Top-Antitop Production at the LHC

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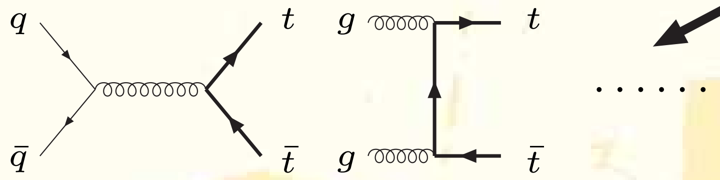
Plan of the Talk

- General Introduction
  - Top Quark at the Tevatron
  - LHC Perspectives
- Status of the Theoretical calculations
  - The General Framework
  - Total Cross Section at NLO
- Analytic Two-Loop QCD Corrections
- Conclusions

Top Quark

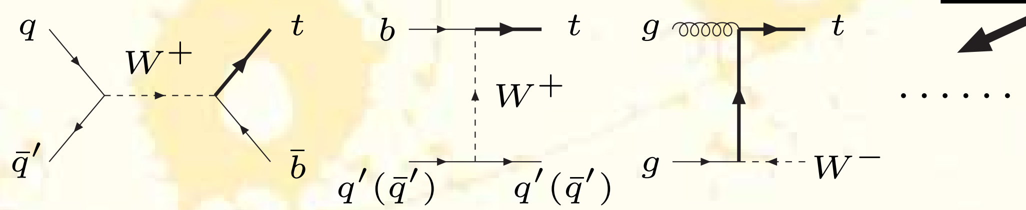
Top Quark

- With a mass of $m_t = 173.1 \pm 1.3$ GeV, the TOP quark (the up-type quark of the third generation) is the heaviest elementary particle produced so far at colliders.
- Because of its mass, top quark is going to play a unique role in understanding the EW symmetry breaking $\Rightarrow$ Heavy-Quark physics crucial at the LHC.
- Two production mechanisms
  - $pp(\bar{p}) \to t\bar{t}$



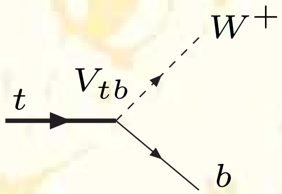
Pair Production

  - $pp(\bar{p}) \to t\bar{b}, tq'(\bar{q}'), tW^-$



Single Top

- Top quark does not hadronize, since it decays in about $5 \cdot 10^{-25}$s (one order of magnitude smaller than the hadronization time) $\Longrightarrow$ opportunity to study the quark as single particle
  - Spin properties
  - Interaction vertices
  - Top quark mass
- Decay products: almost exclusively $t \to W^+ b \quad (|V_{tb}| \gg |V_{td}|, |V_{ts}|)$



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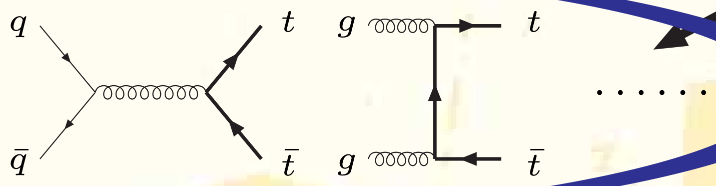

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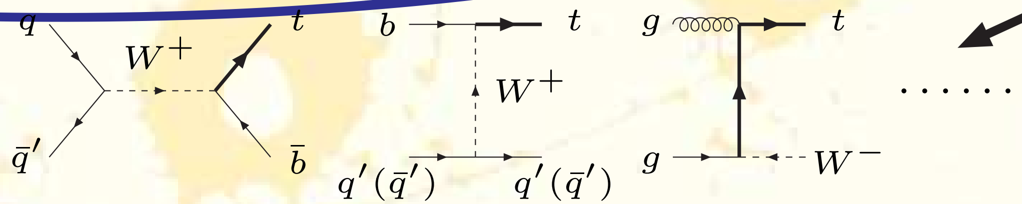

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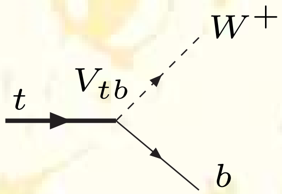

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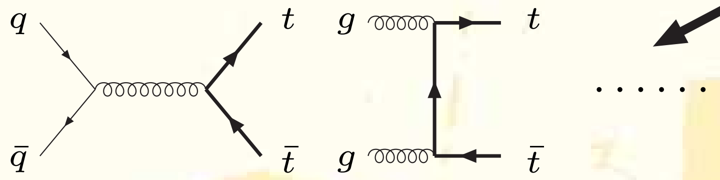

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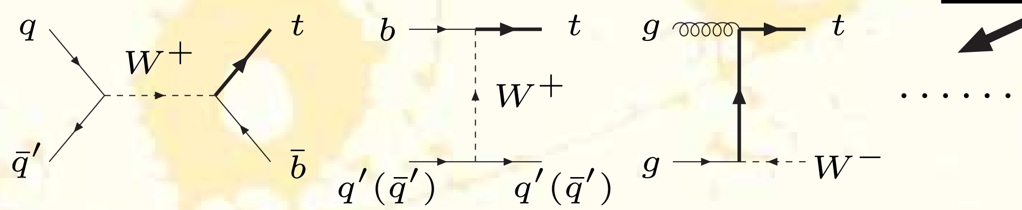

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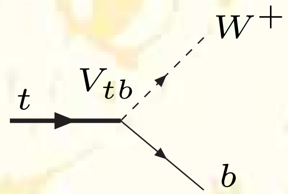
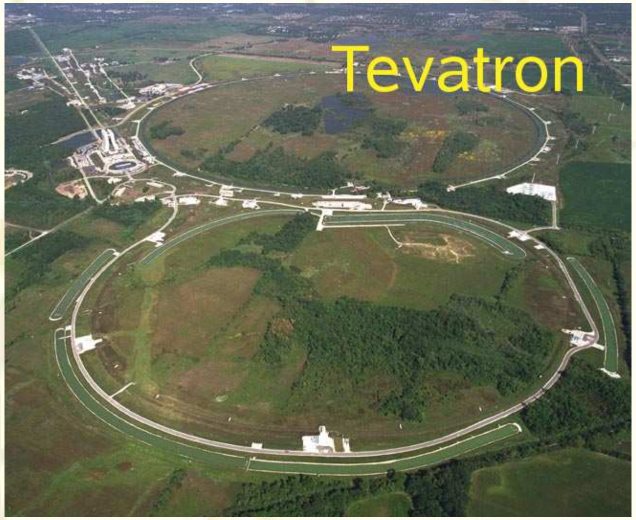


Top Quark

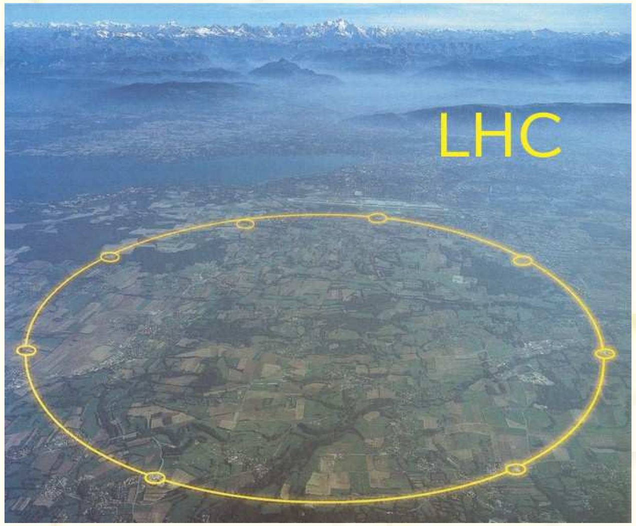
Tevatron

- To date the Top quark could be produced and studied only at the Tevatron (discovery 1995)
- $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV
- $L \sim 6.5$fb^{-1} reached in 2009
- $\mathcal{O}(10^3)$ $t\bar{t}$ pairs produced so far
- Only recently confirmation of single-t



