solved by $$\phi(p, x_5) = \sum_k f_{(k)}(x_5) \, \phi_{(k)}(p)$$

4D field!

with:

$$f_{(k)} = \begin{cases} \cos(k x_5) \\ \sin(k x_5) \end{cases} \Rightarrow \; p^2 = k^2$$

Note that under x5 -> -x5, cos -> + cos while sin -> -sin!
Also, k=0 only allowed for cos!

Gauge bosons

$$S_{\rm gauge} = \int_0^{2\pi} dx_5\, dx_6 \left\{ -\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - \frac{1}{2\xi} \left(\partial_\mu A^\mu - \xi(\partial_5 A_5 + \partial_6 A_6)\right)^2 \right\}$$

gauge fixing term

After solving the Equations of Motion,
and imposing orbifold parities [μ → (++), 5 → (−+), 6 → (−−)]
the spectrum is:

$$p_{KK} = (-1)^{k+l} \qquad m_{(k,l)} = \sqrt{k^2 + l^2}$$

(k,l)	p_{KK}	$A_\mu^{(++)}$	$A_5^{(-+)}$	$A_6^{(--)}$
$(0,0)$	$+$	$\frac{1}{2\pi}$		
$(0,2l)$	$+$	$\frac{1}{\sqrt{2}\pi}\cos 2lx_6$		
$(0,2l-1)$	$-$		$\frac{1}{\sqrt{2}\pi}\sin(2l-1)x_6$	
$(2k,0)$	$+$	$\frac{1}{\sqrt{2}\pi}\cos 2kx_5$		
$(2k-1,0)$	$-$			$\frac{1}{\sqrt{2}\pi}\sin(2k-1)x_5$
$(k,l)_{\rm k+l\ even}$	$+$	$\frac{1}{\pi}\cos kx_5 \cos lx_6$	$\frac{l}{\pi\sqrt{k^2+l^2}}\sin kx_5 \cos lx_6$	$-\frac{k}{\pi\sqrt{k^2+l^2}}\cos kx_5 \sin lx_6$
$(k,l)_{\rm k+l\ odd}$	$-$	$\frac{1}{\pi}\sin kx_5 \sin lx_6$	$\frac{l}{\pi\sqrt{k^2+l^2}}\cos kx_5 \sin lx_6$	$-\frac{k}{\pi\sqrt{k^2+l^2}}\sin kx_5 \cos lx_6$

Splittings I: loops

- Generic loop contributions can be written as:

$$\Pi = \Pi_T + p_g \, \Pi_G + p_r \, \Pi_R + p_g p_r \, \Pi_{G'}$$

Log divergence!

- For gauge scalars, tier (1,0):

$$\begin{aligned}
\delta m_B^2 &= \frac{g'^2}{64\pi^4 R^2}\left[-79T_6 + 14\zeta(3) + \pi^2 n^2 L + \dots\right], \\
\delta m_W^2 &= \frac{g^2}{64\pi^4 R^2}\left[-39T_6 + 70\zeta(3) + 17\pi^2 n^2 L + \dots\right], \\
\delta m_G^2 &= \frac{{g_s}^2}{64\pi^4 R^2}\left[-36T_6 + 84\zeta(3) + 24\pi^2 n^2 L + \dots\right].
\end{aligned}$$

- Divergence localized on singular points and proportional to the tier mass!

- Proportional to the KK mass scale!