LHC

- Running since end 2009
- pp collisions at $\sqrt{s} = 7\,(14)$ TeV
- LHC will be a factory for heavy quarks ($\mathcal{L} \sim 10^{33}$–10^{34}cm^{-2}s^{-1}, $t\bar{t}$ at $\sim$1Hz!)
- Even in the first low-luminosity phase (2 years $\sim$ 1fb^{-1} @ 7 TeV) $\sim \mathcal{O}(10^4)$ registered $t\bar{t}$ pairs



Top Quark @ Tevatron

Top Quark @ Tevatron

Events measured at Tevatron

$\sigma_{t\bar{t}} \sim$ 7pb

- $p\bar{p} \to t\bar{t} \to W^+bW^-\bar{b} \to l\nu l\nu b\bar{b}$ — Dilepton $\sim 10\%$
- $p\bar{p} \to t\bar{t} \to W^+bW^-\bar{b} \to l\nu q\bar{q}'b\bar{b}$ — Lep+jets $\sim 44\%$
- $p\bar{p} \to t\bar{t} \to W^+bW^-\bar{b} \to q\bar{q}'q\bar{q}'b\bar{b}$ — All jets $\sim 46\%$

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- $p\bar{p} \rightarrow t\bar{t} \rightarrow W^+ b W^- \bar{b} \rightarrow l\nu q\bar{q}' b\bar{b}$
- $p\bar{p} \rightarrow t\bar{t} \rightarrow W^+ b W^- \bar{b} \rightarrow q\bar{q}' q\bar{q}' b\bar{b}$

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2 high-p_T lept, $\geq$ 2 jets and ME

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NO lept, $\geq$ 6 jets and low ME

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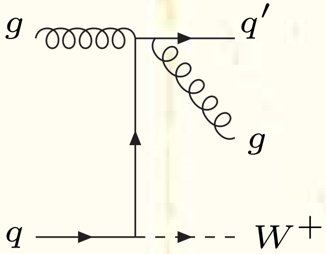
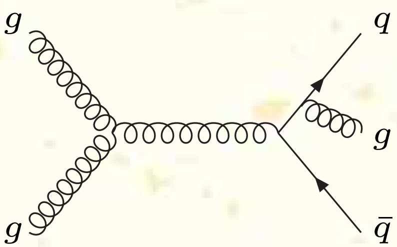
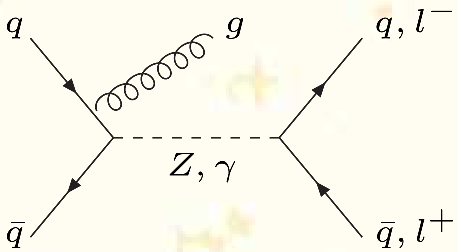
Background Processes



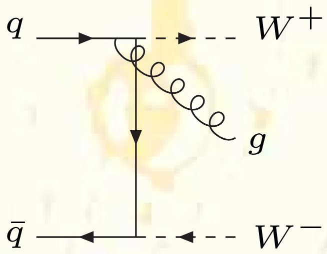
W+jets



QCD



Drell-Yan



Di-boson

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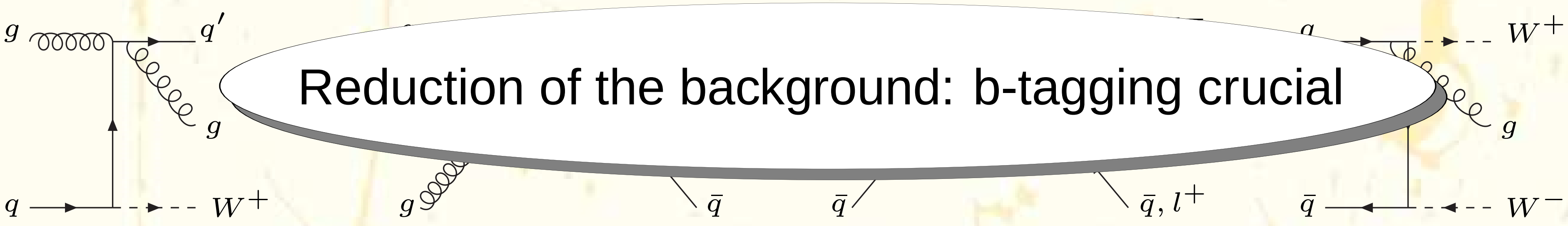


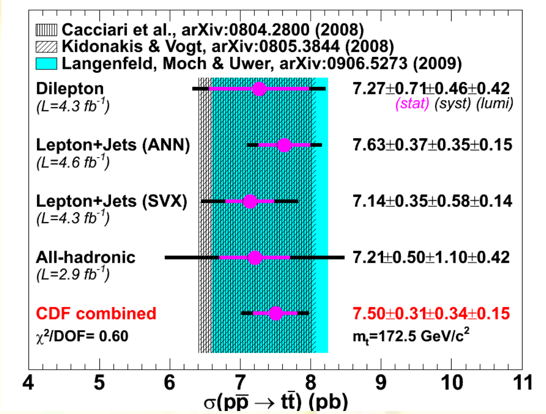
Reduction of the background: b-tagging crucial

W+jets QCD Drell-Yan Di-boson

Top Quark @ Tevatron

- Total Cross Section

$$\sigma_{t\bar{t}} = \frac{N_{data} - N_{bkgr}}{\epsilon\, L}$$

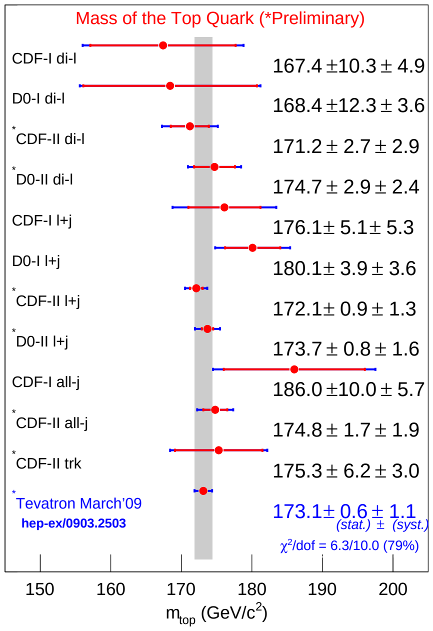


Combination CDF-D0 ($m_t = 175$ GeV)

$$\sigma_{t\bar{t}} = 7.0 \pm 0.6\,\text{pb} \qquad (\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 9\%)$$

- Top-quark Mass
  - Fundamental parameter of the SM. A precise measurement useful to constraint Higgs mass from radiative corrections (Δr)
  - A possible extraction: $\sigma_{t\bar{t}} \Longrightarrow$ need of precise theoretical determination

$$\frac{\Delta m_t}{m_t} \sim \frac{1}{5}\,\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}$$



Combination CDF-D0

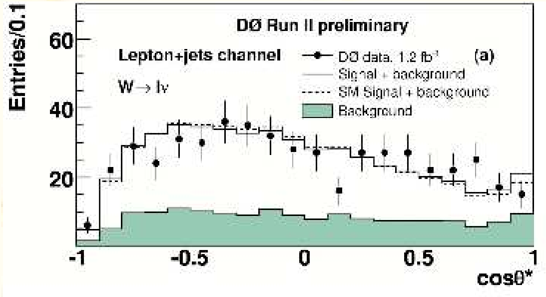
$$m_t = 173.1 \pm 1.3\,\text{GeV}\ (0.75\%)$$

Top Quark @ Tevatron

- W helicity fractions $F_i = B(t \to bW^+(\lambda_W = i))$ $(i = -1, 0, 1)$ measured fitting the distribution in θ^* (the angle between l^+ in the W^+ rest frame and W^+ direction in the t rest frame)

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{3}{4}F_0 \sin^2\theta^* + \frac{3}{8}F_-(1-\cos\theta^*)^2 + \frac{3}{8}F_+(1+\cos\theta^*)^2$$

$$F_0 + F_+ + F_- = 1$$



$$\boxed{F_0 = 0.66 \pm 0.16 \pm 0.05 \quad F_+ = -0.03 \pm 0.06 \pm 0.03}$$

- Spin correlations measured fitting the double distribution

($\theta_1\,(\theta_2)$ is the angle between the dir of flight of $l_1\,(l_2)$ in the $t(\bar{t})$ rest frame and the $t(\bar{t})$ direction in the $t\bar{t}$ rest frame)

$$\frac{1}{N}\frac{d^2N}{d\cos\theta_1\, d\cos\theta_2} = \frac{1}{4}(1 + \kappa\cos\theta_1\cos\theta_2)$$

$$\boxed{-0.455 < \kappa < 0.865\,(68\%\,CL)}$$

- Forward-Backward Asymmetry

$$\boxed{A_{FB} = \frac{N(y_t > 0) - N(y_t < 0)}{N(y_t > 0) + N(y_t < 0)}}$$

$$\boxed{A_{FB} = (19.3 \pm 6.5(\text{sta}) \pm 2.4(\text{sys}))\%}$$

Top Quark @ Tevatron

Tevatron searches of physics BSM in top events

- New production mechanisms via new spin-1 or spin-2 resonances: $q\bar{q} \to Z' \to t\bar{t}$ in lepton+jets and all hadronic events (bumps in the invariant-mass distribution)
- Top charge measurements (recently excluded $Q_t = -4/3$)
- Anomalous couplings

$$L = -\frac{g}{\sqrt{2}}\bar{b}\left\{\gamma^\mu (V_L P_L + V_R P_R) + \frac{i\sigma^{\mu\nu}(p_t - p_b)_\nu}{M_W}(g_L P_L + g_R P_R)\right\} t W_\mu^-$$

  - From helicity fractions
  - From asymmetries in the final state (for instance $A_{FB} = 3/4\,(F_+ - F_-)$)
- Forward-backward asymmetry
- Non SM Top decays. Search for charged Higgs: $t \to H^+ b \to q\bar{q}' b(\tau\nu b)$
- Search for heavy $t' \to W^+ b$ in lepton+jets

Top Quark @ Tevatron

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No Evidence
of New Physics so far

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Top Quark @ Tevatron

	Value	Lum fb^{-1}	SM value	SM-like?
m_t	$173.1 \pm 0.6 \pm 1.1$ GeV	up to 4.8	/	/
$\sigma_{t\bar{t}}$	$7.0 \pm 0.3 \pm 0.4 \pm 0.4$ pb ($m_t = 175$ GeV)	2.8	6.7 pb	YES
W-helicity	$F^0 = 0.66 \pm 0.16 \pm 0.05$ $F^+ = -0.03 \pm 0.06 \pm 0.03$	1.9	$F^0 = 0.7$ $F^+ = 0$	YES
Spin Correlat.	$-0.455 < \kappa < 0.865\,(68\%\,CL)$	2.8	$\kappa = 0.8$	YES
A_{FB}	$0.19 \pm 0.07 \pm 0.02$	3.2	0.05 @NLO	YES
Γ_t	< 13.1 GeV $(95\%\,CL)$	1.0	1.5 GeV	YES
τ_t	$c\tau_t < 52.5\,\mu\mathrm{m}\ (95\%\,CL)$	0.3	$\sim 10^{-16}$ m	YES
BR	$(t \to Wb)/(t \to Wq) > 0.61\ (95\%\,CL)$	0.2	$\sim 100\%$	YES
Charge	Exclude $Q_t = -4/3\ (87\%\,CL)$	1.5	$2/3$	YES

LHC Perspectives

- Cross Section
  - With $100\,\text{pb}^{-1}$ of accumulated data an error of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 15\%$ is expected (dominated by statistics!)
  - After 5 years of data taking an error of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 5\%$ is expected
- Top Mass
  - With $1\,\text{fb}^{-1}$ Mass accuracy: $\Delta m_t \sim 1 - 3$ GeV
- Top Properties
  - W helicity fractions and spin correlations with $10\,\text{fb}^{-1} \Longrightarrow$ 1-5%
  - Top-quark charge. With $1\,\text{fb}^{-1}$ we could be able to determine $Q_t = 2/3$ with an accuracy of $\sim 15\%$
- Sensitivity to new physics
  - all the above mentioned points
  - Narrow resonances: with $1\,\text{fb}^{-1}$ possible discovery of a Z' of $M_{Z'} \sim 700$ GeV with $\sigma_{pp'\to t\bar{t}} \sim 11$ pb

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Top-Anti Top Pair Production

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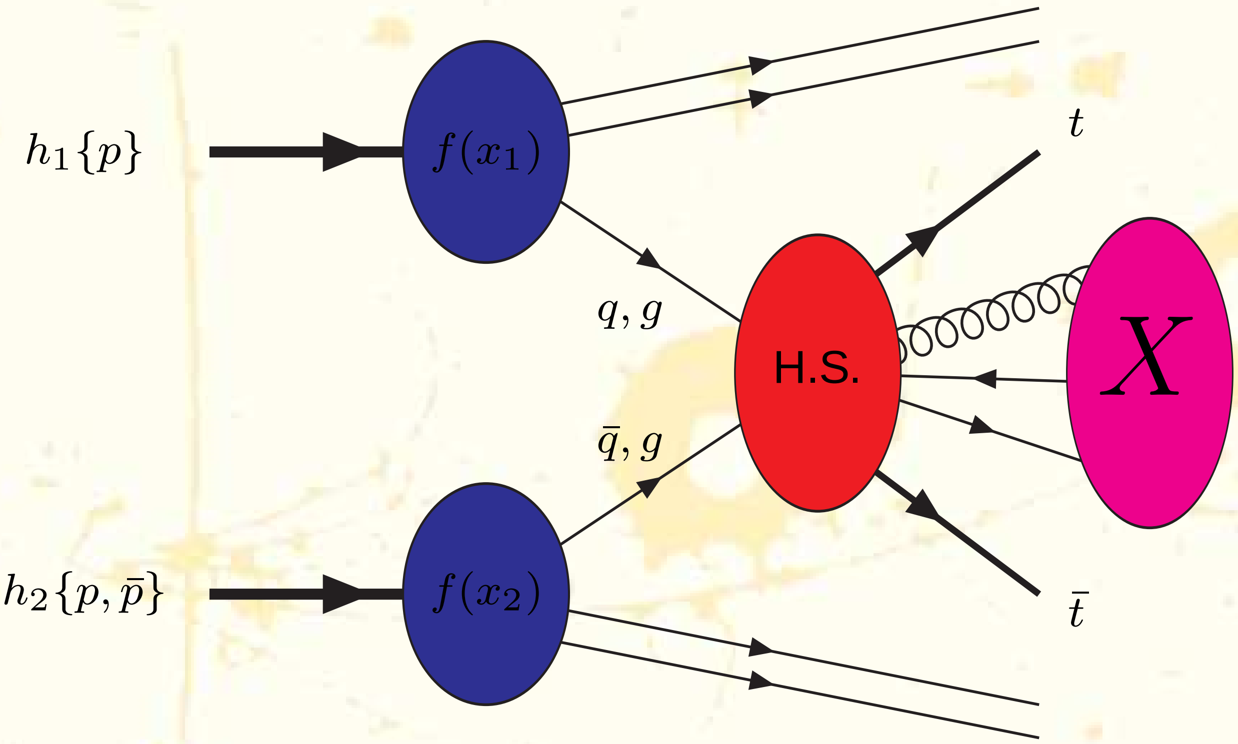
According to the factorization theorem, the process $\boxed{h_1 + h_2 \to t\bar{t} + X}$ can be sketched as in the figure:



$$\sigma^{t\bar{t}}_{h_1,h_2} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{h_1,i}(x_1, \mu_F) f_{h_2,j}(x_2, \mu_F)\ \hat{\sigma}_{ij}\left(\hat{s}, m_t, \alpha_s(\mu_R), \mu_F, \mu_R\right)$$