Splittings I: loops

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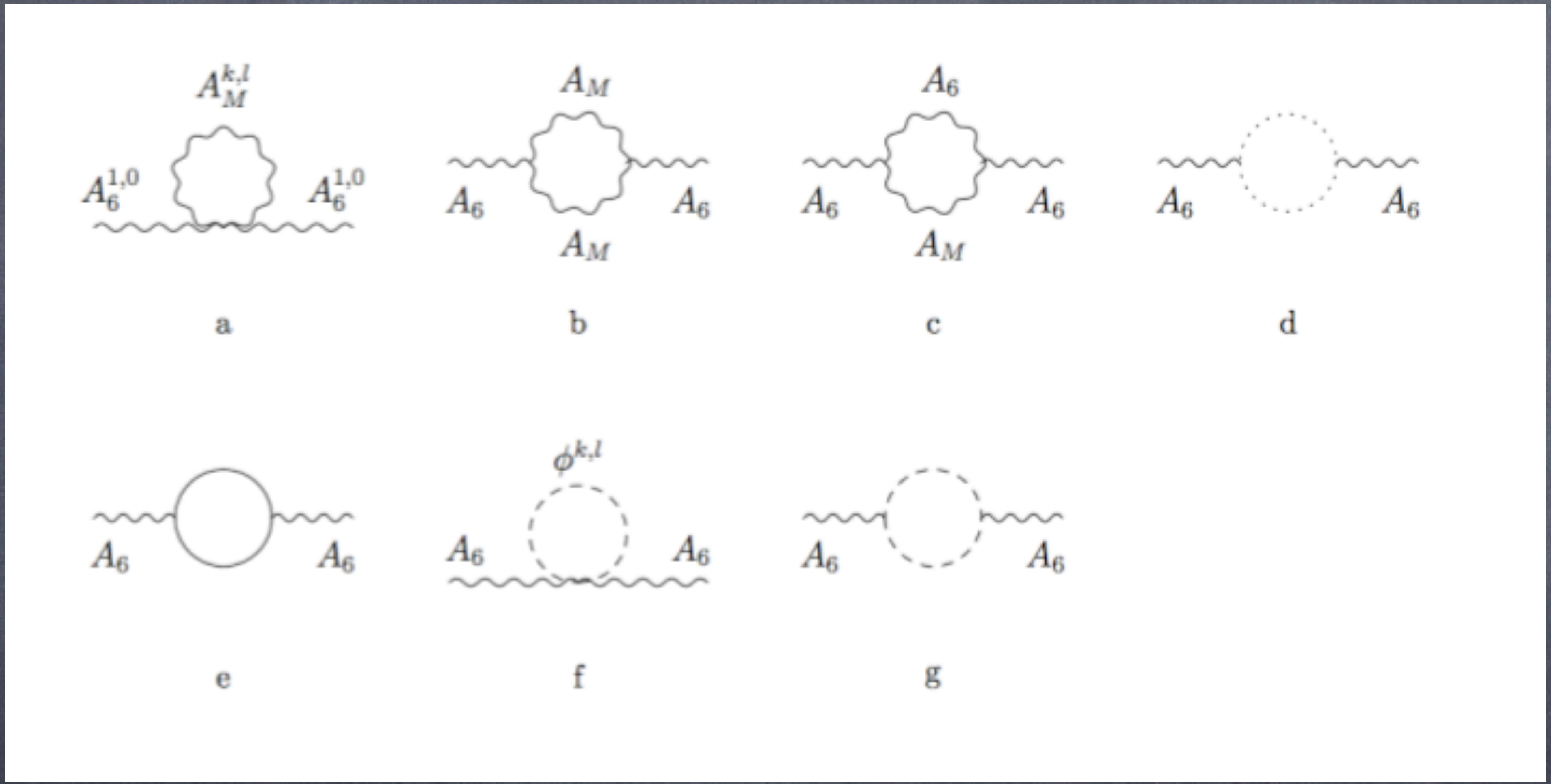

- Divergence localized on singular points and proportional to the tier mass!
- Proportional to the KK mass scale!

Splittings II: Higgs VEV

- The Higgs VEV does not mix tiers (v is constant!)
- At level (0,0), we obtain the Standard Model!
- For massive tiers:

$$m^2_{(k,l)} = (k^2 + l^2) m^2_{KK} + m^2_0$$

- Mixing angle in the neutral gauge boson sector (A-Z): smaller than the Weinberg mixing angle!

$$W_n^3 \quad B_n \;) \cdot \begin{pmatrix} \delta m_W^2 + m_W^2 & -\tan\theta_W m_W^2 \\ -\tan\theta_W m_W^2 & \delta m_B^2 + \tan^2\theta_W m_W^2 \end{pmatrix} \cdot \begin{pmatrix} W_n^3 \\ B_n \end{pmatrix} .$$

Splittings II: Higgs VEV

- The Higgs VEV does
- At level (0,0), we ob
- For massive tiers:

$$\int_0^{2\pi} dx_5 dx_6 \; |\mathcal{D}H|^2 \Rightarrow \int_0^{2\pi} dx_5 dx_6 \; m_W^2 W^2$$

$$\Rightarrow \sum_{k,l} m_W^2 W_{k,l}^2$$

$$m_{(k,l)}^2 = (k^2 + l^2) m_{KK}^2 + m_0^2$$

- Mixing angle in the neutral gauge boson sector (A-Z): smaller than the Weinberg mixing angle!