$$s = \left(p_{h_1} + p_{h_2}\right)^2\ ,\ \hat{s} = x_1 x_2 s$$

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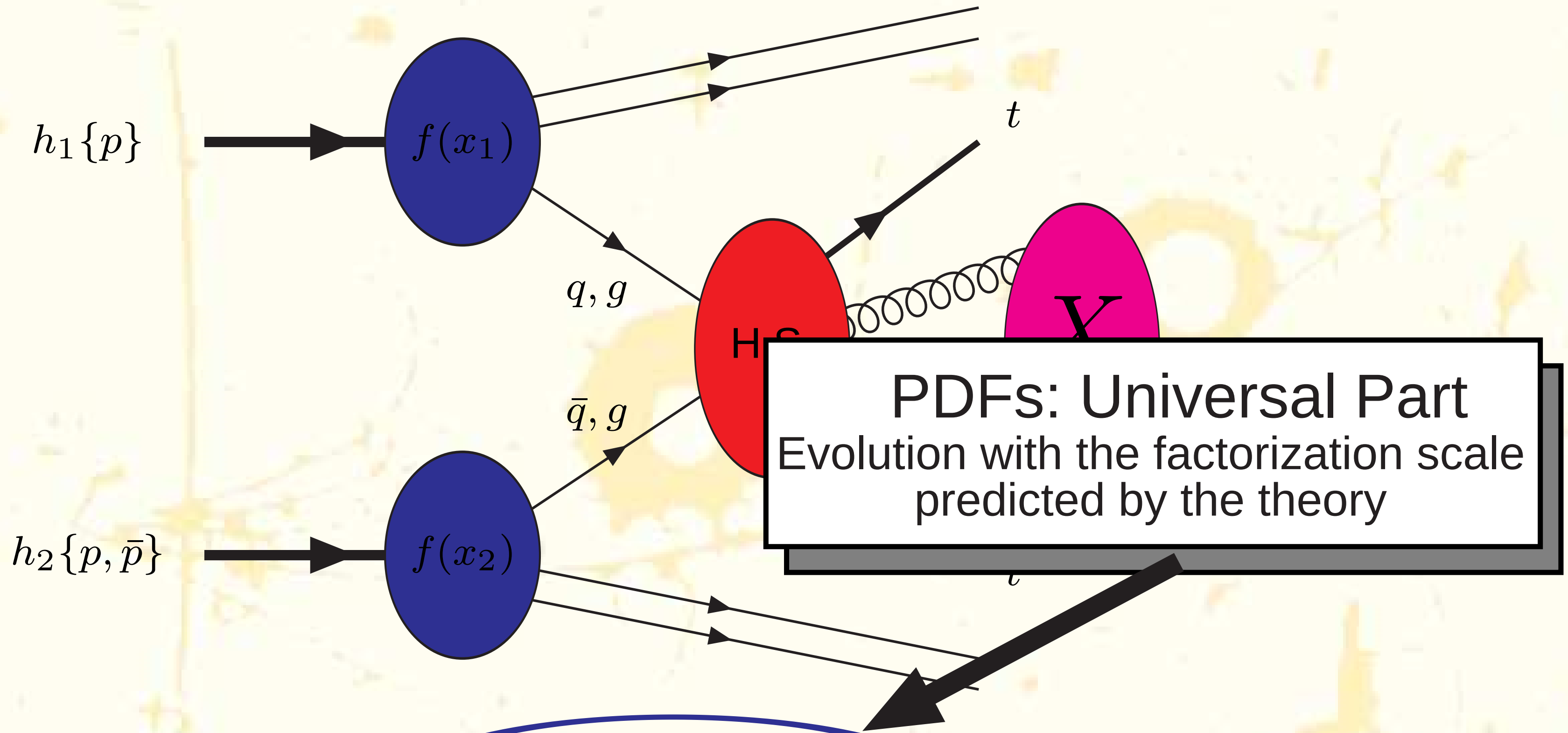

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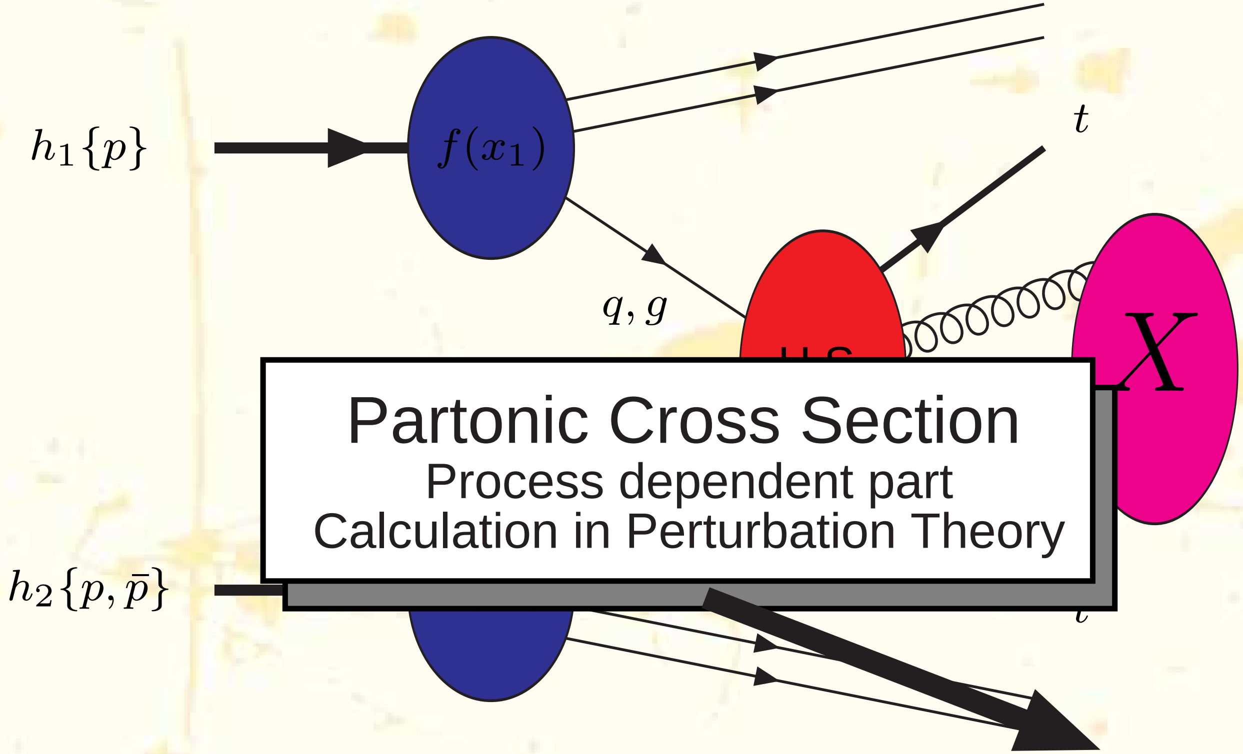

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The Partonic Cross Section: Tree-Level

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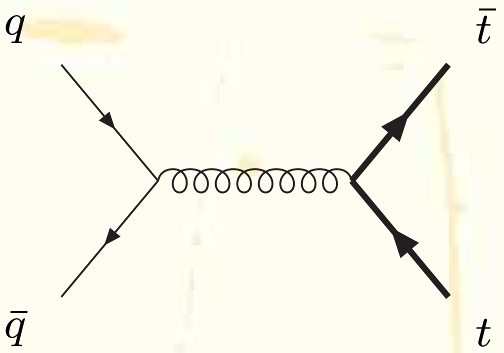
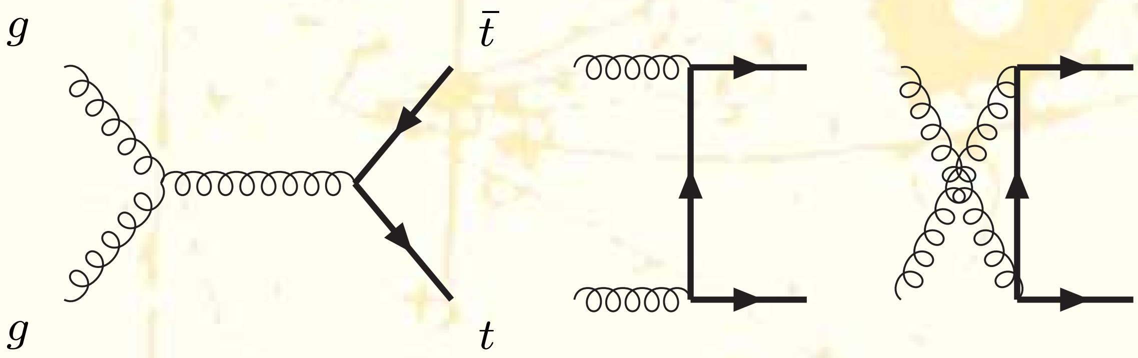
$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



Dominant at Tevatron

$\sim 85\%$

$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



Dominant at LHC

$\sim 90\%$

$\sigma_{t\bar{t}}^{LO}(LHC, m_t = 171\,\text{GeV}) = 583\,\text{pb} \pm 30\%$

$\sigma_{t\bar{t}}^{LO}(Tev, m_t = 171\,\text{GeV}) = 5.92\,\text{pb} \pm 44\%$

The Partonic Cross Section: NLO

The Partonic Cross Section: NLO

Fixed Order

- The NLO QCD corrections are quite sizable: + 25% at Tevatron and +50% at LHC. Reduction of the th error to $\pm 15\%$.

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  Beenakker *et al.* ’94 Bernreuther, Fuecker, and Si ’05-’08
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- Even if $\alpha_S \ll 1$ (perturbative region) we can have at all orders $\alpha_S^n \ln^m (1-\rho) \sim \mathcal{O}(1)$

  Resummation $\Longrightarrow$ improved perturbation theory

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All-order Soft-Gluon Resummation

- Leading-Logs (LL)

  Laenen et al. '92-'95; Berger and Contopanagos '95-'96; Catani et al. '96.

- Next-to-Leading-Logs (NLL)

  Kidonakis and Sterman '97; R. B., Catani, Mangano, and Nason '98-'03.

- Next-to-Next-to-Leading-Logs (NNLL) under study ...

  Moch and Uwer '08; Beneke et al. '09; Czakon et al. '09; Kidonakis '09

NLO+NLL Theoretical Prediction

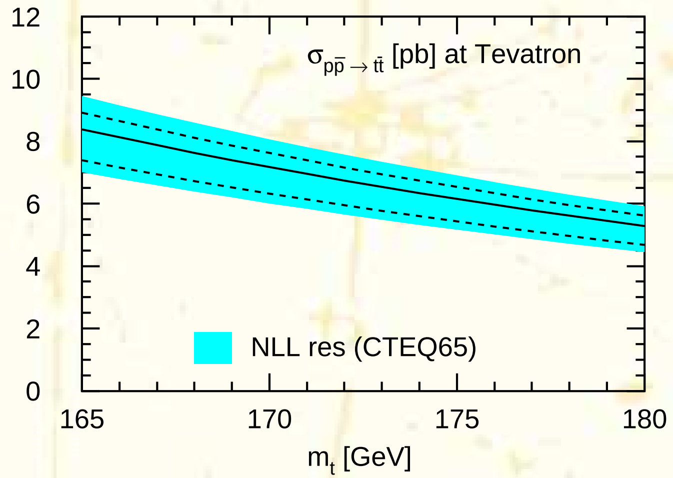
TEVATRON

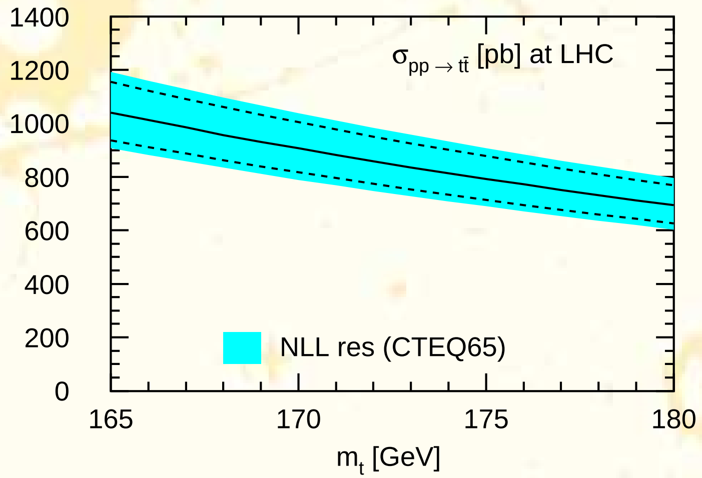
$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{Tev}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 7.61\ {}^{+0.30(3.9\%)}_{-0.53(6.9\%)}\ (\mathrm{scales})\ {}^{+0.53(7\%)}_{-0.36(4.8\%)}\ (\mathrm{PDFs})\ \ \mathrm{pb}$$

LHC

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{LHC}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 908\ {}^{+82(9.0\%)}_{-85(9.3\%)}\ (\mathrm{scales})\ {}^{+30(3.3\%)}_{-29(3.2\%)}\ (\mathrm{PDFs})\ \ \mathrm{pb}$$

M. Cacciari, S. Frixione, M. Mangano, P. Nason, and G. Ridolfi, JHEP 0809:127,2008

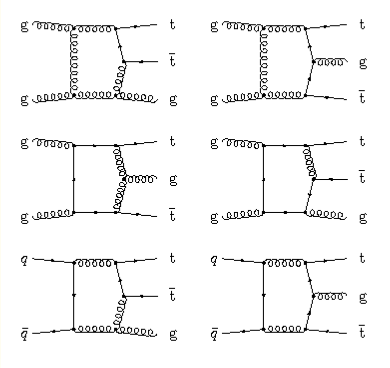




S. Moch and P. Uwer, Phys. Rev. D **78** (2008) 034003

Distributions

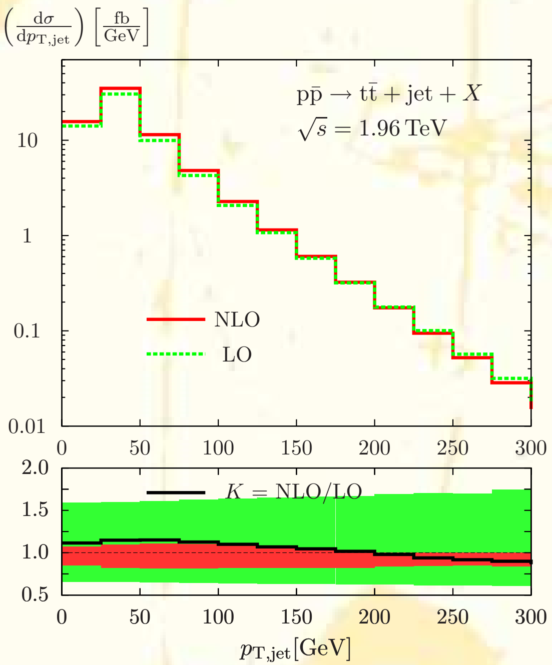
- $p\bar{p} \to t\bar{t} + 1\,jet$
  - Important for a deeper understanding of the $t\bar{t}$ prod (possible structure of the top-quark)
  - Important for the charge asymmetry at Tevatron
  - Technically complex involving multi-leg NLO diagrams

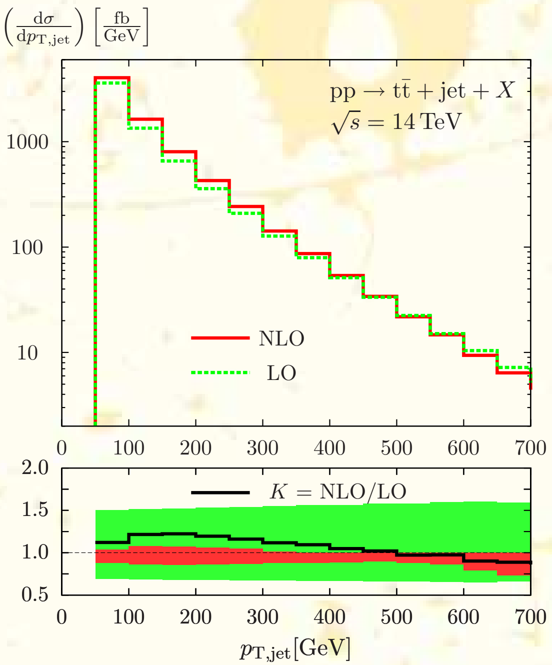


$\sigma_{t\bar{t}+j}$ (LHC) = 376.2^{+17}_{-48} pb (with $p_{T,jet,cut} = 50$ GeV)

confirmed recently by G. Bevilacqua, M. Czakon, C.G. Papadopoulos, M. Worek, arXiv:1002.4009

$t\bar{t} + 2j$





S. Dittmaier, P. Uwer and S. Weinzierl, Eur. Phys. J. C **59** (2009) 625

Measurement Requirements for $\sigma_{t\bar{t}}$

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Experimental requirements for $\sigma_{t\bar{t}}$:

- Tevatron $\Delta\sigma/\sigma \sim 12\% \Longrightarrow$ ok!
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Kidonakis-Vogt, Moch-Uwer, Langenfeld-Moch-Uwer, presented recently "approximated NNLO" results for $\sigma_{t\bar{t}}$ including

- scale dependence at NNLO
- NNLL soft-gluon contributions
- Coulomb corrections

This drastically reduces the uncertainty (factorization/renormalization scale dependence) to the level predicted for LHC: $\boxed{\sim 4 - 6\%}$.