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Splittings III: localized operators

- Can add kinetic terms on the two singular points:

$$\begin{aligned}\delta_0 &= \frac{1}{2}\left(\delta(x_5)\delta(x_6) + \delta(x_5-\pi)\delta(x_6-\pi)\right)\\ \delta_\pi &= \frac{1}{2}\left(\delta(x_5)\delta(x_6-\pi) + \delta(x_5-\pi)\delta(x_6)\right)\end{aligned}$$

$$\mathcal{L}_i = \frac{\delta_i}{\Lambda^2}\left(-\frac{r_{1i}}{4}F_{\mu\nu}^2 - \frac{r_{2i}}{2}(\partial_5 A_6 - \partial_6 A_5)^2\right)$$

Vectors:

$$m^2_{(k,l)} = \sqrt{k^2+l^2}\left(1 - \frac{z_{(k,l)}}{4\pi^2\Lambda^2} + \dots\right)$$

Scalars::

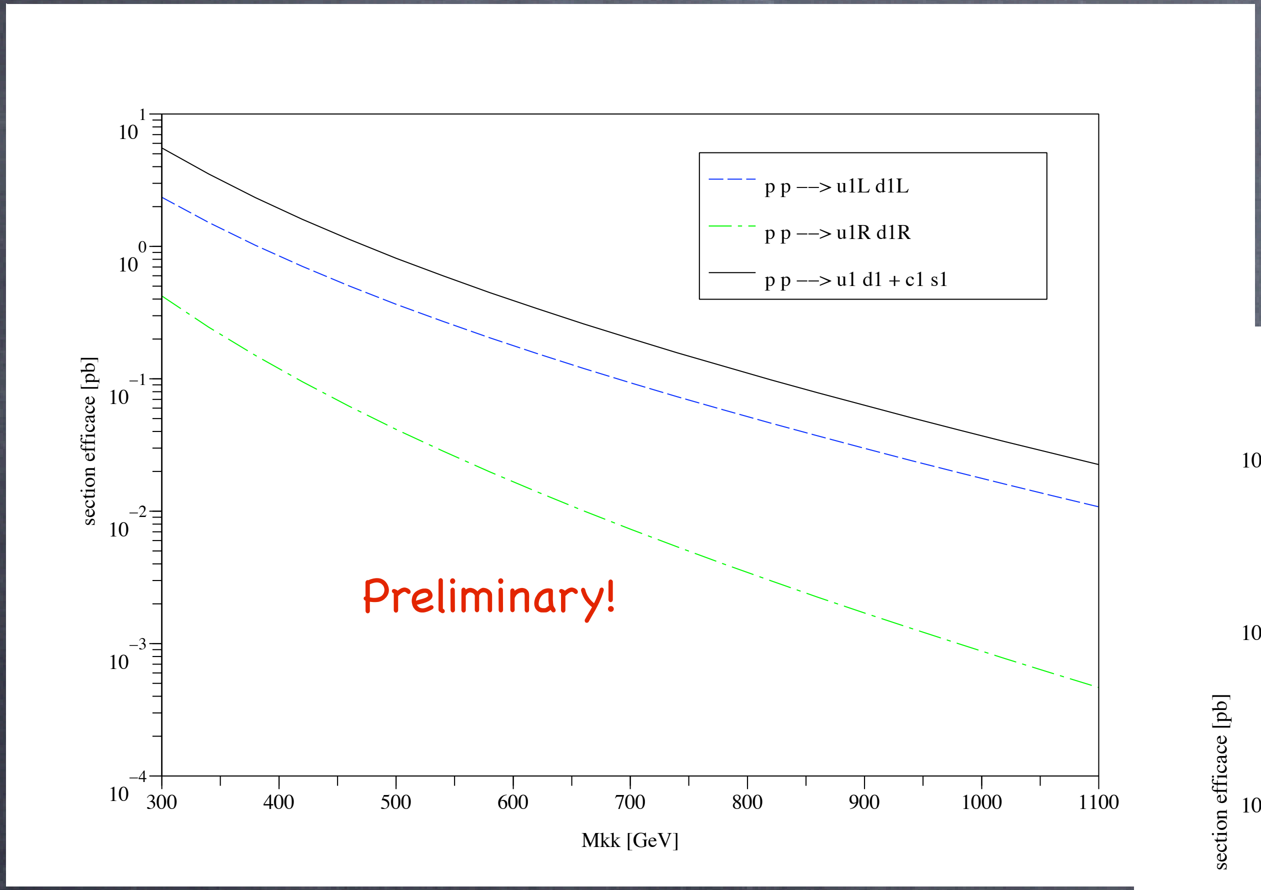
$$\delta m^2_{i,j} = m_i m_j \frac{\delta_{ij}}{4\pi^2\Lambda^2}$$