These results are "approximated" NNLO results.

Nevertheless, they indicate that a COMPLETE NNLO computation is indeed needed in order to match the experimental precision of LHC.

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  - two-loop matrix elements for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
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Dittmaier, Uwer and Weinzierl ’07-’08

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  - Both matrix elements known for $t\bar{t} + j$ calculation, BUT subtraction up to 1 unresolved parton, while in a complete NNLO computation of $\sigma_{t\bar{t}}$ we need subtraction terms with up to 2 unresolved partons.

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$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times0)} + \mathcal{A}_2^{(1\times1)}$$

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  R. B., Ferroglia, Gehrmann, Maitre, and Studerus '08-'09

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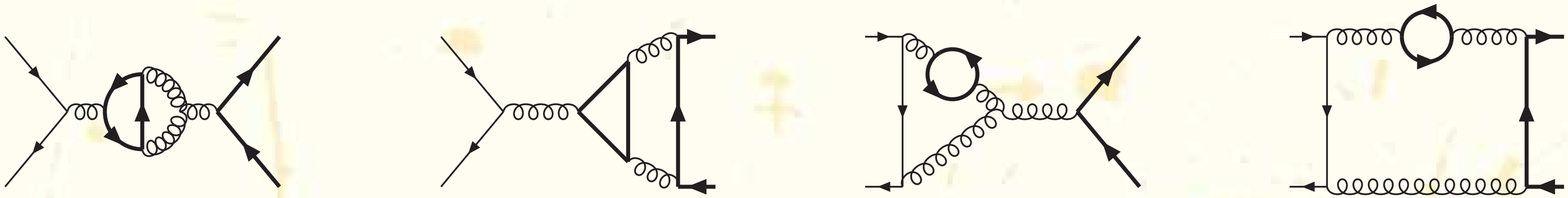
- The poles of $\mathcal{A}_2^{(2\times 0)}$ (and therefore of B and C) are known analytically

  Ferroglia, Neubert, Pecjak, and Li Yang '09

Two-Loop Corrections to $q\bar{q} \rightarrow t\bar{t}$

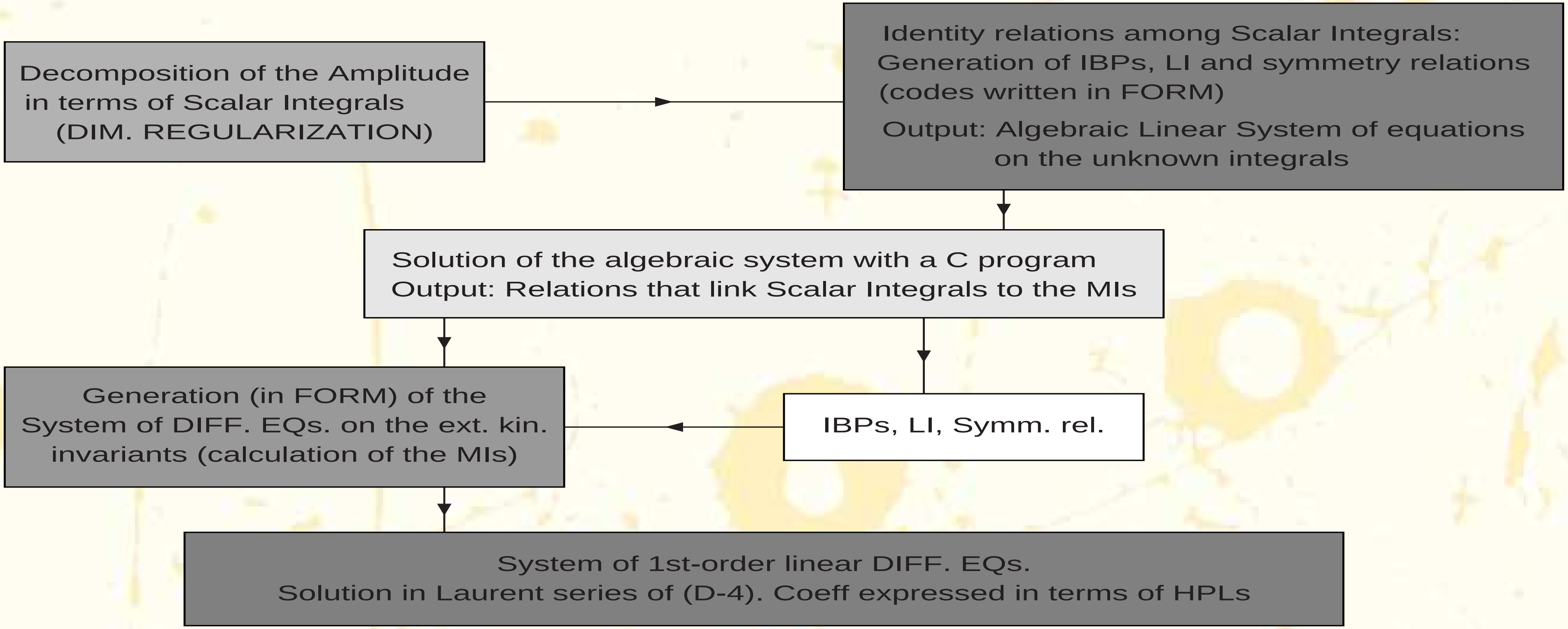
- D_i, E_i, F_i come from the corrections involving a closed (light or heavy) fermionic loop:



- A the leading-color coefficient, comes from the planar diagrams:

- The calculation is carried out analytically using:
  - Laporta Algorithm for the reduction of the dimensionally-regularized scalar integrals (in terms of which we express the $|\mathcal{M}|^2$) to the Master Integrals (MIs)
  - Differential Equations Method for the analytic solution of the MIs

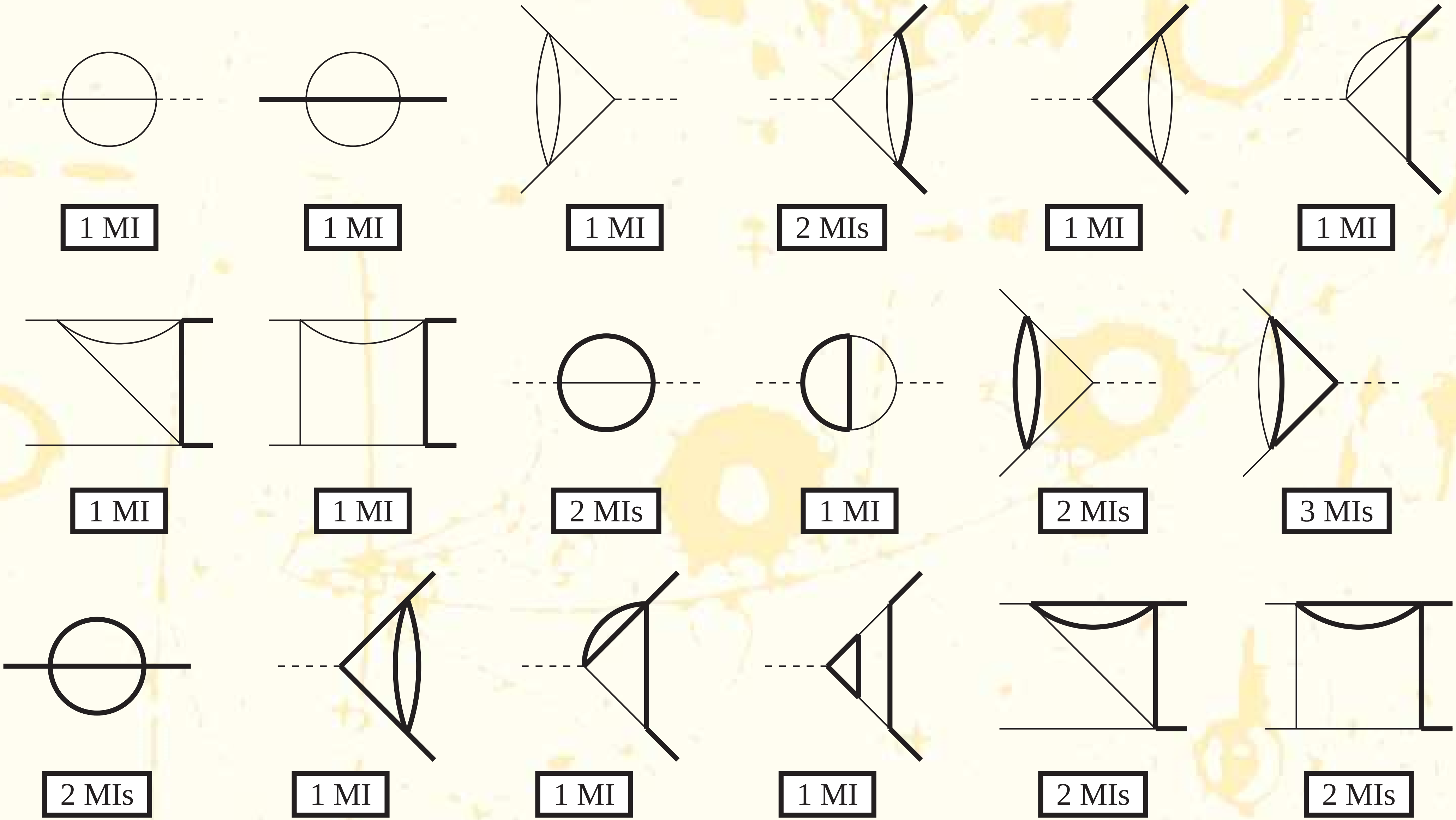
Laporta Algorithm and Diff. Equations



PUBLIC PROGRAMS

- AIR Maple package (C. Anastasiou and A. Lazopoulos, JHEP **0407** (2004) 046)
- FIRE Mathematica package (A. V. Smirnov, JHEP **0810** (2008) 107)
- REDUZE a C++/GiNaC package (C. Studerus, arXiv:0912.2546)

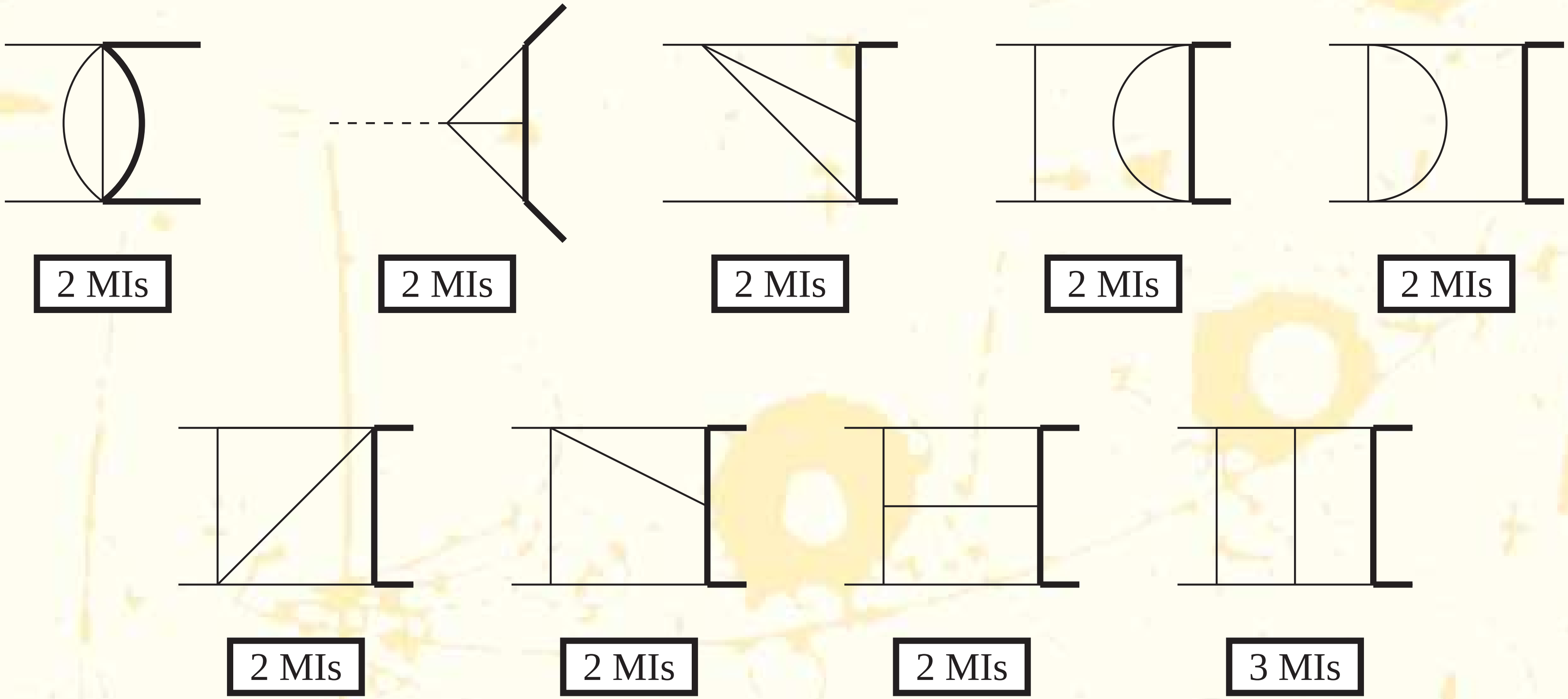
Master Integrals for N_l and N_h



18 irreducible two-loop topologies (26 MIs)

R. B., A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, JHEP **0807** (2008) 129.

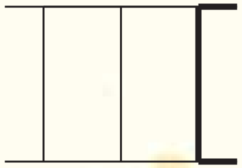
Master Integrals for the Leading Color Coeff



For the leading color coefficient there are 9 additional irreducible topologies (19 MIs)

R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, JHEP **0908** (2009) 067.

Example



$$= \frac{1}{m^6} \sum_{i=-4}^{-1} A_i \epsilon^i + \mathcal{O}(\epsilon^0)$$

$$A_{-4} = \frac{x^2}{24(1-x)^4(1+y)},$$

$$A_{-3} = \frac{x^2}{96(1-x)^4(1+y)}\Big[-10G(-1;y) + 3G(0;x) - 6G(1;x)\Big],$$

$$A_{-2} = \frac{x^2}{48(1-x)^4(1+y)}\Big[-5\zeta(2) - 6G(-1;y)G(0;x) + 12G(-1;y)G(1;x) + 8G(-1,-1;y)\Big],$$

$$\begin{aligned} A_{-1} = \frac{x^2}{48(1-x)^4(1+y)}\Big[& -13\zeta(3) + 38\zeta(2)G(-1;y) + 9\zeta(2)G(0;x) + 6\zeta(2)G(1;x) - 24\zeta(2)G(-1/y;x) \\ & +24G(0;x)G(-1,-1;y) - 24G(1;x)G(-1,-1;y) - 12G(-1/y;x)G(-1,-1;y) \\ & -12G(-y;x)G(-1,-1;y) - 6G(0;x)G(0,-1;y) + 6G(-1/y;x)G(0,-1;y) + 6G(-y;x)G(0,-1;y) \\ & +12G(-1;y)G(1,0;x) - 24G(-1;y)G(1,1;x) - 6G(-1;y)G(-1/y,0;x) + 12G(-1;y)G(-1/y,1;x) \\ & -6G(-1;y)G(-y,0;x) + 12G(-1;y)G(-y,1;x) + 16G(-1,-1,-1;y) - 12G(-1,0,-1;y) \\ & -12G(0,-1,-1;y) + 6G(0,0,-1;y) + 6G(1,0,0;x) - 12G(1,0,1;x) - 12G(1,1,0;x) + 24G(1,1,1;x) \\ & -6G(-1/y,0,0;x) + 12G(-1/y,0,1;x) + 6G(-1/y,1,0;x) - 12G(-1/y,1,1;x) + 6G(-y,1,0;x) \\ & -12G(-y,1,1;x)\Big] \end{aligned}$$