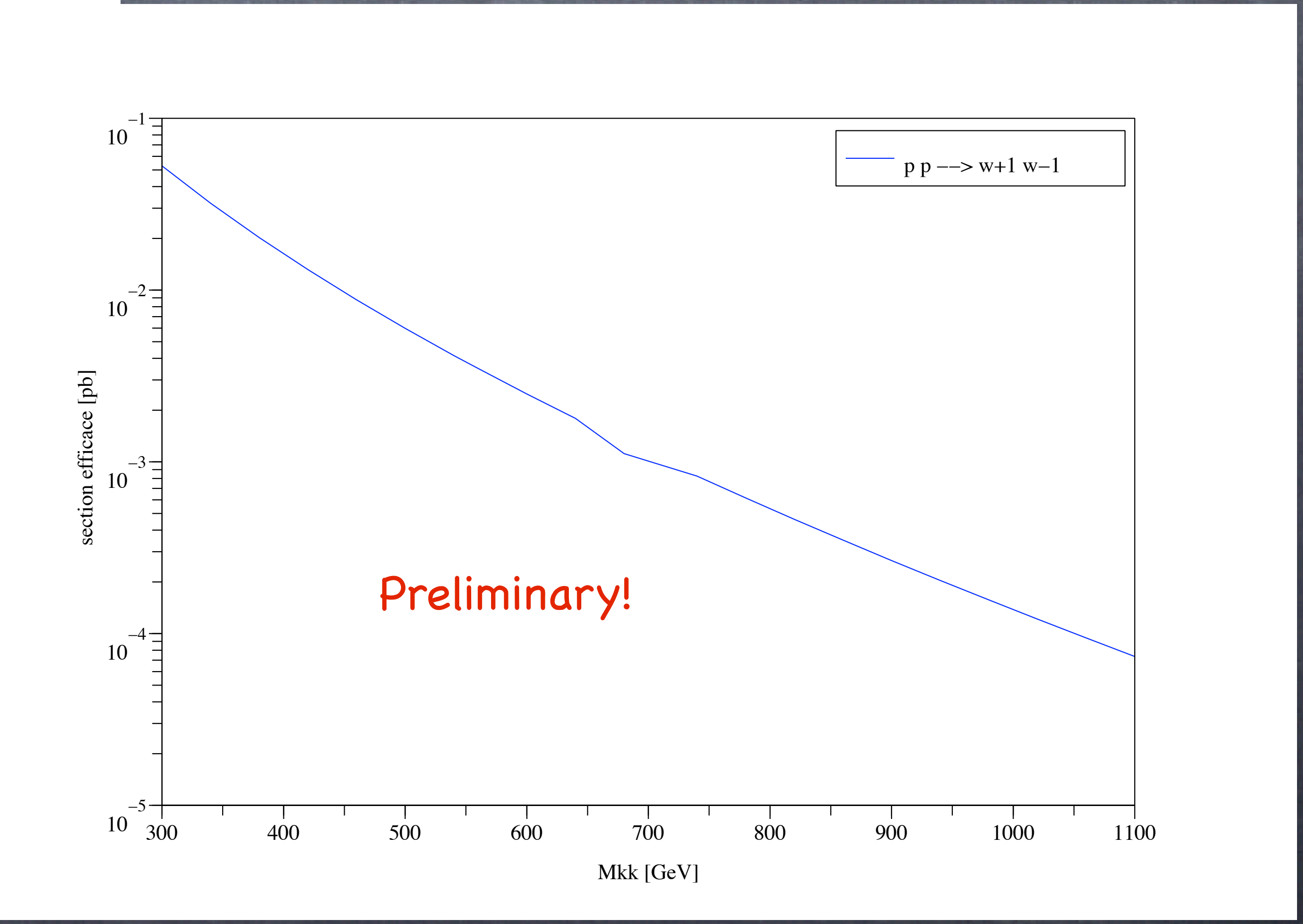
Note: remove degeneracy between (k,l) and (l,k)!

Small and arbitrary corrections: neglect for now!

Phenomenology at the LHC

- Production rates are large:





Thanks to Bogna Kubik-Deriaz

KK parity is not natural!

The typical situation is:

- We start from, say, 1 compact XD...

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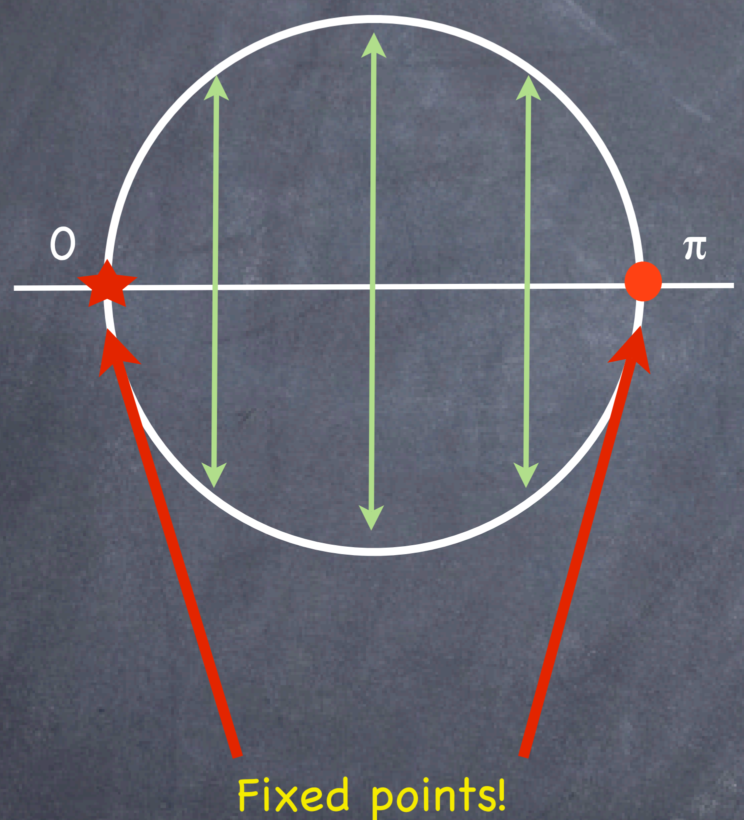

- We start from, say, 1 compact XD...
- We orbifold to obtain chiral fermions...

$x_5 \to -x_5$

KK parity is not natural!

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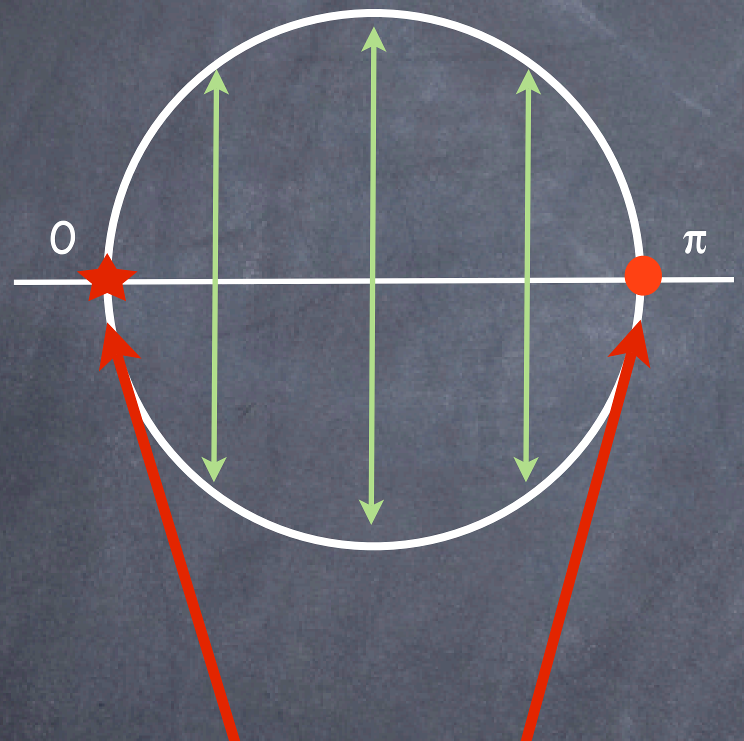