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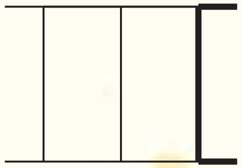

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$$+24G(0;x)G(-1,-1;y) - 24G(1;x)G(-1,-1;y) - 12G(-1/\ldots;x)\,G(-1,-1;y)$$

$$-12G(-y;x)G(-1,-1;y) - 6G(0;x)G(0,-1;y) + 6G(\ldots/y;x)\,G(0,-1;y) + 6G(-y;x)G(0,-1;y)$$

$$+12G(-1;y)G(1,0;x) - 24G(-1;y)G(1,1;x) - 6G(-1;y)G(-1/y,0;x) + 12G(-1;y)G(-1/y,1;x)$$

$$-6G(-1;y)G(-y,0;x) \ldots 12G(-1;y)G(-y,1;x) + 16G(-1,-1,-1;y) - \ldots(-1,0,-1;y)$$

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$$\rho = \frac{4m_t^2}{\hat{s}} \to 1$$

1- and 2-dim GHPLs

GHPLs

- One- and two-dimensional Generalized Harmonic Polylogarithms (GHPLs) are defined as repeated integrations over set of basic functions. In the case at hand

$$f_w(x) = \frac{1}{x-w}\,, \quad \text{with} \quad w \in \left\{0, 1, -1, -y, -\frac{1}{y}, \frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right\}$$

$$f_w(y) = \frac{1}{y-w}\,, \quad \text{with} \quad w \in \left\{0, 1, -1, -x, -\frac{1}{x}, 1 - \frac{1}{x} - x\right\}$$

- The weight-one GHPLs are defined as

$$G(0;x) = \ln x\,, \qquad G(w;x) = \int_0^x dt\, f_w(t)$$

- Higher weight GHPLs are defined by iterated integrations

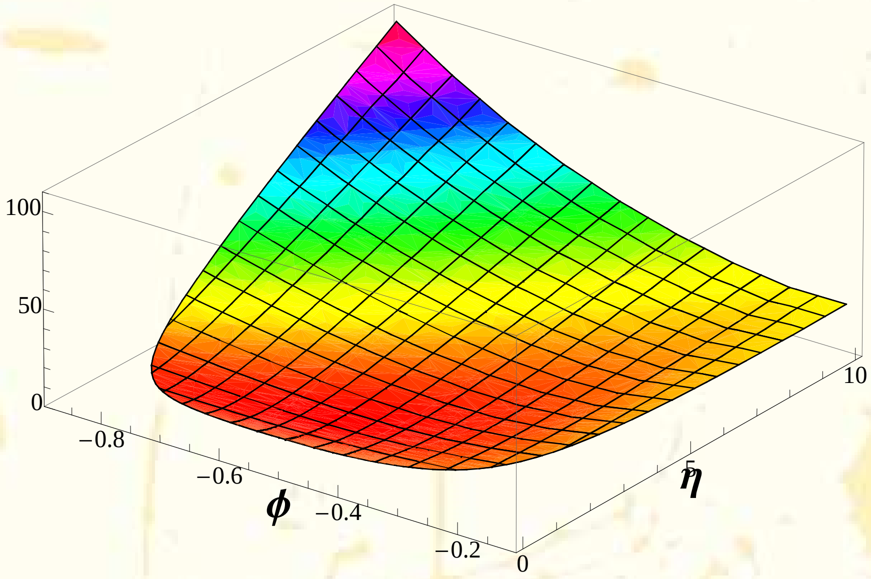
$$G(\underbrace{0,0,\cdots,0}_{n};x) = \frac{1}{n!}\ln^n x\,, \qquad G(w,\cdots;x) = \int_0^x dt\, f_w(t) G(\cdots;t)$$

- Shuffle algebra. Integration by parts identities

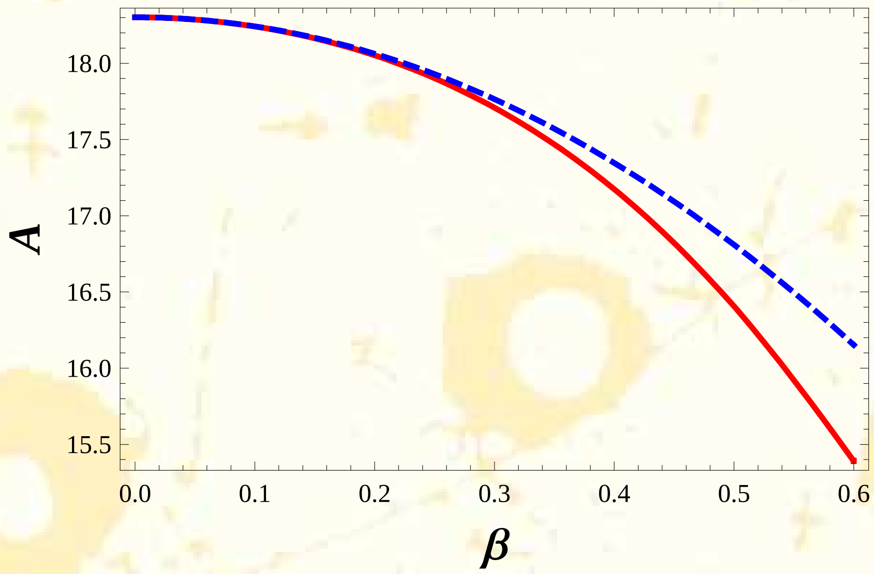
Remiddi and Vermaseren ’99, Gehrmann and Remiddi ’01-’02, Aglietti and R. B. ’03, Vollinga and Weinzierl ’04, R. B., A. Ferroglia, T. Gehrmann, and C. Studerus ’09

Coefficient A

Finite part of A



Threshold expansion versus exact result



$$\eta = \frac{s}{4m^2} - 1\,, \qquad \phi = -\frac{t - m^2}{s}$$

$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

partonic c.m. scattering angle $= \frac{\pi}{2}$

Numerical evaluation of the GHPLs with GiNaC C++ routines.

Vollinga and Weinzierl ’04

Two-Loop Corrections to $gg \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times 0)} \quad = \quad & (N_c^2-1)\Big(N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h \\ & + N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h \\ & + N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh}\Big)\end{aligned}$$

789 two-loop diagrams contribute to 16 different color coefficients

- No numeric result for $\mathcal{A}_2^{(2\times 0)}$ yet
- The poles of $\mathcal{A}_2^{(2\times 0)}$ are known analytically

Ferroglia, Neubert, Pecjak, and Li Yang '09

- The coefficients A, E_l–I_l can be evaluated analytically as for the $q\bar{q}$ channel

R. B., Ferroglia, Gehrmann, and Studerus, in preparation

Conclusions

- In the last 15 years, Tevatron explored top-quark properties reaching a remarkable experimental accuracy. The top mass could be measured with $\Delta m_t / m_t = 0.75\%$ and the production cross section with $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} = 9\%$. Other observables could be measured only with bigger errors.
- At the LHC the situation will further improve. The production cross section of $t\bar{t}$ pairs is expected to reach the accuracy of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} = 5\%$!!
- This experimental precision demands for more accurate theoretical predictions. Quantum corrections have to be unavoidably taken into account.
- For the production cross section, $\sigma_{t\bar{t}}$, a complete NNLO analysis is mandatory in order to reach the experimental accuracy expected in 3-4 years from now.
- In spite of a big activity of different groups, many ingredients are still missing.
- In this talk I briefly reviewed the analytic evaluation of the two-loop matrix elements, afforded using the Laporta algorithm for the reduction to the MIs and the Differential Equations method for their analytic evaluation. To date, the corrections involving a fermionic loop (light or heavy) in the $q\bar{q}$ channel are completed, together with the leading color coefficient. Analogous corrections in the gg channel can be calculated with the same technique and are at the moment under study.