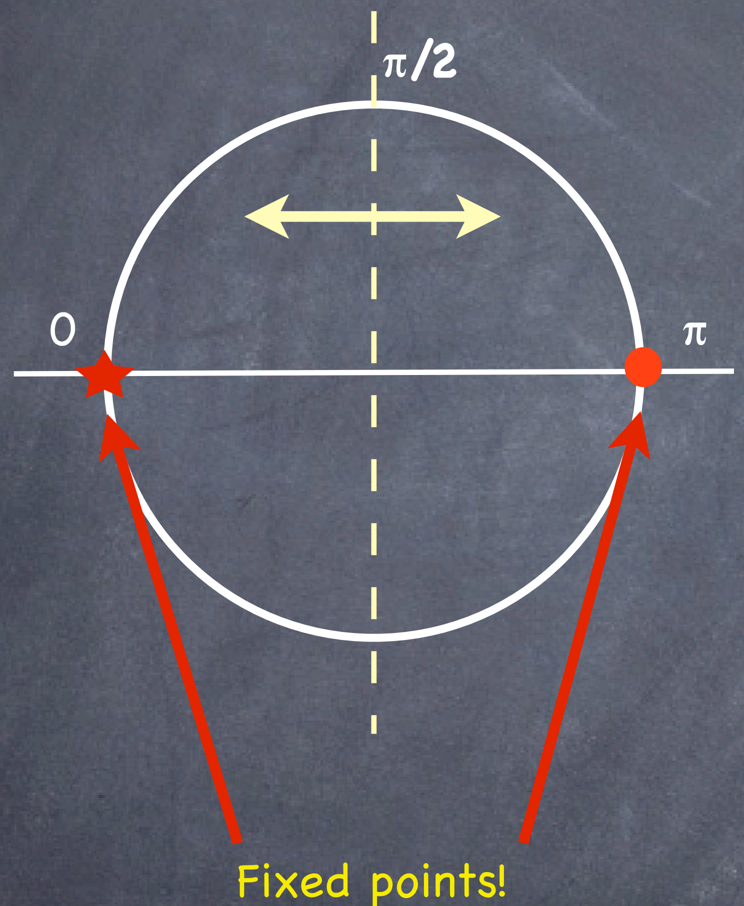
$$\Psi = \begin{pmatrix} \chi \\ \bar{\eta} \end{pmatrix}$$ chiral components

$$S = \int dx_5 \; i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi + i\eta\sigma^\mu \partial_\mu \bar{\eta} - \eta \partial_5 \chi + \bar{\chi} \partial_5 \bar{\eta}$$

different parities for chiral components only under a symmetry that changes sign to all extra coordinate(s)

KK parity is not natural!

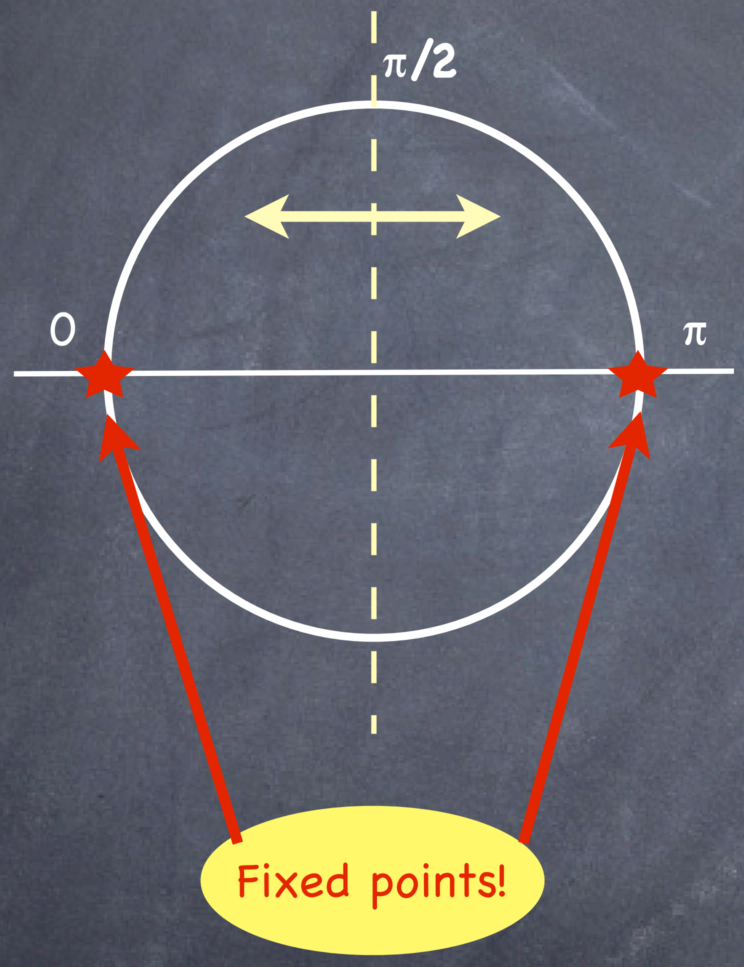
The typical situation is:



- We start from, say, 1 compact XD...
- We orbifold to obtain chiral fermions...
- We impose a discrete parity: Kaluza-Klein parity!

KK parity is not natural!

The typical situation is:



- We start from, say, 1 compact XD...
- We orbifold to obtain chiral fermions...
- We impose a discrete parity: Kaluza-Klein parity!

The KK parity is added ad hoc,
it requires to identify two DIFFERENT fixed points!

Orbifold without fixed points:

- In 2D there are 17 orbifolds (discrete symmetries of the plane)...
- of which 3 do not have fixed points/lines:

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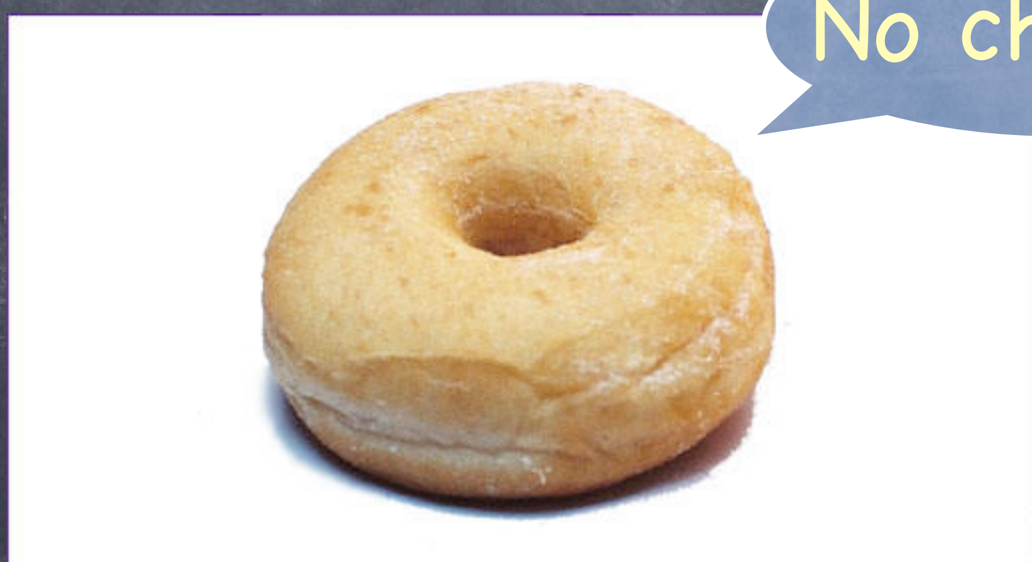
1: torus

No chirality!



2: Klein bottle

3: Real projective plane...

Orbifold without fixed points:

plane)...

3: Real projective plane...

``5D" limit

$p_{KK} = (-1)^{k+l}$	(0,0) m = 0	(1,0) & (0,1) ✗ m = 1	(1,1) ✗ m = 1.41	(2,0) & (0,2) ✗ m = 2	(2,1) ✗ & (1,2) ✗ m = 2.24
Gauge bosons G, A, Z, W	✓		✓	✓	✓
Gauge scalars G, A, Z, W		✓	✓		✓
Higgs boson(s)	✓		✓	✓	✓
Fermions	✓	✓	✓ (x2)	✓	✓ (x2)